6-2 حجوم الأجسام الدورانية

اوجد حجم المجسم الناتج من دوران المنطقة المستوية دورة كاملة حول محور السينات والمحددة بكل من المستقيمات والمنحنيات التالية:

(1) $y_1 = x^2$ ، $y_2 = 0$ ، $x = 2$ ، $x = 0$

$$V = \pi \int_0^2 (x^2)^2 \, dx = \pi \int_0^2 x^4 \, dx$$

$$= \pi \left[\frac{x^5}{5} \right]_0^2 = \pi \left(\frac{2^5}{5} - 0 \right) = \frac{32\pi}{5} \text{ units cupe}$$

(2) $y_1 = \frac{1}{x}$ ، $y_2 = 0$ ، $x = 1$ ، $x = 4$

$$y_1 > 0 \quad \forall x \in [1, 4]$$

$$V = \pi \int_1^4 \left(\frac{1}{x}\right)^2 dx = \pi \int_1^4 x^{-2} \, dx = \pi \left[\frac{-1}{x} \right]_1^4$$

$$= \pi \left(\frac{-1}{4} - \left(\frac{-1}{1}\right) \right) = \frac{3\pi}{4} \text{ units cupe}$$

(3) $y_1 = \sqrt{1 - x^2}$ ، $y_2 = 0$

$\sqrt{1-x^2} = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow$

$x = 1$ ، $x = -1$

$$V = \pi \int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx$$

$$= \pi \int_{-1}^{1} 1 - x^2 \, dx = \pi \left[x - \frac{x^3}{3} \right]_{-1}^{1}$$

$$= \pi \left(1 - \frac{1^3}{3} - \left((-1) - \frac{(-1)^3}{3} \right) \right)$$

$$= \frac{4\pi}{3} \text{ units cupe}$$

(4) $y_1 = x^2+1$, $y_2 = x+3$

$y_1 = y_2 \Rightarrow x^2+1 = x+3 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1)=0$

$x=2$, $x=-1$

$0 \in (-1,2) \Rightarrow y_1(0) = 0^2+1 = 1$

$y_2(0) = 0+3 = 3$ $\Rightarrow y_2 \geq y_1 \geq 0 \ \forall x \in [-1,2]$

$$V = \int_{-1}^{2} (y_2)^2 - (y_1)^2 \, dx = \int_{-1}^{2} (x+3)^2 - (x^2+1)^2 \, dx$$

$$= \int_{-1}^{2} x^2+6x+9-(x^4+2x^2+1)\, dx$$

$$= \int_{-1}^{2} -x^4 - x^2 + 6x + 8 \, dx$$

$$= \left[\frac{-x^5}{5} - \frac{x^3}{3} + 3x^2 + 8x \right]_{-1}^{2}$$

$$= \frac{-(2)^5}{5} - \frac{2^3}{3} + 3(2)^2 + 8(2) - \left(\frac{-(-1)^5}{5} - \frac{(-1)^3}{3} + 3(-1)^2 + 8(-1) \right)$$

$= \frac{117}{5}$ units cupe

(5) $y_1 = \sec x$, $y_2 = \sqrt{2}$, $\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$

$y_1 = y_2 \Rightarrow \sec x = \sqrt{2} \Rightarrow x = \frac{\pi}{4}$ or $x = \frac{-\pi}{4}$

$0 \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \Rightarrow y_1(0) = 1$
$y_2(0) = \sqrt{2} \Rightarrow$

$y_2 \geq y_1 \geq 0 \ \forall x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$$V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (y_2)^2 - (y_1)^2 \, dx$$

$$= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sqrt{2})^2 - (\sec x)^2 \, dx$$

$$= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2 - \sec^2 x \, dx$$

$$= \pi \left[2x - \tan x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \pi \left[2\left(\frac{\pi}{4}\right) - \tan\frac{\pi}{4} - \left(2\left(-\frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{4}\right)\right)\right]$$

$$= \pi \left[\frac{\pi}{2} - 1 + \frac{\pi}{2} + -1\right]$$

$$= \pi \left[\pi - 2\right]$$

$= \pi^2 - 2\pi$ units cupe

(6) $y_1 = x+1$ ، $y_2 = x-1$, $x=1$, $x=4$

y_1 ، y_2 مستقيمان متوازيان : هما غير متقاطعان

$$2 \in (1 ، 4) \Rightarrow \begin{matrix} y_1(2) = 3 \\ y_2(2) = 1 \end{matrix} \Rightarrow$$

$$y_1 \geq y_2 \geq 0 \ \forall \ x \in [1 ، 4]$$

$$V = \pi \int_1^4 (y_1)^2 - (y_2)^2 \, dx$$

$$= \pi \int_1^4 (x+1)^2 - (x-1)^2 \, dx$$

$$= \pi \int_1^4 x^2 + 2x + 1 - (x^2 - 2x + 1) \, dx$$

$$= \pi \int_1^4 4x \, dx = \pi \left[2x^2 \right]_1^4$$

$$= \pi \left(2(4)^2 - 2(1)^2 \right) = 30\pi \text{ units cupe}$$

(7) $y_1 = x$ ، $y_2 = 1$, $x = 0$

$y_1 = y_2 \Rightarrow x = 1$ ، $\frac{1}{2} \in (0 ، 1) \Rightarrow$

$$\begin{matrix} y_1(\frac{1}{2}) = \frac{1}{2} \\ y_2(\frac{1}{2}) = 1 \end{matrix} \Rightarrow y_2 \geq y_1 \geq 0 \quad \forall x \in [0 ، 1]$$

$$V = \pi \int_0^1 (1)^2 - (x)^2 \, dx = \pi \int_0^1 1 - x^2 \, dx =$$

$$= \pi \left[x - \frac{x^3}{3} \right]_0^1$$

$$= \pi \left(1 - \frac{1^3}{3} - 0 \right) = \frac{2\pi}{3} \text{ units cupe}$$

(8) $y_1 = \sqrt{x}$, $y_2 = 0$, $x = 4$

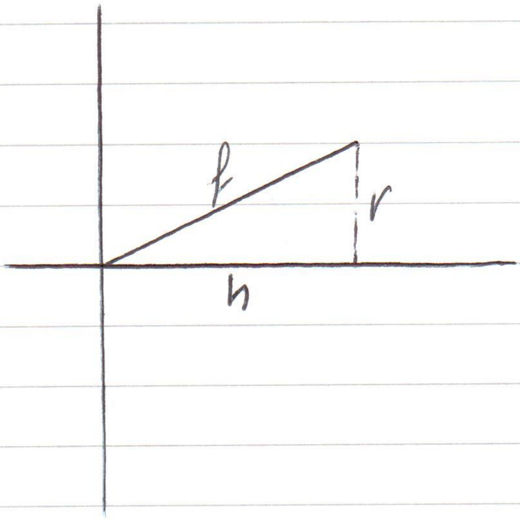
$y_1 = y_2 \Rightarrow \sqrt{x} = 0 \Rightarrow x = 0$

$y_1 \geq y_2 \geq 0 \quad \forall x \in [0, 4]$

$$V = \pi \int_0^4 (\sqrt{x})^2 - 0 \, dx = \pi \int_0^4 x \, dx = \pi \left[\frac{x^2}{2} \right]_0^4$$

$$= \pi \left(\frac{4^2}{2} - 0 \right) = 8\pi \quad \text{units cupe}$$

(9)



$$f(x) = \frac{r}{h} x$$

$$V = \pi \int_0^h (f(x))^2 \, dx$$

$$= \pi \int_0^h \left(\frac{r}{h} x \right)^2 dx$$

$$= \pi \int_0^h \frac{r^2}{h^2} x^2 \, dx$$

$$= \pi \frac{r^2}{h^2} \left[\frac{x^3}{3} \right]_0^h = \frac{\pi}{3} \frac{r^2}{h^2} (h^3 - 0)$$

$$= \frac{1}{3} \pi r^2 h$$