الحركة الدورانية

Linear and Angular Measures			
Quantity	**Linear**	**Angular**	**Relationship**
Displacement	d (m)	θ (rad)	$d = r\theta$
Velocity	v (m/s)	ω (rad/s)	$v = r\omega$
Acceleration	a (m/s^2)	α (rad/s^2)	$a = r\alpha$

$$K_{trans} = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2\omega^2$$

عزم القصور الذاتي لكتلة نقطية (I) يساوي: كتلة الجسم (m) مضروبة في مربع المسافة التي يبعدها الجسم عن محور الدوران (r).

$$I = mr^2$$

مثال : هذا نموذج مبسط لعصا رفيعة تدور. وعلى كل طرف لها جسم دائري . فإذا كان طول العصا 0.66m وكتلة كل جسم 0.30kg . فأوجد عزم القصور الذاتي للعصا عند دورانها حول محور في منتصف المسافة بين الجسمين وعمودي على العصا. (كتلة العصا مهملة) .....

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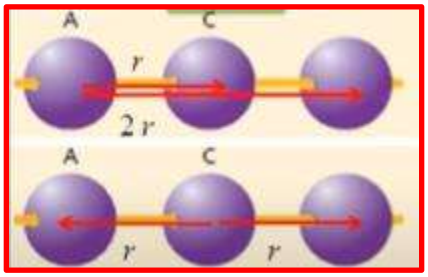
٢٤- التحدي : يوضح الشكل 13 ثلاث أجسام كروية متساوية الكتلة على عصا صغيرة الكتلة . تأمل في عزم القصور الذاتي للنظام .عندما يدورحول الجسم الكروي A أولاً وعندما يدور حول الجسم الكروي C

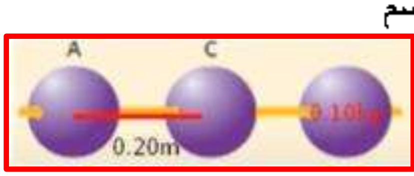


a- هل عزوم القصور الذاتي متماثلة أم مختلفة ؟اشرح إذا كانت عزوم القصور الذاتي مختلفة . ففي أي حالة يكون هذا العزم أكبر ؟

عزم القصور الذاتي عندما يدور النظام حول A

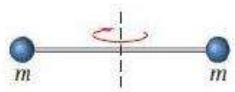
عزم القصور الذاتي عندما يدور النظام حول c

٢٤- التحدي : يوضح الشكل 13 ثلاث أجسام كروية متساوية الكتلة على عصا صغيرة الكتلة . تأمل في عزم القصور الذاتي للنظام .عندما يدورحول الجسم الكروي A أولاً وعندما يدور حول الجسم الكروي c

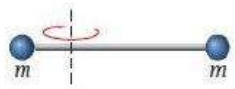


b- إذا كانت كتلة كل جسم كروي 0.10kg وكانت المسافة بين الجسمين الكرويين A وC تساوي 0.20m فاوجدعزم القصور الذاتي في الحالتين التاليتين : الدوران حول الجسم الكروي A والدوران حول الجسم الكروي C .

1- فكِّر في كتلتين متساويتين، m، متصلتين بساق رفيع عديم الكتلة. كما هو موضح بالأشكال، تدور الكتلتان في مستوى أفقي حول محور رأسيٍّ يمثَّل بخط متقطِّع. ما النظام الذي يحظى بأعلى عزم قصور ذاتي؟



(a)



(b)

(c)

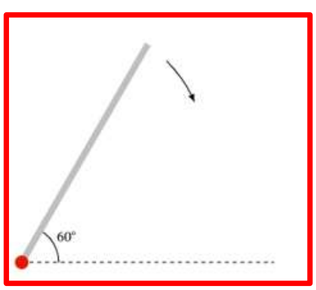
2- افترض أن الأرض جسم كروي صلب ذو كثافة ثابتة، كتلتة 5.98×10^{24}kg ونصف قطره 370 km

- ما عزم القصور الذاتي للأرض، مع اعتبار أنها تدور حول محورها،

- ما هو التردد الزاوي لدوران الأرض

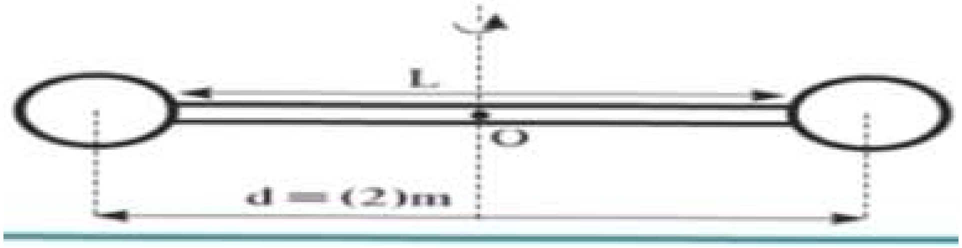
- ما الطاقة الحركية لهذا الدوران المحوري

3- قضيب منتظم كتلته 1.0 kg،وطوله 2.0 m،يدور بشكل حر حول أحد طرفيه، كما هو موضح في الشكل. إذا تحرك القضيب من السكون عند زاوية 60o فوق الأفقي، فما سرعة الطرف الحر للقضيب عند مروره بالوضع الأفقي



احسب القصور الذاتي الدوراني للنظام المؤلف من كرتين من الحديد متماثلتين كتلة الواحد منهما $m = (5)Kg$ ونصف قطرها $r = (\ 5\ )cm$ مثبتتين على طرفي عصا كتلتها $M = (2)Kg$ وطولها L المسافة بين مركزي الكرتين $(\ 2\ )\ m$ يدور النظام حول محور عمودي يمر بنقطة الوسط للعصا كما هو موضح بالشكل المجاور علما ان القصور الدوراني لكل من الأجسام الثلاثة حول محور يمر بمركز الثقل كل منهما

$I_{0\ \ rod} = \frac{1}{12} M\, L^2$



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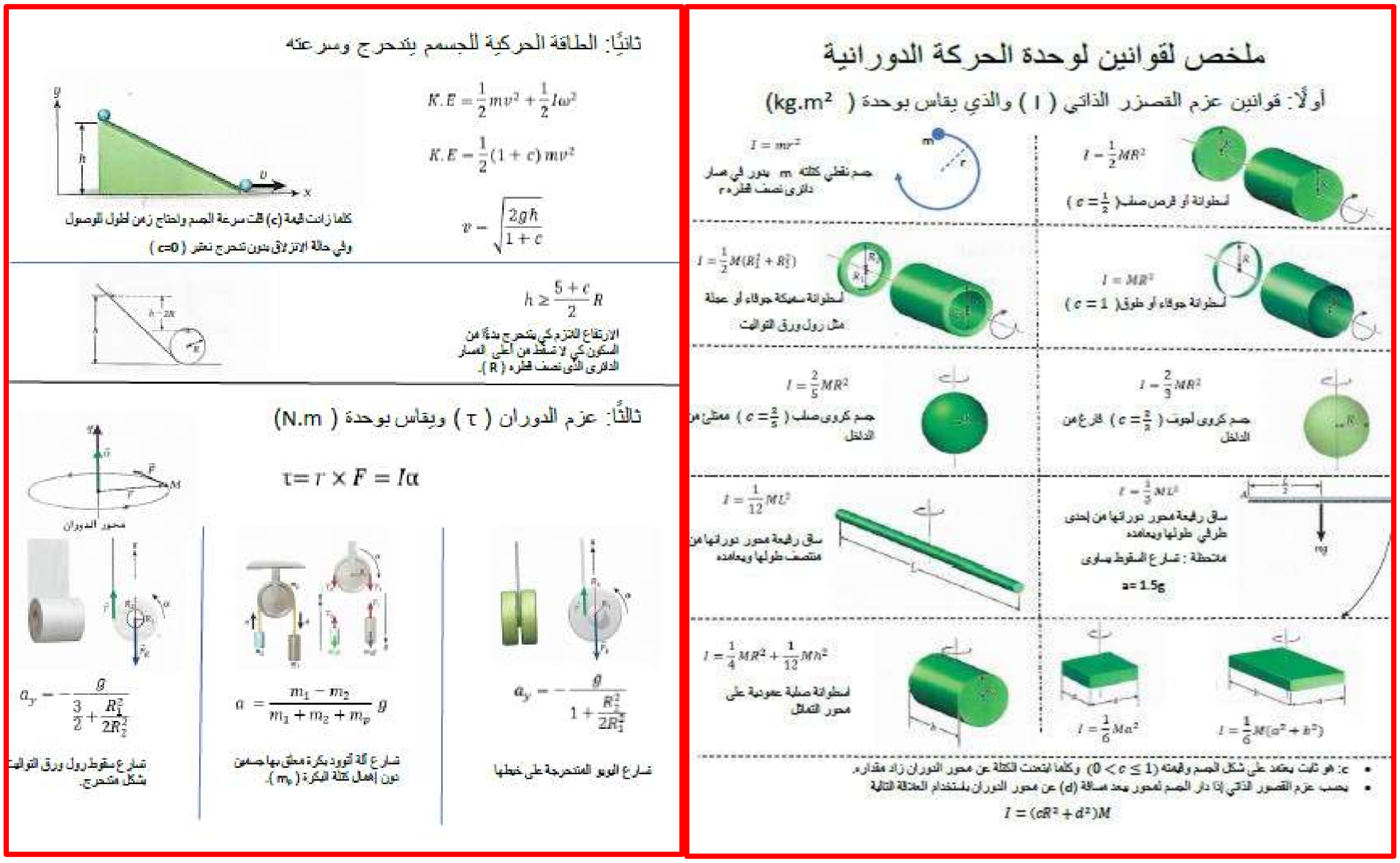


1. A 7.0 kg solid ball, radius 10cm, is rolling at 10m/s. Calculate the kinetic energy and angular momentum of the ball.

$$\omega = \frac{v}{R} = \frac{10m/s}{.10m} = 100/s$$

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(7)(0.1)^2 = 0.028kgm\text{^}2$$

$$L = I\omega = (0.028kgm\text{^}2)(100/s) = 2.8kgm\text{^}2/s$$

$$KE = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2$$

$$KE = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}(\tfrac{2}{5}MR^2)\frac{v^2}{R^2} = \tfrac{1}{2}Mv^2 + \tfrac{1}{5}Mv^2$$

$$KE = \tfrac{7}{10}Mv^2 = \tfrac{7}{10}(7)10^2 = 490J$$

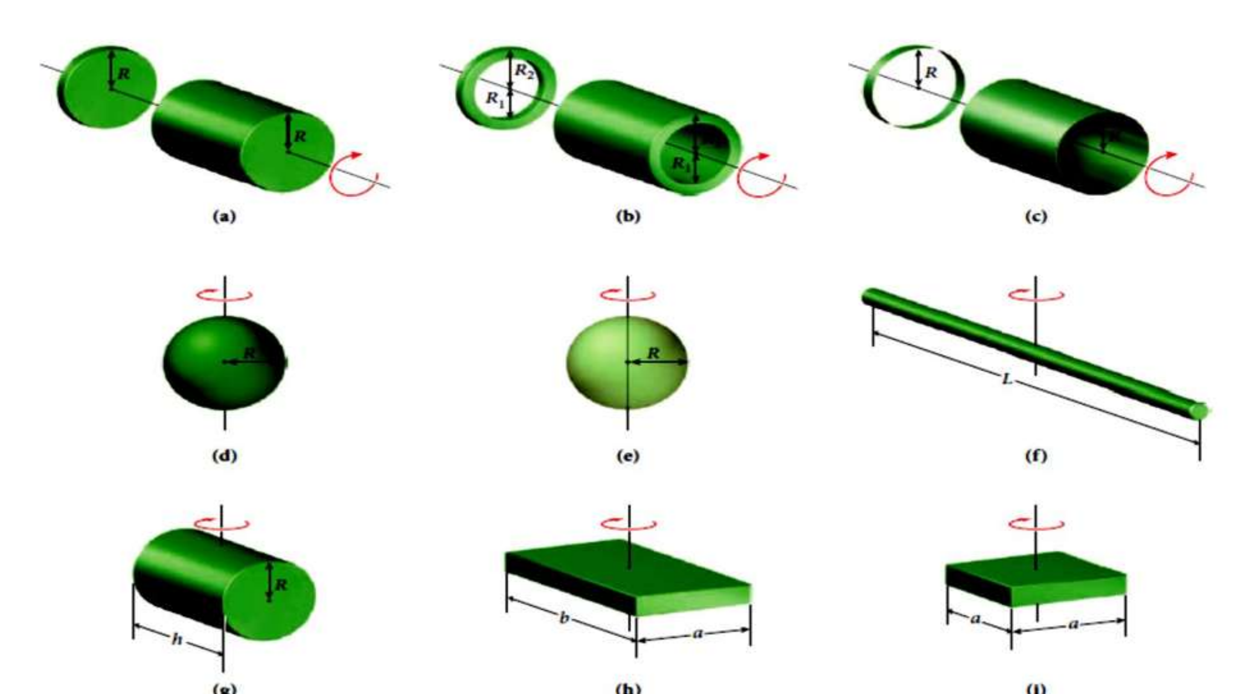
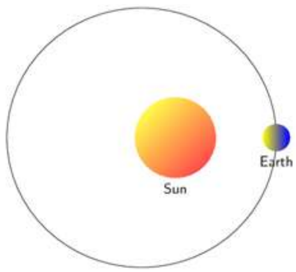
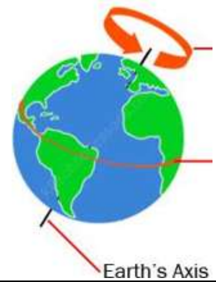


Table 10.1 The Moment of Inertia and Value of Constant *c* for the Objects Shown in Figure 10.10. All Objects Have Mass *M*

Object	I	c
a) Solid cylinder or disk	$\frac{1}{2}MR^2$	$\frac{1}{2}$
b) Thick hollow cylinder or wheel	$\frac{1}{2}M(R_1^2+R_2^2)$	
c) Hollow cylinder or hoop	MR^2	1
d) Solid sphere	$\frac{2}{5}MR^2$	$\frac{2}{5}$
e) Hollow sphere	$\frac{2}{3}MR^2$	$\frac{2}{3}$
f) Thin rod	$\frac{1}{12}Mh^2$	
g) Solid cylinder perpendicular to symmetry axis	$\frac{1}{4}MR^2+\frac{1}{12}Mh^2$	
h) Flat rectangular plate	$\frac{1}{12}M(a^2+b^2)$	
i) Flat square plate	$\frac{1}{6}Ma^2$	

10.1 Kinetic Energy of Rotation

10.1 and 10.2 38-44

Circular Motion	Rotational Motion
	

The kinematic quantities of circular motion.

Angular speed $\omega = \frac{d\theta}{dt}$

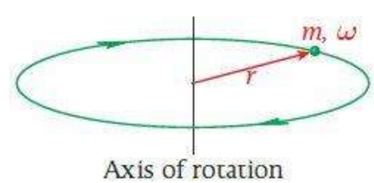
Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

The angular quantities are related to the linear quantities as follows:

Quantity	Linear	Angular	Relationship
Displacement	s	θ	$s = r\theta$
Velocity	v	ω	$v = r\omega$
Acceleration	a	α	$\vec{a} = r\alpha\hat{t} - r\omega^2\hat{r}$ $a_t = r\alpha$ $a_c = \omega^2 r$ $a = \sqrt{(a_c)^2 + (a_t)^2}$

The kinetic energy of a moving object was defined as

$$\boldsymbol{K_{trans}} = \frac{1}{2}mv^2$$



A point particle moving in a circle about the axis of rotation.

➢ *If the motion of this object is circular, we can use the relationship between linear and angular velocity to obtain kinetic energy of rotation for a point particle's motion on the circumference of a circle of radius* $\boldsymbol{r}$ *about a fixed axis, as illustrated in Figure*

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2\omega^2$$

$$\boldsymbol{K_{rot} = \frac{1}{2}mr^2\omega^2}$$

Several Point Particles in Circular Motion

The kinetic energy of a collection of rotating objects is written as the total kinetic energy (K*) as the sum of the individual kinetic energies of all particles and can given by*

$$\boldsymbol{K_{rot}} = \sum_{i=1}^{n} K_i = \sum_{i=1}^{n} \frac{1}{2}m_i v_i^2 = \frac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega_i^2$$

All of the point particles in the system will undergo circular motion around the common axis of rotation with the same angular velocity. With this constraint, the sum of the particles' kinetic energies becomes

$$\boldsymbol{K_{rot}} = \frac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega_i^2 = \frac{1}{2}\sum_{i=1}^{n} m_i r_i^2\, \omega_i^2 = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega_i^2$$

remember ***moment of inertia*** $\boldsymbol{I} = \sum_{i=1}^{n} m_i r_i^2$

$$\boldsymbol{K_{rot}} = \frac{1}{2}I\omega^2$$

Example 10.1 P 292 Rotational Kinetic Energy of Earth

Assume that the Earth is a solid sphere of constant density, with mass $5.98 \times 10^{24}\ kg$, *radius* $6370\ km$ *and its moment of inertia* $\boldsymbol{I} = 9.71 \times 10^{37} kgm^2$

1. *What is the angular frequency of Earth's rotation*

$$\omega = \frac{\Delta\theta}{T} = \frac{2\pi}{1\ day} = \frac{2\pi}{1 \times 24 \times 60 \times 60} = 7.272 \times 10^{-5}\ rad/s$$

2. *What is the kinetic energy of this rotation?*

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 9.71 \times 10^{37} \times (7.272 \times 10^{-5})^2 = 2.568 \times 10^{29} J$$

3. *If the orbital speed of Earth around the Sun (*$v_{t,earth} = 2.97 \times 10^4\ m/s$*), what is the kinetic energy of of Earth's motion around the Sun?*

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 5.98 \times 10^{24} \times (2.97 \times 10^4)^2 = 2.637 \times 10^{33} J$$

4. Which kinetic energy is greater than the other?

$$\frac{2.637 \times 10^{33}}{2.568 \times 10^{29}} = 10259$$

the kinetic energy of Earth's motion around the Sun is larger than the kinetic energy of Earth's motion around its self.

$$K_{tran} > K_{rota}$$

Concept Check 10.1 P 286

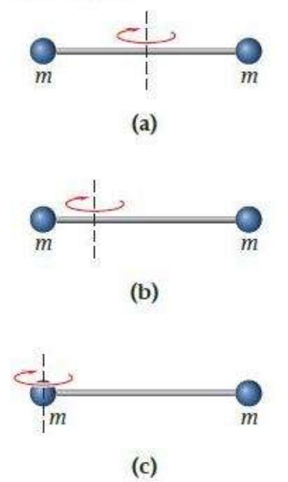
$$\boldsymbol{I} = \sum_{i=1}^{n} m_i r_i^2$$

Consider two equal masses, m, connected by a thin, massless rod. As shown in the figures, the two masses spin in a horizontal plane around a vertical axis represented by the dashed line. Which system has the highest moment of inertia?

a. $\boldsymbol{I} = m_1 r_1^2 + m_2 r_2^2 = md^2 + md^2 = 2md^2$

b. $\boldsymbol{I} = m_1 r_1^2 + m_2 r_2^2 = m\left(\frac{d}{2}\right)^2 + m\left(\frac{3d}{2}\right)^2 = \frac{10}{4}md^2 = 2.5md^2$

c. $\boldsymbol{I} = m_1 r_1^2 + m_2 r_2^2 = m(0)^2 + m(2d)^2 = 4md^2$



10.2 Calculation of Moment of Inertia (Indirect LMS)

We will represent an extended object by a collection of small identical-sized cubes of volume V and (possibly different) mass density (ρ).

$$\rho = \frac{mass}{Volume} \quad \Rightarrow \quad mass = \rho \times Volume \Rightarrow \quad m = \rho \times V$$

$$\boldsymbol{I} = \sum_{i=1}^{n} m_i r_i^2 = \left(\sum_{i=1}^{n} \rho(\vec{r}_i) r_i^2\, V\right)$$

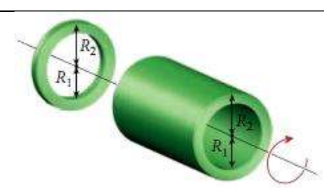
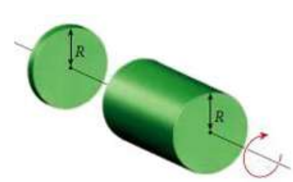
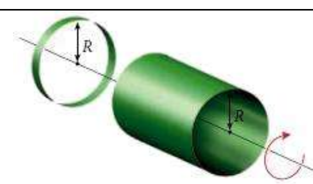
The conventional calculus approach, letting the volume of the cubes approach zero ($v \to 0$) in this limit, the sum in above equation approaches the integral, which gives an expression for the moment of inertia of an extended object:

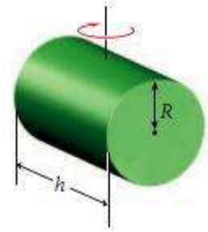
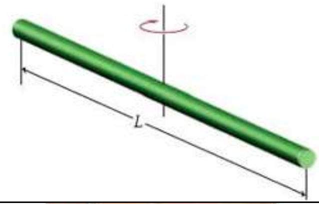
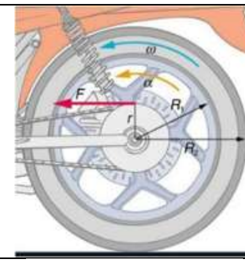
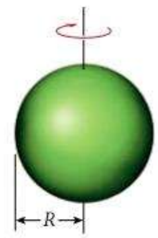
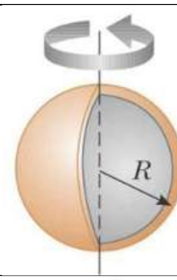
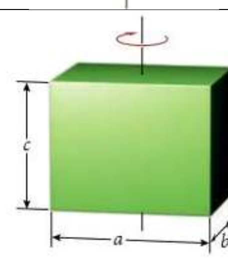
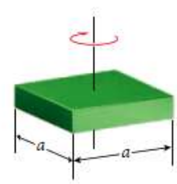
$$\boldsymbol{I} = \boldsymbol{\rho} \int_{V} \boldsymbol{r}_{\perp}^{2}\, \boldsymbol{dV} \qquad\qquad \boldsymbol{M} = \boldsymbol{\rho} \int_{V} \boldsymbol{dV} = \boldsymbol{\rho V}$$

We can now calculate moments of inertia for some objects with particular shapes.

First, we'll assume that the axis of rotation passes through the center of mass of the object

Rotation about an axis through the Center of Mass

Object	I	
A hollow cylinder rotating about its symmetry axis.	$\boldsymbol{I} = \frac{1}{2} M(R_1^2 + R_2^2)$	
A solid cylinder rotating about its symmetry axis	$\boldsymbol{I} = \frac{1}{2} MR^2$ $c = \frac{1}{2} \quad \Rightarrow \boldsymbol{I} = cMR^2$	
A thin cylindrical shell or hoop, for which all the mass is concentrated on the circumference	$\boldsymbol{I} = MR^2$ $c = 1 \quad \Rightarrow \boldsymbol{I} = cMR^2$	

A solid cylinder of height h rotating about an axis through its center of mass but perpendicular to its symmetry axis.	$\boldsymbol{I} = M\left(\frac{1}{4}R^2 + \frac{1}{12}h^2\right)$	
A long thin rod, where R<<l	$\boldsymbol{I} = \frac{1}{12}Ml^2$	
moment of inertia for a wheel	$\boldsymbol{I} = \frac{1}{2}M(R_1^2 + R_2^2)$	
Solid sphere rotating about any axis through its center of mass	$\boldsymbol{I} = \frac{2}{5}MR^2$ $c = \frac{2}{5} \quad \Rightarrow \boldsymbol{I} = cMR^2$	
A thin spherical shell rotating about any axis through its center of mass is	$\boldsymbol{I} = \frac{2}{3}MR^2$ $c = \frac{2}{3} \quad \Rightarrow \boldsymbol{I} = cMR^2$	
A rectangular block with side lengths a, b, and c rotating about an axis through the center of mass and parallel to side c	$\boldsymbol{I} = \frac{1}{12}M(a^2 + b^2)$	
Flat square plate	$\boldsymbol{I} = \frac{1}{6}Ma^2$	

Parallel-axis theorem

Big Question *what is the moment of inertia for rotation about an axis that does not pass through the center of mass?*

The parallel-axis theorem answers this question. It states that the moment of inertia, $I_{\parallel}$, *for rotation of an object of mass M about an axis located a distance d away from the object's center of mass and parallel to an axis through the center of mass, for which the moment of inertia is* I_{cm} *is given by*

$$I_{\parallel} = I_{cm} + Md^2$$

Note that, the moment of inertia with respect to rotation about an arbitrary axis parallel to an axis through the center of mass can be written as

$$I = M(cR^2 + d^2) \qquad\qquad 0 < c \le 1$$

where R is the maximum perpendicular distance of any part of the object from its axis of rotation through the center of mass d is the distance of the rotation axis from a parallel axis through the center of mass.

Example 10.1 P 292 moment of inertia of Earth

Assume that the Earth is a solid sphere of constant density, with mass $5.98 \times 10^{24}\ kg$ *and radius* $6370\ km$*. Calculate* the moment of inertia *of the Earth with respect to rotation about its axis?*

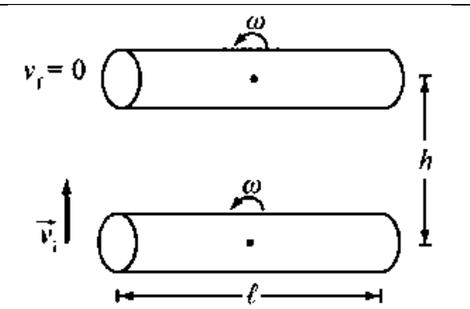
$$\boldsymbol{I} = \frac{2}{5} MR^2$$

$$\boldsymbol{I} = \frac{2}{5} \times 5.98 \times 10^{24} \times (6370 \times 10^3)^2$$

$$\boldsymbol{I} = 9.71 \times 10^{37} kgm^2$$

Q 10.40 P 318

A 24-cm-long pen is tossed up in the air, reaching a maximum height of 1.2 m above its release point. On the way up, the pen makes 1.8 revolutions. Treating the pen as a thin uniform rod, calculate the ratio between the rotational kinetic energy and the translational kinetic energy at the instant the pen is released. Assume that the rotational speed does not change during the toss.

<table>
<tr>
<td>$h = 1.2\ m$
$l = 0.24\ m$
$\Delta\theta = 1.8\ rev$
$v_f = 0\ m/s$
$\boldsymbol{I} = \frac{1}{12} Ml^2$</td>
<td>$\bar{\omega} = ????$
$\frac{K_{rot}}{K_{tran}} = ???$</td>
<td>
</td>
</tr>
</table>

$$\boldsymbol{E_i = E}$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + 0 = 0 + mgh$$

$$v_i = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.2} = 4.85\ m/s$$

$$K_{tran} = \frac{1}{2}mv_i^2 = \frac{1}{2}m \times 4.85^2 = 11.76m\ (J)$$

$$K_{rot} = \frac{1}{2}I\omega_i^2$$

$$K_{rot} = \frac{1}{2} \times \left(\frac{1}{12}ml^2\right) \times \omega_i^2$$

$$\omega =??\quad \Rightarrow \quad \omega = \frac{\Delta\theta}{\Delta t}$$

$$v_f = v_i + a\Delta t$$

$$0 = 4.85 - (9.81\Delta t) \qquad \Rightarrow \qquad \Delta t = 0.495\ s$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{1.8 \times 2\pi}{0.495} = 22.89\ rad/s$$

$$K_{rot} = \frac{1}{2} \times \left(\frac{1}{12}ml^2\right) \times \omega_i^2$$

$$K_{rot} = \frac{1}{2} \times \left(\frac{1}{12}m(0.24)^2\right) \times (22.89)^2 = 1.2578m\ (J)$$

$$\frac{K_{rot}}{K_{tran}} = \frac{1.2578m}{11.76m} = 0.107$$

$$K_{rot} = \frac{1}{2} mr^2 \omega^2$$

$$I = mr^2$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

10.3 Rolling Without Slipping:

10.3 45-46

***Rolling Motion**: is a special case of rotational motion that is performed by round objects of radius R that move across a surface without slipping.*

For rolling motion, we can connect the linear and angular quantities by realizing that: linear distance moved (r) *by the center of mass is the same as the length of corresponding arc* $(R\theta)$ *of the object's circumference.*

$$(r)_{linear\ distance\ by\ the\ center\ of\ mass} = (R\theta)_{the\ lenght\ of\ corresponding\ arc\ of\ the\ object's circumferance}$$

Reminder
- $The\ radius\ R, remains\ constant.$
- $v = R\omega$
- $a = R\alpha$

***The total kinetic energy of an object in rolling motion* is** *the sum of its **translational** and **rotational** kinetic energies:*

Reminder • $I = cMR^2$ $\quad 0 < c \leq 1$

Object	c
Solid cylinder or disk	$\frac{1}{2}$
Hollow cylinder or hoop	1
Solid Sphere	$\frac{2}{5}$
Hollow Sphere	$\frac{2}{3}$

- $U_{potential\ energy} = mgh$

- $E_{total\ mechanical\ energy} = K_{kinetic\ energy} + U_{potential\ energy}$

$$K_{total} = K_{trans} + K_{rot}$$

$$K_{total} = \frac{1}{2}mv^2 + \frac{1}{2}\boldsymbol{I}\,\omega^2$$

$$K_{total} = \frac{1}{2}mv^2 + \frac{1}{2}(\boldsymbol{cmR^2})\left(\frac{v}{R}\right)^2$$

$$K_{total} = \frac{1}{2}mv^2 + \frac{1}{2}(cmv^2)$$

$$\boldsymbol{K_{total} = (1 + c)\frac{1}{2}mv^2}$$

Notice:

- *The kinetic energy of a rolling object is always greater than that of an object that is sliding, provided they have the same mass and linear velocity.*
- *we can apply the concept of conservation of total mechanical energy (the sum of kinetic and potential energy).*

Concept Check 10.2 P 294

A solid sphere, a solid cylinder, and a hollow cylinder have the same mass and radius and are rolling with the same speed. Which one of the following statements is true?

$c_{solid\ sphere} = \frac{2}{5}$ $c_{solid\ cylinder} = \frac{1}{2}$ $c_{hollow\ cylinder} = 1$

$$\boldsymbol{K_{total} = (1 + c)\frac{1}{2}mv^2}$$

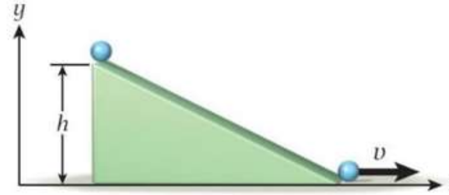
a) The solid sphere has the highest kinetic energy.

b) The solid cylinder has the highest kinetic energy.

c) The hollow cylinder has the highest kinetic energy.

d) All three objects have the same kinetic energy.

Solved Problem 10.1 / Sphere Rolling Down an Inclined Plane

A solid sphere with a mass of 5.15 kg and a radius of 0.340 m starts from rest at a height of 2.10 m above the base of an inclined plane and rolls down without sliding under the influence of gravity. What is the linear speed of the center of mass of the sphere just as it leaves the incline and rolls onto a horizontal surface?	 *Sphere rolling down an inclined plane.*

$m = 5.15\ kg$ $v_i = 0\ m/s$ $\omega_i = 0\ m/s$	$r = 0.34\ m$ $h_i = 2.10\ m$ $v_f = ???$

$$E_i = K_{i,total} + U_i$$

$$K_{i,total} = K_{i,rtational} + K_{i,translational}$$

$$K_{i,total} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = (1+c)\frac{1}{2}mv^2 = 0$$

$$E_i = 0 + mgh \qquad \Rightarrow \qquad \boldsymbol{E_i = mgh}$$

$$E_f = K_{f,total} + U_f$$

$$K_{f,total} = K_{f,rtational} + K_{f,translational}$$

$$K_{i,total} = (1+c) \times \frac{1}{2}mv^2 = \frac{1+c}{2}mv^2$$

$$E_f = K_{f,total} + U_f$$

$$E_f = \frac{1+c}{2}mv^2 + 0 \qquad \Rightarrow \qquad \boldsymbol{E_f = \frac{1+c}{2}mv^2}$$

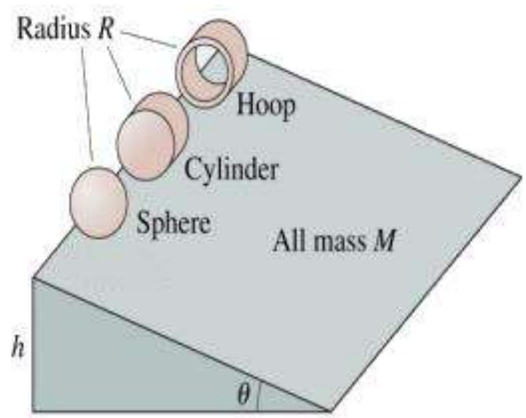
$$\boldsymbol{E_i = E_f}$$

$$mgh = \frac{1+c}{2}mv^2$$

$$\boldsymbol{v = \sqrt{\frac{2gh}{1+c}}}$$

$$v = \sqrt{\frac{2gh}{1+c}} = \sqrt{\frac{2 \times 9.81 \times 2.1}{1+\frac{2}{5}}} = 5.425\ m/s$$

Example 10.2 Race Down an Incline

A solid sphere, a solid cylinder, and a hollow cylinder (a tube), all of the same mass m and the same outer radius R, are released from rest at the top of an incline and start rolling without sliding. In which order do they arrive at the bottom of the incline?		

$$\boldsymbol{E_i = E_f}$$

$$K_{i,total} + U_i = K_{f,total} + U_f$$

$$v_{solid\ sphere} = \sqrt{\frac{2gh}{1+c}} = \sqrt{\frac{2gh}{1+\frac{2}{5}}} = \sqrt{\frac{2gh}{\frac{7}{5}}} = \sqrt{\frac{2gh}{1.4}} \quad (faster)$$

$$v_{solid\ cylinder} = \sqrt{\frac{2gh}{1+c}} = \sqrt{\frac{2gh}{1+\frac{1}{2}}} = \sqrt{\frac{2gh}{1.5}} \quad (middle)$$

$$v_{hollow\ cylinder} = \sqrt{\frac{2gh}{1+c}} = \sqrt{\frac{2gh}{1+1}} = \sqrt{\frac{2gh}{2}} \quad (slower)$$

$$\boldsymbol{K_{rot} = \frac{1}{2} mr^2 \omega^2}$$

$$\boldsymbol{I = mr^2}$$

$$\boldsymbol{K_{rot} = \frac{1}{2} I\, \omega^2}$$

$$K_{total} = (1 + c)\frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2gh}{1 + c}}$$

10.4 *Torque*

We have seen that a force can cause linear motion of an object, which can be described in terms of the motion of the center of mass of the object.

Big question: Where are the force vectors acting on an extended object placed in a free-body diagram?
Ans: A force can be exerted on an extended object at a point away from its center of mass, which can cause the object to rotate as well as move linearly.

Moment Arm

➢*The moment arm is: The perpendicular distance from the line of action of the force to the axis of rotation.*

➢ *Consider the hand attempting to use a wrench to loosen a bolt in. It is clear that*

		
It will be downright impossible to turn the bolt	*It will be quite a bit harder to turn the bolt*	*It will be easiest to turn the bolt*
Three ways to use a wrench to loosen a bolt		

Notice: • ***Force*** *is not the only relevant quantity to rotate object.*

- *The perpendicular distance from the line of action of the force to the axis of rotation, called the* ***moment arm*** $(r_{\perp})$,
- *The* ***angle*** *at which the force is applied, relative to the moment arm.*

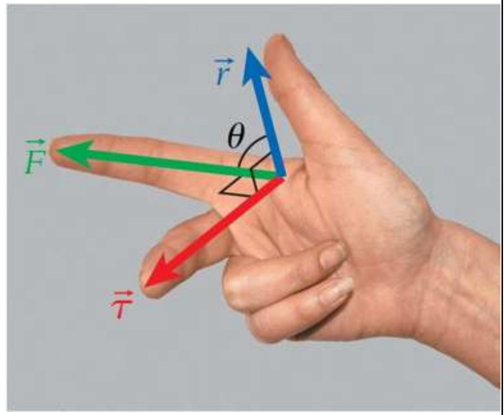
Three considerations ($\vec{F}$), ($\vec{r}$), (θ) are quantified by the concept of ***Torque ($\vec{\tau}$)*** *(also called moment).*

$$\vec{\tau} = \vec{r} \times \vec{F}$$

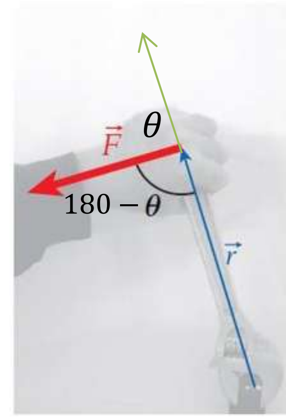
magnitude *of the torque* $\tau = rf \sin\theta$

$$\tau = r_{\perp} f$$

$$(r_{\perp} = r \sin\theta)$$



Right-hand rule for the direction of the torque for a given force and position vector.



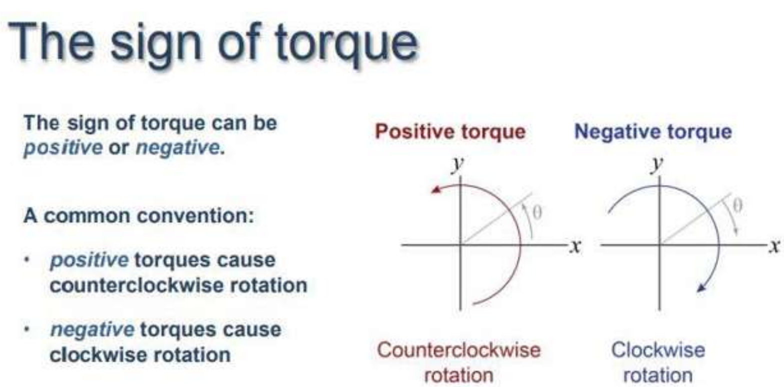
The force F and moment arm r, with the angle θ between them

Notice

- *The position vector ($\vec{r}$) is measured with the origin at the axis of rotation.*
- *The SI unit of torque is Nm, not to be confused with the unit of energy, which is the joule ($J = N.m$)*
- *Torque is an example of an axial vector. (Angular quantities can also be vectors, called axial vectors)*
- *An axial vector is any vector that points along the rotation axis.*
- *The direction of the torque is given by a right-hand rule.*
- *The torque points in a direction perpendicular to the plane spanned by the force and position vectors.*
- *Thus, if the position vector points along the thumb and the force vector points*
- *along the index finger, then the direction of the axial torque vector is the direction of the middle finger, as shown in.*
- *Torque vector is perpendicular to both the force vector and the position vector.*

The net torque is defined as the difference between the sum of all clockwise torques and the sum of all counterclockwise torques:

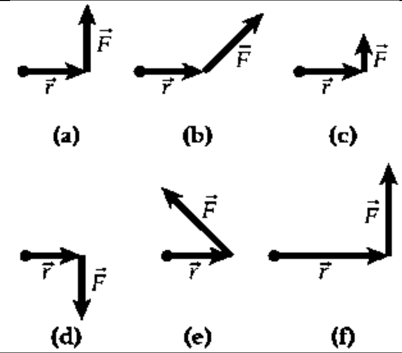
$$\tau_{net} = \sum \tau_{counterclockwise} - \sum \tau_{clockwise}$$



Concept Check 10.4

Choose the combination of position vector, $\vec{r}$, and force vector, $\vec{F}$, that produces the torque of highest magnitude around the point indicated by the black dot.

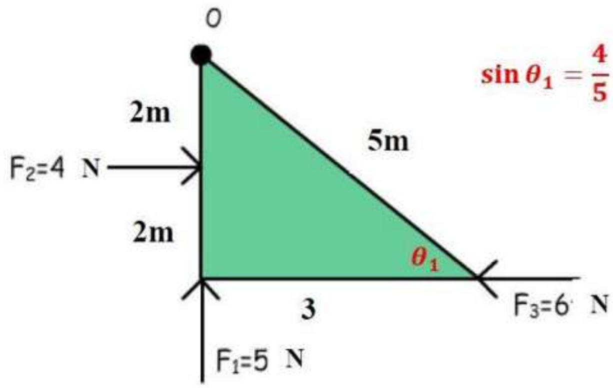
f



Practice

*If the given triangle plate is fixed from the point O and can rotate around this point, find the **net torque** applied by the given forces.*

A. 16 Nm Clockwise	C. 8 Nm Counterclockwise
B. 16 Nm Counterclockwise	D. 8 Nm Clockwise



$\tau = rf\sin\theta$	$\tau = r_\perp f$
$\tau_1 = 0\ N.m$	$\tau_1 = 0\ N.m$
$\tau_2 = r_2 f_2 \sin\theta_{r,f}$ $\tau_2 = 2 \times 4 \times \sin 90^\circ = +8\ N.m$ $counterclockwise\ (ccw)$	$\tau_2 = r_\perp f$ $\tau_2 = 2 \times 4 = +8\ N.m$ $counterclockwise\ (ccw)$
$\tau_3 = r_3 f_3 \sin\theta_{r,f}$ $\tau_3 = 5 \times 6 \times \sin\theta = 5 \times 6 \times \frac{4}{5} = 24\ N.m$ $clockwise\ (cw)$ $\tau_3 = -24\ N.m$	$\tau_2 = r_\perp f$ $\tau_3 = 4 \times 6 = 24\ N.m$ $clockwise\ (cw)$ $\tau_3 = -24\ N.m$

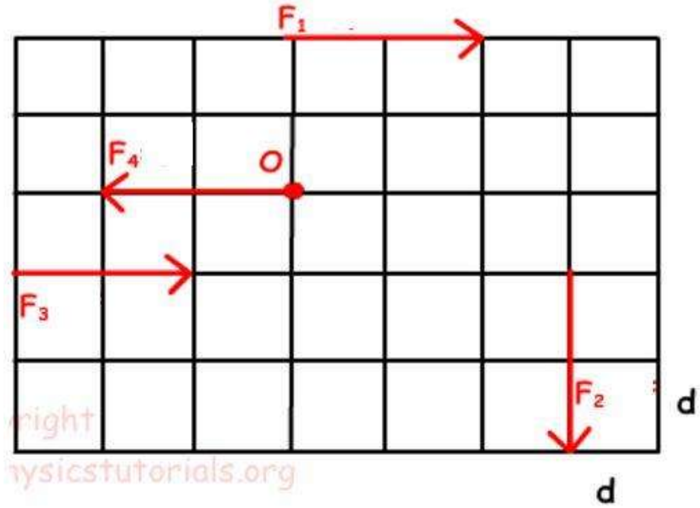
$$\tau_{total} = 0 + 8 + (-24)$$

$$\tau_{total} = -16\ N.m$$

$$\tau_{total} = 16\ N.m \qquad clockwise\ (cw)$$

Practice

If the plate is fixed from the point O, find the net torque of the given forces. (assume that distance d = 0.1 m and $F_1 = 15N$, $F_2 = 10N$, $F_3 = 25N$, $F_4 = 5N$



$\boldsymbol{\tau = rf \sin\theta}$; $\boldsymbol{\theta} = 90^{\circ} \Rightarrow \sin(90^{\circ}) = 1$
$\tau_1 = (2 \times d) \times (15) = (2 \times 0.1) \times (15) = 3\ N.m$ $\tau_1 = (0.2) \times (15) = 3\ N.m$ $clockwise$
$\tau_2 = (3 \times d) \times (10) = (3 \times 0.1) \times (10)$ $\tau_2 = (0.3) \times (10) = 3\ N.m$ $clockwise$
$\tau_3 = (1 \times d) \times (10) = (1 \times 0.1) \times (25)$ $\tau_3 = (0.1) \times (25) = 2.5\ N.m$ $counterclockwise$
$\tau_4 = 0d \times 5 = (0 \times 0.1) \times (5)$ $\tau_4 = (0) \times (5) = 0\ N.m$

$$\tau_{total} = (-3) + (-3) + (+2.5) + 0$$

$$\tau_{total} = -3.5\ N.m$$

$3.5\ N.m \quad clockwise$