ص ٦٦

اوجد قيمة كل من :

(٤) $(\text{جا}\,\theta + \text{جتا}\,\theta)^{2} - ٢\,\text{جا}\,\theta\,\text{جتا}\,\theta$

$= \text{جا}^{2}\theta + ٢\,\text{جا}\,\theta \cdot \text{جتا}\,\theta + \text{جتا}^{2}\theta - ٢\,\text{جا}\,\theta\,\text{جتا}\,\theta$

$= ١$

(٥) $(\text{ظا}^{2}\theta + ١)\,\text{جتا}^{2}\theta$

$= \text{قا}^{2}\theta \times \text{جتا}^{2}\theta = \frac{١}{\text{جتا}^{2}\theta} \times \text{جتا}^{2}\theta = ١$

(٦) $١ + \text{ظا}^{2}(-\theta) - \text{قا}^{2}\theta =$

$= ١ + (-\text{ظا}\,\theta)^{2} - \frac{١}{\text{جتا}^{2}\theta}$

$= ١ + \frac{\text{جا}^{2}\theta}{\text{جتا}^{2}\theta} - \frac{١}{\text{جتا}^{2}\theta} = ١ + \frac{\text{جا}^{2}\theta - ١}{\text{جتا}^{2}\theta}$

$= ١ + \frac{-\text{جتا}^{2}\theta}{\text{جتا}^{2}\theta} = ١ - ١ =$ صفر

(٧) $٩\,\text{قا}^{2}\theta - ٥\,\text{ظا}^{2}\theta - \frac{٤}{\text{جتا}^{2}\theta} =$

$= ٩\,\frac{١}{\text{جتا}^{2}\theta} - ٥\,\frac{\text{جا}^{2}\theta}{\text{جتا}^{2}\theta} - \frac{٤}{\text{جتا}^{2}\theta}$

$= \frac{٥}{\text{جتا}^{2}\theta} - \frac{٥\,\text{جا}^{2}\theta}{\text{جتا}^{2}\theta} = \frac{٥}{\text{جتا}^{2}\theta}(١ - \text{جا}^{2}\theta)$

$= \frac{٥}{\text{جتا}^{2}\theta} \times \text{جتا}^{2}\theta = ٥$

اثبت صحة المتطابقات

٨) $١ + \text{ظتا}^{٢}(-\theta) = \text{قتا}^{٢}\theta$

$\text{الطرف الأول} = ١ + (-\text{ظتا}(\theta))^{٢}$

$= ١ + \text{ظتا}^{٢}\theta = \text{قتا}^{٢}\theta = \text{الطرف الثاني}$

٩) $\text{قا}^{٤}\theta - \text{قا}^{٢}\theta = \text{ظا}^{٤}\theta + \text{ظا}^{٢}\theta$

$\text{الطرف الأول} = \text{قا}^{٢}\theta(\text{قا}^{٢}\theta - ١) = \text{قا}^{٢}\theta\,\text{ظا}^{٢}\theta$

$\text{الطرف الثاني} = \text{ظا}^{٢}\theta(\text{ظا}^{٢}\theta + ١) = \text{ظا}^{٢}\theta\,\text{قا}^{٢}\theta$

الطرفان متساويان

١٠) $(١ - \text{جتا}^{٢}\theta)(١ + \text{ظتا}^{٢}\theta) = ١$

$\text{الطرف الأول} = \text{جا}^{٢}\theta \times \text{قتا}^{٢}\theta$

$= \text{جا}^{٢}\theta \times \frac{١}{\text{جا}^{٢}\theta} = ١$

١١) $٣\,\text{جا}^{٢}\theta + ٤\,\text{جتا}^{٢}\theta = ٣ + \text{جتا}^{٢}\theta$

$\text{الطرف الأول} = ٣\,\text{جا}^{٢}\theta + ٣\,\text{جتا}^{٢}\theta + \text{جتا}^{٢}\theta$

$= ٣(\text{جا}^{٢}\theta + \text{جتا}^{٢}\theta) + \text{جتا}^{٢}\theta$

$= ٣ \times ١ + \text{جتا}^{٢}\theta$

$= ٣ + \text{جتا}^{٢}\theta$

ص ٦٧

(١)

$$\text{قتا}\,\theta \times \text{جتا}\,\theta - \text{ظتا}\,\theta = \frac{1}{\text{جا}\,\theta} \times \text{جتا}\,\theta - \frac{\text{جتا}\,\theta}{\text{جا}\,\theta}$$

$$= \frac{\text{جتا}\,\theta}{\text{جا}\,\theta} - \frac{\text{جتا}\,\theta}{\text{جا}\,\theta} = \text{صفر}$$

(٢)

$$\text{ظتا}^2(-\theta) - \text{قتا}^2\,\theta = (-\text{ظتا}\,\theta)^2 - \text{قتا}^2\,\theta$$

$$= \frac{\text{جتا}^2\,\theta}{\text{جا}^2\,\theta} - \frac{1}{\text{جا}^2\,\theta} = \frac{\text{جتا}^2\,\theta - 1}{\text{جا}^2\,\theta}$$

$$= \frac{-\text{جا}^2\,\theta}{\text{جا}^2\,\theta} = -1$$

(٣)

$$(\text{قا}\,\theta + \text{ظا}\,\theta)(\text{قا}\,\theta - \text{ظا}\,\theta) = \text{قا}^2\,\theta - \text{ظا}^2\,\theta$$

$$= \frac{1}{\text{جتا}^2\,\theta} - \frac{\text{جا}^2\,\theta}{\text{جتا}^2\,\theta} = \frac{\text{جتا}^2\,\theta}{\text{جتا}^2\,\theta} = 1$$

(٤)

$$\text{جا}\,\theta\ \text{قتا}\,\theta - \text{جتا}^2\,\theta - \text{جا}^2\,\theta =$$

$$= \text{جا}\,\theta \times \frac{1}{\text{جا}\,\theta} - (\text{جتا}^2\,\theta + \text{جا}^2\,\theta)$$

$$= 1 - 1 = \text{صفر}$$

(٥)

$$\frac{\text{جا}^2\,\theta}{1 - \text{جتا}\,\theta} - \text{جتا}\,\theta = \frac{\text{جا}^2\,\theta\,(1 + \text{جتا}\,\theta)}{(1 - \text{جتا}\,\theta)(1 + \text{جتا}\,\theta)} - \text{جتا}\,\theta$$

$$= \frac{\text{جا}^2\,\theta\,(1 + \text{جتا}\,\theta)}{1 - \text{جتا}^2\,\theta} - \text{جتا}\,\theta$$

$$= \frac{\cancel{\text{جا}^2\,\theta}\,(1 + \text{جتا}\,\theta)}{\cancel{\text{جا}^2\,\theta}} - \text{جتا}\,\theta = 1 + \text{جتا}\,\theta - \text{جتا}\,\theta = 1$$

(٦) $\text{ظا}\,\theta + \text{ظتا}\,\theta - \text{قا}\,\theta\ \text{قتا}\,\theta =$

$$= \frac{\text{جا}\,\theta}{\text{جتا}\,\theta} + \frac{\text{جتا}\,\theta}{\text{جا}\,\theta} - \frac{1}{\text{جتا}\,\theta} \times \frac{1}{\text{جا}\,\theta}$$

$$= \frac{\text{جا}^2\theta + \text{جتا}^2\theta}{\text{جتا}\,\theta \cdot \text{جا}\,\theta} - \frac{1}{\text{جتا}\,\theta \cdot \text{جا}\,\theta}$$

$$= \frac{1}{\text{جتا}\,\theta \cdot \text{جا}\,\theta} - \frac{1}{\text{جتا}\,\theta \cdot \text{جا}\,\theta} = \text{صفر}$$

(٩) $\text{جا}\,\theta\,(\text{ظتا}\,\theta + \text{ظا}\,\theta) = \text{قا}\,\theta$

الطرف الأول $= \text{جا}\,\theta \left(\dfrac{\text{جتا}\,\theta}{\text{جا}\,\theta} + \dfrac{\text{جا}\,\theta}{\text{جتا}\,\theta} \right)$

$= \text{جا}\,\theta \left(\dfrac{\text{جتا}^2\theta + \text{جا}^2\theta}{\text{جا}\,\theta\ \text{جتا}\,\theta} \right)$

$= \text{جا}\,\theta \left(\dfrac{1}{\text{جا}\,\theta\ \text{جتا}\,\theta} \right)$

$= \dfrac{\text{جا}\,\theta}{\text{جا}\,\theta\ \text{جتا}\,\theta} = \dfrac{1}{\text{جتا}\,\theta} = \text{قا}\,\theta$

(١٠) $\dfrac{\text{جا}\,\theta}{\text{جا}\,\theta - \text{جتا}\,\theta} = \dfrac{1}{1 - \text{ظتا}\,\theta}$

الطرف الثاني $= \dfrac{1}{1 - \text{ظتا}\,\theta} = \dfrac{1}{1 - \dfrac{\text{جتا}\,\theta}{\text{جا}\,\theta}}$

$= \dfrac{1}{\dfrac{\text{جا}\,\theta - \text{جتا}\,\theta}{\text{جا}\,\theta}} = \dfrac{\text{جا}\,\theta}{\text{جا}\,\theta - \text{جتا}\,\theta}$