CHAPTER 10

Reasoning and Proof

Then

You used segment and angle relationships.

Now

In this chapter, you will:

- Make conjectures and find counterexamples for statements.
- Use deductive reasoning to reach valid conclusions.
- Write proofs involving segment and angle theorems.

Why? ▲

SCIENCE AND NATURE Biologists and other scientists use inductive and deductive reasoning to make decisions and draw logical conclusions about animal populations.

Get Ready for the Chapter

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Evaluate each expression for the given value of x.

1. $4x + 7;\ x = 6$

2. $(x - 2)180;\ x = 8$

3. $5x^2 - 3x;\ x = 2$

4. $\frac{x(x-3)}{2};\ x = 5$

5. $x + (x + 1) + (x + 2);\ x = 3$

Write each verbal expression as an algebraic expression.

6. eight less than five times a number

7. three more than the square of a number

Solve each equation.

8. $8x - 10 = 6x$

9. $18 + 7x = 10x + 39$

10. $3(11x - 7) = 13x + 25$

11. $3x + 8 = \frac{1}{2}x + 35$

12. $\frac{2}{3}x + 1 = 5 - 2x$

13. **CLOTHING** Nabila bought 4 shirts at the mall for AED 52. Write and solve an equation to find the average cost of one shirt.

Refer to the figure in Example 3.

14. Identify a pair of vertical angles that appear to be obtuse.

15. Identify a pair of adjacent angles that appear to be complementary.

16. Identify a linear pair.

17. If $m\angle DXB = 116$ and $m\angle EXA = 3x + 2$, find x.

18. If $m\angle BXC = 90$, $m\angle CXD = 6x - 13$, and $m\angle DXE = 10x + 7$, find x.

QuickReview

Example 1

Evaluate $x^2 - 2x + 11$ for $x = 6$.

$x^2 - 2x + 11$	Original expression
$= (6)^2 - 2(6) + 11$	Substitute 6 for x.
$= 36 - 2(6) + 11$	Evaluate the exponent.
$= 36 - 12 + 11$	Multiply.
$= 35$	Simplify.

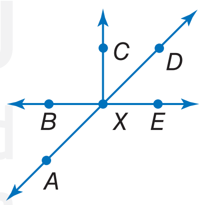
Example 2

Solve $36x - 14 = 16x + 58$.

$36x - 14 = 16x + 58$	Original equation
$36x - 14 - 16x = 16x + 58 - 16x$	Subtract $16x$ from each side.
$20x - 14 = 58$	Simplify.
$20x - 14 + 14 = 58 + 14$	Add 14 to each side.
$20x = 72$	Simplify.
$\frac{20x}{20} = \frac{72}{20}$	Divide each side by 20.
$x = 3.6$	Simplify.

Example 3

If $m\angle BXA = 3x + 5$ and $m\angle DXE = 56$, find x.



$m\angle BXA = m\angle DXE$	Vertical ∡ are ≅.
$3x + 5 = 56$	Substitution
$3x = 51$	Subtract 5 from each side.
$x = 17$	Divide each side by 3.

The term morabaha refers to an amount of money that is paid or received when borrowing or lending money. If a customer borrows money from a bank, the customer pays the bank morabaha for the use of its money. If a customer saves money in a bank account, the bank pays the customer morabaha for the use of his or her money.

The amount of money that is initially borrowed or saved is called the principal. The morabaha rate is a percentage earned or charged during a certain time period. Simple morabaha is the amount of morabaha charged or earned after the morabaha rate is applied to the principal.

Simple morabaha (I) is the product of three values: the principal (P), the morabaha rate written as a decimal number (r), and time (t): $I = P \times r \times t$.

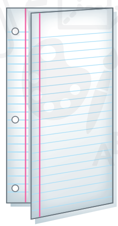
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 11. To get ready, identify important terms and organize your resources.

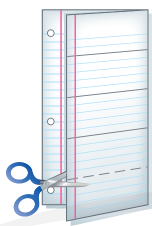
FOLDABLES® StudyOrganizer

Reasoning and Proof Make this Foldable to help you organize your Chapter 11 notes about logic, reasoning, and proof. Begin with one sheet of notebook paper.

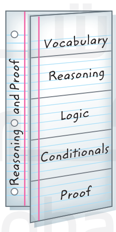
1 **Fold** lengthwise to the holes.



2 **Cut** five tabs in the top sheet.



3 **Label** the tabs as shown.



NewVocabulary

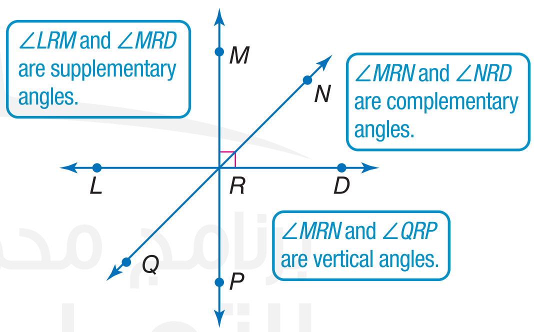
English
postulate
proof
theorem

ReviewVocabulary

complementary angles two angles with measures that have a sum of 90

supplementary angles two angles with measures that have a sum of 180

vertical angles two nonadjacent angles formed by intersecting lines



EXPLORE

10-1 Geometry Lab
Necessary and Sufficient Conditions

We all know that water is a *necessary* condition for plants to survive. However, it is not a *sufficient* condition. For example, plants also need sunlight to survive.

Necessary and sufficient conditions are important in mathematics. Consider the property of having four sides. While *having four sides* is a necessary condition for something being a square, that single condition is not, by itself, a sufficient condition to guarantee that it is a square. Trapezoids are four-sided figures that are not squares.

Condition	Definition	Examples
necessary	A condition *A* is said to be *necessary* for a condition *B*, if and only if the falsity or nonexistence of *A* guarantees the falsity or nonexistence of *B*.	Having opposite sides parallel is a necessary condition for something being a square.
sufficient	A condition *A* is said to be *sufficient* for a condition *B*, if and only if the truth or existence of *A* guarantees the truth or existence of *B*.	Being a square is a sufficient condition for something being a rectangle.

Exercises

Determine whether each statement is *true* or *false*. If false, give a counterexample.

1. Being a square is a necessary condition for being a rectangle.
2. Being a rectangle is a necessary condition for being a square.
3. Being greater than 5 is a necessary condition for being less than 10.
4. Being less than 18 is a sufficient condition for being less than 25.
5. Walking on four legs is a sufficient condition for being a dog.
6. Breathing air is a necessary condition for being a human being.
7. Being an equilateral rectangle is both a necessary and sufficient condition for being a square.

Determine whether I is a *necessary* condition for II, a *sufficient* condition for II, or *both*. Explain.

8. I. Two points are given.
   II. An equation of a line can be written.
9. I. Two planes are parallel.
   II. Two planes do not intersect.
10. I. Two angles are acute.
    II. Two angles are complementary.

LESSON 10-1 Postulates and Paragraph Proofs

Then

- You used deductive reasoning by applying the Law of Detachment and the Law of Syllogism.

Now

1. Identify and use basic postulates about points, lines, and planes.
2. Write paragraph proofs.

Why?

- If a feather and an apple are dropped from the same height in a vacuum chamber, the two objects will fall at the same rate. This demonstrates one of Sir Isaac Newton's laws of gravity and inertia. These laws are accepted as fundamental truths of physics. Some laws in geometry also must be assumed or accepted as true.

NewVocabulary

postulate
axiom
proof
theorem
deductive argument
paragraph proof
informal proof

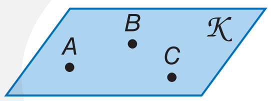
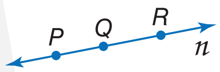
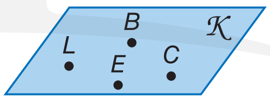
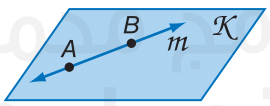
Mathematical Practices

2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.

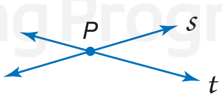
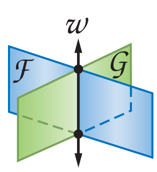
1 Points, Lines, and Planes

A **postulate** or **axiom** is a statement that is accepted as true without proof. Basic ideas about points, lines, and planes can be stated as postulates.

Postulates Points, Lines, and Planes

Words	Example	
11.1 Through any two points, there is exactly one line.		Line n is the only line through points P and R.
11.2 Through any three noncollinear points, there is exactly one plane.		Plane K is the only plane through noncollinear points A, B, and C.
11.3 A line contains at least two points.		Line n contains points P, Q, and R.
11.4 A plane contains at least three noncollinear points.		Plane K contains noncollinear points L, B, C, and E.
11.5 If two points lie in a plane, then the entire line containing those points lies in that plane.		Points A and B lie in plane K, and line m contains points A and B, so line m is in plane K.

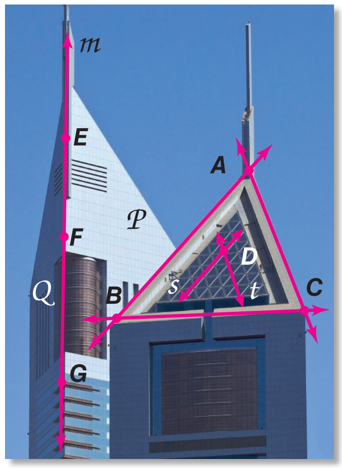
KeyConcept Intersections of Lines and Planes

Words	Example	
11.6 If two lines intersect, then their intersection is exactly one point.		Lines s and t intersect at point P.
11.7 If two planes intersect, then their intersection is a line.		Planes F and G intersect in line w.

These additional postulates form a foundation for proofs and reasoning about points, lines, and planes.

Real-World Example 1 Identifying Postulates

ARCHITECTURE Explain how the picture illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



a. Line *m* contains points *F* and *G*. Point *E* can also be on line *m*.

The edge of the building is a straight line *m*. Points *E*, *F*, and *G* lie along this edge, so they lie along a line *m*. Postulate 11.3, which states that a line contains at least two points, shows that this is true.

b. Lines *s* and *t* intersect at point *D*.

The lattice on the window of the building forms intersecting lines. Lines *s* and *t* of this lattice intersect at only one location, point *D*. Postulate 11.6, which states that if two lines intersect, then their intersection is exactly one point, shows that this is true.

GuidedPractice

1A. Points *A*, *B*, and *C* determine a plane. **1B.** Planes *P* and *Q* intersect in line *m*.

You can use postulates to explain your reasoning when analyzing statements.

Example 2 Analyze Statements Using Postulates

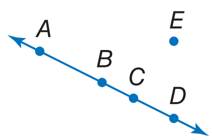
Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

a. If two coplanar lines intersect, then the point of intersection lies in the same plane as the two lines.

Always; Postulate 11.5 states that if two points lie in a plane, then the entire line containing those points lies in that plane. So, since both points lie in the plane, any point on those lines, including their point of intersection, also lies in the plane.

b. Four points are noncollinear.

Sometimes; Postulate 11.3 states that a line contains at least two points. This means that a line can contain two *or more* points. So four points can be noncollinear, like *A*, *E*, *C*, and *D*, or collinear, like points *A*, *B*, *C*, and *D*.



GuidedPractice

2A. Two intersecting lines determine a plane. **2B.** Three lines intersect in two points.

StudyTip

Axiomatic System An axiomatic system is a set of axioms, from which some or all axioms can be used to logically derive theorems.

2 Paragraph Proofs

To prove a conjecture, you use deductive reasoning to move from a hypothesis to the conclusion of the conjecture you are trying to prove. This is done by writing a **proof**, which is a logical argument in which each statement you make is supported by a statement that is accepted as true.

Once a statement or conjecture has been proven, it is called a **theorem**, and it can be used as a reason to justify statements in other proofs.

KeyConcept The Proof Process

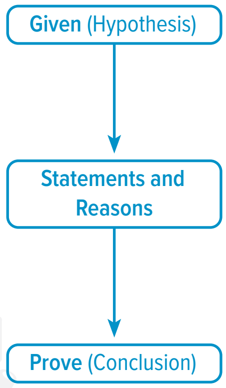
Step 1 List the given information and, if possible, draw a diagram to illustrate this information.

Step 2 State the theorem or conjecture to be proven.

Step 3 Create a **deductive argument** by forming a logical chain of statements linking the given to what you are trying to prove.

Step 4 Justify each statement with a reason. Reasons include definitions, algebraic properties, postulates, and theorems.

Step 5 State what it is that you have proven.



> **StudyTip**
> **Proposition** A *proposition* is a statement that makes an assertion that is either false or true. In mathematics, a proposition is usually used to mean a true assertion and can be synonymous with theorem.

One method of proving statements and conjectures, a **paragraph proof**, involves writing a paragraph to explain why a conjecture for a given situation is true. Paragraph proofs are also called **informal proofs**, although the term *informal* is not meant to imply that this form of proof is any less valid than any other type of proof.

Example 3 Write a Paragraph Proof

Given that M is the midpoint of $\overline{XY}$ write a paragraph proof to show that $\overline{XM} \cong \overline{MY}$.

Steps 1 and 2 →

Given: M is the midpoint of $\overline{XY}$.

Prove: $\overline{XM} \cong \overline{MY}$

Steps 3 and 4 →

If M is the midpoint of $\overline{XY}$, then from the definition of midpoint of a segment, we know that $XM = MY$. This means that $\overline{XM}$ and $\overline{MY}$ have the same measure. By the definition of congruence, if two segments have the same measure, then they are congruent.

Step 5 →

Thus, $\overline{XM} \cong \overline{MY}$.

GuidedPractice

3. Given that C is between A and B and $\overline{AC} \cong \overline{CB}$, write a paragraph proof to show that C is the midpoint of $\overline{AB}$.

> **Problem-SolvingTip**
> **Work Backward** One strategy for writing a proof is to *work backward*. Start with what you are trying to prove, and work backward step by step until you reach the given information.

Once a conjecture has been proven true, it can be stated as a theorem and used in other proofs. The conjecture in Example 3 is known as the Midpoint Theorem.

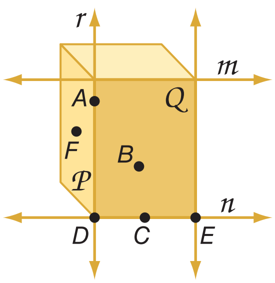
Theorem 11.1 Midpoint Theorem

If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

Check Your Understanding

Example 1

Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



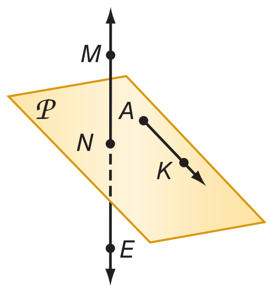
1. Planes P and Q intersect in line r.
2. Lines r and n intersect at point D.
3. Line n contains points C, D, and E.
4. Plane P contains the points A, F, and D.
5. Line n lies in plane Q.
6. Line r is the only line through points A and D.

Example 2

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

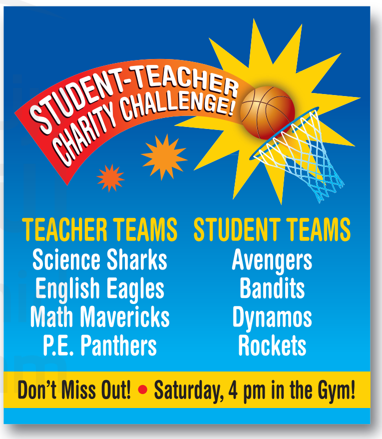
7. The intersection of three planes is a line.
8. Line r contains only point P.
9. Through two points, there is exactly one line.

In the figure, $\overrightarrow{AK}$ is in plane P and M is on $\overleftrightarrow{NE}$. State the postulate that can be used to show each statement is true.



10. M, K, and N are coplanar.
11. $\overleftrightarrow{NE}$ contains points N and M.
12. N and K are collinear.
13. Points N, K, and A are coplanar.

14. **SPORTS** Each year, Rana's school hosts a student vs. teacher basketball tournament to raise money for charity. This year, there are eight teams participating in the tournament. During the first round, each team plays all of the other teams.



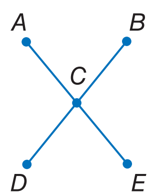
   a. How many games will be played in the first round?

   b. Draw a diagram to model the number of first round games. Which postulate can be used to justify your diagram?

   c. Find a numerical method that you could use regardless of the number of the teams in the tournament to calculate the number of games in the first round.

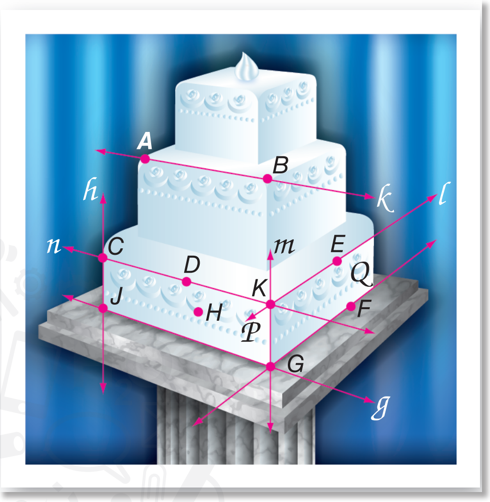
Example 3

15. **ARGUMENTS** In the figure at the right, $\overline{AE} \cong \overline{DB}$ and C is the midpoint of $\overline{AE}$ and $\overline{DB}$. Write a paragraph proof to show that $AC = CB$.



Practice and Problem Solving

Example 1 **CAKES** **Explain how the picture illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.**



16. Lines n and ℓ intersect at point K.
17. Planes P and Q intersect in line m.
18. Points D, K, and H determine a plane.
19. Point D is also on the line n through points C and K.
20. Points D and H are collinear.
21. Points E, F, and G are coplanar.
22. $\overleftrightarrow{EF}$ lies in plane Q.
23. Lines h and g intersect at point J.

Example 2 **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

24. There is exactly one plane that contains noncollinear points A, B, and C.
25. There are at least three lines through points J and K.
26. If points M, N, and P lie in plane X, then they are collinear.
27. Points X and Y are in plane Z. Any point collinear with X and Y is in plane Z.
28. The intersection of two planes can be a point.
29. Points A, B, and C determine a plane.

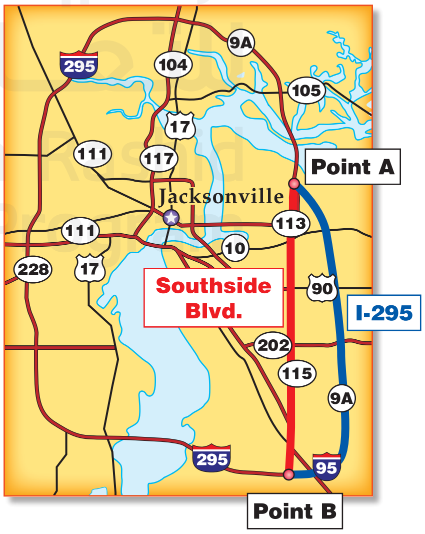
Example 3

30. **PROOF** Point Y is the midpoint of $\overline{XZ}$. Z is the midpoint of $\overline{YW}$. Prove that $\overline{XY} \cong \overline{ZW}$.

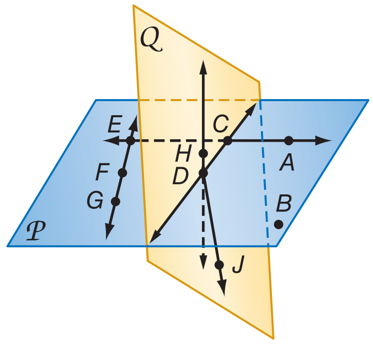
31. **PROOF** Point L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. If $\overline{MK} \cong \overline{JL}$, prove that $\overline{LK} \cong \overline{MK}$.

32. **ARGUMENTS** Last weekend, Ibrahim and his friends spent Saturday afternoon at the park. There were several people there with bikes and skateboards. There were a total of 11 bikes and skateboards that had a total of 36 wheels. Use a paragraph proof to show how many bikes and how many skateboards there were.

33. **DRIVING** Khadija is traveling from point A to point B. Two possible routes are shown on the map. Assume that the speed limit on Southside Boulevard is 55 kilometers per hour and the speed limit on I–295 is 70 kilometers per hour.
    a. Which of the two routes covers the shortest distance? Explain your reasoning.
    b. If the distance from point A to point B along Southside Boulevard is 10.5 kilometers and the distance along I-295 is 11.6 kilometers, which route is faster, assuming that Khadija drives the speed limit?



In the figure at the right, $\overleftrightarrow{CD}$ and $\overleftrightarrow{CE}$ lie in plane P and $\overleftrightarrow{DH}$ and $\overleftrightarrow{DJ}$ lie in plane Q. State the postulate that can be used to show each statement is true.



34. Points C and B are collinear.

35 $\overleftrightarrow{EG}$ contains points E, F, and G.

36. $\overleftrightarrow{DA}$ lies in plane P.

37. Points D and F are collinear.

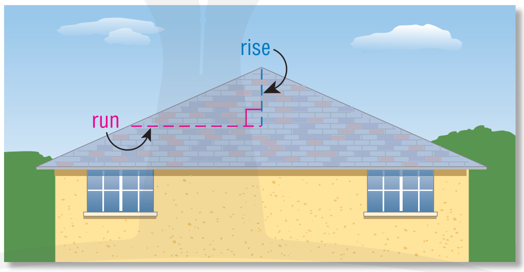
38. Points C, D, and B are coplanar.

39. Plane Q contains the points C, H, D, and J.

40. $\overleftrightarrow{AC}$ and $\overleftrightarrow{FG}$ intersect at point E.

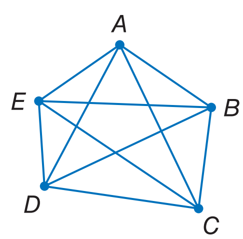
41. Plane P and plane Q intersect at $\overleftrightarrow{CD}$.

42. **ARGUMENTS** Roofs are designed based on the materials used to ensure that water does not leak into the buildings they cover. Some roofs are constructed from waterproof material, and others are constructed for watershed, or gravity removal of water. The pitch of a roof is the rise over the run, which is generally measured in rise per meter of run. Use the statements below to write a paragraph proof justifying the following statement: The pitch of the roof in Manal's design is not steep enough.



- Waterproof roofs should have a minimum slope of $\frac{1}{4}$ centimeter per meter.
- Watershed roofs should have a minimum slope of 4 centimeters per meter.
- Manal is designing a house with a watershed roof.
- The pitch in Manal's design is 2 centimeters per meter.

43. **NETWORKS** Amer is setting up a network of multiple computers so that each computer is connected to every other. The diagram at the right illustrates this network if Amer has 5 computers.



a. Draw diagrams of the networks if Amer has 2, 3, 4, or 6 computers.

b. Create a table with the number of computers and the number of connections for the diagrams you drew.

c. If there are n computers in the network, write an expression for the number of computers to which each of the computers is connected.

d. If there are n computers in the network, write an expression for the number of connections there are.

44. SENSE-MAKING The photo is of the rotunda in the Pantheon in Rome, Italy. A rotunda is a round building, usually covered by a dome.

a. If you were standing in the middle of the rotunda, which arched exit is the closest to you?

b. What information did you use to formulate your answer?

c. What term describes the shortest distance from the center of a circle to a point on the circle?

H.O.T. Problems Use Higher-Order Thinking Skills

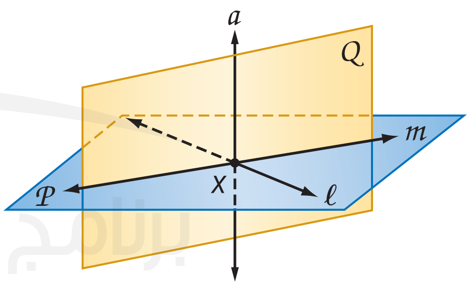
45. ERROR ANALYSIS Obaid and Ali were working on a paragraph proof to prove that if $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear, then B is the midpoint of $\overline{AD}$. Each student started his or her proof in a different way. Is either of them correct? Explain your reasoning.

Obaid	Ali
If B is the midpoint of $\overline{AB}$, then B divides $\overline{AD}$ into two congruent segments.	$\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear.

46. OPEN ENDED Draw a figure that satisfies five of the seven postulates you have learned. Explain which postulates you chose and how your figure satisfies each postulate.

47. CHALLENGE Use the following true statement and the definitions and postulates you have learned to answer each question.

Two planes are perpendicular if and only if one plane contains a line perpendicular to the second plane.



a. Through a given point, there passes one and only one plane perpendicular to a given line. If plane Q is perpendicular to line ℓ at point X and line ℓ lies in plane P, what must also be true?

b. Through a given point, there passes one and only one line perpendicular to a given plane. If plane Q is perpendicular to plane P at point X and line a lies in plane Q, what must also be true?

REASONING Determine if each statement is *sometimes*, *always*, or *never* true. Explain your reasoning or provide a counterexample.

48. Through any three points, there is exactly one plane.

49. Three coplanar lines have two points of intersection.

50. WRITING IN MATH How does writing a proof require logical thinking?

Standardized Test Practice

51. ALGEBRA Which is one of the solutions of the equation $3x^2 - 5x + 1 = 0$?

A $\frac{5 + \sqrt{13}}{6}$

B $\frac{-5 - \sqrt{13}}{6}$

C $\frac{5}{6} - \sqrt{13}$

D $-\frac{5}{6} + \sqrt{13}$

52. GRIDDED RESPONSE Sultan has 20 marbles in a bag, all the same size and shape. There are 8 red, 2 blue, and 10 yellow marbles in the bag. He will select a marble from the bag at random. What is the probability that the marble Sultan selects will be yellow?

53. Which statement *cannot* be true?

F Three noncollinear points determine a plane.

G Two lines intersect in exactly one point.

H At least two lines can contain the same two points.

J A midpoint divides a segment into two congruent segments.

54. SAT/ACT What is the greatest number of regions that can be formed if 3 distinct lines intersect a circle?

A 3

B 4

C 5

D 6

E 7

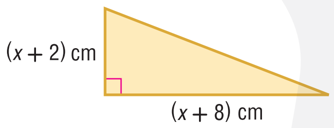
Spiral Review

Solve each equation by completing the square. Round to the nearest tenth if necessary.

55. $x^2 + 4x - 8 = 5$

56. $3x^2 + 5x = 18$

57. Find the value of x in the figure if the area is 36 square centimeters.



Factor each polynomial.

58. $\frac{1}{2}t^2 - 162$

59. $25d^2 - 49d$

60. $196t^2u^3 - 144u^3$

61. $169a^4b^6 - 121c^8$

62. $4g^2 - 1296h^2$

63. $18a^3 + 27a^2 - 50a - 75$

64. BIOLOGY During an experiment, the number of cells of a virus can be modeled by $f(t) = 2^{t-2}$, where t is the time in days and $f(t)$ is the number of cells. Determine how many days have passed if there are 64 virus cells.

Solve each equation by factoring.

65. $2x^2 + x - 10 = 0$

66. $2x^2 + x = 28$

Simplify.

67. $(10 + 3i) + (3 - 7i)$

68. $(2 + i)(2 - i)$

69. $\frac{5}{1 + 3i}$

70. HEIGHT Mazen is 172.7 centimeters tall. How many inches tall is Mazen?

Skills Review

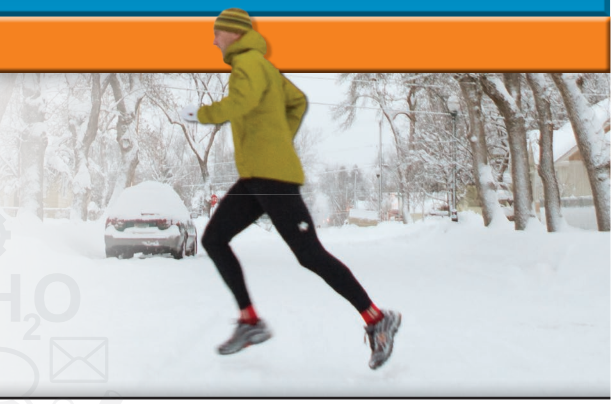
ALGEBRA Solve each equation.

71. $4x - 3 = 19$

72. $\frac{1}{3}x + 6 = 14$

73. $5(x^2 + 2) = 30$

LESSON 10-2 Algebraic Proofs



Then

- You used postulates about points, lines, and planes to write paragraph proofs.

Now

1. Use algebra to write two-column proofs.
2. Use properties of equality to write geometric proofs.

Why?

- The Fahrenheit scale sets the freezing and boiling points of water at 32° and 212°, respectively, while the Celsius scale sets them at 0° and 100°. You can use an algebraic proof to show that if these scales are related by the formula $C = \frac{5}{9}(F - 32)$, then they are also related by the formula $F = \frac{9}{5}C + 32$.

NewVocabulary
algebraic proof
two-column proof
formal proof

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.

1 Algebraic Proof

Algebra is a system with sets of numbers, operations, and properties that allow you to perform algebraic operations. The following table summarizes several properties of real numbers that you studied in algebra.

KeyConcept Properties of Real Numbers

The following properties are true for any real numbers a, b, and c.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then, $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

An **algebraic proof** is a proof that is made up of a series of algebraic statements. The properties of equality provide justification for many statements in algebraic proofs.

Example 1 Justify Each Step When Solving an Equation

Prove that if $-5(x + 4) = 70$, then $x = -18$. Write a justification for each step.

$-5(x + 4) = 70$	Original equation or Given
$-5x + (-5)4 = 70$	Distributive Property
$-5x - 20 = 70$	Substitution Property of Equality
$-5x - 20 + 20 = 70 + 20$	Addition Property of Equality
$-5x = 90$	Substitution Property of Equality
$\frac{-5x}{-5} = \frac{90}{-5}$	Division Property of Equality
$x = -18$	Substitution Property of Equality

GuidedPractice

State the property that justifies each statement.

1A. If $4 + (-5) = -1$, then $x + 4 + (-5) = x - 1$.

1B. If $5 = y$, then $y = 5$.

1C. Prove that if $2x - 13 = -5$, then $x = 4$. Write a justification for each step.

Example 1 is a proof of the conditional statement *If* $-5(x + 4) = 70$, *then* $x = -18$. Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reason for each statement.

In geometry, a similar format is used to prove conjectures and theorems. A **two-column proof** or **formal proof** contains *statements* and *reasons* organized in two columns.

StudyTip

Arguments An *algorithm* is a series of steps for carrying out a procedure or solving a problem. Proofs can be considered a type of algorithm because they go step by step.

Real-World Example 2 Write an Algebraic Proof

SCIENCE If the formula to convert a Fahrenheit temperature to a Celsius temperature is $C = \frac{5}{9}(F - 32)$, then the formula to convert a Celsius temperature to a Fahrenheit temperature is $F = \frac{9}{5}C + 32$. Write a two-column proof to verify this conjecture.

Begin by stating what is given and what you are to prove.

Given: $C = \frac{5}{9}(F - 32)$

Prove: $F = \frac{9}{5}C + 32$

Proof:

Statements	**Reasons**
1. $C = \frac{5}{9}(F - 32)$	**1.** Given
2. $\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$	**2.** Multiplication Property of Equality
3. $\frac{9}{5}C = F - 32$	**3.** Substitution Property of Equality
4. $\frac{9}{5}C + 32 = F - 32 + 32$	**4.** Addition Property of Equality
5. $\frac{9}{5}C + 32 = F$	**5.** Substitution Property of Equality
6. $F = \frac{9}{5}C + 32$	**6.** Symmetric Property of Equality

StudyTip

Mental Math If your teacher permits you to do so, some steps may be eliminated by performing mental calculations. For example, steps 2 and 4 in Example 2 could be omitted. Then the reason for statement 3 would be Multiplication Property of Equality and the reason for statement 5 would be Addition Property of Equality.

GuidedPractice

Write a two-column proof to verify that each conjecture is true.

2A. If $\frac{5x + 1}{2} - 8 = 0$, then $x = 3$.

2B. **PHYSICS** If the distance d moved by an object with initial velocity u and final velocity v in time t is given by $d = t \cdot \frac{u + v}{2}$, then $u = \frac{2d}{t} - v$.

2 Geometric Proof

Since geometry also uses variables, numbers, and operations, many of the properties of equality used in algebra are also true in geometry. For example, segment measures and angle measures are real numbers, so properties from algebra can be used to discuss their relationships as shown in the table below.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Study Tip

Commutative and Associative Properties Throughout this text we shall assume that if a, b, and c are real numbers, then the following properties are true.

Commutative Property of Addition
$a + b = b + a$

Commutative Property of Multiplication
$a \cdot b = b \cdot a$

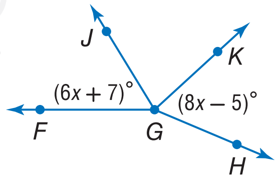
Associative Property of Addition
$(a + b) + c = a + (b + c)$

Associative Property of Multiplication
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

These properties can be used to write geometric proofs.

Example 3 Write a Geometric Proof

If $\angle FGJ \cong \angle JGK$ and $\angle JGK \cong \angle KGH$, then $x = 6$. Write a two-column proof to verify this conjecture.



Given: $\angle FGJ \cong \angle JGK$, $\angle JGK \cong \angle KGH$, $m\angle FGJ = 6x + 7$, $m\angle KGH = 8x - 5$

Prove: $x = 6$

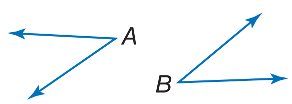
Proof:

Statements	Reasons
1. $m\angle FGH = 6x + 7$, $m\angle KGH = 8x - 5$ $\angle FGJ \cong \angle JGK$; $\angle JGK \cong \angle KGH$	1. Given
2. $m\angle FGJ = m\angle JGK$; $m\angle JGK = m\angle KGH$	2. Definition of congruent angles
3. $m\angle FGJ = m\angle KGH$	3. Transitive Property of Equality
4. $6x + 7 = 8x - 5$	4. Substitution Property of Equality
5. $6x + 7 + 5 = 8x - 5 + 5$	5. Addition Property of Equality
6. $6x + 12 = 8x$	6. Substitution Property of Equality
7. $6x + 12 - 6x = 8x - 6x$	7. Subtraction Property of Equality
8. $12 = 2x$	8. Substitution Property of Equality
9. $\frac{12}{2} = \frac{2x}{2}$	9. Division Property of Equality
10. $6 = x$	10. Substitution Property of Equality
11. $x = 6$	11. Symmetric Property of Equality

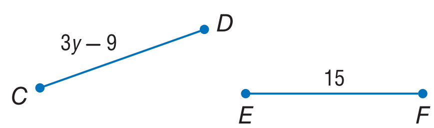
Guided Practice

Write a two-column proof to verify each conjecture.

3A. If $\angle A \cong \angle B$ and $m\angle A = 37$, then $m\angle B = 37$.



3B. If $\overline{CD} \cong \overline{EF}$, then $y = 8$.



Check Your Understanding

Example 1 **State the property that justifies each statement.**

1. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.
2. $XY = XY$
3. If $5 = x$, then $x = 5$.
4. If $2x + 5 = 11$, then $2x = 6$.

Example 2

5. Complete the following proof.

Given: $\dfrac{y+2}{3} = 3$

Prove: $y = 7$

Proof:

Statements	Reasons
a. ?	**a.** Given
b. $3\left(\dfrac{y+2}{3}\right) = 3(3)$	**b.** ?
c. ?	**c.** ?
d. $y = 7$	**d.** Subtraction Property

Examples 2–3 **PROOF** **Write a two-column proof to verify each conjecture.**

6. If $-4(x - 3) + 5x = 24$, then $x = 12$.
7. If $\overline{AB} \cong \overline{CD}$, then $x = 7$.

8. **ARGUMENTS** Maha measures her heart rate whenever she exercises and tries to make sure that she is staying in her target heart rate zone. The American Heart Association suggests a target heart rate of $T = 0.75(220 - a)$, where T is a person's target heart rate and a is his or her age.
   a. Prove that given a person's target heart rate, you can calculate his or her age using the formula $a = 220 - \dfrac{T}{0.75}$.
   b. If Maha target heart rate is 153, then how old is she? What property justifies your calculation?

Practice and Problem Solving

Example 1 **State the property that justifies each statement.**

9. If $a + 10 = 20$, then $a = 10$.
10. If $\dfrac{x}{3} = -15$, then $x = -45$.
11. If $4x - 5 = x + 12$, then $4x = x + 17$.
12. If $\dfrac{1}{5}BC = \dfrac{1}{5}DE$, then $BC = DE$.

State the property that justifies each statement.

13. If $5(x + 7) = -3$, then $5x + 35 = -3$.

14. If $m\angle 1 = 25$ and $m\angle 2 = 25$, then $m\angle 1 = m\angle 2$.

15. If $AB = BC$ and $BC = CD$, then $AB = CD$.

16. If $3\left(x - \frac{2}{3}\right) = 4$, then $3x - 2 = 4$.

Example 2 **ARGUMENTS** **Complete each proof.**

17. **Given:** $\frac{8 - 3x}{4} = 32$

Prove: $x = -40$

Proof:

Statements	Reasons
a. $\frac{8 - 3x}{4} = 32$	**a.** Given
b. $4\left(\frac{8 - 3x}{4}\right) = 4(32)$	**b.** ___?___
c. $8 - 3x = 128$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $x = -40$	**e.** ___?___

18. **Given:** $\frac{1}{5}x + 3 = 2x - 24$

Prove: $x = 15$

Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. ___?___	**b.** Multiplication Property
c. $x + 15 = 10x - 120$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $135 = 9x$	**e.** ___?___
f. ___?___	**f.** Division Property
g. ___?___	**g.** Symmetric Property

Example 3 **PROOF** **Write a two-column proof to verify each conjecture.**

19. If $-\frac{1}{3}n = 12$, then $n = -36$.

20. If $-3r + \frac{1}{2} = 4$, then $r = -\frac{7}{6}$.

21 **SCIENCE** Acceleration a in meters per second squared, distance traveled d in meters, velocity v in meters per second, and time t in seconds are related in the formula $d = vt + \frac{1}{2}at^2$.

a. Prove that if the values for distance, velocity, and time are known, then the acceleration of an object can be calculated using the formula $a = \frac{2d - 2vt}{t^2}$.

b. If an object travels 2850 meters in 30 seconds with an initial velocity of 50 meters per second, what is the acceleration of the object? What property justifies your calculation?

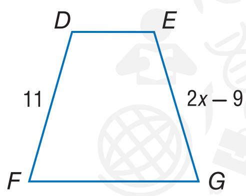
22. **ARGUMENTS** The Ideal Gas Law is given by the formula $PV = nRT$, where P = pressure in atmospheres, V = volume in liters, n = the amount of gas in moles, R is a constant value, and T = temperature in degrees Kelvin.

a. Prove that if the pressure, volume, and amount of the gas are known, then the formula $T = \frac{PV}{nR}$ gives the temperature of the gas.

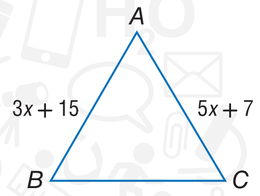
b. If you have 1 mole of oxygen with a volume of 25 liters at a pressure of 1 atmosphere, what is the temperature of the gas? The value of R is 0.0821. What property justifies your calculation?

PROOF **Write a two-column proof.**

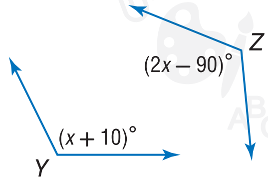
23. If $\overline{DF} \cong \overline{EG}$, then $x = 10$.



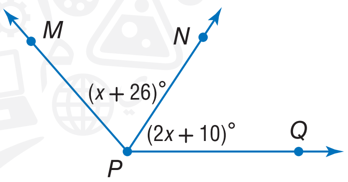
24. If $\overline{AB} \cong \overline{AC}$, then $x = 4$.



25. If $\angle Y \cong \angle Z$, then $x = 100$.



26. If $\angle MPN \cong \angle QPN$, then $x = 16$.



27. **ELECTRICITY** The voltage V of a circuit can be calculated using the formula $V = \frac{P}{I}$, where P is the power and I is the current of the circuit.

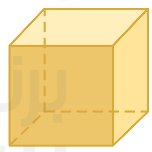
a. Write a proof to show that when the power is constant, the voltage is halved when the current is doubled.

b. Write a proof to show that when the current is constant, the voltage is doubled when the power is doubled.

28. **MULTIPLE REPRESENTATIONS** Consider a cube with a side length of s.

a. **Concrete** Sketch or build a model of cubes with side lengths of 2, 4, 8, and 16 units.

b. **Tabular** Find the volume of each cube. Organize your results into a table like the one shown.



s units

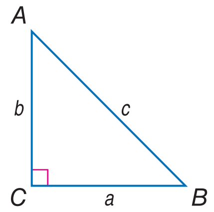
Side Lenth (s)	Volume (V)
2	
4	
8	
16	

c. **Verbal** Use your table to make a conjecture about the change in volume when the side length of a cube is doubled. Express your conjecture in words.

d. **Analytical** Write your conjecture as an algebraic equation.

e. **Logical** Write a proof of your conjecture. Be sure to write the *Given* and *Prove* statements at the beginning of your proof.

29. **PYTHAGOREAN THEOREM** The Pythagorean Theorem states that in a right triangle ABC, the sum of the squares of the measures of the lengths of the legs, a and b, equals the square of the measure of the hypotenuse c, or $a^2 + b^2 = c^2$. Write a two-column proof to verify that $a = \sqrt{c^2 - b^2}$. Use the Square Root Property of Equality, which states that if $a^2 = b^2$, then $a = \pm\sqrt{b^2}$.



An *equivalence relation* is any relationship that satisfies the Reflexive, Symmetric, and Transitive Properties. For real numbers, equality is one type of equivalence relation. Determine whether each relation is an equivalence relation. Explain your reasoning.

30. "has the same birthday as," for the set of all human beings

31. "is taller than," for the set of all human beings

32. "is bluer than" for all the paint colors with blue in them

33. $\neq$, for the set of real numbers

34. $\geq$, for the set of real numbers

35. $\approx$, for the set of real numbers

H.O.T. Problems Use Higher-Order Thinking Skills

36. **OPEN ENDED** Give one real-world *example* and one real-world *non-example* of the Symmetric, Transitive, and Substitution properties.

37. **SENSE-MAKING** Point P is located on $\overline{AB}$. The length of $\overline{AP}$ is $2x + 3$, and the length of $\overline{PB}$ is $\frac{3x + 1}{2}$. Segment AB is 10.5 units long. Draw a diagram of this situation, and prove that point P is located two thirds of the way between point A and point B.

REASONING **Classify each statement below as *sometimes*, *always*, or *never* true. Explain your reasoning.**

38. If a and b are real numbers and $a + b = 0$, then $a = -b$.

39. If a and b are real numbers and $a^2 = b$, then $a = \sqrt{b}$.

40. **CHALLENGE** Alia makes a conjecture that the sum of two odd integers is an even integer.

 a. List information that supports this conjecture. Then explain why the information you listed does not prove that this conjecture is true.

 b. Two odd integers can be represented by the expressions $2n - 1$ and $2m - 1$, where n and m are both integers. Give information that supports this statement.

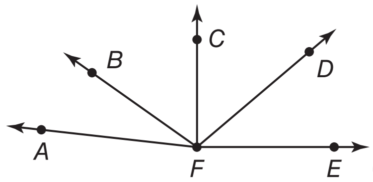
 c. If a number is even, then it is a multiple of what number? Explain in words how you could use the expressions in part **a** and your answer to part **b** to prove Alia's conjecture.

 d. Write an algebraic proof that the sum of two odd integers is an even integer.

41. **WRITING IN MATH** Why is it useful to have different formats that can be used when writing a proof?

Standardized Test Practice

42. In the diagram, $m\angle CFE = 90$ and $\angle AFB \cong \angle CFD$. Which of the following conclusions does not have to be true?



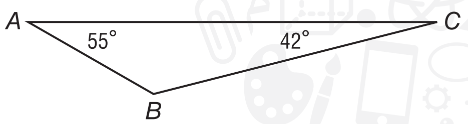
A $m\angle BFD = m\angle BFD$

B $\overline{BF}$ bisects $\angle AFD$.

C $m\angle CFD = m\angle AFB$

D $\angle CFE$ is a right angle.

43. SHORT RESPONSE Find the measure of $\angle B$ when $m\angle A = 55$ and $m\angle C = 42$.



44. ALGEBRA Khawla walk-a-thon supporters have pledged AED 30 plus AED 7.50 for each kilometer she walks. Maysa's supporters have pledged AED 45 plus AED 3.75 for each kilometer she walks. After how many kilometers will Khawla and Maysa have raised the same amount of money?

F 10

G 8

H 5

J 4

45. SAT/ACT When 17 is added to $4m$, the result is $15z$. Which of the following equations represents the statement above?

A $17 + 15z = 4m$

B $(4m)(15z) = 17$

C $4m - 15z = 17$

D $17(4m) = 15z$

E $4m + 17 = 15z$

Spiral Review

Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain. (Lesson 11-1)

46. Four points will lie in one plane.

47. Two obtuse angles will be supplementary.

48. Planes P and Q intersect in line m. Line m lies in both plane P and plane Q.

Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph of each equation.

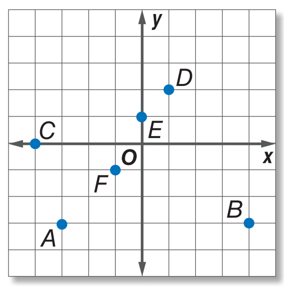
49. $y = 4x^2 + 8x - 5$

50. $y = -2x^2 + 8x + 5$

51. $y = x^2 - 8x + 9$

52. $y = 4x^2 + 16x - 6$

Write the ordered pair for each point shown.



53. A

54. B

55. C

56. D

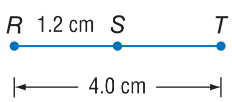
57. E

58. F

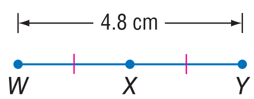
Skills Review

Find the measurement of each segment. Assume that each figure is not drawn to scale.

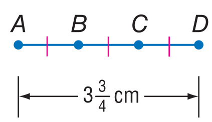
59. $\overline{ST}$



60. $\overline{WX}$



61. $\overline{BC}$



LESSON 10-3

Proving Segment Relationships

Then

- You wrote algebraic and two-column proofs.

Now

- 1 Write proofs involving segment addition.
- 2 Write proofs involving segment congruence.

Why?

- Abeer works at a fabric store after school. She measures a length of fabric by holding the straight edge of the fabric against a yardstick. To measure lengths such as 39 inches, she marks a length of 36 inches. From the end of that mark, she measures an additional length of 3 inches. This ensures that the total length of fabric is 36 + 3 inches or 39 inches.

Mathematical Practices

2 Reason abstractly and quantitatively.

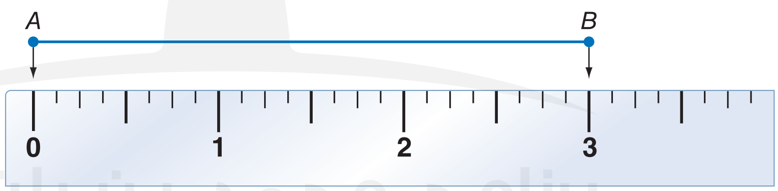
3 Construct viable arguments and critique the reasoning of others.

1 Ruler Postulate

Postulate 11.8 Ruler Postulate

Words The points on any line or line segment can be put into one-to-one correspondence with real numbers.

Symbols Given any two points *A* and *B* on a line, if *A* corresponds to zero, then *B* corresponds to a positive real number.

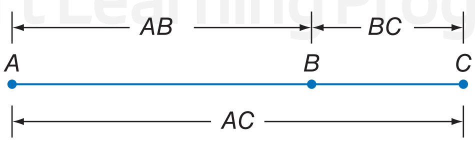


Below is the Segment Addition Postulate.

Postulate 11.9 Segment Addition Postulate

Words If *A*, *B*, and *C* are collinear, then point *B* is between *A* and *C* if and only if $AB + BC = AC$.

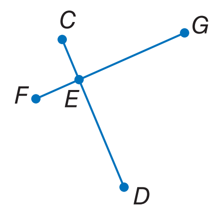
Symbols



The Segment Addition Postulate is used as a justification in many geometric proofs.

Example 1 Use the Segment Addition Postulate

Prove that if $\overline{CE} \cong \overline{FE}$ and $\overline{ED} \cong \overline{EG}$ then $\overline{CD} \cong \overline{FG}$.



Given: $\overline{CE} \cong \overline{FE}$; $\overline{ED} \cong \overline{EG}$

Prove: $\overline{CD} \cong \overline{FG}$

Proof:

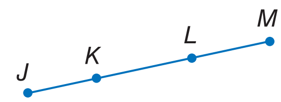
Statements	Reasons
1. $\overline{CE} \cong \overline{FE}$; $\overline{ED} \cong \overline{EG}$	**1.** Given
2. $CE = FE$; $ED = EG$	**2.** Definition of congruence
3. $CE + ED = CD$	**3.** Segment Addition Postulate
4. $FE + EG = CD$	**4.** Substitution (Steps 2 & 3)
5. $FE + EG = FG$	**5.** Segment Addition Postulate
6. $CD = FG$	**6.** Substitution (Steps 4 & 5)
7. $\overline{CD} \cong \overline{FG}$	**7.** Definition of congruence

ReadingMath

Substitution Property The Substitution Property of Equality is often just written as *Substitution*.

GuidedPractice

Copy and complete the proof.



1. **Given:** $\overline{JL} \cong \overline{KM}$

Prove: $\overline{JK} \cong \overline{LM}$

Proof:

Statements	Reasons
a. $\overline{JL} \cong \overline{KM}$	**a.** Given
b. $JL = KM$	**b.** ___?___
c. $JK + KL =$ ___?___ ; $KL + LM =$ ___?___	**c.** Segment Addition Postulate
d. $JK + KL = KL + LM$	**d.** ___?___
e. $JK + KL - KL = KL + LM - KL$	**e.** Subtraction Property of Equality
f. ___?___	**f.** Substitution
g. $\overline{JK} \cong \overline{LM}$	**g.** Definition of congruence

2 Segment Congruence

In Lesson 11-2, you saw that segment measures are reflexive, symmetric, and transitive. Since segments with the same measure are congruent, congruence of segments is also reflexive, symmetric, and transitive.

Theorem 11.2 Properties of Segment Congruence

Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

You will prove the Symmetric and Reflexive Properties in Exercises 6 and 7, respectively.

VocabularyLink

Symmetric

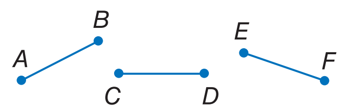
Everyday Use balanced or proportional

Math Use If $a = b$, then $b = a$.

Proof Transitive Property of Congruence

Given: $\overline{AB} \cong \overline{CD}$; $\overline{CD} \cong \overline{EF}$

Prove: $\overline{AB} \cong \overline{EF}$



Paragraph Proof:

Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, $AB = CD$ and $CD = EF$ by the definition of congruent segments. By the Transitive Property of Equality, $AB = EF$. Thus, $\overline{AB} \cong \overline{EF}$ by the definition of congruence.

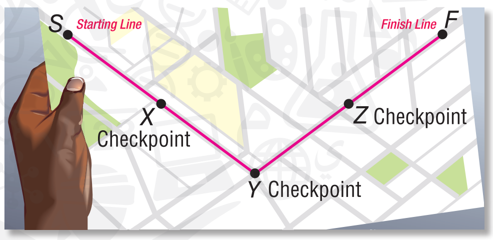
Real-WorldLink

According to a recent poll, 70% of teens who volunteer began doing so before age 12. Others said they would volunteer if given more opportunities to do so.

Source: Youth Service America

Real-World Example 2 Proof Using Segment Congruence

VOLUNTEERING The route for a charity fitness run is shown. Checkpoints X and Z are the midpoints between the starting line and Checkpoint Y and Checkpoint Y and the finish line F, respectively. If Checkpoint Y is the same distance from Checkpoints X and Z, prove that the route from Checkpoint Z to the finish line is congruent to the route from the starting line to Checkpoint X.



Given: X is the midpoint of $\overline{SY}$. Z is the midpoint of $\overline{YF}$. $XY = YZ$

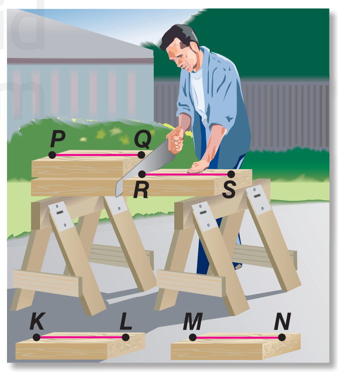
Prove: $\overline{ZF} \cong \overline{SX}$

Two-Column Proof:

Statements	Reasons
1. X is the midpoint of $\overline{SY}$. Z is the midpoint of $\overline{YF}$. $XY = YZ$	1. Given
2. $\overline{SX} \cong \overline{XY}$; $\overline{YZ} \cong \overline{ZF}$	2. Definition of midpoint
3. $\overline{XY} \cong \overline{YZ}$	3. Definition of congruence
4. $\overline{SX} \cong \overline{YZ}$	4. Transitive Property of Congruence
5. $\overline{SX} \cong \overline{ZF}$	5. Transitive Property of Congruence
6. $\overline{ZF} \cong \overline{SX}$	6. Symmetric Property of Congruence

GuidedPractice

2. **CARPENTRY** A carpenter cuts a 20 centimeter by 40 centimeter board to a desired length. He then uses this board as a pattern to cut a second board congruent to the first. Similarly, he uses the second board to cut a third board and the third board to cut a fourth board. Prove that the last board cut has the same measure as the first.



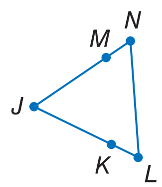
Check Your Understanding

Example 1

1. ARGUMENTS Copy and complete the proof.

Given: $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$

Prove: $\overline{LJ} \cong \overline{NJ}$



Proof:

Statements	Reasons
a. $\overline{LK} \cong \overline{NM}$, $\overline{KJ} \cong \overline{MJ}$	**a.** ___?___
b. ___?___	**b.** Def. of congruent segments
c. $LK + KJ = NM + MJ$	**c.** ___?___
d. ___?___	**d.** Segment Addition Postulate
e. $LJ = NJ$	**e.** ___?___
f. $\overline{LJ} \cong \overline{NJ}$	**f.** ___?___

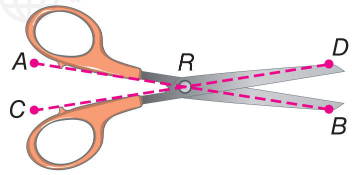
Example 2

2. PROOF Prove the following.

Given: $\overline{WX} \cong \overline{YZ}$

Prove: $\overline{WY} \cong \overline{XZ}$

3 **SCISSORS** Refer to the diagram shown. $\overline{AR}$ is congruent to $\overline{CR}$. $\overline{DR}$ is congruent to $\overline{BR}$. Prove that $AR + DR = CR + BR$.



Practice and Problem Solving

Example 1

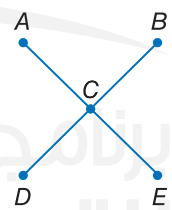
4. ARGUMENTS Copy and complete the proof.

Given: C is the midpoint of $\overline{AE}$.

C is the midpoint of $\overline{BD}$.

$\overline{AE} \cong \overline{BD}$

Prove: $\overline{AC} \cong \overline{CD}$

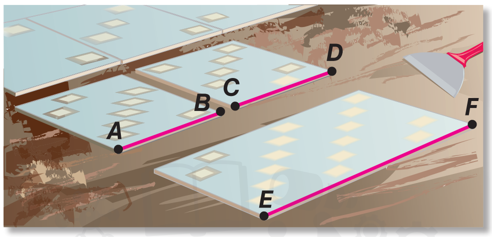


Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. $AC = CE$, $BC = CD$	**b.** ___?___
c. $AE = BD$	**c.** ___?___
d. ___?___	**d.** Segment Addition Postulate
e. $AC + CE = BC + CD$	**e.** ___?___
f. $AC + AC = CD + CD$	**f.** ___?___
g. ___?___	**g.** Simplify.
h. ___?___	**h.** Division Property
i. $\overline{AC} \cong \overline{CD}$	**i.** ___?___

Example 2

5. **TILING** A tile setter cuts a piece of tile to a desired length. He then uses this tile as a pattern to cut a second tile congruent to the first. He uses the first two tiles to cut a third tile whose length is the sum of the measures of the first two tiles. Prove that the measure of the third tile is twice the measure of the first tile.



ARGUMENTS **Prove each theorem.**

6. Symmetric Property of Congruence (Theorem 11.2)

7. Reflexive Property of Congruence (Theorem 11.2)

8. **TRAVEL** Four cities are connected by Interstate 90: City A, City B, City C and City D, City A is the farthest west.

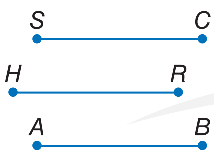
- City C is 126 kilometers from City D and 263 kilometers from City A.
- City A is 137 kilometers from City D and 184 kilometers from City B.

a. Draw a diagram to represent the locations of the cities in relation to each other and the distances between each city. Assume that Interstate 90 is straight.

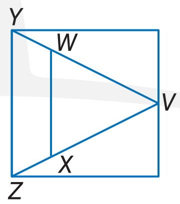
b. Write a paragraph proof to support your conclusion.

PROOF **Prove the following.**

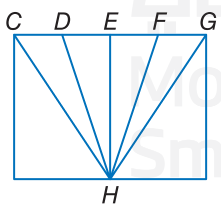
9. If $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$, then $\overline{SC} \cong \overline{AB}$.



10. If $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$, then $\overline{VW} \cong \overline{VX}$.



11. If E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$, then $\overline{CE} \cong \overline{EG}$.

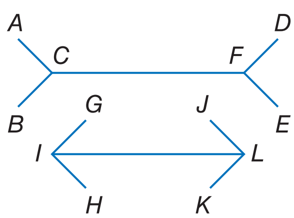


12. If B is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$, then $AE = 4AB$.

13. **OPTICAL ILLUSION** $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, and $AC + CF + FE = GI + IL + LK$.

a. Prove that $\overline{CF} \cong \overline{IL}$.

b. Justify your proof using measurement. Explain your method.

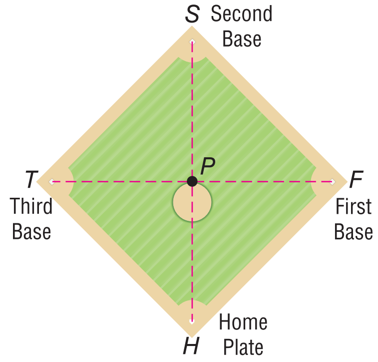


14. **CONSTRUCTION** Construct a segment that is twice as long as $\overline{PQ}$. Explain how the Segment Addition Postulate can be used to justify your construction.

15 **BASEBALL** Use the diagram of a baseball diamond shown.

a. On the diagram, $\overline{SH} \cong \overline{TF}$. P is the midpoint of $\overline{SH}$ and $\overline{TF}$. Using a two-column proof, prove that $\overline{SP} \cong \overline{TP}$.

b. The distance from home plate to second base is 38.8 meters. Calculate the distance from first base to second base?



16. **MULTIPLE REPRESENTATIONS** A is the midpoint of $\overline{PQ}$, B is the midpoint of $\overline{PA}$, and C is the midpoint of $\overline{PB}$.

a. **Geometric** Make a sketch to represent this situation.

b. **Algebraic** Make a conjecture as to the algebraic relationship between PC and PQ.

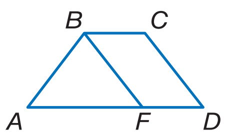
c. **Geometric** Copy segment $\overline{PQ}$ from your sketch. Then construct points B and C on $\overline{PQ}$. Explain how you can use your construction to support your conjecture.

d. **Concrete** Use a ruler to draw a segment congruent to $\overline{PQ}$ from your sketch and to draw points B and C on $\overline{PQ}$. Use your drawing to support your conjecture.

e. **Logical** Prove your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

17. **CRITIQUE** In the diagram, $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$. Examine the conclusions made by Lamya and Sally. Is either of them correct?



Lamya	Sally
Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{AF}$ by the Transitive Property of Congruence	Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{BF}$ by the Reflexive Property of Congruence.

18. **CHALLENGE** $ABCD$ is a square. Prove that $\overline{AC} \cong \overline{BD}$.

19. **WRITING IN MATH** Does there exist an Addition Property of Congruence? Explain.

20. **REASONING** Classify the following statement as *true* or *false*. If false, provide a counterexample.

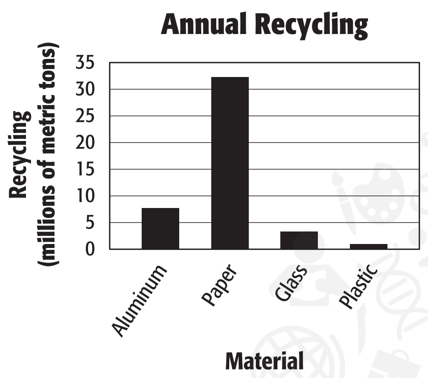
If A, B, C, D, and E are collinear with B between A and C, C between B and D, and D between C and E, and $AC = BD = CE$, *then* $AB = BC = DE$.

21. **OPEN ENDED** Draw a representation of the Segment Addition Postulate in which the segment is two centimeters long, contains four collinear points, and contains no congruent segments.

22. **WRITING IN MATH** Compare and contrast paragraph proofs and two-column proofs.

Standardized Test Practice

23. ALGEBRA The chart below shows annual recycling by material in a country. About how many kilogram of aluminum are recycled each year?



A 7.5 **C** 7,500,000

B 15,000 **D** 15,000,000,000

24. ALGEBRA Which expression is equivalent to $\frac{12x^{-4}}{4x^{-8}}$?

F $\frac{1}{3x^4}$ **H** $8x^2$

G $3x^4$ **J** $\frac{x^4}{3}$

25. SHORT RESPONSE The measures of two complementary angles are in the ratio 4 : 1. What is the measure of the smaller angle?

26. SAT/ACT Hala can word process 40 words per minute. How many minutes will it take Hala to word process 200 words?

A 0.5 **D** 10

B 2 **E** 12

C 5

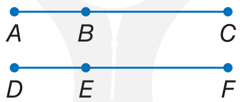
Spiral Review

27. PROOF Write a two-column proof. (Lesson 11-2)

Given: $AC = DF$

$AB = DE$

Prove: $BC = EF$



28. MODELS Bilal is using six squares of cardboard to form a rectangular prism. What geometric figure do the pieces of cardboard represent, and how many lines will be formed by their intersections?

29. LIGHT A light fell 25 feet from a building. The formula $h = -16t^2 + 25$ can be used to approximate the number of seconds it will take the light to hit the ground.

a. How long will it take the light to hit the ground?

b. If you catch it at 4 feet, how long did the light drop?

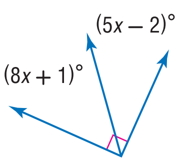
Simplify.

30. $\sqrt{48}$ **31.** $\sqrt{162}$ **32.** $\sqrt{25a^6b^4}$ **33.** $\sqrt{45xy^8}$

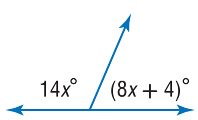
Skills Review

ALGEBRA Find x.

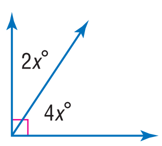
34.



35.



36.



LESSON 10-4 Proving Angle Relationships

Then

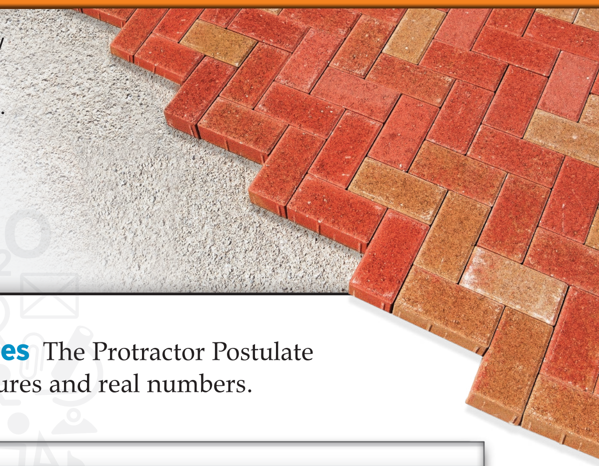
- You identified and used special pairs of angles.

Now

1. Write proofs involving supplementary and complementary angles.
2. Write proofs involving congruent and right angles.

Why?

- Jamal's school is building a walkway that will include bricks with the names of graduates from each class. All of the bricks are rectangular, so when the bricks are laid, all of the angles form linear pairs.



Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

6 Attend to precision.

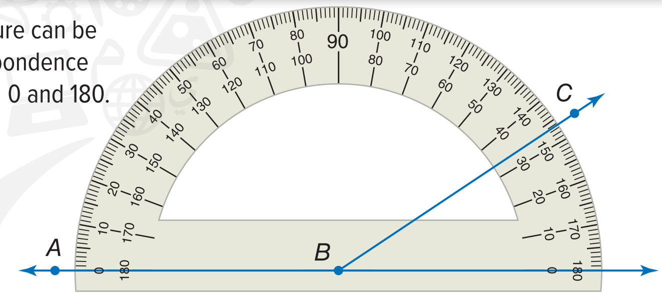
1 Supplementary and Complementary Angles

The Protractor Postulate illustrates the relationship between angle measures and real numbers.

Postulate 11.10 Protractor Postulate

Words Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.

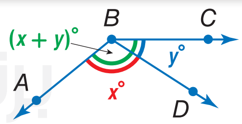
Example If $\overrightarrow{BA}$ is placed along the protractor at 0°, then the measure of $\angle ABC$ corresponds to a positive real number.



In Lesson 11-3, you learned about the Segment Addition Postulate. A similar relationship exists between the measures of angles.

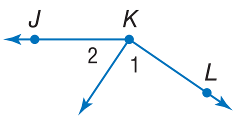
Postulate 11.11 Angle Addition Postulate

D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.



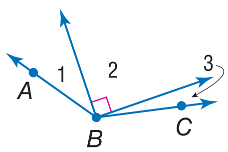
Example 1 Use the Angle Addition Postulate

Find $m\angle 1$ if $m\angle 2 = 56$ and $m\angle JKL = 145$.



$m\angle 1 + m\angle 2 = m\angle JKL$	Angle Addition Postulate
$m\angle 1 + 56 = 145$	$m\angle 2 = 56$ $m\angle JKL = 145$
$m\angle 1 + 56 - 56 = 145 - 56$	Subtraction Property of Equality
$m\angle 1 = 89$	Substitution

Guided Practice



1. If $m\angle 1 = 23$ and $m\angle ABC = 131$, find the measure of $\angle 3$. Justify each step.

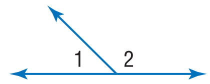
The Angle Addition Postulate can be used with other angle relationships to provide additional theorems relating to angles.

Study Tip

Linear Pair Theorem The Supplement Theorem may also be known as the *Linear Pair Theorem.*

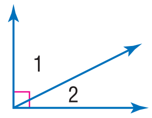
Theorems

11.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.



Example $m\angle 1 + m\angle 2 = 180$

11.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.



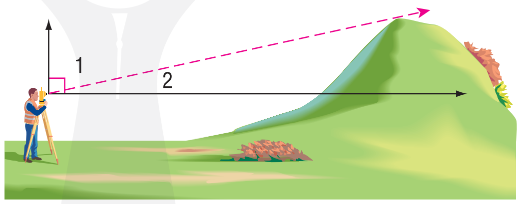
Example $m\angle 1 + m\angle 2 = 90$

You will prove Theorems 11.3 and 11.4 in Exercises 16 and 17, respectively.

Real-World Example 2 Use Supplement or Complement

SURVEYING **Using a transit, a surveyor sights the top of a hill and records an angle measure of about 73°. What is the measure of the angle the top of the hill makes with the horizon? Justify each step.**

Understand Make a sketch of the situation. The surveyor is measuring the angle of his line of sight below the vertical. Draw a vertical ray and a horizontal ray from the point where the surveyor is sighting the hill, and label the angles formed. We know that the vertical and horizontal rays form a right angle.



Plan Since $\angle 1$, and $\angle 2$ form a right angle, you can use the Complement Theorem.

Solve

$m\angle 2 + m\angle 1 = 90$	Complement Theorem
$73 + m\angle 1 = 90$	$m\angle 1 = 73$
$73 + m\angle 1 - 73 = 90 - 73$	Subtraction Property of Equality
$m\angle 1 = 17$	Substitution

The top of the hill makes a 17° angle with the horizon.

Check Since we know that the sum of the angles should be 90, check your math. The sum of 17 and 73 is 90. ✓

Review Vocabulary

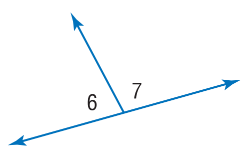
supplementary angles two angles with measures that add to 180

complementary angles two angles with measures that add to 90

linear pair a pair of adjacent angles with noncommon sides that are opposite rays

Guided Practice

2. $\angle 6$ and $\angle 7$ form linear pair. If $m\angle 6 = 3x + 32$ and $m\angle 7 = 5x + 12$, find x, $m\angle 6$, and $m\angle 7$. Justify each step.



2 Congruent Angles

The properties of algebra that applied to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

Theorem 11.5 Properties of Angle Congruence

Reflexive Property of Congruence

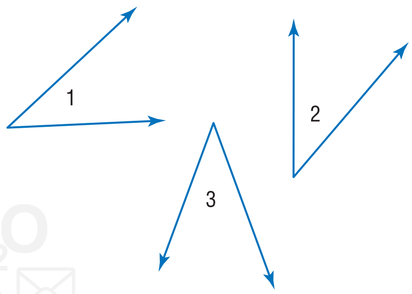
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

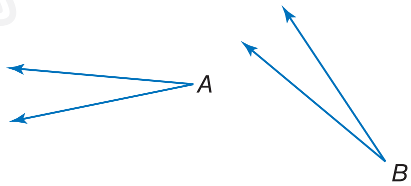


You will prove the Reflexive and Transitive Properties of Congruence in Exercises 18 and 19, respectively.

Proof Symmetric Property of Congruence

Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$



Paragraph Proof:

We are given $\angle A \cong \angle B$. By the definition of congruent angles, $m\angle A = m\angle B$. Using the Symmetric Property of Equality, $m\angle B = m\angle A$. Thus, $\angle B \cong \angle A$ by the definition of congruent angles.

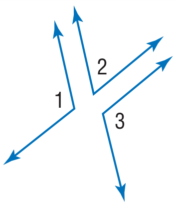
Algebraic properties can be applied to prove theorems for congruence relationships involving supplementary and complementary angles.

Theorems

11.6 Congruent Supplements Theorem
Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation *$\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.*

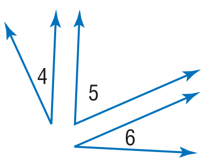
Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



11.7 Congruent Complements Theorem
Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation *$\angle$s compl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.*

Example If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.

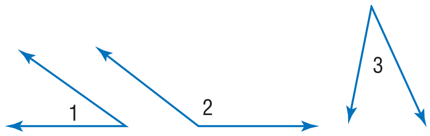


You will prove one case of Theorem 11.6 in Exercise 6.

ReadingMath

Abbreviations and Symbols
The notation $\angle$s means *angles*.

Proof One Case of the Congruent Supplements Theorem

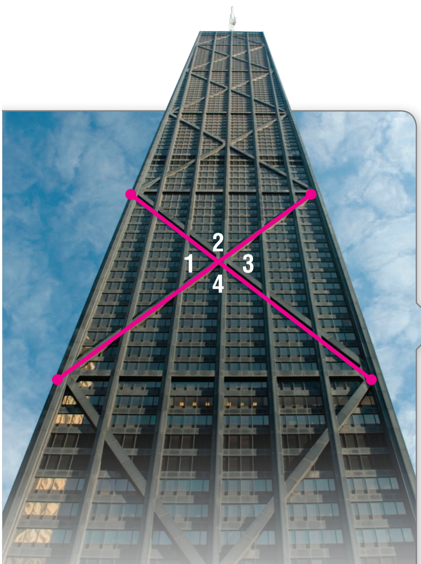


Given: $\angle 1$ and $\angle 2$ are supplementary.
$\angle 2$ and $\angle 3$ are supplementary.

Prove: $\angle 1 \cong \angle 3$

Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180$; $m\angle 2 + m\angle 3 = 180$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	**3.** Substitution
4. $m\angle 2 = m\angle 2$	**4.** Reflexive Property
5. $m\angle 1 = m\angle 3$	**5.** Subtraction Property
6. $\angle 1 \cong \angle 3$	**6.** Definition of congruent angles



Real-WorldLink

The 100-story John Hancock Building uses huge X-braces in its design. These diagonals are connected to the exterior columns, making it possible for strong wind forces to be carried from the braces to the exterior columns and back.

Source: PBS

Example 3 Proofs Using Congruent Comp. or Suppl. Theorems

Prove that vertical angles 2 and 4 in the photo at the left are congruent.

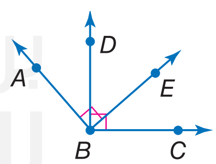
Given: $\angle 2$ and $\angle 4$ are vertical angles.

Prove: $\angle 2 \cong \angle 4$

Proof:

Statements	Reasons
1. $\angle 2$ and $\angle 4$ are vertical angles.	**1.** Given
2. $\angle 2$ and $\angle 4$ are nonadjacent angles formed by intersecting lines.	**2.** Definition of vertical angles
3. $\angle 2$ and $\angle 3$ from a linear pair. $\angle 3$ and $\angle 4$ form a linear pair.	**3.** Definition of a linear pair
4. $\angle 2$ and $\angle 3$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	**4.** Supplement Theorem
5. $\angle 2 \cong \angle 4$	**5.** $\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

GuidedPractice



3. In the figure, $\angle ABE$ and $\angle DBC$ are right angles. Prove that $\angle ABD \cong \angle EBC$.

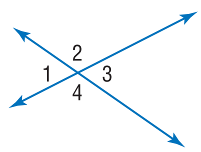
ReviewVocabulary

Vertical Angles two nonadjacent angles formed by intersecting lines

Note that in Example 3, $\angle 1$ and $\angle 3$ are vertical angles. The conclusion in the example supports the following Vertical Angles Theorem.

Theorem 11.8 Vertical Angles Theorem

If two angles are vertical angles, then they are congruent.



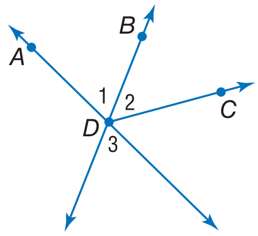
Abbreviation *Vert.* $\angle$s *are* $\cong$.

Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

You will prove Theorem 11.8 in Exercise 28.

Example 4 Use Vertical Angles

Prove that if $\overrightarrow{DB}$ bisects $\angle ADC$, then $\angle 2 \cong \angle 3$.



Given: $\overrightarrow{DB}$ bisects $\angle ADC$.

Prove: $\angle 2 \cong \angle 3$

Proof:

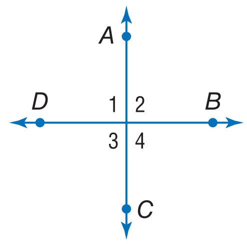
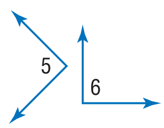
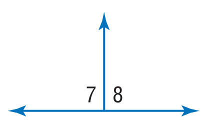
Statements	Reasons
1. $\overrightarrow{DB}$ bisects $\angle ADC$.	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Definition of angle bisector
3. $\angle 1$ and $\angle 3$ are vertical angles.	**3.** Definition of vertical angles
4. $\angle 3 \cong \angle 1$	**4.** Vert. ⦞ are $\cong$.
5. $\angle 3 \cong \angle 2$	**5.** Transitive Property of Congruence
6. $\angle 2 \cong \angle 3$	**6.** Symmetric Property of Congruence

GuidedPractice

4. If $\angle 3$ and $\angle 4$ are vertical angles, $m\angle 3 = 6x + 2$, and $m\angle 4 = 8x - 14$, find $m\angle 3$ and $m\angle 4$. Justify each step.

The theorems in this lesson can be used to prove the following right angle theorems.

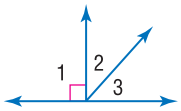
Theorems Right Angle Theorems

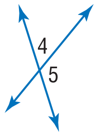
Theorem	Example
11.9 Perpendicular lines intersect to form four right angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. ⦞.	
11.10 All right angles are congruent. **Example** If $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. ⦞, then $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.	
11.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 4$, $\angle 3 \cong \angle 4$, and $\angle 1 \cong \angle 3$.	
11.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If $\angle 5 \cong \angle 6$ and $\angle 5$ is suppl. to $\angle 6$, then $\angle 5$ and $\angle 6$ are rt. ⦞.	
11.13 If two congruent angles form a linear pair, then they are right angles. **Example** If $\angle 7$ and $\angle 8$ form a linear pair, then $\angle 7$ and $\angle 8$ are rt. ⦞.	

You will prove Theorems 11.9–11.13 in Exercises 22–26.

Check Your Understanding

Example 1 **Find the measure of each numbered angle, and name the theorems that justify your work.**





1. $m\angle 2 = 26$

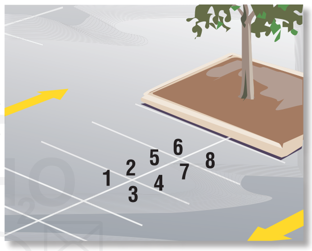
2. $m\angle 2 = x$, $m\angle 3 = x - 16$

3. $m\angle 4 = 2x$, $m\angle 5 = x + 9$

4. $m\angle 4 = 3(x - 1)$, $m\angle 5 = x + 7$

Example 2

5. **PARKING** Refer to the diagram of the parking lot at the right. Given that $\angle 2 \cong \angle 6$, prove that $\angle 4 \cong \angle 8$.

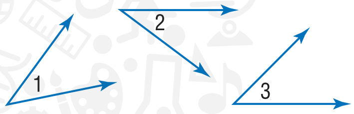


Example 3

6. **PROOF** **Copy and complete the proof of one case of Theorem 11.6.**

Given: $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 3$ are complementary.

Prove: $\angle 1 \cong \angle 2$



Proof:

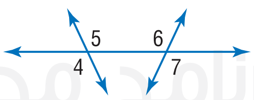
Statements	Reasons
a. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**a.** ___?___
b. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**b.** ___?___
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**c.** ___?___
d. ___?___	**d.** Reflexive Property
e. $m\angle 1 = m\angle 2$	**e.** ___?___
f. $\angle 1 \cong \angle 2$	**f.** ___?___

Example 4

7. **ARGUMENTS** Write a two-column proof.

Given: $\angle 4 \cong \angle 7$

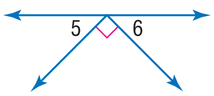
Prove: $\angle 5 \cong \angle 6$



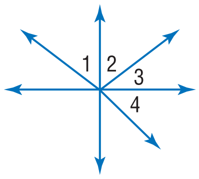
Practice and Problem Solving

Examples 1–3 **Find the measure of each numbered angle, and name the theorems used that justify your work.**

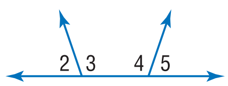
8. $m\angle 5 = m\angle 6$



9. $\angle 2$ and $\angle 3$ are complementary. $\angle 1 \cong \angle 4$ and $m\angle 2 = 28$

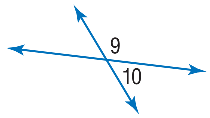


10. $\angle 2$ and $\angle 4$ and $\angle 4$ and $\angle 5$ are supplementary. $m\angle 4 = 105$

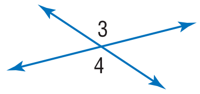


Find the measure of each numbered angle and name the theorems used that justify your work.

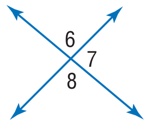
11. $m\angle 9 = 3x + 12$
$m\angle 10 = x - 24$



12. $m\angle 3 = 2x + 23$
$m\angle 4 = 5x - 112$



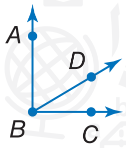
13 $m\angle 6 = 2x - 21$
$m\angle 7 = 3x - 34$



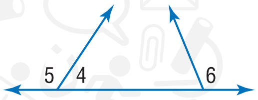
Example 4

PROOF **Write a two-column proof.**

14. **Given:** $\angle ABC$ is a right angle.
Prove: $\angle ABD$ and $\angle CBD$ are complementary.



15. **Given:** $\angle 5 \cong \angle 6$
Prove: $\angle 4$ and $\angle 6$ are supplementary.



Write a proof for each theorem.

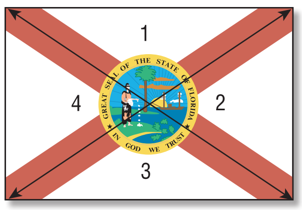
16. Supplement Theorem

17. Complement Theorem

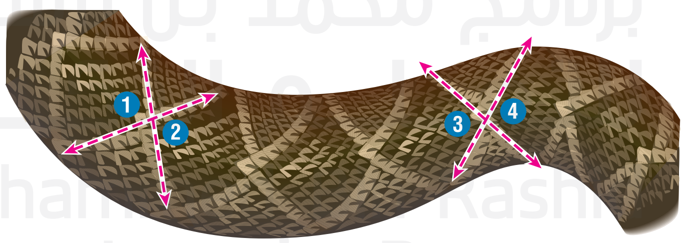
18. Reflexive Property of Angle Congruence

19. Transitive Property of Angle Congruence

20. **FLAGS** Refer to the flag at the right. Prove that the sum of the four angle measures is 360.



21. **ARGUMENTS** The diamondback rattlesnake is a pit viper with a diamond pattern on its back. An enlargement of a skin is shown below. If $\angle 1 \cong \angle 4$, prove that $\angle 2 \cong \angle 3$.



PROOF **Use the figure to write a proof of each theorem.**

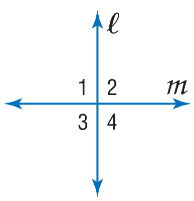
22. Theorem 11.9

23. Theorem 11.10

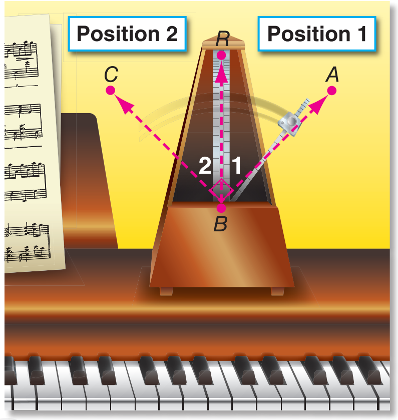
24. Theorem 11.11

25. Theorem 11.12

26. Theorem 11.13



27. **ARGUMENTS** To mark a specific tempo, the weight on the pendulum of a metronome is adjusted so that it swings at a specific rate. Suppose $\angle ABC$ in the photo is a right angle. If $m\angle 1 = 45$, write a paragraph proof to show that $\overrightarrow{BR}$ bisects $\angle ABC$.



28. **PROOF** Write a proof of Theorem 11.8.

29 **GEOGRAPHY** Utah, Colorado, Arizona, and New Mexico all share a common point on their borders called Four Corners. This is the only place where four states meet in a single point. If $\angle 2$ is a right angle, prove that lines ℓ and m are perpendicular.

30. **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle relationships.

a. **Geometric** Draw a right angle ABC. Place point D in the interior of this angle and draw $\overrightarrow{BD}$. Draw $\overrightarrow{KL}$ and construct $\angle JKL$ congruent to $\angle ABD$.

b. **Verbal** Make a conjecture as to the relationship between $\angle JKL$ and $\angle DBC$.

c. **Logical** Prove your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

31. **OPEN ENDED** Draw an angle WXZ such that $m\angle WXZ = 45$. Construct $\angle YXZ$ congruent to $\angle WXZ$. Make a conjecture as to the measure of $\angle WXY$, and then prove your conjecture.

32. **WRITING IN MATH** Write the steps that you would use to complete the proof below.

Given: $\overline{BC} \cong \overline{CD}$, $AB = \frac{1}{2}BD$

Prove: $\overline{AB} \cong \overline{CD}$

33. **CHALLENGE** In this lesson, one case of the Congruent Supplements Theorem was proven. In Exercise 6, you proved the same case for the Congruent Complements Theorem. Explain why there is another case for each of these theorems. Then write a proof of this second case for each theorem.

34. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

If one of the angles formed by two intersecting lines is acute, then the other three angles formed are also acute.

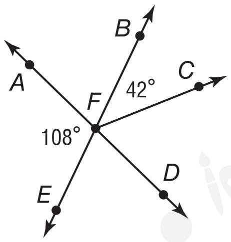
35. **WRITING IN MATH** Explain how you can use your protractor to quickly find the measure of the supplement of an angle.

Standardized Test Practice

36. GRIDDED RESPONSE What is the mode of this set of data?

4, 3, −2, 1 , 4, 0, 1, 4

37. Find the measure of $\angle CFD$.



A 66°

B 72°

C 108°

D 138°

38. ALGEBRA Simplify.

$$4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$$

F $9x^2 + 3x - 14$

G $9x^2 + 13x - 14$

H $27x^2 + 37x - 38$

J $27x^2 + 27x - 26$

39. SAT/ACT On a coordinate grid where each unit represents 1 kilometer, Wafa's house is located at (3, 0) and a mall is located at (0, 4). What is the distance between Wafa's house and the mall?

A 3 kilometers

B 5 kilometers

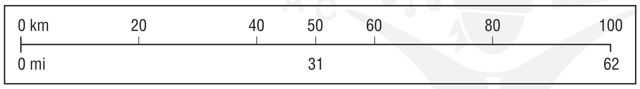
C 12 kilometers

D 13 kilometers

E 25 kilometers

Spiral Review

40. MAPS On a map, there is a scale that lists kilometers on the top and miles on the bottom.



Suppose $\overline{AB}$ and $\overline{CD}$ are segments on this map. If $AB = 100$ kilometers and $CD = 62$ miles, is $\overline{AB} \cong \overline{CD}$? Explain. (Lesson 11-3)

State the property that justifies each statement. (Lesson 11-4)

41. If $y + 7 = 5$, then $y = -2$.

42. If $MN = PQ$, then $PQ = MN$.

43. If $a - b = x$ and $b = 3$, then $a - 3 = x$.

44. If $x(y + z) = 4$, then $xy + xz = 4$.

Graph each function. State the domain and range.

45. $f(x) = 3(4)^x$

46. $f(x) = 2^{3x} - 3$

Solve each equation.

47. $2^{x-1} = 8^{x+3}$

48. $5^{2x+12} = 25^{10x-12}$

Skills Review

Refer to the figure.

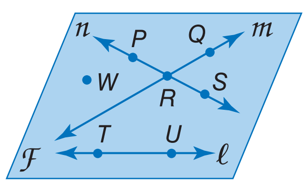
49. Name a line that contains point P.

50. Name the intersection of lines n and m.

51. Name a point not contained in lines ℓ, m, or n.

52. What is another name for line n?

53. Does line ℓ intersect line m or line n? Explain.



My Notes

CHAPTER 10

Study Guide and Review

Study Guide

KeyConcepts

Proof (Lessons 11-1 through 11-4)

Step 1 List the given information and draw a diagram, if possible.

Step 2 State what is to be proved.

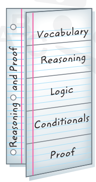
Step 3 Create a deductive argument.

Step 4 Justify each statement with a reason.

Step 5 State what you have proved.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

algebraic proof	paragraph proof
axiom	postulate
deductive argument	proof
formal proof	theorem
informal proof	two-column proof

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. A postulate is a statement that requires proof.

2. A theorem is a statement that is accepted as true without proof.

3. In a two-column proof, the properties that justify each step are called reasons.

4. An informal proof involves writing a paragraph to explain why a conjecture is true.

Study Guide and Review *Continued*

10-1 Postulates and Paragraph Proofs

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

5. Two planes intersect at a point.
6. Three points are contained in more than one plane.
7. If line m lies in plane X and line m contains a point Q, then point Q lies in plane X.
8. If two angles are complementary, then they form a right angle.
9. **NETWORKING** Six people are introduced at a business convention. If each person shakes hands with each of the others, how many handshakes will be exchanged? Include a model to support your reasoning.

Example 1

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

a. ***If points X, Y, and Z lie in plane R, then they are not collinear.***

Sometimes; the fact that X, Y, and Z are contained in plane R has no bearing on whether those points are collinear or not.

b. **For any two points *A* and *B*, there is exactly one line that contains them.**

Always; according to Postulate 11-1, there is exactly one line through any two points.

10-2 Algebraic Proofs

State the property that justifies each statement.

10. If $7(x - 3) = 35$, then $35 = 7(x - 3)$.
11. If $2x + 19 = 27$, then $2x = 8$.
12. $5(3x + 1) = 15x + 5$
13. $7x - 2 = 7x - 2$
14. If $12 = 2x + 8$ and $2x + 8 = 3y$, then $12 = 3y$.
15. Copy and complete the following proof.

Given: $6(x - 4) = 42$

Prove: $x = 11$

Statements	Reasons
a. $6(x - 4) = 42$	a. ___?___
b. $6x - 24 = 42$	b. ___?___
c. $6x = 66$	c. ___?___
d. $x = 11$	d. ___?___

16. Write a two-column proof to show that if $PQ = RS$, $PQ = 5x + 9$, and $RS = x - 31$, then $x = -10$.
17. **GRADES** Faleh received the same quarter grade as Khalifa. Khalifa received the same quarter grade as Hamad. Which property would show that Faleh and Hamad received the same grade?

Example 2

Write a two-column proof.

Given: $\frac{5x - 3}{6} = 2x + 1$

Prove: $x = -\frac{9}{7}$

Proof:

Statements	Reasons
1. $\frac{5x - 3}{6} = 2x + 1$	1. Given
2. $5x - 3 = 6(2x + 1)$	2. Multiplication Property of Equality
3. $5x - 3 = 12x + 6$	3. Distributive Property of Equality
4. $-3 = 7x + 6$	4. Subtraction Property of Equality
5. $-9 = 7x$	5. Subtraction Property of Equality
6. $-\frac{9}{7} = x$	6. Division Property of Equality
7. $x = -\frac{9}{7}$	7. Symmetric Property of Equality

10-3 Proving Segment Relationships

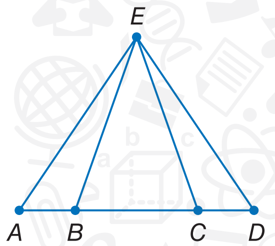
Write a two-column proof.

18. Given: X is the midpoint of $\overline{WY}$ and $\overline{VZ}$.

Prove: $VW = ZY$

19. Given: $AB = DC$

Prove: $AC = DB$



20. GEOGRAPHY Fahd is planning to drive from his house to his grandmother's house along Interstate 35. The map he is using gives the distance from his house to another location as 194 kilometers and from this location to his grandmother's house as 243 kilometers. What allows him to conclude that the distance he will be driving is 437 kilometers from his house to his grandmother's house? Assume that Interstate 35 forms a straight line.

Example 3

Write a two-column proof.

Given: B is the midpoint of $\overline{AC}$.

C is the midpoint of $\overline{BD}$.

Prove: $\overline{AB} \cong \overline{CD}$

Proof:

Statements	Reasons
1. B is the midpoint of $\overline{AC}$.	1. Given
2. $\overline{AB} \cong \overline{BC}$	2. Definition of midpoint
3. C is the midpoint of $\overline{BD}$.	3. Given
4. $\overline{BC} \cong \overline{CD}$	4. Definition of midpoint
5. $\overline{AB} \cong \overline{CD}$	5. Transitive Property of Equality

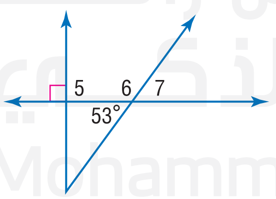
10-4 Proving Angle Relationships

Find the measure of each angle.

21. $\angle 5$

22. $\angle 6$

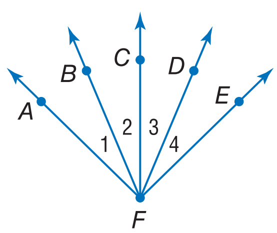
23. $\angle 7$



24. PROOF Write a two-column proof.

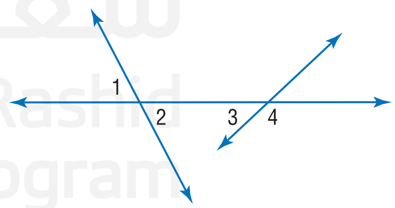
Given: $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$

Prove: $\angle AFC \cong \angle EFC$



Example 4

Find the measure of each numbered angle if $m\angle 1 = 72$ and $m\angle 3 = 26$.



$m\angle 2 = 72$, since $\angle 1$ and $\angle 2$ are vertical angles.

$\angle 3$ and $\angle 4$ form a linear pair and must be supplementary angles.

$26 + m\angle 4 = 180$ Definition of supplementary angles

$m\angle 4 = 154$ Subtract 26 from each side.

CHAPTER 10 Practice Test

1. **PROOF** Copy and complete the following proof.
Given: $3(x - 4) = 2x + 7$
Prove: $x = 19$
Proof:

Statements	Reasons
a. $3(x - 4) = 2x + 7$	**a.** Given
b. $3x - 12 = 2x + 7$	**b.** __?__
c. __?__	**c.** Subtraction Property
d. $x = 19$	**d.** __?__

Determine whether each statement is *always*, *sometimes*, or *never* true.

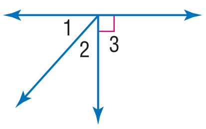
2. Two angles that are supplementary form a linear pair.

3. If B is between A and C, then $AC + AB = BC$.

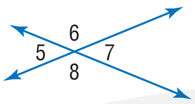
4. If two lines intersect to form congruent adjacent angles, then the lines are perpendicular.

Find the measure of each numbered angle, and name the theorems that justify your work.

5. $m\angle 1 = x$, $m\angle 2 = x - 6$



6. $m\angle 7 = 2x + 15$, $m\angle 8 = 3x$



Write each statement in if-then form.

7. An acute angle measures less than 90.

8. Two perpendicular lines intersect to form right angles.

9. **MULTIPLE CHOICE** If a triangle has one obtuse angle, then it is an obtuse triangle.

Which of the following statements is the contrapositive of the conditional above?

A If a triangle is not obtuse, then it has one obtuse angle.

B If a triangle does not have one obtuse angle, then it is not an obtuse triangle.

C If a triangle is not obtuse, then it does not have one obtuse angle.

D If a triangle is obtuse, then it has one obtuse angle.

Determine the slope of the line that contains the given points.

10. $G(8, 1)$, $H(8, -6)$

11. $A(0, 6)$, $B(4, 0)$

12. $E(6, 3)$, $F(-6, 3)$

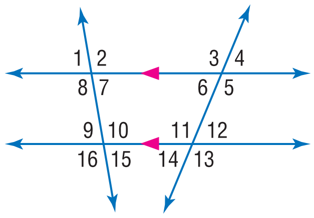
13. $E(5, 4)$, $F(8, 1)$

In the figure, $m\angle 8 = 96$ and $m\angle 12 = 42$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

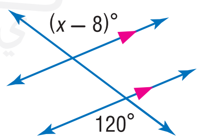
14. $\angle 9$

15. $\angle 11$

16. $\angle 6$



17. Find the value of x in the figure below.

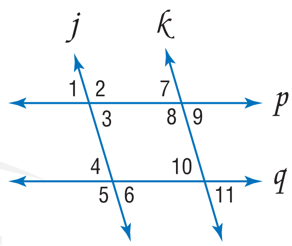


Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

18. $\angle 4 \cong \angle 10$

19. $\angle 9 \cong \angle 6$

20. $\angle 7 \cong \angle 11$



My Notes

Preparing for Standardized Tests

Logical Reasoning

Solving geometry problems frequently requires the use of logical reasoning. You can use the fundamentals of logical reasoning to help you solve problems on standardized tests.

Strategies for Using Logical Reasoning

Step 1

Read the problem to determine what information you are given and what you need to find out in order to answer the question.

Step 2

Determine if you can apply one of the principles of logical reasoning to the problem.

- **Counterexample**: A counterexample contradicts a statement that is known to be true.

  Identify any answer choices that contradict the problem statement and eliminate them.

- **Postulates:** A postulate is a statement that describes a fundamental relationship in geometry.

  Determine if you can apply a postulate to draw a logical conclusion.

Step 3

If you cannot reach a conclusion using only the principles in Step 2, determine if one of the tools below would be helpful.

- **Patterns:** Look for a pattern to make a conjecture.
- **Truth Tables:** Use a truth table to organize the truth values of the statement provided in the problem.
- **Venn Diagrams:** Use a Venn Diagram to clearly represent the relationships between members of groups.
- **Proofs:** Use deductive and inductive reasoning to reach a conclusion in the form of a proof.

Step 4

If you still cannot reach a conclusion using the tools in Step 3, make a **conjecture**, or educated guess, about which answer choice is most reasonable. Then mark the problem so that you can return to it if you have extra time at the end of the exam.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

In a school of 292 students, 94 participate in sports, 122 participate in academic clubs, and 31 participate in both. How many students at the school do not participate in sports or academic clubs?

A 95 **C** 122

B 107 **D** 138

Read the problem carefully. There are no clear counterexamples, and a postulate cannot be used to draw a logical conclusion. Therefore, consider the tools that you can use to organize the information.

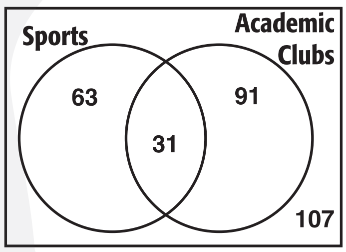
A Venn diagram can be used to show the intersection of two sets. Make a Venn diagram with the information provided in the problem statement.

Determine how many students participate in only sports or academic clubs.

Only sports: $94 - 31 = 63$

Only academic clubs: $122 - 31 = 91$

School Participation



Use the information to calculate the number of students who do not participate in either sports or academic clubs.

$292 - 63 - 91 - 31 = 107$

There are 107 students who do not participate in either sports or academic clubs. The correct answer is B.

Exercises

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Determine the truth of the following statement. If the statement is false, give a counterexample.

The product of two even numbers is even.

A false; $8 \times 4 = 32$

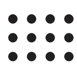
B false; $7 \times 6 = 42$

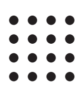
C false; $3 \times 10 = 30$

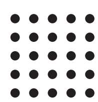
D true

2. Find the next item in the pattern.

F 

G 

H 

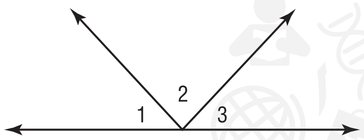
J 

CHAPTER 10 Standardized Test Practice

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. In the diagram below, $\angle 1 \cong \angle 3$.



Which of the following conclusions does not have to be true?

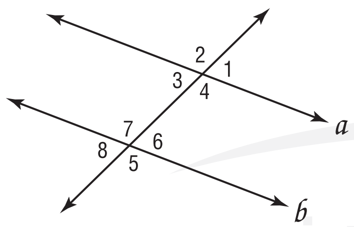
F $m\angle 1 - m\angle 2 + m\angle 3 = 90$

G $m\angle 1 + m\angle 2 + m\angle 3 = 180$

H $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$

J $m\angle 2 - m\angle 1 = m\angle 2 - m\angle 3$

2. If $a \parallel b$ in the diagram below, which of the following may *not* true?



A $\angle 1 \cong \angle 3$

B $\angle 4 \cong \angle 7$

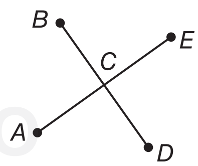
C $\angle 2 \cong \angle 5$

D $\angle 8 \cong \angle 2$

Test-Taking Tip

Question 3 A *counterexample* is an example used to show that a given statement is not always true.

3. In the diagram, $\overline{BD}$ intersects $\overline{AE}$ at C. Which of the following conclusions does *not* have to be true?



A $\angle ACB \cong \angle ECD$

B $\angle ACB$ and $\angle ACD$ form a linear pair.

C $\angle BCE$ and $\angle ACD$ are vertical angles.

D $\angle BCE$ and $\angle ECD$ are complementary angles.

4. What is the effect on the graph of the equation $y = x^2 + 4$ when it is changed to $y = x^2 - 3$?

F The slope of the graph changes.

G The graph widens.

H The graph is the same shape, and the vertex of the graph is moved down.

J The graph is the same shape, and the vertex of the graph is shifted to the left.

5. Which equation will produce the narrowest parabola when graphed?

A $y = 3x^2$

B $y = \frac{3}{4}x^2$

C $y = -6x^2$

D $y = -\frac{3}{4}x^2$

6. What is the effect on the graph of the equation $y = 3x^2$ when the equation is changed to $y = 2x^2$?

F The graph of $y = 2x^2$ is a reflection of the graph of $y = 3x^2$ across the y-axis.

G The graph is rotated 90 degrees about the origin.

H The graph is narrower.

J The graph is wider.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. Use the proof to answer the question.

Given: $\angle A$ is the complement of $\angle B$.
$m\angle B = 46$

Prove: $m\angle A = 44$

Proof:

Statements	Reasons
1. A is the complement of $\angle B$; $m\angle B = 46$.	**1.** Given
2. $m\angle A + m\angle B = 90$	**2.** Def. of comp. angles
3. $m\angle A + 46 = 90$	**3.** Substitution Prop.
4. $m\angle A + 46 - 46 = 90 - 46$	**4.** ___?___
5. $m\angle A = 44$	**5.** Substitution Prop.

What reason can be given to justify Statement 4?

8. HEIGHT The height h of a bouncing ball at time t seconds can be modeled by the equation $h = -16t^2 + 28.3t$.

a. Write the equation that models the height in factored form.

b. What is the height of the ball at 1.5 seconds?

c. How high will the ball bounce?

Extended Response

9. Sultan launches a model rocket from ground level. The rocket's height h in meters is given by the equation $h = -4.9t^2 + 56t$, where t is the time in seconds after the launch.

a. What is the maximum height the rocket will reach? Round to the nearest tenth of a meter. Show each step and explain your method.

b. How long after it is launched will the rocket reach its maximum height? Round to the nearest tenth of a second.