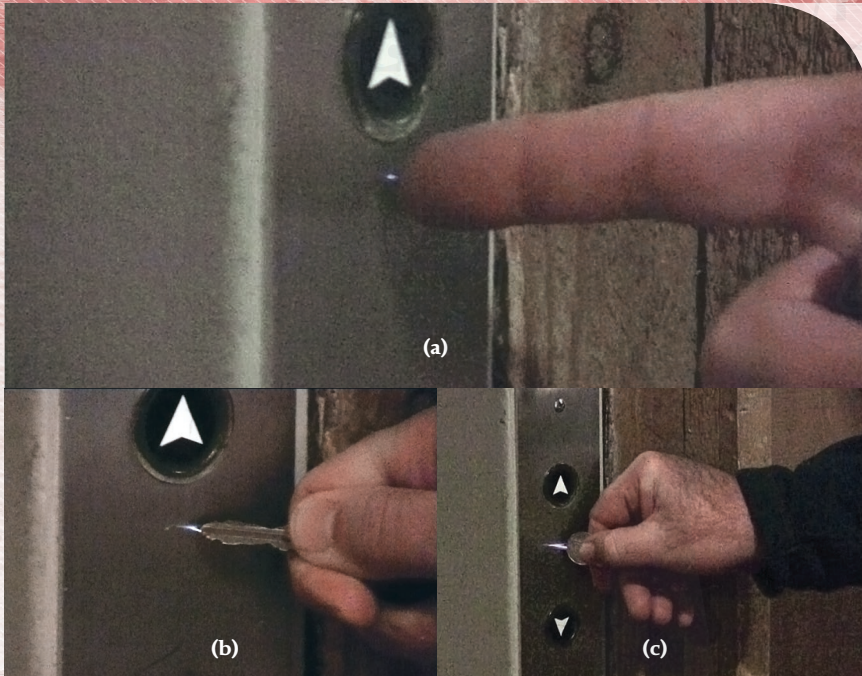# 1

Electrostatics

FIGURE 1.1 (a) A spark due to static electricity occurs between a person's finger and a metal surface near an elevator button. (b) and (c) Similar sparks are generated when the person holds a metal object like a car key or a coin, but are painless because the spark forms between the metal surface and the metal object.

Many people think of static electricity as the annoying spark that occurs when they reach for a metal object like a doorknob on a dry day, after they have been walking on a carpet (Figure 1.1). In fact, many electronics manufacturers place small metal plates on equipment so that users can discharge any spark on the plate and not damage the more sensitive parts of the equipment. However, static electricity is more than just an occasional annoyance; it is the starting point for any study of electricity and magnetism, forces that have changed human society as radically as anything since the discovery of fire or the wheel.

In this chapter, we examine the properties of electric charge. A moving electric charge gives rise to a separate phenomenon, called *magnetism,* which is covered in later chapters. Here we look at charged objects that are not moving–hence the term *electrostatics.* All objects have charge, since charged particles make up atoms and molecules. We often don't notice the effects of electrical charge because most objects are electrically neutral. The forces that hold atoms together and that keep objects separate even when they're in contact, are all electric in nature.

WHAT WE WILL LEARN

- Electric charge gives rise to a force between charged particles or objects.
- Electricity and magnetism together make up the electromagnetic force, one of the four fundamental forces of nature.
- There are two kinds of electric charge, positive and negative. Like charges repel, and unlike charges attract.
- Electric charge is quantized, meaning that it occurs only in integral multiples of a smallest elementary quantity. Electric charge is also conserved.
- Most materials around us are electrically neutral.
- The electron is an elementary particle, and its charge is the smallest observable quantity of electric charge.

- Insulators conduct electricity poorly or not at all. Conductors conduct electricity well but not perfectly–some energy losses occur.
- Semiconductors can be made to change between a conducting state and a nonconducting state.
- Superconductors conduct electricity perfectly.
- Objects can be charged directly by contact or indirectly by induction.
- The force that two stationary electric charges exert on each other is proportional to the product of the charges and varies as the inverse square of the distance between the two charges.
- Electrostatic forces between particles can be added as vectors by the process of superposition.

**FIGURE 1.2** Lightning strikes over a city.

1.1 Electromagnetism

Perhaps no mystery puzzled ancient civilizations more than electricity, which they observed primarily in the form of lightning strikes (Figure 1.2). The destructive force inherent in lightning, which could set objects on fire and kill people and animals, puzzled people because they did not understand what caused it or where the lightening came from.

The ancient Greeks knew that if you rubbed a piece of amber with a piece of cloth, you could attract small, light objects with the amber. We now know that rubbing amber with a cloth transfers negatively charged particles called *electrons* from the cloth to the amber. (The words *electron* and *electricity* derive from the Greek word for amber.) Lightning also consists of a flow of electrons. The early Greeks and others also knew about naturally occurring magnetic objects called *lodestones,* which were found in deposits of magnetite, a mineral consisting of iron oxide. These objects were used to construct compasses as early as 300 BC.

The relationship between electricity and magnetism was not understood until the middle of the 19th century. The following chapters will reveal how electricity and magnetism can be unified into a common framework called *electromagnetism.* However, unification of forces does not stop there. During the early part of the 20th century, two more fundamental forces were discovered: the weak force, which operates in beta decay (in which an electron and a neutrino are spontaneously emitted from certain types of nuclei), and the strong force, which acts inside the atomic nucleus. Currently, the electromagnetic and weak forces are viewed as two aspects of the electroweak force (Figure 1.3). For the phenomena discussed in this and the following chapters, this electroweak unification has no influence; it becomes important in the highest-energy particle collisions. Because the energy scale for the electroweak unification is so high, most textbooks continue to speak of four fundamental forces: gravitational, electromagnetic, weak, and strong.

Today, a large number of physicists believe that the electroweak force and the strong force can also be unified, that is, described in a common framework. Several theories propose ways to accomplish this, but so far experimental evidence is missing. Interestingly, the force that has been known longer than any of the other fundamental forces, gravity, seems to be hardest to shoehorn into a unified framework with the other fundamental forces. Quantum gravity, supersymmetry, and string theory are current foci of cutting-edge physics research in which theorists are attempting to construct this grand unification and discover the (hubristically

Forces of Nature
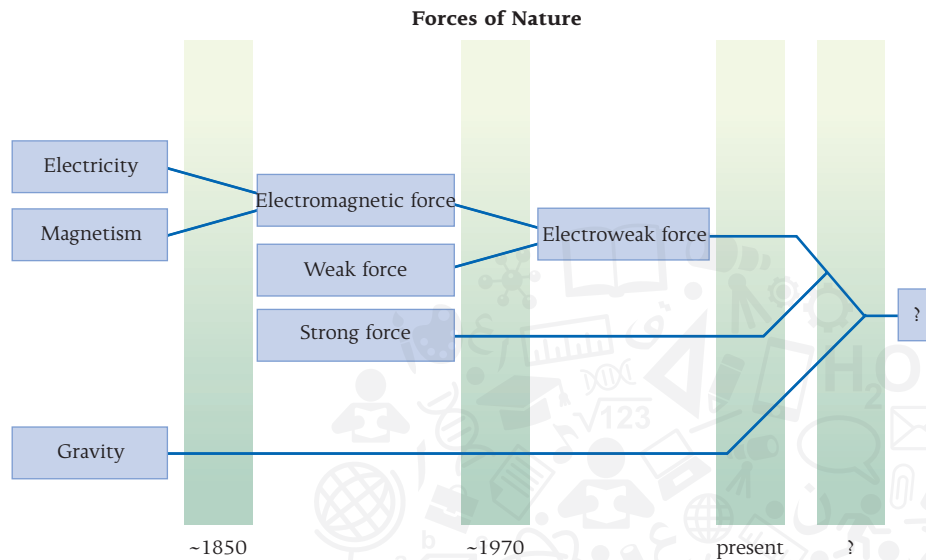
FIGURE 1.3 The history of the unification of fundamental forces.

named) Theory of Everything. They are mainly guided by symmetry principles and the conviction that nature must be elegant and simple.

In this chapter, we consider electric charge, how materials react to electric charge, static electricity, and the forces resulting from electric charges. **Electrostatics** covers situations where charges stay in place and do not move.

1.2 Electric Charge

Let's look a little deeper into the cause of the electric sparks that you occasionally receive on a dry winter day if you walk across a carpet and then touch a metal doorknob. (Electrostatic sparks have even ignited gas fumes while someone is filling the tank at a gas station. This is not an urban legend; a few of these cases have been caught on gas station surveillance cameras.) The process that causes this sparking is called **charging.** Charging consists of the transfer of negatively charged particles, called **electrons,** from the atoms and molecules of the material of the carpet to the soles of your shoes. This charge can move relatively easily through your body, including your hands. The built-up electric charge discharges through the metal of the doorknob, creating a spark.

The two types of electric charge found in nature are **positive charge** and **negative charge.** Normally, objects around us do not seem to be charged; instead, they are electrically neutral. Neutral objects contain roughly equal numbers of positive and negative charges that largely cancel each other. Only when positive and negative charges are not balanced do we observe the effects of electric charge.

If you rub a glass rod with a cloth, the glass rod becomes charged and the cloth acquires a charge of the opposite sign. If you rub a plastic rod with fur, the rod and fur also become oppositely charged. If you bring two charged glass rods together, they repel each other. Similarly, if you bring two charged plastic rods together, they also repel each other. However, a charged glass rod and a charged plastic rod will attract each other. This difference arises because the glass rod and the plastic rod have opposite charge. These observations led to the following law:

Law of Electric Charges
Like charges repel and opposite charges attract.

The unit of electric charge is the **coulomb** (C), named after the French physicist Charles-Augustine de Coulomb (1736–1806). The coulomb is defined in terms of the SI unit for current, the ampere (A), named after another French physicist, André-Marie Ampère (1775–1836). The ampere can not be derived in terms of the other SI units: meter, kilogram, and second. Instead, the ampere is another fundamental SI unit. For this reason, the SI system of units is sometimes called *MKSA (meter-kilogram-second-ampere) system.* The charge unit is defined as

$$1 \text{ C} = 1 \text{ A s} \tag{1.1}$$

The definition of the ampere must wait until we discuss current in later chapters. However, we can define the magnitude of the coulomb by simply specifying the charge of a single electron:

$$q_e = -e \tag{1.2}$$

where q_e is the charge and e has the (currently best accepted and experimentally measured) value

$$e = 1.602176565(35) \times 10^{-19} \text{ C} \tag{1.3}$$

(Usually it is enough to carry only the first two to four significant digits of this mantissa. We will use a value of 1.602 in this chapter, but you should keep in mind that equation 1.3 gives the full accuracy to which this charge has been measured.)

The charge of the electron is an intrinsic property of the electron, just like its mass. The charge of the **proton,** another basic particle of atoms, is exactly the same magnitude as that of the electron, only the proton's charge is positive:

$$q_p = +e \tag{1.4}$$

The choice of which charge is positive and which charge is negative is arbitrary. The conventional choice of $q_e < 0$ and $q_p > 0$ is due to the American statesman, scientist, and inventor Benjamin Franklin (1706–1790), who pioneered studies of electricity.

One coulomb is an extremely large unit of charge. We'll see later in this chapter just how big it is when we investigate the magnitude of the forces of charges on each other. Units of μC (microcoulombs, 10^{-6} C), nC (nanocoulombs, 10^{-9} C), and pC (picocoulombs, 10^{-12} C) are commonly used.

Benjamin Franklin also proposed that charge is conserved. Charge is not created or destroyed, simply moved from one object to another.

> **Law of Charge Conservation**
> The total electric charge of an isolated system is conserved.

This law is the fourth conservation law we have encountered so far, the first three being the conservation laws for total energy, momentum, and angular momentum. Conservation laws are a common thread that runs throughout all of physics and thus throughout this book as well.

It is important to note that there is a conservation law for charge, but *not* for mass. Mass and energy are not independent of each other. What is sometimes described in introductory chemistry as conservation of mass is not an exact conservation law, but only an approximation used to keep track of the number of atoms in chemical reactions. (It is a good approximation to a large number of significant figures but not an exact law, like charge conservation.) Conservation of charge applies to all systems, from the macroscopic system of plastic rod and fur down to systems of subatomic particles.

Elementary Charge

Electric charge occurs only in integral multiples of a minimum size. This is expressed by saying that charge is **quantized.** The smallest observable unit of electric charge is the charge of the electron, which is -1.602×10^{-19} C (as defined in equation 1.3).

The fact that electric charge is quantized was verified in an ingenious experiment carried out in 1910 by American physicist Robert A. Millikan (1868–1953) and known as the *Millikan oil drop experiment* (Figure 1.4). In this experiment, oil drops were sprayed into a chamber where electrons were knocked out of the drops by some form of radiation, usually X-rays. The resulting positively charged drops were allowed to fall between two electrically charged plates. Adjusting the charge of the plates caused the drops to stop falling and allowed their charge to be measured. What Millikan observed was that charge was quantized rather than continuous. (A quantitative analysis of this experiment will be presented in Chapter 3 on electric potential.) That is, this experiment and its subsequent refinements established that charge comes only in integer multiples of the charge of an electron. In everyday experiences with electricity, we do not notice that charge is quantized because most electrical phenomena involve huge numbers of electrons.

You studied earlier that matter is composed of atoms and that an atom consists of a nucleus containing charged protons and neutral neutrons. A schematic drawing of a carbon atom is shown in Figure 1.5. A carbon atom has six protons and (usually) six neutrons in its nucleus. This nucleus is surrounded by six electrons. Note that this drawing is not to scale. In the actual atom, the distance of the electrons from the nucleus is much larger (by a factor on the order of 10,000) than the size of the nucleus. In addition, the electrons are shown in circular orbits, which is also not quite correct. In Chapter 38, we'll see that the locations of electrons in the atom can be characterized only by probability distributions.

As mentioned earlier, a proton has a positive charge with a magnitude that is *exactly* equal to the magnitude of the negative charge of an electron. In a neutral atom, the number of negatively charged electrons is equal to the number of positively charged protons. The mass of the electron is much smaller than the mass of the proton or the neutron. Therefore, most of the mass of an atom resides in the nucleus. Electrons can be removed from atoms relatively easily. For this reason, electrons are typically the carriers of electricity, rather than protons or atomic nuclei.

The electron is a fundamental particle and has no substructure: It is a point particle with zero radius (at least, according to current understanding). However, high-energy probes have been used to look inside the proton. A proton is composed of charged particles called *quarks*, held together by uncharged particles called *gluons*. Quarks have a charge of $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ times the charge of the electron. These fractionally charged particles cannot exist independently and have never been observed directly, despite numerous extensive searches. Just like the charge of an electron, the charges of quarks are intrinsic properties of these elementary particles.

A proton is composed of two *up quarks* (each with charge $+\frac{2}{3}e$) and one *down quark* (with charge $-\frac{1}{3}e$), giving the proton a charge of $q_{\mathrm{p}} = (2)(+\frac{2}{3}e) + (1)(-\frac{1}{3}e) = +e$, as illustrated in Figure 1.6a. The electrically neutral neutron (hence the name!) is composed of an up quark and two down quarks, as shown in Figure 1.6b, so its charge is $q_{\mathrm{n}} = (1)(+\frac{2}{3}e) + (2)(-\frac{1}{3}e) = 0$. There are also much more massive electron-like particles named *muon* and *tau*. But the basic fact remains that all of the matter in everyday experience is made up of electrons (with electrical charge $-e$), up and down quarks (with electrical charges $+\frac{2}{3}e$ and $-\frac{1}{3}e$, respectively), and gluons (zero charge).

It is remarkable that the charges of the quarks inside a proton add up to *exactly* the same magnitude as the charge of the electron. This fact is still a puzzle, pointing to some deep symmetry in nature that is not yet understood.
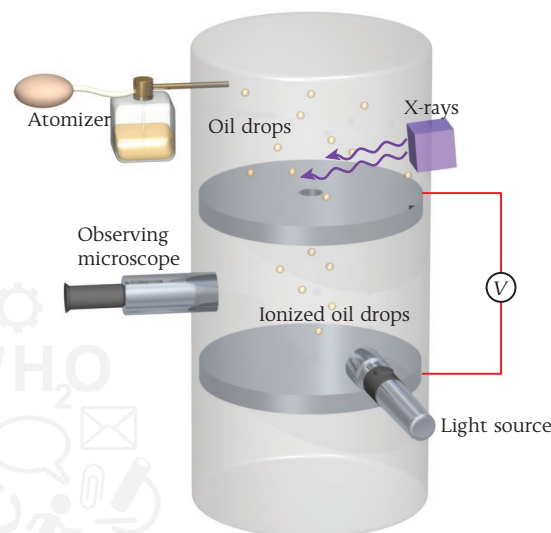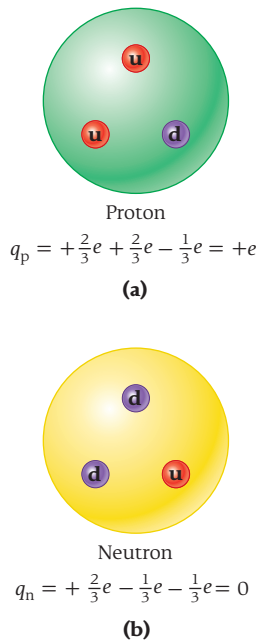

FIGURE 1.4 Schematic drawing of the Millikan oil drop experiment.
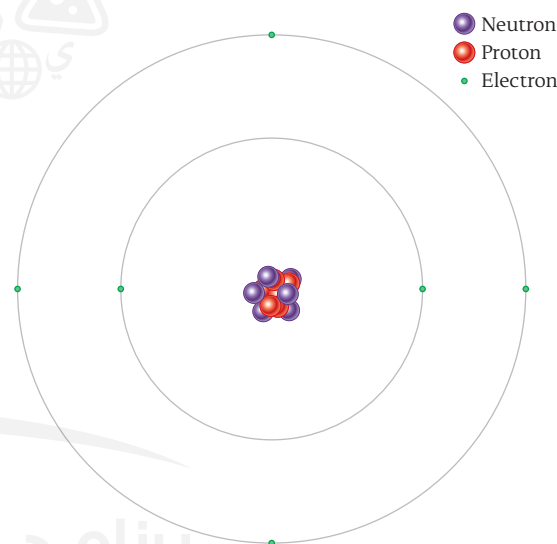
FIGURE 1.5 In a carbon atom, the nucleus contains six neutrons and six protons. The nucleus is surrounded by six electrons. Note that this drawing is schematic and not to scale.

Proton
$$q_p = +\tfrac{2}{3}e + \tfrac{2}{3}e - \tfrac{1}{3}e = +e$$
(a)

Neutron
$$q_n = +\tfrac{2}{3}e - \tfrac{1}{3}e - \tfrac{1}{3}e = 0$$
(b)

FIGURE 1.6 (a) A proton contains two up quarks (u) and one down quark (d). (b) A neutron contains one up quark (u) and two down quarks (d).

Self-Test Opportunity 1.1

Give the charge of the following elementary particles or atoms in terms of the elementary charge $e = 1.602 \times 10^{-19}$ C.

a) proton

b) neutron

c) helium atom (two protons, two neutron, and two electrons)

d) hydrogen atom (one proton and one electron)

e) up quark

f) down quark

g) electron

h) alpha particle (two protons and two neutrons)

Because all macroscopic objects are made of atoms, which in turn are made of electrons and atomic nuclei consisting of protons and neutrons, the charge, q, of any object can be expressed in terms of the sum of the number of protons, N_p, minus the sum of the number of electrons, N_e, that make up the object:

$$q = e\left(N_p - N_e\right) \tag{1.5}$$

EXAMPLE 1.1   Net Charge

PROBLEM

If we wanted a block of iron of mass 3.25 kg to acquire a positive charge of 0.100 C, what fraction of the electrons would we have to remove?

SOLUTION

Iron has mass number 56. Therefore, the number of iron atoms in the 3.25 kg block is

$$N_{atom} = \frac{(3.25 \text{ kg})(6.022 \times 10^{23} \text{ atoms/mole})}{0.0560 \text{ kg/mole}} = 3.495 \times 10^{25} = 3.50 \times 10^{25} \text{ atoms}$$

Note that we have used Avogadro's number, 6.022×10^{23}, and the definition of the mole, which specifies that the mass of 1 mole of a substance in grams is just the mass number of the substance–in this case, 56.

Because the atomic number of iron is 26, which equals the number of protons or electrons in an iron atom, the total number of electrons in the 3.25 kg block is

$$N_e = 26 N_{atom} = (26)(3.495 \times 10^{25}) = 9.09 \times 10^{26} \text{ electrons}$$

We use equation 1.5 to find the number of electrons, $N_{\Delta e}$, that we would have to remove. Because the number of electrons equals the number of protons in the original uncharged object, the difference in the number of protons and electrons is the number of removed electrons, $N_{\Delta e}$:

$$q = e \cdot N_{\Delta e} \Rightarrow N_{\Delta e} = \frac{q}{e} = \frac{0.100 \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 6.24 \times 10^{17}.$$

Finally, we obtain the fraction of electrons we would have to remove:

$$\frac{N_{\Delta e}}{N_e} = \frac{6.24 \times 10^{17}}{9.09 \times 10^{26}} = 6.87 \times 10^{-10}.$$

We would have to remove fewer than one in a billion electrons from the iron block in order to put the sizable positive charge of 0.100 C on it.

1.3   Insulators, Conductors, Semiconductors, and Superconductors

Materials that conduct electricity well are called **conductors.** Materials that do not conduct electricity are called **insulators.** (Of course, there are good and poor conductors and good and poor insulators, depending on the properties of the specific materials.)

The electronic structure of a material refers to the way in which electrons are bound to nuclei, as we'll discuss in later chapters. For now, we are interested in the relative propensity of the atoms of a material to either give up or acquire electrons. For insulators, no free movement of electrons occurs because the material has no loosely bound electrons that can escape from its atoms and thereby move freely throughout the material. Even when external charge is placed on an insulator, this external charge cannot move appreciably. Typical insulators are glass, plastic, and cloth.

On the other hand, materials that are conductors have an electronic structure that allows the free movement of some electrons. The positive charges of the atoms of a conducting material do not move, since they reside in the heavy nuclei. Typical solid conductors are metals. Copper, for example, is a very good conductor and is therefore used in electrical wiring.

Fluids and organic tissue can also serve as conductors. Pure distilled water is not a very good conductor. However, dissolving common table salt (NaCl), for example, in water improves its conductivity tremendously, because the positively charged sodium ions (Na^+) and negatively charged chlorine ions (Cl^-) can move within the water to conduct electricity. In liquids, unlike solids, positive as well as negative charge carriers are mobile. Organic tissue is not a very good conductor, but it conducts electricity well enough to make large currents dangerous to us.

Semiconductors

A class of materials called **semiconductors** can change from being an insulator to being a conductor and back to an insulator again. Semiconductors were discovered only a little more than 50 years ago but are the backbone of the entire computer and consumer electronics industries. The first widespread use of semiconductors was in transistors (Figure 1.7a); modern computer chips (Figure 1.7b) perform the functions of millions of transistors. Computers and basically all modern consumer electronics products and devices (televisions, cameras, video game players, cell phones, etc.) would be impossible without semiconductors. Gordon Moore, cofounder of Intel, famously stated that due to advancing technology, the power of the average computer's CPU (central processing unit) doubles every 18 months, which is an empirical average over the last 5 decades. This doubling phenomenon is known as *Moore's Law.* Physicists have been and will undoubtedly continue to be the driving force behind this process of scientific discovery, invention, and improvement.

Semiconductors are of two kinds: intrinsic and extrinsic. Examples of *intrinsic semiconductors* are chemically pure crystals of gallium arsenide, germanium, or, especially, silicon. Engineers produce *extrinsic semiconductors* by *doping,* which is the addition of minute amounts (typically 1 part in 10^6) of other materials that can act as electron donors or electron receptors. Semiconductors doped with electron donors are called *n-type* (*n* stands for "negative charge"). If the doping substance acts as an electron receptor, the hole left behind by an electron that attaches to a receptor can also travel through the semiconductor and acts as an effective positive charge carrier. These semiconductors are consequently called *p-type* (*p* stand for "positive charge"). Thus, unlike normal solid conductors in which only negative charges move, semiconductors have movement of negative or positive charges (which are really electron holes, that is, missing electrons).

Superconductors

Superconductors are materials that have zero resistance to the conduction of electricity, as opposed to normal conductors, which conduct electricity well but with some losses. Materials are superconducting only at very low temperatures. A typical superconductor is a niobium-titanium alloy that must be kept near the temperature of liquid helium (4.2 K) to retain its superconducting properties. During the last 20 years, new materials called *high-T_c superconductors* (T_c stands for "critical temperature," which is the maximum temperature that allows superconductivity) have been developed. These are superconducting at the temperature at which nitrogen can exist as a liquid (77.3 K). Materials that are superconductors at room temperature (300 K) have not yet been found, but they would be extremely useful. Research directed at developing such materials and theoretically explaining what physical phenomena cause high-T_c superconductivity is currently in progress.
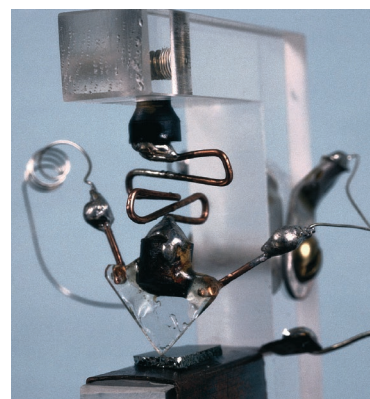
(a)
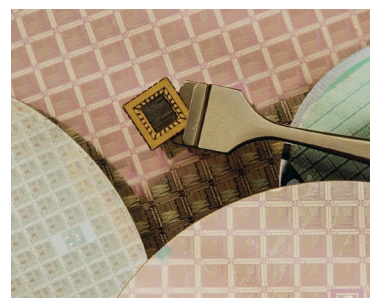
(b)

FIGURE **1.7** (a) Replica of the first transistor, invented in 1947 by John Bardeen, Walter H. Brattain, and William B. Shockley. (b) Modern computer chips made from silicon wafers contain many tens of millions of transistors.

1.4 Electrostatic Charging

Giving a static charge to an object is a process known as **electrostatic charging.** Electrostatic charging can be understood through a series of simple experiments. A power supply serves as a ready source of positive and negative charge. The battery in your car is a similar

FIGURE 1.8 A typical electroscope used in lecture demonstrations.

Concept Check 1.2

The hinged conductor moves away from the fixed conductor if a charge is applied to the electroscope, because

a) like charges repel each other.

b) like charges attract each other.

c) unlike charges attract each other.

d) unlike charges repel each other.

FIGURE 1.9 Inducing a charge: (a) An uncharged electroscope. (b) A negatively charged paddle is brought near the electroscope. (c) The negatively charged paddle is taken away.

power supply; it uses chemical reactions to create a separation between positive and negative charge. Several insulating paddles can be charged with positive or negative charge from the power supply. In addition, a conducting connection is made to the Earth. The Earth is a nearly infinite reservoir of charge, capable of effectively neutralizing electrically charged objects in contact with it. This taking away of charge is called **grounding,** and an electrical connection to the Earth is called a **ground.**

An **electroscope** is a device that gives an observable response when it is charged. You can build a relatively simple electroscope by using two strips of very thin metal foil that are attached at one end and are allowed to hang straight down adjacent to each other from an isolating frame. Kitchen aluminum foil is not suitable, because it is too thick, but hobby shops sell thinner metal foils. For the isolating frame, you can use a Styrofoam coffee cup turned sideways, for example.

The lesson-demonstration-quality electroscope shown in Figure 1.8 has two conductors that in their neutral position are touching and oriented in a vertical direction. One of the conductors is hinged at its midpoint so that it will move away from the fixed conductor if a charge appears on the electroscope. These two conductors are in contact with a conducting ball on top of the electroscope, which allows charge to be applied or removed easily.

An uncharged electroscope is shown in Figure 1.9a. The power supply is used to give a negative charge to one of the insulating paddles. When the paddle is brought near the ball of the electroscope, as shown in Figure 1.9b, the electrons in the conducting ball of the electroscope are repelled, which produces a net negative charge on the conductors of the electroscope. This negative charge causes the movable conductor to rotate because the stationary conductor also has negative charge and repels it. Because the paddle did not touch the ball, the charge on the movable conductors is **induced.** If the charged paddle is then taken away, as illustrated in Figure 1.9c, the induced charge reduces to zero, and the movable conductor returns to its original position, because the total charge on the electroscope did not change in the process.

If the same process is carried out with a positively charged paddle, the electrons in the conductors are attracted to the paddle and flow into the conducting ball. This leaves a net positive charge on the conductors, causing the movable conducting arm to rotate again. Note that the net charge of the electroscope is zero in both cases and that the motion of the conductor indicates only that the paddle is charged. When the positively charged paddle is removed, the movable conductor again returns to its original position. It is important to note that we cannot determine the sign of this charge!
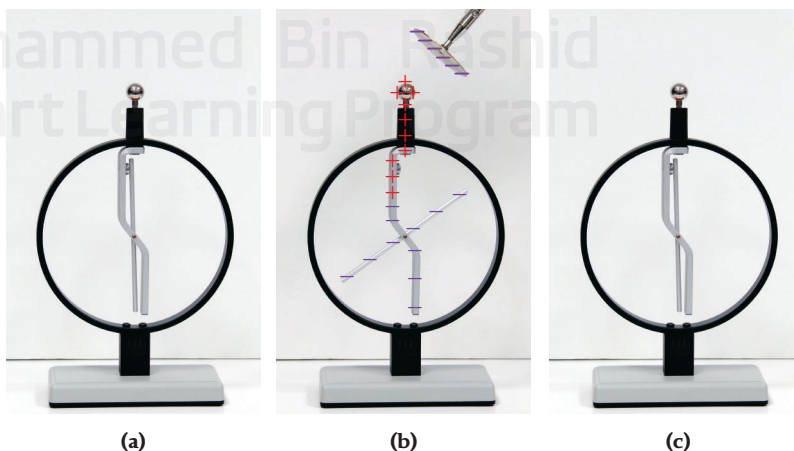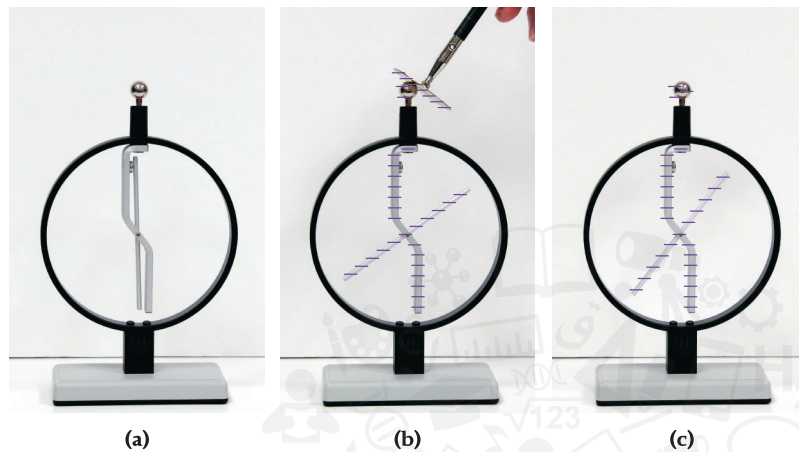
On the other hand, if a negatively charged insulating paddle *touches* the ball of the electroscope, as shown in Figure 1.10b, electrons will flow from the paddle to the conductor, producing a net negative charge. When the paddle is removed, the charge remains and the movable arm remains rotated, as shown in Figure 1.10c. Similarly, if a positively charged insulating paddle touches the ball of the uncharged electroscope, the electroscope transfers electrons to the positively charged paddle and becomes positively charged. Again, both a

(a)   (b)   (c)

positively charged paddle and a negatively charged paddle have the same effect on the electroscope, and we have no way of determining whether the paddles are positively charged or negatively charged. This process is called **charging by contact.**

The two different kinds of charge can be demonstrated by first touching a negatively charged paddle to the electroscope, producing a rotation of the movable arm, as shown in Figure 1.10. If a positively charged paddle is then brought into contact with the electroscope, the movable arm returns to the uncharged position. The charge is neutralized (assuming that both paddles originally had the same absolute value of charge). Thus, there are two kinds of charge. However, because charges are manifestations of mobile electrons, a negative charge is an excess of electrons and a positive charge is a deficit of electrons.

The electroscope can be given a charge without touching it with the charged paddle, as shown in Figure 1.11. The uncharged electroscope is shown in Figure 1.11a. A negatively charged paddle is brought close to the ball of the electroscope but not touching it, as shown in Figure 1.11b. In Figure 1.11c, the electroscope is connected to a ground. Then, while the charged paddle is still close to but not touching the ball of the electroscope, the ground connection is removed in Figure 1.11d. Next, when the paddle is moved away from the electroscope in Figure 1.11e, the electroscope is still positively charged (but with a smaller deflection than in Figure 1.11b). The same process also works with a positively charged paddle. This process is called **charging by induction** and yields an electroscope charge that has the opposite sign from the charge on the paddle.
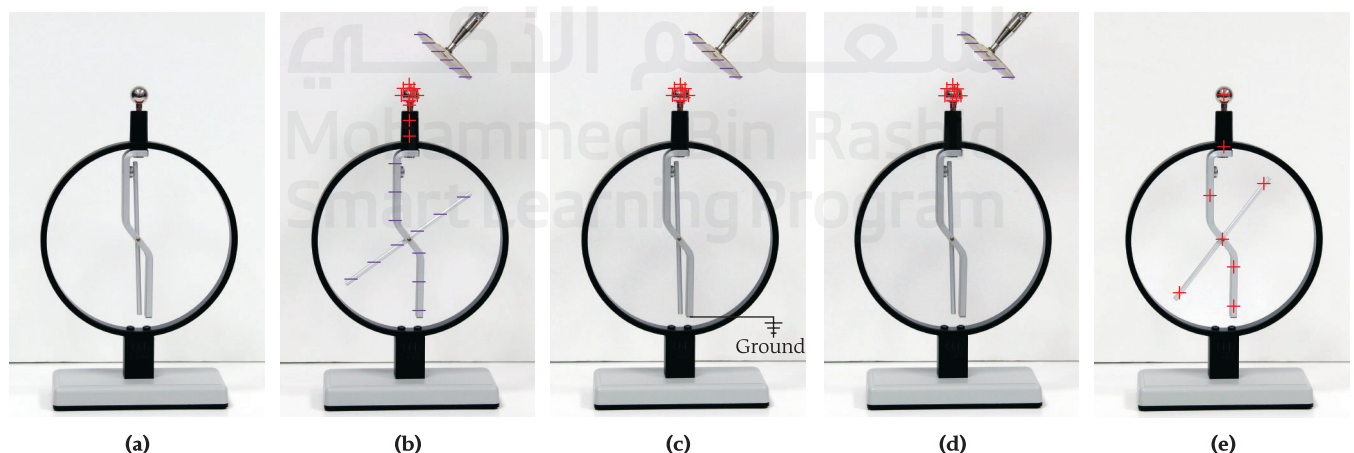
FIGURE 1.11 Charging by induction: (a) An uncharged electroscope. (b) A negatively charged paddle is brought close to the electroscope. (c) A ground is connected to the electroscope. (d) The connection to the ground is removed. (e) The negatively charged paddle is taken away, leaving the electroscope positively charged.
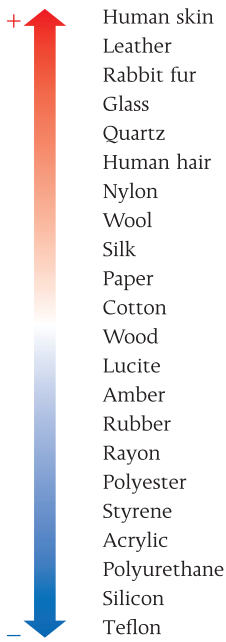
FIGURE 1.12 Triboelectric series for some common materials.

Triboelectric Charging

As mentioned earlier, rubbing two materials together will charge them. We have not addressed two basic questions about this effect: First, what really causes it? And second, which of two given materials gets charged positively and which negatively?

Amazingly, as with many aspects of friction, the microscopic causes of triboelectric charging are still not completely understood. The prevailing theory is that when the surfaces of the two materials involved in the charging process come into contact, adhesion takes place and chemical bonds are formed between atoms at the surfaces. As the surfaces separate, some of these newly formed bonds rupture and leave more of the electrons involved in the bonding on the material with the greater work function.

Recent research results obtained by examining surfaces with atomic force microscopes suggest, however, that triboelectric charging can also occur when two pieces of the *same* material are rubbed against each other. In addition, it has been found that the charging process transfers not only electrons but sometimes also small specks (a few nanometers across) of material.

Which material gets charged positively and which negatively? This question has been answered by a long series of experiments whose results are summarized in Figure 1.12 in the form of a list of some common materials that may be rubbed together. If you rub two materials from this list against each other, the one nearer the top will receive a net positive charge and the other a net negative charge.

Finally, as a rule of thumb, more intense rubbing creates greater charge transfer. This occurs because more intense contact increases friction, which in turn creates more microscopic points of charge transfer on the surfaces of the materials.

1.5   Electrostatic Force—Coulomb's Law
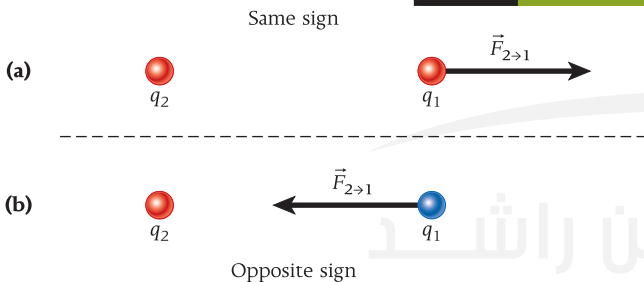
FIGURE 1.13 The force exerted by charge 2 on charge 1: (a) two charges with the same sign; (b) two charges with opposite signs.

The law of electric charges is evidence of a force between any two charges at rest. Experiments show that for the electrostatic force exerted by a charge q_2 on a charge q_1, $\vec{F}_{2\to1}$, the force on q_1 points toward q_2 if the charges have opposite signs and away from q_2 if the charges have like signs (Figure 1.13). This force on one charge due to another charge always acts along the line between the two charges. **Coulomb's Law** gives the magnitude of this force as

$$F = k\frac{|q_1 q_2|}{r^2},\tag{1.6}$$

where q_1 and q_2 are electric charges, $r = \left|\vec{r}_1 - \vec{r}_2\right|$ is the distance between them, and

$$k = 8.99\times10^9 \ \frac{\text{N m}^2}{\text{C}^2}\tag{1.7}$$

is **Coulomb's constant.** You can see that 1 coulomb is a *very* large charge. If two charges of 1 C each were at a distance of 1 m apart, the magnitude of the force they would exert on each other would be 8.99 billion N. For comparison, this force equals the weight of 450 fully loaded space shuttles!

The relationship between Coulomb's constant and another constant, ε_0, called the **electric permittivity of free space,** is
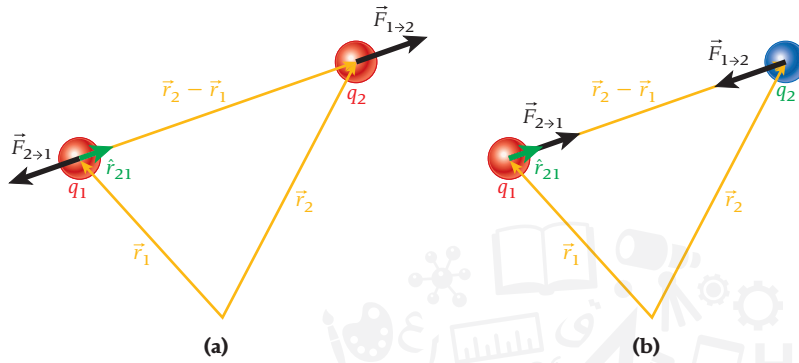
$$k = \frac{1}{4\pi\varepsilon_0}.\tag{1.8}$$

Concept Check 1.3

You place two charges a distance r apart. Then you double each charge and double the distance between the charges. How does the force between the two charges change?

a) The new force is twice as large.

b) The new force is half as large.

c) The new force is four times larger.

d) The new force is four times smaller.

e) The new force is the same.

Consequently, the value of ε_0 is

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N\,m^2}. \tag{1.9}$$

An alternative way of writing equation 1.6 is then

$$F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2}. \tag{1.10}$$

As you'll see in the next few chapters, some equations in electrostatics are more convenient to write with k, while others are more easily written in terms of $1/(4\pi\varepsilon_0)$.

Note that the charges in equations 1.6 and 1.10 can be positive or negative, so the product of the charges can also be positive or negative. Since opposite charges attract and like charges repel, a negative value for the product $q_1 q_2$ signifies attraction and a positive value means repulsion.

Finally, Coulomb's Law for the force due to charge 2 on charge 1 can be written in vector form:

$$\vec{F}_{2\rightarrow 1} = -k\frac{q_1 q_2}{r^3}(\vec{r}_2 - \vec{r}_1) = -k\frac{q_1 q_2}{r^2}\hat{r}_{21}. \tag{1.11}$$

In this equation, $\hat{r}_{21}$ is a unit vector pointing from q_2 to q_1 (see Figure 1.14). The negative sign indicates that the force is repulsive if both charges are positive or both charges are negative. In that case, $\vec{F}_{2\rightarrow 1}$ points away from charge 2, as depicted in Figure 1.14a. On the other hand, if one of the charges is positive and the other negative, then $\vec{F}_{2\rightarrow 1}$ points toward charge 2, as shown in Figure 1.14b.

If charge 2 exerts the force $\vec{F}_{2\rightarrow 1}$ on charge 1, then the force $\vec{F}_{1\rightarrow 2}$ that charge 1 exerts on charge 2 is simply obtained from Newton's Third Law: $\vec{F}_{1\rightarrow 2} = -\vec{F}_{2\rightarrow 1}$.

Superposition Principle

So far in this chapter, we have been dealing with two charges. Now let's consider three point charges, q_1, q_2, and q_3, at positions x_1, x_2, and x_3, respectively, as shown in Figure 1.15. The force exerted by charge 1 on charge 3, $\vec{F}_{1\rightarrow 3}$, is given by

$$\vec{F}_{1\rightarrow 3} = -\frac{kq_1 q_3}{\left(x_3 - x_1\right)^2}\hat{x}.$$
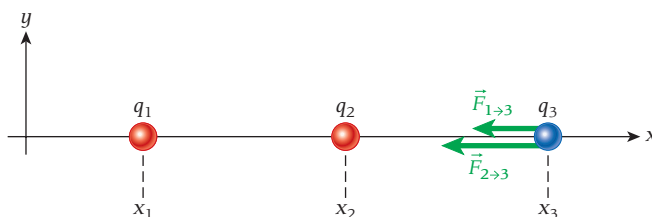
FIGURE 1.15 The forces exerted on charge 3 by charge 1 and charge 2.

The force exerted by charge 2 on charge 3 is

$$\vec{F}_{2\rightarrow 3} = -\frac{kq_2 q_3}{(x_3 - x_2)^2}\hat{x}.$$

The force that charge 1 exerts on charge 3 is not affected by the presence of charge 2. The force that charge 2 exerts on charge 3 is not affected by the presence of charge 1. In addition, the forces exerted by charge 1 and charge 2 on charge 3 add vectorially to produce a net force on charge 3:

$$\vec{F}_{\rightarrow 3} = \vec{F}_{1\rightarrow 3} + \vec{F}_{2\rightarrow 3}.$$

In general, we can express the electrostatic force, $\vec{F}(\vec{r})$, acting on a charge q at position $\vec{r}$ due to a collection of charges, q_i, at positions $\vec{r}_i$ as

$$\vec{F}(\vec{r}) = kq\sum_{i=1}^{n}q_i \frac{\vec{r}_i - \vec{r}}{|\vec{r}_i - \vec{r}|^3}. \tag{1.12}$$

We obtain this result using superposition of the forces and equation 1.11 for each pairwise interaction.

EXAMPLE 1.2   **Electrostatic Force inside the Atom**

PROBLEM 1

What is the magnitude of the electrostatic force that the two protons inside the nucleus of a helium atom exert on each other?

SOLUTION 1

The two protons and two neutrons in the nucleus of the helium atom are held together by the strong force; the electrostatic force is pushing the protons apart. The charge of each proton is $q_p = +e$. A distance of approximately $r = 2\cdot 10^{-15}$ m separates the two protons. Using Coulomb's Law, we can find the force:

$$F = k\frac{|q_p q_p|}{r^2} = \left(8.99\times 10^9 \ \frac{\text{N m}^2}{\text{C}^2}\right)\frac{(+1.6\times 10^{-19}\ \text{C})(+1.6\times 10^{-19}\ \text{C})}{(2\times 10^{-15}\ \text{m})^2} = 58\ \text{N}.$$

Therefore, the two protons in the atomic nucleus of a helium atom are being pushed apart with a force of 58 N (approximately the weight of a small dog). Considering the size of the nucleus, this is an astonishingly large force. Why do atomic nuclei not simply explode? The answer is that an even stronger force, the aptly named strong force, keeps them together.

PROBLEM 2

What is the magnitude of the electrostatic force between a gold nucleus and an electron of the gold atom in an orbit with radius 4.88×10^{-12} m?

SOLUTION 2

The negatively charged electron and the positively charged gold nucleus attract each other with a force whose magnitude is

$$F = k\frac{|q_e q_N|}{r^2},$$

where the charge of the electron is $q_e = -e$ and the charge of the gold nucleus is $q_N = +79e$. The force between the electron and the nucleus is then

$$F = k\frac{|q_e q_N|}{r^2} = \left(8.99\times 10^9 \ \frac{\text{N m}^2}{\text{C}^2}\right)\frac{(1.60\times 10^{-19}\ \text{C})[(79)(1.60\times 10^{-19}\ \text{C})]}{(4.88\times 10^{-12}\ \text{m})^2} = 7.63\times 10^{-4}\ \text{N}.$$

Thus, the magnitude of the electrostatic force exerted on an electron in a gold atom by the nucleus is about 100,000 times less than that between protons inside a nucleus.

Concept Check 1.6

Three charges are arranged on a straight line as shown in the figure. What is the direction of the electrostatic force on the *middle* charge?

a) → $-q$ b) ← q c) ↓ q d) ↑
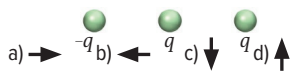
e) There is no force on that charge.

Concept Check 1.7

Three charges are arranged on a straight line as shown in the figure. What is the direction of the electrostatic force on the *right* charge? (Note that the left charge is double what it was in Concept Check 1.6.)

$-2q$ q q

a) → b) ← c) ↓ d) ↑

e) There is no force on that charge.

Note: The gold nucleus has a mass that is approximately 400,000 times that of the electron. But the force the gold nucleus exerts on the electron has exactly the same magnitude as the force that the electron exerts on the gold nucleus. You may say that this is obvious from Newton's Third Law, which is true. But it is worth emphasizing that this basic law holds for electrostatic forces as well.

EXAMPLE 1.3 | Equilibrium Position

PROBLEM

Two charged particles are placed as shown in Figure 1.16: $q_1 = 0.15$ μC is located at the origin, and $q_2 = 0.35$ μC is located on the positive x-axis at $x_2 = 0.40$ m. Where should a third charged particle, q_3, be placed to be at an equilibrium point (such that the forces on it sum to zero)?
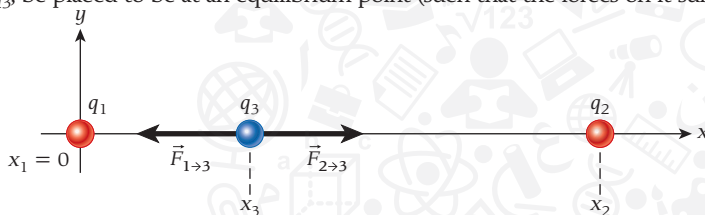
FIGURE 1.16 Placement of three charged particles. The third particle is shown as having a negative charge.

SOLUTION

Let's first determine where *not* to put the third charge. If the third charge is placed anywhere off the x-axis, there will always be a force component pointing toward or away from the x-axis. Thus, we can find an equilibrium point (a point where the forces sum to zero) only *on* the x-axis. The x-axis can be divided into three different segments: $x \le x_1 = 0$, $x_1 < x < x_2$, and $x_2 \le x$. For $x \le x_1 = 0$, the force vectors from both q_1 and q_2 acting on q_3 will point in the positive direction if the charge is negative and in the negative direction if the charge is positive. Because we are looking for a location where the two forces cancel, the segment $x \le x_1 = 0$ can be excluded. A similar argument excludes $x \ge x_2$.

In the remaining segment of the x-axis, $x_1 < x < x_2$, the forces from q_1 and q_2 on q_3 point in opposite directions. We look for the location, x_3, where the absolute magnitudes of both forces are equal and the forces thus sum to zero. We express the equality of the two forces as

$$\left| \vec{F}_{1 \to 3} \right| = \left| \vec{F}_{2 \to 3} \right|,$$

which we can rewrite as

$$k \frac{|q_1 q_3|}{(x_3 - x_1)^2} = k \frac{|q_3 q_2|}{(x_2 - x_3)^2}.$$

We now see that the magnitude and sign of the third charge do not matter because that charge cancels out, as does the constant k, giving us

$$\frac{q_1}{(x_3 - x_1)^2} = \frac{q_2}{(x_2 - x_3)^2}$$

or

$$q_1 (x_2 - x_3)^2 = q_2 (x_3 - x_1)^2. \tag{i}$$

Taking the square root of both sides and solving for x_3, we find

$$\sqrt{q_1} (x_2 - x_3) = \sqrt{q_2} (x_3 - x_1),$$

or

$$x_3 = \frac{\sqrt{q_1} x_2 + \sqrt{q_2} x_1}{\sqrt{q_1} + \sqrt{q_2}}.$$

We can take the square root of both sides of equation (i) because $x_1 < x_3 < x_2$, and so both of the roots, $x_2 - x_3$ and $x_3 - x_1$, are assured to be positive.
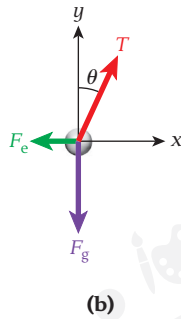
Inserting the numbers given in the problem statement, we obtain

$$x_3 = \frac{\sqrt{q_1} x_2 + \sqrt{q_2} x_1}{\sqrt{q_1} + \sqrt{q_2}} = \frac{\sqrt{0.15 \text{ μC}}(0.4 \text{ m})}{\sqrt{0.15 \text{ μC}} + \sqrt{0.35 \text{ μC}}} = 0.16 \text{ m}.$$

This result makes sense because we expect the equilibrium point to reside closer to the smaller charge.
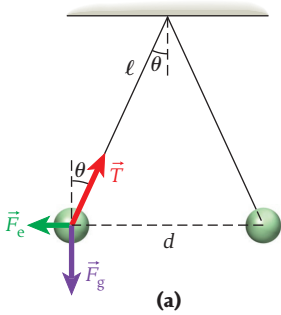
FIGURE 1.17 (a) Two charged balls hanging from the ceiling in their equilibrium position. (b) Free-body diagram for the left-hand charged ball.

SOLVED PROBLEM 1.1 | Charged Balls

PROBLEM
Two identical charged balls hang from the ceiling by insulated ropes of equal length, $\ell = 1.50$ m (Figure 1.17). A charge $q = 25.0$ μC is applied to each ball. Then the two balls hang at rest, and each supporting rope has an angle of 25.0° with respect to the vertical (Figure 1.17a). What is the mass of each ball?

SOLUTION
THINK Each charged ball has three forces acting on it: the force of gravity, the repulsive electrostatic force, and the tension in the supporting rope. We can resolve the components of the three forces and set them equal to zero, allowing us to solve for the mass of the charged balls.

SKETCH A free-body diagram for the left-hand ball is shown in Figure 1.17b.

RESEARCH The condition for static equilibrium says that the sum of the x-components of the three forces acting on the ball must equal zero and the sum of y-components of these forces must equal zero. The sum of the x-components of the forces is

$$T \sin\theta - F_e = 0, \tag{i}$$

where T is the magnitude of the string tension, θ is the angle of the string relative to the vertical, and F_e is the magnitude of the electrostatic force. The sum of the y-components of the forces is

$$T \cos\theta - F_g = 0. \tag{ii}$$

The force of gravity, F_g, is just the weight of the charged ball:

$$F_g = mg, \tag{iii}$$

where m is the mass of the charged ball. The electrostatic force the two balls exert on each other is given by

$$F_e = k\frac{q^2}{d^2}, \tag{iv}$$

where d is the distance between the two balls. We can express the distance between the two balls in terms of the length of the string, ℓ, by looking at Figure 1.17a. We see that

$$\sin\theta = \frac{d/2}{\ell}.$$

We can then express the electrostatic force in terms of the angle with respect to the vertical, θ, and the length of the string, ℓ:

$$F_e = k\frac{q^2}{(2\ell \sin\theta)^2} = k\frac{q^2}{4\ell^2 \sin^2\theta}. \tag{v}$$

SIMPLIFY We divide equation (i) by equation (ii):

$$\frac{T \sin\theta}{T \cos\theta} = \frac{F_e}{F_g},$$

which, after the (unknown) string tension is canceled out, becomes

$$\tan\theta = \frac{F_e}{F_g}.$$

Substituting from equations (iii) and (v) for the force of gravity and the electrostatic force, we get

$$\tan\theta = \frac{k\dfrac{q^2}{4\ell^2 \sin^2\theta}}{mg} = \frac{kq^2}{4mg\ell^2 \sin^2\theta}.$$

Solving for the mass of the ball, we obtain

$$m = \frac{kq^2}{4g\ell^2 \sin^2\theta \tan\theta}.$$

CALCULATE Putting in the numerical values gives

$$m = \frac{\left(8.99\times10^9 \text{ N m}^2/\text{C}^2\right)\left(25.0\,\mu\text{C}\right)^2}{4\left(9.81 \text{ m/s}^2\right)\left(1.50 \text{ m}\right)^2 \left(\sin^2 25.0°\right)\left(\tan 25.0°\right)} = 0.764116 \text{ kg}.$$

ROUND We report our result to three significant figures:

$$m = 0.764 \text{ kg}.$$

DOUBLE-CHECK To double-check, we make the small-angle approximations that $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$. The tension in the string then approaches mg, and we can express the x-components of the forces as

$$T\sin\theta \approx mg\theta = F_e = k\frac{q^2}{d^2} \approx k\frac{q^2}{\left(2\ell\theta\right)^2}.$$

Solving for the mass of the charged ball, we get

$$m = \frac{kq^2}{4g\ell^2\theta^3} = \frac{\left(8.99\times10^9 \text{ N m}^2/\text{C}^2\right)\left(25.0\,\mu\text{C}\right)^2}{4\left(9.81 \text{ m/s}^2\right)\left(1.50 \text{ m}\right)^2 \left(0.436 \text{ rad}\right)^3} = 0.768 \text{ kg},$$

which is close to our answer.
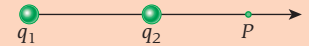
Electrostatic Precipitator

An application of electrostatic charging and electrostatic forces is the cleaning of emissions from coal-fired power plants. A device called an **electrostatic precipitator** (ESP) is used to remove ash and other particulates produced when coal is burned to generate electricity. The operation of this device is illustrated in Figure 1.18.

The ESP consists of wires and plates, with the wires held at a high negative voltage relative to the series of plates held at a positive voltage. (Here the term *voltage* is used colloquially; in Chapter 3, the concept will be defined in terms of electric potential difference.) In Figure 1.18, the exhaust gas from the coal-burning process enters the ESP from the left. Particulates passing near the wires pick up a negative charge. These particles are then attracted to one of the positive plates and stick there. The gas continues through the ESP, leaving the ash and other particulates behind. The accumulated material is then shaken off the plates to a hamper below. This waste can be used for many purposes, including construction materials and fertilizer. Figure 1.19 shows an example of a coal-fired power plant that incorporates an ESP.
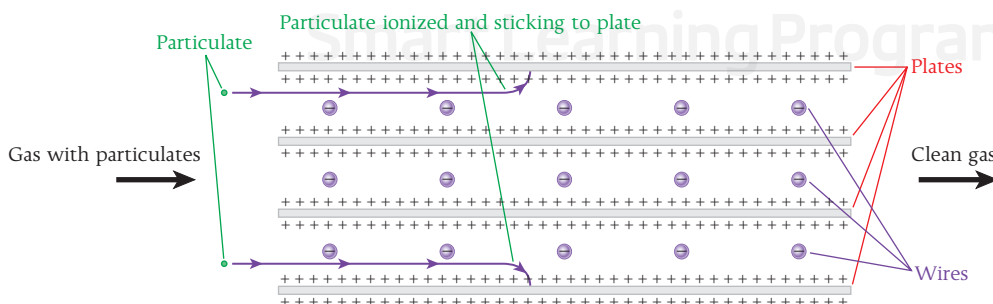
FIGURE 1.18 Operation of an electrostatic precipitator used to clean the exhaust gas of a coal-fired power plant. The view is from the top of the device.
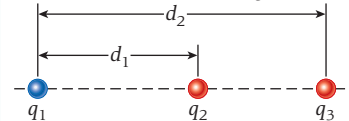
FIGURE 1.19 A coal-fired power plant at Michigan State University that incorporates an electrostatic precipitator to remove particulates from its emissions.

(a)

(b)

FIGURE 1.20 (a) Two charged beads on a wire. (b) Free-body diagram of the forces acting on the second bead.

SOLVED PROBLEM 1.2   Bead on a Wire

PROBLEM

A bead with charge $q_1 = +1.28\ \mu C$ is fixed in place on an insulating wire that makes an angle of $\theta = 42.3°$ with respect to the horizontal (Figure 1.20a). A second bead with charge $q_2 = -5.06\ \mu C$ slides without friction on the wire. At a distance $d = 0.380$ m between the beads, the net force on the second bead is zero. What is the mass, m_2, of the second bead?

SOLUTION

THINK The force of gravity pulling the bead of mass m_2 down the wire is offset by the attractive electrostatic force between the positive charge on the first bead and the negative charge on the second bead. The second bead can be thought of as sliding on an inclined plane.

SKETCH Figure 1.20b shows a free-body diagram of the forces acting on the second bead. We have defined a coordinate system in which the positive x-direction is down the wire. The force exerted on m_2 by the wire can be omitted because this force has only a y-component, and we can solve the problem by analyzing just the x-components of the forces.

RESEARCH The attractive electrostatic force between the two beads balances the component of the force of gravity that acts on the second bead down the wire. The electrostatic force acts in the negative x-direction and its magnitude is given by

$$F_e = k\frac{|q_1 q_2|}{d^2}. \tag{i}$$

The x-component of the force of gravity acting on the second bead corresponds to the component of the weight of the second bead that is parallel to the wire. Figure 1.20b indicates that the component of the weight of the second bead down the wire is given by

$$F_g = m_2 g \sin\theta. \tag{ii}$$

SIMPLIFY For equilibrium, the electrostatic force and the gravitational force are equal: $F_e = F_g$. Substituting the expressions for these forces from equations (i) and (ii) yields

$$k\frac{|q_1 q_2|}{d^2} = m_2 g \sin\theta.$$

Solving this equation for the mass of the second bead gives us

$$m_2 = \frac{k|q_1 q_2|}{d^2 g \sin\theta}.$$

CALCULATE We put in the numerical values and get

$$m_2 = \frac{kq_1 q_2}{d^2 g \sin\theta} = \frac{\left(8.99 \times 10^9\ \text{N m}^2/\text{C}^2\right)(1.28\ \mu C)(5.06\ \mu C)}{(0.380\ \text{m})^2\left(9.81\ \text{m/s}^2\right)(\sin 42.3°)} = 0.0610746\ \text{kg}.$$

ROUND We report our result to three significant figures:

$$m_2 = 0.0611\ \text{kg} = 61.1\ \text{g}.$$

DOUBLE-CHECK To double-check, let's calculate the mass of the second bead assuming that the wire is vertical, that is, $\theta = 90°$. We can then set the weight of the second bead equal to the electrostatic force between the two beads:

$$k\frac{|q_1 q_2|}{d^2} = m_2 g.$$

Solving for the mass of the second bead, we obtain

$$m_2 = \frac{kq_1 q_2}{d^2 g} = \frac{\left(8.99 \times 10^9\ \text{N m}^2/\text{C}^2\right)(1.28\ \mu C)(5.06\ \mu C)}{(0.380\ \text{m})^2\left(9.81\ \text{m/s}^2\right)} = 0.0411\ \text{kg}.$$

As the angle of the wire relative to the horizontal decreases, the calculated mass of the second bead will increase. Our result of 0.0611 kg is somewhat higher than the mass that can be supported with a vertical wire, so it seems reasonable.
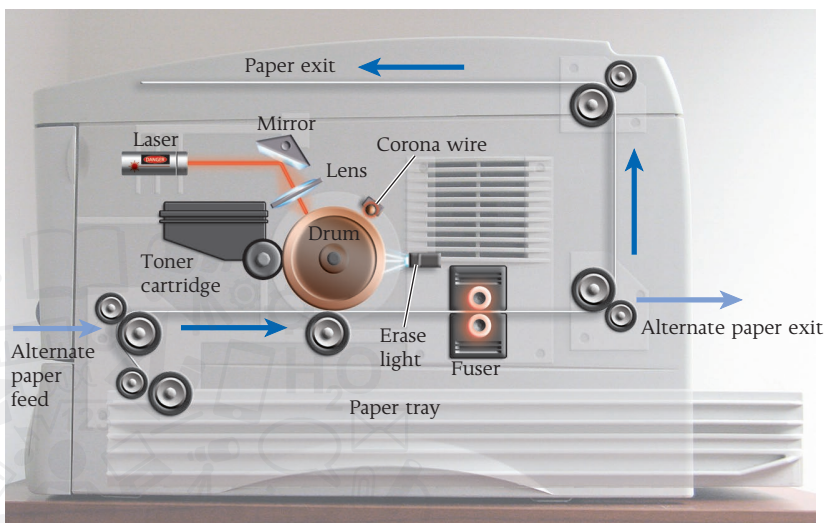
Laser Printer

Another example of a device that applies electrostatic forces is the laser printer. The operation of such a printer is illustrated in Figure 1.21. The paper path follows the blue arrows. Paper is taken from the paper tray or fed manually through the alternate paper feed. The paper passes over a drum where the toner is placed on the surface of the paper and then passes through a fuser that melts the toner and permanently affixes it to the paper.
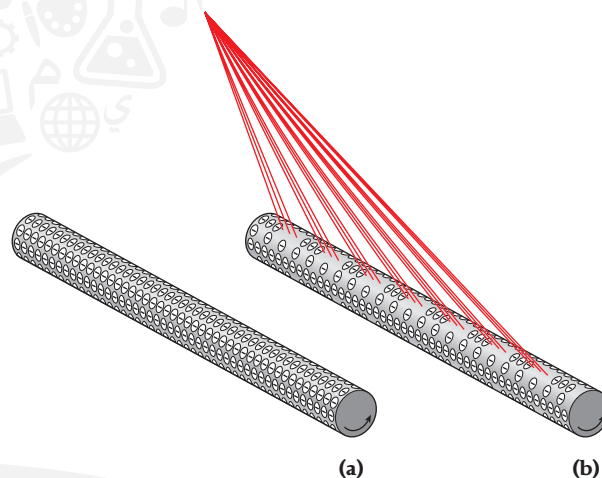
The drum consists of a metal cylinder coated with a special photosensitive material. The photosensitive surface is an insulator that retains charge in the absence of light, but discharges quickly if light is incident on the surface. The drum rotates so that its surface speed is the same as the speed of the moving paper. The basic principle of the operation of the drum is illustrated in Figure 1.22.

FIGURE 1.21 The operation of a typical laser printer.

The drum is negatively charged with electrons using a wire held at high voltage. Then laser light is directed at the surface of the drum. Wherever the laser light strikes the surface of the drum, the surface at that point is discharged. A laser is used because its beam is narrow and remains focused. A line of the image being printed is written one pixel (picture element or dot) at a time using a laser beam directed by a moving mirror and a lens. A typical laser printer can write 600−1200 pixels per inch. The surface of the drum then passes by a roller that picks up toner from the toner cartridge. Toner consists of small, black, insulating particles composed of a plastic-like material. The toner roller is charged to the same negative voltage as the drum. Therefore, wherever the surface of the drum has been discharged, electrostatic forces deposit toner on the surface of the drum. Any portion of the drum surface that has not been exposed to the laser will not pick up toner.

As the drum rotates, it next comes in contact with the paper. The toner is then transferred from the surface of the drum to the paper. As the drum rotates, any remaining toner is scraped off and the surface is neutralized with an erase light or a rotating erase drum in preparation for printing the next image. The paper then continues on to the fuser, which melts the toner, producing a permanent image on the paper.


FIGURE 1.22 (a) The completely charged drum of a laser printer. This drum will produce a blank page. (b) A drum on which one line of information is being recorded by a laser. Wherever the laser strikes the charged drum, the negative charge is neutralized, and the discharged area will attract toner that will produce an image on the paper.

SOLVED PROBLEM 1.3   Four Charged Objects

Consider four charges placed at the corners of a square with side length 1.25 m, as shown in Figure 1.23a.
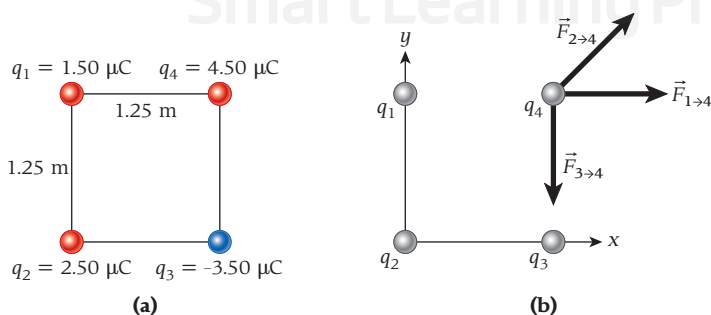
FIGURE 1.23 (a) Four charges placed at the corners of a square. (b) The forces exerted on q_4 by the other three charges.

(a)   **(b)**

– Continued

PROBLEM

What are the magnitude and the direction of the electrostatic force on q_4 resulting from the other three charges?

SOLUTION

THINK The electrostatic force on q_4 is the vector sum of the forces resulting from its interactions with the other three charges. Thus, it is important to avoid simply adding the individual force magnitudes algebraically. Instead we need to determine the individual force components in each spatial direction and add those to find the components of the net force vector. Then we need to calculate the length of that net force vector.

SKETCH Figure 1.23b shows the four charges in an xy-coordinate system with its origin at the location of q_2.

RESEARCH The net force on q_4 is the vector sum of the forces $\vec{F}_{1\to4}$, $\vec{F}_{2\to4}$, and $\vec{F}_{3\to4}$. The x-component of the summed forces is

$$F_x = k\frac{|q_1 q_4|}{d^2} + k\frac{|q_2 q_4|}{\left(\sqrt{2}d\right)^2}\cos 45° = \frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right), \qquad \text{(i)}$$

where d is the length of a side of the square and, as Figure 1.23b indicates, the x-component of $\vec{F}_{3\to4}$ is zero. The y-component of the summed forces is

$$F_y = k\frac{|q_2 q_4|}{\left(\sqrt{2}d\right)^2}\sin 45° - k\frac{|q_3 q_4|}{d^2} = \frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right), \qquad \text{(ii)}$$

where, as Figure 1.23b indicates, the y-component of $\vec{F}_{1\to4}$ is zero.

The magnitude of the net force is given by

$$F = \sqrt{F_x^2 + F_y^2}, \qquad \text{(iii)}$$

and the angle of the net force is given by

$$\tan\theta = \frac{F_y}{F_x}.$$

SIMPLIFY We substitute the expressions for F_x and F_y from equations (i) and (ii) into equation (iii):

$$F = \sqrt{\left[\frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right)\right]^2 + \left[\frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right)\right]^2}.$$

We can rewrite this as

$$F = \frac{kq_4}{d^2}\sqrt{\left(q_1 + \frac{q_2}{2}\cos 45°\right)^2 + \left(\frac{q_2}{2}\sin 45° + q_3\right)^2}.$$

For the angle of the force, we get

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{\frac{kq_4}{d^2}\left(\frac{q_2}{2}\sin 45° + q_3\right)}{\frac{kq_4}{d^2}\left(q_1 + \frac{q_2}{2}\cos 45°\right)}\right) = \tan^{-1}\left(\frac{\left(\frac{q_2}{2}\sin 45° + q_3\right)}{\left(q_1 + \frac{q_2}{2}\cos 45°\right)}\right).$$

CALCULATE Putting in the numerical values, we get

$$\frac{q_2}{2}\sin 45° = \frac{q_2}{2}\cos 45° = \frac{2.50\ \mu C}{2\sqrt{2}} = 0.883883\ \mu C.$$

The magnitude of the force is then

$$F = \frac{\left(8.99\times10^9\ \text{N m}^2/\text{C}^2\right)\left(4.50\ \mu C\right)}{\left(1.25\ \text{m}\right)^2}\sqrt{\left(1.50\ \mu C + 0.883883\ \mu C\right)^2 + \left(0.883883\ \mu C - 3.50\ \mu C\right)^2}$$

$$= 0.0916379\ \text{N}.$$

For the direction of the force, we obtain

$$\theta = \tan^{-1}\left|\frac{\left(\frac{q_2}{2}\sin 45° + q_3\right)}{\left(q_1 + \frac{q_2}{2}\cos 45°\right)}\right| = \tan^{-1}\left|\frac{(0.883883\ \mu C - 3.50\ \mu C)}{(1.50\ \mu C + 0.883883\ \mu C)}\right| = -47.6593°.$$

ROUND We report our results to three significant figures:

$$F = 0.0916\ \text{N}$$

and

$$\theta = -47.7°.$$

DOUBLE-CHECK To double-check our result, we calculate the magnitude of the three forces acting on q_4. For $F_{1\to 4}$, we get

$$F_{1\to 4} = k\frac{|q_1 q_4|}{r_{14}^2} = \frac{(8.99\times 10^9\ \text{N m}^2/\text{C}^2)(1.50\ \mu C)(4.50\ \mu C)}{(1.25\ \text{m})^2} = 0.0388\ \text{N}.$$

For $\vec{F}_{2\to 4}$, we get

$$F_{2\to 4} = k\frac{|q_2 q_4|}{r_{24}^2} = \frac{(8.99\times 10^9\ \text{N m}^2/\text{C}^2)(2.50\ \mu C)(4.50\ \mu C)}{\left[\sqrt{2}(1.25\ \text{m})\right]^2} = 0.0324\ \text{N}.$$

For $F_{3\to 4}$ we get

$$F_{3\to 4} = k\frac{|q_3 q_4|}{r_{34}^2} = \frac{(8.99\times 10^9\ \text{N m}^2/\text{C}^2)(3.50\ \mu C)(4.50\ \mu C)}{(1.25\ \text{m})^2} = 0.0906\ \text{N}$$

All three of the magnitudes of the individual forces are of the same order as our result for the net force. This gives us confidence that our answer is not off by a large factor.

The direction we obtained also seems reasonable, because it orients the resulting force downward and to the right, as could be expected from looking at Figure 1.23b.
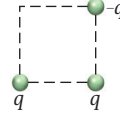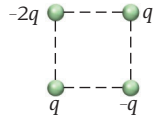

1.6 Coulomb's Law and Newton's Law of Gravitation

Coulomb's Law describing the electrostatic force between two electric charges, F_e, has a form similar to Newton's Law describing the gravitational force between two masses, F_g:

$$F_g = G\frac{m_1 m_2}{r^2} \quad \text{and} \quad F_e = k\frac{|q_1 q_2|}{r^2}$$

where m_1 and m_2 are the two masses, q_1 and q_2 are the two electric charges, and r is the distance of separation. Both forces vary with the inverse square of the distance. The electrostatic force can be attractive or repulsive because charges can have positive or negative signs. (See Figure 1.14a and b.) The gravitational force is always attractive because there is only one kind of mass. (For the gravitational force, only the case depicted in Figure 1.14b is possible.) The relative strengths of the forces are given by the proportionality constants k and G.

EXAMPLE 1.4 | Forces between Electrons

Let's evaluate the relative strengths of the two interactions by calculating the ratio of the electrostatic force and the gravitational force that two electrons exert on each other. This ratio is given by

$$\frac{F_e}{F_g} = \frac{kq_e^2}{Gm_e^2}.$$

Because the dependence on distance is the same for both forces, there is no dependence on distance in the ratio of the two forces–it cancels out. The mass of an electron is $m_e = 9.109\times 10^{-31}$ kg,

– Continued

Concept Check **1.11**

The proton's mass is ~2000 times larger than the electron's mass. Therefore, the ratio F_e/F_g for two protons is _____ the value calculated in Example 1.4 for two electrons.

a) ~4 million times smaller than

b) ~2000 times smaller than

c) the same as

d) ~2000 times larger than

e) ~4 million times larger than

and its charge is $q_e = -1.602 \times 10^{-19}$ C. Using the value of Coulomb's constant given in equation 1.7, $k = 8.99 \times 10^9$ N m²/C², and the value of the universal gravitational constant, $G = 6.67 \times 10^{-11}$ N m²/kg², we find numerically

$$\frac{F_e}{F_g} = \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(9.109 \times 10^{-31} \text{ kg})^2} = 4.17 \times 10^{42}.$$

Therefore, the electrostatic force between electrons is stronger than the gravitational force between them by more than 42 orders of magnitude.

Despite the relative weakness of the gravitational force, it is the only force that matters on the astronomical scale. The reason for this dominance is that all stars, planets, and other objects of astronomical relevance carry no net charge. Therefore, there is no net electrostatic interaction between them, and gravity dominates.

Coulomb's Law of electrostatics applies to macroscopic systems down to the atom, though subtle effects in atomic and subatomic systems require use of a more sophisticated approach called *quantum electrodynamics*. Newton's law of gravitation fails in subatomic systems and also must be modified for some phenomena in astronomical systems, such as the precessional motion of Mercury around the Sun. These fine details of the gravitational interaction are governed by Einstein's theory of general relativity.

The similarities between the gravitational and electrostatic interactions will be covered further in the next two chapters, which address electric fields and electric potential.

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- There are two kinds of electric charge, positive and negative. Like charges repel, and unlike charges attract.

- The quantum (elementary quantity) of electric charge is $e = 1.602 \times 10^{-19}$ C.

- The electron has charge $q_e = -e$, and the proton has charge $q_p = +e$. The neutron has zero charge.

- The net charge of an object is given by e times the number of protons, N_p, minus e times the number of electrons, N_e, that make up the object: $q = e \cdot (N_p - N_e)$.

- The total charge in an isolated system is always conserved.

- Objects can be charged directly by contact or indirectly by induction.

- Coulomb's Law describes the force that two stationary charges exert on each other: $F = k\dfrac{|q_1 q_2|}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1 q_2|}{r^2}$.

- The constant in Coulomb's Law is

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \ \frac{\text{N m}^2}{\text{C}^2}.$$

- The electric permittivity of free space is

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \frac{\text{C}^2}{\text{N m}^2}.$$

ANSWERS TO SELF-TEST OPPORTUNITIES

21.1 a) $+1$ c) 0 e) $+\frac{2}{3}$ g) -1 **22.2** a) true   c) false   e) true

      b) 0 d) 0 f) $-\frac{1}{3}$ h) $+2$ b) false   d) true

PROBLEM-SOLVING GUIDELINES

1. For problems involving Coulomb's Law, drawing a free-body diagram showing the electrostatic force vectors acting on a charged particle is often helpful. Pay careful attention to signs; a negative force between two particles indicates attraction, and a positive force indicates repulsion. Be sure that the directions of forces in the diagram match the signs of forces in the calculations.

2. Use symmetry to simplify your work. However, be careful to take account of charge magnitudes and signs as well as distances. Two charges at equal distances from a third charge do not exert equal forces on that charge if they have different magnitudes or signs.

3. Units in electrostatics often have prefixes indicating powers of 10: Distances may be given in cm or mm; charges may be given in μC, nC, or pC; masses may be given in kg or g. Other units are also common. The best way to proceed is to convert all quantities to basic SI units, to be compatible with the value of k or $1/4\pi\varepsilon_0$.

MULTIPLE-CHOICE QUESTIONS

1.1 When a metal plate is given a positive charge, which of the following is taking place?

a) Protons (positive charges) are transferred to the plate from another object.

b) Electrons (negative charges) are transferred from the plate to another object.

c) Electrons (negative charges) are transferred from the plate to another object, and protons (positive charges) are also transferred to the plate from another object.

d) It depends on whether the object conveying the charge is a conductor or an insulator.

1.2 The force between a charge of 25 μC and a charge of −10 μC is 8.0 N. What is the separation between the two charges?

a) 0.28 m   c) 0.45 m

b) 0.53 m   d) 0.15 m

1.3 A charge Q_1 is positioned on the x-axis at $x = a$. Where should a charge $Q_2 = -4Q_1$ be placed to produce a net electrostatic force of zero on a third charge, $Q_3 = Q_1$, located at the origin?

a) at the origin   c) at $x = -2a$

b) at $x = 2a$ d) at $x = -a$

1.4 Which one of these systems has the most negative charge?

a) 2 electrons

b) 3 electrons and 1 proton

c) 5 electrons and 5 protons

d) N electrons and $N - 3$ protons

e) 1 electron

1.5 Two point charges are fixed on the x-axis: $q_1 = 6.0$ μC is located at the origin, O, with $x_1 = 0.0$ cm, and $q_2 = -3.0$ μC is located at point A, with $x_2 = 8.0$ cm. Where should a third charge, q_3, be placed on the x-axis so that the total electrostatic force acting on it is zero?

a) 19 cm   c) 0.0 cm   e) −19 cm

b) 27 cm   d) 8.0 cm

1.6 Which of the following situations produces the largest net force on the charge Q?

a) Charge $Q = 1$ C is 1 m from a charge of −2 C.

b) Charge $Q = 1$ C is 0.5 m from a charge of −1 C.

c) Charge $Q = 1$ C is halfway between a charge of −1 C and a charge of 1 C that are 2 m apart.

d) Charge $Q = 1$ C is halfway between two charges of −2 C that are 2 m apart.

e) Charge $Q = 1$ C is a distance of 2 m from a charge of −4 C.

1.7 Two protons placed near one another with no other objects close by would

a) accelerate away from each other.

b) remain motionless.

c) accelerate toward each other.

d) be pulled together at constant speed.

e) move away from each other at constant speed.

1.8 Two lightweight metal spheres are suspended near each other from insulating threads. One sphere has a net charge; the other sphere has no net charge. The spheres will

a) attract each other.

b) exert no net electrostatic force on each other.

c) repel each other.

d) do any of these things depending on the sign of the net charge on the one sphere.

1.9 A metal plate is connected by a conductor to a ground through a switch. The switch is initially closed. A charge $+Q$ is brought close to the plate without touching it, and then the switch is opened. After the switch is opened, the charge $+Q$ is removed. What is the charge on the plate then?

a) The plate is uncharged.

b) The plate is positively charged.

c) The plate is negatively charged.

d) The plate could be either positively or negatively charged, depending on the charge it had before $+Q$ was brought near.
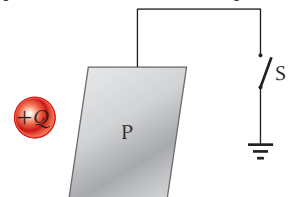
1.10 You bring a negatively charged rubber rod close to a grounded conductor without touching it. Then you disconnect the ground. What is the sign of the charge on the conductor after you remove the charged rod?

a) negative

b) positive

c) no charge

d) cannot be determined from the information given

1.11 When a rubber rod is rubbed with rabbit fur, the rod becomes

a) negatively charged.

b) positively charged.

c) neutral.

d) either negatively charged or positively charged, depending on whether the fur is always moved in the same direction or is moved back and forth.

1.12 When a glass rod is rubbed with a polyester scarf, the rod becomes

a) negatively charged.

b) positively charged.

c) neutral.

d) either negatively charged or positively charged, depending on whether the scarf is always moved in the same direction or is moved back and forth.

1.13 Consider an electron with mass m and charge $-e$ moving in a circular orbit with radius r around a fixed proton with mass M and charge $+e$. The electron is held in orbit by the electrostatic force between itself and the proton. Which one of the following expressions for the speed of the electron is correct?

a) $v = \sqrt{\dfrac{ke^2}{mr}}$ c) $v = \sqrt{\dfrac{2ke^2}{mr^2}}$ e) $v = \sqrt{\dfrac{ke^2}{2Mr}}$

b) $v = \sqrt{\dfrac{GM}{r}}$ d) $v = \sqrt{\dfrac{me^2}{kr}}$

1.14 Consider an electron with mass m and charge $-e$ located a distance r from a fixed proton with mass M and charge $+e$. The electron is released from rest. Which one of the following expressions for the magnitude of the initial acceleration of the electron is correct?

a) $a = \dfrac{2ke^2}{mMr}$ c) $a = \frac{1}{2}me^2k^2$ e) $a = \dfrac{ke^2}{mr^2}$

b) $a = \sqrt{\dfrac{2e^2}{mkr}}$ d) $a = \dfrac{2ke^2}{mr}$

CONCEPTUAL QUESTIONS

1.15 If two charged particles (the charge on each is Q) are separated by a distance d, there is a force F between them. What is the force if the magnitude of each charge is doubled and the distance between them changes to $2d$?

1.16 Suppose the Sun and the Earth were each given an equal amount of charge of the same sign, just sufficient to cancel their gravitational attraction. How many times the charge on an electron would that charge be? Is this number a large fraction of the number of charges of either sign in the Earth?

1.17 It is apparent that the electrostatic force is *extremely* strong, compared to gravity. In fact, the electrostatic force is the basic force governing phenomena in daily life–the tension in a string, the normal forces between surfaces, friction, chemical reactions, and so forth–except weight. Why then did it take so long for scientists to understand this force? Newton came up with his gravitational law long before electricity was even crudely understood.

1.18 Occasionally, people who gain static charge by shuffling their feet on the carpet will have their hair stand on end. Why does this happen?

1.19 Two positive charges, each equal to Q, are placed a distance $2d$ apart. A third charge, $-0.2Q$, is placed exactly halfway between the two positive charges and is displaced a distance $x \ll d$ (that is, x is much smaller than d) perpendicular to the line connecting the positive charges. What is the force on this charge? For $x \ll d$, how can you approximate the motion of the negative charge?

1.20 Why does a garment taken out of a clothes dryer sometimes cling to your body when you wear it?

1.21 Two charged spheres are initially a distance d apart. The magnitude of the force on each sphere is F. They are moved closer to each other such that the magnitude of the force on each of them is $9F$. By what factor has the distance between the two spheres changed?

1.22 How is it possible for one electrically neutral atom to exert an electrostatic force on another electrically neutral atom?

1.23 The scientists who first contributed to the understanding of the electrostatic force in the 18th century were well aware of Newton's law of gravitation. How could they deduce that the force they were studying was *not* a variant or some manifestation of the gravitational force?

1.24 Two charged particles move solely under the influence of the electrostatic forces between them. What shapes can their trajectories have?

1.25 Rubbing a balloon causes it to become negatively charged. The balloon then tends to cling to the wall of a room. For this to happen, must the wall be positively charged?

1.26 Two electric charges are placed on a line, as shown in the figure. Is it possible to place a charged particle (that is free to move) anywhere on the line between the two charges and have it not move?
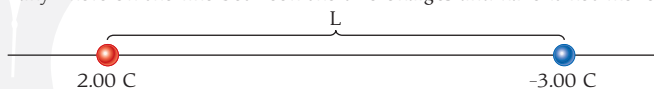
1.27 Two electric charges are placed on a line as shown in the figure. Where on the line can a third charge be placed so that the force on that charge is zero? Does the sign or the magnitude of the third charge make any difference to the answer?
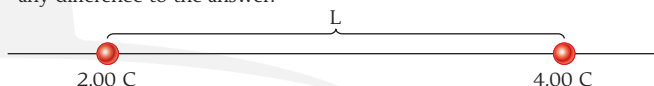
1.28 When a positively charged rod is brought close to a neutral conductor without touching it, will the rod experience an attractive force, a repulsive force, or no force at all? Explain.

1.29 When you exit a car and the humidity is low, you often experience a shock from static electricity created by sliding across the seat. How can you discharge yourself without experiencing a painful shock? Why is it dangerous to get back into your car while fueling your car?

EXERCISES

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

Section 1.2

1.30 How many electrons are required to yield a total charge of 1.00 C?

1.31 The *faraday* is a unit of charge frequently encountered in electrochemical applications and named for the British physicist and chemist Michael Faraday. It consists of exactly 1 mole of elementary charges. Calculate the number of coulombs in 1.000 faraday.

1.32 Another unit of charge is the *electrostatic unit* (esu). It is defined as follows: Two point charges, each of 1 esu and separated by 1 cm, exert a force of exactly 1 dyne on each other: 1 dyne $= 1$ g cm/s$^2 = 1 \times 10^{-5}$ N.

a) Determine the relationship between the esu and the coulomb.

b) Determine the relationship between the esu and the elementary charge.

1.33 A current of 5.00 mA is enough to make your muscles twitch. Calculate how many electrons flow through your skin if you are exposed to such a current for 10.0 s.

•**1.34** How many electrons does 1.00 kg of water contain?

•**1.35** The Earth is constantly being bombarded by cosmic rays, which consist mostly of protons. These protons are incident on the Earth's atmosphere from all directions at a rate of 1245. protons per square meter per second. Assuming that the depth of Earth's atmosphere is 120.0 km, what is the total charge incident on the atmosphere in 5.000 min? Assume that the radius of the surface of the Earth is 6378. km.

•**1.36** Performing an experiment similar to Millikan's oil drop experiment, a student measures these charge magnitudes:

3.26 x10^{-19} C   5.09 x10^{-19} C   1.53 x10^{-19} C

6.39 x10^{-19} C   4.66 x10^{-19} C

Find the charge on the electron using these measurements.

Section 1.3

•**1.37** A silicon sample is doped with phosphorus at 1 part per 1.00x10^6. Phosphorus acts as an electron donor, providing one free electron per atom. The density of silicon is 2.33 g/cm^3, and its atomic mass is 28.09 g/mol.

a) Calculate the number of free (conduction) electrons per unit volume of the doped silicon.

b) Compare the result from part (a) with the number of conduction electrons per unit volume of copper wire, assuming that each copper atom produces one free (conduction) electron. The density of copper is 8.96 g/cm^3, and its atomic mass is 63.54 g/mol.

Section 1.5

1.38 Two charged spheres are 8.00 cm apart. They are moved closer to each other by enough that the force on each of them increases four times. How far apart are they now?

1.39 Two identically charged particles separated by a distance of 1.00 m repel each other with a force of 1.00 N. What is the magnitude of the charges?

1.40 How far apart must two electrons be placed on the Earth's surface for there to be an electrostatic force between them equal to the weight of one of the electrons?

1.41 In solid sodium chloride (table salt), chloride ions have one more electron than they have protons, and sodium ions have one more proton than they have electrons. These ions are separated by about 0.28 nm. Calculate the electrostatic force between a sodium ion and a chloride ion.

1.42 In gaseous sodium chloride, chloride ions have one more electron than they have protons, and sodium ions have one more proton than they have electrons. These ions are separated by about 0.24 nm. Suppose a free electron is located 0.48 nm above the midpoint of the sodium chloride molecule. What are the magnitude and the direction of the electrostatic force the molecule exerts on it?

1.43 Calculate the magnitude of the electrostatic force the two up quarks inside a proton exert on each other if they are separated by a distance of 0.900 fm.

1.44 A −4.00 μC charge lies 20.0 cm to the right of a 2.00 μC charge on the x-axis. What is the force on the 2.00-μC charge?

•**1.45** Two initially uncharged identical metal spheres, 1 and 2, are connected by an insulating spring (unstretched length $L_0 = 1.00$ m, spring constant $k = 25.0$ N/m), as shown in the figure. Charges $+q$ and $-q$ are then placed on the spheres, and the spring contracts to length $L = 0.635$ m. Recall that the force exerted by a spring is $F_s = k\Delta x$, where Δx is the change in the spring's length from its equilibrium length. Determine the charge q. If the spring is coated with metal to make it conducting, what is the new length of the spring?

Before charging   After charging
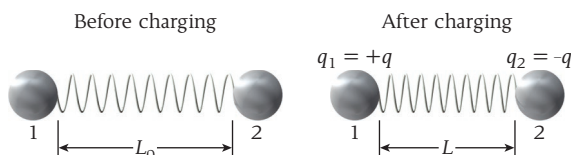
•**1.46** A point charge $+3q$ is located at the origin, and a point charge $-q$ is located on the x-axis at $D = 0.500$ m. At what location on the x-axis will a third charge, q_0, experience no net force from the other two charges?

•**1.47** Identical point charges Q are placed at each of the four corners of a rectangle measuring 2.00 m by 3.00 m. If $Q = 32.0$ μC, what is the magnitude of the electrostatic force on any one of the charges?
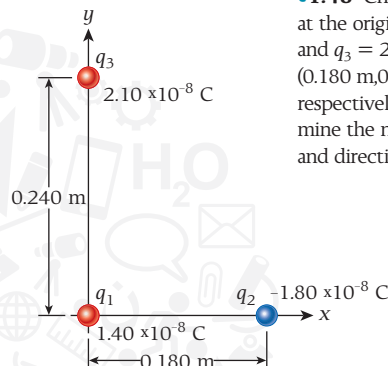
•**1.48** Charge $q_1 = 1.40$x10^{-8} C is placed at the origin. Charges $q_2 = -1.80$x10^{-8} C and $q_3 = 2.10$x10^{-8} C are placed at points (0.180 m,0.000 m) and (0.000 m,0.240 m), respectively, as shown in the figure. Determine the net electrostatic force (magnitude and direction) on charge q_3.

•**1.49** A positive charge Q is on the y-axis at a distance a from the origin, and another positive charge q is on the x-axis at a distance b from the origin.

a) For what value(s) of b is the x-component of the force on q a minimum?

b) For what value(s) of b is the x-component of the force on q a maximum?
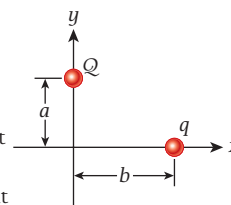
•**1.50** Find the magnitude and direction of the electrostatic force acting on the electron in the figure.
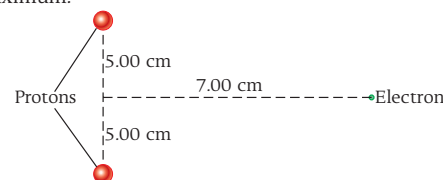
•**1.51** In a region of two-dimensional space, there are three fixed charges: +1.00 mC at (0,0), −2.00 mC at (17.0 mm,−5.00 mm), and +3.00 mC at (−2.00 mm,11.0 mm). What is the net force on the −2.00 mC charge?

•**1.52** Two cylindrical glass beads each of mass $m = 10.0$ mg are set on their flat ends on a horizontal insulating surface separated by a distance $d = 2.00$ cm. The coefficient of static friction between the beads and the surface is $\mu_s = 0.200$. The beads are then given identical charges (magnitude and sign). What is the minimum charge needed to start the beads moving?

•**1.53** A small ball with a mass of 30.0 g and a charge of −0.200 μC is suspended from the ceiling by a string. The ball hangs at a distance of 5.00 cm above an insulating floor. If a second small ball with a mass of 50.0 g and a charge of 0.400 μC is rolled directly beneath the first ball, will the second ball leave the floor? What is the tension in the string when the second ball is directly beneath the first ball?

•**1.54** A +3.00 mC charge and a −4.00 mC charge are fixed in position and separated by 5.00 m.

a) Where can a +7.00 mC charge be placed so that the net force on it is zero?

b) Where can a −7.00 mC charge be placed so that the net force on it is zero?

•**1.55** Four point charges, q, are fixed to the four corners of a square that is 10.0 cm on a side. An electron is suspended above a point at which its weight is balanced by the electrostatic force due to the four electrons, at a distance of 15.0 nm above the center of the square. What is the magnitude of the fixed charges? Express the charge both in coulombs and as a multiple of the electron's charge.

••1.56 The figure shows a uniformly charged thin rod of length L that has total charge Q. Find an expression for the magnitude of the electrostatic force acting on an electron positioned on the axis of the rod at a distance d from the midpoint of the rod.
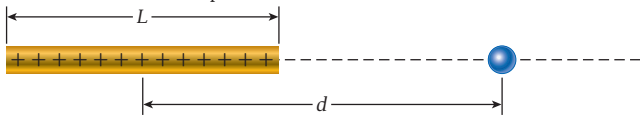
••1.57 A negative charge, $-q$, is fixed at the coordinate (0,0). It is exerting an attractive force on a positive charge, $+q$, that is initially at coordinate $(x,0)$. As a result, the positive charge accelerates toward the negative charge. Use the binomial expansion $(1+x)^n \approx 1 + nx$, for $x \ll 1$, to show that when the positive charge moves a distance $\delta \ll x$ closer to the negative charge, the force that the negative charge exerts on it increases by $\Delta F = 2kq^2\delta/x^3$.

••1.58 Two negative charges ($-q$ and $-q$) of equal magnitude are fixed at coordinates $(-d,0)$ and $(d,0)$. A positive charge of the same magnitude, q, and with mass m is placed at coordinate (0,0), midway between the two negative charges. If the positive charge is moved a distance $\delta \ll d$ in the positive y-direction and then released, the resulting motion will be that of a harmonic oscillator–the positive charge will oscillate between coordinates $(0,\delta)$ and $(0,-\delta)$. Find the net force acting on the positive charge when it moves to $(0,\delta)$ and use the binomial expansion $(1+x)^n \approx 1+nx$, for $x \ll 1$, to find an expression for the frequency of the resulting oscillation. (*Hint*: Keep only terms that are linear in δ.)

Section 1.6

1.59 Suppose the Earth and the Moon carried positive charges of equal magnitude. How large would the charge need to be to produce an electrostatic repulsion equal to 1.00% of the gravitational attraction between the two bodies?

1.60 The similarity of form of Newton's law of gravitation and Coulomb's Law caused some to speculate that the force of gravity is related to the electrostatic force. Suppose that gravitation is entirely electrical in nature–that an excess charge Q on the Earth and an equal and opposite excess charge $-Q$ on the Moon are responsible for the gravitational force that causes the observed orbital motion of the Moon about the Earth. What is the required size of Q to reproduce the observed magnitude of the gravitational force?

•1.61 In the Bohr model of the hydrogen atom, the electron moves around the one-proton nucleus on circular orbits of well-determined radii, given by $r_n = n^2 a_B$, where $n = 1, 2, 3, ...$ is an integer that defines the orbit and $a_B = 5.29 \times 10^{-11}$m is the radius of the first (minimum) orbit, called the *Bohr radius*. Calculate the force of electrostatic interaction between the electron and the proton in the hydrogen atom for the first four orbits. Compare the strength of this interaction to the gravitational interaction between the proton and the electron.

•1.62 Some of the earliest atomic models held that the orbital velocity of an electron in an atom could be correlated with the radius of the atom. If the radius of the hydrogen atom is 5.29×10^{-11} m and the electrostatic force is responsible for the circular motion of the electron, what is the kinetic energy of this orbital electron?

1.63 For the atom described in Problem 1.62, what is the ratio of the gravitational force between electron and proton to the electrostatic force? How does this ratio change if the radius of the atom is doubled?

•1.64 In general, astronomical objects are not exactly electrically neutral. Suppose the Earth and the Moon each carry a charge of -1.00×10^6 C (this is approximately correct; a more precise value is identified in Chapter 2).

a) Compare the resulting electrostatic repulsion with the gravitational attraction between the Moon and the Earth. Look up any necessary data.

b) What effects does this electrostatic force have on the size, shape, and stability of the Moon's orbit around the Earth?

Additional Exercises

1.65 Eight 1.00 μC charges are arrayed along the y-axis located every 2.00 cm starting at $y = 0$ and extending to $y = 14.0$ cm. Find the force on the charge at $y = 4.00$ cm.

1.66 In a simplified Bohr model of the hydrogen atom, an electron is assumed to be traveling in a circular orbit of radius of about 5.29×10^{-11} m around a proton. Calculate the speed of the electron in that orbit.

1.67 The nucleus of a carbon-14 atom (mass = 14 amu) has diameter of 3.01 fm. It has 6 protons and a charge of $+6e$.

a) What is the force on a proton located at 3.00 fm from the surface of this nucleus? Assume that the nucleus is a point charge.

b) What is the proton's acceleration?

1.68 Two charged objects experience a mutual repulsive force of 0.100 N.
If the charge of one of the objects is reduced by half and the distance separating the objects is doubled, what is the new force?

1.69 A particle (charge = $+19.0$ μC) is located on the x-axis at $x = -10.0$ cm, and a second particle (charge = -57.0 μC) is placed on the x-axis at $x = +20.0$ cm. What is the magnitude of the total electrostatic force on a third particle (charge = -3.80 μC) placed at the origin ($x = 0$)?

1.70 Three point charges are positioned on the x-axis: $+64.0$ μC at $x = 0.00$ cm, $+80.0$ μC at $x = 25.0$ cm, and -160.0 μC at $x = 50.0$ cm. What is the magnitude of the electrostatic force acting on the $+64.0-$μC charge?

1.71 From collisions with cosmic rays and from the solar wind, the Earth has a net electric charge of approximately -6.8×10^5 C. Find the charge that must be given to a 1.0 g object for it to be electrostatically levitated close to the Earth's surface.

•1.72 A 10.0 g mass is suspended 5.00 cm above a nonconducting flat plate, directly above an embedded charge of q (in coulombs). If the mass has the same charge, q, how much must q be so that the mass levitates (just floats, neither rising nor falling)? If the charge q is produced by adding electrons to the mass, by how much will the mass be changed?

**•1.73** Four point charges are placed at the following xy-coordinates:

$Q_1 = -1.00$ mC, at $(-3.00$ cm,0.00 cm)

$Q_2 = -1.00$ mC, at $(+3.00$ cm,0.00 cm)

$Q_3 = +1.024$ mC, at $(0.00$ cm,0.00 cm)

$Q_4 = +2.00$ mC, at $(0.00$ cm,-4.00 cm)

Calculate the net force on charge Q4 due to charges Q1, Q2, and Q3.

•1.74 Three 5.00 g Styrofoam balls of radius 2.00 cm are coated with carbon black to make them conducting and then are tied to 1.00 m-long threads and suspended freely from a common point. Each ball is given the same charge, q. At equilibrium, the balls form an equilateral triangle with sides of length 25.0 cm in the horizontal plane. Determine q.

•1.75 Two point charges lie on the x-axis. If one point charge is 6.00 μC and lies at the origin and the other is -2.00 μC and lies at 20.0 cm, at what position must a third charge be placed to be in equilibrium?

•1.76 Two beads with charges $q_1 = q_2 = +2.67$ μC are on an insulating string that hangs straight down from the ceiling as shown in the figure. The lower bead is fixed in place on the end of the string and has a mass $m_1 = 0.280$ kg. The second bead slides without friction on the string. At a distance $d = 0.360$ m between the centers of
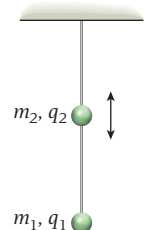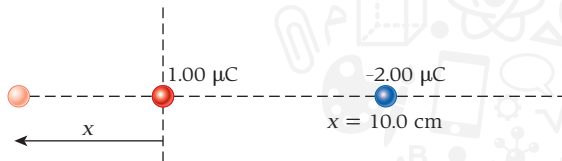
the beads, the force of the Earth's gravity on m_2 is balanced by the electrostatic force between the two beads. What is the mass, m_2, of the second bead? (*Hint:* You can neglect the gravitational interaction between the two beads.)

•**1.77** Find the net force on a +2.00-C charge at the origin of an xy-coordinate system if there is a +5.00 C charge at (3.00 m,0.00) and a −3.00 C charge at (0.00,4.00 m).

•**1.78** Two spheres, each of mass M = 2.33 g, are attached by pieces of string of length L = 45.0 cm to a common point. The strings initially hang straight down, with the spheres just touching one another. An equal amount of charge, q, is placed on each sphere. The resulting forces on the spheres cause each string to hang at an angle of θ = 10.0° from the vertical. Determine q, the amount of charge on each sphere.

•**1.79** A point charge q_1 = 100. nC is at the origin of an xy-coordinate system, a point charge q_2 = −80.0 nC is on the x-axis at x = 2.00 m, and a point charge q_3 = −60.0 nC is on the y-axis at y = −2.00 m. Determine the net force (magnitude and direction) on q_1.

•**1.80** A positive charge q_1 = 1.00 μC is fixed at the origin, and a second charge q_2 = −2.00 μC is fixed at x = 10.0 cm. Where along the x-axis should a third charge be positioned so that it experiences no force?

•**1.81** A bead with charge q_1 = 1.27 μC is fixed in place at the end of a wire that makes an angle of θ = 51.3° with the horizontal. A second bead with mass m_2 = 3.77 g and a charge of 6.79 μC slides without friction on the wire. What is the distance d at which the force of the Earth's gravity on m_2 is balanced by the electrostatic force between the two beads? Neglect the gravitational interaction between the two beads.
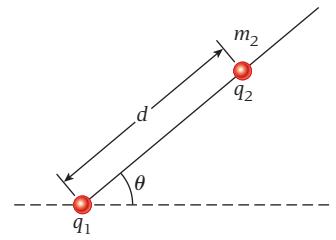
•**1.82** In the figure, the net electrostatic force on charge Q_A is zero. If Q_A = +1.00 nC, determine the magnitude of Q_0.
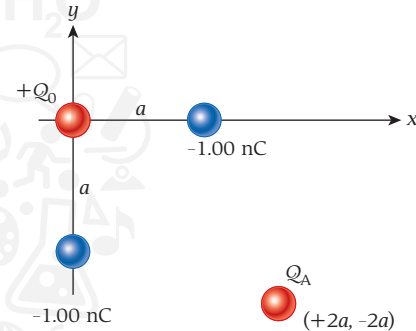
MULTI-VERSION EXERCISES

1.83 Two balls have the same mass, 0.9680 kg, and the same charge, 29.59 μC. They hang from the ceiling on strings of identical length, ℓ, as shown in the figure. If the angle of the strings with respect to the vertical is 29.79°, what is the length of the strings?
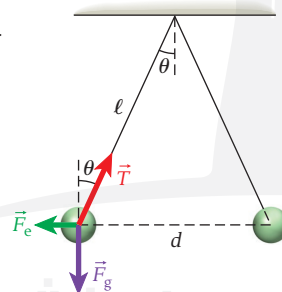
1.84 Two balls have the same mass and the same charge, 15.71 μC. They hang from the ceiling on strings of identical length, ℓ = 1.223 m, as shown in the figure. The angle of the strings with respect to the vertical is 21.07°. What is the mass of each ball?

1.85 Two balls have the same mass, 0.9935 kg, and the same charge. They hang from the ceiling on strings of identical length, ℓ = 1.235 m, as shown in the figure. The angle of the strings with respect to the vertical is 22.35°. What is the charge on each ball?

1.86 As shown in the figure, point charge q_1 is 3.979 μC and is located at x_1 = −5.689 m, and point charge q_2 is 8.669 μC and is located at x_2 = 14.13 m. What is the x-coordinate of the point at which the net force on a point charge of 5.000 μC will be zero?

1.87 As shown in the figure, point charge q_1 is 4.325 μC and is located at x_1, and point charge q_2 is 7.757 μC and is located at x_2 = 14.33 m. The x-coordinate of the point where the net force on a point charge of −3.000 μC is zero is 2.358 m. What is the value of x_1?

1.88 As shown in the figure, point charge q_1 is 4.671 μC and is located at x_1 = −3.573 m, and point charge q_2 is 6.845 μC and is located at x_2. The x-coordinate of the point where the net force on a point charge of −1.000 μC is zero is 4.625 m. What is the value of x_2?