5-2 التكامل بالتعويض

① $\int (2x-3)\sqrt{x^2-3x+5}\,dx$

$u = x^2-3x+5$

$du = 2x-3\,dx$

$= \int (x^2-3x+5)^{\frac{1}{2}}(2x-3)\,dx$

$= \int u^{\frac{1}{2}}\,du = \frac{2}{3}u^{\frac{3}{2}} + c$

$= \frac{2}{3}\sqrt{(x^2-3x+5)^3} + c$

② $\int (4x-5)^8\,dx$

$u = 4x-5$

$du = 4\,dx$

$= \frac{1}{4}\int (4x-5)^8\, 4\,dx$

$= \frac{1}{4}\int u^8\,du = \frac{1}{4}\cdot\frac{1}{9}u^9 + c$

$= \frac{1}{36}(4x-5)^9 + c$

③ $\int (x+2)\sqrt[3]{x^2+4x-1}\,dx$

$u = x^2+4x-1$

$du = 2x+4\,dx$

$= 2(x+2)\,dx$

$= \frac{1}{2}\int (x^2+4x-1)^{\frac{1}{3}}\, 2(x+2)\,dx$

$= \frac{1}{2}\int u^{\frac{1}{3}}\,du = \frac{1}{2}\times\frac{3}{4}u^{\frac{4}{3}} + c$

$= \frac{3}{8}\sqrt[3]{(x^2+4x-1)^4} + c$

4 $\int (x^2-1)\sqrt{x^3-3x+5}\,dx$ u

$u = x^3-3x+5$

$du = 3x^2-3\ dx$

$= \frac{1}{3}\int (x^3-3x+5)^{\frac{1}{2}}\ 3(x^2-1)\,dx$

$= \frac{1}{3}\int u^{\frac{1}{2}}\,du = \frac{1}{3}\,\frac{2}{3}\,u^{\frac{3}{2}} + c$

$= \frac{2}{9}\sqrt{(x^3-3x+5)^3} + c$

5 $\int (x^2-2x)(x^3-3x^2+4)^5\,dx$

$u = x^3-3x^2+4$

$du = 3x^2-6x\ dx$

$= \frac{1}{3}\int (x^3-3x^2+4)^5\ 3(x^2-2x)\,dx$

$= \frac{1}{3}\int u^5\,du = \frac{1}{3}\,\frac{u^6}{6} + c$

$\frac{1}{18}(x^3-3x^2+4)^6 + c$

6 $\displaystyle\int \frac{x^2}{\sqrt[3]{4+x^3}}\,dx = \int (4+x^3)^{\frac{-1}{3}} x^2\,dx$

$u = 4 + x^3$

$du = 3x^2\,dx$

$= \frac{1}{3}\int (4+x^3)^{\frac{-1}{3}}\, 3x^2\,dx$

$= \frac{1}{3}\int u^{\frac{-1}{3}}\,du = \frac{1}{\cancel{3}}\frac{\cancel{3}}{2}u^{\frac{2}{3}} + c$

$= \frac{1}{2}(4+x^3)^{\frac{2}{3}} + c$

7 $\displaystyle\int \frac{dx}{\sqrt[3]{2-3x}} = \int (2-3x)^{\frac{-1}{3}}\,dx$

$u = 2 - 3x$

$du = -3\,dx$

$= \frac{-1}{3}\int (2-3x)^{\frac{-1}{3}}(-3)\,dx = \frac{-1}{3}\int u^{\frac{-1}{3}}\,du$

$= \frac{-1}{\cancel{3}}\frac{\cancel{3}}{2}u^{\frac{2}{3}} + c = \frac{-1}{2}\sqrt[3]{(2-3x)^2}$

8 $\displaystyle\int x(3x+2)^6\,dx$

$u = 3x + 2$

$du = 3\,dx$

$3x = u - 2$

$\frac{1}{3}\frac{1}{3}\int (3x+2)^6\, 3x\, 3\,dx$

$= \frac{1}{9}\int u^6(u-2)\,du = \frac{1}{9}\int u^7 - 2u^6\,du = \frac{1}{9}\left[\frac{u^8}{8} - \frac{2}{7}u^7\right] + c$

$= \frac{1}{72}(3x+2)^8 - \frac{2}{63}(3x+2)^7 + c$

(9) $\int \frac{x}{\sqrt{1+3x}}\,dx = \int (1+3x)^{-\frac{1}{2}}\, x\, dx$

$u = 1+3x$

$du = 3\,dx$

$3x = u-1$

$= \frac{1}{3}\,\frac{1}{3}\int (1+3x)^{-\frac{1}{2}}\, 3x\; 3dx$

$= \frac{1}{9}\int u^{-\frac{1}{2}}(u-1)\,du = \frac{1}{9}\int u^{\frac{1}{2}} - u^{-\frac{1}{2}}\,du$

$= \frac{1}{9}\left[\frac{2}{3}u^{\frac{3}{2}} - \frac{2}{1}u^{\frac{1}{2}}\right] + c$

$= \frac{2}{27}\sqrt{(1+3x)^3} - \frac{2}{9}\sqrt{1+3x} + c$

(10) $\int x^2\sqrt{x-1}\,dx$

$u = x-1$

$du = dx$

$x = u+1$

$= \int u^{\frac{1}{2}}(u+1)^2\,du$

$= \int u^{\frac{1}{2}}(u^2+2u+1)\,du$

$= \int (u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du$

$= \frac{2}{7}u^{\frac{7}{2}} + 2\times\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + c$

$= \frac{2}{7}\sqrt{(x-1)^7} + \frac{4}{5}\sqrt{(x-1)^5} + \frac{2}{3}\sqrt{(x-1)^3} + c$

(11) $\int x^3 \sqrt{x^2-2}\, dx$

$u = x^2 - 2$
$du = 2x\, dx$
$x^2 = u + 2$

$= \int (x^2-2)^{\frac{1}{2}} x^2 \, x\, dx$

$= \frac{1}{2}\int (x^2-2)^{\frac{1}{2}} x^2 \, 2x\, dx$

$= \frac{1}{2}\int u^{\frac{1}{2}} (u+2)\, du$

$= \frac{1}{2}\int (u^{\frac{3}{2}} + 2u^{\frac{1}{2}})\, du$

$= \frac{1}{2}\left[\frac{2}{5} u^{\frac{5}{2}} + 2\times\frac{2}{3} u^{\frac{3}{2}}\right] + C$

$= \frac{1}{5}\sqrt{(x^2-2)^5} + \frac{2}{3}\sqrt{(x^2-2)^3} + C$

(12) $\int x^5 \sqrt[3]{x^3+1}\, dx$

$u = x^3 + 1$
$du = 3x^2\, dx$
$x^3 = u - 1$

$= \frac{1}{3}\int (x^3+1)^{\frac{1}{3}} x^3 \, 3x^2\, dx$

$= \frac{1}{3}\int u^{\frac{1}{3}} (u-1)\, du = \frac{1}{3}\int (u^{\frac{4}{3}} - u^{\frac{1}{3}})\, du$

$= \frac{1}{3}\left[\frac{3}{7} u^{\frac{7}{3}} - \frac{3}{4} u^{\frac{4}{3}}\right] + C$

$= \frac{1}{7}\sqrt[3]{(x^3+1)^7} - \frac{1}{4}\sqrt[3]{(x^3+1)^4} + C$