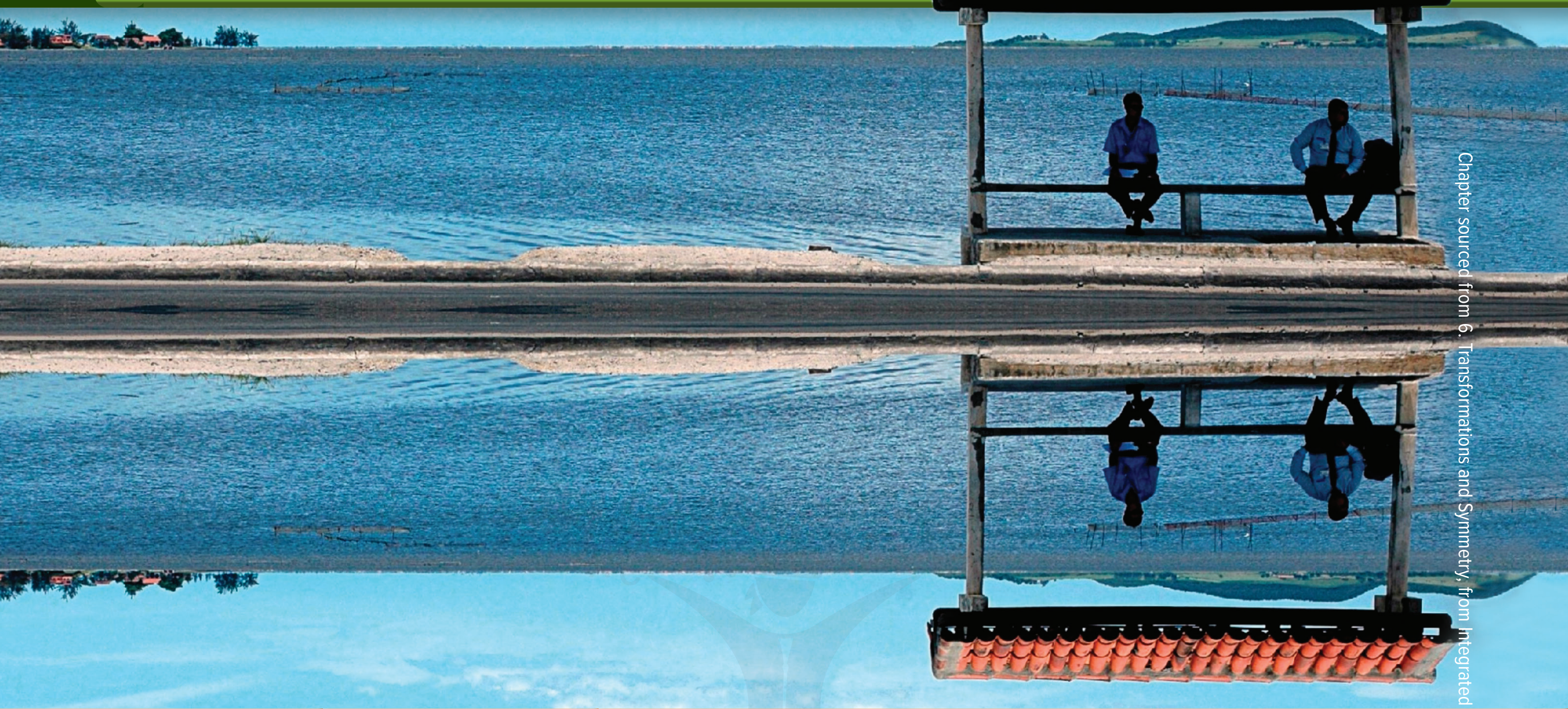
CHAPTER 6

Transformations and Symmetry



Then

You identified reflections, translations, and rotations.

Now

In this chapter, you will:

- Name and draw figures that have been reflected, translated, rotated, or dilated.
- Recognize and draw compositions of transformations.
- Identify symmetry in two- and three-dimensional figures.

Why?

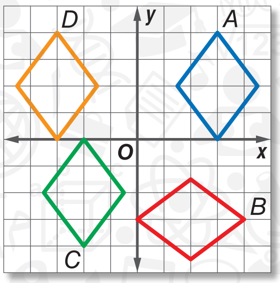
PHOTOGRAPHY Photographers use reflections, rotations, and symmetry to make photographs interesting and visually appealing.

Get Ready for the Chapter

QuickCheck

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

1. *A* to *B*
2. *D* to *A*
3. *A* to *C*

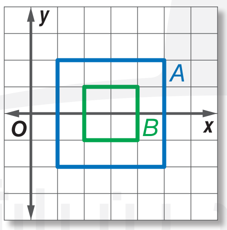


Find the sum of each pair of vectors.

4. $\langle 13, -4 \rangle + \langle -11, 9 \rangle$
5. $\langle 6, -31 \rangle + \langle -22, 3 \rangle$

6. **BAND** During part of a song, the drummer in a marching band moves from (1, 4) to (5, 1). Write the component form of the vector that describes his movement.

7. Determine whether the dilation from *A* to *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

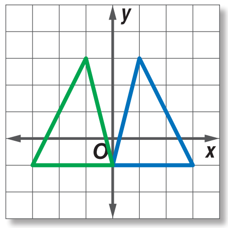


8. **PLAYS** Ahmed is making a model of an ant for a play. Find the scale factor of the model if the ant is 1 centimeter long and the model is $\frac{1}{4}$ m long.

QuickReview

Example 1

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.



Each vertex and its image are the same distance from the *y*-axis. This is a reflection.

Example 2

Write the component form of $\overrightarrow{AB}$ for $A(-1, 1)$ and $B(4, -3)$.

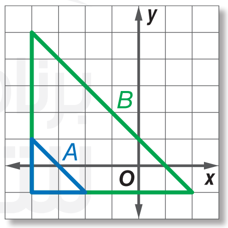
$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$ — Component form of vector

$= \langle 4 - (-1), -3 - 1 \rangle$ — Substitute.

$= \langle 5, -4 \rangle$ — Simplify.

Example 3

Determine whether the dilation from *A* to *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.



B is larger than *A*, so it is an enlargement.

The distance between the vertices of ***A*** is **2** and the corresponding distance for ***B*** is **6**.

The scale factor is $\frac{6}{2}$ or 3.

Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 6. To get ready, identify important terms and organize your resources.

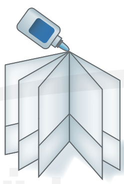
FOLDABLES StudyOrganizer

Transformations and Symmetry Make this Foldable to help you organize your Chapter 6 notes about transformations and symmetry. Begin with three sheets of notebook paper.

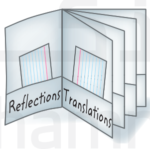
1. **Fold** each sheet of paper in half.

2. **Open** the folded papers and fold each paper lengthwise two inches, to form a pocket.

3. **Glue** the sheets side-by-side to create a booklet.



4. **Label** each of the pockets as shown.



NewVocabulary

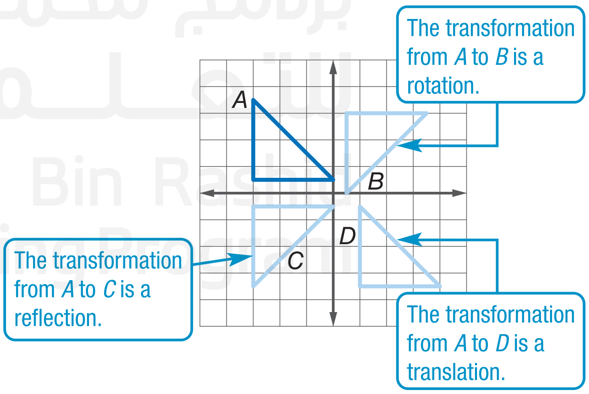
line of reflection
center of rotation
angle of rotation
composition of transformations
symmetry
line symmetry
line of symmetry

ReviewVocabulary

reflection a transformation representing a flip of the figure over a point, line or plane

rotation a transformation that turns every point of a preimage through a specified angle and direction about a fixed point

translation a transformation that moves all points of a figure the same distance in the same direction



LESSON 6-1 Reflections

Then

- You identified reflections and verified them as congruence transformations.

Now

1. Draw reflections.
2. Draw reflections in the coordinate plane.

Why?

- Notice in this water reflection that the distance a point lies above the water line appears the same as the distance its image lies below the water.

NewVocabulary
line of reflection

Mathematical Practices
Use appropriate tools strategically.
Look for and make use of structure.

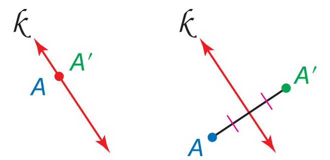
1 Draw Reflections

In an earlier lesson, you learned that a reflection or *flip* is a transformation in a line called the **line of reflection**. Each point of the preimage and its corresponding point on the image are the same distance from this line.

KeyConcept Reflection in a Line

A reflection in a line is a function that maps a point to its image such that

- if the point is on the line, then the image and preimage are the same point, or
- if the point is not on the line, the line is the perpendicular bisector of the segment joining the two points.



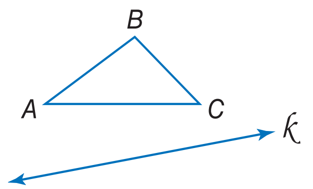
A is on line *k*. *A* is not on line *k*.

A′, *A″*, *A‴*, and so on, name corresponding points for one or more transformations.

To reflect a polygon in a line, reflect each of the polygon's vertices. Then connect these vertices to form the reflected image.

Example 1 Reflect a Figure in a Line

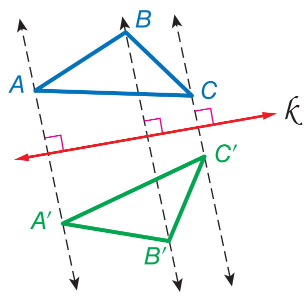
Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.



Step 1 Draw a line through each vertex that is perpendicular to line *k*.

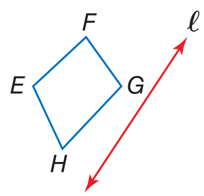
Step 2 Measure the distance from point *A* to line *k*. Then locate *A′* the same distance from line *k* on the opposite side.

Step 3 Repeat Step 2 to locate points *B′* and *C′*. Then connect vertices *A′*, *B′*, and *C′* to form the reflected image.

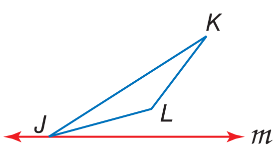


GuidedPractice

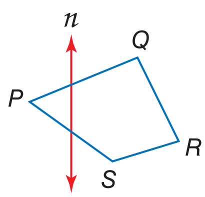
1A.



1B.



1C.



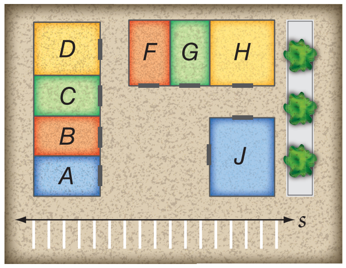
Recall that a reflection is a *congruence transformation* or *isometry*. In the figure in Example 1, $\triangle ABC \cong \triangle A'B'C'$.

Real-WorldCareer

Photographer Photographers take photos for a variety of reasons such as journalism, art, to record an event, or for scientific purposes. In some photography fields such as photojournalism and scientific photography, a bachelor's degree is required. For others, such as portrait photography, technical proficiency is the only requirement.

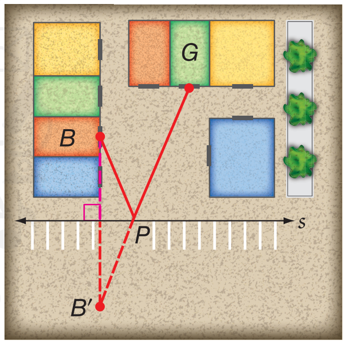
Real-World Example 2 Minimize Distance by Using a Reflection

SHOPPING Suppose you are going to buy clothes in Store B, return to your car, and then buy shoes at Store G. Where along line *s* of parking spaces should you park to minimize the distance you will walk?



Understand You are asked to locate a point P on line s such that $BP + PG$ has the least possible value.

Plan The total distance from B to P and then from P to G is least when these three points are collinear. Use the reflection of point B in line s to find the location for point P.

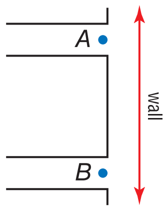


Solve Draw $\overline{B'G}$. Locate P at the intersection of line s and $\overline{B'G}$.

Check Compare the sum $BP + PG$ for each case to verify that the location found for P minimizes this sum.

GuidedPractice

2. TICKET SALES Eiman wants to select a good location to sell tickets for a graduation ceremony. Locate point P such that the distance someone would have to walk from Hallway A, to point P on the wall, and then to their next class in Hallway B is minimized.

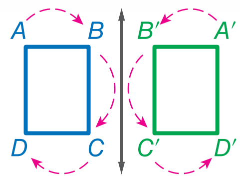


2 Draw Reflections in the Coordinate Plane

Reflections can also be performed in the coordinate plane by using the techniques presented in Example 3.

StudyTip

Characteristics of a Reflection Reflections, like all isometries, preserve distance, angle measure, betweenness of points, and collinearity. The orientation of a preimage and its image, however, are reversed.

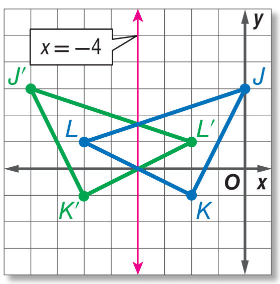


Example 3 Reflect a Figure in a Horizontal or Vertical Line

Triangle *JKL* has vertices $J(0, 3)$, $K(-2, -1)$, and $L(-6, 1)$. Graph $\triangle JKL$ and its image in the given line.

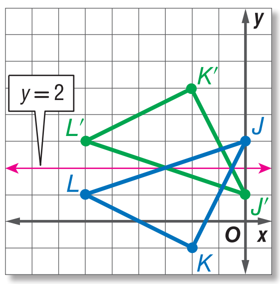
a. $x = -4$

Find a corresponding point for each vertex so that a vertex and its image are equidistant from the line $x = -4$.



b. $y = 2$

Find a corresponding point for each vertex so that a vertex and its image are equidistant from the line $y = 2$.



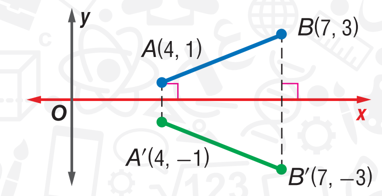
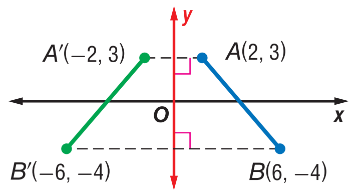
GuidedPractice

Trapezoid *RSTV* has vertices $R(-1, 1)$, $S(4, 1)$, $T(4, -1)$, and $V(-1, -3)$. Graph trapezoid *RSTV* and its image in the given line.

3A. $y = -3$ **3B.** $x = 2$

When the line of reflection is the *x*- or *y*-axis, you can use the following rule.

KeyConcept Reflection in the *x*- or *y*-axis

	Reflection in the *x*-axis		Reflection in the *y*-axis
Words	To reflect a point in the *x*-axis, multiply its *y*-coordinate by −1.	Words	To reflect a point in the *y*-axis, multiply its *x*-coordinate by −1.
Symbols	$(x, y) \rightarrow (x, -y)$	Symbols	$(x, y) \rightarrow (-x, y)$
Example		Example	

ReadingMath

Coordinate Function Notation The expression $P(a, b) \rightarrow P'(a, -b)$ can be read as *point P with coordinates a and b is mapped to new location P prime with coordinates a and negative b.*

Example 4 Reflect a Figure in the *x*- or *y*-axis

Graph each figure and its image under the given reflection.

a. $\triangle ABC$ with vertices $A(-5, 3)$, $B(2, 0)$, and $C(1, 2)$ in the *x*-axis

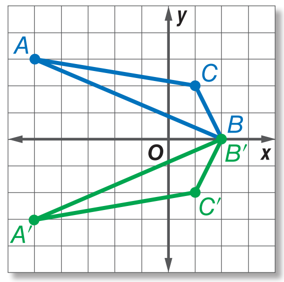
Multiply the *y*-coordinate of each vertex by −1.

$(x, y) \rightarrow (x, -y)$

$A(-5, 3) \rightarrow A'(-5, -3)$

$B(2, 0) \rightarrow B'(2, 0)$

$C(1, 2) \rightarrow C'(1, -2)$



StudyTip

Invariant Points In Example 4a, point *B* is called an *invariant point* because it maps onto itself. Only points that lie on the line of reflection are invariant under a reflection.

b. parallelogram *PQRS* with vertices $P(-4, 1)$, $Q(2, 3)$, $R(2, -1)$, and $S(-4, -3)$ in the *y*-axis

Multiply the *x*-coordinate of each vertex by −1.

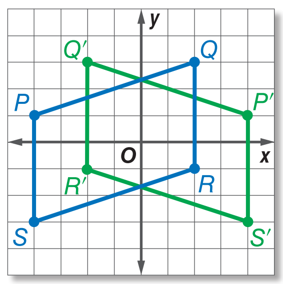
$(x, y) \rightarrow (-x, y)$

$P(-4, 1) \rightarrow P'(4, 1)$

$Q(2, 3) \rightarrow Q'(-2, 3)$

$R(2, -1) \rightarrow R'(-2, -1)$

$S(-4, -3) \rightarrow S'(4, -3)$



GuidedPractice

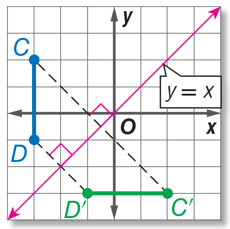
4A. rectangle with vertices $E(-4, -1)$, $F(2, 2)$, $G(3, 0)$, and $H(-3, -3)$ in the *x*-axis

4B. $\triangle JKL$ with vertices $J(3, 2)$, $K(2, -2)$, and $L(4, -5)$ in the *y*-axis

> **ReviewVocabulary**
>
> **Perpendicular Lines**
> Two nonvertical lines are perpendicular if and only if the product of their slopes is −1.

You can also reflect an image in the line $y = x$.

The slope of $y = x$ is 1. In the graph shown, $\overline{CC'}$ is perpendicular to $y = x$, so its slope is −1. From $C(-3, 2)$, move right 2.5 units and down 2.5 units to reach $y = x$. From this point on $y = x$, move right 2.5 units and down 2.5 units to locate $C'(2, -3)$. Using a similar method, the image of $D(-3, -1)$ is found to be $D'(-1, -3)$.



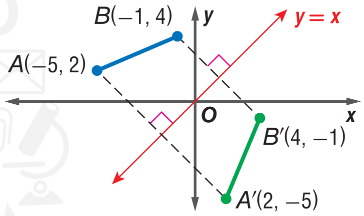
Comparing the coordinates of these and other examples leads to the following rule for reflections in the line $y = x$.

KeyConcept Reflection in Line $y = x$

Words To reflect a point in the line $y = x$, interchange the x- and y-coordinates.

Symbols $(x, y) \rightarrow (y, x)$

Example



> **StudyTip**
>
> **Preimage and Image**
> In this book, the preimage will always be blue and the image will always be green.

Example 5 Reflect a Figure in the Line $y = x$

Quadrilateral $JKLM$ has vertices $J(2, 2)$, $K(4, 1)$, $L(3, -3)$, and $M(0, -4)$. Graph $JKLM$ and its image $J'K'L'M'$ in the line $y = x$.

Interchange the x- and y-coordinates of each vertex.

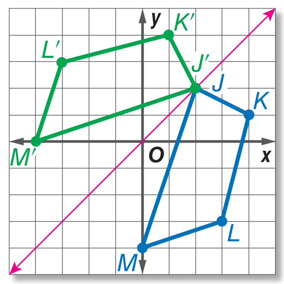
$(x, y) \rightarrow (y, x)$

$J(2, 2) \rightarrow J'(2, 2)$

$K(4, 1) \rightarrow K'(1, 4)$

$L(3, -3) \rightarrow L'(-3, 3)$

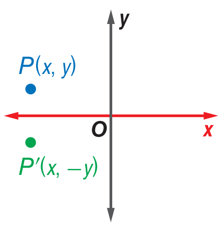
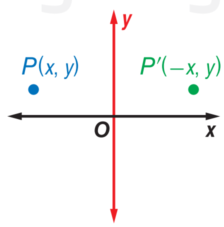
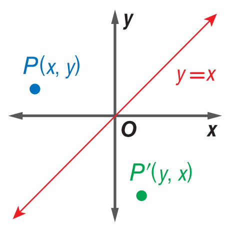
$M(0, -4) \rightarrow M'(-4, 0)$



GuidedPractice

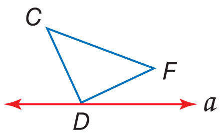
5. $\triangle BCD$ has vertices $B(-3, 3)$, $C(1, 4)$, and $D(-2, -4)$. Graph $\triangle BCD$ and its image in the line $y = x$.

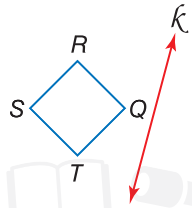
ConceptSummary Reflection in the Coordinate Plane

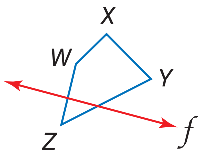
Reflection in the x-axis	Reflection in the y-axis	Reflection in the line $y = x$
		
$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$	$(x, y) \rightarrow (y, x)$

Check Your Understanding

Example 1 **Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.**

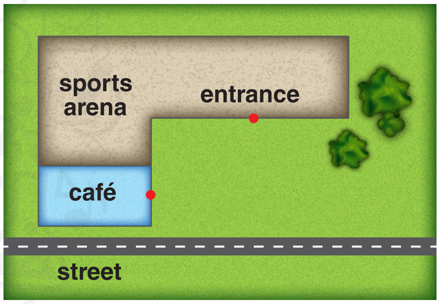
1.


2.


3.


Example 2

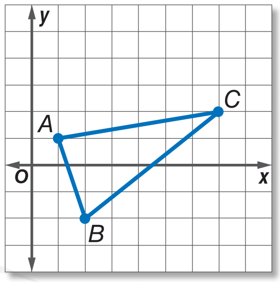
4. **SPORTING EVENTS** Ahmed is waiting at a café for a friend to bring him a ticket to a sold-out sporting event. At what point P along the street should the friend try to stop his car to minimize the distance Ahmed will have to walk from the café, to the car, and then to the arena entrance? Draw a diagram.



Example 3 **Graph $\triangle ABC$ and its image in the given line.**

5. $y = -2$

6. $x = 3$



Examples 4–5 **Graph each figure and its image under the given reflection.**

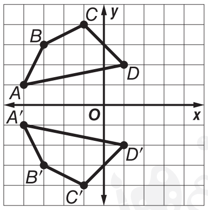
7. $\triangle XYZ$ with vertices $X(0, 4)$, $Y(-3, 4)$, and $Z(-4, -1)$ in the y-axis

8. $\square QRST$ with vertices $Q(-1, 4)$, $R(4, 4)$, $S(3, 1)$, and $T(-2, 1)$ in the x-axis

9. quadrilateral $JKLM$ with vertices $J(-3, 1)$, $K(-1, 3)$, $L(1, 3)$, and $M(-3, -1)$ in the line $y = x$

Practice and Problem Solving

10. $ABCD$ and its image $A'B'C'D'$ in the plane are shown. Which statements can be used to determine the type of transformation that occurred?



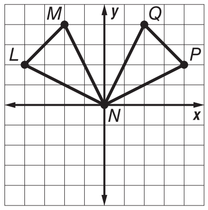
A Slope of $\overline{AB} = 2$; slope of $\overline{B'C'} = -\frac{1}{2}$; since the slopes are negative reciprocals, the transformation is a 90° clockwise rotation.

B The image of each of the points A, B, C, and D is a reflection in the x-axis, so the transformation is a reflection.

C Since B' is six units down from B, the transformation is a translation six units down.

D $CD = 2\sqrt{2}$ and $C'D' = 2\sqrt{2}$; since $CD = C'D'$, the transformation is a dilation with scale factor of 1.

11. $\triangle PQN$ is a transformation of $\triangle LMN$. Which statement verifies that the transformation is a reflection in the y-axis?



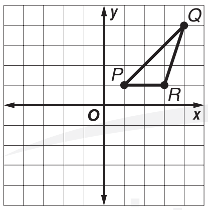
A slope of $\overline{MN}$ • slope of $\overline{NP} = -1$

B slope of $\overline{LN}$ • slope of $\overline{QN} = -1$

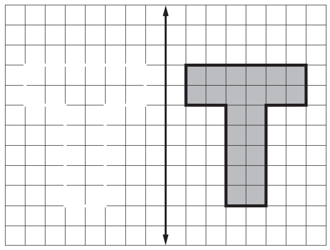
C The image of each point (x, y) is $(-x, y)$.

D $\overline{MN} \cong \overline{QN}$

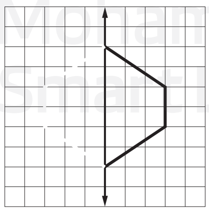
12. If triangle PQR is reflected across the x-axis to become triangle $P'Q'R$, what will be the coordinates of Q'?



13. **GEOMETRY** Draw a figure on the left side of the line so that the given figure and your figure will be symmetric with respect to the line.

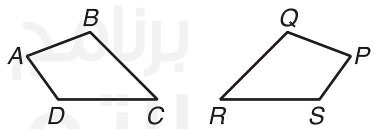


14. **GEOMETRY** Three line segments are shown on the grid below. Draw three more line segments to complete a hexagon that is symmetric with respect to the vertical line.



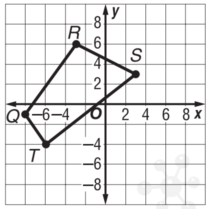
15. In the diagram, quadrilateral $ABCD$ is transformed into quadrilateral $PQRS$.

What is the preimage of $\overleftrightarrow{PS}$?

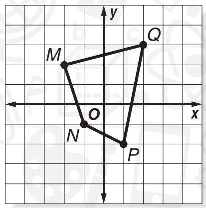


16. Quadrilateral *QRST* is shown below.

If quadrilateral *QRST* is reflected in the *x*-axis and then the *y*-axis to form quadrilateral *Q″R″S″T″*, what will be the coordinates of *T″*?

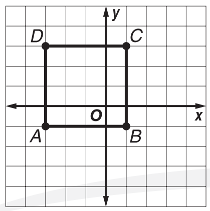


17. The graph of quadrilateral *MNPQ* is shown. What will be the coordinates of *Q*' if the quadrilateral is reflected across the *x*-axis?

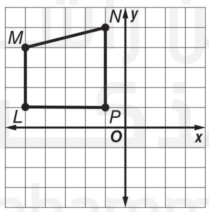


18. Square *ABCD* is shown below.

If *ABCD* is refl ected across the *y*-axis, what will be the coordinates of D'?



19.



If trapezoid *LMNP* is reflected across the *y*-axis, what will be the coordinates of *L*'?

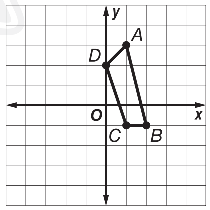
20. The vertices of △*ABC* are *A*(0, 6), *B*(2, 1), and *C* (−3, 4). If the figure is reflected across the *x*-axis to create △*WXY*, what would be the coordinates of the vertices for △*WXY*?

21. Ismail wants to reflect rectangle *HIJK* with vertices *H*(2, 4), *I*(5.5, 4), *J*(5.5, −1), and *K*(2, −1) across the *y*-axis to create rectangle *LMNP*. What will be the coordinates of point *L* if it is the reflection of point *H*?

22. △*UVW* has vertices *U*(−3, 1), *V*(2, 4), and *W*(7, 2), and △*XYZ* has vertices *X*(−3, −1), *Y*(2, −4), and *Z*(7, −2). What kind of transformation can be used to map △*UVW* onto △*XYZ*?

23. If △*LMN* with vertices *L*(−2, 6), *M*(5, 2), and *N*(−6, −1) is reflected across the *x*-axis, what are the coordinates of *L*'?

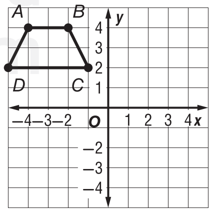
24. Quadrilateral *ABCD* has vertices *A*(1, 3), *B*(2, −1), *C*(1, −1), and *D*(0, 2). *ABCD* is reflected across the line $x = 1$ to make *WXYZ*. Which would be the set of coordinates for *WXYZ*?



25. A triangle has vertices at (1, 0), (1, −1), and (−1, −1). A reflection in what line would place the vertices at (0, 1), (−1, 1) and (−1, −1)?

26. The vertices of △*ABC* are *A*(0, 6), *B*(2, 1), and *C*(–3, 4). If the figure is reflected across the *x*-axis to create △*WXY*, what would be the coordinates of the vertices of △*WXY*?

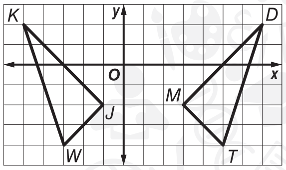
27. What are the coordinates of *B*′ if trapezoid *ABCD* is reflected across the *y*-axis?



28. Which of the following is the reflection of the point $E(-7, 1)$ in the x-axis?

29. The coordinates of the vertices of ΔABC are $A(-3, 1)$, $B(1, 5)$, and $C(7, 0)$. Which are the coordinates of the image, $\Delta A'B'C'$, under the reflection of the triangle in the line $y = x$.

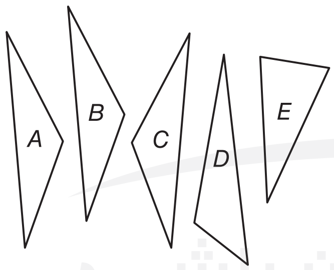
30. In which line is ΔMDT the reflection of ΔJKW?



31. Which is the reflection of $P(-3, 10)$ in the line $y = x$?

32. In which pair of lines is the line segment with endpoints $P''(10, 0)$ and $Q''(12, 4)$ the result of a double reflection of the line segment with endpoints $P(0, 0)$ and $Q(2, 4)$?

33. Which of the following figures appears to be the reflection of Figure A in some line?



34. Which of the following statements is true?

A If $P(x, y)$ is reflected in the y-axis and its image is reflected in the y-axis, the coordinates of the image are $P''(x, -y)$.

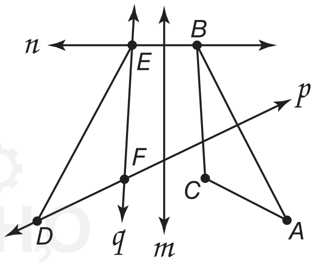
B If $P(x, y)$ is reflected in the y-axis and its image is reflected in the y-axis, the coordinates of the image are $P''(y, -y)$.

C If $P(x, y)$ is reflected in the y-axis and its image is reflected in the y-axis, the coordinates of the image are $P''(x, y)$.

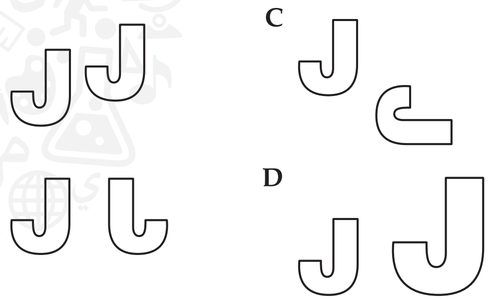
D If $P(x, y)$ is reflected in the y-axis and its image is reflected in the x-axis, the coordinates of the image are $P''(x, -y)$.

35. Under a transformation, hexagon $PQRSTU$ has image $ABRSCD$. Which of the following transformations could accomplish this?

36. Under a reflection, in which line will $\triangle DEF$ be the reflection of $\triangle ABC$?



37. Which image represents a reflection?



38. Which of the following is the reflection of the point $L(-2, -9)$ in the y-axis?

A $L'(-9, -2)$

B $L'(2, 9)$

C $L'(2, -9)$

D $L'(-9, -2)$

39. Under the glide reflection $R_{x=0} \rightarrow T_{x,\,y}$, the image of $A(1, 3)$ is $A'(-1, 6)$. What are the values of x and y?

A $x = -2$ and $y = 3$

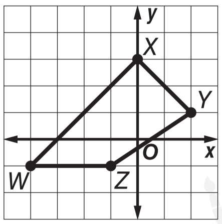
B $x = 0$ and $y = 3$

C $x = 3$ and $y = -2$

D $x = 3$ and $y = 0$

Standardized Test Practice

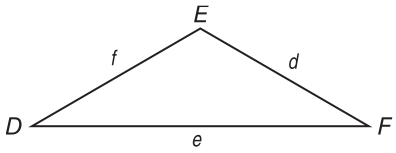
40. SHORT RESPONSE If quadrilateral $WXYZ$ is reflected across the y-axis to become quadrilateral $W'X'Y'Z'$, what are the coordinates of X'?



41. ALGEBRA If the arithmetic mean of $6x$, $3x$, and 27 is 18, then what is the value of x?

A 2
B 3
C 5
D 6

42. In $\triangle DEF$, $m\angle E = 108$, $m\angle F = 26$, and $f = 20$. Find d to the nearest whole number.



F 26 **G** 33 **H** 60 **J** 65

43. SAT/ACT In a coordinate plane, points A and B have coordinates $(-2, 4)$ and $(3, 3)$, respectively. What is the value of AB?

A $\sqrt{50}$
B $(1, 7)$
C $(5, -1)$
D $(1, -1)$
E $\sqrt{26}$

Spiral Review

Find the exact value of each expression if $0° < \theta < 90°$.

44. If $\cos\theta = \frac{3}{5}$, find $\sin\theta$.

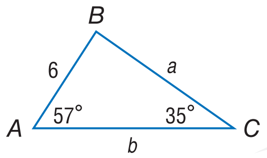
45. If $\tan\theta = 2$, find $\cot\theta$.

46. If $\sin\theta = \frac{\sqrt{5}}{3}$, find $\cos\theta$.

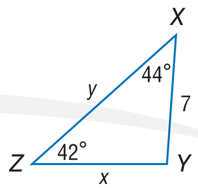
47. If $\csc\theta = \frac{3\sqrt{5}}{5}$, find $\tan\theta$.

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

48.



49.



50. COORDINATE GEOMETRY In $\triangle LMN$, $\overline{PR}$ divides $\overline{NL}$ and $\overline{MN}$ proportionally. If the vertices are $N(8, 20)$, $P(11, 16)$, and $R(3, 8)$ and $\frac{LP}{PN} = \frac{2}{1}$, find the coordinates of L and M.

Solve each equation. Round to the nearest tenth if necessary.

51. $\sin\theta = -0.58$

52. $\cos\theta = 0.32$

53. $\tan\theta = 2.7$

Skills Review

Find the magnitude and direction of each vector.

54. $\overrightarrow{RS}$: $R(-3, 3)$ and $S(-9, 9)$

55. $\overrightarrow{FG}$: $F(-4, 0)$ and $G(-6, -4)$

56. $\overrightarrow{JK}$: $J(8, 1)$ and $K(2, 5)$

57. $\overrightarrow{AB}$: $A(-1, 10)$ and $B(1, -12)$

LESSON 6-2 Translations

Then

- You found the magnitude and direction of vectors.

Now

1. Draw translations.
2. Draw translations in the coordinate plane.

Why?

- Stop-motion animation is a technique in which an object is moved by very small amounts between individually photographed frames. When the series of frames is played as a continuous sequence, the result is the illusion of movement.

NewVocabulary
translation vector

Mathematical Practices
Use appropriate tools strategically.
Model with mathematics.

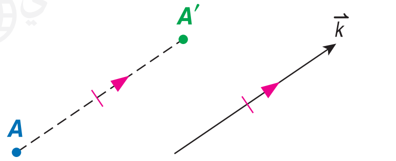
1 Draw Translations

In an earlier lesson, you learned that a translation or *slide* is a transformation that moves all points of a figure the same distance in the same direction. Since vectors can be used to describe both distance and direction, vectors can be used to define translations.

KeyConcept Translation

A translation is a function that maps each point to its image along a vector, called the **translation vector**, such that

- each segment joining a point and its image has the same length as the vector, and
- this segment is also parallel to the vector.



Point A' is a translation of point A along translation vector $\vec{k}$.

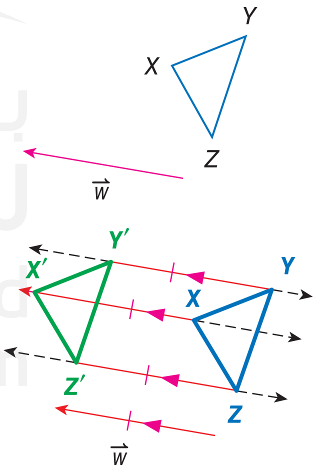
Example 1 Draw a Translation

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

Step 1 Draw a line through each vertex parallel to vector $\vec{w}$.

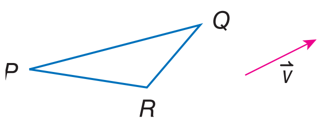
Step 2 Measure the length of vector $\vec{w}$. Locate point X' by marking off this distance along the line through vertex X, starting at X and in the same direction as the vector.

Step 3 Repeat Step 2 to locate points Y' and Z'. Then connect vertices X', Y', and Z' to form the translated image.

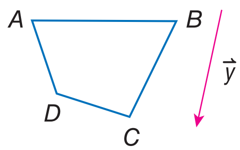


GuidedPractice

1A.

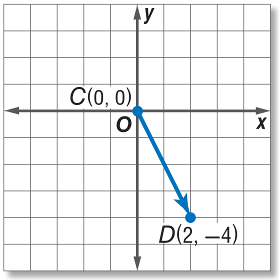


1B.



2 Draw Translations in the Coordinate Plane

Recall that a vector in the coordinate plane can be written as $\langle a, b \rangle$, where a represents the horizontal change and b is the vertical change from the vector's tip to its tail. $\overline{CD}$ is represented by the ordered pair $\langle 2, -4 \rangle$.



Written in this form, called the component form, a vector can be used to translate a figure in the coordinate plane.

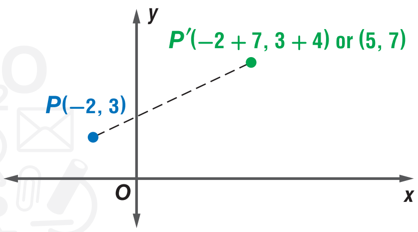
> **ReadingMath**
>
> **Horizontal and Vertical Translations** When the translation vector is of the form $\langle a, 0 \rangle$, the translation is horizontal only. When the translation vector is of the form $\langle 0, b \rangle$, the translation is vertical only.

KeyConcept Translation in the Coordinate Plane

Words To translate a point along vector $\langle a, b \rangle$, add a to the x-coordinate and b to the y-coordinate.

Symbols $(x, y) \rightarrow (x + a, y + b)$

Example The image of $P(-2, 3)$ translated along vector $\langle 7, 4 \rangle$ is $P'(5, 7)$.



A translation is another type of congruence transformation or isometry.

Example 2 Translations in the Coordinate Plane

Graph each figure and its image along the given vector.

a. $\triangle EFG$ **with vertices** $E(-7, -1)$, $F(-4, -4)$, **and** $G(-3, -1)$; $\langle 2, 5 \rangle$

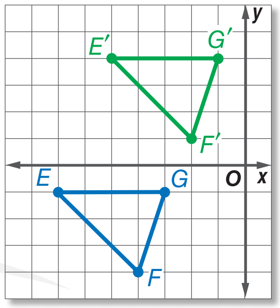
The vector indicates a translation 2 units right and 5 units up.

$(x, y) \rightarrow (x + 2, y + 5)$

$E(-7, -1) \rightarrow E'(-5, 4)$

$F(-4, -4) \rightarrow F'(-2, 1)$

$G(-3, -1) \rightarrow G'(-1, 4)$



b. square $JKLM$ **with vertices** $J(3, 4)$, $K(5, 2)$, $L(7, 4)$, **and** $M(5, 6)$; $\langle -3, -4 \rangle$

The vector indicates a translation 3 units left and 4 units down.

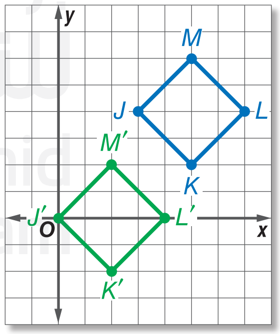
$(x, y) \rightarrow (x + (-3), y + (-4))$

$J(3, 4) \rightarrow J'(0, 0)$

$K(5, 2) \rightarrow K'(2, -2)$

$L(7, 4) \rightarrow L'(4, 0)$

$M(5, 6) \rightarrow M'(2, 2)$



GuidedPractice

2A. $\triangle ABC$ with vertices $A(2, 6)$, $B(1, 1)$, and $C(7, 5)$; $\langle -4, -1 \rangle$

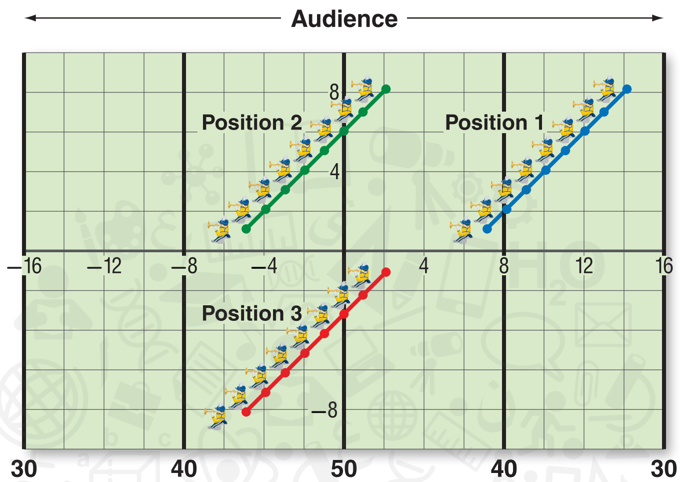
2B. quadrilateral $QRST$ with vertices $Q(-8, -2)$, $R(-9, -5)$, $S(-4, -7)$, and $T(-4, -2)$; $\langle 7, 1 \rangle$

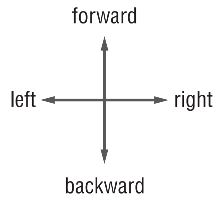
Real-WorldLink

Marching bands often make use of a series of formations that can include geometric shapes. Usually, each band member has an assigned position in each formation. *Floating* is the movement of a group of members together without changing the shape or size of their formation.

Real-World Example 3 Describing Translations

MARCHING BAND **In one part of a marching band's performance, a line of trumpet players starts at position 1, marches to position 2, and then to position 3. Each unit on the graph represents one step.**





a. Describe the translation of the trumpet line from position 1 to position 2 in function notation and in words.

One point on the line in position 1 is (14, 8). In position 2, this point moves to (2, 8). Use the translations function $(x, y) \rightarrow (x + a, y + b)$ to write and solve equations to find a and b.

$$(14 + a, 8 + b) \quad \text{or} \quad (2, 8)$$

$14 + a = 2$ $\qquad$ $8 + b = 8$

$a = -12$ $\qquad$ $b = 0$

function notation: $(x, y) \rightarrow (x + (-12), y + 0)$

So, the trumpet line is translated 12 steps *left* but no steps forward or backward from position 1 to position 2.

b. Describe the translation of the line from position 1 to position 3 using a translation vector.

$$(14 + a, 8 + b) \quad \text{or} \quad (2, -1)$$

$14 + a = 2$ $\qquad$ $8 + b = -1$

$a = -12$ $\qquad$ $b = -9$

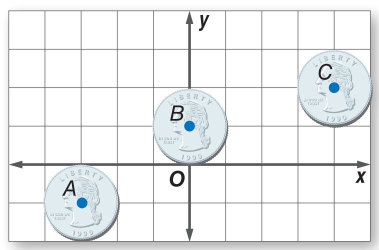
translation vector: $\langle -12, -9 \rangle$

GuidedPractice

3. ANIMATION A coin is filmed using stop-motion animation so that it appears to move.

A. Describe the translation from A to B in function notation and in words.

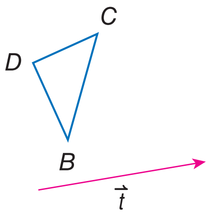
B. Describe the translation from A to C using a translation vector.



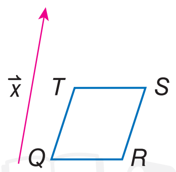
Check Your Understanding

Example 1 **Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.**

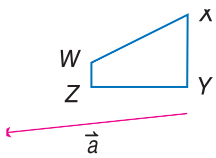
1.



2.



3.



Example 2 **Graph each figure and its image along the given vector.**

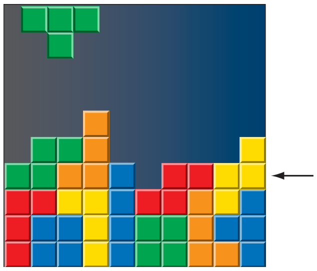
4. trapezoid $JKLM$ with vertices $J(2, 4)$, $K(1, 1)$, $L(5, 1)$ and $M(4, 4)$; $\langle 7, 1\rangle$

5. $\triangle DFG$ with vertices $D(-8, 8)$, $F(-10, 4)$, and $G(-7, 6)$; $\langle 5, -2\rangle$

6. parallelogram $WXYZ$ with vertices $W(-6, -5)$, $X(-2, -5)$, $Y(-1, -8)$, and $Z(-5, -8)$; $\langle -1, 4\rangle$

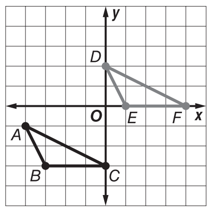
Example 3

7. VIDEO GAMES The object of the video game shown is to manipulate the colored tiles left or right as they fall from the top of the screen to completely fill each row without leaving empty spaces. If the starting position of the tile piece at the top of the screen is (x, y), use function notation to describe the translation that will fill the indicated row.



Practice and Problem Solving

8. Triangle ABC and its image, triangle DEF, are shown. Which statement describes the type of transformation that occurred?



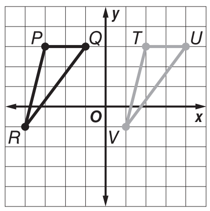
A Slope of $\overline{AC}$ = slope of $\overline{DF}$; since the slopes are the same, the transformation is a rotation.

B Each of the points A, B, and C is reflected in the x-axis.

C For points A, B, and C, each x-coordinate increases by 4 units, and each y-coordinate increases by 3 units. So, the transformation is a translation.

D Since $BC \neq DF$, the transformation is a dilation with a scale factor of 1.

9. PQR and its image TUV are shown. Which statement describes the type of transformation that occurred?



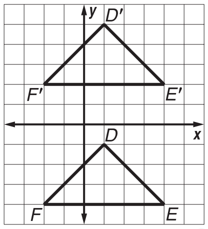
A Since the x-coordinates of points P, Q, and R are each increased by 5 units, the transformation is a translation.

B The image of each of the points P, Q, and R is a reflection in the y-axis.

C $R = (-4, -1)$; $U = (4, 3)$; since the x-coordinates are opposites, the transformation is a reflection in the x-axis.

D Since $QR = UV$, the transformation is a dilation with scale factor of 1.

10. In the illustration, triangle $D'E'F'$ is formed by adding 6 units to the y-coordinate of each vertex of triangle DEF. The best term for describing triangle $D'E'F'$ is



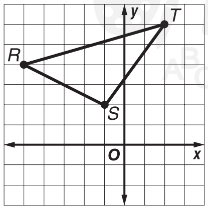
A a rotation of $\triangle DEF$.

B a reflection of $\triangle DEF$.

C similar to $\triangle DEF$.

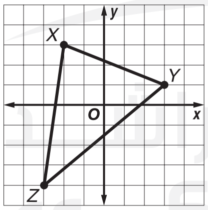
D congruent to $\triangle DEF$.

11. Triangle RST has coordinates $R(-5, 4)$, $S(-1, 2)$ and $T(2, 6)$. What will be the new coordinates of point T if the triangle is translated 3 units to the right and 5 units down?



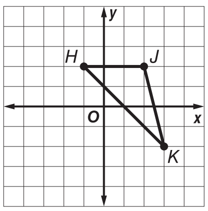
12. $\triangle XYZ$ is shown on the coordinate grid.

If $\triangle XYZ$ is translated so that point X is on the y-axis and point Y is at $(5, -3)$, what will be the new coordinates of point Z?



13. Triangle HJK below is translated so that the coordinates of the new vertices are $H'(-2, 4)$, $J'(1, 4)$, and $K'(2, 0)$.

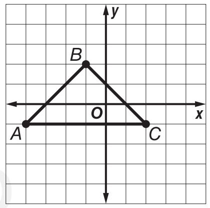
What statement describes this transformation?



14. The vertices of parallelogram $ABCD$ are $A(-3, 0)$, $B(-1, 3)$, $C(-1, -2)$, and $D(-3, -5)$. If the figure is translated 4 units to the right and 2 units up, what are the coordinates of vertex B'?

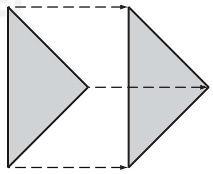
15. Triangle ABC is to be translated to $\triangle A'B'C'$ by using the following rule. $(x, y) \rightarrow (x - 2, y + 3)$

What will be the coordinates of point B'?



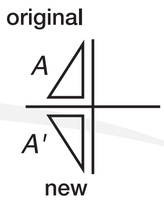
16. The vertices of $\triangle ABC$ are $A(0.5, 8)$, $B(7.5, 7)$, and $C(4.2, 2)$. Which set of coordinates are those of the vertices of the image that results from a translation of $\triangle ABC$ 3.5 units down?

17. Which of the following transformations is shown in the figure?

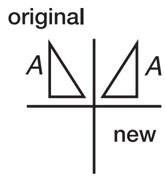


18. Which diagram shows a translation of figure A?

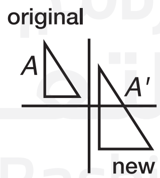
A



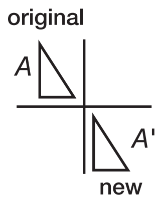
B



C

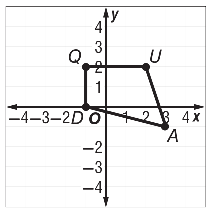


D



19. Quadrilateral $QUAD$ has vertices as shown in the coordinate plane below.

Which transformation will place two vertices at $(5, 2)$ and $(6, -1)$?

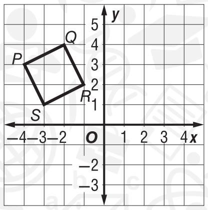


20. The vertices of $\triangle LMN$ are $L(5, 6)$, $M(2, 0)$, and $N(-8, 8)$. If the figure is translated and the image has vertices in random order at $(-2, 0)$, $(1, 6)$, and $(-12, 8)$, then which rule describes the translation?

21. Right triangle GHI has vertices $G(0, 0)$, H $(3, 0)$, and $I(0, 4)$. The triangle is transformed so that H' has coordinates $(3, 2)$. Which could be the transformation applied to $\triangle GHI$?

22. Square $PQRS$ below is to be translated to square $P'Q'R'S'$ by the following motion rule.

$$(x, y) \longrightarrow (x + 2, y - 6)$$

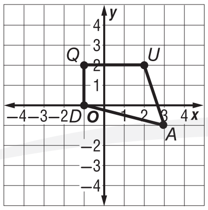


What will be the coordinates of vertex P´?

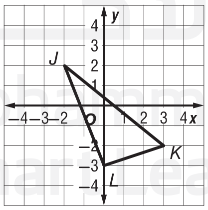
23. The vertices of parallelogram $ABCD$ are $A(-3, 0)$, $B(-1, 3)$, $C(-1, -2)$, and $D(-3, -5)$. If the figure is translated 4 units to the right and 2 units up, what are the coor dinates of vertex B'?

24. Quadrilateral $QUAD$ is translated 4 units to the left and 3 units up.

What are the coordinates of vertex A'?

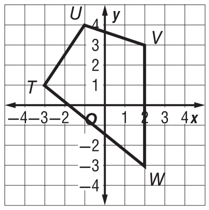


25. $\triangle JKL$ is translated 3 units left and 2 units up to create $\triangle J'K'L'$. What are the coordinates of the vertices?



26. The vertices of $\triangle LMN$ are $L(5, 6)$, $M(2, 0)$, and $N(-8, 8)$. If the figure is translated, and the new vertices are $L'(1, 6)$, $M'(-2, 0)$, and $N'(-12, 8)$, which rule describes the transformation?

27. Quadrilateral $TUVW$ is translated so that the new vertices are $T'(-1, 0)$, $U'(1, 3)$, and $V'(4, 2)$. What are the coordinates of W'?



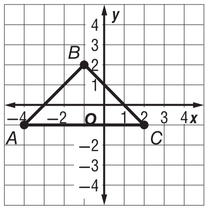
A $(0, -3)$ **C** $(4, -3)$

B $(0, -4)$ **D** $(4, -4)$

28. $\triangle ABC$ is to be translated to $\triangle A'B'C'$ by the following motion rule.

$$(x, y) \longrightarrow (x - 2, y + 3)$$

What will be the coordinates of point B'?



29. The vertices of quadrilateral $ABCD$ are $A(-2, 1)$, $B(-2, 5)$, $C(3, 5)$, and $D(3, 1)$. If $ABCD$ is translated 6 units down and 5 units to the right to create $D'E'F'G'$, what are the coordinates of the vertices of $D'E'F'G'$?

30. Which are the coordinates of the image, P', of the point $P(4, 1)$ under $T_{-3, -3}$?

31. Under which translation will $B(-2, 5)$ be the translation of $A(-7, 8)$?

32. The coordinates of the vertices of ΔRST are $R(3, 1)$, $S(5, 4)$, and $T(7, 11)$. What are the coordinates of the vertices of the image, $\Delta R'S'T'$, under $T_{-6, 1}$?

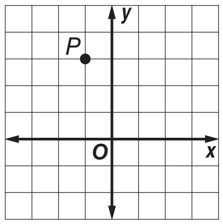
33. Which are the coordinates of the image, H', of the point $H(-8, 3)$ under $T_{8,7}$?

34. Which transformation would produce the image $P'(-4, 2)$ from the point $P(2, -1)$?

35. Which transformation preserves area and orientation?

Standardized Test Practice

36. Identify the location of point P under translation $(x + 3, y + 1)$.



A (0, 6) **C** (2, −4)
B (0, 3) **D** (2, 4)

37. SHORT RESPONSE Which vector best describes the translation of $A(3, -5)$ to $A'(-2, -8)$?

38. ALGEBRA Over the next four days, Maysoun plans to drive 160 miles, 235 miles, 185 miles, and 220 miles. If her car gets an average of 32 miles per gallon of gas, how many gallons of gas should she expect to use in all?

F 25 **G** 30 **H** 35 **J** 40

39. SAT/ACT A bag contains 5 red marbles, 2 blue marbles, 4 white marbles, and 1 yellow marble. If two marbles are chosen in a row, without replacement, what is the probability of getting 2 white marbles?

A $\frac{1}{66}$ **C** $\frac{1}{9}$ **E** $\frac{2}{5}$
B $\frac{1}{11}$ **D** $\frac{5}{33}$

Spiral Review

Graph each figure and its image under the given reflection. (Lesson 6-1)

40. $\overline{DJ}$ with endpoints $D(4, 4)$, $J(-3, 2)$ in the y-axis

41. $\triangle XYZ$ with vertices $X(0, 0)$, $Y(3, 0)$, and $Z(0, 3)$ in the x-axis

42. $\triangle ABC$ with vertices $A(-3, -1)$, $B(0, 2)$, and $C(3, -2)$, in the line $y = x$

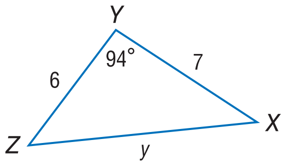
43. quadrilateral $JKLM$ with vertices $J(-2, 2)$, $K(3, 1)$, $L(4, -1)$, and $M(-2, -2)$ in the origin

Solve each equation if $0° \leq \theta \leq 360°$.

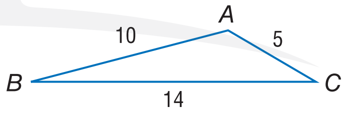
44. $2 \sin \theta = 1$ **45.** $2 \cos \theta + 1 = 0$ **46.** $4 \cos^2 \theta - 1 = 0$

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

47.



48.

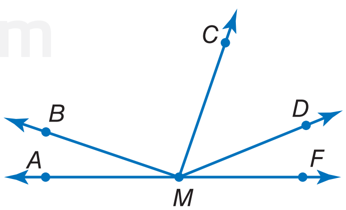


Solve each equation. Round to the nearest tenth if necessary.

49. $\sin \theta = -0.58$ **50.** $\cos \theta = 0.32$ **51.** $\tan \theta = 2.7$

Skills Review

Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



52. $\angle AMC$ **53.** $\angle FMD$

54. $\angle BMD$ **55.** $\angle CMB$

EXPLORE

6-3 Geometry Lab
Rotations

A rotation is a type of transformation that moves a figure about a fixed point, or center of rotation, through a specific angle and in a specific direction. In this activity you will use tracing paper to explore the properties of rotations.

Activity Explore Rotations by Using Patty Paper

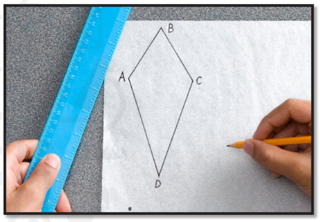
Step 1 On a piece of tracing paper, draw quadrilateral *ABCD* and a point *P*.

Step 2 On another piece of tracing paper, trace quadrilateral *ABCD* and point *P*. Label the new quadrilateral *A′B′C′D′* and the new point *P*.

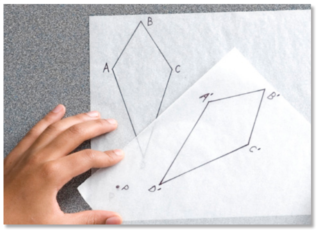
Step 3 Position the tracing paper so that both points *P* coincide. Rotate the paper so that *ABCD* and *A′B′C′D′* do not overlap. Tape the two pieces of tracing paper together.

Step 4 Measure the distance between *A*, *B*, *C*, and *D* to point *P*. Repeat for quadrilateral *A′B′C′D′*. Then copy and complete the table below.

Quadrilateral	Length			
ABCD	*AP*	*BP*	*CP*	*DP*
A′B′C′D′	*A′P*	*B′P*	*C′P*	*D′P*



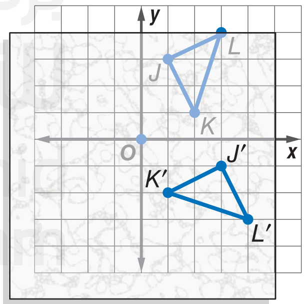
Step 1



Steps 2 and 3

Exercises

1. Graph △*JKL* with vertices *J*(1, 3), *K*(2, 1), and *L*(3, 4) on a coordinate plane, and then trace on tracing paper.
   a. Use a protractor to rotate each vertex 90° clockwise about the origin as shown in the figure at the right. What are the vertices of the rotated image?
   b. Rotate △*JKL* 180° about the origin. What are the vertices of the rotated image?
   c. Use the Distance Formula to find the distance from points *J*, *K*, and *L* to the origin. Repeat for *J′K′L′* and *J″K″L″*.



2. **WRITING IN MATH** If you rotate point (4, 2) 90° and 180° about the origin, how do the *x*- and *y*-coordinates change?

3. **MAKE A PREDICTION** What are the new coordinates of a point (*x*, *y*) that is rotated 270°?

4. **MAKE A CONJECTURE** Make a conjecture about the distances from the center of rotation *P* to each corresponding vertex of *ABCD* and *A′B′C′D′*.

LESSON 6-3

Rotations

:: Then	:: Now	:: Why?
You identified rotations and verified them as congruence transformations.	1 Draw rotations. 2 Draw rotations in the coordinate plane.	Modern windmill technology may be an important alternative to fossil fuels. Windmills convert the wind's energy into electricity through the rotation of turbine blades.

New Vocabulary
center of rotation
angle of rotation

Mathematical Practices
Reason abstractly and quantitatively.
Use appropriate tools strategically.

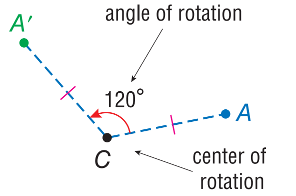
1 Draw Rotations

In an earlier lesson, you learned that a rotation or *turn* moves every point of a preimage through a specified angle and direction about a fixed point.

Key Concept Rotation

A rotation about a fixed point, called the **center of rotation**, through an angle of $x°$ is a function that maps a point to its image such that

- if the point is the center of rotation, then the image and preimage are the same point, or
- if the point is not the center of rotation, then the image and preimage are the same distance from the center of rotation and the measure of the **angle of rotation** formed by the preimage, center of rotation, and image points is x.

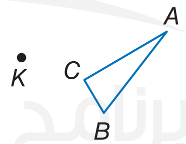


A′ is the image of *A* after a 120° rotation about point *C*.

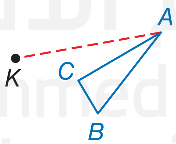
The direction of a rotation can be either clockwise or counterclockwise. Assume that all rotations are counterclockwise unless stated otherwise.

Example 1 Draw a Rotation

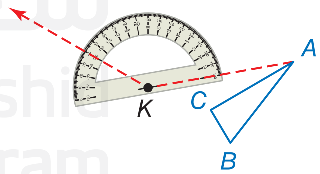
Copy △*ABC* and point *K*. Then use a protractor and ruler to draw a 140° rotation of △*ABC* about point *K*.



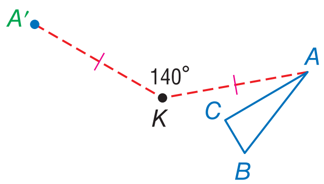
Step 1 Draw a segment from *A* to *K*.



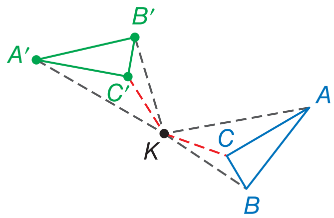
Step 2 Draw a 140° angle using $\overline{KA}$.



Step 3 Use a ruler to draw *A′* such that $KA' = KA$.



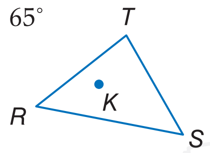
Step 4 Repeat Steps 1–3 for vertices *B* and *C* and draw △*A′B′C′*.



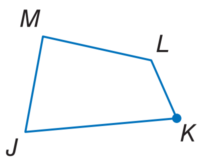
GuidedPractice

Copy each figure and point *K*. Then use a protractor and ruler to draw a rotation of the figure the given number of degrees about *K*.

1A. 65°



1B. 170°

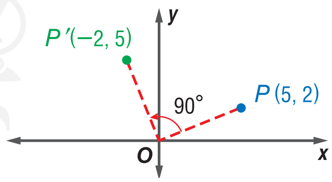
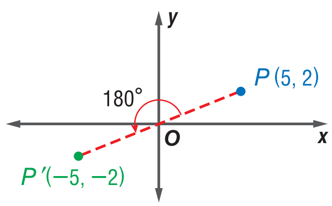
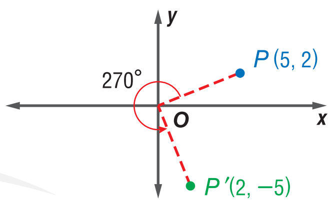


> **StudyTip**
> **Clockwise Rotation** Clockwise rotation can be designated by a negative angle measure. For example a rotation of −90° about the origin is a rotation 90° clockwise about the origin.

2 Draw Rotations in the Coordinate Plane

When a point is rotated 90°, 180°, or 270° counterclockwise about the origin, you can use the following rules.

KeyConcept Rotations in the Coordinate Plane

90° Rotation To rotate a point 90° counterclockwise about the origin, multiply the *y*-coordinate by −1 and then interchange the *x*- and *y*-coordinates. **Symbols** $(x, y) \rightarrow (-y, x)$	**Example** 
180° Rotation To rotate a point 180° counterclockwise about the origin, multiply the *x*- and *y*-coordinates by −1. **Symbols** $(x, y) \rightarrow (-x, -y)$	**Example** 
270° Rotation To rotate a point 270° counterclockwise about the origin, multiply the *x*-coordinate by −1 and then interchange the *x*- and *y*-coordinates. **Symbols** $(x, y) \rightarrow (y, -x)$	**Example** 

> **StudyTip**
> **360° Rotation** A rotation of 360° about a point returns a figure to its original position. That is, the image under a 360° rotation is equal to the preimage.

Example 2 Rotations in the Coordinate Plane

Triangle *PQR* has vertices *P*(1, 1), *Q*(4, 5), and *R*(5, 1). Graph △*PQR* and its image after a rotation 90° about the origin.

Multiply the *y*-coordinate of each vertex by −1 and interchange.

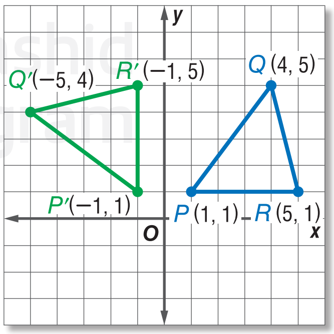
$(x, y) \rightarrow (-y, x)$

$P(1, 1) \rightarrow P'(-1, 1)$

$Q(4, 5) \rightarrow Q'(-5, 4)$

$R(5, 1) \rightarrow R'(-1, 5)$

Graph △*PQR* and its image △*P′Q′R′*.

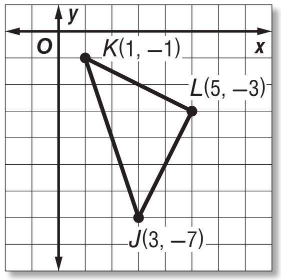


GuidedPractice

2. Parallelogram *FGHJ* has vertices *F*(2, 1), *G*(7, 1), *H*(6, −3), and *J*(1, −3). Graph *FGHJ* and its image after a rotation 180° about the origin.

Standardized Test Example 3 Rotations in the Coordinate Plane

Triangle *JKL* is shown at the right. What is the image of point *J* after a rotation 270° counterclockwise about the origin?



A (−3, −7)

B (−7, 3)

C (−7, −3)

D (7, −3)

Read the Test Item

You are given that △*JKL* has coordinates *J*(3, −7), *K*(1, −1), and *L*(5, −3) and are then asked to identify the coordinates of the image of point *J* after a 270° counterclockwise rotation about the origin.

Solve the Test Item

To find the coordinates of point *J* after a 270° counterclockwise rotation about the origin, multiply the *x*-coordinate by −1 and then interchange the *x*- and *y*-coordinates.

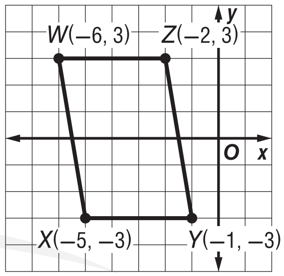
$(x, y) \rightarrow (y, -x)$ $(3, -7) \rightarrow (-7, -3)$

The answer is choice C.

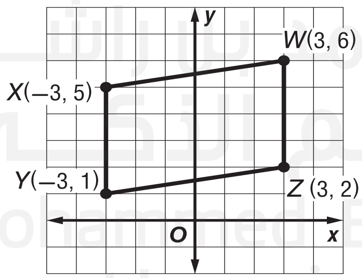
> **StudyTip**
>
> **270° Rotation** You can complete a 270° rotation by performing a 90° rotation and a 180° rotation in sequence.

GuidedPractice

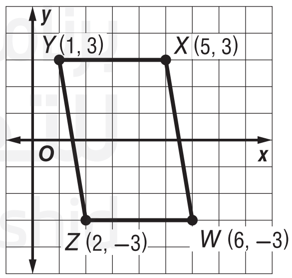
3. Parallelogram *WXYZ* is rotated 180° counterclockwise about the origin. Which of these graphs represents the resulting image?



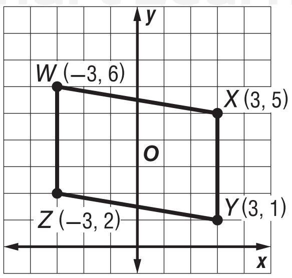
F



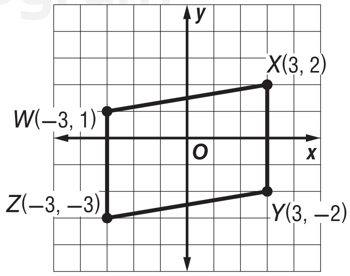
H



G



J



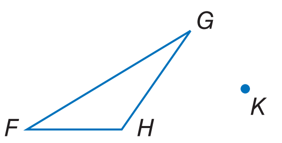
> **Test-TakingTip**
>
> **Sense-Making**
>
> Instead of checking all four vertices of parallelogram *WXYZ* in each graph, check just one vertex, such as *X*.

Check Your Understanding

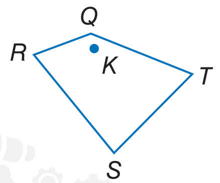
Example 1

TOOLS **Copy each polygon and point *K*. Then use a protractor and ruler to draw the specified rotation of each figure about point *K*.**

1. 45°



2. 120°

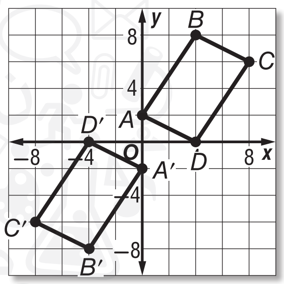


Example 2

3 Triangle *DFG* has vertices *D*(−2, 6), *F*(2, 8), and *G*(2, 3). Graph △*DFG* and its image after a rotation 180° about the origin.

Example 3

4. **MULTIPLE CHOICE** For the transformation shown, what is the measure of the angle of rotation of *ABCD* about the origin?



A 90°

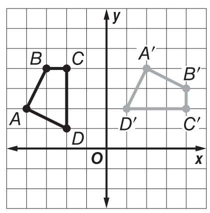
B 180°

C 270°

D 360°

Practice and Problem Solving

5. *ABCD* and its image *A′B′C′D′* in the plane are shown. Which statements describe the type of transformation that occurred?



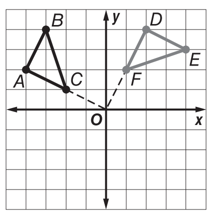
A Slope of $\overleftrightarrow{DO} = -\frac{1}{2}$; slope of $\overleftrightarrow{D'O} = 2$; since the slopes are negative reciprocals, the transformation is a clockwise rotation of 90°.

B *C* = (−2, 4); *A′* = (2, 4); since *A′* is the image of *C* in the *y*-axis, the transformation is a reflection in the *y*-axis.

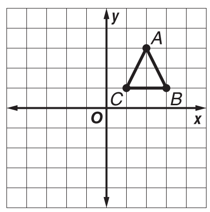
C *A* = (−4, 2); *A′* = (2, 4); the transformation is a translation 6 units to the right and 2 units up.

D $\overline{CD} = 3$ and $B'C' = 1$; since $B'C'$ is one-third the length of $\overline{CD}$, the transformation is a dilation with scale factor $\frac{1}{3}$.

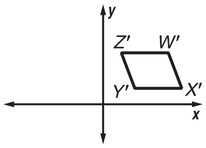
6. △*DEF* is a rotation of △*ABC* in the plane.
Which statement verifies that the angle of rotation is 90°?



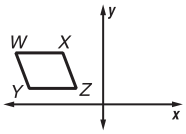
7. If triangle *ABC* is rotated 90° clockwise about the origin to make triangle *A′B′C′*, what are the coordinates of the vertex *A′*?



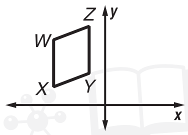
8. Which preimage of quadrilateral $W'X'Y'Z'$ shows that the transformation $WXYZ \rightarrow W'X'Y'Z'$ is a rotation?



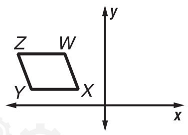
A



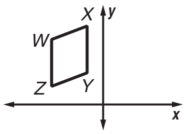
B



C

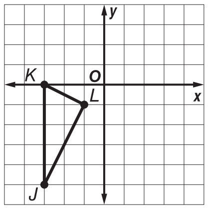


D

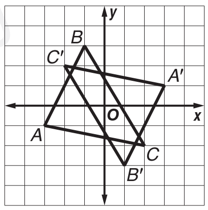


H.O.T. Problems Use Higher-Order Thinking Skills

9. Triangle JKL is graphed on the coordinate plane as shown below. If $\triangle JKL$ is rotated 180° about the origin, what are the coordinates of J'?



On the coordinate plane below, $\triangle ABC$ has been rotated 180° about the origin to create $\triangle A'B'C'$.

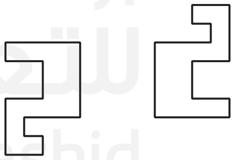


Complete the table below to compare the coordinates of the vertices of $\triangle ABC$ to those of the corresponding vertices on $\triangle A'B'C'$.

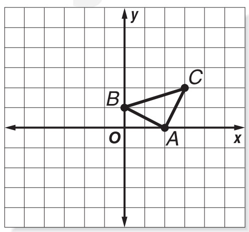
$\triangle ABC$	$\triangle A'B'C'$
$A(-3, -1)$	A'
$B(-1, 3)$	B'
$C(2, -2)$	C'

Choose the coordinates for the vertices of another triangle, $\triangle XYZ$, and write them in the table below. Use the pattern you discovered in the table above to find the coordinates of the vertices of $\triangle X'Y'Z'$, the image of $\triangle XYZ$ after a 180° rotation about the origin. Explain how you used the pattern to complete the table below.

10. What type of transformation was applied to the left figure to form the figure on the right?

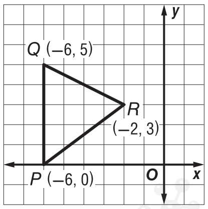


11. If ABC is rotated 90° clockwise about point B, then what are the coordinates of B'?



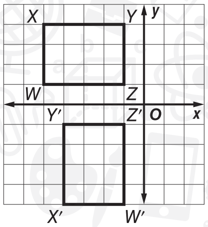
12. Triangle PQR has vertices $P(-6, 0)$, $Q(-6, 5)$, and $R(-2,3)$ as shown below.

What is the image of point R after a 270° rotation counterclockwise about the origin?



13. Look at the transformation below.

What is the measure of the angle of rotation of $WXYZ$ about the origin in a counterclockwise direction?



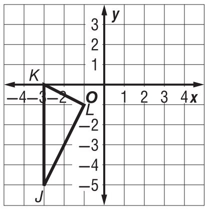
14. If triangle JKL is rotated 180 degrees about the origin, what are the coordinates of J'?

A (5, 3)

B (3, 0)

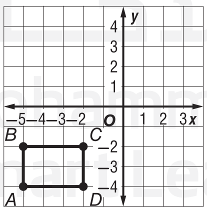
C (3, 5)

D (3, –5)



15. Triangle JKL has vertices at $J(0, 1)$, $K(2, 3)$, and $L(4, 0)$. If the triangle is rotated 180° about the origin, what will be the coordinates of K'?

16. What are the coordinates of C' if rectangle $ABCD$ is rotated 90° clockwise about the origin?

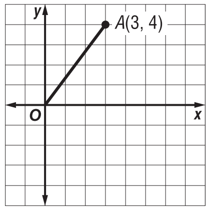


17. Which is the image of $P(0, 7)$ under a 90°-counterclockwise rotation?

18. Which is the image of $Q(-3, 0)$ under a 90°-clockwise rotation?

19. Point $R(4, -2)$ is rotated about the origin 90°-counterclockwise. In which quadrant will the image of that point lie?

20. One vertex of a square is point A in the diagram below. The square is rotated 180° about the origin. What are the coordinates of A', the image of A under the rotation?



21. Under which rotation about the origin will $P'(-6, 1)$ be the image of $P(1, 6)$?

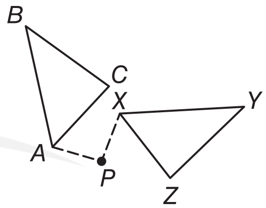
22. The image of $P(x, y)$ under a rotation about the origin O and through x° counterclockwise is $P'(x', y')$. Under which rotation about O can you rotate $P'(x', y')$ so that the image is $P(x, y)$?

23. A point in the first quadrant is rotated 90° counterclockwise. In which quadrant will the image of that point be located?

24. Point $P(x, y)$ is a point in the second quadrant. Under which rotation about the origin will the coordinates of the image be $P(-y, x)$?

25. Which point is the image a 90° counterclockwise rotation of point $P(-4.7, 3.5)$ about the origin?

26. One triangle is a rotation of the other about P. Which statement is **not** true?



A The triangles are congruent.

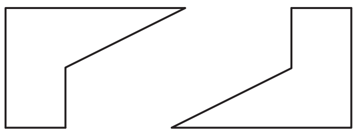
B The orientation of one triangle is different from that of the other triangle.

C Each of A, B, and C is rotated the same number of degrees to form ΔXYZ.

D $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\angle C \cong \angle Z$

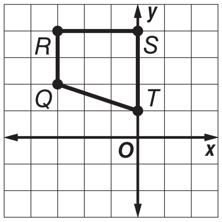
27. Which is the image of $P(-5, 12)$ under a 90°-counterclockwise rotation?

28. The polygons shown below are congruent. Which transformation could be used to demonstrate their congruence?



Standardized Test Practice

29. What rotation of trapezoid $QRST$ creates an image with point R' at (4, 3)?



A 270° counterclockwise about point T

B 185° counterclockwise about point T

C 180° clockwise about the origin

D 90° clockwise about the origin

31. SHORT RESPONSE $\triangle XYZ$ has vertices $X(1, 7)$, $Y(0, 2)$, and $Z(-5, -2)$. What are the coordinates of X' after a rotation 270° counterclockwise about the origin?

30. ALGEBRA The population of the United States in July of 2007 was estimated to have surpassed 301,000,000. At the same time the world population was estimated to be over 6,602,000,000. What percent of the world population, to the nearest tenth, lived in the United States at this time?

F 3.1% | **H** 4.2%

G 3.5% | **J** 4.6%

32. SAT/ACT An 18-foot ladder is placed against the side of a house. The base of the ladder is positioned 8 feet from the house. How high up on the side of the house, to the nearest tenth of a foot, does the ladder reach?

A 10.0 ft | **D** 22.5 ft

B 16.1 ft | **E** 26.0 ft

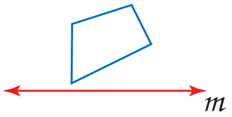
C 19.7 ft

Spiral Review

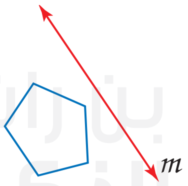
33. VOLCANOES A cloud of dense gas and dust from a volcano blows 64 kilometers west and then 48 kilometers north. Make a sketch to show the translation of the dust particles. Then find thedistance of the shortest path that would take the particles to the same position. (Lesson 6-2)

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler. (Lesson 6-1)

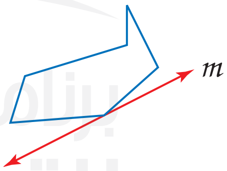
34.



35.



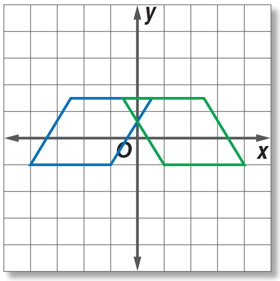
36.



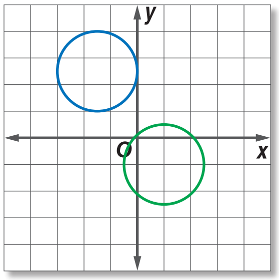
Skills Review

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

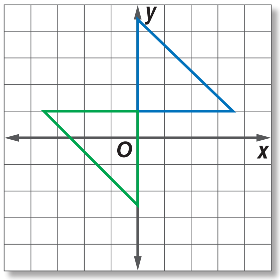
37.



38.



39.



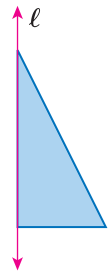
EXTEND 6-3

Geometry Lab
Solids of Revolution

A **solid of revolution** is a three-dimensional figure obtained by rotating a plane figure or curve about a line.

Activity 1

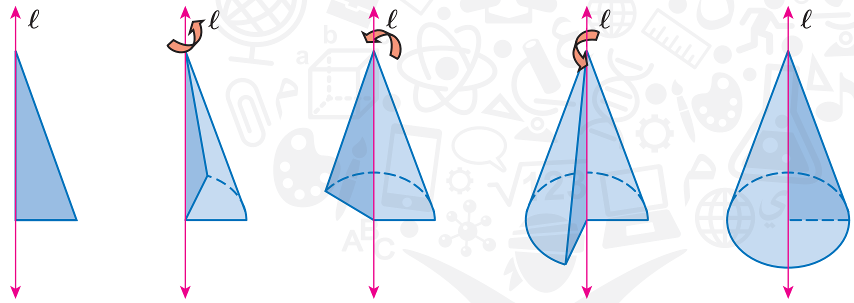
Identify and sketch the solid formed by rotating the right triangle shown about line ℓ.



Step 1 Copy the triangle onto card stock or heavy construction paper and cut it out.

Step 2 Use tape to attach the triangle to a dowel rod or straw.

Step 3 Rotate the end of the straw quickly between your hands and observe the result.

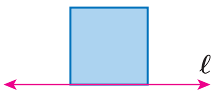


The blurred image you observe is that of a cone.

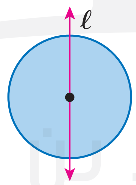
Model and Analyze

Identify and sketch the solid formed by rotating the two-dimensional shape about line ℓ.

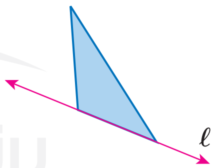
1.



2.



3.



4. Sketch and identify the solid formed by rotating the rectangle shown about the line containing

a. side $\overline{AB}$.

b. side $\overline{AD}$.

c. the midpoints of sides $\overline{AB}$ and $\overline{AD}$.

5. DESIGN Draw a two-dimensional figure that could be rotated to form the vase shown, including the line in which it should be rotated.

6. REASONING *True* or *false:* All solids can be formed by rotating a two-dimensional figure. Explain your reasoning.

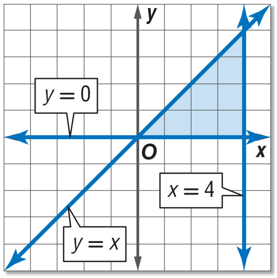
Solids of Revolution *Continued*

In calculus, you will be asked to find the volumes of solids generated by revolving a region on the coordinate plane about the *x*- or *y*-axis. An important first step in solving these problems is visualizing the solids formed.

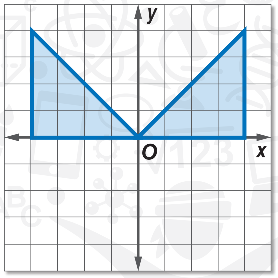
Activity 2

Sketch the solid that results when the region enclosed by $y = x$, $x = 4$, and $y = 0$ is revolved about the y-axis.

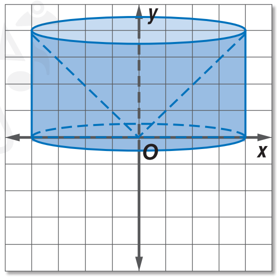
Step 1 Graph each equation to find the region to be rotated.



Step 2 Reflect the region about the y-axis.



Step 3 Connect the vertices of the right triangles using curved lines.



The solid is a cylinder with a cone cut out of its center.

Model and Analyze

Sketch the solid that results when the region enclosed by the given equations is revolved about the y-axis.

7. $y = -x + 4$
$x = 0$
$y = 0$

8. $y = x^2$
$y = 4$

9. $y = x^2$
$y = 2x$

Sketch the solid that results when the region enclosed by the given equations is revolved about the x-axis.

10. $y = -x + 4$
$x = 0$
$y = 0$

11. $y = x^2$
$y = 0$
$x = 2$

12. $y = x^2$
$y = 2x$

13. **OPEN ENDED** Graph a region in the first quadrant of the coordinate plane.

a. Sketch the graph of the region when revolved about the y-axis.

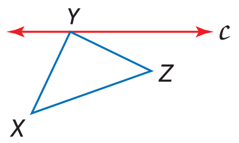
b. Sketch the graph of the region when revolved about the x-axis.

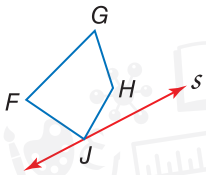
14. **CHALLENGE** Find equations that enclose a region such that when rotated about the x-axis, a solid is produced with a volume of 18π cubic units.

Mid-Chapter Quiz

Lessons 6-1 through 6-3

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler. (Lesson 6-1)

1. 

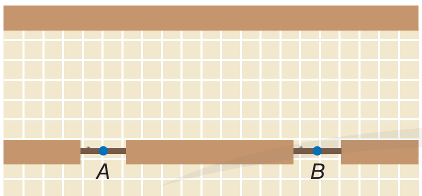
2. 

Graph each figure and its image after the specified reflection. (Lesson 6-1)

3. △*FGH* has vertices *F*(−4, 3), *G*(−2, 0), and *H*(−1, 4); in the *y*-axis

4. rhombus *QRST* has vertices *Q*(2, 1), *R*(4, 3), *S*(6, 1), and *T*(4, −1); in the *x*-axis

5. **CLUBS** The drama club is selling candy during the intermission of a school play. Locate point *P* along the wall to represent the candy table so that people coming from either door *A* or door *B* would walk the same distance to the table. (Lesson 6-1)

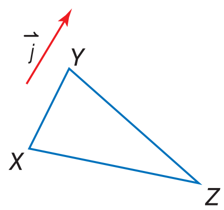


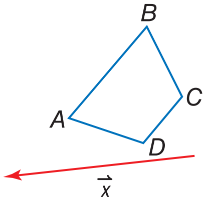
Graph each figure and its image after the specified translation. (Lesson 6-2)

6. △*ABC* with vertices *A*(0, 0), *B*(2, 1), *C*(1, −3); ⟨3, −1⟩

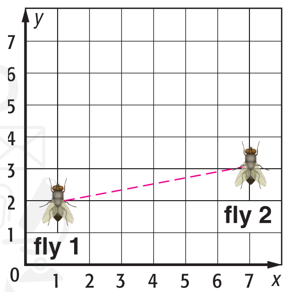
7. rectangle *JKLM* has vertices *J*(−4, 2), *K*(−4, −2), *L*(−1, −2), and *M*(−1, 2); ⟨5, −3⟩

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector. (Lesson 6-2)

8. 

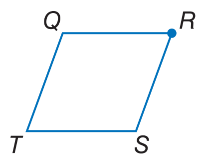
9. 

10. **COMICS** Faris is making a comic. He uses graph paper to make sure the dimensions of his drawings are accurate. If he draws a coordinate plane with two flies as shown below, what vector represents the movement from fly 1 to fly 2? (Lesson 6-2)

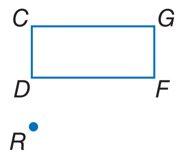


Copy each polygon and point *R*. Then use a protractor and ruler to draw the specified rotation of each figure about point *R*. (Lesson 6-3)

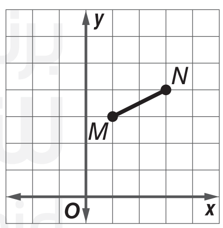
11. 45°



12. 60°



13. **MULTIPLE CHOICE** What is the image of point *M* after a rotation of 90° about the origin? (Lesson 6-3)



A (−3, 1) **C** (−1, −3)

B (−3, −1) **D** (3, 1)

Graph each figure and its image after the specified rotation. (Lesson 6-3)

14. △*RST* has vertices *R*(−3, 0), *S*(−1, −4), and *T*(0, −1); 90°

15. square *JKLM* has vertices *J*(−1, 2), *K*(−1, −2), *L*(3, −2), and *M*(3, 2); 180°

EXPLORE

6-4 Geometry Software Lab
Compositions of Transformations

In this lab, you will use Geometer's Sketchpad to explore the effects of performing multiple transformations on a figure.

Activity

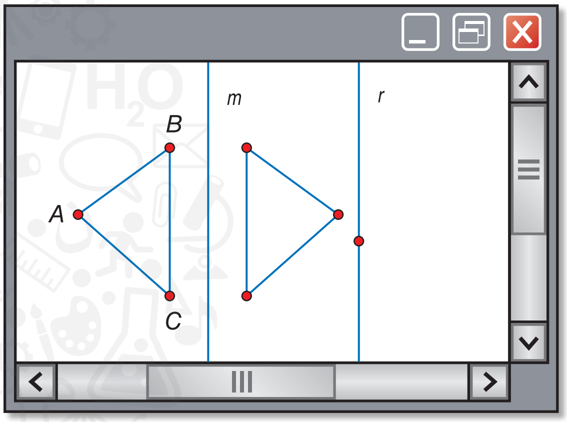
Reflect a figure in two vertical lines.

Step 1 Use the line segment tool to construct a triangle with one vertex pointing to the left so that you can easily see changes as you perform transformations. Label the triangle *ABC*.

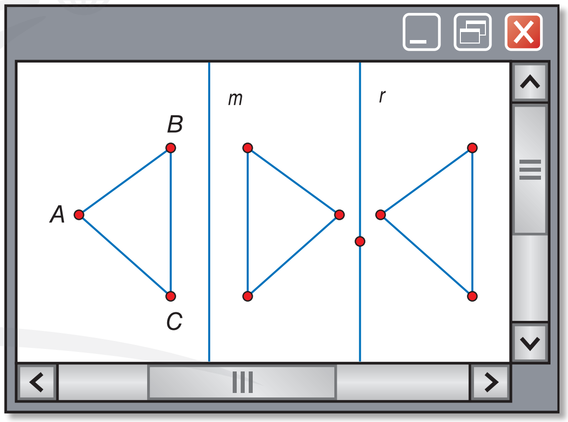
Step 2 Insert and label a line *m* to the right of $\triangle ABC$. Insert a point so that the distance from the point to line *m* is greater than the width of $\triangle ABC$. Draw the line parallel to line *m* through the point and label the new line *r*.

Step 3 Select line *m* and choose **Mark Mirror** from the **Transform** menu. Select all sides and vertices of $\triangle ABC$ and choose **Reflect** from the **Transform** menu.

Step 4 Repeat the process you used in Step 3 to reflect the new image in line *r*.



Steps 1–3



Step 4

Analyze the Results

1. How are the original figure and the final figure related?
2. What single transformation could be used to produce the final figure?
3. If you move line *m*, what happens? if you move line *r*?
4. **MAKE A CONJECTURE** If you reflected the figure in a third line, what single transformation do you think could be used to produce the final figure? Explain your reasoning.
5. Repeat the activity for a pair of perpendicular lines. What single transformation could be used to produce the same final figure?
6. **MAKE A CONJECTURE** If you reflected the figure from Exercise 5 in a third line perpendicular to the second line, what single transformation do you think could be used to produce the final figure? Explain your reasoning.

LESSON 6-4

Compositions of Transformations

Then

- You drew reflections, translations, and rotations.

Now

1. Draw glide reflections and other compositions of isometries in the coordinate plane.
2. Draw compositions of reflections in parallel and intersecting lines.

Why?

- The pattern of footprints left in the sand after a person walks along the edge of a beach illustrates the composition of two different transformations—translations and reflections.

NewVocabulary
composition of transformations
glide reflection

Mathematical Practices
Make sense of problems and persevere in solving them.
Model with mathematics.

1 Glide Reflections

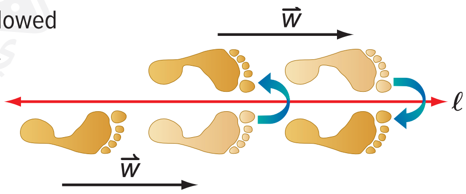
When a transformation is applied to a figure and then another transformation is applied to its image, the result is called a **composition of transformations**. A glide reflection is one type of composition of transformations.

KeyConcept Glide Reflection

A **glide reflection** is the composition of a translation followed by a reflection in a line parallel to the translation vector.

Example

The glide reflection shown is the composition of a translation along $\vec{w}$ followed by a reflection in line ℓ.



Example 1 Graph a Glide Reflection

Triangle *JKL* has vertices *J*(6, −1), *K*(10, −2), and *L*(5, −3). Graph △*JKL* and its image after a translation along ⟨0, 4⟩ and a reflection in the *y*-axis.

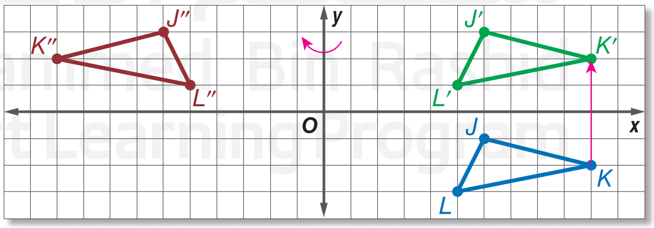
Step 1 translation along ⟨0, 4⟩

(x, y)	→	$(x, y + 4)$
$J(6, -1)$	→	$J'(6, 3)$
$K(10, -2)$	→	$K'(10, 2)$
$L(5, -3)$	→	$L'(5, 1)$

Step 2 reflection in the *y*-axis

(x, y)	→	$(-x, y)$
$J'(6, 3)$	→	$J''(-6, 3)$
$K'(10, 2)$	→	$K''(-10, 2)$
$L'(5, 1)$	→	$L''(-5, 1)$

Step 3 Graph △*JKL* and its image △*J″K″L″*.



GuidedPractice

Triangle *PQR* has vertices *P*(1, 1), *Q*(2, 5), and *R*(4, 2). Graph △*PQR* and its image after the indicated glide reflection.

1A. Translation: along ⟨−2, 0⟩
Reflection: in *x*-axis

1B. Translation: along ⟨−3, −3⟩
Reflection: in $y = x$

In Example 1, $\triangle JKL \cong \triangle J'K'L'$ and $\triangle J'K'L' \cong \triangle J''K''L''$. By the Transitive Property of Congruence, $\triangle JKL \cong \triangle J''K''L''$. This suggests the following theorem.

Theorem 6.1 Composition of Isometries

The composition of two (or more) isometries is an isometry.

You will prove one case of Theorem 6.1 in Exercise 30.

StudyTip

Rigid Motions Glide reflections, reflections, translations, and rotations are the only four *rigid motions* or isometries in a plane.

So, the composition of two or more isometries—reflections, translations, or rotations—results in an image that is congruent to its preimage.

Example 2 Graph Other Compositions of Isometries

The endpoints of $\overline{CD}$ are $C(-7, 1)$ and $D(-3, 2)$. Graph $\overline{CD}$ and its image after a reflection in the x-axis and a rotation 90° about the origin.

Step 1 reflection in the x-axis

$(x, y) \rightarrow (x, -y)$

$C(-7, 1) \rightarrow C'(-7, -1)$

$D(-3, 2) \rightarrow D'(-3, -2)$

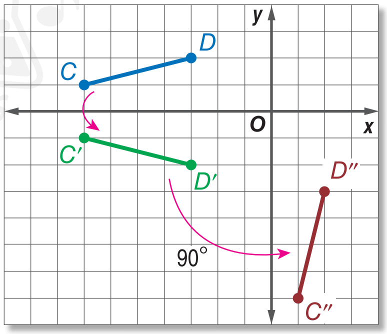
Step 2 rotation 90° about origin

$(x, y) \rightarrow (-y, x)$

$C'(-7, -1) \rightarrow C''(1, -7)$

$D'(-3, -2) \rightarrow D''(2, -3)$

Step 3 Graph $\overline{CD}$ and its image $\overline{C''D''}$.



ReadingMath

Double Primes Double primes are used to indicate that a vertex is the image of a second transformation.

GuidedPractice

Triangle ABC has vertices $A(-6, -2)$, $B(-5, -5)$, and $C(-2, -1)$. Graph $\triangle ABC$ and its image after the composition of transformations in the order listed.

2A. Translation: along $\langle 3, -1 \rangle$
Reflection: in y-axis

2B. Rotation: 180° about origin
Translation: along $\langle -2, 4 \rangle$

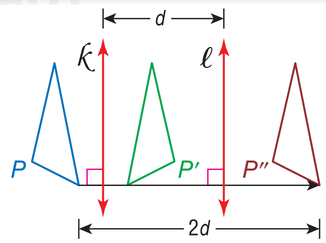
2 Compositions of Two Reflections

The composition of two reflections in parallel lines is the same as a translation.

Theorem 6.2 Reflections in Parallel Lines

The composition of two reflections in parallel lines can be described by a translation vector that is

- perpendicular to the two lines, and
- twice the distance between the two lines.



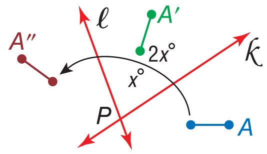
You will prove Theorem 6.2 in Exercise 28.

The composition of two reflections in intersecting lines is the same as a rotation.

Theorem 6.3 Reflections in Intersecting Lines

The composition of two reflections in intersecting lines can be described by a rotation

- about the point where the lines intersect and
- through an angle that is twice the measure of the acute or right angle formed by the lines.



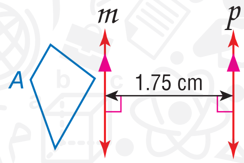
You will prove Theorem 6.3 in Exercise 29.

> **WatchOut!**
> **Order of Composition**
> Be sure to compose two transformations according to the order in which they are given.

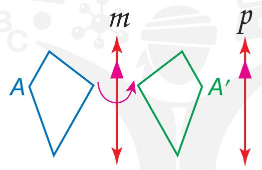
Example 3 Reflect a Figure in Two Lines

Copy and reflect figure *A* in line *m* and then line *p*. Then describe a single transformation that maps *A* onto *A*″.

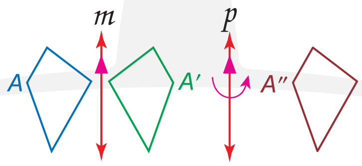
a.



Step 1 Reflect *A* in line *m*.

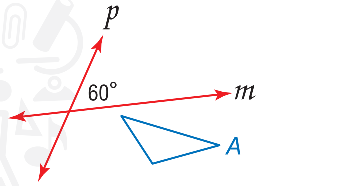


Step 2 Reflect *A*′ in line *p*.

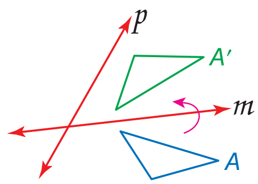


By Theorem 6.2, the composition of two reflections in parallel vertical lines *m* and *p* is equivalent to a horizontal translation right 2 • 1.75 or 3.5 centimeters.

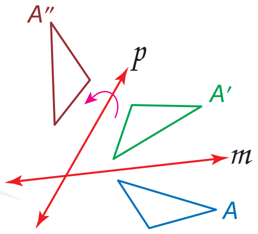
b.



Step 1



Step 2

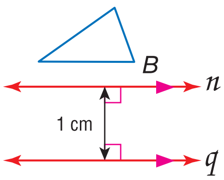


By Theorem 6.3, the composition of two reflections in intersecting lines *m* and *p* is equivalent to a 2 • 60° or 120° counterclockwise rotation about the point where lines *m* and *p* intersect.

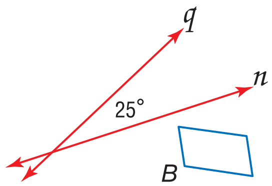
GuidedPractice

Copy and reflect figure *B* in line *n* and then line *q*. Then describe a single transformation that maps *B* onto *B*″.

3A.



3B.



Many patterns in the real world are created using compositions of transformations.

Real-WorldLink

In carpets, border patterns result when any of several basic transformations are repeated in one direction. There are seven possible combinations: translations, horizontal reflections, vertical reflections, vertical followed by horizontal reflections, glide reflections, rotations, and reflections followed by glide reflections.

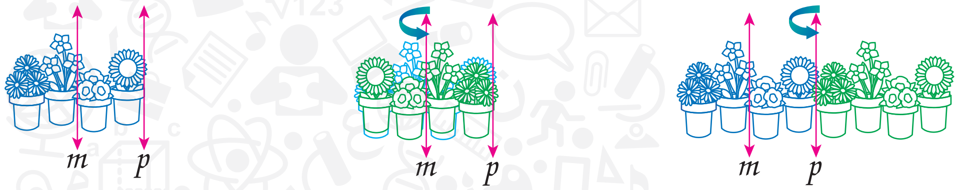
Source: The Textile Museum

Real-World Example 4 Describe Transformations

STATIONERY BORDERS Describe the transformations that are combined to create each stationery border shown.

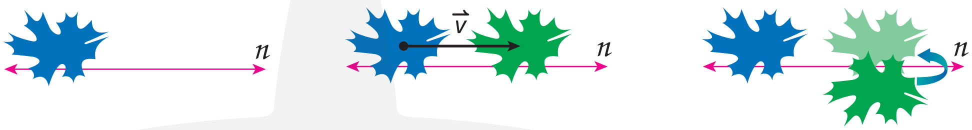
a.

The pattern is created by successive translations of the first four potted plants. So this pattern can be created by combining two reflections in lines m and p as shown. Notice that line m goes through the center of the preimage.



b.

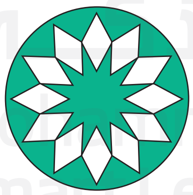
The pattern is created by glide reflection. So this pattern can be created by combining a translation along translation vector $\vec{v}$ followed by a reflection over horizontal line n as shown.



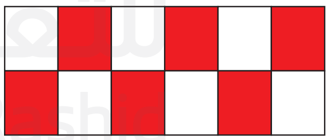
GuidedPractice

4. CARPET PATTERNS Describe the transformations that are combined to create each carpet pattern shown.

A.



B.



ConceptSummary Compositions of Translations

Glide Reflection	Translation	Rotation
the composition of a reflection and a translation	the composition of two reflections in parallel lines	the composition of two reflections in intersecting lines

Check Your Understanding

Example 1 **Triangle *CDE* has vertices $C(-5, -1)$, $D(-2, -5)$, and $E(-1, -1)$. Graph $\triangle CDE$ and its image after the indicated glide reflection.**

1. Translation: along $\langle 4, 0 \rangle$
Reflection: in *x*-axis

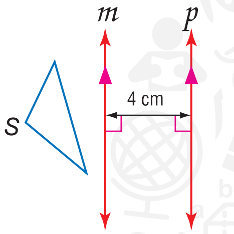
2. Translation: along $\langle 0, 6 \rangle$
Reflection: in *y*-axis

Example 2

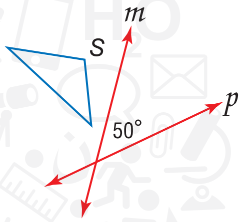
3. The endpoints of $\overline{JK}$ are $J(2, 5)$ and $K(6, 5)$. Graph $\overline{JK}$ and its image after a reflection in the *x*-axis and a rotation 90° about the origin.

Example 3 **Copy and reflect figure *S* in line *m* and then line *p*. Then describe a single transformation that maps *S* onto *S″*.**

4.

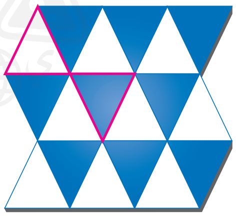


5.



Example 4

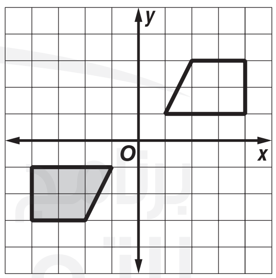
6. **TILE PATTERNS** Ismail is creating a pattern for the top of a table with tiles in the shape of isosceles triangles. Describe the transformation combination that was used to create the pattern.



Practice and Problem Solving

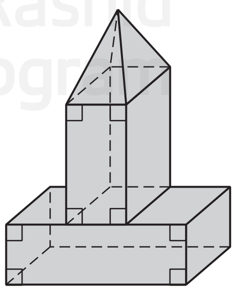
Example 2

7. Which two transformations could have been used to change the shaded figure to the figure that is not shaded?

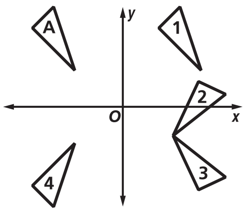


Example 2

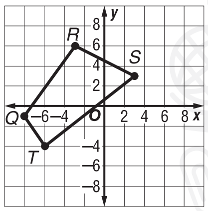
8. This sketch represents a building located across from the Pioneer Hotel in Seattle. Which figures are represented in the sketch?



9. If Figure A is transformed by a rotation and then a reflection, which figure could be the final image?



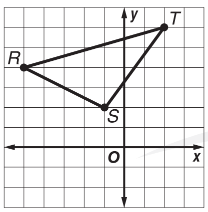
10. If quadrilateral $QRST$ is reflected over the x-axis and then reflected over the y-axis, in which quadrants will the final image be located?



A I, III, and IV **C** I and II only

B II, III, and IV **D** II and IV only

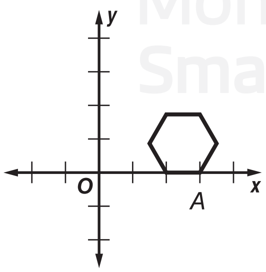
11. Triangle RST has coordinates $R(-5, 4)$,$S(-1, 2)$ and $T(2, 6)$. What will be the new coordinates of point T if the triangleis translated 5 units down and reflected over the y-axis?



A $(-2, 1)$ **C** $(2, -1)$

B $(-1, 2)$ **D** $(2, 1)$

12. A regular hexagon lies in the coordinate plane with point A at $(3, 0)$.

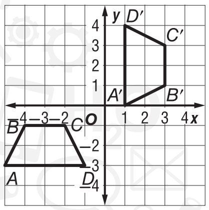


What are the coordinates of vertex A after a reflection across the y-axis and a translation up 2 units?

13. Triangle JKL is dilated by scale factor 1.5, reflected across the y-axis, and translated 2 units left. What will be the new coordinates of vertex J after the three transformations?

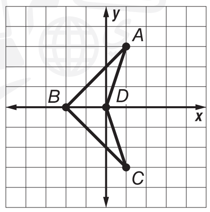
14. Trapezoid $ABCD$ has vertices as shown in the coordinate plane below.

$ABCD$ is transformed to create a congruent image. What transformations occurred to create $A'B'C'D'$?



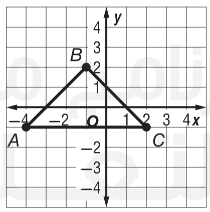
15. Quadrilateral $ABCD$ is rotated and translated to create an image with vertices $A'(-3, 3)$ and $B'(0, 0)$.

What are the coordinates of D'?



16. Triangle ABC is dilated about the origin with a scale factor of 2 and then translated so that the midpoint of $\overline{A'B'}$ has the same coordinates as the midpoint of $\overline{AB}$.

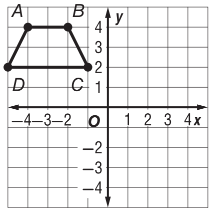
What are the coordinates of C'?



17. A triangle is formed by the points $P(2, -2)$, $Q(-2, -4)$, and $R(6, -2)$. The triangle is dilated by a scale factor of $\frac{1}{2}$ and then translated four units right and four units up.

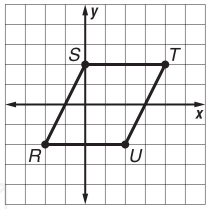
What are the coordinates of $\triangle P'Q'R'$?

18. Trapezoid $ABCD$ has vertices as shown in the coordinate plane below. If $ABCD$ is reflected across the y-axis and then rotated 90° counterclockwise about the origin, what would be the coordinates of vertex C'?

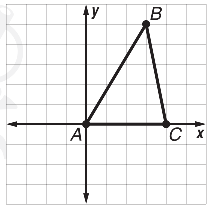


19. Triangle STU has vertices $S(-5, -2)$, $T(-1, 4)$, and $U(6, 3)$. If this triangle is translated 3 units right and 5 units down and then reflected over the x-axis, what will be the coordinates of T', the image of T?

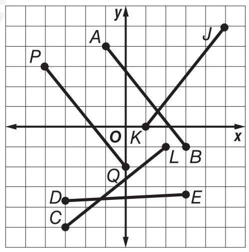
20. If parallelogram $RSTU$ is translated 5 units to the left and 3 units up and then reflected over the y-axis, what will be the coordinates of T', the image of T, under these transformations?



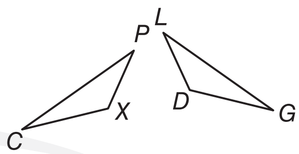
21. If $\triangle ABC$ as shown is rotated 180° counterclockwise about the origin, its image will be $\triangle A'B'C'$. Which of the following transformations or combinations of transformations of $\triangle ABC$ will produce a different image from $\triangle A'B'C'$?



22. Which line segment represents the image of $\overline{PQ}$ under a glide reflection?



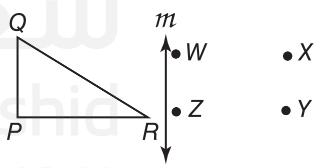
23. Which type of transformation can you use to show that $\triangle CXP \cong \triangle GDL$?



24. The vertices of a triangle lie in the second quadrant. In which quadrant will the image of the triangle under the glide reflection $T_{0,\,4} \rightarrow R_{x\,=\,0}$ lie?

25. Point $P(x, y)$ is reflected across the y-axis and then its image is translated vertically a units, where $a > 0$. Which of the following gives the coordinates of the final image P'?

26. Which of the following sets of points could be the vertices of the image of $\triangle PQR$ under a glide reflection with line m as the line of reflection?



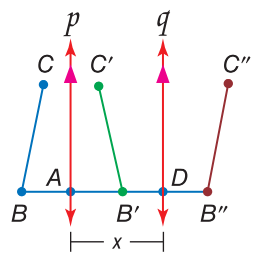
27. A triangle has vertices at $(-1, 3)$, $(2, 5)$, and $(0, 1)$. If the triangle is translated left 4 units, then dilated by a factor of 3, what are the coordinates of the image of the triangle?

28. **PROOF** Write a two-column proof of Theorem 6.2.

Given: A reflection in line p maps $\overline{BC}$ to $\overline{B'C'}$.
A reflection in line q maps $\overline{B'C'}$ to $\overline{B''C''}$.
$p \parallel q$, $AD = x$

Prove: **a.** $\overline{BB''} \perp p$, $\overline{BB''} \perp q$

b. $BB'' = 2x$

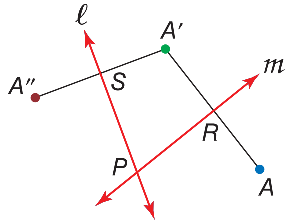


29. **PROOF** Write a paragraph proof of Theorem 6.3.

Given: Lines ℓ and m intersect at point P. A is any point not on ℓ or m.

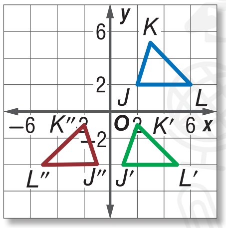
Prove: **a.** If you reflect point A in m, and then reflect its image A' in ℓ, A'' is the image of A after a rotation about point P.

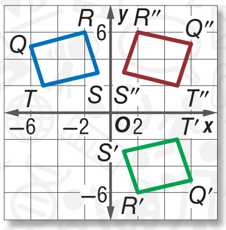
b. $m\angle APA'' = 2(m\angle SPR)$



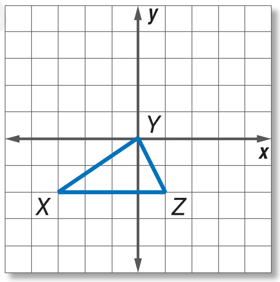
H.O.T. Problems Use Higher-Order Thinking Skills

Describe the transformations that combined to map each figure.

30. 

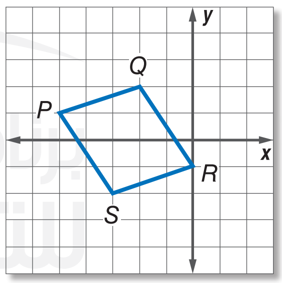
31. 

32. **ERROR ANALYSIS** Asma and Amani are translating $\triangle XYZ$ along $\langle 2, 2\rangle$ and reflecting it in the line $y = 2$. Asma says that the transformation is a glide reflection. Amani disagrees and says that the transformation is a composition of transformations. Is either of them correct? Explain your reasoning.



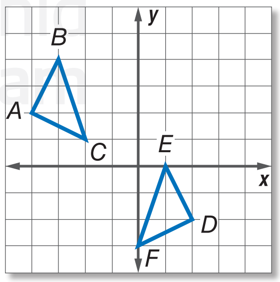
33. **WRITING IN MATH** Do any points remain invariant under glide reflections? under compositions of transformations? Explain.

34. **CHALLENGE** If $PQRS$ is translated along $\langle 3, -2\rangle$, reflected in $y = -1$, and rotated 90° about the origin, what are the coordinates of $P'''Q'''R'''S'''$?



35. **ARGUMENTS** If an image is to be reflected in the line $y = x$ and the x-axis, does the order of the reflections affect the final image? Explain.

36. **OPEN ENDED** Write a glide reflection or composition of transformations that can be used to transform $\triangle ABC$ to $\triangle DEF$.

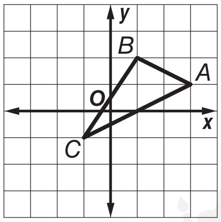


37. **REASONING** When two rotations are performed on a single image, does the order of the rotations *sometimes*, *always*, or *never* affect the location of the final image? Explain.

38. **WRITING IN MATH** Compare and contrast glide reflections and compositions of transformations.

Standardized Test Practice

39. $\triangle ABC$ is translated along the vector $\langle -2, 3 \rangle$ and then reflected in the x-axis. What are the coordinates of A' after the transformation?



A $(1, -4)$
B $(1, 4)$
C $(-1, 4)$
D $(-1, -4)$

40. SHORT RESPONSE What are the coordinates of D'' if $\overline{CD}$ with vertices $C(2, 4)$ and $D(8, 7)$ is translated along $\langle -6, 2 \rangle$ and then reflected over the y-axis?

41. ALGEBRA Write $\frac{18x^2 - 2}{3x^2 - 5x - 2}$ in simplest terms.

F $\frac{18}{3x + 1}$ **H** $\frac{2(3x - 1)}{x - 2}$
G $\frac{2(3x + 1)}{x - 2}$ **J** $2(3x - 1)$

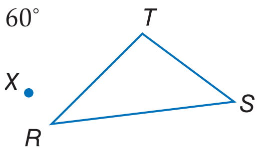
42. SAT/ACT If $f(x) = x^3 - x^2 - x$, what is the value of $f(-3)$?

A -39 **D** -15
B -33 **E** -12
C -21

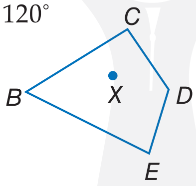
Spiral Review

Copy each polygon and point *X*. Then use a protractor and ruler to draw the specified rotation of each figure about point *X*. (Lesson 6-3)

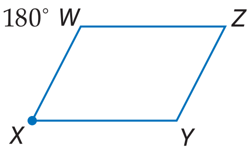
43. 60°



44. 120°



45. 180°



Graph each figure and its image along the given vector. (Lesson 6-2)

46. $\triangle FGH$ with vertices $F(1, -4)$, $G(3, -1)$, and $H(7, -1)$; $\langle 2, 6 \rangle$

47. quadrilateral $ABCD$ with vertices $A(-2, 7)$, $B(-1, 4)$, $C(2, 3)$, and $D(2, 7)$; $\langle -3, -5 \rangle$

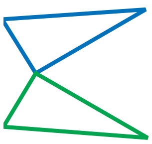
48. SAILS One side of a triangular sail of a sailboat is 7.5 meters long. The angle opposite this side is 40°. Another angle formed by the sail measures 55°. What is the perimeter of the sail to the nearest tenth?

49. GARDENING A triangular flower bed has sides measuring 1.35 meters, 1.8 meters, and 2.25 meters. Find the measure of the smallest angle.

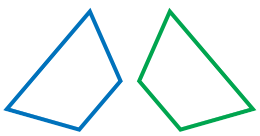
Skills Review

Each figure shows a preimage and its reflected image in some line. Copy each figure and draw the line of reflection.

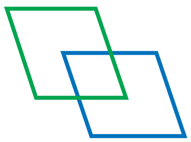
50.



51.



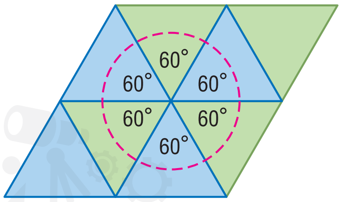
52.



EXTEND 6-4

Geometry Lab
Tessellations

A **tessellation** is a pattern of one or more figures that covers a plane so that there are no overlapping or empty spaces. The sum of the angles around the vertex of a tessellation is 360°.



A **regular tessellation** is formed by only one type of regular polygon. A regular polygon will tessellate if it has an interior angle measure that is a factor of 360. A **semi-regular tessellation** is formed by two or more regular polygons.

Activity 1 Regular Tessellation

Determine whether each regular polygon will tessellate in the plane. Explain.

a. hexagon

Let x represent the measure of an interior angle of a regular hexagon.

$x = \frac{180(n-2)}{n}$ Interior Angle Formula

$= \frac{180(6-2)}{6}$ $n = 6$

$= 120$ Simplify.

Since 120 is a factor of 360, a regular hexagon will tessellate in the plane.

b. decagon

Let x represent the measure of an interior angle of a regular decagon.

$x = \frac{180(n-2)}{n}$ Interior Angle Formula

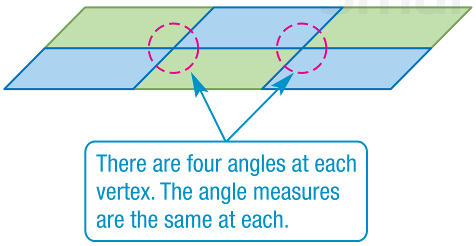
$= \frac{180(10-2)}{10}$ $n = 10$

$= 144$ Simplify.

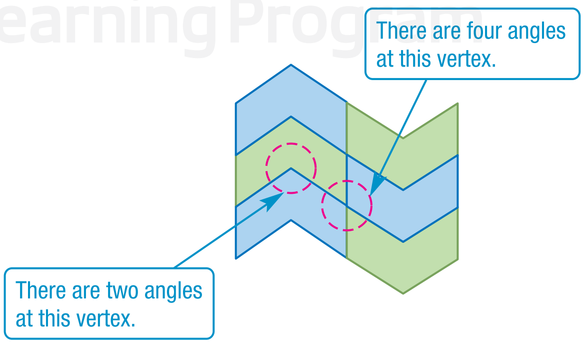
Since 144 is not a factor of 360, a regular decagon will not tessellate in the plane.

A tessellation is **uniform** if it contains the same arrangement of shapes and angles at each vertex.

Uniform



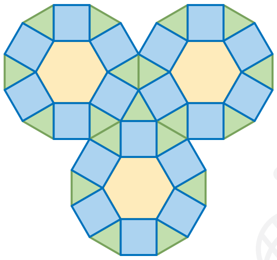
Not Uniform



Activity 2 Classify Tessellations

Determine whether each pattern is a tessellation. If so, describe it as *regular*, *semi-regular*, or *neither* and *uniform* or *not uniform*.

a.

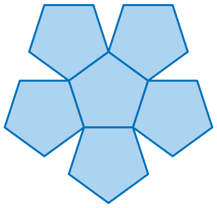


There is no unfilled space, and none of the figures overlap, so the pattern is a **tessellation**.

The tessellation consists of regular hexagons, squares and equilateral triangles, so it is **semi-regular**.

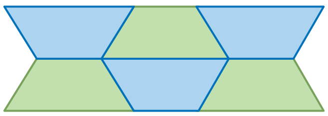
There are four angles around some of the vertices and five around others, so it is **not uniform**.

b.



There is unfilled space, so the pattern is a **not a tessellation**.

c.



There is no unfilled space, and none of the figures overlap, so the pattern is a **tessellation**.

The tessellation consists of trapezoids, which are not regular polygons, so it is **neither** regular nor semi-regular.

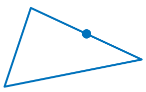
There are four angles around each of the vertices and the angle measures are the same at each vertex, so it is **uniform**.

You can use the properties of tessellations to design and create tessellations.

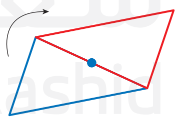
Activity 3 Draw a Tessellation

Draw a triangle and use it to create a tessellation.

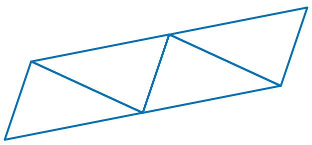
Step 1 Draw a triangle and find the midpoint of one side.



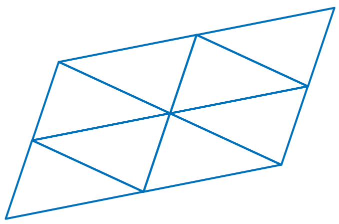
Step 2 Rotate the triangle 180° about the point.



Step 3 Translate the pair of triangles to make a row.



Step 4 Translate the row to make a tessellation.



Activity 4 Tessellations using Technology

Use Geometer's Sketchpad to create a tessellation.

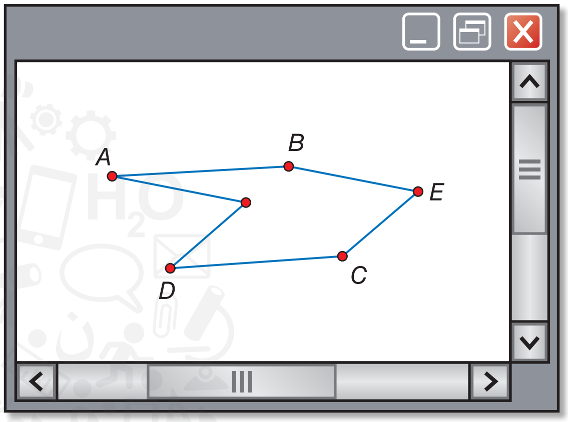
Step 1 Insert three points and construct a line through two of the points. Then construct the line parallel to the first line through the third point using the **Parallel Line** option from the **Construct** menu. Complete the parallelogram and label the points A, B, C, and D. Hide the lines.

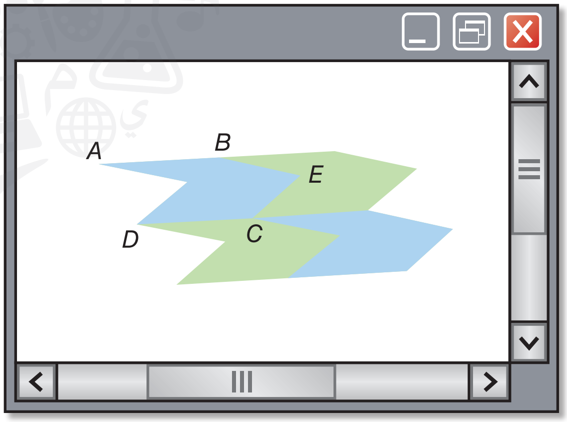
Step 2 Insert another point E on the exterior of the parallelogram. Draw the segments between A and B, B and E, E and C, and C and D.

Step 3 Highlight B and then A. From the **Transform** menu, choose **Mark Vector**. Select the $\overline{BE}$, $\overline{EC}$, and point E. From the **Transform** menu, choose **Translate**.

Step 4 Starting with A, select all of the vertices around the perimeter of the polygon. Choose **Hexagon Interior** from the **Construct** menu.

Step 5 Choose point A and then point B and mark the vector as you did in Step 3. Select the interior of the polygon and choose **Translate** from the **Transform** menu. Continue the tessellation by marking vectors and translating the polygon. You can choose **Color** from the **Display** menu to create a color pattern.



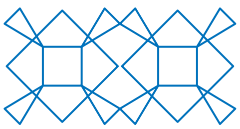
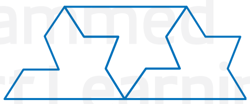
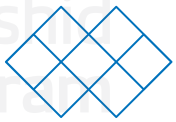


Exercises

Determine whether each regular polygon will tessellate in the plane. Write *yes* or *no*. Explain.

1. triangle
2. pentagon
3. 16-gon

Determine whether each pattern is a tessellation. Write *yes* or *no*. If so, describe it as *regular*, *semi-regular*, or *neither* and *uniform* or *not uniform*.

4. 
5. 
6. 

Draw a tessellation using the following shape(s).

7. octagon and square
8. hexagon and triangle
9. right triangle
10. trapezoid and a parallelogram
11. **WRITING IN MATH** Find examples of the use of tessellations in architecture, mosaics, and artwork. For each example, explain how tessellations were used.
12. **MAKE A CONJECTURE** Describe a figure that you think will tessellate in three-dimensional space. Explain your reasoning.

LESSON 6-5 Symmetry

Then

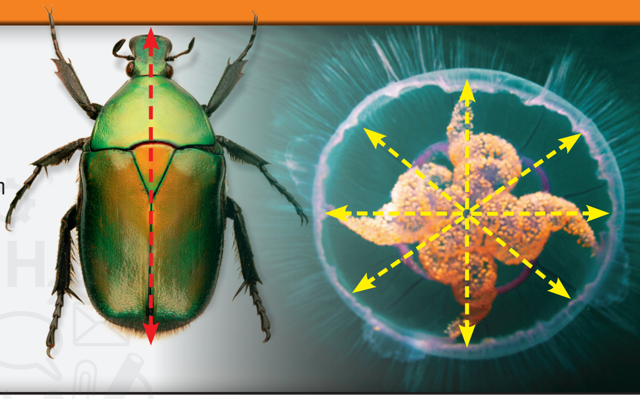
- You drew reflections and rotations of figures.

Now

1. Identify line and rotational symmetries in two-dimensional figures.
2. Identify plane and axis symmetries in three-dimensional figures.

Why?

- In the animal kingdom, the symmetry of an animal's body is often an indication of the animal's complexity. Animals displaying line symmetry, such as insects, are usually more complex life forms than those displaying rotational symmetry, like a jellyfish.



NewVocabulary

symmetry
line symmetry
line of symmetry
rotational symmetry
center of symmetry
order of symmetry
magnitude of symmetry
plane symmetry
axis symmetry

Mathematical Practices

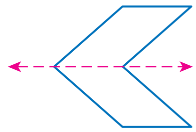
Model with mathematics.
Look for and express regularity in repeated reasoning.

1 Symmetry in Two-Dimensional Figures

A figure has **symmetry** if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself. One type of symmetry is line symmetry.

KeyConcept Line Symmetry

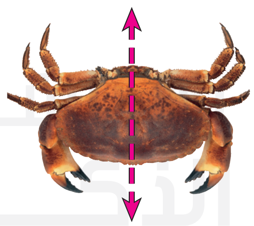
A figure in the plane has **line symmetry** (or *reflection symmetry*) if the figure can be mapped onto itself by a reflection in a line, called a **line of symmetry** (or *axis of symmetry*).



Real-World Example 1 Identify Line Symmetry

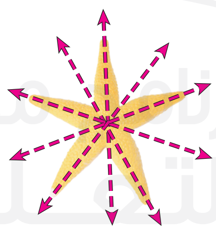
BEACHES **State whether the object appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.**

a.



Yes; the crab has one line of symmetry.

b.



Yes; the starfish has five lines of symmetry.

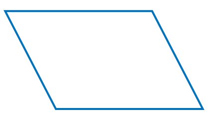
c.

No; there is no line in which the oyster shell can be reflected so that it maps onto itself.

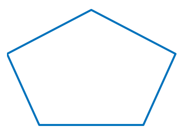
GuidedPractice

State whether the figure has line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

1A.



1B.



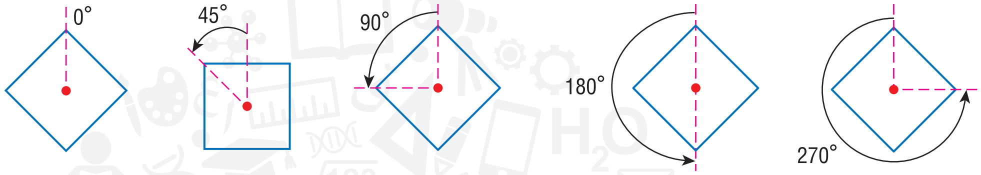
1C.

Another type of symmetry is rotational symmetry.

KeyConcept Rotational Symmetry

A figure in the plane has **rotational symmetry** (or *radial symmetry*) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure, called the **center of symmetry** (or *point of symmetry*).

Examples The figure below has rotational symmetry because a rotation of 90°, 180°, or 270° maps the figure onto itself.



The number of times a figure maps onto itself as it rotates from 0° to 360° is called the **order of symmetry**. The **magnitude of symmetry** (or angle of rotation) is the smallest angle through which a figure can be rotated so that it maps onto itself. The order and magnitude of a rotation are related by the following equation.

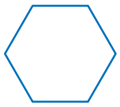
$$\text{magnitude} = 360° \div \text{order}$$

The figure above has rotational symmetry of order 4 and magnitude 90°.

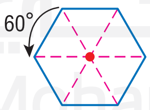
Example 2 Identify Rotational Symmetry

State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

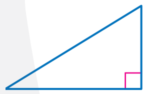
a.



Yes; the regular hexagon has order 6 rotational symmetry and magnitude 360° ÷ 6 or 60°. The center is the intersection of the diagonals.

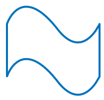


b.

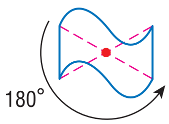


No; no rotation between 0° and 360° maps the right triangle onto itself.

c.



Yes; the figure has order 2 rotational symmetry and magnitude 360° ÷ 2 or 180°. The center is the intersection of the diagonals.



GuidedPractice

FLOWERS **State whether the flower appears to have rotational symmetry. Write *yes* or *no*. If so, copy the flower, locate the center of symmetry, and state the order and magnitude of symmetry.**

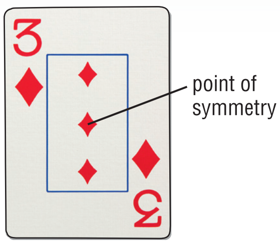
2A.

2B.

2C.

StudyTip

Point Symmetry A figure has *point symmetry* if the figure can be mapped onto itself by a rotation of 180°. The flag of the United Kingdom exhibits point symmetry. It looks the same right-side up as upside down.



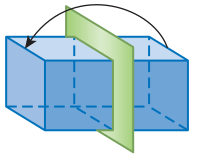
2 Symmetry in Three-Dimensional Figures

Three-dimensional figures can also have symmetry.

KeyConcept Three-Dimensional Symmetries

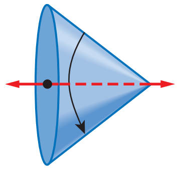
Plane Symmetry

A three-dimensional figure has **plane symmetry** if the figure can be mapped onto itself by a reflection in a plane.



Axis Symmetry

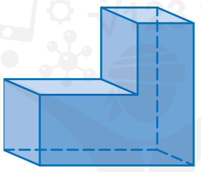
A three-dimensional figure has **axis symmetry** if the figure can be mapped onto itself by a rotation between 0° and 360° in a line.



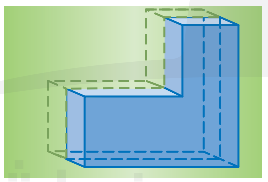
Example 3 Three-Dimensional Symmetry

State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

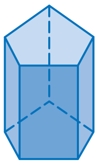
a. L-shaped prism



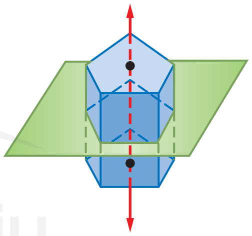
plane symmetry



b. regular pentagonal prism



both plane symmetry and axis symmetry



GuidedPractice

SPORTS **State whether each piece of sports equipment appears to have *plane* symmetry, *axis* symmetry, *both*, or *neither* (ignoring the equipment's stitching or markings).**

3A.

3B.

3C.

3D.

ReviewVocabulary

prism a polyhedron with two parallel congruent bases connected by parallelogram faces

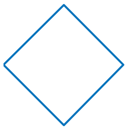
Real-WorldLink

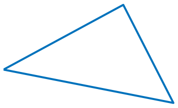
Aerodynamically designed to spin after it is thrown, an American football's shape is a prolate spheroid. This means that one axis of symmetry is longer than its other axes.

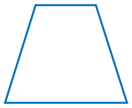
Source: *Complete Idiot's Guide to Football*

Check Your Understanding

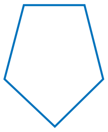
Example 1 **State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.**

1. 

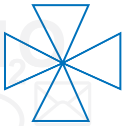
2. 

3. 

Example 2 **State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.**

4. 

5. 

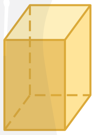
6. 

Examples 1–2 **7.** **U.S. CAPITOL** Completed in 1863, the dome is one of the most recent additions to the United States Capitol. It is supported by 36 iron ribs and has 108 windows, divided equally among three levels.

a. Excluding the spire of the dome, how many horizontal and vertical lines of symmetry does the dome appear to have?

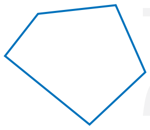
b. Does the dome have rotational symmetry? If so, state the order and magnitude of symmetry.

Example 3 **8.** State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

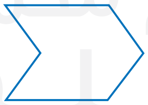


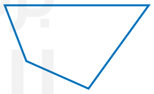
Practice and Problem Solving

Example 1 **REGULARITY** **State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.**

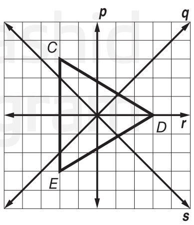
9. 

10. 

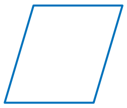
11. 

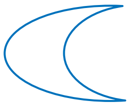
12. 

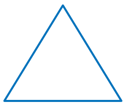
13. Triangle *CDE* is drawn in the plane. Which of the lines is a line of symmetry?

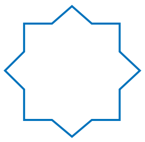


State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

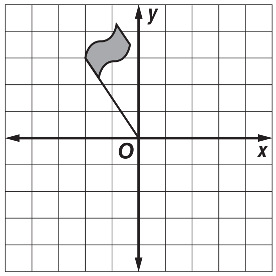
14. 

15. 

16. 

17. 

18. A flag is rotated 180° in the plane. Which statement is true?



A The figure is symmetric about the point (0, 0).

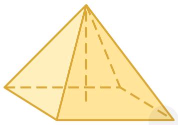
B The figure is symmetric about the y-axis.

C The figure is symmetric about the x-axis.

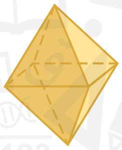
D The figure is symmetric about the point (−3, 2).

State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

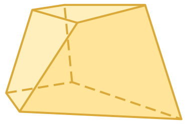
19.



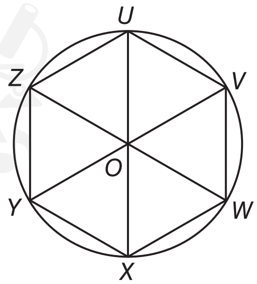
20.



21.

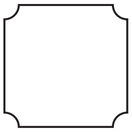


22. Hexagon $UVWXYZ$ is inscribed in a circle for a tile design. Which point shows the location of point U after a 120° clockwise rotation around the center O?

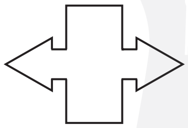


23. A graphic artist wants to design a logo with lines of symmetry. Which logo does not have exactly 4 lines of symmetry?

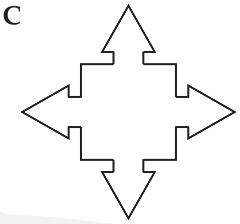
A



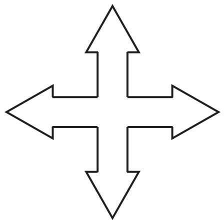
B



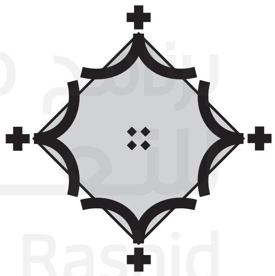
C



D



24. Amal is looking at sweater designs. Which statement describes the symmetry in the design?



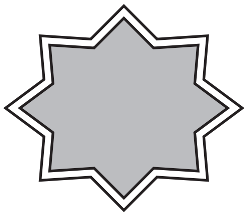
A The design has exactly 4 lines of symmetry.

B The design has exactly 3 lines of symmetry.

C The design has exactly 2 lines of symmetry

D The design has exactly 1 line of symmetry

25. Ahmed is designing a logo for his club. Which statement describes the symmetry in the design?



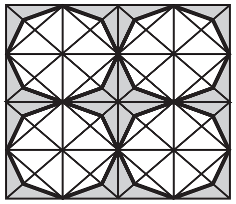
A The design has only 1 line of symmetry.

B The design has only 2 lines of symmetry.

C The design has only 3 lines of symmetry.

D The design has only 4 lines of symmetry.

26. An artist created a tessellation by drawing lines of symmetry in a square and then using them to draw a design. What is the sum of the measures of the interior angles of one convex octagon in the design?



COORDINATE GEOMETRY **Determine whether the figure with the given vertices has *line* symmetry and/or *rotational* symmetry.**

27. $A(-4, 0)$, $B(0, 4)$, $C(4, 0)$, $D(0, -4)$

28. $R(-3, 3)$, $S(-3, -3)$, $T(3, 3)$

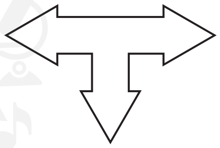
ALGEBRA **Graph the function and determine whether the graph has *line* and/or *rotational* symmetry. If so, state the order and magnitude of symmetry, and write the equations of any lines of symmetry.**

29. $y = x$

30. $y = x^2 + 1$

31. $y = -x^3$

32. Ismail is designing a logo for his club. Which statement describes the symmetry in the design?



A The design has only 1 line of symmetry.

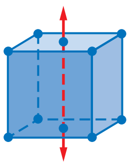
B The design has only 2 lines of symmetry.

C The design has only 3 lines of symmetry.

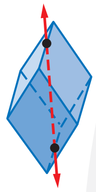
D The design has only 4 lines of symmetry.

CRYSTALLOGRAPHY **Determine whether the crystals below have *plane* symmetry and/or *axis* symmetry. If so, state the magnitude of symmetry.**

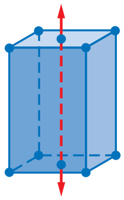
33.



34.



35.



H.O.T. Problems Use Higher-Order Thinking Skills

36. **CRITIQUE** Usama says that Figure A has only line symmetry, and Ayman says that Figure A has only rotational symmetry. Is either of them correct? Explain your reasoning.

Figure A

37. **CHALLENGE** A quadrilateral in the coordinate plane has exactly two lines of symmetry, $y = x - 1$ and $y = -x + 2$. Find possible vertices for the figure. Graph the figure and the lines of symmetry.

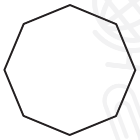
38. **REASONING** A regular polyhedron has axis symmetry of order 3, but does not have plane symmetry. What is the figure? Explain.

Standardized Test Practice

39. How many lines of symmetry can be drawn on the picture of the Canadian flag below?

A 0 **C** 2
B 1 **D** 4

40. GRIDDED RESPONSE What is the order of symmetry for the figure below?



41. ALGEBRA A computer company ships computers in wooden crates that each weigh 45 kilograms when empty. If each computer weighs no more than 13 kilograms, which inequality *best* describes the total weight in kilograms w of a crate of computers that contains c computers?

F $c \leq 13 + 45w$ **H** $w \leq 13c + 45$
G $c \geq 13 + 45w$ **J** $w \geq 13c + 45$

42. SAT/ACT What is the slope of the line determined by the linear equation $5x - 2y = 10$?

A -5 **D** $\frac{2}{5}$
B $-\frac{5}{2}$ **E** $\frac{5}{2}$
C $-\frac{2}{5}$

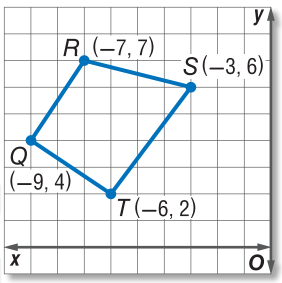
Spiral Review

Triangle *JKL* has vertices *J*(1, 5), *K*(3, 1), and *L*(5, 7). Graph △*JKL* and its image after the indicated transformation. (Lesson 6-4)

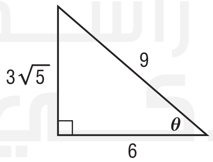
43. Translation: along $\langle -7, -1 \rangle$
Reflection: in x-axis

44. Translation: along $\langle 1, 2 \rangle$
Reflection: in y-axis

45. Quadrilateral *QRST* is shown at the right. What is the image of point *R* after a rotation 180° counterclockwise about the origin? (Lesson 6-3)



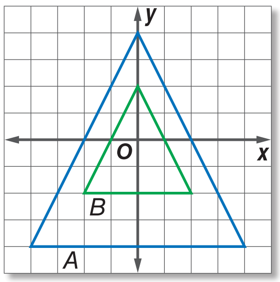
46. CONSTRUCTION A window has the dimensions shown below. Use the measures of the sides of the triangle to show that $\sin^2\theta + \cos^2\theta = 1$.



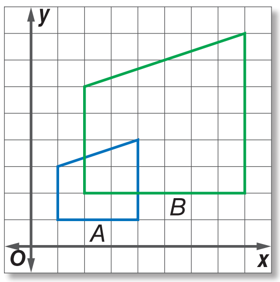
Skills Review

Determine whether the dilation from Figure A to Figure B is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

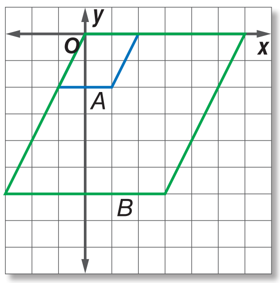
47.



48.



49.



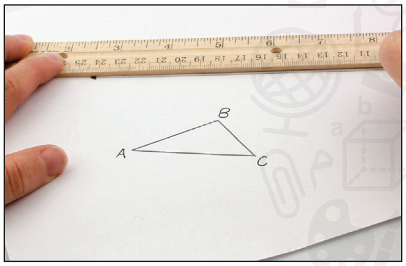
EXTEND 6-5 Geometry Lab
Exploring Constructions with a Reflective Device

A reflective device is a tool made of semitransparent plastic that reflects objects. It works best if you lay it on a flat service in a well-lit room. You can use a reflective device to transform geometric objects.

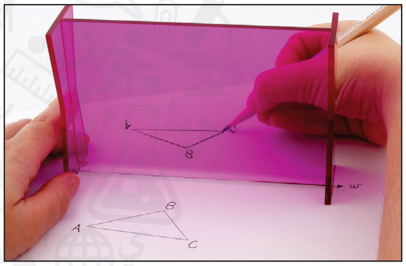
Activity 1 Reflect a Triangle

Use a reflective device to reflect $\triangle ABC$ in w. Label the reflection $\triangle A'B'C'$.

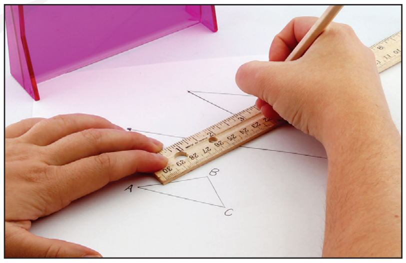
Step 1 Draw $\triangle ABC$ and the line of reflection w.



Step 2 With the reflective device on line w, draw points for the vertices of the reflection.



Step 3 Use a straightedge to connect the points to form $\triangle A'B'C'$.

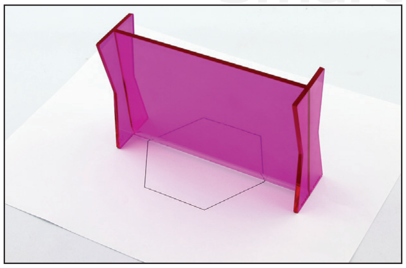


We have used a compass, straightedge, string, and paper folding to make geometric constructions. You can also use a reflective device for constructions.

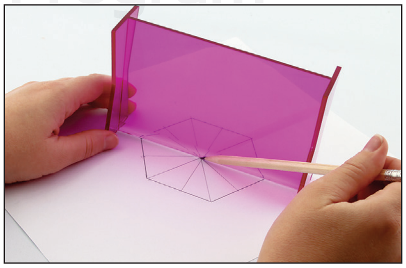
Activity 2 Construct Lines of Symmetry

Use a reflective device to construct the lines of symmetry for a regular hexagon.

Step 1 Draw a regular hexagon. Place the reflective device on the shape and move it until one half of the shape matches the reflection of the other half. Draw the line of symmetry.



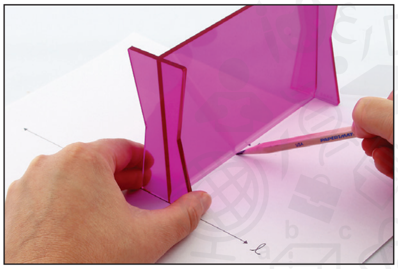
Step 2 Repeat Step 1 until you have found all the lines of symmetry.



Activity 3 Construct a Parallel line

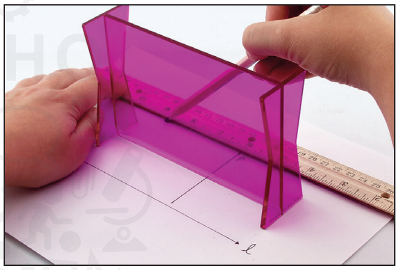
Use a reflective device to reflect line ℓ to line m that is parallel and passes through point P.

Step 1



Draw line ℓ and point P. Place a short side of the reflective device on line ℓ and the long side on point P. Draw a line. This line is perpendicular to ℓ through P.

Step 2



Place the reflective device so that the perpendicular line coincides with itself and the reflection of line ℓ passes through point P. Use a straightedge to draw the parallel line m through P.

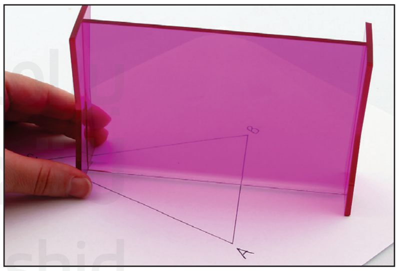
In an earlier Explore, we constructed perpendicular bisectors with paper folding. You can also use a reflective device to construct perpendicular bisectors of a triangle.

Activity 4 Construct Perpendicular Bisectors

Use a reflective device to find the circumcenter of $\triangle ABC$.

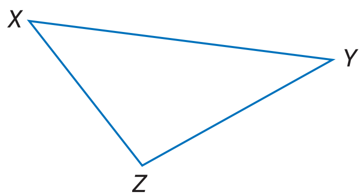
Step 1 Draw $\triangle ABC$. Place the reflective device between A and B and adjust it until A and B coincide. Draw the line of symmetry.

Step 2 Repeat Step 1 for sides $\overline{AC}$ and $\overline{BC}$. Then place a point at the intersection of the three perpendicular bisectors. This is the circumcenter of the triangle.



Model and Analyze

1. How do you know that the steps in Activity 4 give the actual perpendicular bisector and the circumcenter of $\triangle ABC$?



2. Construct the angle bisectors and find the incenter of $\triangle XYZ$. Describe how you used the reflective device for the construction.

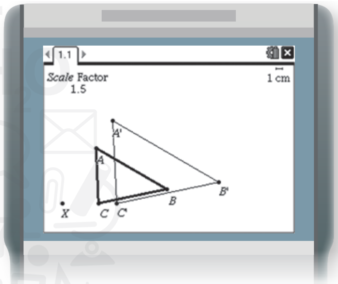
EXPLORE 6-6

Graphing Technology Lab
Dilations

You can use a Graphing Calculator to explore properties of dilations.

Activity 1 Dilation of a Triangle

Dilate a triangle by a scale factor of 1.5.



Step 1 Add a new **Geometry** page. Then, from the **Points & Lines** menu, use the **Point** tool to add a point and label it *X*.

Step 2 From the **Shapes** menu, select **Triangle** and specify three points. Label the points *A*, *B*, and *C*.

Step 3 From the **Actions** menu, use the **Text** tool to separately add the text *Scale Factor* and *1.5* to the page.

Step 4 From the **Transformation** menu, select **Dilation**. Then select point *X*, $\triangle ABC$, and the text *1.5*.

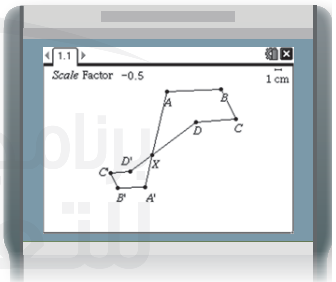
Step 5 Label the points on the image A', B', and C'.

Analyze the Results

1. Using the **Slope** tool on the **Measurement** menu, describe the effect of the dilation on $\overline{AB}$. That is, how are the lines through $\overline{AB}$ and $\overline{A'B'}$ related?
2. What is the effect of the dilation on the line passing through side $\overline{CA}$?
3. What is the effect of the dilation on the line passing through side $\overline{CB}$?

Activity 2 Dilation of a Polygon

Dilate a polygon by a scale factor of −0.5.



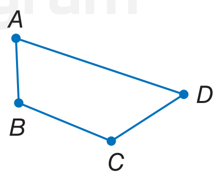
Step 1 Add a new **Geometry** page and draw polygon *ABCDX* as shown. Add the text *Scale Factor* and *−0.5* to the page.

Step 2 From the **Transformation** menu, select **Dilation**. Then select point *X*, polygon *ABCDX*, and the text *−0.5*.

Step 3 Label the points on the image A', B', C', and D'.

Model and Analyze

4. Analyze the effect of the dilation in Activity 2 on sides that contain the center of the dilation.
5. Analyze the effect of a dilation of trapezoid *ABCD* shown with a scale factor of 0.75 and the center of the dilation at *A*.

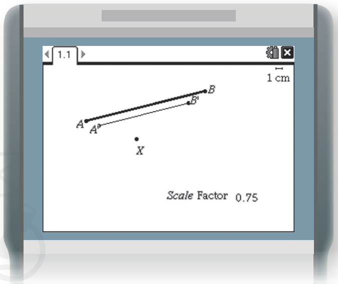


6. **MAKE A CONJECTURE** Describe the effect of a dilation on lines that pass through the center of a dilation and lines that do not pass through the center of a dilation.

Activity 3 Dilation of a Segment

Dilate a segment $\overline{AB}$ by the indicated scale factor.

a. scale factor: 0.75



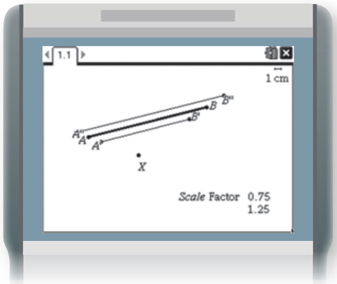
Step 1 On a new **Geometry** page, draw a line segment using the **Points & Lines** menu. Label the endpoints A and B. Then add and label a point X.

Step 2 Add the text *Scale Factor* and *0.75* to the page.

Step 3 From the **Transformation** menu, select **Dilation**. Then select point X, $\overline{AB}$, and the text *0.75*.

Step 4 Label the dilated segment $\overline{A'B'}$.

b. scale factor: 1.25



Step 1 Add the text *1.25* to the page.

Step 2 From the **Transformation** menu, select **Dilation**. Then select point X, $\overline{AB}$, and the text 1.25.

Step 3 Label the dilated segment $\overline{A''B''}$.

Model and Analyze

7. Using the **Length** tool on the **Measurement** menu, find the measures of $\overline{AB}$, $\overline{A'B'}$, and $\overline{A''B''}$.

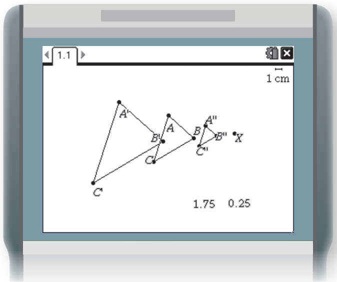
8. What is the ratio of $A'B'$ to AB? What is the ratio of $A''B''$ to AB?

9. What is the effect of the dilation with scale factor 0.75 on segment $\overline{AB}$? What is the effect of the dilation with scale factor 1.25 on segment $\overline{AB}$?

10. Dilate segment $\overline{AB}$ in Activity 3 by scale factors of −0.75 and −1.25. Describe the effect on the length of each dilated segment.

11. **MAKE A CONJECTURE** Describe the effect of a dilation on the length of a line segment.

12. Describe the dilation from $\overline{AB}$ to $\overline{A'B'}$ and $\overline{A'B'}$ to $\overline{A''B''}$ in the triangles shown.



LESSON 6-6 Dilations

Then

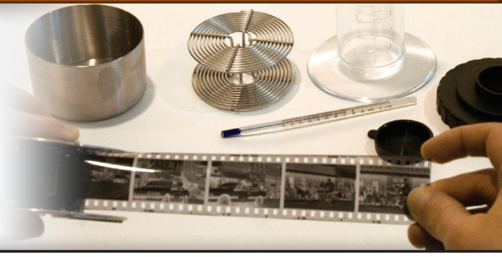
- You identified dilations and verified them as similarity transformations.

Now

- **1** Draw dilations.
- **2** Draw dilations in the coordinate plane.

Why?

- Some photographers still prefer traditional cameras and film to produce negatives. From these negatives, photographers can create scaled reproductions.



Mathematical Practices
Make sense of problems and persevere in solving them.
Use appropriate tools strategically.

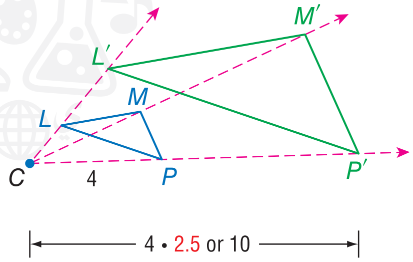
1 Draw Dilations

A dilation or *scaling* is a similarity transformation that enlarges or reduces a figure proportionally with respect to a *center* point and a *scale* factor.

KeyConcept Dilation

A dilation with center C and positive scale factor k, $k \neq 1$, is a function that maps a point P in a figure to its image such that

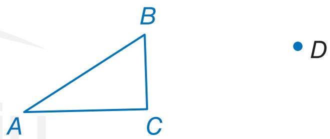
- if point P and C coincide, then the image and preimage are the same point, or
- if point P is not the center of dilation, then P' lies on $\overrightarrow{CP}$ and $CP' = k(CP)$.



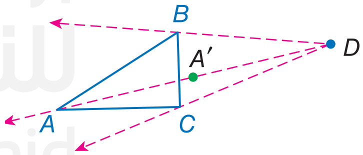
$\triangle L'M'P'$ is the image of $\triangle LMP$ under a dilation with center C and scale factor **2.5**.

Example 1 Draw a Dilation

Copy $\triangle ABC$ and point D. Then use a ruler to draw the image of $\triangle ABC$ under a dilation with center D and scale factor $\frac{1}{2}$.

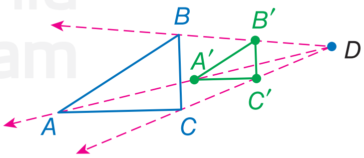


Step 1 Draw rays from D though each vertex.



Step 2 Locate A' on $\overrightarrow{DA}$ such that $DA' = \frac{1}{2}DA$.

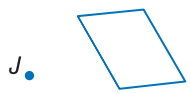
Step 3 Locate B' on $\overrightarrow{DB}$ and C' on $\overrightarrow{DC}$ in the same way. Then draw $\triangle A'B'C'$.



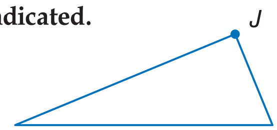
GuidedPractice

Copy the figure and point J. Then use a ruler to draw the image of the figure under a dilation with center J and the scale factor k indicated.

1A. $k = \frac{3}{2}$



1B. $k = 0.75$

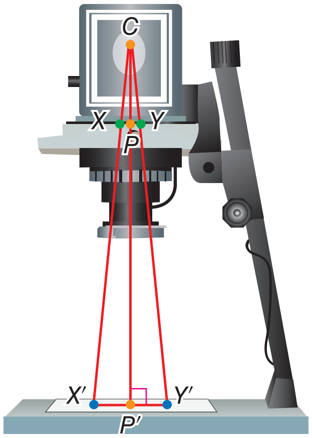


You also learned that if $k > 1$, then the dilation is an *enlargement*. If $0 < k < 1$, then the dilation is a *reduction*. Since $\frac{1}{2}$ is between 0 and 1, the dilation in Example 1 is a reduction.

A dilation with a scale factor of 1 is called an *isometry dilation*. It produces an image that coincides with the preimage. The two figures are congruent.

Real-World Example 2 Find the Scale Factor of a Dilation

PHOTOGRAPHY To create different-sized prints, you can adjust the distance between a film negative and the enlarged print by using a photographic enlarger. Suppose the distance between the light source C and the negative is 45 millimeters (CP). To what distance PP' should you adjust the enlarger to create a 22.75-centimeter wide print ($X'Y'$) from a 35-millimeter wide negative (XY)?



Understand This problem involves a dilation. The center of dilation is C, $XY = 35$ mm, $X'Y' = 22.75$ cm or 227.5 mm, and $CP = 45$ mm. You are asked to find PP'.

Plan Find the scale factor of the dilation from the preimage XY to the image $X'Y'$. Use the scale factor to find CP' and then use CP and CP' to find PP'.

Solve The scale factor k of the enlargement is the ratio of a length on the image to a corresponding length on the preimage.

$k = \frac{\text{image length}}{\text{preimage length}}$	Scale factor of image
$= \frac{X'Y'}{XY}$	image $= X'Y'$, preimage $= XY$
$= \frac{227.5}{35}$ or 6.5	Divide.

Use this scale factor of 6.5 to find CP'.

$CP' = k(CP)$	Definition of dilation
$= 6.5(45)$	$k = 6.5$ and $CP = 45$
$= 292.5$	Multiply.

Use CP' and CP to find PP'.

$CP + PP' = CP'$	Segment Addition
$45 + PP' = 292.5$	$CP = 45$ and $CP' = 292.5$
$PP' = 247.5$	Subtract 45 from each side.

So the enlarger should be adjusted so that the distance from the negative to the enlarged print (PP') is 247.5 millimeters or 24.75 centimeters.

Check Since the dilation is an enlargement, the scale factor should be greater than 1. Since $6.5 > 1$, the scale factor found is reasonable. ✓

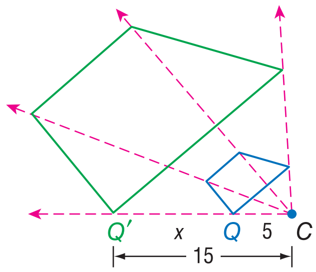
Problem-Solving Tip

Perseverance

To prevent careless errors in your calculations, estimate the answer to a problem before solving. In Example 2, you can estimate the scale factor of the dilation to be about $\frac{240}{40}$ or 6. Then CP' would be about $6 \cdot 50$ or 300 and PP' about $300 - 50$ or 250 millimeters, which is 25 centimeters. A measure of 24.75 centimeters is close to this estimate, so the answer is reasonable.

GuidedPractice

2. Determine whether the dilation from Figure Q to Q' is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and x.



2 Dilations in the Coordinate Plane

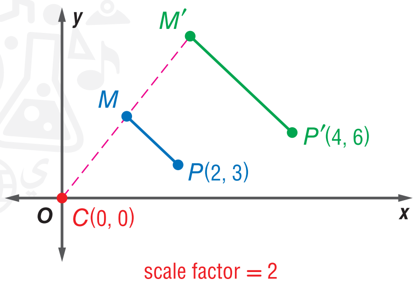
Dilations in the Coordinate Plane You can use the following rules to find the image of a figure after a dilation centered at the origin.

KeyConcept Dilations in the Coordinate Plane

Words To find the coordinates of an image after a dilation centered at the origin, multiply the x- and y-coordinates of each point on the preimage by the scale factor of the dilation, k.

Symbols $(x, y) \rightarrow (kx, ky)$

Example



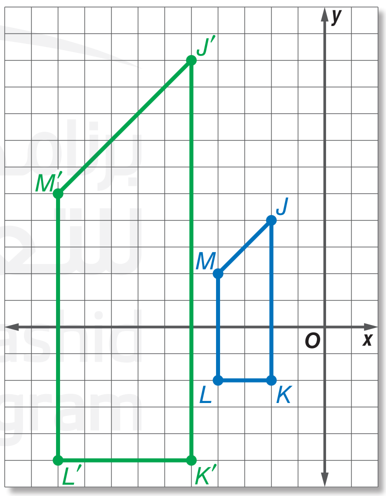
Example 3 Dilations in the Coordinate Plane

Quadrilateral $JKLM$ has vertices $J(-2, 4)$, $K(-2, -2)$, $L(-4, -2)$, and $M(-4, 2)$. Graph the image of $JKLM$ after a dilation centered at the origin with a scale factor of 2.5.

Multiply the x- and y-coordinates of each vertex by the scale factor, 2.5.

(x, y)	$\rightarrow$	$(2.5x, 2.5y)$
$J(-2, 4)$	$\rightarrow$	$J'(-5, 10)$
$K(-2, -2)$	$\rightarrow$	$K'(-5, -5)$
$L(-4, -2)$	$\rightarrow$	$L'(-10, -5)$
$M(-4, 2)$	$\rightarrow$	$M'(-10, 5)$

Graph $JKLM$ and its image $J'K'L'M'$.



GuidedPractice

Find the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

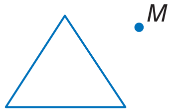
3A. $Q(0, 6)$, $R(-6, -3)$, $S(6, -3)$; $k = \frac{1}{3}$

3B. $A(2, 1)$, $B(0, 3)$, $C(-1, 2)$, $D(0, 1)$; $k = 2$

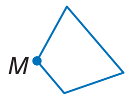
Check Your Understanding

Example 1 **Copy the figure and point *M*. Then use a ruler to draw the image of the figure under a dilation with center *M* and the scale factor *k* indicated.**

1. $k = \frac{1}{4}$

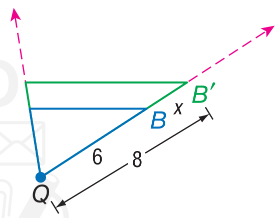


2. $k = 2$

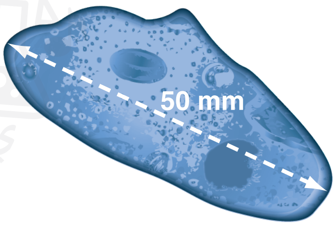


Example 2

3 Determine whether the dilation from Figure *B* to *B′* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and *x*.



4. **BIOLOGY** Under a microscope, a single-celled organism 200 microns in length appears to be 50 millimeters long. If 1 millimeter = 1000 microns, what magnification setting (scale factor) was used? Explain your reasoning.



Example 3 **Graph the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.**

5. $W(0, 0), X(6, 6), Y(6, 0); k = 1.5$

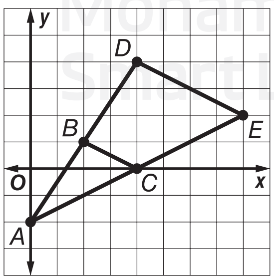
6. $Q(-4, 4), R(-4, -4), S(4, -4), T(4, 4); k = \frac{1}{2}$

7. $A(-1, 4), B(2, 4), C(3, 2), D(-2, 2); k = 2$

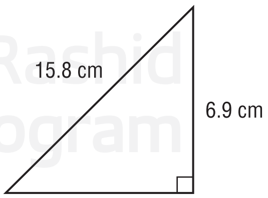
8. $J(-2, 0), K(2, 4), L(8, 0), M(2, -4); k = \frac{3}{4}$

Practice and Problem Solving

9. $\triangle ADE$ is a dilation of $\triangle ABC$ in the plane. Write a statement that can be used to verify that $\overline{DE} \parallel \overline{BC}$.



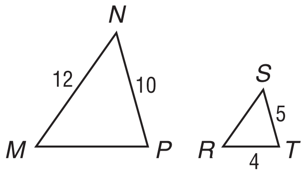
10. Consider the following diagram.



The triangle is dilated such that the perimeter of the new triangle is 82.4 centimeters. What is the length of the missing side in the new triangle?

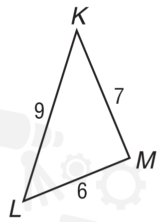
11. In the figure below, triangle *MNP* is similar to triangle *RST*.

Which scale factor was used to transform triangle $\triangle MNP$ to $\triangle RST$?

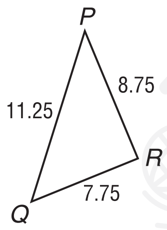


12. $\triangle KLM$ is shown below.

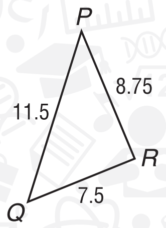
Which of the following shows $\triangle KLM$ dilated by a scale factor of $\frac{5}{4}$ to create a similar triangle $\triangle PQR$?



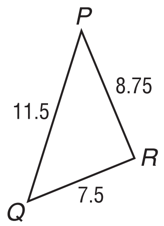
A



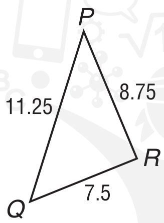
C



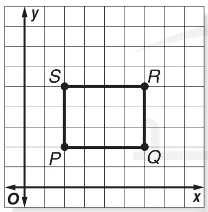
B



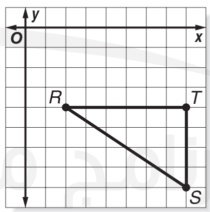
D



13. Rectangle *PQRS* is shown. If the rectangle is dilated by a scale factor of 2, with the origin as its center of dilation, find the new coordinates of *R*′.



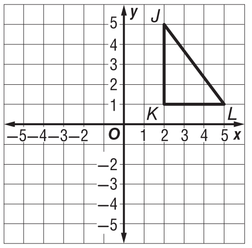
14. $\triangle RST$ is shown. If it is dilated by a scale factor of 2 and has the origin as the center of dilation, which are the coordinates of S′?



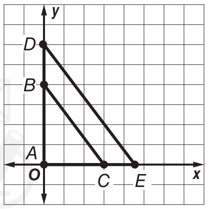
15. Badr is animating a cartoon character on the coordinate plane, using a dilation with scale factor 2. If *A*(1, 3), *B*(3, 4), and *C*(2, −3) are three points on Puff the Blowfish before he inflates, what are the coordinates of the corresponding points *D*, *E*, and *F* on the inflated image?

16. Which type of transformation preserves orientation but not size?

17. Right triangle *JKL* is dilated to form image $\triangle J'K'L'$. If the perimeter of $\triangle J'K'L'$ is 36 centimeters, what is the area of the image?



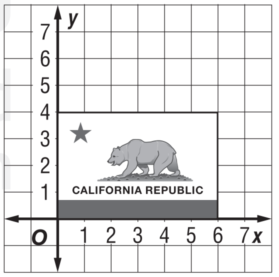
18. Triangle *ABC* with vertices $A(0, 0)$, $B(0, 4)$, and $C(3, 0)$ is dilated to form triangle *ADE*. What is the length of $\overline{DE}$ if *D* has coordinates (0, 5)?



19. Square *JKLM* has vertices $J(1, 0)$, $K(2, 1)$, $L(3, 0)$, and $M(2, -1)$. If the figure is dilated with a center at the origin and with a scale factor of $\sqrt{2}$, then what is the length of each side of the dilated square?

20. Isosceles trapezoid *LMNO* has vertices $L(-4, -3)$, $M(-4, 0)$, N$(-2, 1)$,and $O(-2, -4)$. If the figure is dilated with the center at the origin and with a scale factor of 1.5, what is the length of $\overline{L'M'}$ of the dilated isosceles trapezoid?

21. The state flag of California is shown on the grid below. Suppose the flag were enlarged so that the vertices of the new flag were (0, 0), (0, 6), (9, 6), and (9, 0). What is the ratio of the perimeter of the original flag to that of the enlarged flag?



22. Under a dilation, $\triangle XYZ$ is the image of $\triangle ABC$ and $XY = \frac{5}{8}AB$. What is the scale factor?

23. Which of the following are the coordinates of the image of $A(4, -12)$ under a dilation with center at the origin and scale factor 0.25?

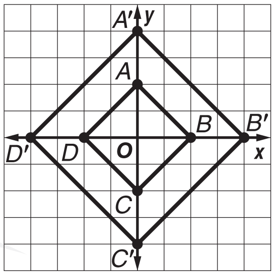
24. By what scale factor r will $Q(-20, 8)$ be the image of $P(-5, 2)$?

25. Under a dilation, the image of square $ABCD$ is square $AXYZ$. Which of the following is the center of the dilation?

26. The endpoints of $\overline{AB}$ are $A(3, -7)$ and $B(7, -12)$. The image of $\overline{AB}$ under a dilation with center at the origin is $\overline{A'B'}$. The coordinates of A' are $A'(9, -21)$. What are the coordinates of B'?

H.O.T. Problems Use Higher-Order Thinking Skills

27. Look at the figures on the grid at the right.

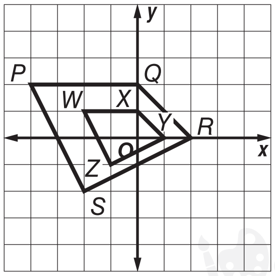


A Describe the transformation of quadrilateral $ABCD$ that yielded quadrilateral $A'B'C'D'$.

B Describe the result of rotating quadrilateral $ABCD$ 90° around the origin in the clockwise direction.

Standardized Test Practice

28. EXTENDED RESPONSE Quadrilateral *PQRS* was dilated to form quadrilateral *WXYZ*.



a. Is the dilation from *PQRS* to *WXYZ* an enlargement or reduction?

b. Which number *best* represents the scale factor for this dilation?

29. ALGEBRA How many grams of pure water must a pharmacist add to 50 grams of a 15% saline solution to make a solution that is 10% saline?

A 25 **C** 15

B 20 **D** 5

30. Buthaina wants to replicate a painting in an art museum. The painting is 0.90 meters wide and 1.80 meters long. She decides on a dilation reduction factor of 0.25. What size paper should she use?

F 10 cm × 20 cm **H** 20 cm × 40 cm

G 15 cm × 30 cm **J** 25 cm × 50 cm

31. SAT/ACT For all x, $(x - 7)^2 = ?$

A $x^2 - 49$ **D** $x^2 - 14x + 49$

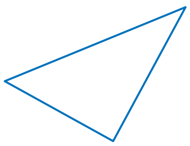
B $x^2 + 49$ **E** $x^2 + 14x - 49$

C $x^2 - 14x - 49$

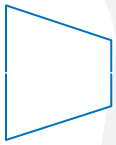
Spiral Review

State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number. (Lesson 6-5)

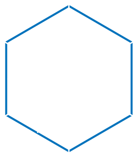
32.



33.

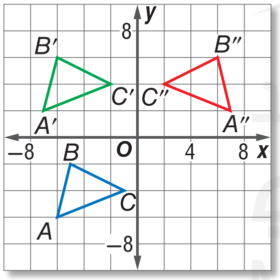


34.

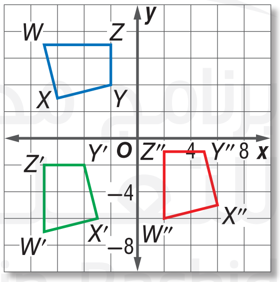


Describe the transformations that combined to map each figure. (Lesson 6-4)

35.



36.



37. ASSEMBLIES The number of assemblies at West High School each year is normally distributed with a mean of 12.4 and a standard deviation of 1.6.

a. What is the probability that there will be more than 10 assemblies in a given year?

b. If the school has existed for 30 years, in how many of those years were there between 11 and 13 assemblies?

Skills Review

Find the value of *x* to the nearest tenth.

38. $58.9 = 2x$

39. $\frac{108.6}{\pi} = x$

40. $228.4 = \pi x$

41. $\frac{336.4}{x} = \pi$

CHAPTER 6

Study Guide and Review

Study Guide

KeyConcepts

Reflections (Lesson 6-1)

- A reflection is a transformation representing a flip of a figure over a point, line, or plane.

Translations (Lesson 6-2)

- A translation is a transformation that moves all points of a figure the same distance in the same direction.
- A translation maps each point to its image along a translation vector.

Rotations (Lesson 6-3)

- A rotation turns each point in a figure through the same angle about a fixed point.

Compositions of Transformations (Lesson 6-4)

- A translation can be represented as a composition of reflections in parallel lines and a rotation can be represented as a composition of reflections in intersecting lines.

Symmetry (Lesson 6-5)

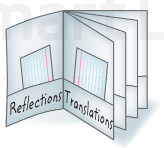
- The line of symmetry in a figure is a line where the figure could be folded in half so that the two halves match exactly.
- The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.
- The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

Dilations (Lesson 6-6)

- Dilations enlarge or reduce figures proportionally.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

- angle of rotation
- axis symmetry
- center of rotation
- composition of transformations
- glide reflection
- line of reflection
- line of symmetry
- line symmetry
- magnitude of symmetry
- order of symmetry
- plane symmetry
- rotational symmetry
- symmetry
- translation vector

VocabularyCheck

Choose the term that best completes each sentence.

1. When a transformation is applied to a figure, and then another transformation is applied to its image, this is a(n) (composition of transformations, order of symmetries).
2. If a figure is folded across a straight line and the halves match exactly, the fold line is called the (line of reflection, line of symmetry).
3. A (dilation, glide reflection) enlarges or reduces a figure proportionally.
4. The number of times a figure maps onto itself as it rotates from 0° to 360° is called the (magnitude of symmetry, order of symmetry).
5. A (line of reflection, translation vector) is the same distance from each point of a figure and its image.
6. A figure has (a center of rotation, symmetry) if it can be mapped onto itself by a rigid motion.
7. A glide reflection includes both a reflection and a (rotation, translation).
8. To rotate a point (90°, 180°) counterclockwise about the origin, multiply the y-coordinate by −1 and then interchange the x- and y-coordinates.
9. A (vector, reflection) is a congruence transformation.
10. A figure has (plane symmetry, rotational symmetry) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

Lesson-by-Lesson Review

6-1 Reflections

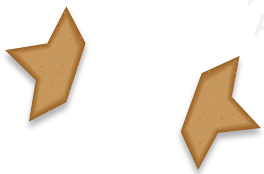
Graph each figure and its image under the given reflection.

11. rectangle *ABCD* with *A*(2, −4), *B*(4, −6), *C*(7, −3), and *D*(5, −1) in the *x*-axis

12. triangle *XYZ* with *X*(−1, 1), *Y*(−1, −2), and *Z*(3, −3) in the *y*-axis

13. quadrilateral *QRST* with *Q*(−4, −1), *R*(−1, 2), *S*(2, 2), and *T*(0, −4) in the line $y = x$

14. **ART** Badria is making the two-piece sculpture shown for a memorial garden. In her design, one piece of the sculpture is a reflection of the other, to be placed beside a sidewalk that would be located along the line of reflection. Copy the figures and draw the line of reflection.



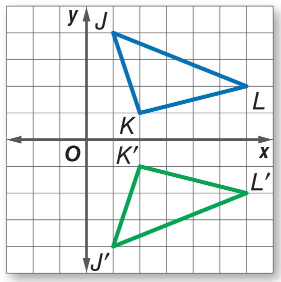
Example 1

Graph △*JKL* with vertices *J*(1, 4), *K*(2, 1), and *L*(6, 2) and its reflected image in the *x*-axis.

Multiply the *y*-coordinate of each vertex by −1.

(*x*, *y*)	→	(*x*, −*y*)
J(1, 4)	→	*J*′(1, −4)
K(2, 1)	→	*K*′(2, −1)
L(6, 2)	→	*L*′(6, −2)

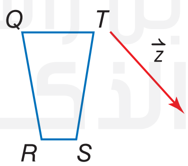
Graph △*JKL* and its image △*J*′*K*′*L*′.



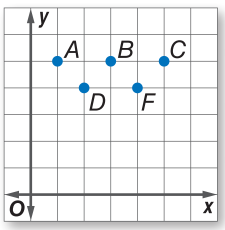
6-2 Translations

15. Graph △*ABC* with vertices *A*(0, −1), *B*(2, 0), *C*(3, −3) and its image along ⟨−5, 4⟩.

16. Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.



17. **PERFORMANCE** Five performers are positioned onstage as shown. *B*, *F*, and *C* move along ⟨0, −2⟩, while *A* moves along ⟨5, −1⟩. Draw the final positions.



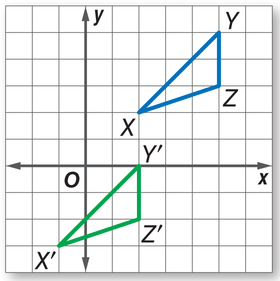
Example 2

Graph △*XYZ* with vertices *X*(2, 2), *Y*(5, 5), *Z*(5, 3) and its image along ⟨−3, −5⟩.

The vector indicates a translation 3 units left and 5 units down.

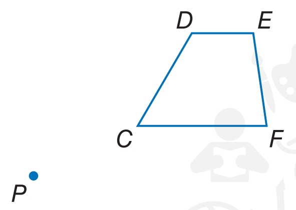
(*x*, *y*)	→	(*x* − 3, *y* − 5)
X(2, 2)	→	*X*′(−1, −3)
Y(5, 5)	→	*Y*′(2, 0)
Z(5, 3)	→	*Z*′(2, −2)

Graph △*XYZ* and its image △*X*′*Y*′*Z*′.



6-3 Rotations

18. Copy trapezoid *CDEF* and point *P*. Then use a protractor and ruler to draw a 50° rotation of *CDEF* about point *P*.



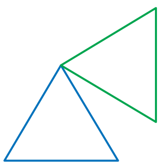
Graph each figure and its image after the specified rotation about the origin.

19. △*MNO* with vertices *M*(−2, 2), *N*(0, −2), *O*(1, 0); 180°

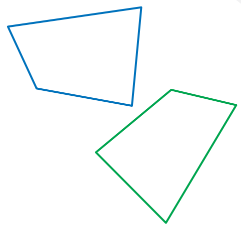
20. △*DGF* with vertices *D*(1, 2), *G*(2, 3), *F*(1, 3); 90°

Each figure shows a preimage and its image after a rotation about a point *P*. Copy each figure, locate point *P*, and find the angle of rotation.

21.



22.



Example 3

Triangle *ABC* has vertices *A*(−4, 0), *B*(−3, 4), and *C*(−1, 1). Graph △*ABC* and its image after a rotation 270° about the origin.

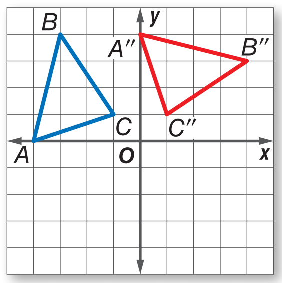
One method to solve this is to combine a 180° rotation with a 90° rotation. Multiply the *x*- and *y*-coordinates of each vertex by −1.

(*x*, *y*)	→	(−*x*, −*y*)
A(−4, 0)	→	*A*′(4, 0)
B(−3, 4)	→	*B*′(3, −4)
C(−1, 1)	→	*C*′(1, −1)

Multiply the *y*-coordinate of each vertex by −1 and interchange.

(−*x*, −*y*)	→	(*y*, −*x*)
A′(4, 0)	→	*A*″(0, 4)
B′(3, −4)	→	*B*″(4, 3)
C′(1, −1)	→	*C*″(1, 1)

Graph △*ABC* and its image △*A*″*B*″*C*″.



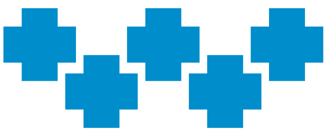
6-4 Compositions of Transformations

Graph each figure with the given vertices and its image after the indicated transformation.

23. $\overline{CD}$: *C*(3, 2) and *D*(1, 4)
Reflection: in $y = x$
Rotation: 270° about the origin.

24. $\overline{GH}$: *G*(−2, −3) and *H*(1, 1)
Translation: along ⟨4, 2⟩
Reflection: in the *x*-axis

25. PATTERNS Jassim is creating a pattern for the border of a poster using a stencil. Describe the transformation combination that he used to create the pattern below.



Example 4

The endpoints of $\overline{RS}$ are *R*(4, 3) and *S*(1, 1). Graph $\overline{RS}$ and its image after a translation along ⟨−5, −1⟩ and a rotation 180° about the origin.

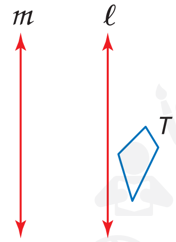
Step 1 translation along ⟨−5, −1⟩

(*x*, *y*)	→	(*x* − 5, *y* − 1)
R(4, 3)	→	*R*′(−1, 2)
S(1, 1)	→	*S*′(−4, 0)

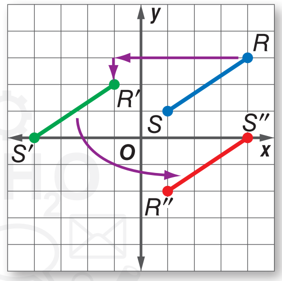
Step 2 rotation 180° about origin

(*x*, *y*)	→	(−*x*, −*y*)
R′(−1, 2)	→	*R*″(1, −2)
S′(−4, 0)	→	*S*″(4, 0)

26. Copy and reflect figure T in line ℓ and then line m. Then describe a single transformation that maps T onto T''.



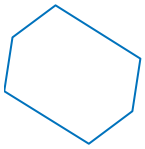
Step 3 Graph $\overline{RS}$ and its image $\overline{R''S''}$.



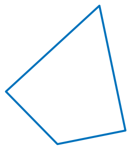
6-5 Symmetry

State whether each figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

27.

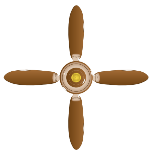


28.



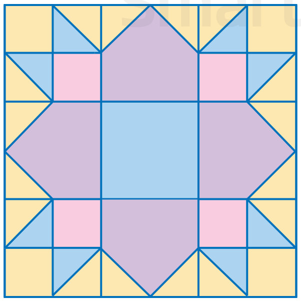
State whether each figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

29.



30.

31. KNITTING Hessa is creating a pattern for a scarf she is knitting for her friend. How many lines of symmetry are there in the pattern?



Example 5

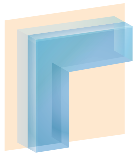
State whether each figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

a.

The light bulb has both plane and axis symmetry.

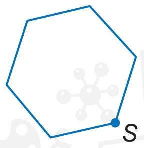
b.

The prism has plane symmetry.

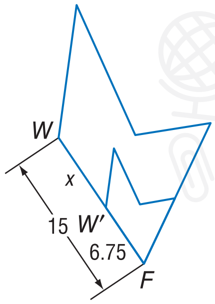


6-6 Dilations

32. Copy the figure and point *S*. Then use a ruler to draw the image of the figure under a dilation with center *S* and scale factor $r = 1.25$.



33. Determine whether the dilation from figure *W* to *W′* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and *x*.



34. **CLUBS** The members of the Math Club use an overhead projector to make a poster. If the original image was 15 centimeters wide, and the image on the poster is 1.2 meters wide, what is the scale factor of the enlargement?

Example 6

Square *ABCD* has vertices *A*(0, 0), *B*(0, 8), *C*(8, 8), and *D*(8, 0). Find the image of *ABCD* after a dilation centered at the origin with a scale factor of 0.5.

Multiply the *x*- and *y*-coordinates of each vertex by the scale factor, 0.5.

$(x, y) \rightarrow (0.5x, 0.5y)$

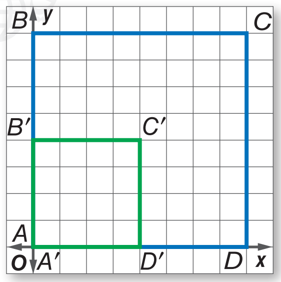
$A(0, 0) \rightarrow A'(0, 0)$

$B(0, 8) \rightarrow B'(0, 4)$

$C(8, 8) \rightarrow C'(4, 4)$

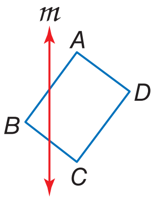
$D(8, 0) \rightarrow D'(4, 0)$

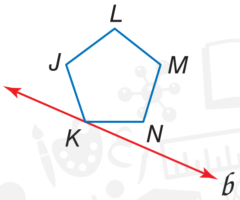
Graph *ABCD* and its image *A′B′C′D′*.



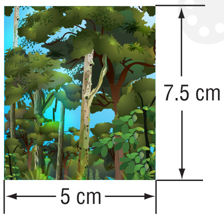
Practice Test

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

1. 

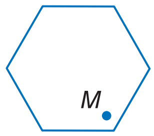
2. 

3. **PROJECTS** Jamal wants to enlarge the picture below to 10 centimeters by 15 centimeters for a school project. If his school's copy machine can only enlarge up to 150% by whole number percents, find two whole number percents by which he can enlarge the piece and get as close to 10 centimeters by 15 centimeters or less.



Copy the figure and point *M*. Then use a ruler to draw the image of the figure under a dilation with center *M* and the scale factor *r* indicated.

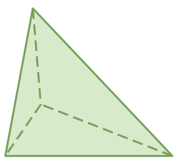
4. $r = 1.5$

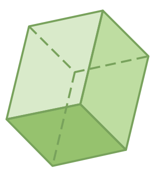


5. $r = \frac{1}{3}$

6. **PARKS** Halima is on a ride at an amusement park that slides the rider to the right, and then rotates counterclockwise about its own center 60° every 2 seconds. How many seconds pass before Halima returns to her starting position?

State whether each figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

7. 

8. 

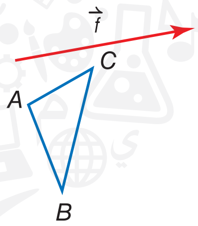
Graph each figure and its image under the given transformation.

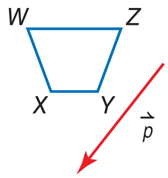
9. ▱$FGHJ$ with vertices $F(-1, -1)$, $G(-2, -4)$, $H(1, -4)$, and $J(2, -1)$ in the x-axis

10. $\triangle ABC$ with vertices $A(0, -1)$, $B(2, 0)$, $C(3, -3)$; $\langle -5, 4 \rangle$

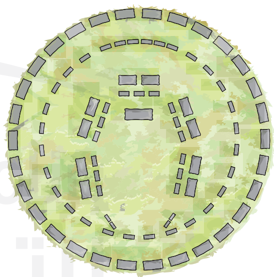
11. quadrilateral $WXYZ$ with vertices $W(2, 3)$, $X(1, 1)$, $Y(3, 0)$, $Z(5, 2)$; 180° about the origin

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

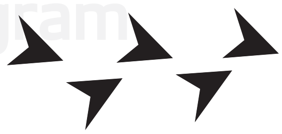
12. 

13. 

14. **ART** An artist's rendition of what Stonehenge, a famous archeological site in England, would have looked like before the stones fell or were removed, is shown below. What is the order and magnitude of symmetry for the outer ring?



15. **MULTPLE CHOICE** What transformation or combination of transformations does the figure below represent?



A dilation
B glide reflection
C rotation
D translation

CHAPTER 6 Preparing for Standardized Tests

Work Backward

In most problems, a set of conditions or facts is given and you must find the end result. However, some problems give you the end result and ask you to find something that happened earlier in the process. To solve problems like this, you must work backward.

Strategies for Working Backward

Step 1

Look for keywords that indicate you will need to work backward to solve the problem.

Sample Keywords:

- What was the **original**...?
- What was the value **before**...?
- Where was the **starting** or **beginning**...?

Step 2

Undo the steps given in the problem statement to solve.

- List the sequence of steps from the beginning to the end result.
- Begin with the end result. Retrace the steps in reverse order.
- "Undo" each step using inverses to get back to the original value.

Step 3

Check your solution if time permits.

- Make sure your answer makes sense.
- Begin with your answer and follow the steps in the problem statement forward to see if you get the same end result.

Standardized Test Example

Solve the problem below. Responses will be graded using the short-response scoring rubric shown.

Hamdah is using a geometry software program to experiment with transformations on the coordinate grid. She began with a point and translated it 4 units up and 8 units left. Then she reflected the image in the x-axis. Finally, she dilated this new image by a scale factor of 0.5 with respect to the origin to arrive at $(-1, -4)$. What were the original coordinates of the point?

Scoring Rubric

Criteria	Score
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Read the problem statement carefully. You are given a sequence of transformations of a point on a coordinate grid. You know the coordinates of the final image and are asked to find the original coordinates. Undo each transformation in reverse order to work backward and solve the problem.

Example of a 2-point response:

original point → translation → reflection → dilation → end result

Begin with the coordinates of the end result and work backward.

Dilate by 2 to undo the dilation by 0.5:

$(-1, -4) \rightarrow (-1 \times 2, -4 \times 2) = (-2, -8)$

Reflect back across the *x*-axis to undo the reflection:

$(-2, -8) \rightarrow (-2, 8)$

Translate 4 units down and 8 units right to undo the translation:

$(-2, 8) \rightarrow (-2 + 8, 8 - 4) = (6, 4)$

The original coordinates of the point were (6, 4).

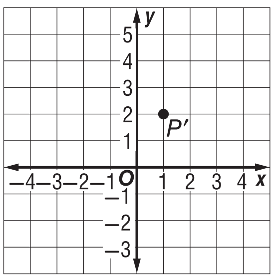
The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

Exercises

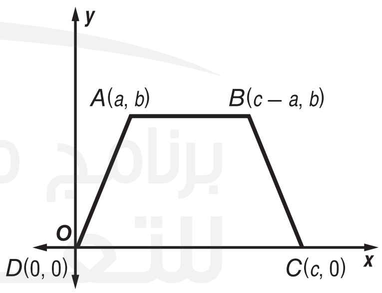
Solve each problem. Show your work. Responses will be graded using the short-response scoring rubric given at the beginning of the lesson.

1. A flea landed on a coordinate grid. The flea hopped across the *x*-axis and then across the *y*-axis in the form of two consecutive reflections. Then it walked 9 units to the right and 4 units down. If the flea's final position was at (4, −1), what point did it originally land on?

2. The coordinate grid below shows the final image when a point was rotated 90° clockwise about the origin, dilated by a scale factor of 2, and shifted 7 units right. What were the original coordinates?



3. Figure *ABCD* is an isosceles trapezoid.



Which of the following are the coordinates of an endpoint of the median of *ABCD*?

A $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$

B $\left(\frac{2c-a}{2}, \frac{b}{2}\right)$

C $\left(\frac{c}{2}, 0\right)$

D $\left(\frac{c}{2}, b\right)$

4. If the measure of an interior angle of a regular polygon is 108, what type of polygon is it?

F octagon

G hexagon

H pentagon

J triangle