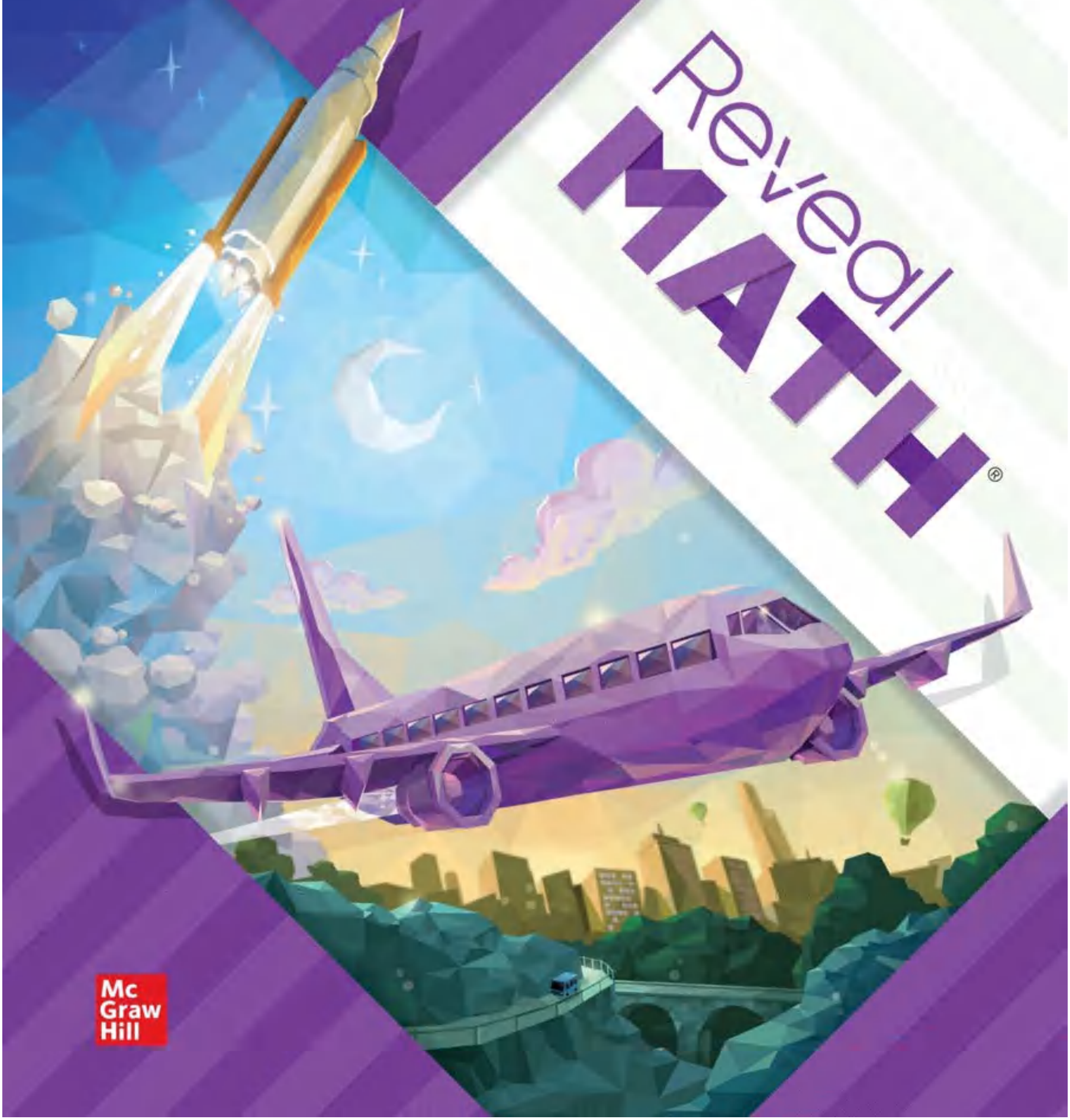
Teacher Edition
Grade 5 • Volume 2

Reveal MATH®



McGraw Hill

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Use the image on the back cover to spark student curiosity about slope. Here are some questions to help guide the conversation as students describe what they notice and wonder about this takeoff.

- What story could you tell about this image?
- What could you use to find out the angle of the plane's take-off?

Back cover: guvendemir/E+/Getty Images

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Contents in Brief

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Welcome to *Reveal Math*

We are excited to share with you the *Reveal Math* program.

In developing *Reveal Math,* we had a clear vision for elementary math instruction. It was important that the program we developed incorporated key findings from recent research on best practices in math instruction. It was also important that the program reflect an emphasis on building students' social and emotional competencies as well ensuring their academic growth.

We also thought extensively about your needs teaching math and your expectations for a high-quality math curriculum. It was important to us that the program provide flexibility in instructional and implementation options to meet the range of instructional settings and the range of learners.

We were purposeful about the organization of concepts and the scope and sequence to make sure students build deep conceptual understanding and develop proficiency with essential concepts and skills.

We are confident that *Reveal Math* incorporates all these goals.

- The lesson model offers two instructional options for each lesson: a guided exploration that is teacher-guided and an activity-based exploration that has students exploring concepts through small group activities and drawing generalizations and understanding from the activities.
- The lesson model incorporates an initial sense-making activity that builds students' proficiency with problem solving. By focusing systematically on sense-making, students develop and refine not just their observation and questioning skills, but the foundation for mathematical modeling.
- Both instructional options focus on fostering mathematical language and rich mathematical discourse by including probing questions and prompts.
- The Math is... unit builds student agency for mathematics. Students consider their strengths in mathematics, the thinking habits of proficient "doers of mathematics," and the classroom norms that are important to a productive learning environment.
- The scope and sequence reflects the learning progressions recommended by leading mathematicians and mathematics educators. It emphasizes developing deep understanding of the grade-level concepts and fluency with skills, while also providing rich opportunities to apply concepts to solve problems.

Thank you for using *Reveal Math*.
The *Reveal Math* author team

The *Reveal Math* Authorship

McGraw Hill's Learning Scientists teamed up with expert authors to create a program guided by validated academic research and classroom best practices.

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Facilitator of strategies that drive informed instructional decisions.

Dinah Zike, M.Ed.
Creator of learning tools that make connections through visual-kinesthetic techniques.

Unit
1

Math Is...

Unit 2

Volume

Unit
3

Place Value and Number Relationships

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Unit 5

Multiply Multi-Digit Whole Numbers

Unit 6

Multiply Decimals

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Add and Subtract Fractions

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Unit 11

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Measurement and Data

Unit 13

Geometry

Algebraic Thinking

Divide Decimals

PACING: 10 days

LESSON		MATH OBJECTIVE	LANGUAGE OBJECTIVE	SOCIAL AND EMOTIONAL LEARNING OBJECTIVE
Unit Opener	IGNITE! *Lemonade Stand* Explore division of whole numbers by decimals using informal strategies.			
8-1	Division Patterns with Decimals and Powers of 10	Students use place-value patterns to determine the quotient of a decimal divided by a power of 10. Students use the relationship between place-value positions to explain patterns when dividing decimals by powers of 10.	Students talk about place-value patterns when dividing decimals by powers of 10 while answering *Wh-* questions and using the term *shift*.	Students determine the strategies and analyses necessary to make informed decisions when engaging in mathematical practices.
8-2	Estimate Quotients of Decimals	Students estimate quotients of decimals using the same strategies used to estimate quotients of whole numbers. Students use estimated quotients to make predictions about a calculated solution. Students use estimated quotients to assess the reasonableness of a calculated solution.	Students discuss estimating the quotients of decimals while answering *Wh-* and Yes/No questions and using terms such as *could* and *would*.	Students practice strategies for persisting at a mathematical task, such as setting a small goal or setting timers for remaining focused.
8-3	Represent Division of Decimals by a Whole Number	Students represent division of decimals with equal sharing or equal grouping.	Students discuss how to divide decimals by whole numbers while answering *Wh-* questions and using the modal *might*.	Students engage in active listening and work collaboratively with a partner to complete mathematical tasks.
8-4	Divide Decimals by Whole Numbers	Students use place-value understanding and equivalent representations to divide a decimal by a whole number.	Students explain how to divide a decimal by a whole number by answering multiple *How* questions using can.	Students identify and discuss the emotions experienced during math learning.
8-5	Divide Whole Numbers by Decimals	Students use decimal grids to represent and solve a division equation. Students multiply by a power of 10 to write an equivalent expression with a whole-number divisor to solve a division equation.	Students discuss finding quotients of whole numbers using division grids and powers of 10, answering *How* and *Why*.	Students recognize and work to understand the emotions of others and practice empathetic responses.
8-6	Divide Decimals by Decimals	Students multiply the dividend and the divisor by a power of 10 to write an equivalent equation contining whole numbers to solve a division equation.	Students discuss multiple strategies to find quotients of decimals while answering *Wh-* questions.	Students set learning goals and initiate work on tasks to accomplish their goals.
Math Probe	***Decimal Division*** Select the correct quotient for division with a decimal dividend and divisor.			
Fluency Practice **Unit Review**				
Unit Assessment **Performance Task**				

FOCUS QUESTION:
What strategies can I use to divide decimals?

LESSON	KEY VOCABULARY: Math Terms	KEY VOCABULARY: Academic Terms	MATERIALS TO GATHER		RIGOR FOCUS	STANDARD
8-1	power of 10	expand reflect suggest	• base-ten blocks • calculators • hundred grids		Conceptual Understanding, Procedural Skill and Fluency	5.NBT.A.2
8-2	dividend divisor estimate quotient	negate variation	• calculator • number cubes		Conceptual Understanding, Procedural Skill and Fluency	5.NBT.B.7
8-3	decimal dividend divisor	analyze suggest	• bills and coins manipulatives • index cards	• *Tenths and Hundredths Representations* Teaching Resource	Conceptual Understanding, Procedural Skill and Fluency	5.NBT.B.7
8-4	dividend divisor place value quotient	infer transition	• number cubes		Conceptual Understanding, Procedural Skill and Fluency	5.NBT.B.7
8-5	dividend divisor power of 10 quotient	reflect address	• *10 × 10 Grids* Teaching Resource		Conceptual Understanding, Procedural Skill and Fluency	5.NBT.B.7
8-6	dividend divisor partial quotients power of 10 quotient	advantage assert disadvantage	• *Tenths and Hundredths Representations*		Conceptual Understanding, Procedural Skill and Fluency	5.NBT.B.7

Unit Overview

Focus

Dividing Decimals

Dividing by a (positive) decimal less than 1 is not always intuitive. When children first learn about the effect of each operation, they see that addition and multiplication have answers that are generally greater than the first addend or factor, whereas subtraction and division generally have answers that are less than the minuend or dividend.

This unit opens with use of different-sized glasses and jugs to illustrate division by decimals. This is an important opportunity to help students form a concrete understanding that dividing does *not* always "make smaller." When you ask, for example, how many small (0.2-liter) glasses are in a 2-liter jug, students better understand the nature of division: not simply making numbers smaller—but finding out how many of one quantity "fit into" another.

As the unit progresses, even when size is not involved, provide frequent "how many fit into" prompts, to consistently remind students that they are to find out how many of a certain decimal are in a given number.

Models may be helpful when making informal connections to dividing a whole number by a decimal. Models may help students understand that dividing something into smaller-size pieces produces *more*. For example, a ruler can be used to show that twenty 0.1-centimeter segments fit into 2 centimeters. Thus $2 \div 0.1 = 20$ is an example where the quotient is greater than the dividend.

A strategy used to divide a number by a decimal is to multiply the dividend and the divisor by the same power of 10 so that the divisor becomes a whole number. Students can use representations to show that the quotient does not change as a result of this process. For example, students can use decimal grids to show that $1 \div 0.2 = 5$ (5 groups of 0.2 are in 1 full grid), and $0.6 \div 0.15 = 4$ (4 groups of 0.15 are in 0.6 grid). Those quotients can then be compared to $10 \div 2 = 5$ and $60 \div 15 = 4$ for students to conclude that the respective quotients (based on decimals and whole numbers) are the same.

Coherence

What Students Have Learned

- Students divide 4-digit dividends by 1-digit divisors and know how to make sense of remainders. (Grade 4)
- Students use place-value understanding and properties of operations to perform multi-digit division. (Grade 4)
- Students use partial-quotient strategies to divide multi-digit numbers with single-digit divisors. (Grade 4)
- Students illustrate and explain calculations using equations, rectangular arrays, and/or area models. (Grade 4)
- Students solve real-world division problems with one-digit divisors. (Grade 4)

What Students Are Learning

- Students use strategies based on place value to divide decimals.
- Students estimate quotients of decimals.
- Students represent division of decimals by a whole number.
- Students use place-value understanding and equivalent representations to divide decimals by whole numbers.
- Students divide whole numbers by decimals using decimal grids and equivalent equations.
- Students divide decimals by decimals using area models to find partial quotients for equivalent equations.

What Students Will Learn

- Students fluently divide decimals using the standard algorithm (Grade 6).

Rigor

Conceptual Understanding

Students develop understanding of

- dividing decimals by powers of 10 using strategies based on place value, properties of operations, and patterns in the quotients of powers of 10;
- using representations and the relationship between multiplication and division to better understand division of decimals by whole numbers and decimals.

Procedural Skill and Fluency

Students build proficiency with

- writing an explanation describing patterns used when dividing with decimals;
- using decimal place value, basic facts, and division strategies by expanding their skills to include division of decimals by whole numbers;
- using decimal place value, basic facts, and division strategies by expanding their skills to include division of decimals by decimals.

Application

Students apply their knowledge of

- estimating quotients to successfully solve contextual, real-world problems;
- understanding of decimal division to solve problems with real-world contexts.

Application is not a targeted element of rigor for this standard.

Effective Teaching Practices

Elicit and Use Evidence of Student Thinking

As students progress through the unit, listen carefully to their answers and reasoning. Whether students get a correct solution is important, but the thought process that gets them to a solution—either correct or incorrect—is a window that provides an opportunity for greater growth and more targeted instruction.

As students learn about operations with decimals, there are multiple possibilities for errors related to understanding and execution.

Students may have misconceptions about the nature of decimals less than 1. They may have persistent calculation errors. They may have misconceptions about place-value patterns for decimal division.

Ask frequent questions, especially those that require reasoning. Use students' responses to inform instruction and determine what kinds of practice and review might be necessary.

For example, if students begin to move in the wrong place-value direction when they divide with decimals, ask them to consider what the quantities represent. Return to the use of concrete and visual models if students do not seem comfortable describing operations and values.

Lessons 8-1 and 8-2 provide a foundation in understanding decimal division that should be revisited when students' reasoning does not show solid understanding. The base-ten place-value systemis concretely represented in cents and dollars, and estimation helps students assess the reasonableness of their calculations.

Estimation provides a wonderful window into student thinking. Students should be encouraged to share how they used compatible numbers and rounding to form estimates. They should be expected to explain, say, that although an estimate of 50 may not be very close to an exact answer of 42, it does provide a convincing argument that answers such as 4.2 or 1.42 are not reasonable.

Math Practices and Processes

Look for and Make Use of Structure

It is possible for many students to simply memorize the rules for division with decimals and then carry out the operations successfully.

However, analyzing and understanding the structure of decimal operations provides greater reliability as students solve problems, as well as greater transference of learning to increasingly complex challenges as they progress in mathematics.

Encourage students to see division with decimals in concrete terms. Refer back to the first lesson in this unit, with powers of 10, if students become confused. Use concrete modeling to help students remember the structure behind the numbers and equations.

Focus students' attention on the models presented in this unit. When students see equations next to the charts, they will especially recognize the place-value pattern when they divide by a decimal and then apply the pattern with increasing competence.

Use whole numbers to reinforce structure. Have students use whole-number estimation and comparisons. Encourage students to reason and generalize about how whole-number and decimal division and the same and different.

Provide consistent opportunities for students to focus on structure.

Some suggestions include:

- Students work with partners to analyze a division problem with decimals. One partner makes a concrete model or drawing to show the division as it is written. The other partner makes a model or drawing to show a compatible division with whole numbers. Students compare and discuss their representations and answers.
- Students use place-value charts to record and analyze a division expression and explain its structure to the class.

Social and Emotional Learning

- **Responsible Decision-Making – Analyze Situations** (Lesson 8-1): Students make sense through analysis, which helps them make informed decisions.
- **Self-Management – Self-Discipline** (Lesson 8-2): Self-disciplined students can manage their impulses to focus on a mathematical task.
- **Relationship Skills – Build Relationships** (Lesson 8-3): Building positive relationships can help establish a strong classroom community.
- **Self-Awareness – Identify Emotions** (Lesson 8-4): Students who can identify and understand their own feelings and emotions can better manage the reactions to those feelings and emotions.
- **Social Awareness – Empathy** (Lesson 8-5): Students who can empathize with others are more able to build positive relationships.
- **Self-Management – Self-Motivation** (Lesson 8-6): Students who self-motivate can take initiative and persevere through challenging tasks.

Language of Math

Vocabulary

Students will be using these key terms in this unit.

- **Dividend –** (Lesson 8-2): Students were introduced to this term in the context of division fluency. It is the number that gets divided in a division problem.
- **Divisor –** (Lesson 8-2): Students were introduced to this term in the context of division fluency. It is the number that divides another number in a division problem.
- **Estimate –** (Lesson 8-2): Students were introduced to this term in the context of solving word problems involving all four operations. Have students discuss how estimation can help them evaluate the reasonableness of a solution.
- **Powers of 10 –** (Lesson 8-1): This term was introduced in Unit 3 in the context of observing place-value patterns when multiplying by numbers such as 100, 10, 0.1, and 0.01. In this unit, students multiply the divisor and the dividend by the same power of 10 so that the divisor is a whole number.
- **Quotient –** (Lesson 8-2): Students were introduced to this term in the context of division. This is the result of dividing one number by another number.

Math Language Development

A Focus on Division Language

Students may reach grade 5 with knowledge of division terminology but without having integrated the terms into their functional vocabulary.

Emphasize the terms *dividend*, *divisor*, and *quotient* as you relate division expressions and equations to their corresponding representations. Help students recognize that using the correct word for each part of an expression or equation is much more efficient than using phrases such as *the number that is dividing the other number* (divisor), and more specific than *the answer* (quotient).

To help reinforce division terminology, present division problems and have students identify the terms associated with each value.

dividend ÷ divisor = quotient

The word *dividend* comes from the word *divide*. Students should be able to recognize that this is the value that they are going to divide.

Advise students that the word *divisor* includes the suffix "-or," which students can recognize as an indicator of action.

Present common nouns that end in "-or" or "-er" such as *actor, sailor, worker,* and *teacher.* Ask students to say what each noun means, to elicit answers such as "an actor acts" and "a teacher teaches." Then ask, "What does a divisor do?" Students should be able to reply, "A divisor divides."

Some students might note that a "divider" also divides.

Advise students that the word *quotient* comes from the Latin word *quotiens* which means "how many times". Discuss the fact that the answer to a division problem also describes the relationship in terms of *how many times* one number fits into another.

English Language Learner

In this unit, students are provided with a number of scaffolds to support their comprehension of the language used to present and explain dividing decimals. Because many of the words and phrases used in this section are likely unfamiliar or unknown to ELs, students are supported in understanding and using these words.

Lesson 8-1 – *long*

Lesson 8-2 – *both*

Lesson 8-3 – *evenly*

Lesson 8-4 – *amount*

Lesson 8-5 – *[3] dollars in quarters*

Lesson 8-6 – *each*

Unit Routines

Number Routines

Build Fluency The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the thinking habits of mind that are important for proficient doers of math.

Decompose It

Purpose: Build flexibility with numbers.
Overview: Students generate multiple (at least 3) ways to decompose given numbers and share their thinking for each decomposition. The teacher records decompositions and then facilitates a discussion of patterns in the decompositions.

About How Much

Purpose: Build estimating skills.
Overview: Students estimate the value of expressions (with operations) shown, explaining their strategies and thinking. The teacher records students' estimates, then reveals the value of the expression. Students analyze the estimates and discuss which are closest to the actual value of the expression.

Where Does It Go?

Purpose: Build estimating skills using benchmarks.
Overview: Students place a target number on number lines with different endpoints and justify their placement. In some instances, as a challenge, the target number may not actually belong on one of the number lines.

Sense-Making Routines

- **Is It Always True?** (Lesson 8-1) Students discuss whether or not division always makes a number smaller.
- **Notice & Wonder: What question could you ask?** (Lesson 8-2) Students are presented with an image of fabric and a statement about the theater department using the fabric for costumes.
- **Notice & Wonder: What question could you ask?** (Lesson 8-3) Students are presented with an image of bills, coins, and piggy banks. Students might wonder about equal sharing and how to divide the money among the banks when the amount is not a whole number.
- **Which Doesn't Belong?** (Lesson 8-4) Students are presented with four numbers. Students might determine the equivalent values based on the number of different size parts.
- **Notice & Wonder: What do you notice? What do you wonder?** (Lesson 8-5) Students are presented with an image of a pile of quarters. Students might wonder if the whole pile is quarters and the value of the money in the pile.
- **Notice & Wonder: What do you notice? What do you wonder?** (Lesson 8-6) Students see four equations. Students might wonder how the quotients are the same with different dividends and divisors.

Math Language Routines

The Mathematical Language Routines used in this unit give teachers a structured, yet adaptable format for amplifying and developing students' social and academic language. For more information on the Mathematical Language Routines, see the Appendix.

- Lesson 8-1 – Students participate in MLR2: Collect and Display and MLR4: Info Gap so that students' oral words and phrases can be captured into a stable, collective reference and so that they have a structured and interactive opportunity to work together to solve how to represent unknown values using division.
- Lesson 8-2 – Students participate in MLR6: Three Reads so that they have a structured and interactive opportunity to list the steps they must complete in order to estimate a quotient.
- Lesson 8-3 – Students' meta-awareness can be fostered as they discuss how the tools and strategies they've learned in the past can help them solve a problem using division and decimal grids and so that students' oral and written output can be fostered as they compare and contrast different ways to solve division problems.
- Lesson 8-4 – Students participate in MLR1: Stronger and Clearer Each Time so that they have a structured and interactive opportunity to revise and refine both their ideas and their verbal and written output while dividing a decimal by a whole number.
- Lesson 8-5 – Students participate in MLR7: Compare and Connect so that students' meta-awareness can be fostered as they compare and contrast different ways to solve such problems.
- Lesson 8-6 – Students participate in MLR5: Co-Craft Questions and Problems so that they can produce the language of mathematical questions related to solving division problems that include dividing decimals by powers of 10.

Unit 8
How Ready Am I?

Name ____________________

1. Which is the quotient of 144 ÷ 6?
   A. 150 B. 138
   C. 27 **D.** 24
2. Which is the quotient of 143 ÷ 5?
   A. 28 R1 **B.** 28 R3
   C. 280 D. 286
3. Which is the quotient of 1,596 ÷ 42?
   A. 37 R32 **B.** 38
   C. 38 R10 D. 43
4. Gerard spent $184 on 23 items. Each item costs the same amount. How much was each item?
   A. $161
   B. $8
   C. $4,232
   D. $7
5. Which is the product of 5.7 × 1,000?
   A. 57
   B. 570
   C. 5,700
   D. 57,000
6. Which is the unknown? 0.035 × ? = 35
   A. 0.001
   B. 0.01
   C. 10
   D. 1,000

Assessment Resource Book 139

7. A baker makes 180 plain bagels. He plans to place the bagels equally in 60 bags for a fundraiser. How many bagels will he place in each bag?
   A. 30 bagels
   B. 20 bagels
   C. 3 bagels
   D. 2 bagels
8. Mary plans to give an equal number of her 48 stickers to 7 of her friends and keep any leftover stickers for herself. Which statement correctly describes how Mary will share her stickers?
   A. Each friend should receive 7 stickers, leaving Mary with 1 sticker to keep.
   B. Each friend should receive 7 stickers, leaving Mary with 0 stickers to keep.
   C. Each friend should receive 6 stickers, leaving Mary with 6 stickers to keep.
   D. Each friend should receive 6 stickers, leaving Mary with 3 stickers to keep.
9. Yolanda knows that 43 × 0.56 = 24.08. Which is 4.3 × 0.56?
   A. 2.408
   B. 24.08
   C. 240.8
   D. 2,408
10. A rectangular flower garden is 12 feet long and 6.4 feet wide. What is the area of the flower garden?
   A. 36.8 square feet
   B. 72 square feet
   C. 76.8 square feet
   D. 768 square feet

140 Assessment Resource Book

Administer the Readiness Diagnostic to determine your students' readiness for this unit.

Targeted Intervention

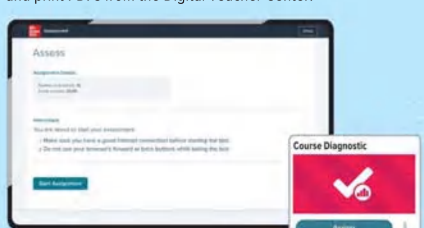
Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	1	Division of 3-digit by 1-digit number	Three-Digit Dividends (Partial Quotients)	4.NBT.B.6
2	1	Division of 3-digit by 1-digit number with remainder	Three-Digit Dividends (Partial Quotients)	4.NBT.B.6
3	1	Division of 4-digit by 2-digit number	Divide by 2-Digit (Partial Quotients)	5.NBT.B.6
4	2	Division of 3-digit by 2-digit number in a word problem	Divide by 2-Digit (Partial Quotients)	5.NBT.B.6
5	1	Multiply decimal by power of 10	Multiply by Powers of 10 (Decimal Point)	5.NBT.A.2
6	1	Multiply decimal by whole number	Multiply Decimals by Whole Numbers-Model	5.NBT.B.7
7	2	Making equal groups with division by multiples of 10	Divide by Multiples of 10	5.NBT.B.6
8	2	Interpret remainders in division	Interpret Remainders in Word Problems	4.OA.A.3
9	1	Patterns in decimal multiplication	Multiply Decimal Numbers (Patterns)	5.NBT.B.7
10	2	Multiply whole number by decimal	Multiply Decimals by Whole Numbers-Model	5.NBT.B.7

GO ONLINE

Assign the digital Readiness Diagnostic to students or download and print PDFs from the Digital Teacher Center.



Focus Question

Introduce the Focus Question, *What strategies can I use to divide decimals?*

Ask students to think about what they know about decimals, place value, and division.

- What do you already know about decimals? What do you already know about place value?
- When do you think you might need to divide with decimals?
- What do you think you will be learning in this unit?

Remind students that at the end of the unit, they will reflect back on what they learned in this unit.

Family Letter

Each letter presents an overview of the math in the unit and home activities to support student learning.

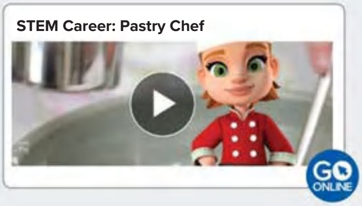
STEM in Action

Videos

Students can watch the two STEM videos.

STEM Career: Pastry Chef Saffron talks about her aspirations to be a pastry chef.

Shopping for Baking Supplies Saffron uses division to determine the price she pays for different baking supplies.

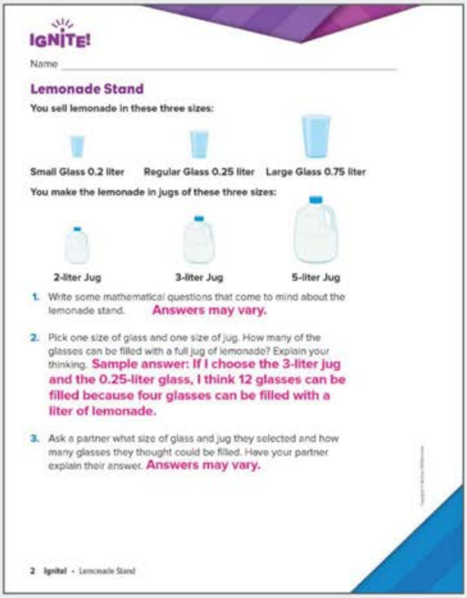


STEM Project

Students can complete the STEM Project Card during their workstation time.

STEM Adventure

Students can complete the STEM Adventure during their workstation time.



IGNITE!

Name

Lemonade Stand

You sell lemonade in these three sizes:

Small Glass 0.2 liter | Regular Glass 0.25 liter | Large Glass 0.75 liter

You make the lemonade in jugs of these three sizes:

2-liter Jug | 3-liter Jug | 5-liter Jug

1. Write some mathematical questions that come to mind about the lemonade stand. **Answers may vary.**

2. Pick one size of glass and one size of jug. How many of the glasses can be filled with a full jug of lemonade? Explain your thinking. **Sample answer: If I choose the 3-liter jug and the 0.25-liter glass, I think 12 glasses can be filled because four glasses can be filled with a liter of lemonade.**

3. Ask a partner what size of glass and jug they selected and how many glasses they thought could be filled. Have your partner explain their answer. **Answers may vary.**

2 Ignite! • Lemonade Stand

Ignite!

Lemonade Stand

This Ignite! sets the stage for decimal division by using “friendly” decimals to allow students to apply informal strategies, such as drawing pictures or repeated subtraction, to obtain the answers.

1. Have students observe the sizes of glasses of lemonade they will sell and the sizes of jugs in which they will make the lemonade. Then have them work in pairs to do problem 1.
   - Share some of your questions with the class.
2. Ask the questions below, and have students record the results in the table with problem 2. Remind students that they can draw pictures, use knowledge about fractions, and other tools to answer the questions.
   - How many small glasses can be made from each of the three jug sizes? How much lemonade will be left over in each jug? Explain.
   - How many regular glasses can be made from each jug? How much lemonade will be left over in each jug? Explain.
   - How many large glasses can be made from each jug? How much lemonade will be left over in each jug? Explain.
3. The following questions have students explore quotients involving decimals by considering related problems involving whole numbers.
   - How many 2-liter containers could be filled from a 20-liter tank?
   - How does that answer compare to the number of 0.2-liter glasses that are in a 2-liter jug that you found in the table?
   - How many 2-liter containers could be filled from a 30-liter tank? from a 50-liter tank?

Unit Resources At-A-Glance

Workstations

Reveal Math offers rich and varied resources that teachers can use to differentiate and enrich students' instructional experiences with the unit content. The table presents an overview of the resources available for the unit with recommendations for when to use.

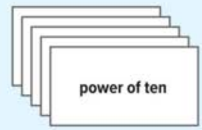
	Activity	Description	Use After Lesson
Game Station	Game Station	Students build proficiency in dividing with decimals.	
		• **Divide by 0.1 and 0.01 Race**	8-1
		• **Estimating Quotients Bump**	8-2
		• **Represent Decimal Division Four in a Row**	8-3
		• **Dividing Decimals by Whole Numbers Task Cards**	8-4
		• **Divide by Whole Numbers Tic Tac Toe**	8-5
		• **Divide Decimals by Decimals Task Cards**	8-6
Digital Station	Digital Game	**Factory Sort** Students add and subtract within 1,000,000.	8-1
Application Station	**Have students complete at least one of the Use It! activities for this unit.**		
	STEM Project Card	**That is Tasty!** Students use decimal division to size up and down a recipe.	8-6
	Connection Card	**Leave a Trail!** Students create a poster showing how they divide decimals while making trail mix.	8-6
	Real World Card	**Red Block White Block Road Block** Students use a looping code to move a dog through a maze and then make their own maze and code.	8-1

Additional Resources

Use the resources below to provide additional support for this unit.

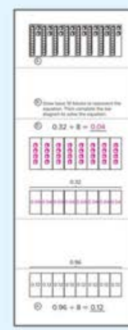
Vocabulary

Use the vocabulary cards to help students learn the vocabulary in this unit. Encourage students to write their own definitions of the new terms on the front side of the card.



Foldables

Use the unit foldable with Lessons 8-1, 8-3, and 8-4.



Spiral Review

Students can complete the Spiral Review at any point during the unit as either a paper-and-pencil or digital activity.

Lesson	Standard
8-1	5.MD.C.3
8-2	5.NBT.B.6
8-3	5.NBT.A.1
8-4	5.NBT.A.2
8-5	5.MD.C.4
8-6	5.NBT.B.7

LESSON **8-1**

Division Patterns with Decimals and Powers of 10

Learning Targets

- I can use place-value patterns to divide a decimal by a power of 10.
- I can explain patterns when dividing a decimal by a power of 10.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NBT.A** Understand the place value system.

◇ **5.NBT.A.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Term	Academic Terms
power of 10	expand
	reflect
	suggest

Materials

The materials may be for any part of the lesson.

- base-ten blocks
- calculator
- hundreds grids

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use place-value patterns to determine the quotient of a decimal divided by a power of 10. • Students use the relationship between place-value positions to explain patterns when dividing decimals by powers of 10.	• Students talk about place-value patterns when dividing decimals by powers of 10 while answering *Wh-* questions and using the term *shift*. • To support maximizing linguistic and cognitive meta-awareness and optimizing output, ELs participate in MLR2: Collect and Display and MLR4: Info Gap.	• Students determine the strategies and analyses necessary to make informed decisions when engaging in mathematical practices.

Coherence

Previous	Now	Next
• Students recognized that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right (Grade 4). • Students divided milti-digit whole numbers (Unit 7).	• Students use place-value patterns to divide decimals by powers of 10.	• Students estimate quotients involving decimals (Unit 8). • Students write and evaluate expressions involving whole-number exponents (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students develop understanding of dividing decimals by powers of 10 using strategies based on place value, properties of operations, and patterns in the quotients of powers of 10.	• Students write an explanation describing patterns used when dividing with decimals.	• Students apply their understanding of dividing decimals by powers of 10 to solve contextual problems. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Decompose It 5–7 min

Build Fluency Students build number sense as they decompose the number 12.36 in 3 ways.

Remind students that there will be many different possible answers to the problem. As solutions are given, record them for students to evaluate and compare.

These prompts encourage students to talk about their reasoning:

- What do you notice about the number?
- How did you determine different decompositions of 12.36?
- How did you know that your decompositions were reasonable?
- What is another way to think about how to decompose the number?

Launch 5–7 min

Sense-Making Routine

Purpose Students think about the meanings of division to challenge some overgeneralizations that some students may have made. This lesson is their first experience with dividing by a number less than 1.

Is It Always True?

- Is the statement always true?

See the Appendix for a full description of the sense-making routines.

Teaching Tip Some students may benefit from using manipulatives such as base-ten blocks to test their theories. Visual learners may also draw on hundred grids to explore different divisions.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of how division affects a value and are based on possible comments and questions that students may make during the share out.

- If you find an example where division makes numbers less, does that mean the statement is always true?
- How many one-half oranges are in 6 oranges?
- What types of numbers do you think you can divide by? What types of numbers do you think you can divide?

Math is... Mindset

- What helps you understand a problem situation?

SEL Responsible Decision Making: Flexible Thinking

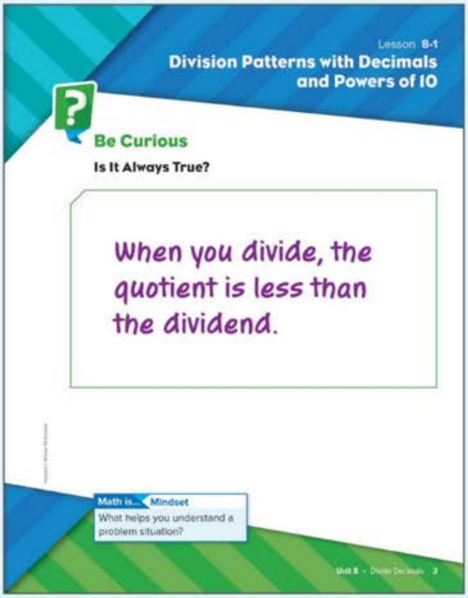
As students work through the Is It Always True? routine, encourage them to analyze or think about the situation critically before they take steps toward solving. Having them consider what information they have, what question is being asked, and what tools they may use can help students make sense of the situation. This analysis can help students make more informed decisions.

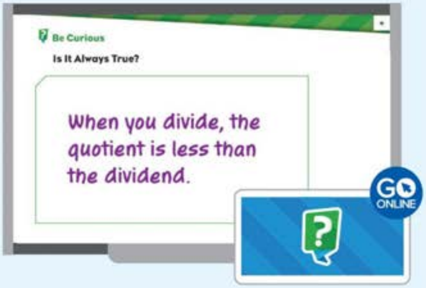
Transition to Explore & Develop

Have students think about what division represents. Guide them toward discussing how the relationship between the dividend and the divisor affects the quotient.

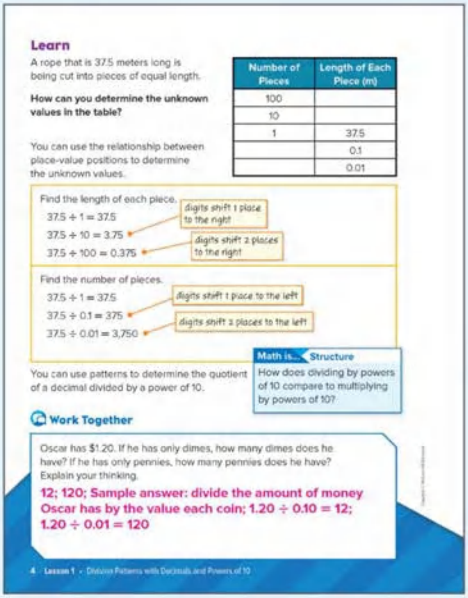
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how place-value patterns can help us solve problems involving division of decimals by powers of 10.





Explore & Develop 20 min



Learn

A rope that is 37.5 meters long is being cut into pieces of equal length.

How can you determine the unknown values in the table?

Number of Pieces	Length of Each Piece (m)
100	
10	
1	37.5
	0.1
	0.01

You can use the relationship between place-value positions to determine the unknown values.

Find the length of each piece.

$37.5 \div 1 = 37.5$

$37.5 \div 10 = 3.75$ digits shift 1 place to the right

$37.5 \div 100 = 0.375$ digits shift 2 places to the right

Find the number of pieces.

$37.5 \div 1 = 37.5$

$37.5 \div 0.1 = 375$ digits shift 1 place to the left

$37.5 \div 0.01 = 3{,}750$ digits shift 2 places to the left

You can use patterns to determine the quotient of a decimal divided by a power of 10.

Math is... Structure
How does dividing by powers of 10 compare to multiplying by powers of 10?

Work Together

Oscar has $1.20. If he has only dimes, how many dimes does he have? If he has only pennies, how many pennies does he have? Explain your thinking.

12; 120; Sample answer: divide the amount of money Oscar has by the value each coin; $1.20 \div 0.10 = 12$; $1.20 \div 0.01 = 120$

4 Lesson 1 • Division Patterns with Decimals and Powers of 10

1 Pose the Problem

MLR Collect and Display

As students discuss the questions, write key words and phrases you hear, such as *shift*, *unknown values*, *relationship*, *pieces*, and *power of 10*. Display the words and phrases for student reference and use the student generated expressions to help make connections between student language and math vocabulary. Update the collection with new understandings as the lesson progresses.

ETP Pose Purposeful Questions

- What is this problem about? How can you restate the problem in your own words?
- When have you seen problems like this before?
- How might the length of each piece and the numbers of pieces of string be related?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Info Gap

Pair students. Provide Partner A with a problem like the one on the Learn page. Provide Partner B with the information to solve the problem. Instruct Partner B to ask A what information they need, and for A to respond, explaining why they need it. Have students continue until the problem is completed.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How would you describe the place-value patterns when dividing a decimal by a power of 10?

Key Takeaway

- The relationship between place-value positions can explain patterns when dividing decimals by powers of 10.

Work Together

Students use their knowledge of place-value patterns when dividing by 0.1 or 0.01 to find the number of dimes and pennies in a given amount. Students can work on the activity in pairs before sharing their work.

Common Error Students may relate the number of decimal places in the divisor and/or dividend to the number of decimal places in the quotient. Invite students to ask, "Does this make sense?" if they are making this error.

LOM Language of Math

Point out to students that *powers of 10* are not only numbers like 100 and 1,000 that can be written as a product of 10 multiplied by itself a number of times, but also numbers like $\frac{1}{100}$ and $\frac{1}{1{,}000}$ which can be written as a product of $\frac{1}{10}$ multiplied by itself a number of times.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore place-value patterns when dividing by powers of 10 and use their patterns to divide decimals by powers of 10.

Materials: calculator

Directions: Students enter any decimal number on the calculator (e.g. 24.1), then divide their decimal by 10. Have them guess the quotient before they press the equal key. Students continue dividing by 10 mentally, challenging themselves to predict the quotient before they press the equal key. After repeatedly dividing by 10, have them repeat the process by repeatedly dividing by 0.1.

ETP Implement Tasks That Promote Reasoning and Problem Solving

- How did you use mental math to predict the quotient?
- What happens to the digits in the quotient each time the number is divided by 10?
- What happens to the digits in the quotient each time the number is divided by 0.1?
- How could you summarize the results to predict how dividing a decimal by a power of 10 affects the decimal value?

Math is... Structure

- How does dividing by powers of 10 compare to multiplying by powers of 10?

Students connect dividing by powers of 10 to multiplying by them and expand their understanding of the structure of the place value system.

Activity Debrief: Have students share their findings when repeatedly dividing by 10 or 0.1. Discuss patterns in the number of places the digits of the dividend shifted.

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the missing values in the table?

Guided Exploration

Students use the relationship between place-value positions to solve division problems involving decimals divided by powers of 10.

ETP Use and Connect Mathematical Representations

- Why should you use division to find the length of each piece?

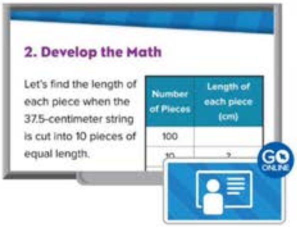
Have students justify the thinkng about dividing by 10 and 100. For example, ask:

- How can you write 37.5 ÷ 10 = ? as a related multiplication equation?
- How can you use what you know about multiplying decimals by powers of 10 to solve for the unknown in that equation?
- How can you use the solution to that equation to write a related division equation to solve the original equation?
- Is it possible to use a representation to solve 37.5 ÷ 100? What would be some reasons why a representation may not be the most efficient strategy?
- **Think About It:** How can you use multiplication to check if your answers when dividing decimals by powers of 10 are correct?

Math is... Structure

- How does dividing by powers of 10 compare to multiplying by powers of 10?

Students connect dividing by powers of 10 to multiplying by them and deepen their understanding of the structure of the place value system.



EL English Learner Scaffolds

Entering/Emerging Support students' understanding of the word *long* as it pertains to length. Using a ruler, measure the length of a pencil. Say *This pencil is [18] centimeters long*. Repeat the task, measuring a notebook. Say *This notebook is [21] centimeters long*. Then ask students to find the values on the Learn table that represent how long something is.

Developing/Expanding Support students' understanding of the word *long* as it pertains to length. Using a ruler, measure the length of a pencil. Say *This pencil is [18] centimeters long*. Repeat the task, measuring a notebook. Say *This notebook is [21] centimeters long*. Ask students to repeat the task, measuring an object of their choice, using *long* in their sentence. Provide sentence frames for students who need more guidance.

Bridging/Reaching Ask students to explain how they can use the relationship between place-value positions to find the length of each piece in the problem. Listen for key words and phrases such as *shift/place(s), to the right, and long*. Allow students to interject, pointing out any mistakes that they may catch in meaning or understanding.

Practice & Reflect 10 min

On My Own

Name ________________

What are the quotients? Use a pattern to solve and explain your thinking.

1. $64.2 \div 100 =$ **0.642**
   $64.2 \div 10 =$ **6.42**
   $64.2 \div 1 =$ **64.2**
   $64.2 \div 0.1 =$ **642**
   $64.2 \div 0.01 =$ **6,420**

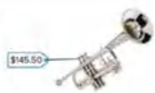
2. $7.5 \div 100 =$ **0.075**
   $7.5 \div 10 =$ **0.75**
   $7.5 \div 1 =$ **7.5**
   $7.5 \div 0.1 =$ **75**
   $7.5 \div 0.01 =$ **750**

1–2 Sample answer: As the divisor decreases by a power of 10, the quotient increases by a power of 10.

What is the quotient?

3. $91.4 \div 0.1 =$ **914**
4. $55.8 \div 0.01 =$ **5,580**
5. $50.5 \div 0.01 =$ **5,050**
6. $33.2 \div 0.1 =$ **332**
7. $16.4 \div 10 =$ **1.64**
8. $444.8 \div 100 =$ **4.448**

9. Elisha is buying a trumpet. She will make 10 equal payments to pay for the trumpet. How much will each payment be?



$14.55

10. Danny walked 567.3 miles in 100 days. Michelle walked 567.3 miles by walking 0.1 mile each day. Who walked for more days? Who walked farther each day? Explain.

Michelle; Danny; Student explanations will vary. Sample response: Danny walked 5.673 miles each day ($567.3 \div 100$). Michelle walked for 5,673 days ($567.3 \div 0.1$).

Unit 8 • Divide Decimals 5

11. Bryson's bicycling club goes on a long ride 10 Saturdays of every year. What was the average distance they rode each trip last year?

34.55 miles

12. **Error Analysis** Paul has 32.4 milliliters of solution. He uses 0.1 milliliter of solution for each experiment. Paul states that he can complete 3.24 trials using all of his solution. How do you respond to him?

Sample answer: I disagree with Paul. Paul should move the digits to the left: $32.4 \div 0.1 = 324$. He can complete 324 trials with the solution.

13. **Extend Your Thinking** Find the value of x that makes the equation true. Explain how you know.

$7 \div 0.01 = 700 \div x$

$x = 1$; Sample answer: You can multiply 7 and 0.01 by 100 to get 700 and 1. Both expressions will result in the same quotient.

Reflect

How does the relationship between place-value positions help you divide decimals by powers of 10?

Answers may vary.

Math is... Mindset
What helped you understand a problem situation today?

6 Lesson 1 • Division Patterns with Decimals and Powers of 10

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 1 Students may apply the same pattern for dividing by powers of 10 as they do for multiplying by powers of 10. You may wish to remind them of the inverse relationship between multiplication and division.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill and Fluency
9–11	2	Application
12–13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How does the relationship between place-value positions help you divide decimals by powers of 10?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you understand a problem situation today?

Students reflect on how they practiced responsible decision-making.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can use place-value patterns to divide a decimal by a power of 10.
- I can explain patterns when dividing a decimal by a power of 10.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Divide decimals by powers of 10	5.NBT.A.2
2	2	Divide decimals by powers of 10	5.NBT.A.2
3	1	Divide decimals by powers of 10	5.NBT.A.2
4	2	Divide decimals by powers of 10	5.NBT.A.2

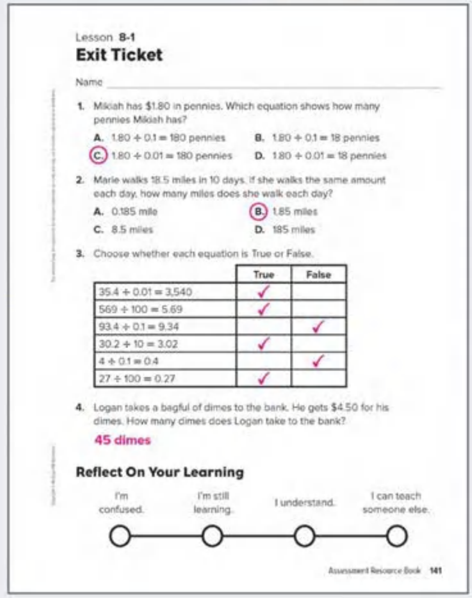
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 8-1

Exit Ticket

Name ____________

1. Mikiah has $1.80 in pennies. Which equation shows how many pennies Mikiah has?
   A. 1.80 ÷ 0.1 = 180 pennies B. 1.80 ÷ 0.1 = 18 pennies
   C. 1.80 ÷ 0.01 = 180 pennies D. 1.80 ÷ 0.01 = 18 pennies

2. Marie walks 18.5 miles in 10 days. If she walks the same amount each day, how many miles does she walk each day?
   A. 0.185 mile B. 1.85 miles
   C. 8.5 miles D. 185 miles

3. Choose whether each equation is True or False.

	True	False
35.4 ÷ 0.01 = 3,540	✓	
569 ÷ 100 = 5.69	✓	
93.4 ÷ 0.1 = 9.34		✓
30.2 ÷ 10 = 3.02	✓	
4 ÷ 0.1 = 0.4		✓
27 ÷ 100 = 0.27	✓	

4. Logan takes a bagful of dimes to the bank. He gets $4.50 for his dimes. How many dimes does Logan take to the bank?
   45 dimes

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 141

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Write It

Work with students in pairs as they create a diagram explaining the patterns that can be used to show decimal division problems using powers of ten as divisors. They should provide a sample problem and conclude with a summarizing sentence. Make sure students recognize that the quotient is greater than the dividend when the divisor is between 0 and 1.

GO ONLINE

Take Another Look Lesson

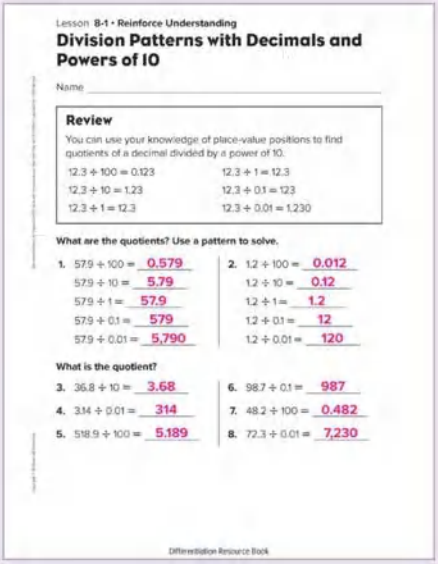
Assign the interactive lesson to reinforce targeted skills.

- Divide by Powers of 10 (Decimal Point)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 77



Lesson 8-1 • Reinforce Understanding

Division Patterns with Decimals and Powers of 10

Name

Review

You can use your knowledge of place-value positions to find quotients of a decimal divided by a power of 10.

12.3 ÷ 100 = 0.123	12.3 ÷ 1 = 12.3
12.3 ÷ 10 = 1.23	12.3 ÷ 0.1 = 123
12.3 ÷ 1 = 12.3	12.3 ÷ 0.01 = 1,230

What are the quotients? Use a pattern to solve.

1. 57.9 ÷ 100 = 0.579
   57.9 ÷ 10 = 5.79
   57.9 ÷ 1 = 57.9
   57.9 ÷ 0.1 = 579
   57.9 ÷ 0.01 = 5,790

2. 1.2 ÷ 100 = 0.012
   1.2 ÷ 10 = 0.12
   1.2 ÷ 1 = 1.2
   1.2 ÷ 0.1 = 12
   1.2 ÷ 0.01 = 120

What is the quotient?

3. 36.8 ÷ 10 = 3.68
4. 3.14 ÷ 0.01 = 314
5. 518.9 ÷ 100 = 5.189
6. 98.7 ÷ 0.1 = 987
7. 48.2 ÷ 100 = 0.482
8. 72.3 ÷ 0.01 = 7,230

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Divide by 0.1 and 0.01 Race

Students practice solving decimal division problems with divisors of 0.1 and 0.01.

GO ONLINE

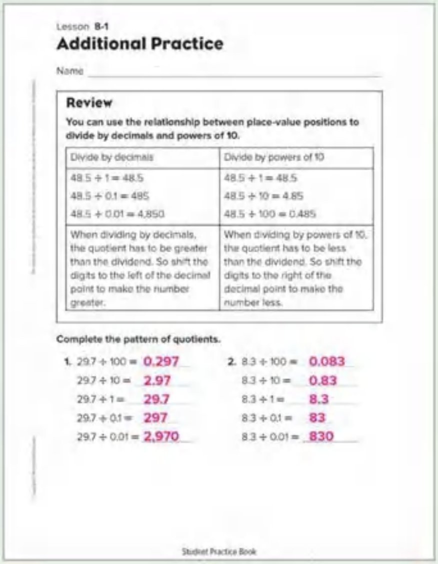
Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 77–78



Lesson 8-1

Additional Practice

Name

Review

You can use the relationship between place-value positions to divide by decimals and powers of 10.

Divide by decimals	Divide by powers of 10
48.5 ÷ 1 = 48.5 48.5 ÷ 0.1 = 485 48.5 ÷ 0.01 = 4,850	48.5 ÷ 1 = 48.5 48.5 ÷ 10 = 4.85 48.5 ÷ 100 = 0.485
When dividing by decimals, the quotient has to be greater than the dividend. So shift the digits to the left of the decimal point to make the number greater.	When dividing by powers of 10, the quotient has to be less than the dividend. So shift the digits to the right of the decimal point to make the number less.

Complete the pattern of quotients.

1. 29.7 ÷ 100 = 0.297
   29.7 ÷ 10 = 2.97
   29.7 ÷ 1 = 29.7
   29.7 ÷ 0.1 = 297
   29.7 ÷ 0.01 = 2,970

2. 8.3 ÷ 100 = 0.083
   8.3 ÷ 10 = 0.83
   8.3 ÷ 1 = 8.3
   8.3 ÷ 0.1 = 83
   8.3 ÷ 0.01 = 830

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with addition and subtraction within 1,000,000.

Assign

Use It! Application Station

Red Block White Block Road Block

Students use a looping code to move a dog through a maze and then make their own maze and code.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

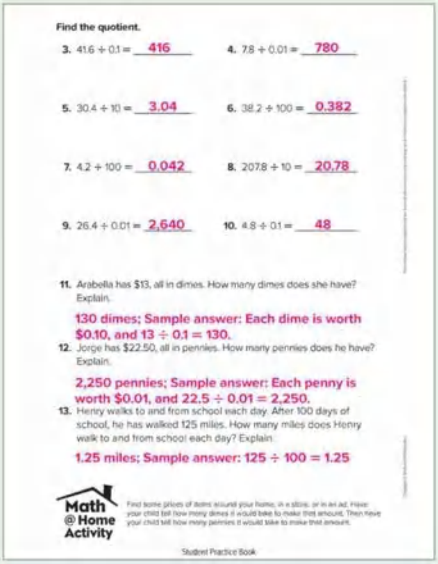
STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 77–78



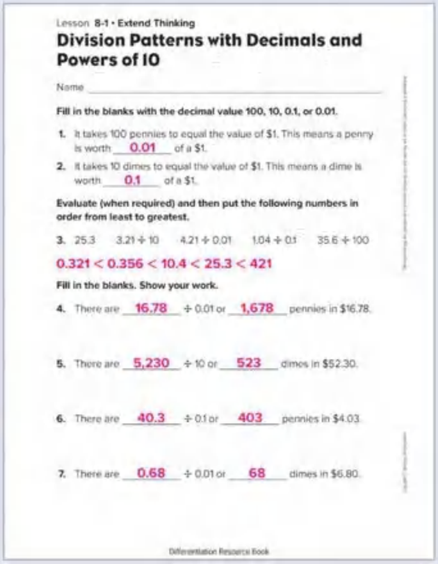
Find the quotient.

3. $41.6 \div 0.1 =$ 416
4. $7.8 \div 0.01 =$ 780
5. $30.4 \div 10 =$ 3.04
6. $38.2 \div 100 =$ 0.382
7. $4.2 \div 100 =$ 0.042
8. $207.8 \div 10 =$ 20.78
9. $26.4 \div 0.01 =$ 2,640
10. $4.8 \div 0.1 =$ 48
11. Arabella has \$13, all in dimes. How many dimes does she have? Explain.
    130 dimes; Sample answer: Each dime is worth \$0.10, and $13 \div 0.1 = 130$.
12. Jorge has \$22.50, all in pennies. How many pennies does he have? Explain.
    2,250 pennies; Sample answer: Each penny is worth \$0.01, and $22.5 \div 0.01 = 2,250$.
13. Henry walks to and from school each day. After 100 days of school, he has walked 125 miles. How many miles does Henry walk to and from school each day? Explain.
    1.25 miles; Sample answer: $125 \div 100 = 1.25$

Math @Home Activity: Find some prices of items around your home, in a store, or in an ad. Have your child tell how many dimes it would take to make that amount. Then have your child tell how many pennies it would take to make that amount.

Student Practice Book

Differentiation Resource Book, p. 78



Lesson 8-1 • Extend Thinking

Division Patterns with Decimals and Powers of 10

Name ____________

Fill in the blanks with the decimal value 100, 10, 0.1, or 0.01.

1. It takes 100 pennies to equal the value of \$1. This means a penny is worth 0.01 of a \$1.
2. It takes 10 dimes to equal the value of \$1. This means a dime is worth 0.1 of a \$1.

Evaluate (when required) and then put the following numbers in order from least to greatest.

3. 25.3 $\quad 3.21 \div 10 \quad 4.21 \div 0.01 \quad 1.04 \div 0.1 \quad 35.6 \div 100$
   $0.321 < 0.356 < 10.4 < 25.3 < 421$

Fill in the blanks. Show your work.

4. There are 16.78 $\div$ 0.01 or 1,678 pennies in \$16.78.
5. There are 5,230 $\div$ 10 or 523 dimes in \$52.30.
6. There are 40.3 $\div$ 0.1 or 403 pennies in \$4.03.
7. There are 0.68 $\div$ 0.01 or 68 dimes in \$6.80.

Differentiation Resource Book

LESSON **8-2**

Estimate Quotients of Decimals

Learning Targets

- I can explain how to estimate quotients of decimals.
- I can estimate quotients of decimals to determine if calculations are reasonable.
- I can use an estimated quotient to make predictions about a calculated solution.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NBT.B** Perform operations with multi-digit whole numbers and with decimals to hundredths.

◇ **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Math Practices and Processes

MPP Reason abstractly and quantitatively.

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	Academic Terms
dividend	negate
divisor	variation
estimate	
quotient	

Materials

The materials may be for any part of the lesson.

- calculators
- number cubes

Focus

Content Objectives	Language Objectives	SEL Objective
• Students estimate quotients of decimals using the same strategies used to estimate quotients of whole numbers. • Students use estimated quotients to make predictions about and assess the reasonableness of a calculated solution.	• Students discuss estimating the quotients of decimals while answering *Wh-* and Yes/No questions and using terms such as *could* and *would*. • To support sense-making, ELs participate in MLR6: Three Reads.	• Students practice strategies for persisting at a mathematical task, such as setting a small goal or setting timers for remaining focused.

Coherence

Previous	Now	Next
• Students found whole-number quotients and remainders (Grade 4). • Students used place-value patterns to divide decimals by powers of 10 (Unit 8).	• Students estimate quotients involving decimals.	• Students represent division of decimals by a whole number (Unit 8). • Students add, subtract, multiply, and divide using the standard algorithm (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students gain an understanding of estimation as a method to help determine the reasonableness of calculations involving decimal quotients.	• Students build their proficiency with division with decimals as they use estimation to develop skill in evaluating the reasonableness of quotients.	• Students estimate decimal division using measurement in real-world contexts. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
About How Much?

5–7 min

Build Fluency Students build number sense as they estimate the value of multiplication expressions.

Remind students that this routine involves mental math only. Students should not solve for the product.

These prompts encourage students to talk about their reasoning:

- How did you find your estimate? How did you determine whether your estimates were reasonable?
- How did you round each factor? How can you round the numbers differently so that the multiplication is easier?
- Why is more than one estimate reasonable?

Purpose Students read a numberless word problem as they engage in contextual sense-making. They share thoughts on what math they see in the problem, describe the relationship between the quantities, and think about different ways to solve a comparison.

Numberless Word Problem

- What could you ask?
- What math do you use in this problem?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students work in pairs as they make sense of the context. Encourage them to think about quantities and their relationship.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' thinking about estimating quotients and are based on possible comments and questions that students may make during the share out.

- How do you know what operation to use to solve the problem?
- What do you need to know to determine the answer to your question?

Math is... Mindset

- What do you do to stay focused on your work?

SEL Self-Management: Self-Discipline

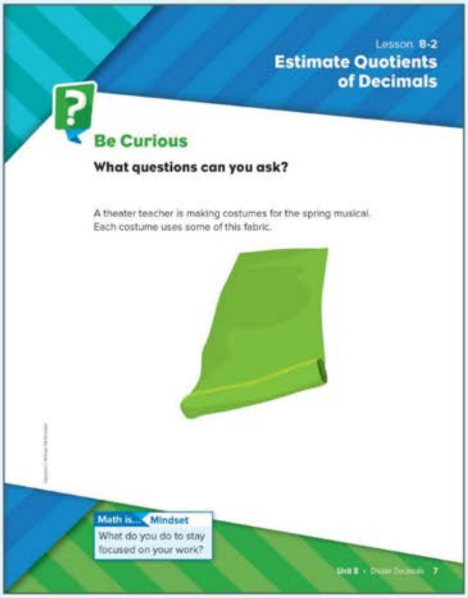
Help students develop strong learning habits by providing them with opportunities to practice self-management. Before beginning the Numberless Word Problem routine, discuss ways that students will manage distractions and stay focused on their work estimating quotients of decimals. Some students may benefit from setting a goal to remain on task for a set time before taking a mental break, or considering what routines they follow that might help with their work.

Transition to Explore & Develop

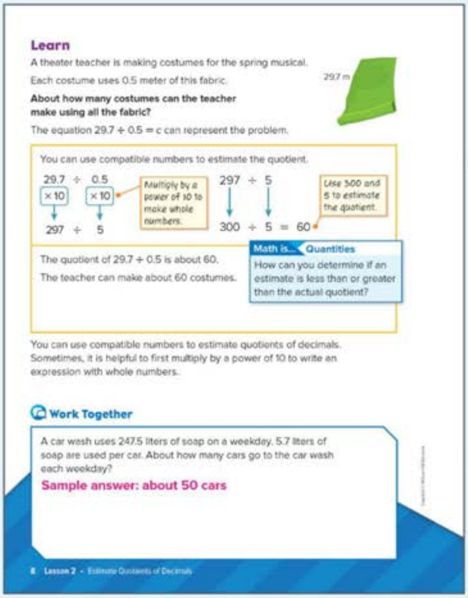
Have students share their thoughts on what they think the quantities may be in the problem. Ask them to describe the relationship between the quantities in the problem. Explain that they are going to consider the same problem but with numbers this time.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can use estimation strategies that we already know for estimation of quotients of whole numbers to estimate quotients of decimals.



Explore & Develop 20 min


Learn

A theater teacher is making costumes for the spring musical.

Each costume uses 0.5 meter of this fabric.

29.7 m

About how many costumes can the teacher make using all the fabric?

The equation $29.7 \div 0.5 = c$ can represent the problem.

You can use compatible numbers to estimate the quotient.

$29.7 \div 0.5$

$\boxed{\times 10}$ $\boxed{\times 10}$

$297 \div 5$

Multiply by a power of 10 to make whole numbers.

$297 \div 5$

$300 \div 5 = 60$

Use 300 and 5 to estimate the quotient.

The quotient of $29.7 \div 0.5$ is about 60.

The teacher can make about 60 costumes.

Math is... Quantities

How can you determine if an estimate is less than or greater than the actual quotient?

You can use compatible numbers to estimate quotients of decimals. Sometimes, it is helpful to first multiply by a power of 10 to write an expression with whole numbers.

Work Together

A car wash uses 247.5 liters of soap on a weekday. 5.7 liters of soap are used per car. About how many cars go to the car wash each weekday?

Sample answer: about 50 cars

8 Lesson 2 • Estimate Quotients of Decimals

1 Pose the Problem

ETP Pose Purposeful Questions

- What are the important facts in the problem? How did you determine which facts are important?
- Do you need an exact answer? How do you know?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Three Reads

1st Read: Have students underline the key numbers that will be used to solve the problem.

2nd Read: Have students write the meaning of each number in context.

3rd Read: Have students work in pairs to create mathematical expressions estimating the quotient.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How is estimating quotients of decimals similar to estimating quotients of whole numbers? How is it different?
- Why is estimating quotients useful?

Key Takeaways

- Estimating quotients can help make predictions about a calculated solution.
- Estimating quotients helps assess the reasonableness of a calculated solution.
- Strategies used to estimate quotients of whole numbers, such as compatible numbers, can also be used to estimate quotients involving decimals.

Work Together

Students work together to solve a word problem involving estimating the quotient of decimals.

Common Error If students multiply both the dividend and divisor by 10, make sure they understand they do not have to then divide their estimate by 10.

LOM Language of Math

As students solve problems, make sure they refer to their estimates as *estimates* or *estimated quotients* rather than *quotients* or *calculated quotients*. It is important to use precise language in mathematics, and that is one of the practices detailed in SMP 6.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore estimating quotients by extending estimation strategies used for estimating quotients of whole numbers.

Directions: Work with students to write the division expression that represents the Pose the Problem. Have students work in pairs or small groups to estimate the quotient.

ETP Support Productive Struggle

- Do you think the quotient will be greater than or less than the dividend? Explain why.
- What could you do to make the numbers easier to work with?
- What estimatation strategies have you used to estimate the quotient of whole numbers? How can you extend those strategies to decimals?

Provide students with two possible calculated quotients, one that is correct and one that is incorrect. Ask students to use their estimate to determine which quotient is reasonable.

Math is... Quantities

- How can you determine if an estimate is less than or greater than the actual quotient?

Students make sense of quantities and their relationships in problem situations.

Activity Debrief: Have students share their strategies for estimating the quotient. Facilitate a discussion to ensure students understand that there are multiple estimation strategies, such as using compatible numbers and thinking about the relationship between multiplication and division.

Guided Exploration

Students use compatible numbers to estimate the quotients of decimals.

ETP Facilitate Mathematical Meaningful Discourse

- What estimation strategies have you used before?

Have students justify the strategy using place value and the relationship between multiplication and division. Ask:

- How can you write the equation $29.7 \div 0.5 = q$ as a related multiplication equation?
- Explain how you use place-value patterns and the equation $q \times 0.5 = 29.7$ to determine the product $q \times 5$?
- How can you write $q \times 5 = 297$ as a related division equation?
- Since $29.7 \div 0.5 = q$ and $297 \div 5 = q$, what does that tell you about these expressions?
- **Think About It:** Why did you multiply by 10 and not a different power of 10?

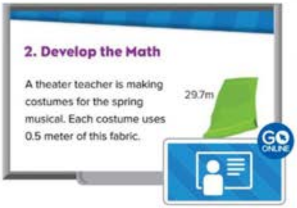
Have the students estimate the solution to $297 \div 5$. Ask:

- What compatible numbers will you use to estimate the solution? Why?

Math is... Quantities

- How can you determine if an estimate is less than or greater than the actual quotient?

Students make sense of quantities and their relationships in problem situations.



EL English Learner Scaffolds

Entering/Emerging Support students in understanding the term *both*. Pick up two classroom objects. Say, *I'm holding both*. Repeat the task with another set of classroom objects to demonstrate *both*. Then ask students to find the example of *both* on the Learn page. Point out how there are two decimals.

Developing/Expanding Support students in understanding the term *both*. Pick up two classroom objects. Say, *I'm holding both*. Repeat the task with another set of classroom objects to demonstrate *both*. Ask students to use *both* in a sentence, using classroom objects. Then ask students to find the example of *both* on the Learn page. Have them say what both refers to (the two decimals).

Bridging/Reaching Ask students to use *both* in a sentence, demonstrating with two classroom objects. Then instruct students to work in groups to come up with synonyms for it, such as *one and the other*, *the couple*, *the pair*, and *the two*, and to share their list with the class. Allow students to use a dictionary or thesaurus if preferred.

Practice & Reflect 10 min

On My Own

Name ______

Estimate the quotient. Sample answers given.

1. $4.42 \div 0.81 = x$ **5**
2. $36.8 \div 5.7 = d$ **7**
3. $19.73 \div 3.21 = c$ **7**
4. $5.4 \div 0.25 = m$ **18**

Which is a reasonable calculated quotient for each expression?

5. $7.78 \div 0.84 = d$
   - A. 92
   - B. 9.2
   - C. 0.92
   - D. 1.92
6. $23.4 \div 3.2 = s$
   - A. 7.3
   - B. 73.3
   - C. 70.3
   - D. 780.3
7. $4.2 \div 0.96 = b$
   - A. 43.75
   - B. 33.75
   - C. 4.3
   - D. 0.43
8. $13.2 \div 7.4 = p$
   - A. 1.7
   - B. 10.7
   - C. 17.2
   - D. 170.3

Unit 8 • Divide Decimals 9

9. Laraine has $13 to spend.
   a. If she buys only songs, about how many songs can she download? **about 6 songs**
   b. If she buys only games, about how many games can she download? **about 3 games**

10. **Error Analysis** Tess calculated that the quotient for the division expression $10.5 \div 2.1$ is 0.5. She says that her calculation is reasonable. How do you respond to Tess?
    No, her calculation is not reasonable. I can use compatible numbers 10 and 2 to estimate that the quotient is about 5, which is not close to 0.5.
11. Janet has $15.37 to spend on bus fare for school. Each bus ride she takes costs $2.25. About how many bus rides can she take with the amount of money she has?
    Sample answer: about 6 bus rides
12. **Extend Your Thinking** Write a division expression with decimals that has an estimated quotient of 6.
    Sample answer: $32.4 \div 5.1$

Reflect

How did you apply what you already know about estimation to estimating the quotients of decimals?
Answers may vary.

Math is... Mindset
What helped you stay focused on your work?

10 Lesson 2 • Estimate Quotients of Decimals

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Misconception: Exercises 1–8 As students find estimates for each equation, make sure they remember that they can, and should, use the numbers easiest for them. For example, when they see two digits to the right of the decimal point in 4.42 and 0.81, they may multiply by 100, and use the compatible numbers 442 and 81. Or they may find it is simpler to multiply by 10, use 44.2 and 8.1, and round the numbers from there.

Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill and Fluency
5–8	2	Procedural Skill and Fluency
9	2	Application
10	3	Conceptual Understanding
11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How did you apply what you already know about estimation to estimating the quotients of decimals?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you stay focused on your work?

Students reflect on how they practiced self-regulation.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can explain how to estimate quotients of decimals.
- I can estimate quotients of decimals to determine if calculations are reasonable.
- I can use an estimated quotient to make predictions about a calculated solution.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Estimate decimal quotients	5.NBT.B.7
2	1	Estimate decimal quotients	5.NBT.B.7
3	2	Estimate decimal quotients	5.NBT.B.7

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking

Lesson 8-2
Exit Ticket

Name ______

1. Which is the most reasonable estimate for 41.53 ÷ 0.632?
   A. 7 B. 9
   C. 70 D. 90
2. Which equations show a reasonable estimate for the quotient 5.32 ÷ 0.09 using powers of 10 and compatible numbers? Choose all that apply.
   A. 5 ÷ 0.1 = 50
   B. 540 ÷ 9 = 60
   C. 54 ÷ 9 = 6
   D. 50 ÷ 10 = 5
3. Olivia buys beads to make a necklace.
   - She pays $17.98 for 6 star beads.
   - She pays $8.98 for 2 heart beads.
   - She pays $16.16 for 9 glitter beads.
   - She pays $12.32 for 3 striped beads.

   Using estimation, which type of bead costs the least per bead?
   A. heart bead B. striped bead
   C. star bead D. glitter bead

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

142 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Roll to Round

Work with students in pairs. Students roll two number cubes and create a 2-digit dividend by putting the greater digit in the ones place and the lesser digit in the tenths place. Then roll one number cube to create a 1-digit divisor with the digit in tenths. Students estimate an answer. Students check their estimates by finding the quotient with a calculator. Discuss with students how the estimates related to the actual quotient.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Estimate Quotients (Decimal Numbers)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 79

Lesson 8-2 • Reinforce Understanding

Estimate Quotients of Decimals

Name ______

Review

You can use place value and powers of 10 to help you estimate the quotient.

54.2 ÷ 0.91

First, multiply both numbers by 10, 100, or 1000.	54.2 ÷ 0.91 (54.2 × **100**) ÷ (0.91 × **100**) 5,420 ÷ 91
Next, use compatible numbers or rounding.	5,420 ÷ 91 ↓ ↓ 5,400 ÷ 90

A possible estimate for 54.2 ÷ 0.91 is 60.

Which is a reasonable quotient? Estimate the quotient of each expression and determine the reasonable calculated quotient. Show your work.

1. 90.6 ÷ 2.9 Sample: 900 ÷ 30 = 30
   A. 0.312
   B. 3.12
   C. 31.2
   D. 312

2. 5.58 ÷ 0.82 Sample: 560 ÷ 80 = 7
   A. 0.68
   B. 6.8
   C. 68
   D. 680

3. 2.99 ÷ 0.59 Sample: 300 ÷ 60 = 50
   A. 0.507
   B. 5.07
   C. 50.7
   D. 507

4. 66.2 ÷ 0.61 Sample: 6,600 ÷ 600 = 110
   A. 0.1085
   B. 1.085
   C. 10.85
   D. 108.5

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Estimating Quotients Bump

Students practice estimating quotients of decimals.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 79–80

Lesson 8-2

Additional Practice

Name ______

Review

You can use compatible numbers to estimate a quotient.

Estimate the quotient 31.6 ÷ 0.6. When dividing by a decimal less than 1, multiply the dividend and divisor so that the divisor is a whole number, then look for compatible numbers.	Estimate the quotient 31.6 ÷ 8.4. When dividing by a decimal greater than 1, look for compatible numbers.
To make 0.6 into a whole number, multiply by 10. Also, multiply the dividend, 31.6, by 10. 31.6 ÷ 0.6 316 ÷ 6	An estimate for 31.6 ÷ 8.4 is 32 ÷ 8 = 4
Now use compatible numbers. An estimate for 316 ÷ 6 is 300 ÷ 6 = 50 So 31.6 ÷ 0.6 is about 50.	So 31.6 ÷ 8.4 is about 4.

Which is the quotient?

1. 31.28 ÷ 0.46
   a. 0.68
   b. 6.8
   c. 68
   d. 680

2. 43.99 ÷ 8.3
   a. 0.53
   b. 5.3
   c. 53
   d. 530

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with addition and subtraction within 1,000,000.

Assign

Use It! Application Station

Red Block White Block Road Block

Students use a looping code to move a dog through a maze and then make their own maze and code.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 79–80

Estimate the quotient. Show your work. Sample answers given

3. 24.45 ÷ 0.8
   30; 240 ÷ 8
4. 73.4 ÷ 7.7
   9; 72 ÷ 8
5. 12.83 ÷ 0.21
   60; 1,200 ÷ 20
6. 9.1 ÷ 2.8
   3; 9 ÷ 3
7. 83.24 ÷ 9.06
   9; 81 ÷ 9
8. 65.2 ÷ 0.87
   70; 630 ÷ 9
9. Harriet spends $12.58 on some stickers. Each sticker costs $0.06. About how many stickers did Harriet buy? Explain.
   Sample answer: about 200 stickers; 12.58 ÷ 0.06 is equivalent to 1,258 ÷ 6, which can be estimated by 1,200 ÷ 6 = 200.
10. A bicycle race covers a distance of 64.5 kilometers. There are water stations every 7.6 kilometers. About how many water stations are there along the course? Explain.
    Sample answer: about 8 water stations; 64.5 ÷ 7.6 can be estimated by 64 ÷ 8 = 8.

Math @Home Activity Practice estimating quotients with your child. While preparing dinner or packing lunches, look for situations where it would be natural to find an estimate for a quotient. For example, if a jar of peanut butter contains 11.7 ounces, and each sandwich uses 0.38 ounce, about how many sandwiches can be made? Allow them to use a calculator to check the estimate.

Student Practice Book

Differentiation Resource Book, p. 80

Lesson 8-2 • Extend Thinking

Estimate Quotients of Decimals

Name

Write a division expression with decimals that has the estimated quotient. The first one is done for you.

Sample answers given. Check students' work.

	Division Expression	Estimated Division Expression	Estimated Quotient
1.	5.8 ÷ 2.1	6 ÷ 2	3
2.	5.39 ÷ 0.62	5.4 ÷ 0.6	9
3.	3.198 ÷ 0.401	3.2 ÷ 0.4	8
4.	0.3599 ÷ 0.0289	0.36 ÷ 0.03	12
5.	75.01 ÷ 14.9	75 ÷ 15	5
6.	1.189 ÷ 0.204	1.2 ÷ 0.2	6
7.	49.1 ÷ 6.8	49 ÷ 7	7
8.	559.91 ÷ 141.04	560 ÷ 140	4

Differentiation Resource Book

LESSON **8-3**

Represent Division of Decimals by a Whole Number

Learning Target

- I can represent division of decimals by whole numbers using equal sharing or equal grouping.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NBT.B** Perform operations with multi-digit whole numbers and with decimals to hundredths.

◇ **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Math Practices and Processes

MPP Model with mathematics.

MPP Look for and express regularity in repeated reasoning.

Vocabulary

Math Terms	Academic Terms
decimal	analyze
dividend	suggest
divisor	

Materials

The materials may be for any part of the lesson.

- bills and coins manipulatives
- index cards
- *Tenths and Hundredths Representations* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students represent division of decimals with equal sharing or equal grouping.	• Students discuss how to divide decimals by whole numbers while answering *Wh-* questions and using the modal *might*. • To support maximizing linguistic and cognitive meta-awareness and optimize output, ELs participate in MLR7: Compare and Connect.	• Students engage in active listening and work collaboratively with a partner to complete mathematical tasks.

Coherence

Previous	Now	Next
• Students found whole-number quotients and remainders (Grade 4). • Students estimated quotients involving decimals (Unit 8).	• Students represent division of decimals by a whole number.	• Students use place-value understanding and modeling to divide decimals by whole numbers (Unit 8). • Students add, subtract, multiply, and divide using the standard algorithm (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students use representations and the relationship between multiplication and division to better understand division of decimals by whole numbers.	• Students build their proficiency with division as they expand their skills to include division of decimals by whole numbers.	• Students divide decimals by whole numbers in problems with real-world contexts. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
About How Much?

5–7 min

Build Fluency Students build number sense as they estimate the value of multiplication expressions.

Remind students that this routine involves mental math only. Students should not solve for the product.

These prompts encourage students to talk about their reasoning:

- How did you find your estimate?
- How do you determine if an estimate is reasonable?
- How did you round each factor? How did you choose to round to the nearest hundred or ten?
- Why is more than one estimate reasonable?

Launch 5–7 min

Sense-Making Routine

Purpose Students think about equal sharing and how they might divide money among banks when the amount of money is not a whole number.

Notice & Wonder

- What question could you ask?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students work in small groups as they notice and wonder. Encourage students to build on one another's thinking as they discuss any questions they can ask about the image.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of representing dividing decimals by whole numbers and are based on possible comments and questions that students may make during the share out.

- If there were no coins, how could you share the dollar bills?
- If there will no dollar bills, how could you share the coins?

Math is… Mindset

- What can you do today to help build a relationship with a classmate?

SEL Relationship Skills: Build Relationships

Invite students to work with a partner, possibly a new or less familiar peer for those who feel comfortable, to complete the Notice & Wonder routine. Encourage students to actively and respectfully listen to one another as they explore and collaborate to represent division of decimals by a whole number.

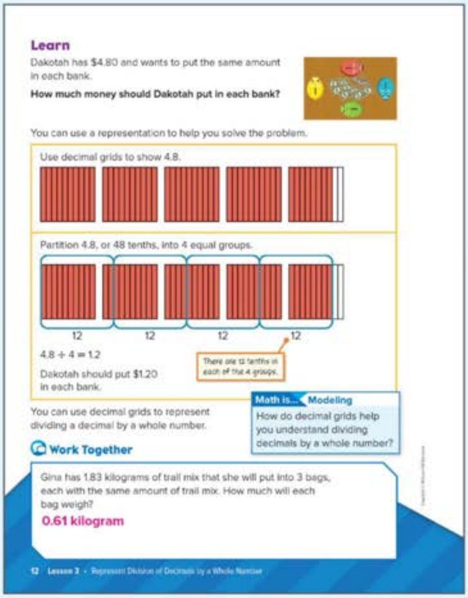
Transition to Explore & Develop

Ask questions that get students thinking about real-world situations involving dividing money in a group of people. Encourage students to think of situations where the amount of money is not a whole number, such as $5.00, but a decimal like $5.50. Have students think about how they would determine how to split up that amount of money in a group.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can represent dividing decimals by whole numbers.

Explore & Develop 20 min



Learn

Dakotah has $4.80 and wants to put the same amount in each bank.

How much money should Dakotah put in each bank?

You can use a representation to help you solve the problem.

Use decimal grids to show 4.8.

Partition 4.8, or 48 tenths, into 4 equal groups.

12 12 12 12

$4.8 \div 4 = 1.2$

Dakotah should put $1.20 in each bank.

There are 12 tenths in each of the 4 groups.

You can use decimal grids to represent dividing a decimal by a whole number.

Math is... Modeling

How do decimal grids help you understand dividing decimals by a whole number?

Work Together

Gina has 1.83 kilograms of trail mix that she will put into 3 bags, each with the same amount of trail mix. How much will each bag weigh?

0.61 kilogram

12 Lesson 3 • Represent Division of Decimals by a Whole Number

❶ Pose the Problem

MLR Discussion Supports

Prompt students to think about the tools and strategies they already know that can help them answer the questions. List the information that students provide on the board, and then prompt students to use the information to help them solve the problem.

ETP Pose Purposeful Questions

- How does the picture help you understand the problem?
- What are the important quantities in this problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and assign them a division problem to solve. Have them work alone, one solving it using a division equation and the other solving it using a decimal grid. Have them compare their answers and approaches. Revisit this activity throughout the lesson to help students build proficiency.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do using equations and decimal grids help you represent dividing a decimal by a whole number?

Key Takeaway

- Division of decimals can be represented by equal sharing or equal grouping.

Work Together

Students work together to solve a division word problem involving a decimal. Encourage students to estimate the quotient before they begin solving.

Common Error Students have only used decimal grids that are split into tenths at this point in the lesson. Make sure students study the place value of the decimal in the Work Together problem to understand that they should use decimal grids split into hundredths instead.

LOM Language of Math

Looking at the *-or* suffix may help students to remember which is the *divisor.* Words ending in *-or* are people or things that perform actions, like collectors who collect, or excavators that excavate. Divisors divide, dividends get divided.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using representations to divide a decimal by a whole number.

Materials: *Tenths and Hundredths Representations* Teaching Resource, bills and coins manipulatives

Directions: Make available copies of *Tenths and Hundredths Representations* Teaching Resource and bills and coins manipulatives. Have students work together to solve the Pose the Problem. Students may use any tool they choose to help them solve the problem.

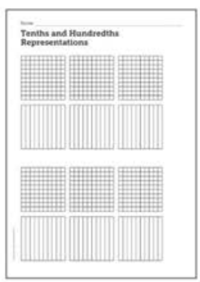
ETP Support Productive Struggle

- How can you determine the operation you need to use to solve the problem?
- What tools can you use to help you determine the quotient?
- What are some strategies you can use to determine the quotient?
- Is the quotient greater than or less than 1? Explain why.

Math is... Modeling

- How does a representation help you understand dividing decimals by a whole number?

Students reflect on if using a representation has served its purpose to help them understand dividing a decimal by a whole number.



Activity Debrief: Have students share their strategies for determining the quotient. Encourage students to look for similarities and differences between the strategies.

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students use decimal grids to solve a division word problem with a decimal as the dividend and a whole number as the divisor.

ETP Use and Connect Mathematical Representations

Have the students estimate the solution. Ask:

- What power of ten will you multiply by? Why?
- What compatible numbers will you use? Why?
- How will basic facts and place-value patterns help you?
- **Think About It:** Why are there 5 decimal grids? Why are the decimal grids showing tenths?

Have students complete their own decimal grids to determine an equivalent representation of 12 tenths. Ask:

- Will you need more than one decimal grid to determine the equivalent representation of 12 tenths?

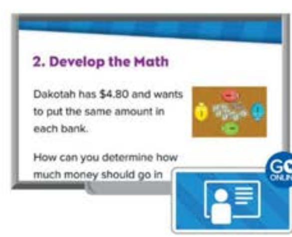
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

Math is... Modeling

- How do decimal grids help you understand dividing decimals by a whole number?

Students reflect on if using decimal grids as a model has served its purpose to help them understand dividing a decimal by a whole number.



EL English Learner Scaffolds

Entering/Emerging Support students' understanding of the adverb *evenly* using classroom manipulatives. Set ten counting chips on the desk. Say *I'm going to split these evenly*. Split into two groups of five. Demonstrate again with classroom manipulatives. Finally, test comprehension by demonstrating the task two more times, one correctly splitting the items up evenly, and one time not. Ask after each demonstration, *Did I split these evenly?*

Developing/Expanding Support students' understanding of the adverb *evenly* using classroom manipulatives. Set ten counting chips on the desk. Say *I'm going to split these evenly*. Split into two groups of five. Demonstrate again with classroom manipulatives. Finally, ask students to use *evenly* in a sentence, demonstrating with manipulatives. Provide sentence frames for students who need more guidance.

Bridging/Reaching Ask students to use *evenly* in a sentence, demonstrating with manipulatives. Then instruct students to work in groups, using a dictionary to come up with similar-meaning words, such as *equally* and *uniformly*, and to share their list with the class.

Practice & Reflect 10 min

On My Own

Name ______

What is the quotient? Use decimal grids to solve. Check students' grids.

1. $3.5 \div 7 =$ 0.5
2. $4.53 \div 3 =$ 1.51
3. $2.04 \div 4 =$ 0.51
4. $2.8 \div 2 =$ 1.4
5. $3.9 \div 3 =$ 1.3
6. $6.9 \div 3 =$ 2.3
7. $0.72 \div 8 =$ 0.09
8. $2.4 \div 4 =$ 0.6

9. Six friends are going to run a relay race that is 3.12 miles long. Each friend will run an equal distance. How many miles will each friend run?
   0.52 mile
10. A street is 6.3 miles long. Workers partition the street into 3 equal parts for a renovation project. How long is each part?
    2.1 miles
11. **STEM Connection** Saffron measured out 6.5 cups of flour. She plans to use an equal amount of flour to make 5 batches of cupcakes. How many cups of flour will be in each batch?
    1.3 cups

12. **Extend Your Thinking** How does knowing how to divide whole numbers help you divide a decimal by a whole number? Explain your thinking.
    Sample answer: I can use what I know about dividing whole numbers and relating division to multiplication and apply it to dividing decimals by whole numbers.

Reflect

How is dividing a decimal by a whole number similar to or different from dividing whole numbers?

Answers may vary.

Math is... Mindset

What helped you to build a relationship with a classmate today?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 3 Some students may incorrectly represent a decimal containing 0, like 2.04, as 2.4. Suggest that students check their answers by either using multiplication or repeated addition.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill and Fluency
9–11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How is dividing a decimal by a whole number similar to or different from dividing whole numbers?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you to build a relationship with a classmate today?

Students reflect on how they developed stronger relationship skills.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can represent division of decimals by whole numbers using equal sharing or equal grouping.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1a	2	Represent division of decimals by a whole number	5.NBT.B.7
1b	2	Represent division of decimals by a whole number	5.NBT.B.7
1c	2	Represent division of decimals by a whole number	5.NBT.B.7
2	1	Represent division of decimals by a whole number	5.NBT.B.7
3	2	Represent division of decimals by a whole number	5.NBT.B.7

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

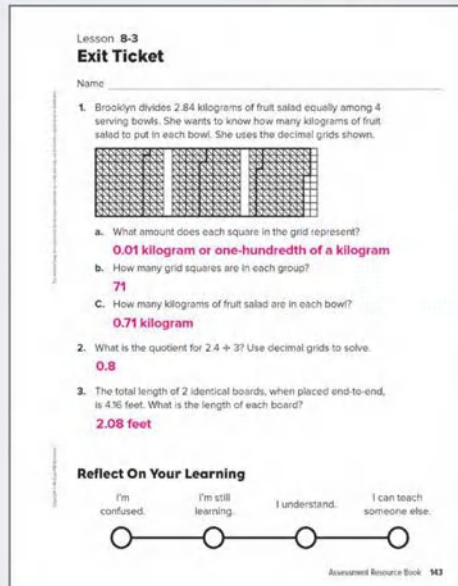
If students score	Then have students do
5 of 5	Additional Practice or any of the B or E activities
4 of 5	*Take Another Look* or any of the B activities
3 or fewer of 5	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 8-3

Exit Ticket

Name ______

1. Brooklyn divides 2.84 kilograms of fruit salad equally among 4 serving bowls. She wants to know how many kilograms of fruit salad to put in each bowl. She uses the decimal grids shown.

a. What amount does each square in the grid represent?

0.01 kilogram or one-hundredth of a kilogram

b. How many grid squares are in each group?

71

c. How many kilograms of fruit salad are in each bowl?

0.71 kilogram

2. What is the quotient for $2.4 \div 3$? Use decimal grids to solve.

0.8

3. The total length of 2 identical boards, when placed end-to-end, is 4.16 feet. What is the length of each board?

2.08 feet

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 143

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Solve It, Keep It

Work with students in pairs. Provide each pair with expressions on index cards such as these: 3.5 ÷ 5, 2.4 ÷ 3, 3.6 ÷ 6, 4.8 ÷ 4. One student chooses a card and uses a representation to solve the problem. Encourage students to estimate the quotient to check for reasonableness. If the pair agrees on the quotient, the student keeps the card. Then the other student takes a turn until all cards have been collected.

GO ONLINE

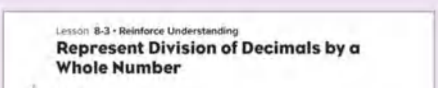
Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Divide Decimals by Whole Numbers-Model

INDEPENDENT WORK

Differentiation Resource Book, p. 81


Lesson 8-3 • Reinforce Understanding

Represent Division of Decimals by a Whole Number

Name ____________

Review

You can use decimal grids to help you find quotients.

To determine 3.6 ÷ 9, we divide 3.6 into 9 groups.

There are 4 tenths in each of the 9 groups.

3.6 ÷ 9 = 0.4

What is the quotient? Use decimal grids to solve.

1. 4.2 ÷ 6 = 0.7
2. 6.8 ÷ 4 = 1.7
3. 0.45 ÷ 3 = 0.15
4. 0.81 ÷ 3 = 0.27

Differentiation Resource Book

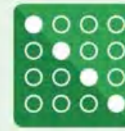
B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Represent Decimal Division Four in a Row

Students practice representing the division of decimals by whole numbers.



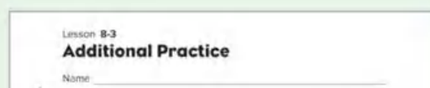
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 81–82


Lesson 8-3

Additional Practice

Name ____________

Review

You can use tenths or hundredths grids to show how to divide a decimal by a whole number.

Molly has $2.60. She wants to divide the money into 4 equal groups. How much money will be in each group?

Use hundredths decimal grids to represent the division.

Divide 260 hundredths into 4 equal groups. Each group has 65 hundredths.

There will be $0.65 in each group.

Write the equation shown by the decimal grids.

1. 1.75 ÷ 5 = 0.35
2. 3.2 ÷ 8 = 0.4

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with addition and subtraction within 1,000,000.

Assign

Use It! Application Station

That is Tasty! Students use decimal division to size up and down a recipe. *The content of this card has concepts covered later in Lesson 8-6. You may want to assign this card to students ready to explore content covered later in this unit.*

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 81–82

What is the quotient? Use decimal grids to solve.

3. 4.2 ÷ 6
0.7

4. 1.86 ÷ 3
0.62

5. 3.36 ÷ 8
0.42

6. 4.32 ÷ 4
1.08

7. 1.8 ÷ 9
0.2

8. 6.37 ÷ 7
0.91

9. Miriam has a length of string that is 6.5 inches long. She cuts it into 5 equal lengths. How long is each piece of string?
1.3 inches

10. Along a hike, Jon takes 6 pictures. He takes 1 picture at equal distances along the 3.72-mile trail. How far did Jon hike between pictures?
0.62 mile(s)

Math @ Home Activity On a sheet of paper, write a similar problem to the ones in this lesson involving the division of a decimal by a whole number. Have your child show you how to use decimal grids to find the quotient. Repeat with a different division problem.

Student Practice Book

Differentiation Resource Book, p. 82

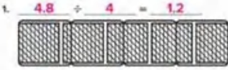
Lesson 8-3 • Extend Thinking

Represent Division of Decimals by a Whole Number

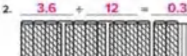
Name ______

Use the decimal grids to write the equation and solution.

1. 4.8 ÷ 4 = 1.2



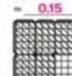
2. 3.6 ÷ 12 = 0.3



3. 5.6 ÷ 7 = 0.8

4. 0.96 ÷ 8 = 0.12

5. 0.75 ÷ 5 = 0.15



Write the solutions in order from greatest to least.

1.2 > 0.8 > 0.3 > 0.15 > 0.12

Differentiation Resource Book

LESSON **8-4**

Divide Decimals by Whole Numbers

Learning Target

- I can use place-value understanding and equivalent representations to divide a decimal by a whole number.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NBT.B** Perform operations with multi-digit whole numbers and with decimals to hundredths.

◇ **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Math Practices and Processes

MPP Use appropriate tools strategically.

MPP Look for and express regularity in repeated reasoning.

Vocabulary

Math Terms	Academic Terms
dividend	infer
divisor	transition
place value	
quotient	

Materials

The materials may be for any part of the lesson.

- number cubes

Focus

Content Objective

- Students use place-value understanding and equivalent representations to divide a decimal by a whole number.

Language Objectives

- Students explain how to divide a decimal by a whole number by answering multiple *How* questions using *can*.
- To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.

SEL Objective

- Students identify and discuss the emotions experienced during math learning.

Coherence

Previous

- Students found whole-number quotients and remainders (Grade 4).
- Students represented division of decimals by a whole number (Unit 8).

Now

- Students use place-value understanding and equivalent representations to divide decimals by whole numbers.

Next

- Students divide whole numbers by decimals using decimal grids and equivalent equations (Unit 8).
- Students add, subtract, multiply, and divide using the standard algorithm (Grade 6).

Rigor

Conceptual Understanding

- Students build on their understanding of dividing decimals as they begin to notice generalizable patterns through visual representations.

Procedural Skill & Fluency

- Students build their proficiency for decimal place value, basic facts, and division strategies by expanding their skills to include division of decimals by whole numbers.

Application

- Students apply their understanding of dividing decimals by whole numbers to solve problems with real-world contexts.

Application is not a targeted element of rigor for this standard.

Number Routine About How Much?

GO ONLINE

5–7 min

Build Fluency Students build number sense as they estimate the value of each multiplication expression.

Remind students that this routine involves mental math only. Students should not solve for the product.

These prompts encourage students to talk about their reasoning:

- How did you find your estimate? Which estimates were reasonable?
- How do you determine if an estimate is reasonable?
- How can you estimate an answer using only mental math?
- How did you think about each factor?
- Why is more than one estimate reasonable?

Launch 5–7 min

Sense-Making Routine

Purpose Students think about equivalent representations of numbers (for example, 1.2 is equivalent to 12 tenths).

Which Doesn't Belong?

- Which doesn't belong?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students work on their own to determine which doesn't belong so that they can compare their individual answers with other students' and learn about each other's thinking.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' understanding of equivalent representations and are based on possible comments and questions that students may make during the share out.

- How are the numbers similar?
- How can you represent the numbers?
- Do any of the given numbers have the same value? How do you know?

Math is... Mindset

- How can you understand your feelings?

SEL **Self-Awareness: Identify Feelings and Emotions**

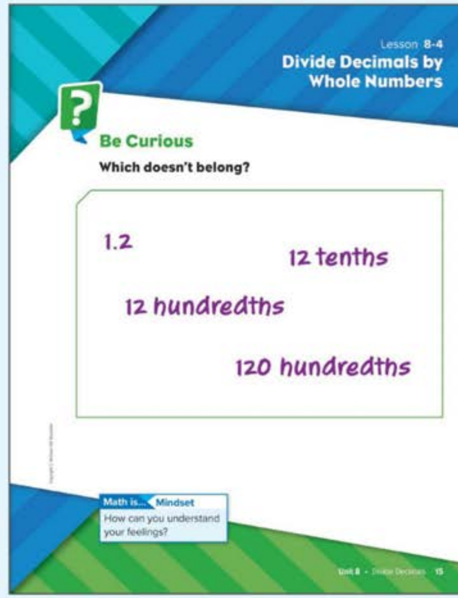
After students participate in the Which Doesn't Belong? routine, invite them to share the emotions they were experiencing. Encourage students to focus on the feelings they experience when they were successful as well as when they were not. Their work throughout the lesson with dividing decimals by whole numbers may be challenging, and they may feel upset or frustrated. Remind students that these feelings are neither right nor wrong, but how we deal with our feelings can affect success with math work. Sharing and listening can help students build understanding of their own emotions as well as the emotions of others.

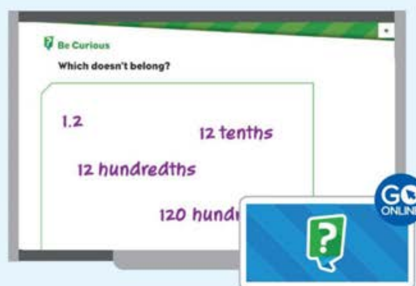
Transition to Explore & Develop

Make sure students understand that they have already used equivalent representations to help them solve problems before, and now they will expand the use of them to division involving decimals.

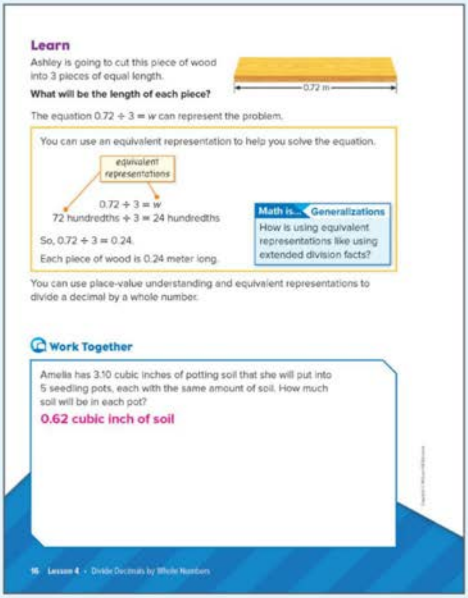
ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about how we can use place-value understanding and equivalent representations to divide a decimal by a whole number.





Explore & Develop 20 min



Learn

Ashley is going to cut this piece of wood into 3 pieces of equal length.

What will be the length of each piece?

0.72 m

The equation $0.72 \div 3 = w$ can represent the problem.

You can use an equivalent representation to help you solve the equation.

equivalent representations

$0.72 \div 3 = w$

72 hundredths ÷ 3 = 24 hundredths

So, $0.72 \div 3 = 0.24$.

Each piece of wood is 0.24 meter long.

Math is... Generalizations
How is using equivalent representations like using extended division facts?

You can use place-value understanding and equivalent representations to divide a decimal by a whole number.

Work Together

Amelia has 3.10 cubic inches of potting soil that she will put into 5 seedling pots, each with the same amount of soil. How much soil will be in each pot?

0.62 cubic inch of soil

16 Lesson 4 • Divide Decimals by Whole Numbers

❶ Pose the Problem

ETP Pose Purposeful Questions

- What information is given to you in the problem?
- Are you dividing a decimal by a whole number or a whole number by a decimal?
- How is the problem similar to other problems you have solved?
- What ways do you know to represent dividing a decimal by a whole number?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them solve a division problem. Ask them to work individually and write about the different steps they took to determine the quotient. Then have students compare their writing with their partner, correcting when necessary. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you write an equivalent representation of a decimal?
- How does understanding place value help you solve problems that involve dividing a decimal by a whole number?

Key Takeaway

- When dividing a decimal by a whole number, it is helpful to think about different ways to represent the dividend. For example, $1.2 \div 4$ can be thought of as 12 tenths ÷ 4.

Work Together

Students work together to solve a word problem that involves dividing a decimal by a whole number. Encourage students to estimate the quotient before they begin solving the problem.

Common Error Students may "get stuck" with this problem if they think of 3.1 as 31 tenths and try to share 31 in 5 groups. Remind them that can also write 3.1 as 3.10 and share 310 hundredths in 5 groups.

LOM Language of Math

Remind students that *equivalent* means "equal in value or amount." Writing 1.2 as 12 tenths means the representations represent the same amount and have the same solution, but one is using a representation that allows us to divide more easily.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students analyze related division expressions to develop a strategy for dividing decimals by whole numbers.

Directions: Present students with expressions, such as 36 ÷ 3, 3.6 ÷ 3, and 0.36 ÷ 3. In pairs or small groups, have students predict the similarities and differences of the *quotients* and discuss how they came to these conclusions.

After students have had time to record their prediction, have them find the quotients using previously taught strategies. Students should determine whether their prediction was true, and discuss why. Ask each student or group to create a generalization statement about dividing decimals by whole numbers to share with the class.

ETP Support Productive Struggle

- How are the expressions similar? How are they different?
- How can you use the meaning of division to help you determine the quotient?
- How can you use equivalent representations of the decimal number to help you determine the quotient?

Math is... Generalizations

- Are there generalizations you can make to help you work more efficiently?

Students attend to the detail of looking for repeated reasoning while solving problems.

Activity Debrief: Facilitate a discussion about using equivalent representations of the decimal number as a strategy for dividing a decimal by a whole number.

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the length of each piece?

Guided Exploration

Students use what they already know about equivalent representations and place-value understanding to solve an equation that involves dividing a decimal by a whole number.

ETP Use and Connect Mathematical Representations

Have the students create the equation. Ask:
- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:
- What power of ten will you multiply the dividend and divisor by? Why?
- What compatible numbers will you use to estimate the solution? Why?
- How will basic facts and place-value patterns help you estimate the solution?

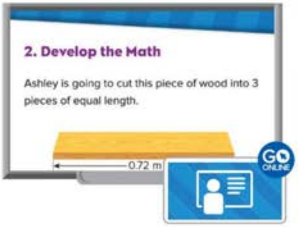
- **Think About It:** How can using decimal grids help you represent 0.72 and 0.24 in equivalent ways?

Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:
- Is the calculated solution reasonable? Why or why not?

Math is... Generalizations

- How is using equivalent representations like using extended division facts?

Students attend to the detail of looking for repeated reasoning while solving problems.



EL English Learner Scaffolds

Entering/Emerging Support students in understanding the word *amount*. Using materials such as sand or a liquid, fill half a jar or glass. Then, repeat, filling half of another jar or glass of the same size. Say *This [jar] has the same amount of [sand] as that jar*. Repeat twice, once filling the objects with the same amount of sand or liquid, and once with a different amount. Ask *Is this the same amount?*

Developing/Expanding Support students in understanding the word *amount*. Using materials such as sand or a liquid, fill half a jar or glass. Then, repeat, filling half of another jar or glass of the same size. Say *This [jar] has the same amount of [sand] as that jar*. Ask students to repeat the task, using *amount* in a sentence. Provide sentence frames for students who need more guidance.

Bridging/Reaching Provide students with classroom materials such as sand or water (something that can't be counted) as well as math manipulatives such as counting chips. Have them sort the materials and manipulatives into different containers and discuss the amounts using *amount* and *number*; for example, *These two boxes have a different amount of sand. These two containers have the same number of counters.*

Practice & Reflect 10 min

On My Own

Name

1. Which shows an equivalent representation of the expression 0.36 ÷ 3?
   A. 36 ÷ 3
   B. 36 tenths ÷ 3
   C. 36 hundredths ÷ 3
   D. 0.36 ÷ 0.3
2. Which shows an equivalent representation of the expression 2.16 ÷ 4?
   A. 216 ÷ 4
   B. 21.6 ÷ 4
   C. 216 tenths ÷ 4
   D. 216 hundredths ÷ 4

What is the quotient?

3. 0.24 ÷ 8 = **0.03**
4. 0.63 ÷ 9 = **0.07**
5. 0.96 ÷ 6 = **0.16**
6. 0.84 ÷ 4 = **0.21**
7. 1.26 ÷ 7 = **0.18**
8. 2.25 ÷ 5 = **0.45**
9. 3.18 ÷ 3 = **1.06**
10. 4.52 ÷ 4 = **1.13**

Unit 8 • Divide Decimals 17

11. Three friends equally split the cost of a large bag of popcorn. The total cost was $6.12. How much did each person have to pay? **$2.04**
12. **STEM Connection** Saffron is using 4.5 cups of sugar to make cookies. She is making 5 batches. How many cups of sugar will be in each batch? **0.9 cup**

13. A piece of ribbon is 0.64 meter long. Kylie is going to cut it into 4 equal pieces. How long will each piece be? **0.16 meter**
14. **Extend Your Thinking** Solve the following equations.
    0.24 ÷ 3 = **0.08**
    0.24 ÷ 30 = **0.008**
    0.24 ÷ 300 = **0.0008**
    What do you notice about the divisors and quotients?
    Sample answer: The quotient decreases proportionally in place value as the divisor increases.

Reflect

How can you use your understanding of place value and equivalent representations to divide decimals by whole numbers?

Answers may vary.

Math is... Mindset
How have you worked to understand your feelings?

18 Lesson 4 • Divide Decimals by Whole Numbers

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–6 Make sure students understand that once they have determined the quotient using equivalent representations, they must use their understanding of place value to represent the quotient as a decimal. For example, students may see 0.24 ÷ 8 as 24 ÷ 8 and give an answer of 3. Encourage them to think about the quotient in terms of hundredths instead of simply the value of the basic division fact quotient.

Item Analysis

Item	DOK	Rigor
1–2	1	Conceptual Understanding
3–8	2	Procedural Skill and Fluency
9–10	2	Conceptual Understanding
11–13	2	Application
14	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you use your understanding of place value and equivalent representations to divide decimals by whole numbers?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How have you worked to understand your feelings?

Students reflect on how they practiced self-awareness.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can use place-value understanding and equivalent representations to divide a decimal by a whole number.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Divide decimals by whole numbers	5.NBT.B.7
2	1	Divide decimals by whole numbers	5.NBT.B.7
3	1	Divide decimals by whole numbers	5.NBT.B.7
4	2	Divide decimals by whole numbers	5.NBT.B.7

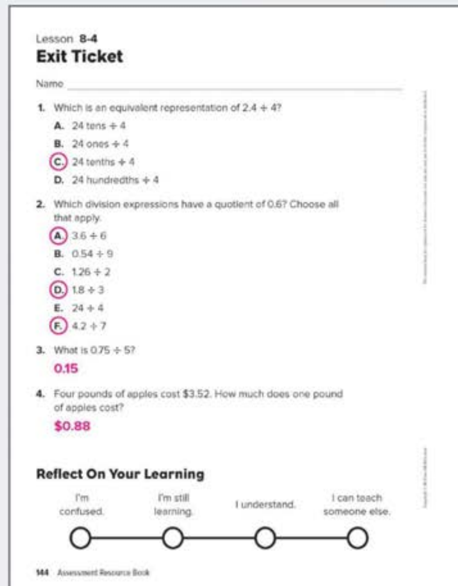
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 8-4

Exit Ticket

Name

1. Which is an equivalent representation of 2.4 ÷ 4?
   - A. 24 tens ÷ 4
   - B. 24 ones ÷ 4
   - C. 24 tenths ÷ 4
   - D. 24 hundredths ÷ 4
2. Which division expressions have a quotient of 0.6? Choose all that apply.
   - A. 3.6 ÷ 6
   - B. 0.54 ÷ 9
   - C. 1.26 ÷ 2
   - D. 1.8 ÷ 3
   - E. 24 ÷ 4
   - F. 4.2 ÷ 7
3. What is 0.75 ÷ 5?
   0.15
4. Four pounds of apples cost $3.52. How much does one pound of apples cost?
   $0.88

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

144 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Swap It Out

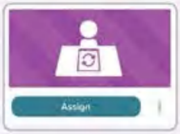
Provide partners with two number cubes. Partners roll a 2-digit factor and a 1-digit factor, multiply, then rewrite the multiplication equation as a division expression with the dividend in the hundredths and a 1-digit whole-number divisor. Partners work together to solve the division expression. Repeat as time permits.

GO ONLINE

Take Another Look Lessons

Assign the interactive lesson to reinforce targeted skills.

- Divide Decimals by Whole Numbers



INDEPENDENT WORK

Differentiation Resource Book, p. 83

Lesson 8-4 • Reinforce Understanding

Divide Decimals by Whole Numbers

Name

Review

You can use place-value understanding and equivalent representations to divide a decimal by a whole number.

$1.82 \div 7 = 182$ hundredths $\div 7$

$= 26$ hundredths

$= 0.26$

$1.82 \div 7 = 0.26$

Write equivalent representations for the equations and then solve.

1. $0.65 \div 13 =$ 0.05
   65 hundredths ÷ 13
   = 5 hundredths
2. $5.1 \div 3 =$ 1.7
   51 tenths ÷ 3 =
   17 tenths
3. $1.44 \div 9 =$ 0.16
   144 hundredths ÷ 9
   = 16 hundredths
4. $6.4 \div 8 =$ 0.8
   64 tenths ÷ 8
   = 8 tenths
5. $0.96 \div 8 =$ 0.12
   96 hundredths ÷ 8
   = 12 hundredths
6. $4.02 \div 2 =$ 2.01
   402 hundredths ÷ 2
   = 201 hundredths
7. $3.15 \div 3 =$ 1.05
   315 hundredths ÷ 3
   = 105 hundredths
8. $34.2 \div 6 =$ 5.7
   342 tenths ÷ 6
   = 57 tenths

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Dividing Decimals by Whole Numbers Task Cards

Students practice solving problems involving dividing decimals by whole numbers.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 83–84

Lesson 8-4

Additional Practice

Name

Review

You can use place-value understanding and equivalent representations to divide a decimal by a whole number.

Michael has a board that is 1.44 meters long. He wants to cut it into 3 pieces of equal length. How long should Michael cut each board?

Write a division equation to represent the problem.

$1.44 \div 3 = b$

Write an equivalent representation.

144 hundredths $\div 3 = b$

144 hundredths $\div 3 = 48$ hundredths

Each board will be 0.48 meters long.

1. Which is an equivalent representation of $3.5 \div 5$?
   A. 35 tens ÷ 5
   B. 35 ones ÷ 5
   C. 35 tenths ÷ 5
   D. 35 hundredths ÷ 5
2. Which is an equivalent representation of $2.16 \div 4$?
   A. 216 tens ÷ 4
   B. 216 ones ÷ 4
   C. 216 tenths ÷ 4
   D. 216 hundredths ÷ 4

Write an equivalent representation for the division. Then find the quotient.

3. $1.86 \div 6$
   186 hundredths ÷ 6; 0.31
4. $0.72 \div 4$
   72 hundredths ÷ 4; 0.18

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games:**

Assign the digital game to develop fluency with addition and subtraction within 1,000,000.

Assign

Use It! Application Station

Leave a Trail! Students create a poster showing how they divided decimals while making trail mix. *The content of this card has concepts covered later in Lesson 8-6. You may want to assign this card to students ready to explore content covered later in this unit.*

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 83–84

Write an equivalent representation for the division. Then find the quotient.

5. $5.6 \div 8$
56 tenths ÷ 8; 0.7

6. $4.32 \div 3$
432 hundredths ÷ 3; 1.44

7. $1.15 \div 5$
115 hundredths ÷ 5; 0.23

8. $14.8 \div 4$
148 tenths ÷ 4; 3.7

9. Greta buys 5 pens for $3.45. How much does each pen cost? Explain how you can use an equivalent representation to help you solve.
$0.69; Sample answer: I wrote the equation $3.45 \div 5 = p$; this is equivalent to 345 hundredths ÷ 5, which is 69 hundredths, or 0.69. So each pen costs $0.69.

10. Jack buys 8 pounds of apples for $9.52. How much does 1 pound of apples cost?
$1.19

11. A length of ribbon is 0.8 meter long. Justine cuts the ribbon into 4 equal lengths to wrap presents. How long is each piece of ribbon?
0.2 meter

Math @ Home Activity With your child, be alert to examples of decimal numbers around your home, at the store, or just in your everyday experiences. Suggest a whole-number divisor, and have your child tell you an equivalent representation of the division, and tell (or estimate) the quotient. Repeat for other examples.

Student Practice Book

Differentiation Resource Book, p. 84

Lesson **8-4 • Extend Thinking**

Divide Decimals by Whole Numbers

Name ____________

Sort the equivalent representations based on the solutions. Show you work. The first one has been done for you.

Expression	Work and Solution
A. $0.72 \div 2$	*72 hundredths ÷ 2 = 36 hundredths, or 0.36*
B. $3.84 \div 12$	384 hundredths ÷ 12 = 32 hundredths, or 0.32
C. $19.52 \div 122$	1,952 hundredths ÷ 122 = 16 hundredths, or 0.16
D. $37.5 \div 15$	375 tenths ÷ 15 = 25 tenths, or 2.5
E. $1.44 \div 9$	144 hundredths ÷ 9 = 16 hundredths, or 0.16
F. $22.4 \div 14$	224 tenths ÷ 14 = 16 tenths, or 1.6
G. $2.56 \div 8$	256 hundredths ÷ 8 = 32 hundredths, or 0.32
H. $1.08 \div 3$	108 hundredths ÷ 3 = 36 hundredths, or 0.36
I. $57.5 \div 23$	575 tenths ÷ 23 = 25 tenths, or 2.5
J. $2.75 \div 11$	275 hundredths ÷ 11 = 25 hundredths, or 0.25
K. $4.75 \div 19$	475 hundredths ÷ 19 = 25 hundredths, or 0.25
L. $57.6 \div 36$	576 tenths ÷ 36 = 16 tenths, or 1.6

The solution is 0.16. C E	The solution is 0.25. J K	The solution is 0.32. B G
The solution is 0.36. A H	The solution is 1.6. F L	The solution is 2.5. D I

Differentiation Resource Book

LESSON 8-5

Divide Whole Numbers by Decimals

Learning Targets

- I can use decimal grids to represent and solve a division equation.
- I can write an equivalent equation with a whole-number divisor to solve a division equation.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NBT.B** Perform operations with multi-digit whole numbers and with decimals to hundredths.

◇ **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Math Practices and Processes

MPP Model with mathematics.

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
dividend	address
divisor	reflect
power of 10	
quotient	

Materials

The materials may be for any part of the lesson.

- *10 × 10 Grids* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use decimal grids to represent and solve a division equation. • Students multiply by a power of 10 to write an equivalent equation with a whole-number divisor to solve a division equation.	• Students discuss finding quotients of whole numbers using division grids and powers of 10, answering *How* and *Why*. • To support cultivating conversation and maximizing linguistic and cognitive meta-awareness, ELs participate in MLR7: Compare and Connect.	• Students recognize and work to understand the emotions of others and practice empathetic responses.

Coherence

Previous	Now	Next
• Students found whole-number quotients and remainders (Grade 4). • Students used place-value understanding and equivalent representations to divide decimals by whole numbers (Unit 8).	• Students divide whole numbers by decimals using decimal grids and equivalent equations.	• Students divide decimals by decimals using area models to find partial quotients for equivalent equations (Unit 8). • Students add, subtract, multiply, and divide using the standard algorithm (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of place value as they relate different strategies to dividing whole numbers by decimals.	• Students build proficiency with dividing whole numbers by decimals.	• Students apply their understanding of dividing whole numbers by decimals to solve problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine

Where Does It Go?

5–7 min

Build Fluency Students build number sense as they determine where a given number would be located on two number lines with different endpoints.

These prompts encourage students to talk about their reasoning:

- How did you determine where the number would be located on the first number line?
- How did you use the first number line to determine where the number would go on the second number line?
- How is the whole on the second number line different from the whole on the first number line? Why does it matter?
- How can you double-check your work?

Sense-Making Routine

Purpose Students think about ways to determine how many "groups" there are in different scenarios.

Notice & Wonder

- What do you notice? What do you wonder?

See the Appendix for a full description of the sense-making routines.

Teaching Tip Have students Think Pair Share. Have them individually think about the image of a pile of coins, then pair with another student and share thoughts about what they notice and wonder before sharing ideas with the class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of dividing by a decimal, and are based on possible comments and questions that students may make during the share out.

- If you know the amount of money in the pile in dollars and cents, how could you find the amount of money in the plie in cents only?
- A quarter is worth $0.25. How could you represent $0.25 using cents only?
- If you know the amount of money in the pile in cents how could you find how many groups of 25 cents are in the pile?

Math is... Mindset

- How can you recognize and understand how others are feeling?

SEL Social Awareness: Empathy

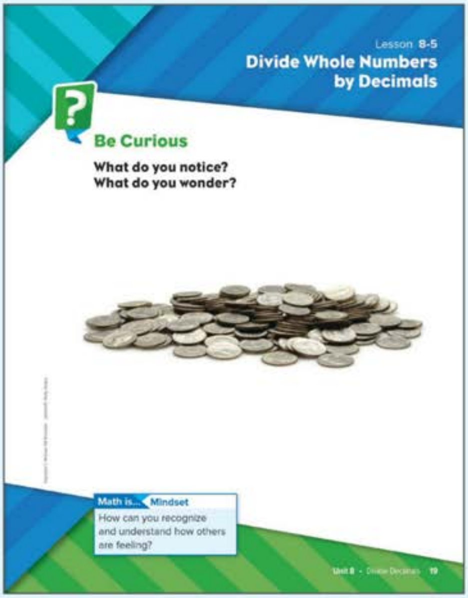
Establish a classroom culture that welcomes openness and empathy by encouraging students to share and discuss their emotions and recognize the emotions of others. After students participate in the Notice & Wonder routine, invite them to share the emotions they were experiencing. Encourage students to think about their own experiences with the emotions being shared by others. Their work throughout the lesson with dividing whole numbers by decimals may be challenging, and they may feel emotions such as happy, excited, or frustrated. Sharing and listening can help students build understanding of their own emotions as well as empathy for others.

Transition to Explore & Develop

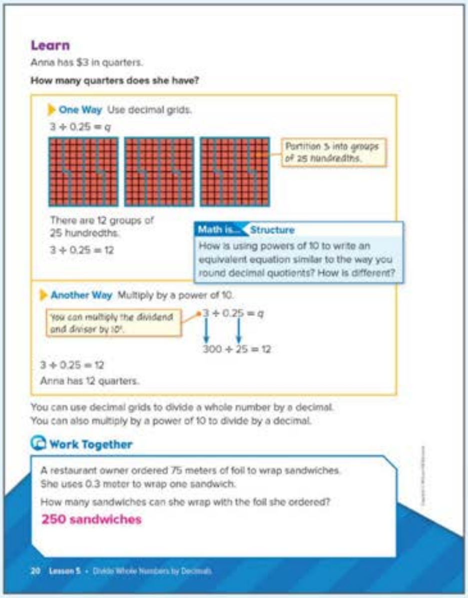
Ask questions that get students thinking about strategies and models used to divide by a decimal. Guide students to think about how they can explain strategies used to solve real-world problems involving decimal divisors.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can use decimal grids and equivalent equations to divide a whole number by a decimal.



Explore & Develop 20 min



Learn

Anna has $3 in quarters.

How many quarters does she have?

One Way Use decimal grids.

$3 \div 0.25 = q$

Partition 3 into groups of 25 hundredths.

There are 12 groups of 25 hundredths.

$3 \div 0.25 = 12$

Math is... Structure

How is using powers of 10 to write an equivalent equation similar to the way you round decimal quotients? How is it different?

Another Way Multiply by a power of 10.

You can multiply the dividend and divisor by 10^2.

$3 \div 0.25 = q$

$300 \div 25 = 12$

$3 \div 0.25 = 12$

Anna has 12 quarters.

You can use decimal grids to divide a whole number by a decimal. You can also multiply by a power of 10 to divide by a decimal.

Work Together

A restaurant owner ordered 75 meters of foil to wrap sandwiches. She uses 0.3 meter to wrap one sandwich.

How many sandwiches can she wrap with the foil she ordered?

250 sandwiches

20 Lesson 5 • Divide Whole Numbers by Decimals

❶ Pose the Problem

ETP Pose Purposeful Questions

- What problems like this have you seen before?
- What strategy did you use to solve those problems? Can you use that strategy here?
- Based on what you know about mathematics, can you make a conjecture about how many quarters there are?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and assign them the same problem. Have one student solve it using decimal grids and the other using powers of 10. Then have them compare their answers and approaches. Revisit this activity throughout the lesson to help students build proficiency.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do decimal grids help you understand division by decimals?
- How can you rewrite an equivalent division equation using powers of 10?
- How can an equivalent equation help you solve a division equation?

Key Takeaways

- One strategy to divide a whole number by a decimal is to use decimal grids to represent and solve the problem.
- Another strategy to divide a whole number by a decimal is to multiply by a power of 10 to write an equivalent equation with a whole-number divisor to solve.

Work Together

The Work Together activity can be used as a formative assessment opportunity to check students' understanding of how to find the quotient of a whole number and a decimal. Have students work on the activity individually or in pairs before asking them to explain how they found their answers.

Common Misconception Students may think that the quotient of two numbers must be less than both numbers, but that is not true when the divisor is a number between 0 and 1.

LOM Language of Math

The word *equivalent* has several meanings in mathematics. Equations that have the same solution are called equivalent. A square and a triangle that have the same area are called equivalent. It is also used in chemistry and cartography (the science of making or drawing maps).

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using a representation to divide whole numbers by decimals. They use their representation to develop other strategies for dividing whole numbers by decimals.

Materials: *10 × 10 Grids* Teaching Resource

Directions: Provide copies of the *10 × 10 Grids* Teaching Resource. Display division expressions, such as 3 ÷ 0.12 and 3 ÷ 0.2. Have students work in pairs or small groups to explore using a representation to determine each quotient.

ETP Support Productive Struggle

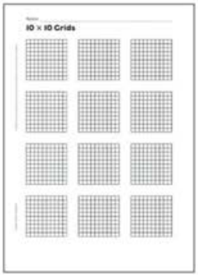
- How can you use the meaning of division to help you use a representation to solve?
- How can you represent the dividend? How can you represent the divisor? How is the quotient shown in the representation?

Math is... Structure

- What ideas have you learned before that were helpful in solving these problems?

Students look for and apply place-value structure to solve problems.

Activity Debrief: Have students share their representations and thinking. Facilitate a discussion to ensure students see that both the dividend and divisor were represented using an equivalent value written as hundredths, for example, 3 as 300 hundredths, 0.12 as 12 hundredths, and 0.2 as 20 hundredths. And that these values can be found by multiplying the dividend and the divisor by a power of 10.



Have students revisit the Pose the Problem question and discuss answers.

- How can you determine how many quarters Anna has?

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students build on their understanding of division to find quotients of whole numbers divided by decimals using decimal grids.

ETP Use and Connect Mathematical Representations

Have the students estimate the solution. Ask:

- What power of ten will you multiply by? Why?
- What compatible numbers will you use? Why?
- How will basic facts and place-value patterns help you?

- **Think About It:** Why is it useful to use hundredths decimal grids instead of tenths decimal grids as a tool to solve this problem?
- When have you used powers of 10 to write an equivalent representation before? Why did you do it?

Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

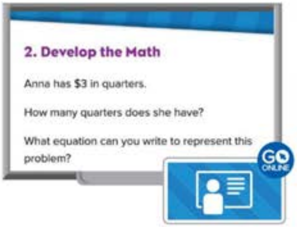
Have the students use mental math and/or a related multiplication equation to check the calculated solution. Ask:

- What related multiplication could you use? Why?
- How many quarters make 3 dollars? How do you know?

Math is... Structure

- How is using powers of 10 to write an equivalent equation similar to the way you round decimal quotients? How is it different?

Students step back for an overview and shift perspective.



EL English Learner Scaffolds

Entering/Emerging Support students' understanding of the phrase *[3] dollars in quarters*. Put four quarters on the table. Say *A quarter equals 25 cents. I have one dollar in quarters*. Put four more quarters on the table. First, ask *Do I have eight dollars in quarters?* Then ask *Do I have two dollars in quarters?* Finally, work with students to find the decimal number on the Learn page that represents a quarter.

Developing/Expanding Support students' understanding of the phrase *[3] dollars in quarters*. Put four quarters on the table. Say *A quarter equals 25 cents. I have one dollar in quarters*. Put four more quarters on the table and ask students to tell you how much money you have in quarters. Provide sentence frames for students who may need more guidance.

Bridging/Reaching Ask students to explain the strategy they prefer to divide whole numbers by decimal numbers. Listen for key words and phrases such as *$3 in quarters, decimal grids, powers of 10,* and *groups of*. Validate and provide correction as needed.

Practice & Reflect
10 min

On My Own

Name ____________

What is the quotient? Tell which strategy you used. Check students' strategy.

1. $6 \div 0.2 =$ 30
2. $84 \div 1.2 =$ 70
3. $9 \div 0.6 =$ 15
4. $56 \div 3.5 =$ 16

Solve each problem. Then, explain your solution.

5. Darren has a cooler with 9 liters of lemonade. He pours 0.3 liter of lemonade into each glass. How many glasses of lemonade can Darren fill?
   30 glasses; To divide 9 by 0.3, multiply both by 10, then divide. $9 \div 0.3 = 90 \div 3 = 30$
6. Mr. Ramirez bought a watermelon that weighs 12 pounds for a picnic. He cuts it into pieces that each weigh 1.5 pounds. How many pieces of watermelon can Mr. Ramirez cut?
   8 pieces; To divide 12 by 1.5, multiply both by 10, then divide. $12 \div 1.5 = 120 \div 15 = 8$
7. A grocery store got a delivery of 24 pounds of almonds. They package the almonds into containers with 0.75 pound of almonds in each. How many containers can they fill with almonds?
   32 containers; To divide 24 by 0.75, multiply both by 100, then divide. $24 \div 0.75 = 2{,}400 \div 75 = 32$
8. Melissa has \$30 to spend on apples from a local apple orchard. How many pounds of apples can Melissa buy?
   24 pounds; To divide 30 by 1.25, multiply both by 100, then divide. $30 \div 1.25 = 3{,}000 \div 125 = 24$

9. **Error Analysis** Mario says that $28 \div 0.7 = 0.4$. Do you agree or disagree? Explain how you know.
   I disagree; Sample answer: $28 \div 0.7 = 40$; Multiply both 28 and 0.7 by 10, and then divide 280 by 7.
10. A car drove 104 miles in 1.6 hours. If the speed of the car was the same for the entire trip, how fast did the car go? How do you know?
    65 miles per hour; Sample answer: Multiply both 104 and 1.6 by 10 to get $1{,}040 \div 16 = 65$.
11. Write a real-life problem that involves dividing a decimal by a whole number. Solve the problem using a representation.
    Check students' work.
12. **Extend Your Thinking** Is the quotient of $52 \div 1.04$ less than or greater than 52? How do you know? What is the quotient?
    Less than; Sample answer: Because 1.04 is greater than 1, the quotient is less than 52. If the divisor was less than 1, the quotient would be greater than 52; 50.

Reflect

How do powers of 10 help you divide by a decimal?
Answers may vary.

Math is... Mindset
What helped you recognize and understand how others were feeling?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–4 Students may multiply the divisor by a power of 10 rather than multiplying both the dividend and the divisor by a power of 10. In Exercise 1, the student may multiply 0.2 by 10 only and obtain the quotient $6 \div 2 = 3$. Remind students that both the dividend and the divisor must be multiplied by the same nonzero number.

Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill and Fluency
5–8	2	Application
9	3	Conceptual Understanding
10–11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How do powers of 10 help you divide by a decimal?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you recognize and understand how others were feeling?

Students reflect on how they practiced social awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can use decimal grids to represent and solve a division equation.
- I can write an equivalent equation with a whole-number divisor to solve a division equation.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Divide whole numbers by decimals	5.NBT.B.7
2	1	Divide whole numbers by decimals	5.NBT.B.7
3	2	Divide whole numbers by decimals	5.NBT.B.7

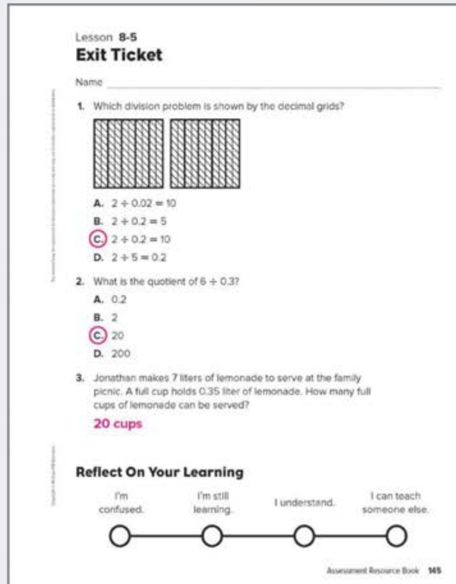
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 8-5

Exit Ticket

Name ______________________

1. Which division problem is shown by the decimal grids?

A. $2 \div 0.02 = 10$
B. $2 \div 0.2 = 5$
C. $2 \div 0.2 = 10$
D. $2 \div 5 = 0.2$

2. What is the quotient of $6 \div 0.3$?

A. 0.2
B. 2
C. 20
D. 200

3. Jonathan makes 7 liters of lemonade to serve at the family picnic. A full cup holds 0.35 liter of lemonade. How many full cups of lemonade can be served?

20 cups

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 145

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Compare It!

Work with students in pairs. Provide each student with a division problem with a 1- or 2-digit dividend and a decimal divisor such that there will be no remainder. One student should divide using decimal grids and the other should divide using powers of 10. Have students compare results and if they have different results have them discuss. Repeat with other division problems and have partners switch roles.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Divide Whole Numbers by Decimals-Model
- Divide Whole Numbers by Decimals

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 85

Lesson 8-5 • Reinforce Understanding

Divide Whole Numbers by Decimals

Name ____________________

Review

You can use powers of 10 to help you find quotients when dividing whole numbers by decimals.

$45 \div 0.75 = (45 \times 100) \div (0.75 \times 100)$
$= 4{,}500 \div 75$
$= 60$

$45 \div 0.75 = 60$

Use powers of 10 to help you solve to find the quotient. Show your work.

1. $8 \div 0.4 =$ 20
   $80 \div 4 = 20$
2. $54 \div 0.9 =$ 60
   $540 \div 9 = 60$
3. $6 \div 0.25 =$ 24
   $600 \div 25 = 24$
4. $30 \div 0.15 =$ 200
   $3{,}000 \div 15 = 200$
5. $100 \div 2.5 =$ 40
   $1{,}000 \div 25 = 40$
6. $78 \div 6.5 =$ 12
   $780 \div 65 = 12$
7. $81 \div 2.7 =$ 30
   $810 \div 27 = 30$
8. $36 \div 0.45 =$ 80
   $3600 \div 45 = 80$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Divide by Whole Numbers Tic Tac Toe

Students practice dividing whole numbers by decimals.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 85–86

Lesson 8-5

Additional Practice

Name ____________________

Review

You can use powers of 10 to help you divide a whole number by a decimal.

A deli has 6 pounds of ham to make sandwiches. Each sandwich uses 0.3 pound of ham. How many sandwiches can be made?

Write a division equation to represent the problem.

$$6 \div 0.3 = h$$

Use a power of 10 so that the divisor, 0.3, is a whole number:

$0.3 \times 10 = 3$

Multiply the dividend by the same power of 10: $6 \times 10 = 60$

Write an equivalent equation with the new dividend and divisor:

$60 \div 3 = h$

Since $60 \div 3 = 20$, it must be that $6 \div 0.3 = 20$.

The deli can make 20 sandwiches.

Write an equivalent division so that the divisor is a whole number. Then find the quotient.

1. $4 \div 0.05$
   $400 \div 5$; 80
2. $12 \div 0.4$
   $120 \div 4$; 30
3. $10 \div 0.25$
   $1{,}000 \div 25$; 40
4. $3 \div 0.2$
   $30 \div 2$; 15

Student Practice Book

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with addition and subtraction within 1,000,000.

Assign

E Extend Thinking

Use It! Application Station

That is Tasty! Students use decimal division to size up and down a recipe. *The content of this card has concepts covered later in Lesson 8-6. You may want to assign this card to students ready to explore content covered later in this unit.*

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 85–86

Write an equivalent division so that the divisor is a whole number. Then find the quotient.

5. 8 ÷ 0.16
800 ÷ 16; 50

6. 14 ÷ 0.35
1,400 ÷ 35; 40

7. 91 ÷ 1.4
910 ÷ 14; 65

8. 72 ÷ 4.5
720 ÷ 45; 16

9. Nancy has $6 to spend at the fruit stand. She uses all of her money to buy oranges that cost $0.75 per pound. How many pounds of oranges did Nancy buy?
8 pounds

10. A sandwich shop has 30 pounds of shredded lettuce to use on its sandwiches. On each sandwich, 0.15 pound of lettuce is used. How many sandwiches can be made with the available lettuce?
200 sandwiches

11. A ball of yarn contains 60 feet of yarn. Tina needs 1.2 feet of yarn to make a bow for a package. How many packages can Tina make a bow for with the yarn she has?
50 packages

Math @Home Activity With your child, be alert to examples of whole numbers around your home, at the store, or just in your everyday experiences, that can be divided by a decimal, such as 2 pounds of lunch meat. Suggest a decimal-number divisor, such as using 0.25 pound of lunch meat in each sandwich, and have your child tell you an equivalent representation of the division with a whole-number divisor, and tell (or estimate) the quotient. Repeat for other examples.

Student Practice Book

Differentiation Resource Book, p. 86

Lesson 8-5 • Extend Thinking

Divide Whole Numbers by Decimals

Name ____________

Find Quotient A and B. Show your work. Then determine the comparison (<, =, or >) between A and B. The first one is done for you as an example.

	Quotient A	<, =, or >	Quotient B
1.	24 ÷ 0.8 *240 ÷ 8* *30*	<	16 ÷ 0.5 *160 ÷ 5* *32*
2.	18 ÷ 0.15 1800 ÷ 15 = 120	>	36 ÷ 2.25 3,600 ÷ 225 = 16
3.	60 ÷ 7.5 600 ÷ 75 = 8	=	28 ÷ 3.5 280 ÷ 35 = 8
4.	6 ÷ 0.025 6,000 ÷ 25 = 240	<	21 ÷ 0.75 2,100 ÷ 75 = 28
5.	24 ÷ 0.6 240 ÷ 6 = 40	>	12 ÷ 0.40 120 ÷ 4 = 30 or 1,200 ÷ 40
6.	66 ÷ 2.2 660 ÷ 22 = 30	<	18 ÷ 0.2 180 ÷ 2 = 90
7.	35 ÷ 0.07 3,500 ÷ 7 = 500	=	3 ÷ 0.006 3,000 ÷ 6 = 500

Differentiation Resource Book

LESSON 8-6

Divide Decimals by Decimals

Learning Target

- I can write an equivalent equation containing whole numbers to solve a division equation.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NBT.B** Perform operations with multi-digit whole numbers and with decimals to the hundredths.

◇ **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

MPP Attend to precision.

Vocabulary

Math Terms	Academic Terms
dividend	advantage
divisor	assert
partial quotients	disadvantage
power of 10	
quotient	

Materials

The materials may be for any part of the lesson.

- *Tenths and Hundredths Representations* Teaching Resource

Focus

Content Objective

- Students multiply the dividend and the divisor by a power of 10 to write an equivalent equation containing whole numbers to solve a division equation.

Language Objectives

- Students discuss multiple strategies to find quotients of decimals while answering *Wh-* questions.
- To support optimizing output, ELs participate in MLR5: Co-Craft Questions and Problems.

SEL Objective

- Students set learning goals and initiate work on tasks to accomplish their goals.

Coherence

Previous

- Students found whole-number quotients and remainders (Grade 4).
- Students divided whole numbers by decimals using decimal grids and equivalent equations (Unit 8).

Now

- Students divide decimals by decimals using area models to find partial quotients for equivalent equations.

Next

- Students add and subtract fractions (Unit 9).
- Students add, subtract, multiply, and divide using the standard algorithm (Grade 6).

Rigor

Conceptual Understanding

- Students build on their understanding of division as they notice and use patterns in dividing a decimal by a decimal.

Procedural Skill & Fluency

- Students build proficiency with strategies for dividing a decimal by a decimal.

Application

- Students apply their understanding of division with decimals to solve problems with real-world contexts.

Application is not a targeted element of rigor for this standard.

Number Routine

GO ONLINE

Where Does It Go?

5–7 min

Build Fluency Students develop number sense as they determine where a given number would be located on two number lines with different endpoints.

These prompts encourage students to talk about their reasoning:

- How did you determine where the number would be located on the first number line?
- How did you use the first number line to determine where the number would go on the second number line?
- What did you do differently with the second number line than with the first number line?
- How can you double-check your work?

Launch 5–7 min

Sense-Making Routine

Purpose Students should focus on the fact that changing both the divisor and the dividend by the same factor results in the same quotient.

Notice & Wonder

- What do you notice? What do you wonder?

See the Appendix for a full description of the sense-making routines.

Teaching Tip Have students examine the equations in multiple ways. One way could be to use equivalent representations to state each equation, such as, “There are 2 groups of 5 tenths in 10 tenths.” Another could be to find the power of 10 needed to make the last equation equivalent to each of the other equations.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students’ understanding of dividing decimals and are based on possible comments and questions that students may make during the share out.

- What is similar about these equations?
- What is different about the equations?
- What patterns do you see?
- What other equations could you write that would continue the pattern shown?

Math is... Mindset

- What helps you want to do your best work?

SEL Self-Regulation: Working Memory

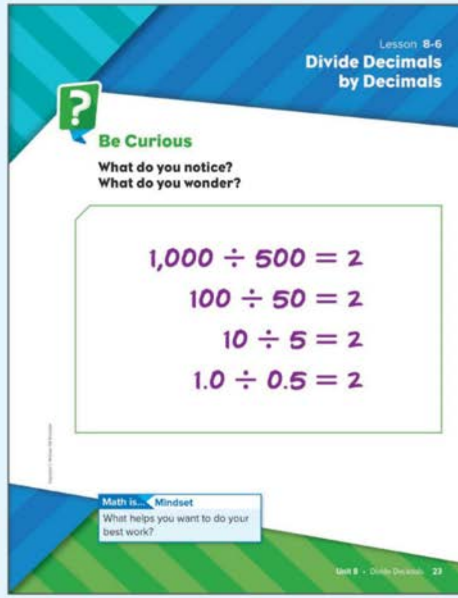
Before beginning the Notice & Wonder routine, guide students to make their own specific and attainable goal for the day. Goals may be centered around dividing decimals by decimals or may be focused on strong behaviors, such as active listening or staying on task. Creating a personal goal can allow students to practice self-motivation as they work toward achieving that goal.

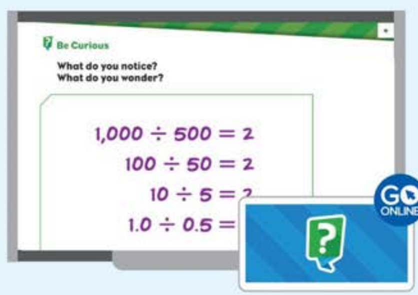
Transition to Explore & Develop

Ask questions that get students thinking about patterns in division. Guide students to think about how they can explain strategies used to divide decimals by decimals.

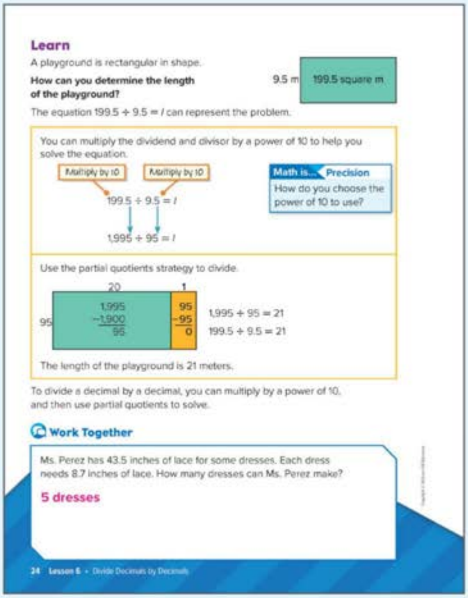
ETP Establish Mathematics Goals to Focus Learning

- Let’s think about how we can divide a decimal by a decimal by solving an equivalent equation.





Explore & Develop 20 min



Learn

A playground is rectangular in shape.

How can you determine the length of the playground?

9.5 m | 199.5 square m

The equation $199.5 \div 9.5 = l$ can represent the problem.

You can multiply the dividend and divisor by a power of 10 to help you solve the equation.

Multiply by 10 Multiply by 10

$199.5 \div 9.5 = l$

$1{,}995 \div 95 = l$

Math is... Precision
How do you choose the power of 10 to use?

Use the partial quotients strategy to divide.

20 1

95 | 1,995 − 1,900 = 95 | 95 − 95 = 0

$1{,}995 \div 95 = 21$

$199.5 \div 9.5 = 21$

The length of the playground is 21 meters.

To divide a decimal by a decimal, you can multiply by a power of 10, and then use partial quotients to solve.

Work Together

Ms. Perez has 43.5 inches of lace for some dresses. Each dress needs 8.7 inches of lace. How many dresses can Ms. Perez make?

5 dresses

24 Lesson 6 • Divide Decimals by Decimals

❶ Pose the Problem

ETP Pose Purposeful Questions

- How is this similar to other problems you have solved?
- How will the length of the playground be related to its area?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Co-Craft Questions and Problems

Pair students and have them co-create a problem similar to the one on the Learn page. Have them work together to solve their problem and then trade their problem with another pair. After each pair solves the other pair's problems, have them form a group of four to check solutions and correct any mistakes that may have been made. Revisit the task throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you use partial quotients as a strategy to divide decimals?

Key Takeaway

- One strategy to divide a decimal by a decimal is to multiply the dividend and the divisor by a power of 10 to write an equivalent equation containing whole numbers to solve.

Work Together

The Work Together activity can be used as a formative assessment opportunity to check students' understanding of how to solve a problem involving division of a decimal by a decimal. Have students work on the activity individually or in pairs before asking them to explain how they found their answers.

Common Error Watch for students who only multiply the divisor by a power of 10. Stress that to create an equivalent equation, both the divisor and dividend must be multiplied by the same power of 10.

LOM Language of Math

The word *area* comes from the Latin word *area*, meaning "an empty piece of flat gound." *Area* can mean the amount of space a figure takes up, like "the area of a rectangle," or a geographic region, like "the hilly area of Wisconsin," or an idea or activity, like "we are becoming experts in the area of mathematics."

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using a representation to divide decimals. They use their representation to develop other strategies for dividing whole numbers by decimals.

Materials: *Tenths and Hundredths Representations* Teaching Resource

Directions: Provide copies of the *Tenths and Hundredths Representations* Teaching Resource. Display division expressions, such as 1.2 ÷ 0.4 and 2.4 ÷ 1.2. Have students work in pairs or small groups to explore using a representation to determine each quotient.

ETP Support Productive Struggle

- How did you use a representation to divide a whole number by a decimal? How can you extend that to dividing two decimals?
- How did you determine whether to use the tenths grid or the hundredths grid?
- How do the quotients of these division problems relate to the quotients of 12 ÷ 4 and 24 ÷ 12? Why do you think this happens?

Activity Debrief: Have students share their representations and thinking. Facilitate a discussion to ensure students see that both the dividend and divisor were represented using an equivalent value as hundredths, such as 12 tenths and 4 tenths. And that these values can be found by multiplying the dividend and the divisor by a power of 10.

Math is... Precision

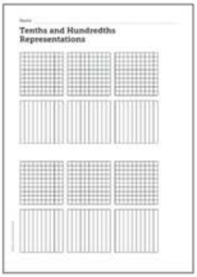
- How do you choose the power of 10 to use?

Students give carefully formulated explanations of their reasoning.

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the length of the playground?

A PDF of the Teaching Resource is available in the Digital Teacher Center.



Guided Exploration

Students build on their understanding of division to find quotients of decimals divided by decimals using powers of 10, area models, and partial quotients.

ETP Use and Connect Mathematical Representations

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- What power of ten will you multiply the dividend and divisor by? Why?
- What compatible numbers will you use to estimate the solution? Why?
- How will basic facts and place-value patterns help you estimate the solution?
- Why do you multiply by 10^1?

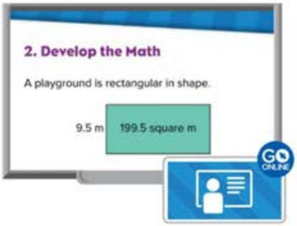
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?
- **Think About It:** How would this problem be different if the width was missing?

Math is... Precision

- How do you choose the power of 10 to use?

Students give carefully formulated explanations of their reasoning.



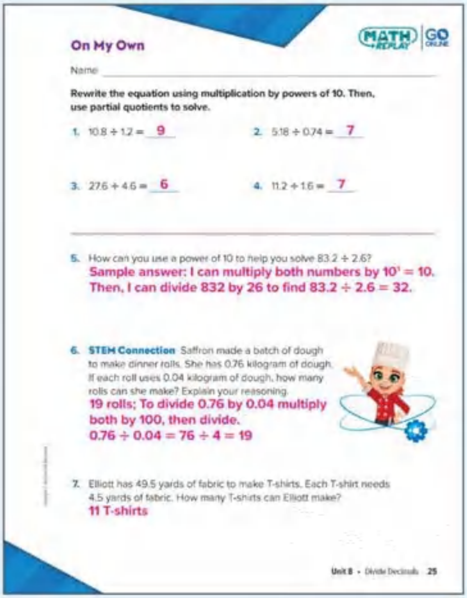
EL English Learner Scaffolds

Entering/Emerging Support students' understanding of the word *each*. Use classroom objects to demonstrate. First, write \$5.50 on three notebooks, for example. Say *Each notebook is \$5.50*. Repeat twice, once labeling objects the same price, and once, not. Ask, *Is each [book \$3.25]?*

Developing/Expanding Support students' understanding of the word *each*. Use classroom objects to demonstrate. First, write \$5.50 on three notebooks, for example. Say *Each notebook is \$5.50*. Repeat twice, once labeling objects the same price, and once, not. Ask, *Is each [book \$3.25]?* Finally, ask students to repeat the task, using *each* in a sentence. Provide sentence frames for students who need more guidance.

Bridging/Reaching Ask students to explain the meaning of the word *each*, and to use it in a sentence. Allow students to interject, giving their opinions, agreeing or disagreeing, and explaining why, as well as pointing out any mistakes that they may catch. For example, *No, that's not correct....* or *No, each means....*

Practice & Reflect 10 min


On My Own

Name

Rewrite the equation using multiplication by powers of 10. Then, use partial quotients to solve.

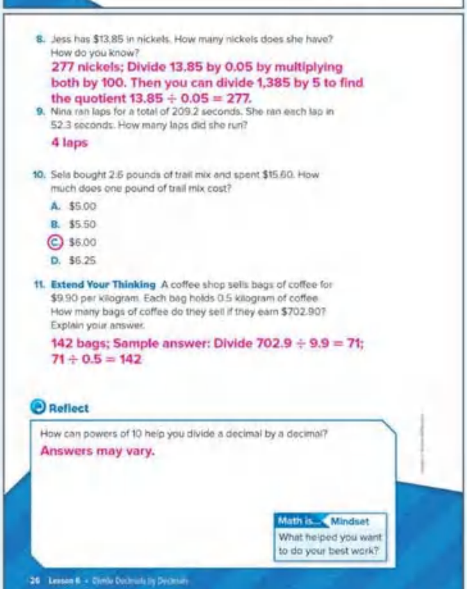
1. $10.8 \div 1.2 =$ **9**
2. $5.18 \div 0.74 =$ **7**
3. $27.6 \div 4.6 =$ **6**
4. $11.2 \div 1.6 =$ **7**

5. How can you use a power of 10 to help you solve $83.2 \div 2.6$?
Sample answer: I can multiply both numbers by $10^1 = 10$. Then, I can divide 832 by 26 to find $83.2 \div 2.6 = 32$.

6. **STEM Connection** Saffron made a batch of dough to make dinner rolls. She has 0.76 kilogram of dough. If each roll uses 0.04 kilogram of dough, how many rolls can she make? Explain your reasoning.
19 rolls; To divide 0.76 by 0.04 multiply both by 100, then divide. $0.76 \div 0.04 = 76 \div 4 = 19$

7. Elliott has 49.5 yards of fabric to make T-shirts. Each T-shirt needs 4.5 yards of fabric. How many T-shirts can Elliott make?
11 T-shirts

Unit 8 • Divide Decimals 25


8. Jess has $13.85 in nickels. How many nickels does she have? How do you know?
277 nickels; Divide 13.85 by 0.05 by multiplying both by 100. Then you can divide 1,385 by 5 to find the quotient $13.85 \div 0.05 = 277$.

9. Nina ran laps for a total of 209.2 seconds. She ran each lap in 52.3 seconds. How many laps did she run?
4 laps

10. Sela bought 2.6 pounds of trail mix and spent $15.60. How much does one pound of trail mix cost?
A. $5.00
B. $5.50
C. $6.00
D. $6.25

11. **Extend Your Thinking** A coffee shop sells bags of coffee for $9.90 per kilogram. Each bag holds 0.5 kilogram of coffee. How many bags of coffee do they sell if they earn $702.90? Explain your answer.
142 bags; Sample answer: Divide $702.9 \div 9.9 = 71$; $71 \div 0.5 = 142$

Reflect

How can powers of 10 help you divide a decimal by a decimal?
Answers may vary.

Math is... Mindset
What helped you want to do your best work?

26 Lesson 6 • Divide Decimals by Decimals

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 2 Students may automatically multiply by 10, not taking into account what they are trying to accomplish. Remind them that the goal is to write an equivalent equation that contains only whole numbers. That may involve multiplying by 10 or 10^3 or 10^4, etc., depending upon the greatest number of decimal places that appear in the equation.

Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill and Fluency
5	2	Conceptual Understanding
6–10	2	Application
11	3	Conceptual Understanding

Reflect

Students complete the Reflect question.
- How can powers of 10 help you divide a decimal by a decimal?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you want to do your best work?

Students reflect on how they practiced self-motivation.

Learning Target

Ask students to reflect on the Learning Target of the lesson.
- I can write an equivalent equation containing whole numbers to solve a division equation.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Divide decimals by decimals	5.NBT.B.7
2	1	Divide decimals by decimals	5.NBT.B.7
3	2	Divide decimals by decimals	5.NBT.B.7
4	2	Divide decimals by decimals	5.NBT.B.7

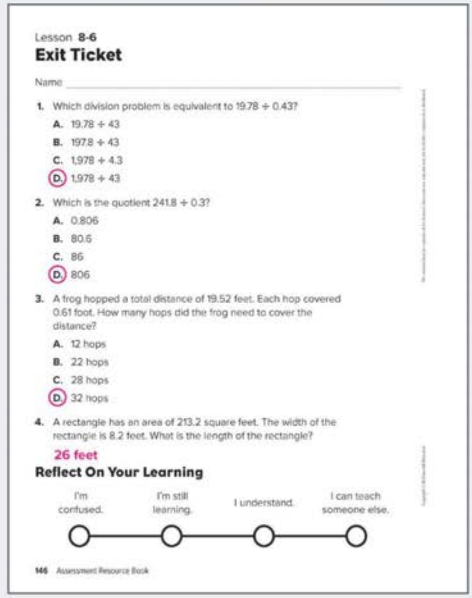
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 8-6
Exit Ticket

Name

1. Which division problem is equivalent to 19.78 ÷ 0.43?
   A. 19.78 ÷ 43
   B. 197.8 ÷ 43
   C. 1,978 ÷ 4.3
   D. 1,978 ÷ 43

2. Which is the quotient 241.8 ÷ 0.3?
   A. 0.806
   B. 80.6
   C. 86
   D. 806

3. A frog hopped a total distance of 19.52 feet. Each hop covered 0.61 foot. How many hops did the frog need to cover the distance?
   A. 12 hops
   B. 22 hops
   C. 28 hops
   D. 32 hops

4. A rectangle has an area of 213.2 square feet. The width of the rectangle is 8.2 feet. What is the length of the rectangle?
   26 feet

Reflect On Your Learning

I'm confused. I'm still learning. I understand. I can teach someone else.

146 Assessment Resource Book

 10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Divide with Decimals

Work with students in pairs. Students write two 2-digit numbers and multiply the numbers. Students write the product divided by one of the factors. Then they insert a decimal point before the last digit in each number, trade papers, and divide using powers of 10, area models, and partial quotients. Help students understand how to use the place value of the dividend and divisor to determine the power of ten by which to multiply before labeling the area model.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Divide Decimals-Model
- Divide Decimals

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 87

Lesson 8-6 • Reinforce Understanding

Divide Decimals by Decimals

Name ______________________

Review

You can use place value and powers of 10 to help you estimate the quotient.

$9.75 \div 0.25$

First, multiply both numbers by 10, 100, or 1000.	$9.75 \div 0.25$ $(9.75 \times \mathbf{100}) \div (0.25 \times \mathbf{100})$ $975 \div 25$
Next, use an area model and partial quotients to find the quotient.	30, 9 25: $975 - 750 = 225$; $225 - 225 = 0$

$9.75 \div 0.25 = 39$

What is the quotient? Use an area model and partial quotients to solve.

1. $5.4 \div 0.36 =$ **15**
   $540 \div 36 = 15$
   10, 5
   36: $540 - 360 = 180$; $180 - 180 = 0$

2. $14.11 \div 0.17 =$ **83**
   $1{,}411 \div 17 = 83$
   80, 3
   17: $1{,}411 - 1{,}360 = 51$; $51 - 51 = 0$

3. $52.2 \div 1.8 =$ **29**
   $522 \div 18 = 29$
   20, 9
   18: $522 - 360 = 162$; $162 - 162 = 0$

4. $12.48 \div 0.24 =$ **52**
   $1{,}248 \div 24 = 52$
   50, 2
   24: $1{,}248 - 1{,}200 = 48$; $48 - 48 = 0$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Divide Decimals by Decimals Task Cards

Students practice dividing decimals.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 87–88

Lesson 8-6

Additional Practice

Name ______________________

Review

You can use powers of 10 to help you divide a decimal number by another decimal number.

The area of a rectangular garden is 51.2 square meters. The width of the garden is 6.4 meters. What is the length of the garden?

Write a division equation to represent the problem.

$51.2 \div 6.4 = \ell$

Use a power of 10 so that the divisor, 6.4, is a whole number.

$6.4 \times 10 = 64$

Multiply the dividend by the same power of 10: $51.2 \times 10 = 512$

Write an equivalent equation with the new dividend and divisor.

$512 \div 64 = \ell$

Since $512 \div 64 = 8$, it must be that $51.2 \div 6.4 = 8$.

The garden is 8 meters long.

Write an equivalent division so that the divisor is a whole number. Then find the quotient.

1. $2.7 \div 0.9$
   $27 \div 9$; 3
2. $3.2 \div 0.4$
   $32 \div 4$; 8
3. $9.4 \div 4.7$
   $94 \div 47$; 2
4. $24.94 \div 0.58$
   $2{,}494 \div 58$; 43

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with addition and subtraction within 1,000,000.

Assign

Use It! Application Station

Leave a Trail! Students create a poster showing how they divided decimals while making trail mix.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 87–88

Write an equivalent division so that the divisor is a whole number. Then find the quotient.

5. 16.8 ÷ 4.2
168 ÷ 42; 4

6. 14.35 ÷ 0.35
1,435 ÷ 35; 41

7. 33.06 ÷ 0.87
3,306 ÷ 87; 38

8. 170.1 ÷ 2.7
1,701 ÷ 27; 63

9. Dexter is designing a rectangular bedroom. The area of the bedroom is to be 137.5 square feet, and the length is to be 12.5 feet. What is the width of the bedroom?
11 feet

10. Amelia has 0.96 pound of grated cheese. She uses 0.06 pound of cheese on each of her salads. How many salads can Amelia make?
16 salads

11. A fence post is placed every 4.2 feet. How many fence posts are needed for a fence that is 176.4 feet long?
42 fence posts

Math @ Home Activity Look for situations around your home where dividing two decimals would be natural. Ask your child to identify a situation and explain how to find the quotient. For example, if there are 2.5 pounds of strawberries in a container and they are divided in 0.5-pound bags, how many bags can be filled?

Student Practice Book

Differentiation Resource Book, p. 88

Lesson 8-6 • Extend Thinking

Divide Decimals by Decimals

Name ______

Fill in the blanks for the area models, partial quotients, and equations.

1. 6.48 ÷ 0.24 = 27
648 ÷ 24 = 27

	20	7
24	648 −480 168	168 −168 0

2. 36.05 ÷ 0.35 = 103
3,605 ÷ 35 = 103

	100	3
35	3,605 −3,500 105	105 −105 0

3. 98.8 ÷ 2.6 = 38
988 ÷ 26 = 38

	30	8
26	988 −780 208	208 −208 0

4. 7.2 ÷ 0.45 = 16
720 ÷ 45 = 16

	10	6
45	720 −450 270	270 −270 0

Differentiation Resource Book

Unit 8

Decimal Division

MATH PROBES

Name ______________________

For each problem, use what you know about place value and division to select the correct quotient. Do not actually calculate the division.

1. What is the quotient for
   21.76 ÷ 0.80?

   Circle your answer.

   a. 0.0272
   b. 0.272
   c. 2.72
   d. 27.2

Explain or show your answer.
Explanations may vary.

2. Which provides the answer to
   3.0 ÷ 3.75?

   Circle your answer.

   a. 80
   b. 8.0
   c. 0.80
   d. 0.08

Explain or show your answer.
Explanations may vary.

Unit 8 • Divide Decimals 27

For each problem, use what you know about place value and division to select the correct quotient. Do not actually calculate the division.

3. Which provides the answer to
   0.036 ÷ 0.24?

   Circle your answer.

   a. 0.15
   b. 0.015
   c. 1.50
   d. 15.0

Explain or show your answer.
Explanations may vary.

4. Which provides the answer to
   80.4 ÷ 0.67?

   Circle your answer.

   a. 0.12
   b. 1.2
   c. 12
   d. 120

Explain or show your answer.
Explanations may vary.

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

28 Math Probe | Decimal Division

Analyze the Probe ✓ Formative Assessment

Targeted Concept Determine the quotient of a decimal divided by a decimal by using reasoning about the magnitude of the dividend and divisor and the meaning of division.

Targeted Misconceptions Some students do not apply place-value ideas to determine quotients when a decimal is divided by a decimal. They may think division always "makes smaller" without considering the magnitude of the dividend and divisor. Some students have difficulty applying strategies for whole number division and estimation to reason about the location of the decimal point in the quotient.

Authentic Student Work

Below are examples of correct student work and explanations.

Sample A

2. Which provides the answer to
   3.0 ÷ 3.75?

   Circle your answer.

   a. 80
   b. ~~8.0~~
   c. 0.80
   d. 0.08

Explain or show your answer.

3 ÷ 3 = 1
3 ÷ 4 = .75
0.80 is between
1 and .75

Sample B

4. Which provides the answer to
   80.4 ÷ 0.67?

   Circle your answer.

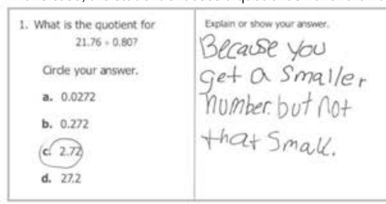
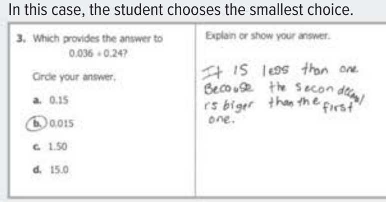
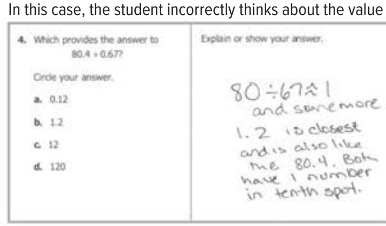
   a. 0.12
   b. 1.2
   c. 12
   d. 120

Explain or show your answer.

pretend that it
is 80 ÷ .5
That would be 160.
The answer has to be
120. None of the others
make sense.

Collect and Assess Student Work

Collect and review student response to determine possible misconceptions. See examples in If-Then chart.

IF incorrect...	THEN the student likely...	Sample Misconceptions
1. a, b, c 2. d 3. b 4. a, b, c	overgeneralizes that the quotient is always less than the dividend. Note that this overgeneralization could lead to a correct answer for Exercise 2 (choice c).	In this case, the student chooses a quotient smaller than the dividend.  1. What is the quotient for 21.76 ÷ 0.8? Circle your answer. a. 0.0272 b. 0.272 c. 2.72 d. 27.2 Explain or show your answer. Because you get a Smaller number but not that Small.
1. a 2. d 3. b 4. a	thinks that when decimals are divided, the answer is a very small decimal (generally the smallest decimal among the answer choices).	In this case, the student chooses the smallest choice.  3. Which provides the answer to 0.036 ÷ 0.24? Circle your answer. a. 0.15 b. 0.015 c. 1.50 d. 15.0 Explain or show your answer. It is less than one Because the second decimal is biger than the first one.
1. c 2. b, d 3. b, c 4. a, b	overgeneralizes that when decimals are divided, the quotient will have the decimal point in the same place as the dividend or the divisor. Note that this overgeneralization could lead to correct answers for Exercise 2 (choice c) and for Exercise 3 (choice a).	In this case, the student incorrectly thinks about the value of the dividend and divisor.  4. Which provides the answer to 80.4 ÷ 0.67? Circle your answer. a. 0.12 b. 1.2 c. 12 d. 120 Explain or show your answer. 80 ÷ 67 ≈ 1 and some more 1.2 is closest and is also like the 80.4. Both have 1 number in tenth spot.

1. Many of the above difficulties result in a combination of correct and incorrect responses. For correct responses, be sure to check for sound reasoning.

Take Action

Choose from the following resources or suggestions:

- Revisit decimal division activities in Lessons 8-2 through 8-6.
- Apply whole number division ideas to decimal division. For example, point out that just as 10 ÷ 2 = ? can be thought of as "finding how many 2s fit into 10," you can think of 10 ÷ 0.2 = ? as "finding how many two-tenths fit into 10."
- Use decimal language that reinforces place-value understanding. For example, ensure that students read 0.35 as "thirty-five hundredths" rather than as, say, "point three five."
- Encourage students to call to mind visual images of concrete materials as a strategy for reasoning about decimal division and the placement of the decimal point.
- Do estimation activities in groups, allowing students to discuss their strategies.

Revisit the Probe After additional instruction, have students review their initial answers. Use these questions for discussion:

- *Are there any answers you would like to change? Explain.*
- *Are there any questions that you still have about any of the exercises on this probe?*

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Unit Review

Unit Review Name ______

Vocabulary Review

Choose the correct word(s) to complete each sentence.

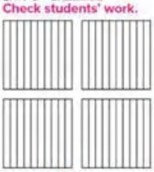
dividend	estimate	quotient
divisor	power of 10	

1. To find an approximate value in a calculation is to **estimate**. (Lesson 8-2)
2. In a division problem, the **dividend** is divided by the **divisor**. (Lesson 8-2)
3. The **quotient** is the result of dividing one number by another. (Lesson 8-2)
4. Place value can be represented using a **power of 10**. (Lesson 8-1)

Unit 8 • Divide Decimals 29

Review

5. Use the decimal grids to solve $2.4 \div 6 = d$. (Lesson 8-3)
Check students' work.



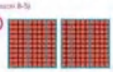
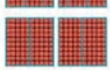
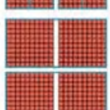
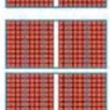
$2.4 \div 6 =$ **0.4**

6. Use a pattern to find the quotients. (Lesson 8-1)
$32.8 \div 100 =$ **0.328**
$32.8 \div 10 =$ **3.28**
$32.8 \div 1 =$ **32.8**
$32.8 \div 0.1 =$ **328**
$32.8 \div 0.01 =$ **3,280**

7. Mr. Blare earns $12.40 an hour. Last week, he earned $471.20. How many hours did he work last week? (Lesson 8-6) **38 h**

8. What is the quotient? (Lesson 8-5)
$82 \div 0.2 = b$ **410**

9. A quarter is about 0.2 centimeters thick. About how many quarters are needed so that a stack of quarters is 10.8 centimeters tall? (Lesson 8-2)
A. 10
B. 20
C. 50
D. 80

10. Which model shows $2 \div 0.4$? (Lesson 8-5)
A.

B.

C.

D.


30 Unit 8 • Review

Students can complete the **Unit Review** to prepare for the **Unit Assessment.** Students may complete the Review in their Interactive eBook in the Digital Student Center.

Vocabulary Review

Item Analysis

Item	Lesson
1	8-2
2	8-2
3	8-2
4	8-1

Review

Item Analysis

Item	DOK	Lesson	Standard
5	1	8-3	5.NBT.B.7
6	1	8-1	5.NBT.A.2
7	2	8-6	5.NBT.B.7
8	1	8-5	5.NBT.B.7
9	2	8-2	5.NBT.B.7
10	1	8-5	5.NBT.B.7

To review the lessons in this unit, have students watch the Math Replay video in their Digital Student Center.

Assign the Unit Review practice to students from the Digital Teacher Center.

GO ONLINE
MATH REPLAY

Item Analysis (continued)

Item	DOK	Lesson	Standard
11	1	8-4	5.NBT.B.7
12	1	8-5	5.NBT.B.7
13	1	8-5	5.NBT.B.7
14	1	8-6	5.NBT.B.7
15	1	8-4	5.NBT.B.7
16	2	8-4	5.NBT.B.7
17	1	8-4	5.NBT.B.7

Performance Task

Standard: 5.NBT.B.7

Rubric (4 points)

Parts A and B (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with dividing whole numbers by decimals. The student's quotients are accurate.
1 POINT	Student's work reflects developing proficiency with dividing whole numbers by decimals. One of the quotients is accurate.
0 POINTS	Student's work reflects a poor understanding of dividing whole numbers by decimals. No quotients are correct.

Part C (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with dividing decimals. The solution is accurate.
1 POINT	Student's work reflects developing proficiency with dividing decimals. The solution is incorrect due to computational errors, not conceptual weakness.
0 POINTS	Student's work reflects a poor understanding of dividing decimals. The solution is incorrect.

Reflect

The Reflect question provides an opportunity for students to express their understanding of the unit level focus question.

MATH REPLAY | GO ONLINE

11. What is the quotient? (Lesson 8-4)

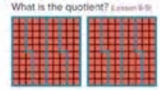
$9.72 \div 3 = d$

3.24

12. Which equivalent expression uses powers of 10 to help you solve $96 \div 1.2$? (Lesson 8-5)

A. $96 \div 12$
B. $96 \div 120$
C. $960 \div 12$
D. $960 \div 120$

13. The model represents $2 \div 0.25$. What is the quotient? (Lesson 8-5)



8

14. Which equivalent expression uses powers of 10 to help you solve $52.71 \div 0.21$? (Lesson 8-6)

A. $5{,}271 \div 21$
B. $5{,}271 \div 0.21$
C. $52.71 \div 21$
D. $52.71 \div 2.1$

15. What is the quotient? (Lesson 8-4)

$2.5 \div 5 = b$

A. 0.05
B. 0.5
C. 5
D. 50

16. Danny cuts a 2.7-meter long wire into 3 equal pieces. How long is each piece of wire? (Lesson 8-4)

0.9 m

17. Which representation can you use to find the quotient of $0.36 \div 6$? (Lesson 8-4)

A. 3.6 tenths $\div$ 6 = 6 tenths
B. 36 tenths $\div$ 6 = 6 tenths
C. 3.6 hundredths $\div$ 6 = 6 hundredths
D. 36 hundredths $\div$ 6 = 6 hundredths

Unit 8 • Divide Decimals 31

Performance Task

Chef Malory is making salads.

Chef Malory's Special Salad

- 10 oz spinach
- $\frac{1}{8}$ small onion
- 6 sliced strawberries
- 0.7 oz walnuts
- 1.4 oz blue cheese
- 0.5 oz vinaigrette

Part A: Chef Malory is buying the blue cheese. He found two containers of blue cheese, one that is 4 oz and another one 7 oz. Which container will allow him to use the entire container and how many salads can he make?

7 oz container; 5 salads

Part B: Chef Malory has a 1.75 pound of walnuts. How many salads could he make? Will he use the whole bag?

40 salads because 1.75 pounds is 28 ounces; yes because $28 \div 0.7 = 40$

Part C: If Chef Malory wanted to use his entire bag of walnuts, how much would he need of the other ingredients?

400 oz spinach; 5 small onions; 240 sliced strawberries; 56 oz blue cheese; 20 oz vinaigrette

Reflect

How is dividing decimals the same as dividing whole numbers?

Answers may vary.

32 Unit 8 • Performance Task

Unit 8

Fluency Practice

Name ______________________

Fluency Strategy

You can use place value and properties of operations to multiply by multiples of 100.

600 × 4 = 6 × 100 × 4 — Write 600 as 6 × 100.

= 6 × 4 × 100 — Use properties to change the order of the factors without changing the product.

= 24 × 100 — Multiply.

= 2,400

1. Use place value and properties to multiply.

900 × 3 = 9 × 100 × 3

= 9 × 3 × 100

= 27 × 100

= 2,700

Fluency Flash

Use the models to complete the multiplication equations.

2.

2 × 5 = 10   2 × 500 = 1,000

Unit 8 • Divide Decimals 33

Fluency Check

What is the difference or product?

3. 4 × 800 = 3,200	10. 900 × 8 = 7,200
4. 6 × 30 = 180	11. 7 × 600 = 4,200
5. 600 × 2 = 1,200	12. 90 × 5 = 450
6. 4 × 90 = 360	13. 2,478 − 247 = 2,231
7. 553 − 151 = 402	14. 200 × 4 = 800
8. 20 × 8 = 160	15. 8 × 700 = 5,600
9. 300 × 7 = 2,100	16. 961 − 432 = 529

Fluency Talk

Explain how you can use properties of operations to find the product of a number and a multiple of 100.

Explanations may vary.

What patterns do you use when you multiply a number by a multiple of 10?

Explanations may vary.

34 Unit 8 • Fluency Practice

Fluency practice helps students develop procedural fluency, that is, the "ability to apply procedures accurately, efficiently, and flexibly." Because there is no expectation of speed, students should not be timed when completing the practice activity.

Build Fluency **Objective** Students practice multiplying by multiples of 100.

Fluency Progression

Unit	Skill	Standard
1	Use Partial Sums to Add	4.NBT.B.4
2	Decompose by Place Value to Subtract	4.NBT.B.4
3	Use an Algorithm to Add	4.NBT.B.4
4	Use an Algorithm to Subtract	4.NBT.B.4
5	Choose a Strategy to Add	4.NBT.B.4
6	Choose a Strategy to Subtract	4.NBT.B.4
7	Multiply by Multiples of 10	5.NBT.B.5
8	**Multiply by Multiples of 100**	**5.NBT.B.5**
9	Divide Multiples of 10	5.NBT.B.6
10	Divide Multiples of 100	5.NBT.B.6
11	Use an Algorithm to Multiply (2- and 3-Digit Numbers by 1-Digit Numbers)	5.NBT.B.5
12	Use an Algorithm to Multiply (2-Digit Numbers by 2-Digit Numbers)	5.NBT.B.5
13	Choose a Strategy to Multiply	5.NBT.B.5
14	Choose a Strategy to Multiply	5.NBT.B.5

Fluency Expectations

Grade 4

- Add and subtract within 1,000,000.

Grade 5

- Multiply multi-digit whole numbers.

Grade 6

- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Performance Task

Baseball

Students draw on their understanding of dividing decimals. Use the rubric shown to evaluate students' work.

Standards: 5.NBT.A.2, 5.NBT.B.7

Rubric (10 points)

Part A (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with dividing with decimals. The student was able to accurately provide 4 or 5 of the 5 answers.
1 POINT	Student's work reflects a developing proficiency with dividing with decimals. The student was able to accurately provide 2 or 3 of the 5 answers.
0 POINTS	Student's work reflects a weak proficiency with dividing with decimals. The student was able to accurately provide 1 or 0 of the 5 answers.

Part B (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with dividing with decimals. The student was able to accurately calculate the answer.
1 POINT	Student's work reflects a developing proficiency with dividing with decimals. A minor error in calculation resulted in an inaccurate answer.
0 POINTS	Student's work reflects a weak proficiency with dividing with decimals. Multiple errors in calculation resulted in an inaccurate answer.

Parts C and D (DOK 2) – 4 points	
4 POINTS	Student's work reflects a proficiency with dividing with decimals. The student was able to accurately calculate the answer.
2 POINTS	Student's work reflects a developing proficiency with dividing with decimals. A minor error in calculation resulted in an inaccurate answer.
0 POINTS	Student's work reflects a weak proficiency with dividing with decimals. Multiple errors in calculation resulted in an inaccurate answer.

Part E (DOK 3) – 2 points	
2 POINTS	Student's work reflects a proficiency with how division affects the final answer. The student was able to accurately respond.
1 POINT	Student's work reflects a developing proficiency with how division affects the final answer. The student was able to respond but with errors.
0 POINTS	Student's work reflects a weak proficiency with how division affects the final answer. The student was not able to accurately respond.

Unit 8

Performance Task

Name ____________________

Baseball

A baseball player's batting average can be found using the following formula.

$$\text{batting average} = \frac{\text{number of hits}}{\text{number of times at bat}}$$

We can also transform (or rearrange) this formula to find additional relationships between these quantities.

$$\text{number of hits} = \text{batting average} \times \text{number of times at bat}$$

$$\text{number of times at bat} = \frac{\text{number of hits}}{\text{batting average}}$$

Part A

In the table below, fill in the missing information for each player. Show your work. Batting averages are written rounded to three decimal places.

Player	number of Hits	number of times at Bat	Batting Average
1	52	**52 ÷ 0.250 = 208**	0.250
2	96	**96 ÷ 0.320 = 300**	0.320
3	49	**49 ÷ 0.200 = 245**	0.200
4	48	**48 ÷ 0.400 = 120**	0.400
5	31	**31 ÷ 0.155 = 200**	0.155

Assessment Resource Book 147

Part B

A 6th player had 36 hits resulting in a batting average of 0.178. Estimate the player's number of at bats.

Sample answer: 36 ÷ 0.180 = 3600 ÷ 18
= 200 at bats

Part C

Suppose Player 5 wants to increase their batting average to 0.275. If they get 35 more hits, how many times at bat will the player need to have?

Sample answer: 31 + 35 = 66 number of hits;
66 ÷ 0.275 = 240 total number of times at bat.

Part D

Suppose Player 3 makes 32 more hits and now has a batting average of 0.270. Calculate Player 3's number of times at bat.

Sample answer: 49 + 32 = 81 number of hits;
81 ÷ 0.270 = 300 total number of times at bat.

Part E

Player 1 thinks his batting average increases because of the amount of times at bat. How would you respond to player 1?

Sample answer: The amount of hits when at bat should make the batting average increase.

148 Assessment Resource Book

Unit Assessments

Two forms of the Unit Assessment, Form A and Form B, are available for either print or digital administration. The items on the two assessments are parallel items, assessing the same concept and standard. The table below provides the item analysis for both forms.

Both Unit Assessments are available in the Assessment Resource Book or as downloadable files from the Digital Teacher Center.

Data When students complete the Unit Assessment in the Digital Student Center, their responses are auto-scored.

Item Analysis

Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	2	8-1	Divide by Powers of 10 (Decimal Point)	5.NBT.A.2
2	2	8-1	Divide by Powers of 10 (Decimal Point)	5.NBT.A.2
3	1	8-2	Estimate Quotients (Decimal Numbers)	5.NBT.B.7
4	2	8-2	Estimate Quotients (Decimal Numbers)	5.NBT.B.7
5	1	8-3	Divide Decimals by Whole Numbers-Model	5.NBT.B.7
6	1	8-4	Divide Decimals by Whole Numbers	5.NBT.B.7
7	1	8-4	Divide Decimals by Whole Numbers	5.NBT.B.7
8	2	8-4	Divide Decimals by Whole Numbers	5.NBT.B.7
9	1	8-5	Divide Whole Numbers by Decimals	5.NBT.B.7
10	2	8-5	Divide Whole Numbers by Decimals	5.NBT.B.7
11	1	8-6	Divide Decimals	5.NBT.B.7
12	2	8-6	Divide Decimals	5.NBT.B.7
13	3	8-5	Divide Whole Numbers by Decimals	5.NBT.B.7
14	3	8-5	Divide Whole Numbers by Decimals	5.NBT.B.7
15	3	8-1, 8-5	Divide Whole Numbers by Decimals	5.NBT.A.2, 5.NBT.B.7

GO ONLINE

Assign the digital Unit Assessment (Form A or B) to students or download and print PDFs from the Digital Teacher Center.

Unit 8

Unit Assessment, Form A

Name

1. Owen has $2.80 in dimes. Which equation shows how many dimes Owen has?
   A. 2.80 ÷ 0.01 = 280 dimes
   B. 2.80 ÷ 0.1 = 28 dimes
   C. 2.80 ÷ 0.01 = 28 dimes
   D. 2.80 ÷ 0.1 = 280 dimes

2. Melanie rides her bicycle 42.4 miles in 10 days. If she rides the same amount of miles each day, how many miles does she ride each day?
   A. 0.424 mile
   B. 4.24 miles
   C. 42.4 miles
   D. 424 miles

3. Which equations show a reasonable estimate for the quotient 7.41 ÷ 0.08 using powers of 10 and compatible numbers? Choose all that apply.
   A. 70 ÷ 1 = 70
   B. 720 ÷ 8 = 90
   C. 72 ÷ 8 = 9
   D. 70 ÷ 10 = 7

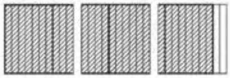
4. Sam is selecting a new cell phone plan. The cell phone company offers different monthly plans based on the number of gigabytes (GB) of data available.

	Plan 1	Plan 2	Plan 3
Cost per Month	$49.99	$75.99	$34.99
Amount of Data	6 GB	9 GB	3 GB

   Which plan has the lowest cost per gigabyte?
   A. All the plans are the same.
   B. Plan 1
   C. Plan 2
   D. Plan 3

Assessment Resource Book 149

5. Which equation is represented by the decimal grids shown?



   A. 2.8 ÷ 0.7 = 40
   B. 0.7 ÷ 2.8 = 4
   C. 0.8 ÷ 4 = 0.7
   D. 2.8 ÷ 4 = 0.7

6. What is the quotient of 0.20 ÷ 5?
   A. 40
   B. 4
   C. 0.4
   D. 0.04

7. Which is an equivalent representation of 1.8 ÷ 3?
   A. 18 tens ÷ 3
   B. 18 ones ÷ 3
   C. 18 tenths ÷ 3
   D. 18 hundredths ÷ 3

8. Lidia uses 1.35 pounds of meat to make 5 sandwiches. She uses the same amount of meat in each sandwich. How much meat does Lidia use to make each sandwich?
   0.27 pound

150 Assessment Resource Book

Unit 8
Unit Assessment, Form A (continued)

Name ______________________

9. What is the quotient of 2 ÷ 0.4?

A. 0.5
B. 5
C. 50
D. 500

10. Darius runs 15 miles. He runs 0.2 mile each minute. How many minutes does it take Darius to run 15 miles?

A. 7.5 minutes
B. 30 minutes
C. 75 minutes
D. 150 minutes

11. Which division problem is equivalent to 8.96 ÷ 0.32?

A. 896 ÷ 32
B. 896 ÷ 3.2
C. 89.6 ÷ 32
D. 8,960 ÷ 32

12. Matthew completed a 26.4 mile race in 2.2 hours. If he ran the same amount of miles each hour, how many miles did Matthew run each hour?

12 miles

Assessment Resource Book 151

13. Randy says 9 ÷ 0.06 has the same quotient as 90 ÷ 6. How do you respond to him?

Sample answer: If 9 changes to 90, then 0.06 needs to change to 0.6, not 6. These two expressions will not have the same quotient.

14. A pizzeria has 4 pounds of tomato sauce to use on the pizzas they make. Their recipe calls for 0.25 pound of tomato sauce for each pizza. The pizzeria sells 10 pizzas. Will the pizzeria have enough tomato sauce to make this number of pizzas or will they run out? Explain your answer.

They will have enough; Sample answer: To solve 4 ÷ 0.25, I wrote the equivalent expression 400 ÷ 25. The pizzeria will have enough for 16 pizzas, so the pizzeria will be able to make 10 pizzas with the available sauce.

15. Ken wants to buy tiles to cover 20 square yards of flooring. He can choose from three sizes of tiles.

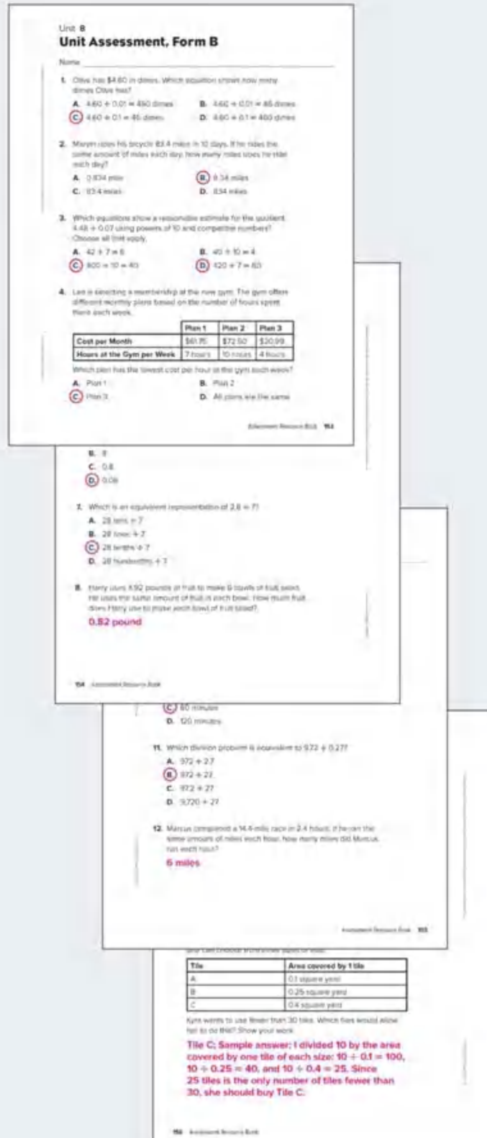
Tile	Area covered by 1 tile
A	0.1 square yard
B	0.25 square yard
C	0.4 square yard

Ken wants to use fewer than 60 tiles. Which tiles would allow him to do this? Show your work.

Tile C; Sample answer: I divided 20 by the area covered by one tile of each size: 20 ÷ 0.1 = 200, 20 ÷ 0.25 = 80, and 20 ÷ 0.4 = 50. Since 50 tiles is the only number of tiles fewer than 60, he should buy Tile C.

152 Assessment Resource Book

Form B


Unit 8
Unit Assessment, Form B

Name

0.82 pound

6 miles

Tile C; Sample answer: I divided 10 by the area covered by one tile of each size: 10 ÷ 0.1 = 100, 10 ÷ 0.25 = 40, and 10 ÷ 0.4 = 25. Since 25 tiles is the only number of tiles fewer than 30, she should buy Tile C.

UNIT 9 PLANNER

Add and Subtract Fractions

PACING: 13 days

LESSON		MATH OBJECTIVE	LANGUAGE OBJECTIVE	SOCIAL AND EMOTIONAL LEARNING OBJECTIVE
Unit Opener	IGNITE! Fraction Wall Students use a fraction wall to explore ways to make a fraction using fractions with different denominators.			
9-1	Estimate Sums and Differences of Fractions	Students use benchmark numbers to estimate sums and differences of fractions. Students explain how to use an estimate to predict or check the reasonableness of a calculated sum or difference of fractions.	Students talk about benchmark numbers to estimate the sums and differences of fractions using *greater than* and *less than*.	Students determine how they can break a problem down to make it easier to solve.
Math Probe	Make an Estimate of the Sum Students use strategies to reason about the magnitude of and addition of fractions.			
9-2	Represent Addition of Fractions with Unlike Denominators	Students use and explain how to use a representation to add fractions with unlike denominators.	Students explain how to use a representation to add fractions with unlike denominators using *can*.	Students exchange ideas for mathematical problem-solving with a peer and provide thoughtful and constructive feedback.
9-3	Add Fractions with Unlike Denominators	Students add and explain how to add fractions with unlike denominators.	Students explain how to add fractions with unlike denominators using *should*.	Students actively listen without interruption as peers describe how they approached a complex mathematical task.
9-4	Represent Subtraction of Fractions with Unlike Denominators	Students use and explain how to use a representation to subtract fractions with unlike denominators.	Students explain how to use a representation to subtract fractions with unlike denominators using *can*.	Students employ self-calming techniques that can be used to help manage reactions to potentially frustrating situations.
9-5	Subtract Fractions with Unlike Denominators	Students subtract and explain how to subtract fractions with unlike denominators.	Students explain how to subtract fractions with unlike denominators using *can* and *should*.	Students practice staying focused on a mathematical problem for a set time.
9-6	Add Mixed Numbers with Unlike Denominators	Students add and explain how to add mixed numbers with unlike denominators.	Students talk about adding mixed numbers with unlike denominators using *can* and *use*.	Students identify multiple possible solutions for a given math problem.
9-7	Subtract Mixed Numbers with Unlike Denominators	Students subtract and explain how to subtract mixed numbers with unlike denominators.	Students talk about subtracting mixed numbers with unlike denominators using *can*, *should*, *same*, and *different*.	Students practice segmenting a complex mathematical task into smaller achievable tasks.
9-8	Add and Subtract Mixed Numbers with Regrouping	Students add and subtract mixed numbers with regrouping.	Students talk about adding and subtracting mixed numbers with regrouping using *rearrange* and *rename*.	Students work toward completing a mathematical task independently using prior knowledge or understanding of mathematical concepts.
9-9	Solve Problems Involving Fractions and Mixed Numbers	Students solve word problems involving fractions.	Students explain how to solve word problems with fractions using *can*, *should*, *reasonable*, and *estimate*.	Students identify a problem and execute the steps necessary to solve the problem.
Unit Review				
Fluency Practice				
Performance Task				
Unit Assessment				

FOCUS QUESTION:
How do I add and subtract fractions?

LESSON	KEY VOCABULARY: Math Terms	KEY VOCABULARY: Academic Terms	MATERIALS TO GATHER		RIGOR FOCUS	STANDARD
9-1	benchmark number estimate	eliminate suggest	• fraction circles • fraction tiles • number cubes • *Benchmark Fraction Number Line* Teaching Resource		Application	5.NF.A.2
9-2	denominator equivalent fractions fraction tiles like denominators numerator	correspond suggest	• *Blank Open Number Line* Teaching Resource • fraction tiles • ruler		Conceptual Understanding, Procedural Skill & Fluency	5.NF.A.1
9-3	equivalent fractions like denominator multiple	accurate condition	• fraction tiles • number cubes		Conceptual Understanding, Procedural Skill & Fluency	5.NF.A.1
9-4	denominator equivalent fractions	establish valid	• *Fraction Number Lines* Teaching Resource • fraction tiles		Conceptual Understanding, Procedural Skill & Fluency	5.NF.A.1
9-5	denominator equivalent fractions	reflect suggest	• fractions tiles • index cards		Conceptual Understanding, Procedural Skill & Fluency	5.NF.A.1
9-6	equivalent fractions mixed number	establish relevant	• fraction tiles • index cards		Conceptual Understanding, Procedural Skill & Fluency	5.NF.A.1
9-7	equivalent fractions mixed number	accurate assert	• blank spinner • fraction tiles		Conceptual Understanding, Procedural Skill & Fluency	5.NF.A.1
9-8	equivalent fractions mixed number	debate eliminate	• fraction tiles • index cards • rulers	• *Explain and Show Your Strategies* Teaching Resource	Conceptual Understanding, PSF	5.NF.A.1
9-9	equivalent fractions mixed number	reflect suggest	• fraction circles • fraction tiles • number cubes	• *Problem-Solving Tool* Teaching Resource	Application	5.NF.A.2

Unit Overview

Focus

Equivalent Fractions and Like Denominators

Students begin this unit by estimating sums and differences of fractions. Estimates are used by mathematically proficient students to check the reasonableness of answers. Since halves and ones are easy to work with, students use these benchmarks to help them estimate. They can round each number they are working with to a benchmark and get a quick idea of what the sum or difference should be. Students learn that they can round to a half if the numerator is about half of the denominator. If the numerator is very close to the denominator, the fraction rounds to 1. If the numerator is much less than the denominator, the fraction rounds to 0.

Students extend the work they did in Grade 4 as they work with fractions with unlike denominators. To start, they apply concepts from Grade 3 to determine how the fractions can be written using a single denominator. They know the denominator tells the unit fraction used to build it. Fractions with unlike denominators are built using different unit fractions. Students get a sense of this and what to do about it by working with physical representations, such as fraction circles and fraction tiles.

Moving from the visual/physical to symbolic manipulations, students use what they learned in Grade 4 about equivalent fractions. They use this relationship to obtain equivalent fractions with common denominators. They are exposed to problems for which only one of the fractions must be rewritten with a common denominator as well as those where both fractions need to be rewritten with a common denominator. The student's task is to find a common multiple of the denominators. After fractions are written with a common denominator, students can add and subtract fractions as they have done previously, by adding and subtracting the numerators, respectively.

They further apply this process with mixed numbers, recognizing that the fractional parts need to be expressed with a common denominator to add or subtract. However, regrouping might be necessary if the sum of the fractional parts is greater than 1 or the first fractional part in a subtraction expression is less than the second fractional part. In these cases, students regroup 1 from the fractional part or the whole part as necessary.

Coherence

What Students Have Learned

- Students compared fractions by creating common denominators or numerators. (Grade 4)
- Students added and subtracted mixed numbers with like denominators. (Grade 4)
- Students solved word problems involving addition and subtraction of fractions. (Grade 4)
- Student expressed a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100. (Grade 4)

What Students Are Learning

- Students add and subtract fractions and mixed numbers with unlike denominators.
- Students solve word problems involving addition and subtraction of fractions and mixed numbers.
- Students use benchmark numbers to estimate the reasonableness of sums and differences of fractions.

What Students Will Learn

- Students solve real-world and mathematical problems by writing and solving equations with nonnegative rational constants and coefficients. (Grade 6)

Rigor

Conceptual Understanding

Students develop understanding of
- estimating sums and differences of fractions;
- adding and subtracting fractions with unlike denominators;
- adding and subtracting mixed numbers with unlike denominators.

Procedural Skill & Fluency

Students build proficiency with
- identifying benchmark fractions and making estimates;
- finding the sums and difference of fractions with unlike denominators;
- finding the sums and differences of mixed numbers with unlike denominators.

Application

Students apply their knowledge of
- determining the reasonableness of answers in real-world contexts;
- adding and subtracting fractions and mixed numbers to solve problems in real-world contexts.

Effective Teaching Practices

Facilitate Meaningful Mathematical Discourse

A rich learning environment includes meaningful discourse among students. In the mathematics classroom, student discourse is an interactive process of collaborative exploration, exchange of ideas, argumentation, and building of shared understanding. It is engaging, commands participation, and promotes deep learning. It is a tool the teacher uses to both ensure and confirm that learning is taking place.

Discourse can be verbal or written and enhanced through visuals. It takes place in a variety of settings—for example, whole class, small groups, and pairs—and it calls for student-student and student-teacher interaction. Student-student discourse allows students to take responsibility for their own learning and the learning of their peers. The student-teacher dynamic has the teacher playing more of a supporting role, making sure that the process follows a productive path.

Classroom discourse can be a means of engaging any element of the teaching and mathematical practices—for example, constructing arguments and critiquing the reasoning of others. It can also be used to enrich the classroom experience for English language learners.

Each lesson in this program calls for discourse using the think-pair-share model and whole-class discussion.

The teacher is a facilitator and performs the following actions—

- During pair and small-group activities, engages students as they explore and share ideas and strategies with each other.
- Observes and gathers information about what students are doing and saying and forms a plan for the whole-class discussion.
- Facilitates student-student and class discourse by creating an environment in which students are willing and eager to take ownership of their ideas.
- Makes sure the discourse stays relevant to the lesson's goals.

Math Practices and Processes

Use Appropriate Tools Strategically

Mathematical tools include physical and virtual manipulatives; drawings, graphs, and diagrams; expressions, equations, and other symbolic constructions; and calculators, math software, and other technology.

When mathematically proficient students look at a problem, they decide which tools they will use to represent and solve it. They are flexible with the choices and know the benefits of one tool versus another and of combining tools to better understand problems and justify solutions.

During the course of a lesson, the teacher should let students explore a problem on their own or in pairs using tools and approaches of their own choosing. The teacher facilitates the exploration using strategic questions while allowing students to interact, exchange ideas, and make discoveries.

A goal in teaching is to have students be able to switch between tools and/or transition from one tool to another—for example, from concrete manipulatives to equations. This flexibility is an indication that a student understands the problem and concept being explored.

- For example, in this unit, the goal is for students to be able to represent problems using fraction circles and/or tiles and sketches and to be able to manipulate these using their knowledge of addition, subtraction, and equivalent fractions. Further, students should be able to apply these representations to the writing and manipulation of equations or other symbolic/numeric constructions and to understand the connections between this work and the physical representations.

Social and Emotional Learning

- **Self-Awareness – Recognize Strengths** (Lesson 9-1): When students recognize their own strengths, they can see themselves as resourceful and may be more willing to attempt to problem solve and help others.
- **Social Awareness – Respect Others** (Lesson 9-2): When students are respectful of one another, they strengthen their class community.
- **Relationship Skills – Communication** (Lesson 9-3): Students who can communicate effectively are more likely to build strong relationships and contribute to a positive classroom culture.
- **Self-Management – Control Impulses** (Lesson 9-4): Students who can regulate their impulses and reactions are better able to navigate and solve problems.
- **Social Awareness – Appreciate Diversity** (Lesson 9-5): When students appreciate diversity, they create a stronger, more inclusive classroom community.
- **Responsible Decision-Making – Identify Problems** (Lesson 9-6): A key step in problem solving is analyzing information to identify the task.
- **Relationship Skills – Teamwork** (Lesson 9-7): When students work effectively as a team, they establish a stronger learning community.
- **Responsible Decision-Making – Solve Problems** (Lesson 9-8): Efficient problem solvers can make informed decisions that lead to solutions.
- **Self-Awareness – Self-Efficacy** (Lesson 9-9): Students with high self-efficacy are more likely to persevere to complete a challenging task.

Language of Math

Mathematical Nouns

Students will be using these key terms in this unit:

- **Benchmark number*** (Lesson 9-1): Students were introduced to this term in the context of comparing fractions in Grade 4. In this unit, the benchmark numbers 0, $\frac{1}{2}$ and 1 are used to estimate sums and differences of fractions and mixed numbers by finding and using estimates of the numbers being added or subtracted.
- **Like denominator*** (Lesson 9-2): Students were introduced to *denominator* in the context of fractions in Grade 3, and they worked with *like* denominators in Grade 4. In Grade 5, students generalize their skills with addition and subtraction of fractions, and they make and hear reference to *unlike* denominators, which are denominators that are not the same.
- **Mixed number*** (Lesson 9-6): Students were introduced to this term in the context of addition and subtraction with mixed numbers in Grade 4. It is a number that has a whole-number part and a fraction part.
- **Multiple*** (Lesson 9-3): Students were introduced to this term in the context of recognizing that a whole number is a multiple of each of its factors in Grade 4. In Grade 5, students find common multiples of unlike denominators as they rewrite fractions to obtain like denominators.

*This is a new term.

Math Language Development

A Focus on Speaking

We learned our first language by listening, speaking, reading, and writing, in that order. Mathematics is like a second language, because we don't start speaking it until a few years after we start speaking our first language, and by then reading and writing get as much attention as listening and speaking. This is natural, since we are by then learning to read and write. Still, a deliberate effort should be made to engage students in speaking about mathematics right away, from the very beginning of their instruction in mathematics.

Acknowledgement of this reality is reflected in the current initiatives in math education. Discourse is cited as a critical component in our effort to give students more ownership of their learning. Teachers are called to probe student thinking and elicit verbal responses. We are further advised that our probing questions be open-ended. In this way, we ask students to dig deeper into their thoughts and their abilities to verbalize them.

By speaking their thoughts, students process their understanding more thoroughly. This helps them understand more deeply, and it builds their recall of concepts.

In this unit, seek opportunities to elicit verbal responses from students.

- There are frequent opportunities to ask students how they will represent a problem. Have them explain the representation to you before (or instead of) showing you. Math is different from everyday language in that we are accustomed to *showing* our thinking. We are inclined, for example, to write equations or draw pictures or graphs to explain our ideas.
- Students will be asked which equivalent fractions they can use and how they can rename fractions and mixed numbers. Have students speak their responses with explanations in descriptive terms, as if in connection to physical representations.

English Language Learner

Making Inputs Comprehensible

In this unit, students are provided with a number of scaffolds to support their comprehension of the language used to present and explain strategies related to adding and subtracting fractions. Because many of the words (would, since) and phrases (same-sized parts, started with, in all, spend [time]), and sentence structures (To ___, do ___ so that ___, When ___, you can ___, You can use ___ to ____.) in this unit could prove challenging to ELs, they are supported in understanding and using them so that the instruction is more accessible.

- Lesson 9-1 – *would*
- Lesson 9-2 – *same-sized part*
- Lesson 9-3 – *To___, do ___ so that___.*
- Lesson 9-4 – *When ___, you can ___.*
- Lesson 9-5 – *started with*
- Lesson 9-6 – *in all*
- Lesson 9-7 – *You can use ___ to ___.*
- Lesson 9-8 – *since*
- Lesson 9-9– *spend [time]*

Unit Routines

Number Routines

Build Fluency The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the thinking habits of mind that are important for proficient doers of math.

What's Another Way to Write It?

Purpose: Build flexibility with number sense and mental math operations.

Overview: Given a number, students generate expressions using operations that, when evaluated, have the same value as the number. The teacher records expressions as students share. Students then look for relationships amongst the expressions.

Which Benchmark Is It Closest To?

Purpose: Enhance rounding and reasoning skills.

Overview: Students determine to which benchmark the given number is closest and explain their reasoning.

Would You Rather?

Purpose: Build flexibility with number sense and mental math operations; enhance decision-making.

Overview: Students choose between two options, both of which require mental computation. Students explain their choice and their rationale for their choice.

Sense-Making Routines

- **Which doesn't belong?** (Lessons 9-1, 9-3) In Lesson 9-1, students compare and contrast fractions to determine which does not belong. In Lesson 9-3, students identify which number in the set does not belong.
- **Notice and Wonder: How are they the same? How are they different?** (Lesson 9-2) Students are presented with sets of fraction tiles that each represent the same part of a whole. They describe how the sets are the same and how they are different.
- **Is it always true?** (Lesson 9-4) Students think about whether the relationship holds true in all cases or if it is a unique relationship.
- **Notice and Wonder: What do you notice? What do you wonder?** (Lessons 9-5, 9-8) In Lesson 9-5, students make observations about rows of packages of party favors. Each package has 5 items in it. In Lesson 9-8, students explore the possibility of regrouping using one whole.
- **Notice and Wonder: What questions can you ask?** (Lessons 9-6, 9-9) In Lesson 9-6, students start thinking about wholes first, and then counting part of the whole. In Lesson 9-9, students consider what questions could be asked about a weekly planner.
- **Notice and Wonder: What question could it be?** (Lesson 9-7) Students look at two representations of mixed numbers and determine what question could be answered using them.

Math Language Routines

The Mathematical Language Routines used in this unit give teachers a structured, yet adaptable format for amplifying and developing students' social and academic language. These routines can also be used as formative assessment opportunities as students develop proficiency in English and mathematical language. They can be used in ways that support real-time-, peer-, and self-assessment. For more information on the Math Language Routines, see the Appendix.

- Lesson 9-1: To support conversation, students participate in MLR8: Discussion Supports.
- Lessons 9-2, 9-8: To optimize output, students participate in MLR1: Stronger and Clearer Each Time.
- Lessons 9-3, 9-7: To optimize output, students participate in MLR7: Compare and Connect.
- Lesson 9-4: To support sense-making, students participate in MLR2: Collect and Display.
- Lesson 9-5: In order to support sense-making, students participate in MLR8: Discussion Supports.
- Lesson 9-6: To maximize linguistic and cognitive meta-awareness, students participate in MLR5: Co-Craft Questions and Problems.
- Lesson 9-9: In order to support sense-making, students participate in MLR6: Three Reads.

Readiness Diagnostic

Unit 9

How Ready Am I?

Name ______________________

1. Which fraction is equivalent to $\frac{2}{3}$?

   A. $\frac{8}{12}$ B. $\frac{3}{4}$
   C. $\frac{7}{8}$ D. $\frac{3}{2}$

2. Which sentence is true?

   A. $\frac{3}{6} < \frac{3}{8}$ B. $\frac{5}{8} < \frac{5}{6}$
   C. $\frac{3}{4} < \frac{1}{4}$ D. $\frac{6}{8} < \frac{3}{8}$

3. Which pair of fractions are both less than $\frac{1}{2}$?

   A. $\frac{1}{3}$ and $\frac{2}{3}$ B. $\frac{3}{8}$ and $\frac{3}{4}$
   C. $\frac{1}{6}$ and $\frac{2}{6}$ D. $\frac{1}{4}$ and $\frac{2}{4}$

4. What is the sum?

   $2\frac{3}{5} + 3\frac{1}{5} = ?$

   A. $1\frac{2}{5}$ B. $5\frac{4}{10}$
   C. $5\frac{3}{5}$ D. $5\frac{4}{5}$

5. Which is equivalent to $\frac{3}{5}$?

   A. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$
   B. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$
   C. $2 + 1 + \frac{1}{5}$
   D. $\frac{1}{2} + \frac{2}{3}$

6. Which is the sum $\frac{3}{12} + \frac{4}{12}$?

   A. $\frac{1}{12}$ B. $\frac{7}{24}$
   C. $\frac{7}{12}$ D. $\frac{12}{12}$

7. What is the difference?

   $\frac{7}{5} - \frac{2}{5} = ?$

   A. $\frac{3}{10}$ B. $\frac{5}{10}$
   C. $\frac{9}{15}$ D. $\frac{5}{5}$

8. Which is $\frac{4}{10} + \frac{3}{100}$?

   A. $\frac{43}{1,000}$ B. $\frac{7}{110}$
   C. $\frac{7}{100}$ D. $\frac{43}{100}$

9. Joe jumped $8\frac{3}{4}$ feet. Jill jumped $6\frac{1}{2}$ feet. How much farther did Joe jump than Jill?

   A. $1\frac{3}{4}$ feet B. $2\frac{1}{4}$ feet
   C. $2\frac{1}{2}$ feet D. $2\frac{2}{2}$ feet

10. Ali plants a flower that is $3\frac{3}{10}$ inches tall. After two weeks, the plant has grown $1\frac{2}{5}$ inches. How tall is the plant now?

    A. $2\frac{1}{10}$ inches B. $4\frac{5}{15}$ inches
    C. $4\frac{1}{2}$ inches D. $4\frac{7}{10}$ inches

Administer the Readiness Diagnostic to determine your students' readiness for this unit.

Targeted Intervention

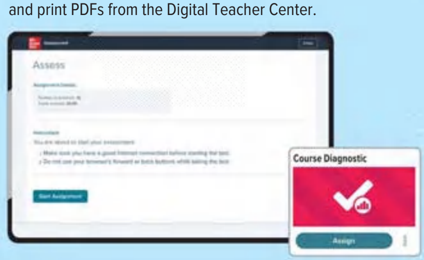
Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	1	Identify equivalent fractions	Equivalent Fractions	4.NF.A.1
2	2	Compare fractions	Compare Using Benchmark One Half	4.NF.A.2
3	2	Compare fractions	Compare Using Benchmark One Half	4.NF.A.2
4	2	Add mixed numbers	Add Mixed Numbers	4.NF.B.3.c
5	2	Decomposing fractions	Build Fractions from Unit Fractions	4.NF.B.3.b
6	2	Add fractions	Add Like Fractions	4.NF.B.3.a
7	2	Subtract fractions	Subtract Like Fractions	4.NF.B.3.a
8	2	Add fractions with denominators 10 and 100	Add Fractions in 10ths and 100ths	4.NF.C.5.a
9	1	Subtract mixed number in a word problem	Subtract Mixed Numbers	4.NF.B.3.c
10	2	Add mixed number in a word problem	Add Mixed Numbers	4.NF.B.3.c

GO ONLINE

Assign the digital Readiness Diagnostic to students or download and print PDFs from the Digital Teacher Center.



Unit Opener

Focus Question

Introduce the Focus Question: *How can I add and subtract fractions?*

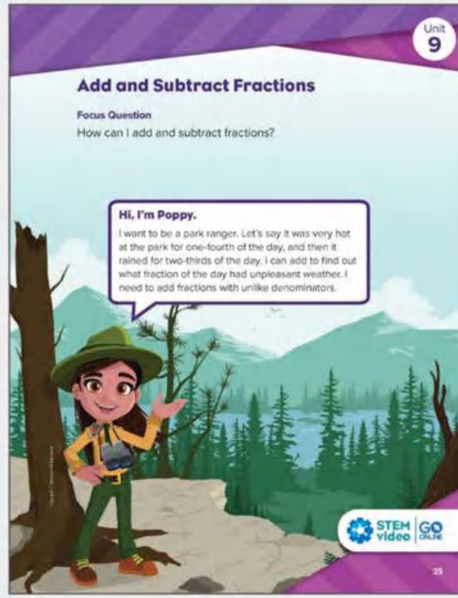
Ask students to think about what they know about adding and subtracting fractions.

- What do you already know about adding fractions?
- What do you already know about subtracting fractions?

Remind students that at the end of the unit, they will reflect back on what they learned.

Family Letter

Each letter presents an overview of the math in the unit and home activities to support student learning.



STEM in Action

Videos

Students can watch the two STEM videos.

STEM Career: Park Ranger Students watch the animation about Poppy and her aspirations to be a park ranger.

Poppy Adds Fractions Students see how Poppy uses addition of fractions to find the part of the day that has unpleasant weather.

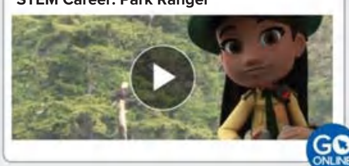
STEM Project Card

Students can complete the STEM Project Card during their workstation time.

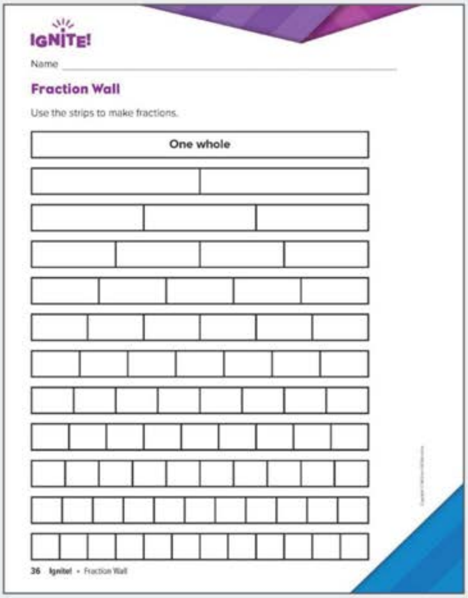
STEM Adventure

Students can complete the STEM Adventure during their workstation time.

STEM Career: Park Ranger



Poppy Adds Fractions



Ignite!

Fraction Wall

Students use fraction strips to explore equivalent fractions. They use the strips to informally discover the need to consider parts of the same size before adding fractions.

Materials: color pencils

1. Direct students to the student page.
   - What do you notice about the strips on the page?
2. Have students explore fractions equivalent to $\frac{1}{2}$.
   - How can you represent $\frac{1}{2}$ using some of the other strips?
3. Have students explore fractions equivalent to $\frac{1}{3}$.
   - How can you represent $\frac{1}{3}$ using some of the other strips?
4. Have students work in pairs to explore how to make a sum of $\frac{1}{2}$ by using two parts of different sizes. Allow students time to explore and discover.
   - Find as many ways as you can to make a sum of $\frac{1}{2}$ using two parts of different sizes. Shade the strips to make sure that they show $\frac{1}{2}$.
5. Help students notice that they have just added fractions with different denominators to make $\frac{1}{2}$.

Workstations

Reveal Math offers rich and varied resources that teachers can use to differentiate and enrich students' instructional experiences with the unit content. The table presents an overview of the resources available for the unit with recommendations for when to use.

Station	Activity	Description	Use After Lesson
Game Station	Game Station	Students build proficiency with adding and subtracting fractions and mixed numbers.	
		• **Estimating Sums and Differences Race**	9-1
		• **Adding Fractions Task Cards**	9-2
		• **Fraction Addition Concentration**	9-3
		• **Subtracting Fractions Task Cards**	9-4
		• **Fraction Subtraction Tic Tac Toe**	9-5
		• **Mixed Number Addition Concentration**	9-6
		• **Fraction and Mixed Number Addition Race**	9-7
		• **Mixed Number Addition Concentration**	9-8
		• **Subtracting Mixed Numbers Task Cards**	9-9
Digital Station	Digital Game	**Submarine Plunge** Students practice dividing multi-digit numbers.	9-1
Application Station	**Have students complete at least one of the Use It! activities for this unit.**		
	STEM Project Card	**Get Moving** Students design a car and measure the distance it travels.	9-8
	Connection Card	**How Do You Say—Fractions?** Students research to find how to say words related to adding and subtracting fractions in other languages.	9-4
	Real World Card	**Create and Solve** Students create a multi-step problem that adds and subtracts mixed numbers to solve. Then they use a digital tool to present the problem.	9-9

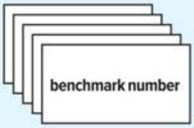
Additional Resources

Use the resources below to provide additional support for this unit.

GO ONLINE

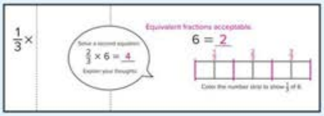
Vocabulary

Use the vocabulary cards to help students learn the vocabulary in this unit. Encourage students to write their own definitions of the key terms on the front side of the card.



Foldables

Use the unit foldables with Lessons 9-2 through 9-4.



Spiral Review Practice

Students can complete the Spiral Review Practice at any point during the unit as either a paper-and-pencil or digital activity.

Lesson	Standard
9-1	5.NBT.B.6
9-2	5.MD.C.5
9-3	5.NBT.A.3
9-4	5.NBT.B.5
9-5	5.NF.A.2
9-6	5.NF.A.1
9-7	5.NBT.A.4
9-8	5.NBT.B.6
9-9	5.NF.A.2

LESSON 9-1

Estimate Sums and Differences of Fractions

Learning Targets

- I can use benchmark numbers to estimate the sums and differences of fractions.
- I can explain how to use an estimate to predict a calculated solution.
- I can explain how to use an estimate to check the reasonableness of a calculated solution.

Standards

◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Math Practices and Processes

MPP Construct viable arguments and critique the reasoning of others.

Vocabulary

Math Terms	Academic Terms
benchmark number	eliminate
estimate	suggest

Materials

The materials may be for any part of the lesson.

- fraction circles
- fraction tiles
- number cubes
- *Benchmark Fraction Number Line* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use benchmark numbers to estimate sums and differences of fractions. • Students explain how to use an estimate to predict or check the reasonableness of a calculated sum or difference of fractions.	• Students talk about benchmark numbers to estimate the sums and differences of fractions using *greater than* and *less than*. • To support cultivating conversation, ELs participate in MLR8: Discussion Supports.	• Students determine how they can break a problem down to make it easier to solve.

Coherence

Previous	Now	Next
• Students compared two fractions having different numerators and different denominators (Grade 4). • Students added and subtracted decimals (Unit 4).	• Students use benchmark fractions to estimate the sums and differences of fractions and assess the reasonableness of calculated solutions.	• Students use representations to understand addition of fractions having unlike denominators (Unit 9).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding of estimation as they estimate sums and differences of fractions and determine the reasonableness of proposed answers.	• Students develop proficiency making estimates. *Procedural Skill & Fluency is not a targeted element of rigor for this standard.*	• Students estimate sums and differences of fractions and determine the reasonableness of proposed answers in real-world contexts.

GO ONLINE

Number Routine

Which Benchmark Is It Closest To? 5–7 min

Build Fluency Students build number sense as they determine which benchmark number each given fraction is closest to and explain their reasoning.

Students exchange ideas with each other. After each fraction is discussed, it can be moved to the appropriate benchmark. These prompts encourage students to talk about their reasoning:

- How did you look at each fraction? What did you consider about the numerator? What did you consider about the denominator?
- What did you consider about the sizes of the numerator and denominator compared to each other?

Launch 5–7 min

Sense-Making Routine

Purpose Students compare and contrast fractions to identify which doesn't belong.

Which Doesn't Belong?

- Which doesn't belong?

Teaching Tip You may want to suggest that students write the fractions that are not already simplified in their simplest form before discussing them as a class.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of using benchmark fractions to estimate the sums and differences of fractions and are based on possible comments and questions that students may make during the share out.

- How can you determine if any of the fractions are equivalent to one another?
- How can you determine which fractions are close in value to one another?

Math is... Mindset

- What are your strengths in math?

SEL **Self-Awareness – Recognize Strengths**

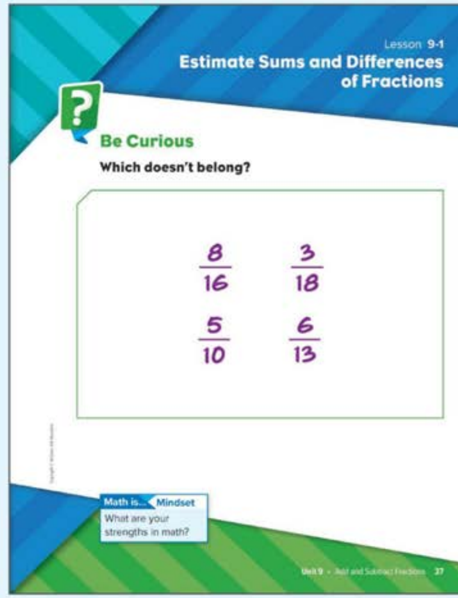
As students determine which fraction doesn't belong, encourage them to recognize their own strengths, so they can see themselves as resourceful and may be more willing to attempt to problem solve and help others. Invite students to share their reasoning for how they identified their solutions and remind them that some questions can have more than one correct answer. As students work with estimating the sums and differences of fractions throughout the lesson, encourage them to work with other students to capitalize on the strengths of each group member.

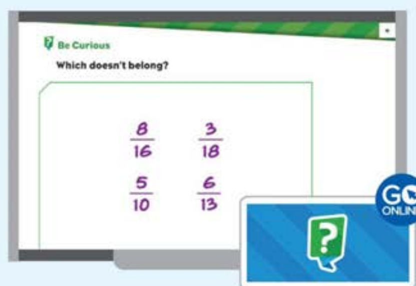
Transition to Explore & Develop

Ask questions that get students thinking about benchmark fractions and how certain fractions are easier to use when estimating sums and differences of fractions.

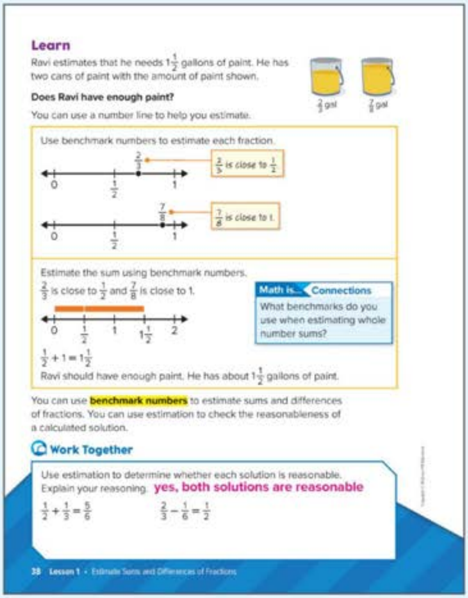
ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about how we can estimate the sums and differences of fractions using benchmark fractions.





Explore & Develop 20 min



Learn

Ravi estimates that he needs $1\frac{1}{2}$ gallons of paint. He has two cans of paint with the amount of paint shown.

$\frac{2}{3}$ gal $\frac{7}{8}$ gal

Does Ravi have enough paint?

You can use a number line to help you estimate.

Use benchmark numbers to estimate each fraction.

$\frac{2}{3}$ is close to $\frac{1}{2}$.

$\frac{7}{8}$ is close to 1.

Estimate the sum using benchmark numbers.

$\frac{2}{3}$ is close to $\frac{1}{2}$ and $\frac{7}{8}$ is close to 1.

$\frac{1}{2} + 1 = 1\frac{1}{2}$

Ravi should have enough paint. He has about $1\frac{1}{2}$ gallons of paint.

Math is... Connections
What benchmarks do you use when estimating whole number sums?

You can use **benchmark numbers** to estimate sums and differences of fractions. You can use estimation to check the reasonableness of a calculated solution.

Work Together

Use estimation to determine whether each solution is reasonable. Explain your reasoning. yes, both solutions are reasonable

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$

38 Lesson 1 • Estimate Sums and Differences of Fractions

❶ Pose the Problem

MLR Discussion Supports

As students talk about what they'll be trying to do to solve the problem, have them pay attention to others' understandings in order to increase their ability to estimate sums and differences of fractions. Restate statements they make as a question to seek clarification and provide vocabulary or grammar prompts for students who need more guidance.

ETP Pose Purposeful Questions

- Is an estimate sufficient to answer the question, or is an exact answer needed? How do you know?

❷ Develop the Math

Choose the option that best meets your instructional goals.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you use benchmark fractions to help you estimate the sums and differences of fractions?
- How can an estimate help you determine if a sum or difference is reasonable?

Key Takeaway

- Benchmark numbers can be used to estimate sums and differences of fractions.

Work Together

Students check the reasonableness of a calculated sum and a calculated difference by using estimates. Make sure students are able to justify their conclusions.

Common Error: Students may think that the answers are not reasonable based on the differing denominators. Remind students that fractions can be equivalent with different denominators.

LOM Language of Math

Explain to students that a *benchmark* is a point of reference against which to compare or assess something. Builders may use a benchmark on a wall or pillar to measure all other distances from.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using benchmark fractions to estimate the sum of two fractions.

Materials: *Benchmark Fraction Number Line* Teaching Resource

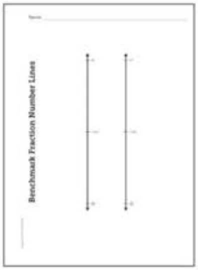
Directions: Provide copies of the *Benchmark Fraction Number Line* Teaching Resource. Have students solve the Pose the Problem.

ETP Support Productive Struggle

- How can you use the number line to help you estimate?
- How can you determine what benchmark number $\frac{1}{4}$ is close to on the number line?
- How can you determine what benchmark number $\frac{2}{3}$ is close to on the number line?
- How can you determine what benchmark number $\frac{7}{8}$ is close to on the number line?
- How can knowing which benchmark numbers the fractions are close to help you find an estimate?
- How can you use the number line to determine if the sum of the fractions will be less than or greater than 1?

Activity Debrief: Have groups of students share their work with the class. Encourage students to discuss how they used the number lines and benchmark numbers to determine their estimates.

A PDF of the Teaching Resource is available in the Digital Teacher Center.



Guided Exploration

Students use benchmark fractions on a number line to estimate a sum of fractions to solve a problem.

ETP Facilitate Meaningful Mathematical Discourse

- **Think About It:** What are some strategies you know for estimating sums?

Have the students use number sense while estimating the sum of $\frac{2}{3}$ and $\frac{1}{4}$. Ask:

- How much less than 1 is $\frac{2}{3}$? Why?
- Is $\frac{1}{4}$ lesser than or greater than $\frac{1}{3}$? Why?
- Is the sum of $\frac{1}{4}$ and $\frac{2}{3}$ less than or greater than 1? Why?

Have the students use number sense while estimating the sum of $\frac{1}{4}$ and $\frac{7}{8}$. Ask:

- Is $\frac{7}{8}$ lesser than or greater than 1? Why?
- Is $\frac{1}{4}$ lesser than or greater than $\frac{1}{2}$? Why?
- Is the sum of $\frac{1}{4}$ and $\frac{7}{8}$ lesser than or greater than $1\frac{1}{2}$? Why?

Have the students use number sense while estimating the sum of $\frac{2}{3}$ and $\frac{7}{8}$. Ask:

- How much less than $1\frac{1}{2}$ (or $\frac{12}{8}$) is $\frac{7}{8}$? Why?
- Is $\frac{2}{3}$ greater than $\frac{5}{8}$? How do you know?
- Is the sum of $\frac{2}{3}$ and $\frac{7}{8}$ less than or greater than $1\frac{1}{2}$? Why?

Math is... Explaining

- How do you know you have checked all the pairs of cans?

Students justify their conclusions, communicate them to others, and respond to the arguments of others.

EL English Learner Scaffolds

Entering/Emerging

Explain *which [two]*. Put four containers on the desk, two with counters, and two without. Say, *Let's see which two [cups] have counters*. Check each container and indicate which two have counters. Repeat, putting different items in various numbers of containers and using *which [three]* as above. Repeat again, grouping the [four] with items on one side and the [four] without on the other. Ask, *Which [four] [cups] have [crayons]?* Allow pointing.

Developing/Expanding

Explain *which [two]*. Put four containers on the desk, two with counters, and two without. Say, *Let's see which two [cups] have counters*. Check each container and indicate which two have counters. Repeat, putting different items in various numbers of containers and using *which [three]* as above. Repeat again, grouping the [four] with items on one side and the [four] without on the other. Ask, *Which [four] [cups] have [crayons]?*

Bridging/Reaching

Instruct students to read the word problem at the top of the Learn page, focusing on *which two*. Discuss with students what information a question with *which* is asking for (one or more things from a set). Discuss with students other question words and *when* we use them (e.g., *when for time, where* for place, etc.).

Practice & Reflect 10 min

On My Own

Name ____________________

Will the sum be *greater than 1* or *less than 1*? Use the number line and explain how you can use benchmark numbers to justify.

1. $\frac{3}{4} + \frac{2}{3}$

   0 — $\frac{1}{2}$ — 1

   greater than 1; Sample answer: $\frac{3}{4}$ and $\frac{2}{3}$ are both close to 1

2. $\frac{3}{5} + \frac{1}{4}$

   0 — $\frac{1}{2}$ — 1

   less than 1; Sample answer: $\frac{3}{5}$ is close to $\frac{1}{2}$ and $\frac{1}{4}$ is close to 0

3. $\frac{1}{3} + \frac{5}{8}$

   0 — $\frac{1}{2}$ — 1

   less than 1; Sample answer: $\frac{5}{8}$ is close to $\frac{1}{2}$ and $\frac{1}{3}$ is close to 0

4. $\frac{3}{10} + \frac{4}{5}$

   0 — $\frac{1}{2}$ — 1

   greater than 1; Sample answer: $\frac{3}{10}$ is close to $\frac{1}{2}$ and $\frac{4}{5}$ is close to 1

Is the sum or difference reasonable? Use estimation to check.

5. $\frac{1}{4} + \frac{5}{6} = \frac{3}{4}$

   Sample answer: no, the sum should be greater than 1

6. $\frac{2}{5} + \frac{1}{2} = \frac{9}{10}$

   Sample answer: yes, the sum should be close to 1 but less than 1

7. $\frac{3}{4} - \frac{3}{8} = \frac{2}{3}$

   Sample answer: no, the difference should be less than $\frac{1}{2}$

8. $\frac{7}{10} - \frac{2}{5} = \frac{1}{2}$

   Sample answer: yes, the difference should be close to $\frac{1}{2}$

9. Dan waters his plants with $\frac{2}{3}$ cup of water on Monday and $\frac{2}{3}$ cup of water on Friday. Does Dan use more than 1 cup of water in all? Explain why or why not.

   Yes, $\frac{2}{3}$ is greater than $\frac{1}{2}$, so the sum would be greater than 1.

10. There is $\frac{7}{8}$ gallon of milk in Zelda's refrigerator. Zelda and her brother drink $\frac{1}{3}$ gallon of milk. About how much milk is left? Explain your answer.

    About $\frac{1}{2}$ gallon of milk is left; $\frac{7}{8}$ is close to 1 but not quite and $\frac{1}{3}$ is close to $\frac{1}{2}$.

11. **STEM Connection** Poppy is helping clean up a park. Her group is cleaning up $\frac{2}{5}$ of the park. Another group is cleaning up $\frac{1}{4}$ of the park. About how much of the park should a third group clean up so that they cover the entire park?

    Sample answer: about $\frac{1}{4}$ of the park

12. **Extend Your Thinking** How can you apply estimating the sums and differences of fractions that are less than 1 to fractions that are greater than 1?

    Sample answer: Use benchmark fractions and whole numbers that are greater than 1 such as $1\frac{1}{2}$, 2, etc.

Reflect

Why is estimating the sums and differences of fractions useful?

Answers may vary.

Math is... Mindset

How did you use your strengths in math today?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Misconception: Exercises 5–8 Make sure students are estimating to check the reasonableness of the solutions rather than trying to calculate the solutions themselves.

Item Analysis

Item	DOK	Rigor
1–4	1	Conceptual Understanding
5–8	2	Procedural Skill & Fluency
9–11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- Why is estimating the sums and differences of fractions useful?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you use your strengths in math today?

Students reflect on how they practiced self-awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can use benchmark numbers to estimate the sums and differences of fractions.
- I can explain how to use an estimate to predict a calculated solution.
- I can explain how to use an estimate to check the reasonableness of a calculated solution.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Estimate sums of fractions	5.NF.A.2
2	1	Estimate differences of fractions	5.NF.A.2
3	2	Estimate sums of fractions	5.NF.A.2
4	2	Estimate differences of fractions	5.NF.A.2

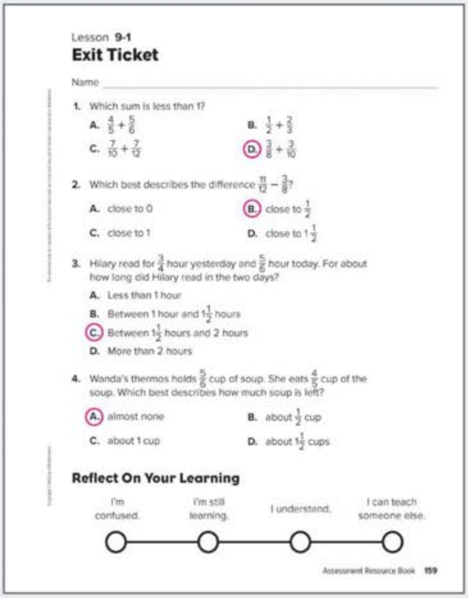
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 9-1

Exit Ticket

Name ____________________

1. Which sum is less than 1?

   A. $\frac{4}{5} + \frac{5}{6}$ B. $\frac{1}{2} + \frac{2}{3}$

   C. $\frac{7}{10} + \frac{7}{12}$ D. $\frac{3}{8} + \frac{3}{10}$

2. Which best describes the difference $\frac{11}{12} - \frac{3}{8}$?

   A. close to 0 B. close to $\frac{1}{2}$

   C. close to 1 D. close to $1\frac{1}{2}$

3. Hilary read for $\frac{3}{4}$ hour yesterday and $\frac{5}{6}$ hour today. For about how long did Hilary read in the two days?

   A. Less than 1 hour

   B. Between 1 hour and $1\frac{1}{2}$ hours

   C. Between $1\frac{1}{2}$ hours and 2 hours

   D. More than 2 hours

4. Wanda's thermos holds $\frac{5}{6}$ cup of soup. She eats $\frac{4}{5}$ cup of the soup. Which best describes how much soup is left?

   A. almost none B. about $\frac{1}{2}$ cup

   C. about 1 cup D. about $1\frac{1}{2}$ cups

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 159

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Benchmark Numbers

Work with students in pairs. Each student rolls two number cubes and makes a fraction with the lesser number as the numerator. Students find the benchmark number (0, $\frac{1}{2}$, or 1) closest to each fraction and add their benchmark numbers to estimate the sum of the fractions. If students have difficulty identifying the benchmark, help them to use fraction circles or fraction tiles to represent the fractions and compare them to the benchmarks.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Estimate & Check (Benchmark Fractions)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 89

Lesson 9-1 • Reinforce Understanding

Estimate Sums and Differences of Fractions

Name ____________________

Review

You can use benchmark fractions to estimate sums and differences of fractions.

Is the Sum Greater than 1?	Estimate the Sum
Consider $\frac{8}{9} + \frac{9}{11}$ $\frac{8}{9}$ is greater than $\frac{1}{2}$. $\frac{9}{11}$ is greater than $\frac{1}{2}$. This means $\frac{8}{9} + \frac{9}{11}$ is greater than 1.	Consider $\frac{8}{9} + \frac{9}{11}$ $\frac{8}{9}$ is close to 1. $\frac{9}{11}$ is close to 1. This means $\frac{8}{9} + \frac{9}{11}$ is close to 2.

Will the sum be greater than 1 or less than 1? Use benchmark fractions to justify your answer.

1. $\frac{5}{9} + \frac{4}{5}$
greater than 1; both fractions are greater than $\frac{1}{2}$

2. $\frac{2}{5} + \frac{3}{7}$
less than 1; both fractions are less than $\frac{1}{2}$

Use estimation to determine whether each solution is reasonable.

3. $\frac{1}{2} + \frac{2}{3} = \frac{5}{6}$
no; the sum should be greater than 1

4. $\frac{5}{6} - \frac{1}{3} = \frac{1}{2}$
yes; the difference should be close to $\frac{1}{2}$

5. $\frac{3}{4} + \frac{1}{6} = \frac{11}{12}$
yes; the sum should be close to 1 but less than 1

6. $\frac{3}{5} - \frac{3}{10} = \frac{1}{2}$
no; the difference should be less than $\frac{1}{2}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Estimating Sums and Differences Race

Students practice estimating sums and differences of fractions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 89–90

Lesson 9-1

Additional Practice

Name ____________________

Review

You can use the benchmarks 0, $\frac{1}{2}$, and 1 to estimate sums and differences of fractional amounts.

Archie uses $\frac{1}{4}$ foot of blue-striped ribbon and $\frac{4}{5}$ foot of solid blue ribbon. He says that he uses $\frac{5}{9}$ foot of ribbon in all. Is Archie's estimate reasonable?

$\frac{1}{4} + \frac{4}{5}$

$\frac{1}{4}$ is less than $\frac{1}{2}$.

$\frac{4}{5}$ is more than $\frac{1}{2}$.

The sum should be close to 1. Archie's sum of $\frac{5}{9}$ is a little more than $\frac{1}{2}$, so his estimate is not reasonable.

Match the sum or difference with its best estimate.

1. $\frac{1}{3} + \frac{7}{8}$ — close to 0
2. $\frac{9}{10} + \frac{5}{6}$ — close to $\frac{1}{2}$
3. $\frac{8}{9} - \frac{2}{3}$ — close to 1
4. $\frac{6}{7} - \frac{1}{4}$ — close to 2

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Assign

Use It! Application Station

How Do You Say—Fractions? Students research to find how to say words related to adding and subtracting fractions in other languages.

The content of this card has concepts covered later in Lesson 9-4. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 89–90

Use estimation with benchmark fractions to answer each question.

5. Kelli swims $\frac{2}{3}$ mile in the morning and $\frac{1}{6}$ mile in the evening. About how many mile(s) does Kelli swim in one day? Explain.

   Sample answer: About $\frac{1}{2}$ mile; $\frac{2}{3}$ is close to $\frac{1}{2}$, and $\frac{1}{6}$ is close to 0.

6. Garry practices playing piano for $\frac{3}{5}$ hour on Wednesday and $\frac{1}{2}$ hour on Thursday. He says that he practices for $\frac{1}{2}$ hour more on Wednesday than on Thursday. Is he correct? Explain.

   No; Sample answer: $\frac{3}{5}$ is close to $\frac{1}{2}$. Since one fraction is $\frac{1}{2}$ and the other is close to $\frac{1}{2}$, the difference should be close to.

7. Norma mixes $\frac{11}{12}$ gallon of water and $\frac{4}{5}$ gallon of lemon juice to make a beverage. She needs 2 gallons of the beverage to serve at a picnic. Does Norma have enough of the beverage? Explain.

   No; Sample answer: $\frac{11}{12}$ is close to 1 and $\frac{4}{5}$ is close to 1, but since both are less than 1, the sum is close to, but less than, 2. So she does not have enough.

8. Wilbert's cooler holds $\frac{5}{6}$ pound of ice. He fills the cooler with $\frac{1}{5}$ pound of ice and says that the cooler is almost full. Is he correct? Explain.

   No; Sample answer: $\frac{5}{6}$ is closer to 1 and $\frac{1}{5}$ is closer to 0, so $\frac{1}{5}$ pound of ice is not almost $\frac{5}{6}$ pound of ice.

Math @ Home Activity Fractions are commonly used in recipes. While preparing a recipe, ask your child to estimate the values used and then find the difference between two different fractional amounts of ingredients used. Then have them estimate the sum of two fractional ingredients used. Allow your child to use different measuring cups to show his or her work.

Student Practice Book

Differentiation Resource Book, p. 90

Lesson 9-1 • Extend Thinking

Estimate Sums and Differences of Fractions

Name ______________________

Match the estimates in Column A to the actual sum or difference in Column B.

Column A	Column B
A fraction that is slightly less than $\frac{1}{2}$ is added to a fraction that is slightly less than $\frac{1}{2}$.	$1\frac{4}{5}+\frac{2}{3}$
A fraction that is slightly less than 1 is added to a fraction that is slightly less than 1.	$\frac{4}{5}+\frac{3}{4}$
A fraction that is slightly less than $\frac{1}{2}$ is subtracted from a fraction that is slightly less than $\frac{1}{2}$.	$1\frac{5}{13}-\frac{4}{7}$
A fraction that is slightly less than 2 is added to a fraction that is slightly less than 1.	$\frac{3}{7}-\frac{2}{9}$
A fraction that is slightly more than $\frac{1}{2}$ is subtracted from a fraction that is slightly less than $1\frac{1}{2}$.	$\frac{2}{5}+\frac{1}{3}$
A fraction that is slightly more than $1\frac{1}{2}$ is added to a fraction that is slightly more than $1\frac{1}{2}$.	$2\frac{1}{9}-\frac{10}{11}$
A fraction that is slightly less than 1 is subtracted from a fraction that is slightly more than 2.	$1\frac{3}{5}+1\frac{7}{12}$

Differentiation Resource Book

Unit 9

Make an Estimate of the Sum

Name

Without actually calculating, use what you know about fractions to estimate the sum.

1. $\frac{1}{7} + \frac{1}{9}$

Circle the best estimate.

a. $\frac{1}{8}$
b. $\frac{1}{4}$
c. $\frac{1}{2}$
d. 2
e. 16

Explain your choice.
Explanations may vary.

2. $\frac{5}{6} + \frac{13}{14}$

Circle the best estimate.

a. $\frac{1}{2}$
b. 1
c. 2
d. 18
e. 20

Explain your choice.
Explanations may vary.

Unit 9 • Add and Subtract Fractions 41

Without actually calculating, use what you know about fractions to estimate the sum.

3. $\frac{4}{7} + \frac{6}{11}$

Circle the best estimate.

a. $\frac{1}{2}$
b. 1
c. 2
d. 10
e. 18

Explain your choice.
Explanations may vary.

4. $\frac{9}{20} + \frac{1}{4}$

Circle the best estimate.

a. $\frac{1}{2}$
b. $\frac{3}{4}$
c. 1
d. 10
e. 24

Explain your choice.
Explanations may vary.

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

42 Math Probe • Make an Estimate of the Sum

Analyze The Probe

Formative Assessment

Targeted Concept Use strategies to reason about the magnitude of fractions and the impact of combining two fractions by addition.

Targeted Misconceptions Some students apply ideas involving whole number addition by adding the numerators and/or the denominators as if they were whole numbers. Some students are unable reason about the magnitude of fractions in comparison to common benchmark fractions as part of a process to find an estimated sum.

Authentic Student Work

Below are examples of students' explanations.

Sample A

1. $\frac{1}{7} + \frac{1}{9}$

Circle the best estimate.

a. $\frac{1}{8}$
b. $\frac{1}{4}$
c. $\frac{1}{2}$
d. 2
e. 16

Explain your choice.

$\frac{1}{7} \rightarrow \frac{1}{8}$ about
$\frac{1}{9} \rightarrow \frac{1}{8}$ about
$\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

Sample B

3. $\frac{4}{7} + \frac{6}{11}$

Circle the best estimate.

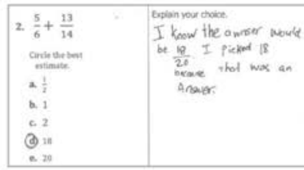
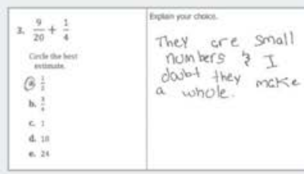
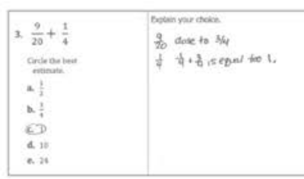
a. $\frac{1}{2}$
b. 1
c. 2
d. 10
e. 18

Explain your choice.

I chose this because they are both near 1/2.
1/2 + 1/2 = 1

Collect and Assess Student Work

Collect and review student responses to determine possible misconceptions. See examples in If-Then chart.

IF incorrect...	THEN the student likely...	Sample Misconceptions
1. d 2. d 3. d 4. d	adds the two numerators and uses that result to be the estimated sum of the two fractions.	In this case, the student adds the numerators and adds the denominators but then focuses on the numerator. 
1. a 2. b 3. a 4. a	finds the sum by adding the numerators and adding the denominators. The student then simplifies the sum (in Exercise 1) and/or chooses the nearest benchmark to determine the estimate (in Exercises 2–4).	In this case, the student adds the numerators and adds the denominators but then focuses on the closest benchmark. 
2. a 3. a	thinks that the estimated sum of two fractions must be a fraction and cannot be a whole number.	In this case, the student doesn't consider the sizes of the fractions when determining the sum. 

Many of the above difficulties result in a combination of correct and incorrect responses.
For correct responses, be sure to check for sound reasoning.

Take Action

Choose from the following resources or suggestions:

- Revisit estimation in Lesson 9-1.
- Review that a fraction is a number with (1) a precise magnitude that can be shown using an area model and (2) a precise location on the number line. With an area model, the denominator relates to the size of pieces; the numerator relates to the number of selected pieces. With number lines, the interval length relates to the denominator; the number of jumps relates to the numerator.
- Use concrete materials to build visual images to help students compare fractions to common benchmarks. Have students select the one that best represents the fraction.
- Have students estimate sums and discuss how they determine a good estimate.

Revisit the Probe After additional instruction, have students review their initial answers to the probe. Use these questions for discussion:

- Are there any answers you would like to change? Explain.
- Are there any questions that you still have about any of the items on this probe?

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

LESSON **9-2**

Represent Addition of Fractions with Unlike Denominators

Learning Targets

- I can use a representation to add fractions with unlike denominators.
- I can explain how to use a representation to add fractions with unlike denominators.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. $\left(\text{In general, } \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}\right)$.

Math Practices and Processes

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	**Academic Terms**
denominator	correspond
equivalent fractions	suggest
fraction tiles	
like denominators	
numerator	

Materials

The materials may be for any part of the lesson.

- *Blank Open Number Line* Teaching Resource
- fraction tiles
- ruler

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use a representation to add fractions with unlike denominators. • Students explain how to use a representation to add fractions with unlike denominators.	• Students explain how to use a representation to add fractions with unlike denominators using *can*. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students exchange ideas for mathematical problem-solving with a peer and provide thoughtful and constructive feedback.

Coherence

Previous	Now	Next
• Students understood, recognized, and generated equivalent fractions (Grade 4). • Students used benchmark fractions to estimate the sums and differences of fractions and assess the reasonableness of calculated solutions (Unit 9).	• Students use representations to understand addition of fractions having unlike denominators.	• Students use equivalent fractions to add fractions having unlike denominators (Unit 9). • Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students develop their understanding of adding fractions with unlike denominators.	• Students build proficiency with equivalent fractions as they represent addition of fractions with unlike denominators.	• Students explore addition of fractions in real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Which Benchmark Is It Closest To? 5–7 min

Build Fluency Students build numer sense as they determine which benchmark number each given fraction is closest to and explain their reasoning.

Remind students that this is a mental activity and they should not need to write anything down.

These prompts encourage students to talk about their reasoning:

- How could you tell if the fraction was between 0 and 1, between 1 and 2, greater than 2, or equal to one of the benchmarks?
- How did you check your answers?

Purpose Students are presented with fraction tiles that all represent the same portion of the whole $\left(\frac{7}{12}\right)$ and consider how the images are similar and different. The students share similarities and differences, some of which may be mathematical in nature, others non-mathematical.

Notice & Wonder™

- How are they the same?
- How are they different?

Teaching Tip You may want to have students replicate the representations with fraction tiles on their own so they can explore more thoroughly how the representations are similar and different.

Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' noticing and wondering about representing addition of fractions having unlike denominators and are based on possible comments and questions that students may make during the share out.

- How can you determine the value of the representations?
- How do you know if the representations represent the same values?
- What operation do you think you would use to determine the values? Why?

Math is... Mindset

- How can you show others that you respect their ideas?

Social Awareness: Respect Others

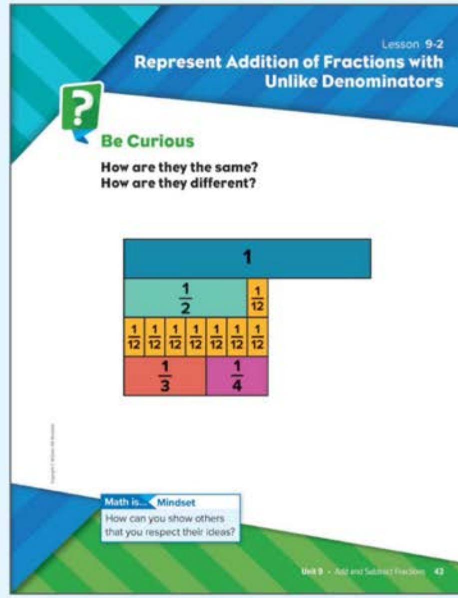
As students work with partners to complete the Notice & Wonder routine, remind them to show respect by listening attentively when others are sharing their ideas. Provide models of constructive and respectful feedback to guide students. As students share what they noticed and wondered, encourage classmates to provide thoughtful feedback to one another. Remind students that respecting others is an important part of being a member of the class community.

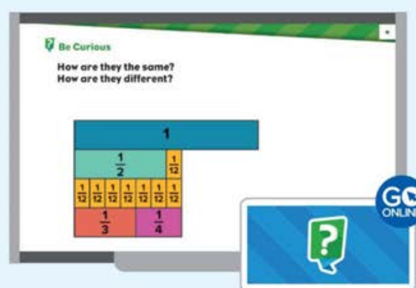
Transition to Explore & Develop

Ask questions that get students thinking about how they can represent addition of fractions having different denominators.

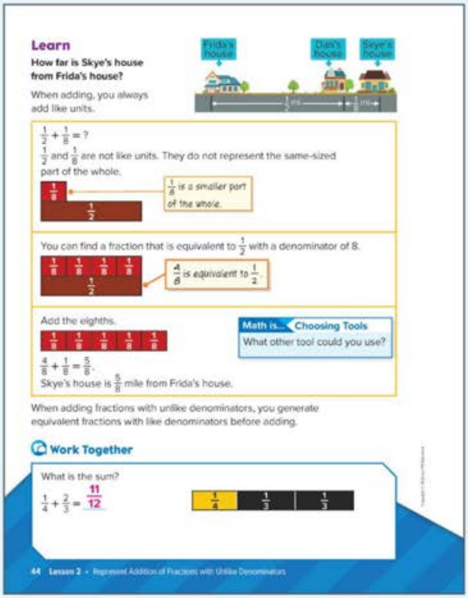
Establish Mathematics Goals to Focus Learning

- Let's think about how we can represent addition of fractions having unlike denominators.





Explore & Develop
20 min


Learn

How far is Skye's house from Frida's house?

When adding, you always add like units.

$\frac{1}{2} + \frac{1}{8} = ?$

$\frac{1}{2}$ and $\frac{1}{8}$ are not like units. They do not represent the same-sized part of the whole.

$\frac{1}{8}$ is a smaller part of the whole.

You can find a fraction that is equivalent to $\frac{1}{2}$ with a denominator of 8.

$\frac{4}{8}$ is equivalent to $\frac{1}{2}$.

Add the eighths.

$\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$.

Skye's house is $\frac{5}{8}$ mile from Frida's house.

Math is... Choosing Tools

What other tool could you use?

When adding fractions with unlike denominators, you generate equivalent fractions with like denominators before adding.

Work Together

What is the sum?

$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$

44 Lesson 2 • Represent Addition of Fractions with Unlike Denominators

❶ Pose the Problem

ETP Pose Purposeful Questions

- When have you seen problems like this before?
- What tools might be helpful in solving the problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about the steps they need to take to solve it. Then have them share their work with their partner and compare. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you add fractions having unlike denominators?
- Why do you have to use like denominators to add fractions having unlike denominators?

Key Takeaways

- When adding fractions with unlike denominators, use equivalent fractions with like denominators before adding.
- You can only add fractions that refer to the same sized part of the whole.

Work Together

Students solve an addition equation involving fractions with unlike denominators using fraction tiles.

Common Error: Students may initially be confused because the equation in Work Together involves changing both fractions instead of just one. Encourage students to think about which numbers 3 and 4 are both factors of.

LOM Language of Math

Tell students that *equivalent* means "equal in value." The word comes from the Latin *aequus* which means "equal" and *valere* which means "to be worthy." You can find the root *aequus* in *equal*, *equity*, and *equation*. You can find the root *valere* in *valiant*, *valor*, and *valid*. Knowing the meaning of common roots can help you determine the definitions of unknown words.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using a representation to add fractions that have unlike denominators.

Materials: *Blank Open Number Line*, ruler, fraction tiles

Directions: Have students work in small groups. Present them with the expression $\frac{3}{4} + \frac{1}{2}$. Invite students to use tools to help them represent and solve the problem.

ETP Implement Tasks That Promote Reasoning and Problem Solving

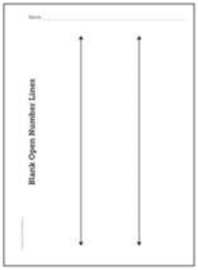
- How does your representation show the meaning of addition?
- How can you use the benchmark number 1 to help you solve this addition?
- How does knowing that $\frac{3}{4}$ can be broken apart into $\frac{1}{2}$ and $\frac{1}{4}$ help you?
- How does knowing that $\frac{1}{2}$ can be broken apart into $\frac{1}{4}$ and $\frac{1}{4}$ help you?

Math is... Choosing Tools

- Why was it helpful to use a tool to solve this problem?

Students make sound decisions about when tools might be helpful, recognizing both the insight to be gained and their limitations.

Activity Debrief: Have students share their representations and solutions. Facilitate a discussion to ensure students understand that fractions can only be added when they have like denominators. If the fractions have unlike denominators, they need to use equivalent fractions with like denominators to add.



Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the distance from Frida's house to Skye's house?

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students use a representation to understand addition of fractions having unlike denominators.

ETP Use and Connect Mathematical Representations

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- What benchmark numbers will you use to estimate the solution? Why?
- Why do you rename $\frac{1}{2}$ instead of $\frac{1}{8}$?
- How can you use the relationship between 2 and 8 to determine an equivalent fraction?
- How can you add once you have renamed $\frac{1}{2}$ as $\frac{4}{8}$?

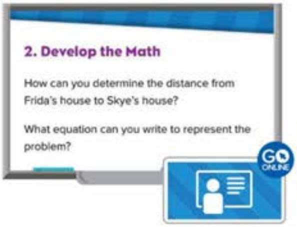
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

Math is... Choosing Tools

- Why was it helpful to use fraction tiles to solve this equation?

Students make sound decisions about when tools might be helpful, recognizing both the insight to be gained and their limitations.



EL English Learner Scaffolds

Entering/Emerging

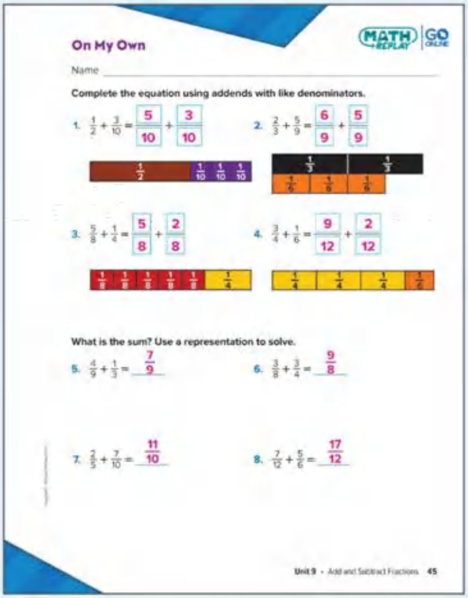
Explain *same-sized part.* Show students a tens rod. Say, *This has ten same-sized parts.* Point to each part to confirm. Then, draw a square. Divide it into three same-sized parts. Say, *This has three same-sized parts.* Finally, draw two more squares. Divide one into four same-sized parts and one into multiple-sized parts. Show each, asking, *Does this show same-sized parts?*

Developing/Expanding

Explain *same-sized part.* Show students a tens rod. Say, *This has ten same-sized parts.* Point to each part to confirm. Then, draw a square. Divide it into three same-sized parts. Say, *This has three same-sized parts.* Finally, draw two more squares. Divide one into four same-sized parts and one into multiple-sized parts. Ask students to tell you which shows same-sized parts.

Bridging/Reaching

Guide students to the Learn page and ask them to review the use of *same-sized part* in the table. Ask students to use the phrase in their own sentence, demonstrating with prompts. For example: *This [tens rod] has ten same-sized parts.* Then ask students to talk about what it means if something is a same-sized part, and how a same-sized part relates to a whole (Pieces equal in size, that together form a whole).



On My Own

Name ______

Complete the equation using addends with like denominators.

1. $\frac{1}{2}+\frac{3}{10}=\frac{5}{10}+\frac{3}{10}$

2. $\frac{2}{3}+\frac{5}{9}=\frac{6}{9}+\frac{5}{9}$

3. $\frac{5}{8}+\frac{1}{4}=\frac{5}{8}+\frac{2}{8}$

4. $\frac{3}{4}+\frac{1}{6}=\frac{9}{12}+\frac{2}{12}$

What is the sum? Use a representation to solve.

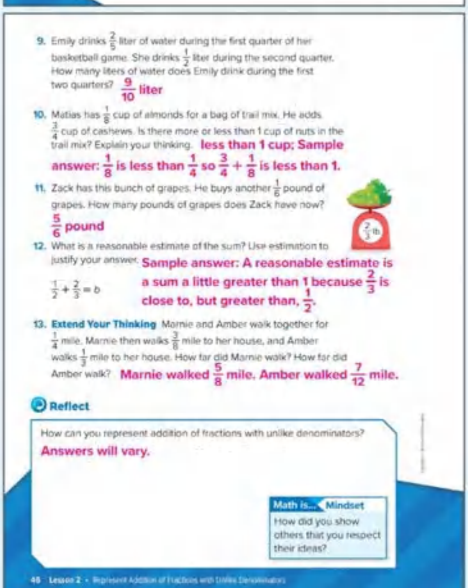
5. $\frac{4}{9}+\frac{1}{3}=\frac{7}{9}$

6. $\frac{3}{8}+\frac{3}{4}=\frac{9}{8}$

7. $\frac{2}{5}+\frac{7}{10}=\frac{11}{10}$

8. $\frac{7}{12}+\frac{5}{6}=\frac{17}{12}$

Unit 9 • Add and Subtract Fractions 45



9. Emily drinks $\frac{2}{5}$ liter of water during the first quarter of her basketball game. She drinks $\frac{1}{2}$ liter during the second quarter. How many liters of water does Emily drink during the first two quarters? $\frac{9}{10}$ liter

10. Matias has $\frac{1}{8}$ cup of almonds for a bag of trail mix. He adds $\frac{3}{4}$ cup of cashews. Is there more or less than 1 cup of nuts in the trail mix? Explain your thinking. less than 1 cup; Sample answer: $\frac{1}{8}$ is less than $\frac{1}{4}$ so $\frac{3}{4}+\frac{1}{8}$ is less than 1.

11. Zack has this bunch of grapes. He buys another $\frac{1}{6}$ pound of grapes. How many pounds of grapes does Zack have now? $\frac{5}{6}$ pound

12. What is a reasonable estimate of the sum? Use estimation to justify your answer. Sample answer: A reasonable estimate is a sum a little greater than 1 because $\frac{2}{3}$ is close to, but greater than, $\frac{1}{2}$.

$\frac{1}{2}+\frac{2}{3}=b$

13. **Extend Your Thinking** Marnie and Amber walk together for $\frac{1}{4}$ mile. Marnie then walks $\frac{3}{8}$ mile to her house, and Amber walks $\frac{1}{3}$ mile to her house. How far did Marnie walk? How far did Amber walk? Marnie walked $\frac{5}{8}$ mile. Amber walked $\frac{7}{12}$ mile.

Reflect

How can you represent addition of fractions with unlike denominators?

Answers will vary.

Math is... Mindset

How did you show others that you respect their ideas?

46 Lesson 2 • Represent Addition of Fractions with Unlike Denominators

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–13 Because students know the importance of rewriting fractions so that they have like denominators, they may sometimes forget to rewrite the numerator as well. Remind students that whatever number they multiply the denominator by, they must also multiply the numerator by that number.

Item Analysis

Item	DOK	Rigor
1–4	1	Conceptual Understanding
5–8	2	Procedural Skill & Fluency
9–12	2	Application
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you represent addition of fractions with unlike denominators?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you show others that you respect their ideas?

Students reflect on how they practiced social awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can use a representation to add fractions with unlike denominators.
- I can explain how to use a representation to add fractions with unlike denominators.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Represent addition of fractions with unlike denominators	5.NF.A.1
2	2	Represent addition of fractions with unlike denominators	5.NF.A.1
3	1	Represent addition of fractions with unlike denominators	5.NF.A.1

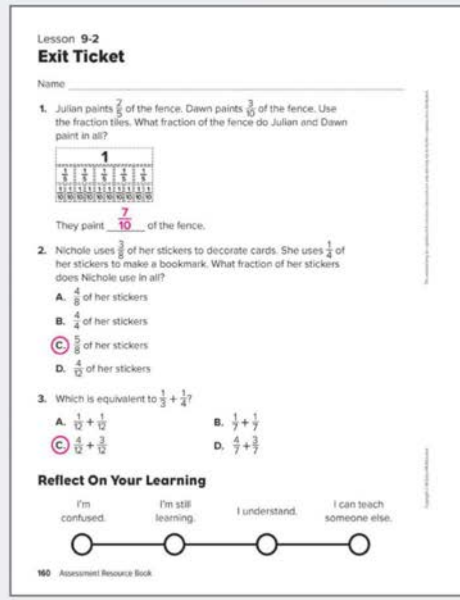
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 9-2

Exit Ticket

Name ________________

1. Julian paints $\frac{2}{5}$ of the fence. Dawn paints $\frac{3}{10}$ of the fence. Use the fraction tiles. What fraction of the fence do Julian and Dawn paint in all?

They paint $\frac{7}{10}$ of the fence.

2. Nichole uses $\frac{3}{8}$ of her stickers to decorate cards. She uses $\frac{1}{4}$ of her stickers to make a bookmark. What fraction of her stickers does Nichole use in all?

A. $\frac{4}{8}$ of her stickers
B. $\frac{4}{4}$ of her stickers
C. $\frac{5}{8}$ of her stickers
D. $\frac{4}{12}$ of her stickers

3. Which is equivalent to $\frac{1}{3} + \frac{1}{4}$?

A. $\frac{1}{12} + \frac{1}{12}$
B. $\frac{1}{7} + \frac{1}{7}$
C. $\frac{4}{12} + \frac{3}{12}$
D. $\frac{4}{7} + \frac{3}{7}$

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

160 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

What's in Common?

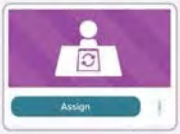
Work with students in groups. Provide fraction tiles. Students choose two tiles with different denominators. Students choose a fraction model to find common denominators and make equivalent fractions. Repeat with different denominator combinations. Make sure students recognize that they must represent the fractions with common denominators before adding them, and that they should add only the numerators while keep the denominator the same.

GO ONLINE

Take Another Look Lessons

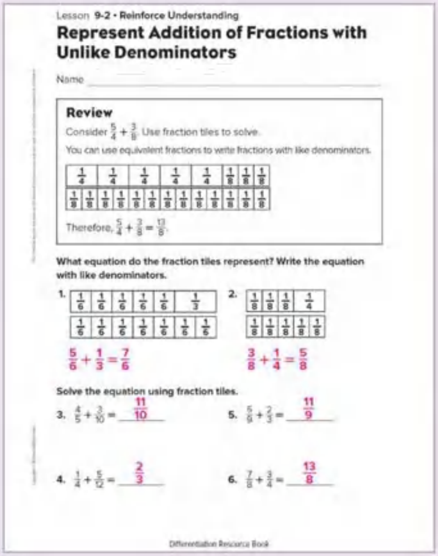
Assign the interactive lessons to reinforce targeted skills.

- Add Fractions (with/without Models)
- Add Fractions-Rename Both (Models)



INDEPENDENT WORK

Differentiation Resource Book, p. 91



Lesson 9-2 • Reinforce Understanding

Represent Addition of Fractions with Unlike Denominators

Name

Review

Consider $\frac{5}{4}+\frac{3}{8}$. Use fraction tiles to solve.

You can use equivalent fractions to write fractions with like denominators.

Therefore, $\frac{5}{4}+\frac{3}{8}=\frac{13}{8}$.

What equation do the fraction tiles represent? Write the equation with like denominators.

1. $\frac{5}{6}+\frac{1}{3}=\frac{7}{6}$

2. $\frac{3}{8}+\frac{1}{4}=\frac{5}{8}$

Solve the equation using fraction tiles.

3. $\frac{4}{5}+\frac{3}{10}=\frac{11}{10}$

4. $\frac{1}{4}+\frac{5}{12}=\frac{2}{3}$

5. $\frac{5}{9}+\frac{2}{3}=\frac{11}{9}$

6. $\frac{7}{8}+\frac{3}{4}=\frac{13}{8}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Adding Fractions Task Cards

Students practice adding fractions.

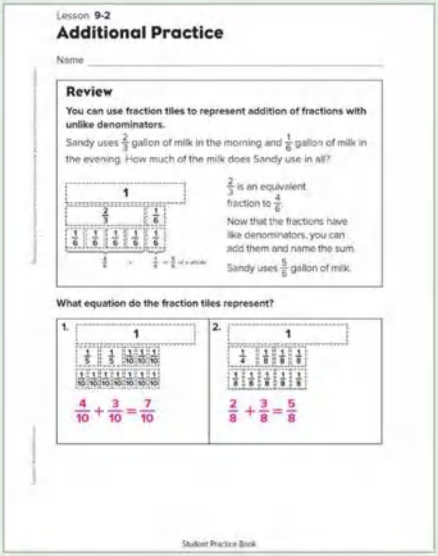
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 91–92



Lesson 9-2

Additional Practice

Name

Review

You can use fraction tiles to represent addition of fractions with unlike denominators.

Sandy uses $\frac{2}{3}$ gallon of milk in the morning and $\frac{1}{6}$ gallon of milk in the evening. How much of the milk does Sandy use in all?

$\frac{2}{3}$ is an equivalent fraction to $\frac{4}{6}$.

Now that the fractions have like denominators, you can add them and name the sum.

Sandy uses $\frac{5}{6}$ gallon of milk.

What equation do the fraction tiles represent?

1. $\frac{4}{10}+\frac{3}{10}=\frac{7}{10}$

2. $\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Use It! Application Station

Get Moving Students design a car and measure the distance it travels.

The content of this card has concepts covered later in Lesson 9-8. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review Practice

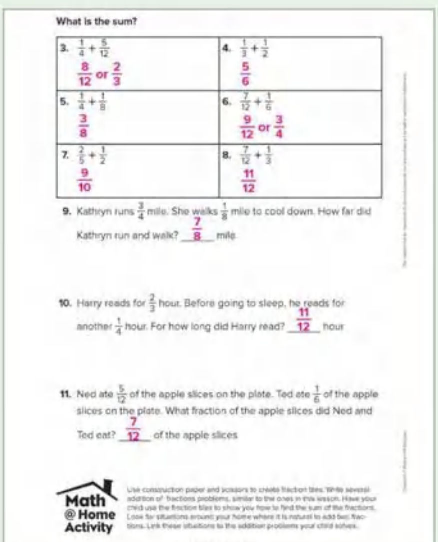
Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

INDEPENDENT WORK

Student Practice Book, pp. 91–92



What is the sum?

3. $\frac{1}{4}+\frac{5}{12}$ $\frac{8}{12}$ or $\frac{2}{3}$	4. $\frac{1}{3}+\frac{1}{2}$ $\frac{5}{6}$
5. $\frac{1}{4}+\frac{1}{8}$ $\frac{3}{8}$	6. $\frac{7}{12}+\frac{1}{6}$ $\frac{9}{12}$ or $\frac{3}{4}$
7. $\frac{2}{5}+\frac{1}{2}$ $\frac{9}{10}$	8. $\frac{7}{12}+\frac{1}{3}$ $\frac{11}{12}$

9. Kathryn runs $\frac{3}{4}$ mile. She walks $\frac{1}{8}$ mile to cool down. How far did Kathryn run and walk? $\underline{\frac{7}{8}}$ mile

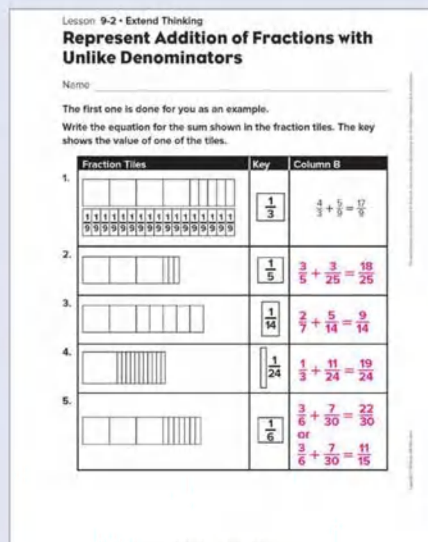
10. Harry reads for $\frac{2}{3}$ hour. Before going to sleep, he reads for another $\frac{1}{4}$ hour. For how long did Harry read? $\underline{\frac{11}{12}}$ hour

11. Ned ate $\frac{5}{12}$ of the apple slices on the plate. Ted ate $\frac{1}{6}$ of the apple slices on the plate. What fraction of the apple slices did Ned and Ted eat? $\underline{\frac{7}{12}}$ of the apple slices

Math @ Home Activity Use construction paper and scissors to create fraction tiles. Write several addition of fractions problems, similar to the ones in this lesson. Have your child use the fraction tiles to show you how to find the sum of the fractions. Look for situations around your home where it is natural to add two fractions. Link these situations to the addition problems your child solves.

Student Practice Book

Differentiation Resource Book, p. 92



Lesson 9-2 • Extend Thinking

Represent Addition of Fractions with Unlike Denominators

Name ______________________

The first one is done for you as an example.

Write the equation for the sum shown in the fraction tiles. The key shows the value of one of the tiles.

	Fraction Tiles	Key	Column B
1.		$\frac{1}{3}$	$\frac{4}{3}+\frac{5}{9}=\frac{17}{9}$
2.		$\frac{1}{5}$	$\frac{3}{5}+\frac{3}{25}=\frac{18}{25}$
3.		$\frac{1}{14}$	$\frac{2}{7}+\frac{5}{14}=\frac{9}{14}$
4.		$\frac{1}{24}$	$\frac{1}{3}+\frac{11}{24}=\frac{19}{24}$
5.		$\frac{1}{6}$	$\frac{3}{6}+\frac{7}{30}=\frac{22}{30}$ or $\frac{3}{6}+\frac{7}{30}=\frac{11}{15}$

Differentiation Resource Book

LESSON **9-3**

Add Fractions with Unlike Denominators

Learning Targets

- I can add fractions with unlike denominators.
- I can explain how to add fractions with unlike denominators.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. $\left(\text{In general, } \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}\right)$.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

Vocabulary

Math Terms	Academic Terms
equivalent fractions	accurate
like denominator	condition
multiple	

Materials

The materials may be for any part of the lesson.

- fraction tiles
- number cubes

Focus

Content Objectives

- Students add fractions with unlike denominators.
- Students explain how to add fractions with unlike denominators.

Language Objectives

- Students explain how to add fractions with unlike denominators using *should*.
- To support optimizing output, ELs participate in MLR7: Compare and Connect.

SEL Objective

- Students actively listen without interruption as peers describe how they approached a complex mathematical task.

Coherence

Previous

- Students understood, recognized, and generated equivalent fractions (Grade 4).
- Students used representations to understand addition of fractions having unlike denominators (Unit 9).

Now

- Students use equivalent fractions to add fractions having unlike denominators.

Next

- Students use representations to understand subtraction of fractions having unlike denominators (Unit 9).
- Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ (Grade 6).

Rigor

Conceptual Understanding

- Students build on their understanding of operations with fractions.

Procedural Skill & Fluency

- Students build proficiency with equivalent fractions and develop general skills and strategies for adding fractions.

Application

- Students solve problems with real-world contexts.

Application is not a targeted element of rigor for this standard.

GO ONLINE

Number Routine

Which Benchmark Is It Closest To? 5–7 min

Build Fluency Students build number sense as they determine which benchmark each fraction value is closest to and explain their reasoning.

These prompts encourage students to talk about their reasoning:

- How did you determine which benchmark each fraction is closest to? Which benchmark number is $\frac{1}{2}$ closest to? How did you determine which benchmark each fraction is closest to?
- Why would it be useful to use benchmark numbers to estimate the size of a fraction?
- Which benchmark number is $\frac{1}{2}$ closest to?

Purpose Students compare and contrast numbers to determine which doesn't belong.

Which Doesn't Belong?

- Which doesn't belong?

Teaching Tip You may want to have students discuss their observations and thoughts with a partner before discussing as a whole class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of using equivalent fractions to add fractions having unlike denominators and are based on possible comments and questions that students may make during the share out.

- How is 12 different from the other numbers?
- How can you make 27 or 45 using 9?

Math is... Mindset

- How do you make sure you share your thinking clearly?

SEL Relationship Skills: Communication

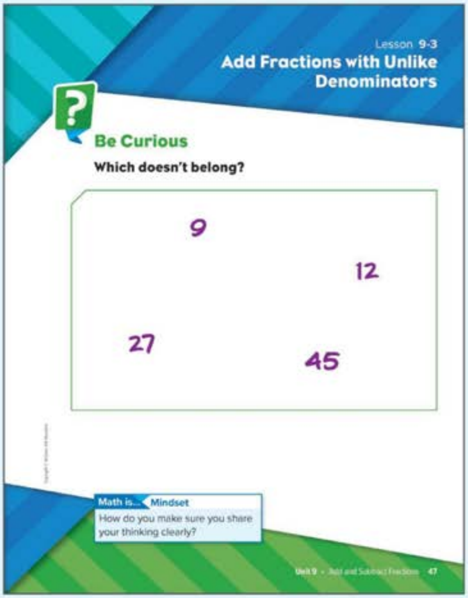
As students engage in collaborative discourse around the Which Doesn't Belong? routine, encourage them to actively and respectfully listen to one another. Invite students to think about and share what active listening looks and sounds like. As students discuss which number doesn't belong, encourage classmates to listen as well as provide thoughtful feedback. Capitalize on opportunities to also model these behaviors when students are speaking.

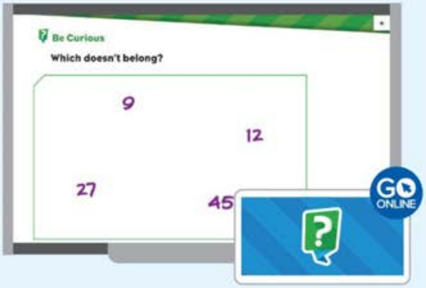
Transition to Explore & Develop

Ask questions that get students thinking about using equivalent fractions to add fractions having unlike denominators.

ETP Establish Mathematics Goals to Focus Learning

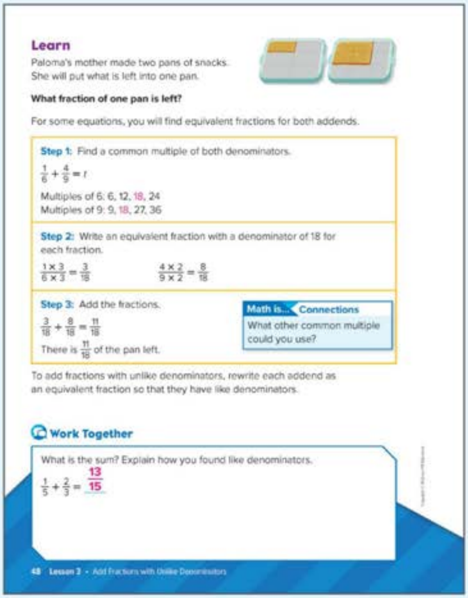
- Let's think about how we can use equivalent fractions to add fractions having unlike denominators.





Explore & Develop

20 min



Learn

Paloma's mother made two pans of snacks. She will put what is left into one pan.

What fraction of one pan is left?

For some equations, you will find equivalent fractions for both addends.

Step 1: Find a common multiple of both denominators.

$\frac{1}{6} + \frac{4}{9} = r$

Multiples of 6: 6, 12, 18, 24
Multiples of 9: 9, 18, 27, 36

Step 2: Write an equivalent fraction with a denominator of 18 for each fraction.

$\frac{1 \times 3}{6 \times 3} = \frac{3}{18}$ $\frac{4 \times 2}{9 \times 2} = \frac{8}{18}$

Step 3: Add the fractions.

$\frac{3}{18} + \frac{8}{18} = \frac{11}{18}$

There is $\frac{11}{18}$ of the pan left.

Math is... Connections
What other common multiple could you use?

To add fractions with unlike denominators, rewrite each addend as an equivalent fraction so that they have like denominators.

Work Together

What is the sum? Explain how you found like denominators.

$\frac{1}{5} + \frac{2}{3} = \frac{13}{15}$

48 Lesson 3 • Add Fractions with Unlike Denominators

1 Pose the Problem

ETP Pose Purposeful Questions

- Do the fractions remaining in each pan have like or unlike denominators? How do you know?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and give them a problem similar to the one on the Learn page. Ask each to work individually to solve the equation, and then have them compare their strategies. Revisit this routine throughout the lesson to help students build proficiency.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- Why do you have to use fractions with like denominators when adding?
- Explain two strategies you can use to find like denominators.

Key Takeaways

- When adding fractions with unlike denominators, use equivalent fractions with like denominators before adding.
- One way to generate equivalent fractions is by multiplying the denominators to determine a like denominator.
- Another way to generate equivalent fractions is by determining a common multiple to identify a like denominator.

Work Together

Students solve an addition problem by rewriting the fractions as equivalent fractions with like denominators.

Common Error: In the Work Together problem, the least common denominator is the product of the two denominators, 15. Make sure students understand that there is no common multiple of 3 and 5 that is less than 15.

LOM Language of Math

Explain to students that *like* means "having similar qualities or characteristics." They are familiar with the verb *like*, but in the context of like denominators it is an adjective. They can use the word in this context outside of class by, for example, saying they are of like mind as someone else with the same opinion.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore strategies for adding fractions that have unlike denominators.

Directions: Have students work together to solve the Pose the Problem. Encourage students to find at least two different ways to solve the problem.

Support Productive Struggle

- What operation should you use to solve the problem?
- How do you need to rewrite the fractions in order to add them?
- How can you use multiplication to write equivalent fractions?
- Is there more than one pair of equivalent fractions you can use to find the sum?

Activity Debrief: Have students share the methods they used to solve the problem. Facilitate a discussion to ensure students understand that one method of finding like denominators is by multiplying the denominators of the addends. Another method is to look for other common multiples of the denominators.

Math is... Exploring

- What is the same about these methods? What is different?

Students identify correspondences between different approaches to solving complex problems.

Guided Exploration

Students add fractions having unlike denominators by writing equivalent fractions with like denominators.

Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- What benchmark numbers will you use to estimate the solution? Why?
- Why do you need to find like denominators in order to add?

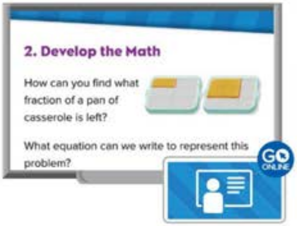
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?
- Why are the sums equivalent when you used different fractions?
- Which method of finding like denominators do you prefer? Why?
- **Think About It:** Are there other common denominators that could have been used? Explain.

Math is... Exploring

- What is the same about these methods? What is different?

Students identify correspondences between different approaches to solving complex problems.



English Learner Scaffolds

Entering/Emerging

Explain *To ___, do ___ so that ___.* Place 20 chips on the desk. Say, *I want to show four equal groups.* Then say, *To make four equal groups, divide the chips so that each group has five chips.* Ungroup the chips and say, *I want to show five equal groups*. Repeat the steps. Finally, start over and ask, *To make two equal groups, should I divide the chips so that each group has two chips or four chips?*

Developing/Expanding

Explain *To ___, do ___ so that ___.* Place 20 chips on the desk. Say, *I want to show four equal groups.* Then say, *To make four equal groups, divide the chips so that each group has five chips.* Ungroup and repeat using five equal groups. Finally, say, *I want to make two equal groups.* Ask students to complete the sentence: *___ make two equal groups, ___ the chips ___ that each group has ___ chips.*

Bridging/Reaching

Guide students to the sentence below the table on the Learn page. Discuss its meaning. Then show students 20 chips. Ask, *How can I make four equal groups?* Instruct them to respond using *To make..., do ___ so that.* Then ask them to explain how to do something else using the same structure. Allow students to interject with their opinions/corrections, i.e., *No, that's not correct. To make...*

On My Own

Name ______________________

Which multiple can you use as a like denominator to add the fractions? Choose all correct answers.

1. $\frac{2}{3}+\frac{3}{4}$
   A. 6
   B. 8
   C. 12
   D. 24

2. $\frac{1}{6}+\frac{3}{8}$
   A. 12
   B. 16
   C. 24
   D. 30

Complete the equation using addends with like denominators. Sample answers shown.

3. $\frac{3}{5}+\frac{1}{4}=\boxed{\frac{12}{20}}+\boxed{\frac{5}{20}}$

4. $\frac{2}{3}+\frac{1}{6}=\boxed{\frac{4}{6}}+\boxed{\frac{1}{6}}$

What is the sum?

5. $\frac{2}{9}+\frac{5}{12}=\frac{23}{36}$

6. $\frac{3}{8}+\frac{1}{3}=\frac{17}{24}$

7. $\frac{5}{8}+\frac{3}{10}=\frac{37}{40}$

8. $\frac{2}{7}+\frac{1}{2}=\frac{11}{14}$

9. A club ordered two same-sized vegetable pizzas cut into different numbers of pieces. What fraction of a whole pizza is left? $\frac{13}{24}$

10. Oliver uses $\frac{1}{6}$ gallon of water for his outdoor plants. He uses $\frac{1}{4}$ gallon of water for his indoor plants. How many gallons of water does Oliver use on all of his plants?
$\frac{5}{12}$ gallon

11. Heather uses $\frac{2}{3}$ foot of yarn for her art project. She adds another $\frac{1}{12}$ foot to complete the project. How much yarn does Heather use in all?
$\frac{9}{12}$ foot of yarn

12. **Error Analysis** Mia found the sum of $\frac{2}{9}+\frac{3}{4}$.
How can you help Mia correct her mistake?
$\frac{2\times 2}{2\times 9}+\frac{3\times 4}{4\times 4}=\frac{4}{18}+\frac{12}{18}=\frac{16}{18}$
Sample answer: $4 \times 4 = 16$, not 18; the correct answer is $\frac{35}{36}$.

13. **Extend Your Thinking** Solve the equation using two different like denominators. Is the sum the same when you use different denominators? Explain why or why not.
$\frac{3}{8}+\frac{7}{10}=?$ $\frac{3}{8}+\frac{7}{10}=\frac{30}{80}+\frac{56}{80}=\frac{86}{80}=\frac{43}{40}$
$\frac{3}{8}+\frac{7}{10}=\frac{15}{40}+\frac{28}{40}=\frac{43}{40}$
Even though the denominators are different, the fractions still represent equivalent amounts.

Reflect

How can equivalent fractions help you add fractions with unlike denominators?
Answers may vary.

Math is... Mindset
How did you make sure you shared your thinking clearly?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 4 When finding like denominators, students may focus on common multiples such as 12 or 18. Make sure students understand that sometimes, one of the denominators can serve as a like denominator.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–11	2	Application
12–13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can equivalent fractions help you add fractions with unlike denominators?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you make sure you shared your thinking clearly?

Students reflect on how they developed stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can add fractions with unlike denominators.
- I can explain how to add fractions with unlike denominators.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Add fractions with unlike denominators	5.NF.A.1
2	1	Add fractions with unlike denominators	5.NF.A.1
3	2	Add fractions with unlike denominators	5.NF.A.1
4	2	Add fractions with unlike denominators	5.NF.A.1

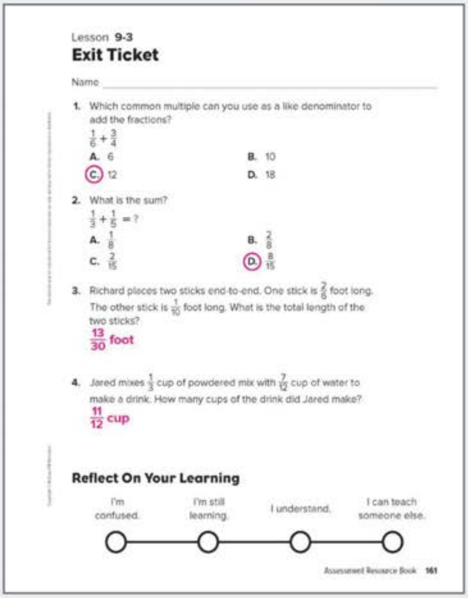
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 9-3
Exit Ticket

Name

1. Which common multiple can you use as a like denominator to add the fractions?
$\frac{1}{6} + \frac{3}{4}$
A. 6 B. 10
C. 12 D. 18

2. What is the sum?
$\frac{1}{3} + \frac{1}{5} = ?$
A. $\frac{1}{8}$ B. $\frac{2}{8}$
C. $\frac{2}{15}$ D. $\frac{8}{15}$

3. Richard places two sticks end-to-end. One stick is $\frac{2}{6}$ foot long. The other stick is $\frac{1}{10}$ foot long. What is the total length of the two sticks?
$\frac{13}{30}$ foot

4. Jared mixes $\frac{1}{3}$ cup of powdered mix with $\frac{7}{12}$ cup of water to make a drink. How many cups of the drink did Jared make?
$\frac{11}{12}$ cup

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 161

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Add Them Up!

Work with students in pairs. Each student rolls two fraction cubes, writes an equation to add, and solves it. If students have difficulty, help them to use fraction tiles to represent the fractions, find equivalent fractions, and then write the renamed fractions in an equation.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Add Fractions (Rename Both)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 93

Lesson **9-3 • Reinforce Understanding**

Add Fractions with Unlike Denominators

Name ______________________

Review

You can make a table to help find the lowest common multiple for the denominator.

Consider $\frac{5}{12} + \frac{3}{8}$. It has denominators 12 and 8.

The first number that appears in both rows is 24. Make equivalent fractions with like denominators of 24.

	× 1	× 2	× 3	× 4
12	12	24	36	48
8	8	16	24	32

$\frac{5}{12} + \frac{3}{8} = \frac{19}{24}$

$$\frac{5}{12} + \frac{3}{8} = \frac{5 \times 2}{12 \times 2} + \frac{3 \times 3}{8 \times 3} = \frac{10}{24} + \frac{9}{24} = \frac{19}{24}$$

What is the sum? Use a table to find the lowest common multiple.

1. $\frac{5}{6} + \frac{2}{5} =$ ______ $\frac{37}{30}$
2. $\frac{2}{9} + \frac{3}{4} =$ ______ $\frac{35}{36}$ or $\frac{11}{8}$
3. $\frac{2}{7} + \frac{1}{3} =$ ______ $\frac{13}{21}$
4. $\frac{3}{8} + \frac{1}{2} =$ ______ $\frac{7}{8}$
5. $\frac{3}{10} + \frac{5}{6} =$ ______ $\frac{34}{30}$ or $\frac{17}{15}$
6. $\frac{5}{6} + \frac{4}{15} =$ ______ $\frac{33}{30}$ or $\frac{11}{10}$
7. $\frac{7}{12} + \frac{5}{8} =$ ______ $\frac{29}{24}$
8. $\frac{2}{11} + \frac{1}{4} =$ ______ $\frac{19}{44}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Addition Concentration

Students practice adding fractions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 93–94

Lesson **9-3**

Additional Practice

Name ______________________

Review

You can add fractions with unlike denominators by finding a common multiple of the denominators to use as a common denominator.

Ollie's math notebook weighs $\frac{3}{8}$ pound. His science notebook weighs $\frac{1}{6}$ pound. How much do Ollie's two notebooks weigh together?

To solve, find $\frac{3}{8} + \frac{1}{6}$.

For the denominators 8 and 6, a common multiple is 24. Write equivalent fractions using 24 as the denominator.

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$ $\qquad$ $\frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$

$\frac{3}{8} + \frac{1}{6} = \frac{9}{24} + \frac{4}{24} = \frac{13}{24}$

Ollie's two notebooks weigh $\frac{13}{24}$ pound together.

Find the sum. Show your work. Check students' work.

1. $\frac{1}{6} + \frac{1}{3} =$ $\frac{3}{6}$ or $\frac{1}{2}$
2. $\frac{1}{5} + \frac{2}{3} =$ $\frac{13}{15}$
3. $\frac{1}{6} + \frac{4}{9} =$ $\frac{11}{18}$
4. $\frac{3}{4} + \frac{1}{6} =$ $\frac{11}{12}$
5. $\frac{1}{4} + \frac{7}{10} =$ $\frac{19}{20}$
6. $\frac{2}{3} + \frac{1}{4} =$ $\frac{11}{12}$

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Assign

Use It! Application Station

Create and Solve Students create a multi-step problem that adds and subtracts mixed numbers to solve. Then they use a digital tool to present the problem.

The content of this card has concepts covered later in Lesson 9-9. You may want to assign this card to students ready to explore content covered later in this unit.

WORKSTATIONS

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

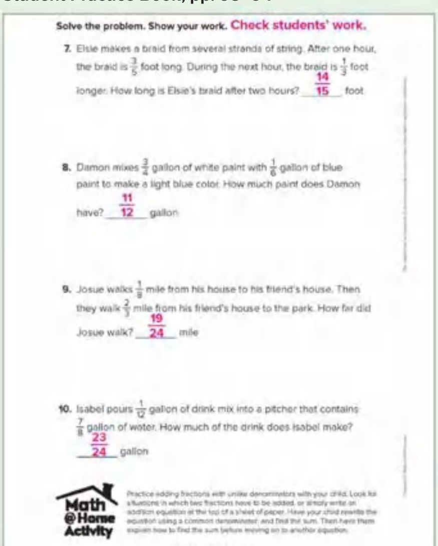
Assign a digital simulation to apply skills and extend thinking.

Assign

GO ONLINE

INDEPENDENT WORK

Student Practice Book, pp. 93–94



Solve the problem. Show your work. Check students' work.

7. Elsie makes a braid from several strands of string. After one hour, the braid is $\frac{3}{5}$ foot long. During the next hour, the braid is $\frac{1}{3}$ foot longer. How long is Elsie's braid after two hours? $\frac{14}{15}$ foot

8. Damon mixes $\frac{3}{4}$ gallon of white paint with $\frac{1}{6}$ gallon of blue paint to make a light blue color. How much paint does Damon have? $\frac{11}{12}$ gallon

9. Josue walks $\frac{1}{8}$ mile from his house to his friend's house. Then they walk $\frac{2}{3}$ mile from his friend's house to the park. How far did Josue walk? $\frac{19}{24}$ mile

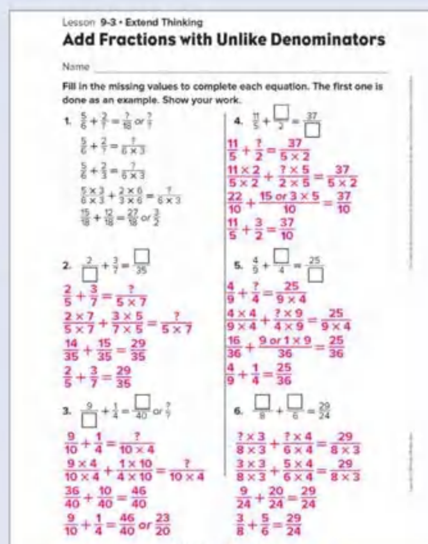
10. Isabel pours $\frac{1}{12}$ gallon of drink mix into a pitcher that contains $\frac{7}{8}$ gallon of water. How much of the drink does Isabel make? $\frac{23}{24}$ gallon

Math @Home Activity

Practice adding fractions with unlike denominators with your child. Look for situations in which two fractions have to be added, or simply write an addition equation at the top of a sheet of paper. Have your child rewrite the equation using a common denominator, and find the sum. Then have them explain how to find the sum before moving on to another equation.

Student Practice Book

Differentiation Resource Book, p. 94



Lesson 9-3 • Extend Thinking

Add Fractions with Unlike Denominators

Name ____________

Fill in the missing values to complete each equation. The first one is done as an example. Show your work.

1. $\frac{5}{6}+\frac{2}{?}=\frac{?}{18}$ or $\frac{?}{?}$
$\frac{5}{6}+\frac{2}{?}=\frac{?}{6\times3}$
$\frac{5}{6}+\frac{2}{3}=\frac{?}{6\times3}$
$\frac{5\times3}{6\times3}+\frac{2\times6}{3\times6}=\frac{?}{6\times3}$
$\frac{15}{18}+\frac{12}{18}=\frac{27}{18}$ or $\frac{3}{2}$

2. $\frac{2}{\square}+\frac{3}{7}=\frac{\square}{35}$
$\frac{2}{5}+\frac{3}{7}=\frac{?}{5\times7}$
$\frac{2\times7}{5\times7}+\frac{3\times5}{7\times5}=\frac{?}{5\times7}$
$\frac{14}{35}+\frac{15}{35}=\frac{29}{35}$
$\frac{2}{5}+\frac{3}{7}=\frac{29}{35}$

3. $\frac{9}{\square}+\frac{1}{4}=\frac{\square}{40}$ or $\frac{?}{?}$
$\frac{9}{10}+\frac{1}{4}=\frac{?}{10\times4}$
$\frac{9\times4}{10\times4}+\frac{1\times10}{4\times10}=\frac{?}{10\times4}$
$\frac{36}{40}+\frac{10}{40}=\frac{46}{40}$
$\frac{9}{10}+\frac{1}{4}=\frac{46}{40}$ or $\frac{23}{20}$

4. $\frac{11}{5}+\frac{\square}{2}=\frac{37}{\square}$
$\frac{11}{5}+\frac{?}{2}=\frac{37}{5\times2}$
$\frac{11\times2}{5\times2}+\frac{?\times5}{2\times5}=\frac{37}{5\times2}$
$\frac{22}{10}+\frac{15 \text{ or } 3\times5}{10}=\frac{37}{10}$
$\frac{11}{5}+\frac{3}{2}=\frac{37}{10}$

5. $\frac{4}{9}+\frac{\square}{4}=\frac{25}{\square}$
$\frac{4}{9}+\frac{?}{4}=\frac{25}{9\times4}$
$\frac{4\times4}{9\times4}+\frac{?\times9}{4\times9}=\frac{25}{9\times4}$
$\frac{16}{36}+\frac{9 \text{ or } 1\times9}{36}=\frac{25}{36}$
$\frac{4}{9}+\frac{1}{4}=\frac{25}{36}$

6. $\frac{\square}{8}+\frac{\square}{6}=\frac{29}{24}$
$\frac{?\times3}{8\times3}+\frac{?\times4}{6\times4}=\frac{29}{8\times3}$
$\frac{3\times3}{8\times3}+\frac{5\times4}{6\times4}=\frac{29}{8\times3}$
$\frac{9}{24}+\frac{20}{24}=\frac{29}{24}$
$\frac{3}{8}+\frac{5}{6}=\frac{29}{24}$

Differentiation Resource Book

LESSON 9-4

Represent Subtraction of Fractions with Unlike Denominators

Learning Targets

- I can use a representation to subtract fractions with unlike denominators.
- I can explain how to use a representation to subtract fractions with unlike denominators.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. $\left(\text{In general, } \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}\right)$.

Math Practices and Processes

MPP Model with mathematics.

MPP Look for and express regularity in repeated reasoning.

Vocabulary

Math Terms	Academic Terms
denominator	establish
equivalent fractions	valid

Materials

The materials may be for any part of the lesson.

- *Fraction Number Lines* Teaching Resource
- fraction tiles

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use a representation to subtract fractions with unlike denominators. • Students explain how to use a representation to subtract fractions with unlike denominators.	• Students explain how to use a representation to subtract fractions with unlike denominators using *can*. • To support sense-making, ELs participate in MLR2: Collect and Display.	• Students employ self-calming techniques that can be used to help manage reactions to potentially frustrating situations.

Coherence

Previous	Now	Next
• Students understood, recognized, and generated equivalent fractions (Grade 4). • Students used equivalent fractions to add fractions having unlike denominators (Unit 9).	• Students use representations to understand subtraction of fractions having unlike denominators.	• Students use equivalent fractions to subtract fractions having unlike denominators (Unit 9). • Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students interpret representations to develop their understanding of subtracting fractions with unlike denominators.	• Students build proficiency through repeated use of representations, such as pictures, tools, and equations.	• Students explore subtraction of fractions in real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine

What's Another Way to Write It? 5–7 min

Build Fluency Students build number sense as they write three multiplication expressions to represent the number 2.5. Record solutions for classroom discussion.

These prompts encourage students to talk about their reasoning:

- Which expression makes the most sense? Which expressions are related?
- Do you notice any patterns? Explain.
- Can you create a new expression using one of the expressions on the board? How?
- Did you use division as a strategy? Explain.

Launch 5–7 min

Sense-Making Routine

Purpose Students are presented with a statement and think about the truth of the statement. Students consider whether the relationship always holds or whether it is unique to the situation.

Is It Always True?

- Is the statement always true?

Teaching Tip You may want to provide students with some specific fractions, such as $\frac{1}{3}$ or $\frac{2}{3}$, to help them think about the question.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of using representations to understand subtraction of fractions having unlike denominators and are based on possible comments and questions that students may make during the share out.

- How can you rewrite a fraction having 3 as its denominator as a fraction having 6 as its denominator?
- Can you think of any fractions for which the statement would not be true?
- How many examples would you need to find to show that the statement is not always true? Why?
- How many examples would you need to find to show that the statement is always true? Why?

Math is... Mindset

- What helps you stay focused in class?

SEL Self-Management – Control Impulses

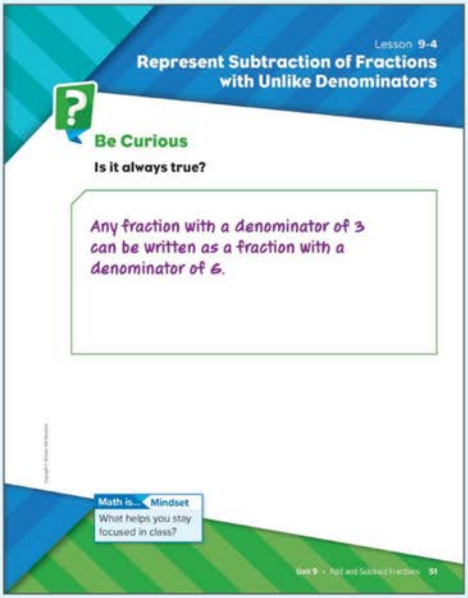
Provide opportunities for students to practice impulse control. As you transition from the Is It Always True? routine, brainstorm strategies that can help students express emotionally and behaviorally appropriate responses in times of frustration or disappointment. As students work with representing subtraction of fractions with unlike denominators, invite them to practice deep-breathing techniques or take movement breaks when necessary.

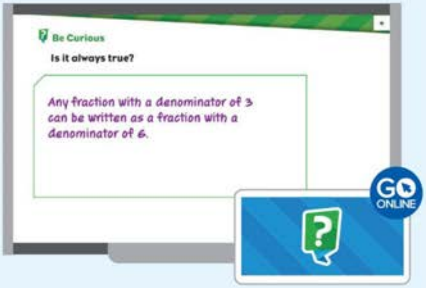
Transition to Explore & Develop

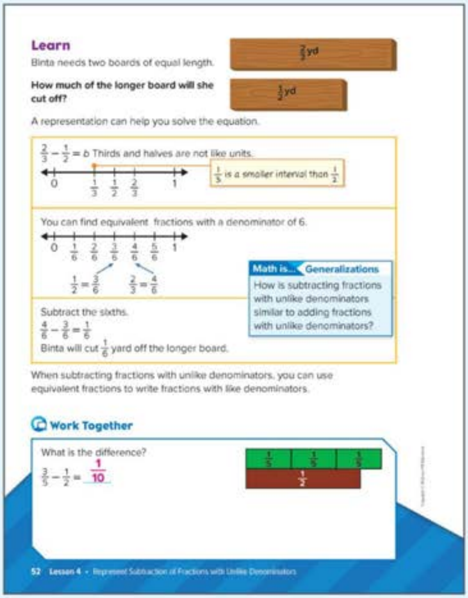
Ask questions that get students thinking about how they might represent subtraction of fractions having unlike denominators.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can represent subtraction of fractions having unlike denominators.







Learn

Binta needs two boards of equal length.

How much of the longer board will she cut off?

A representation can help you solve the equation.

$\frac{2}{3} - \frac{1}{2} = b$ Thirds and halves are not like units.

$\frac{1}{3}$ is a smaller interval than $\frac{1}{2}$

You can find equivalent fractions with a denominator of 6.

$\frac{1}{2} = \frac{3}{6}$ $\frac{2}{3} = \frac{4}{6}$

Subtract the sixths.

$\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

Binta will cut $\frac{1}{6}$ yard off the longer board.

Math is... Generalizations

How is subtracting fractions with unlike denominators similar to adding fractions with unlike denominators?

When subtracting fractions with unlike denominators, you can use equivalent fractions to write fractions with like denominators.

Work Together

What is the difference?

$\frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

52 Lesson 4 • Represent Subtraction of Fractions with Unlike Denominators

❶ Pose the Problem

MLR Collect and Display

As students discuss the questions, record relevant words and phrases they may use such as *representation, like/unlike denominators, fraction tiles,* and *sixths.* Display the words for student reference. Use the student-generated expressions to help students make connections between student language and math vocabulary.

ETP Pose Purposeful Questions

- What are the important quantites in this problem?
- What do the quantities in this problem represent?

❷ Develop the Math

Choose the option that best meets your instructional goals.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How could you explain to a friend how to subtract fractions having unlike denominators?

Key Takeaway

- When subtracting fractions with unlike denominators, it is necessary to generate equivalent fractions with like denominators before subtracting.

Work Together

Students solve a subtraction equation by representing the equation with fraction tiles and writing equivalent fractions having like denominators.

Common Error: Students may be focused on finding like denominators and forget to rewrite the numerator. Remind students to rewrite the numerator after rewriting the denominator.

LOM Language of Math

Explain to students that the word *denominator* comes from *de-*, which means “from,” and *nominare*, which means “to name”. The denominator names the number of equal parts. You can find the word *nominare* in other words such as *nominate*, which means to appoint someone, or *nominally*, which means in name only.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using a representation to subtract fractions that have unlike denominators.

Materials: *Fraction Number Lines* Teaching Resource, fraction tiles

Directions: Have students work in small groups. Present them with the expression $\frac{5}{6} - \frac{1}{3}$. Invite students to use tools to help them represent and solve the problem.

Implement Tasks That Promote Reasoning and Problem Solving

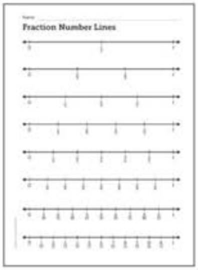
- How does your representation show the meaning of subtraction?
- How does knowing that $\frac{5}{6}$ can be broken apart into $\frac{2}{6}$, $\frac{2}{6}$, and $\frac{1}{6}$ help you?
- How does knowing that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ help you?

Math is... Generalizations

- How is subtracting fractions having unlike denominators similar to adding fractions having unlike denominators?

Students notice if calculations are repeated, and look both for general methods and for shortcuts.

Activity Debrief: Have students share their representations and solutions. Facilitate a discussion to ensure students understand that fractions can only be subtracted when they have like denominators. If the fractions have unlike denominators, they need to use equivalent fractions with like denominators to subtract.



Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the difference of the lengths of these boards?

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students use a representation to understand subtraction of fractions having unlike denominators.

Use and Connect Mathematical Representations

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- What benchmark numbers will you use to estimate the solution? Why?
- How do the fraction tiles help you understand the equation?
- Do you need to rewrite one or both of the fractions? How do you know?
- Why can you use 6 as the denominator?
- How can you subtract once you have written the equivalent fractions with like denominators?

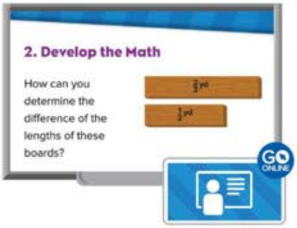
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

Math is... Generalizations

- How is subtracting fractions having unlike denominators similar to adding fractions having unlike denominators?

Students notice if calculations are repeated, and look both for general methods and for shortcuts.



English Learner Scaffolds

Entering/Emerging

Explain *When ___, you can ___.* Draw a triangle and square. Say, *When comparing shapes, you can count the number of sides to determine the shape.* Demonstrate. Then show students two more examples, one where the sentence structure correctly explains how to do something, and one where it doesn't. Ask each time, *Yes or No?*

Developing/Expanding

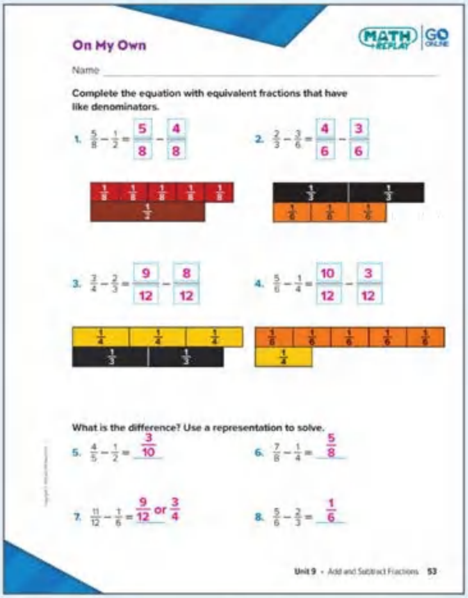
Explain *When ___, you can ___.* Draw a triangle and square. Say, *When comparing shapes, you can count the number of sides to help determine the name of each shape.* Demonstrate. Then ask students to repeat the task with their own sentence, using *When ___, you can ___.* Provide sentence frames for students who need more guidance.

Bridging/Reaching

Guide students to the Learn page and point to the sentence below the table. Confirm comprehension of the sentence structure and then ask students to refer back to Lesson 3's sentence structure, *To ___, do ___ so that ___.* Ask students how *When ___, you can ___.* is similar to *To ___, do ___ so that ___.* (Both are explanations about how to do something./Both explain how to do something.)

Practice & Reflect

10 min



On My Own

Name ______

Complete the equation with equivalent fractions that have like denominators.

1. $\frac{5}{8} - \frac{1}{2} = \frac{5}{8} - \frac{4}{8}$

2. $\frac{2}{3} - \frac{3}{6} = \frac{4}{6} - \frac{3}{6}$

3. $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12}$

4. $\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12}$

What is the difference? Use a representation to solve.

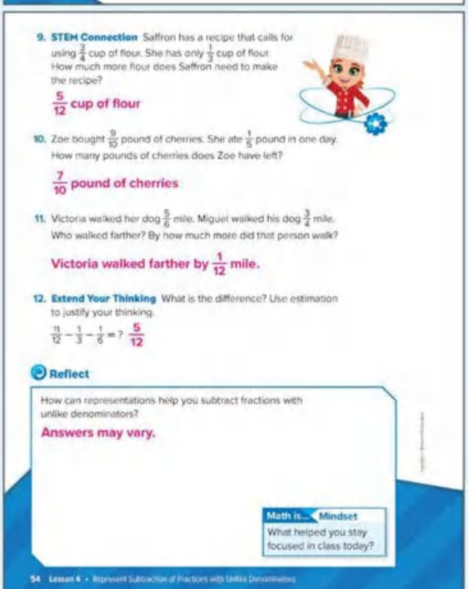
5. $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

6. $\frac{7}{8} - \frac{1}{4} = \frac{5}{8}$

7. $\frac{11}{12} - \frac{1}{6} = \frac{9}{12}$ or $\frac{3}{4}$

8. $\frac{5}{6} - \frac{2}{3} = \frac{1}{6}$

Unit 9 • Add and Subtract Fractions 53



9. **STEM Connection** Saffron has a recipe that calls for using $\frac{3}{4}$ cup of flour. She has only $\frac{1}{3}$ cup of flour. How much more flour does Saffron need to make the recipe?

$\frac{5}{12}$ cup of flour

10. Zoe bought $\frac{9}{10}$ pound of cherries. She ate $\frac{1}{5}$ pound in one day. How many pounds of cherries does Zoe have left?

$\frac{7}{10}$ pound of cherries

11. Victoria walked her dog $\frac{5}{6}$ mile. Miguel walked his dog $\frac{3}{4}$ mile. Who walked farther? By how much more did that person walk?

Victoria walked farther by $\frac{1}{12}$ mile.

12. **Extend Your Thinking** What is the difference? Use estimation to justify your thinking.

$\frac{11}{12} - \frac{1}{3} - \frac{1}{6} = ?$ $\frac{5}{12}$

Reflect

How can representations help you subtract fractions with unlike denominators?

Answers may vary.

Math is... Mindset

What helped you stay focused in class today?

54 Lesson 4 • Represent Subtraction of Fractions with Unlike Denominators

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 5–8 Students may forget to write the fractions with like denominators before subtracting.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can representations help you subtract fractions with unlike denominators?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you stay focused in class today?

Students reflect on how they practiced self-management.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can use a representation to subtract fractions with unlike denominators.
- I can explain how to use a representation to subtract fractions with unlike denominators.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Represent subtraction of fractions with unlike denominators	5.NF.A.1
2	1	Represent subtraction of fractions with unlike denominators	5.NF.A.1
3	2	Represent subtraction of fractions with unlike denominators	5.NF.A.1

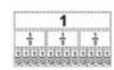
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 9-4

Exit Ticket

Name ____________________

1. Connor measures a caterpillar and a worm. The caterpillar is $\frac{1}{3}$ foot long. The worm is $\frac{7}{12}$ foot long. How much longer is the worm than the caterpillar? Use the fraction tiles.

   A. $\frac{6}{12}$ foot

   B. $\frac{11}{12}$ foot

   C. $\frac{3}{12}$ foot

   D. $\frac{8}{12}$ foot

2. Which equation is shown by the fraction tiles?

   A. $\frac{5}{6} - \frac{3}{4} = \frac{1}{6}$

   B. $\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$

   C. $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

   D. $\frac{5}{6} - \frac{3}{4} = \frac{1}{10}$

3. Roberta has two lengths of string. One string is $\frac{1}{2}$ foot long. The other string is $\frac{1}{5}$ foot long. How much longer is the first length of string?

   $\frac{3}{10}$ foot

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

162 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Make Me Equivalent

Work with students in pairs. Provide partners with fraction tiles. Students choose two tiles with different denominators. Have students write a subtraction equation using the fractions, then solve by determining like denominators and writing equivalent fractions.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Subtract Fractions (with/without Models)
- Subtract Fractions (Model and Rename)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 95

Lesson 9-4 • Reinforce Understanding

Represent Subtraction of Fractions with Unlike Denominators

Name

Review

Consider $\frac{5}{4} - \frac{3}{8}$. Use fraction tiles to solve.

You can use equivalent fractions to write fractions with like denominators.

Therefore, $\frac{5}{4} - \frac{3}{8} = \frac{7}{8}$.

What difference equation do the fraction tiles represent. Write the equation with like denominators.

1. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$ or $\frac{1}{2}$

2. $\frac{6}{8} - \frac{5}{8} = \frac{1}{8}$

What is the difference? Use fraction tiles to help you subtract.

3. $\frac{3}{4} - \frac{7}{12} = \frac{2}{12}$ or $\frac{1}{6}$

4. $\frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

5. $\frac{5}{9} - \frac{1}{6} = \frac{7}{18}$

6. $\frac{7}{8} - \frac{3}{4} = \frac{1}{8}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Subtracting Fractions Task Cards

Students practice subtracting fractions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 95–96

Lesson 9-4

Additional Practice

Name

Review

You can use fraction tiles to represent subtraction of fractions with unlike denominators.

Felipe has $\frac{5}{6}$ foot of string. Tristen has $\frac{1}{2}$ foot of string. How much more string does Felipe have than Tristen?

Felipe has $\frac{2}{6}$ foot, or $\frac{1}{3}$ foot, more string than Tristen.

What subtraction equation do the fraction tiles represent?

1. $\frac{2}{5} - \frac{3}{10} = \frac{1}{10}$

2. $\frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Assign

Use It! Application Station

How Do You Say—Fractions? Students research to find how to say words related to adding and subtracting fractions in other languages.

GO ONLINE

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 95–96

What is the difference?

3. $\frac{5}{12} - \frac{1}{4}$ $\frac{2}{12}$ or $\frac{1}{6}$	4. $\frac{1}{2} - \frac{1}{3}$ $\frac{1}{6}$
5. $\frac{3}{4} - \frac{1}{8}$ $\frac{5}{8}$	6. $\frac{7}{12} - \frac{1}{6}$ $\frac{5}{12}$
7. $\frac{1}{2} - \frac{2}{5}$ $\frac{1}{10}$	8. $\frac{7}{12} - \frac{1}{3}$ $\frac{3}{12}$ or $\frac{1}{4}$

9. Kasey walks $\frac{2}{3}$ mile to get to school. She walks $\frac{1}{4}$ mile to get to her friend's house. How much farther does Kasey walk to school than to her friend's house? $\underline{\frac{5}{12}}$ mile

10. Emmaline creates a tower of blocks that is $\frac{11}{12}$ foot tall. Jed's tower of blocks is $\frac{2}{3}$ foot tall. How much taller is Emmaline's tower than Jed's tower? $\underline{\frac{3}{12} \text{ or } \frac{1}{4}}$ foot

Math @ Home Activity

Student Practice Book

Differentiation Resource Book, p. 96

Lesson 9-4 • Extend Thinking

Represent Subtraction of Fractions with Unlike Denominators

Name ______________________

What is the subtraction equation for the fraction tiles? The first one is done as an example.

	Fraction Tiles	Key	Column B
1.		$\frac{1}{10}$	$\frac{4}{5} - \frac{3}{10} = \frac{5}{10}$ or $\frac{4}{5} - \frac{3}{10} = \frac{1}{2}$
2.		$\frac{1}{2}$	$\frac{3}{2} - \frac{5}{6} = \frac{4}{6}$ or $\frac{3}{2} - \frac{5}{6} = \frac{2}{3}$
3.		$\frac{1}{3}$	$\frac{4}{3} - \frac{7}{12} = \frac{9}{12}$ or $\frac{4}{3} - \frac{7}{12} = \frac{3}{4}$
4.		$\frac{1}{15}$	$\frac{2}{3} - \frac{2}{15} = \frac{8}{15}$
5.		$\frac{1}{3}$	$\frac{5}{3} - \frac{8}{9} = \frac{7}{9}$

Differentiation Resource Book

LESSON **9-5**

Subtract Fractions with Unlike Denominators

Learning Targets

- I can subtract fractions with unlike denominators.
- I can explain how to subtract fractions with unlike denominators.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. $\left(\text{In general, } \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}\right)$.

Math Practices and Processes

MPP Reason abstractly and quantitatively.

Vocabulary

Math Terms	Academic Terms
denominator	reflect
equivalent fractions	suggest

Materials

The materials may be for any part of the lesson.

- fraction tiles
- index cards

Focus

Content Objectives	Language Objectives	SEL Objective
• Students subtract fractions with unlike denominators. • Students explain how to subtract fractions with unlike denominators.	• Students explain how to subtract fractions with unlike denominators using *can* and *should*. • To support sense-making, ELs participate in MLR8: Discussion Supports.	• Students practice staying focused on a mathematical problem for a set time.

Coherence

Previous	Now	Next
• Students understood, recognized, and generated equivalent fractions (Grade 4). • Students used representations to understand subtraction of fractions having unlike denominators (Unit 9).	• Students use equivalent fractions to subtract fractions having unlike denominators.	• Students decompose addends to add mixed numbers (Unit 9). • Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students extend on their basic understanding of operations with fractions.	• Students build proficiency with equivalent fractions and develop general skills and strategies for subtracting fractions.	• Students solve problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine

What's Another Way to Write It? 5–7 min

Build Fluency Students build number sense as they write three different addition expressions that are equivalent to 32.5.

As solutions are given, record them for students to evaluate and compare. These prompts encourage students to talk about their reasoning:

- What type of numbers might you typically think about first?
- How do you show the sum of the values of the number's digits?
- Which two expressions are related? Do you notice any patterns? Explain.

Purpose Students think about real-world examples of using multiples of a certain number.

Notice & Wonder

- What do you notice?
- What do you wonder?

Teaching Tip You may want to have students write down their thoughts on their own before beginning the class discussion.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' noticing and wondering about using equivalent fractions to subtract fractions having unlike denominators and are based on possible comments and questions that students may make during the share out.

- What would you need to know to determine how many toys there are?
- How many toys would there be if the number of toys in each package and the number of packages was switched?

Math is... Mindset

- How do different ideas and viewpoints help you learn better?

SEL Social Awareness – Appreciate Diversity

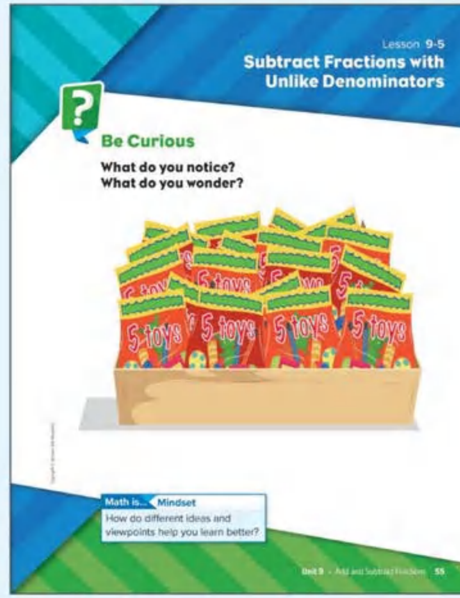
Encourage students to appreciate diversity, so that they create a stronger, more inclusive classroom community. Invite students to set a class Focus Goal for the Notice & Wonder routine by agreeing on a set time that they will, in diverse groups, focus on noticing and wondering. As students work through this time, remind them to be mindful of their collective goal.

Transition to Explore & Develop

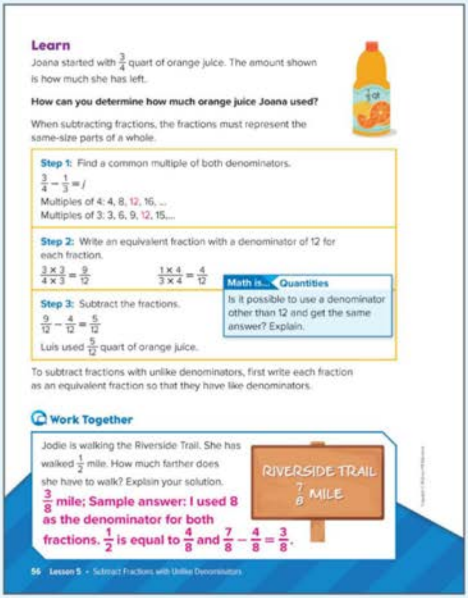
Ask questions that get students thinking about how they can use equivalent fractions to subtract fractions having unlike denominators.

ETP Establish Goals to Focus Learning

- Let's think about how we can use equivalent fractions to subtract fractions having unlike denominators.



Explore & Develop
20 min



Learn

Joana started with $\frac{3}{4}$ quart of orange juice. The amount shown is how much she has left.

How can you determine how much orange juice Joana used?

When subtracting fractions, the fractions must represent the same-size parts of a whole.

Step 1: Find a common multiple of both denominators.

$\frac{3}{4} - \frac{1}{3} = /$

Multiples of 4: 4, 8, 12, 16, ...

Multiples of 3: 3, 6, 9, 12, 15,...

Step 2: Write an equivalent fraction with a denominator of 12 for each fraction.

$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

Step 3: Subtract the fractions.

$\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$

Luis used $\frac{5}{12}$ quart of orange juice.

Math is... Quantities

Is it possible to use a denominator other than 12 and get the same answer? Explain.

To subtract fractions with unlike denominators, first write each fraction as an equivalent fraction so that they have like denominators.

Work Together

Jodie is walking the Riverside Trail. She has walked $\frac{1}{2}$ mile. How much farther does she have to walk? Explain your solution.

$\frac{3}{8}$ mile; Sample answer: I used 8 as the denominator for both fractions. $\frac{1}{2}$ is equal to $\frac{4}{8}$ and $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$.

56 Lesson 5 • Subtract Fractions with Unlike Denominators

1 Pose the Problem

MLR Discussion Supports

As students talk about what they'll be trying to do to solve the problem, have them pay attention to others' understandings in order to increase their ability to add and subtract fractions. Restate statements they make as a question to seek clarification and provide vocabulary or grammar prompts for students who need more guidance.

ETP Pose Purposeful Questions

- What are some representations you could you use to to help you understand this problem?
- What will those representations tell you?

2 Develop the Math

Choose the option that best meets your instructional goals.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How is subtracting fractions having unlike denominators similar to adding fractions having unlike denominators? How is it different?

Key Takeaways

- When subtracting fractions with unlike denominators, it is necessary to generate equivalent fractions with like denominators before subtracting.
- One way to generate equivalent fractions is by multiplying the denominators to determine a common denominator.
- Another way to generate equivalent fractions is by determining a common multiple to identify a common denominator.

Work Together

Students solve a subtraction equation that involves fractions with unlike denominators. Suggest that students use different strategies to determine like denominators. Discuss with them why it is possible to use different like denominators while still determining the same difference.

Common Misconception: Students may think they have to find a denominator that is greater than both 2 and 8. Remind them that one of the existing denominators may already be a common multiple that they can use as a like denominator.

LOM Language of Math

Students may also be aware of multiple as an adjective meaning "more than one," as in "multple choice test."

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore strategies for subtracting fractions that have unlike denominators.

Directions: Have students work together to solve the Pose the Problem. Encourage students to find at least two different ways to solve the problem.

ETP Support Productive Struggle

- What operation should you use to solve the problem?
- How do you need to rewrite the fractions in order to subtract them?
- How can you use multiplication to write equivalent fractions?
- Is there more than one pair of equivalent fractions you can use to find the difference?

Math is... Quantities

- Is it possible to use a denominator other than 12 and get the same answer? Explain.

Students make sense of quantities and their relationships in problem situations.

Activity Debrief: Have students share the methods they used to solve the problem. Facilitate a discussion to ensure students understand that one method of finding like denominators is by multiplying the denominators of the fractions. Another method is to look for other common multiples of the denominators.

Guided Exploration

Students subtract fractions having unlike denominators by writing equivalent fractions with like denominators.

ETP Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- What benchmark numbers will you use to estimate the solution? Why?
- What multiples do 3 and 4 have in common?
- How can you write equivalent fractions?
- How can you subtract once you have written fractions with like denominators?

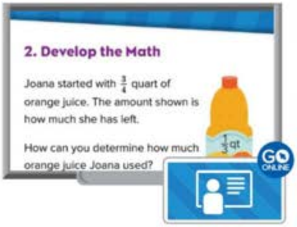
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

Math is... Quantities

- Is it possible to use a denominator other than 12 and get the same answer? Explain.

Students make sense of quantities and their relationships in problem situations.



EL English Learner Scaffolds

Entering/Emerging

Explain *started with.* Put twenty chips on the desk. Take five and count them, and then put them away. Say, *I started with twenty chips. I used five chips. I have fifteen chips left.* Repeat, this time taking away seven chips. Ask, *How many chips did I start with: seven or twenty? Did I use seven or thirteen chips?* and *Do I have seven or thirteen chips* left?

Developing/Expanding

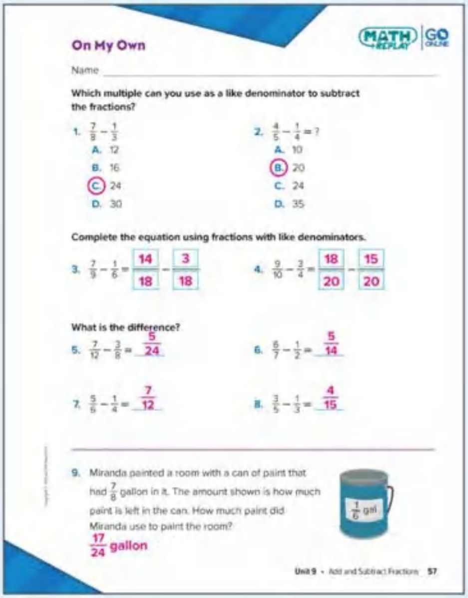
Explain *started with.* Put twenty chips on the desk. Take five and count them, and then put them away. Say, *I started with twenty chips. I used five chips. I have fifteen chips left.* Repeat, this time taking away seven chips. Ask students to complete the following sentences: *I __ (started with) twenty chips. I __ (used) seven chips. I have thirteen chips __ (left).*

Bridging/Reaching

Guide students to the Learn page and ask them to read the word problem, focusing on the words *started with, left,* and *used.* Put twenty chips on the desk. Take five and count them, and then put them away. Ask students to tell you how many chips you started with/used/have left. Then ask students to restate the meaning of the sentences in other words *(You had..., You removed..., You have...remaining.).*

Practice & Reflect

10 min



On My Own

Name ____________________

Which multiple can you use as a like denominator to subtract the fractions?

1. $\frac{7}{8} - \frac{1}{3}$
   A. 12
   B. 16
   C. 24
   D. 30

2. $\frac{4}{5} - \frac{1}{4} = ?$
   A. 10
   B. 20
   C. 24
   D. 35

Complete the equation using fractions with like denominators.

3. $\frac{7}{9} - \frac{1}{6} = \frac{14}{18} - \frac{3}{18}$

4. $\frac{9}{10} - \frac{3}{4} = \frac{18}{20} - \frac{15}{20}$

What is the difference?

5. $\frac{7}{12} - \frac{3}{8} = \frac{5}{24}$

6. $\frac{6}{7} - \frac{1}{2} = \frac{5}{14}$

7. $\frac{5}{6} - \frac{1}{4} = \frac{7}{12}$

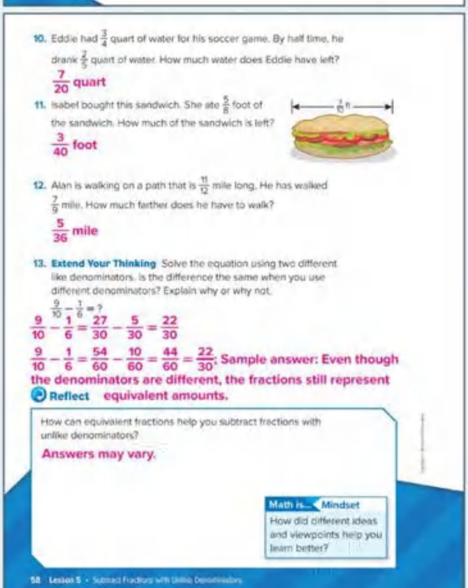
8. $\frac{3}{5} - \frac{1}{3} = \frac{4}{15}$

9. Miranda painted a room with a can of paint that had $\frac{7}{8}$ gallon in it. The amount shown is how much paint is left in the can. How much paint did Miranda use to paint the room?

$\frac{1}{6}$ gal

$\frac{17}{24}$ gallon

Unit 9 • Add and Subtract Fractions 57



10. Eddie had $\frac{3}{4}$ quart of water for his soccer game. By half time, he drank $\frac{2}{5}$ quart of water. How much water does Eddie have left?

$\frac{7}{20}$ quart

11. Isabel bought this sandwich. She ate $\frac{5}{8}$ foot of the sandwich. How much of the sandwich is left?

$\frac{7}{10}$ ft

$\frac{3}{40}$ foot

12. Alan is walking on a path that is $\frac{11}{12}$ mile long. He has walked $\frac{7}{9}$ mile. How much farther does he have to walk?

$\frac{5}{36}$ mile

13. **Extend Your Thinking** Solve the equation using two different like denominators. Is the difference the same when you use different denominators? Explain why or why not.

$\frac{9}{10} - \frac{1}{6} = ?$

$\frac{9}{10} - \frac{1}{6} = \frac{27}{30} - \frac{5}{30} = \frac{22}{30}$

$\frac{9}{10} - \frac{1}{6} = \frac{54}{60} - \frac{10}{60} = \frac{44}{60} = \frac{22}{30}$; Sample answer: Even though the denominators are different, the fractions still represent equivalent amounts.

Reflect

How can equivalent fractions help you subtract fractions with unlike denominators?

Answers may vary.

Math is... Mindset
How did different ideas and viewpoints help you learn better?

58 Lesson 5 • Subtract Fractions with Unlike Denominators

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Misconception: Exercises 1–8 Students may only find like denominators by multiplying the denominators by each other. While that is not an error, encourage students to think about multiples that both denominators have in common that may be less than the product of the two denominators, because it can make working with the numerators easier.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–12	2	Application
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can equivalent fractions help you subtract fractions with unlike denominators?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did different ideas and viewpoints help you learn better?

Students reflect on how they practiced social awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can subtract fractions with unlike denominators.
- I can explain how to subtract fractions with unlike denominators.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Subtract fractions with unlike denominators	5.NF.A.1
2	1	Subtract fractions with unlike denominators	5.NF.A.1
3	2	Subtract fractions with unlike denominators	5.NF.A.1
4	2	Subtract fractions with unlike denominators	5.NF.A.1

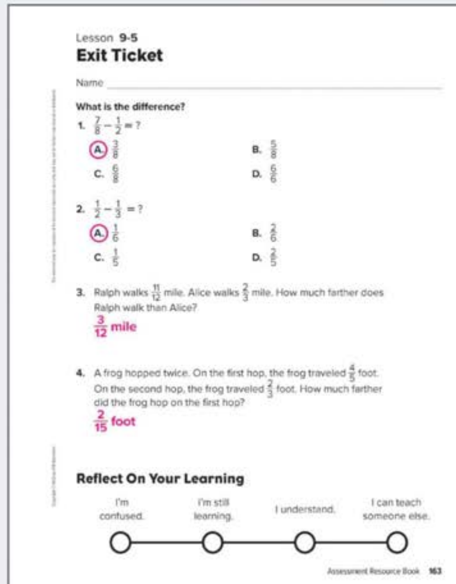
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking


Lesson 9-5

Exit Ticket

Name ____________________

What is the difference?

1. $\frac{7}{8} - \frac{1}{2} = ?$

A. $\frac{3}{8}$ B. $\frac{5}{8}$
C. $\frac{6}{8}$ D. $\frac{6}{6}$

2. $\frac{1}{2} - \frac{1}{3} = ?$

A. $\frac{1}{6}$ B. $\frac{2}{6}$
C. $\frac{1}{5}$ D. $\frac{2}{5}$

3. Ralph walks $\frac{11}{12}$ mile. Alice walks $\frac{2}{3}$ mile. How much farther does Ralph walk than Alice?

$\frac{3}{12}$ mile

4. A frog hopped twice. On the first hop, the frog traveled $\frac{4}{5}$ foot. On the second hop, the frog traveled $\frac{2}{3}$ foot. How much farther did the frog hop on the first hop?

$\frac{2}{15}$ foot

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 163

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Down to Zero

Work with students in groups. Provide students fraction tiles and cards that show fractions less than 1. One student draws two fraction cards with unlike denominators to show the group. Students use the given fractions in a subtraction equation and solve the equation. If students have difficulty, help them find equivalent fractions using fraction tiles, then use the renamed fractions to write and solve a subtraction equation.

GO ONLINE

Take Another Look Lesson

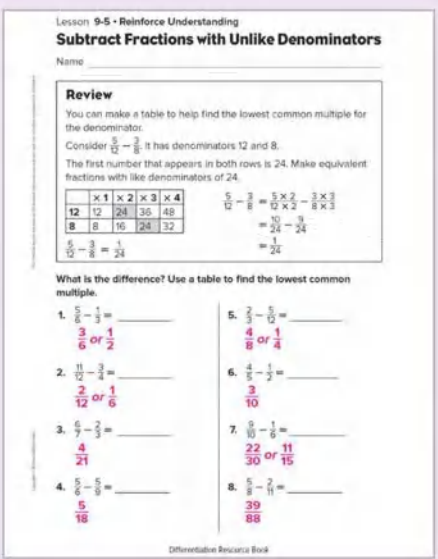
Assign the interactive lesson to reinforce targeted skills.

- Subtract Fractions (Rename Both)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 97



Lesson 9-5 • Reinforce Understanding

Subtract Fractions with Unlike Denominators

Name ____________

Review

You can make a table to help find the lowest common multiple for the denominator.

Consider $\frac{5}{12} - \frac{3}{8}$. It has denominators 12 and 8.

The first number that appears in both rows is 24. Make equivalent fractions with like denominators of 24.

	× 1	× 2	× 3	× 4
12	12	24	36	48
8	8	16	24	32

$\frac{5}{12} - \frac{3}{8} = \frac{1}{24}$

$$\frac{5}{12} - \frac{3}{8} = \frac{5 \times 2}{12 \times 2} - \frac{3 \times 3}{8 \times 3} = \frac{10}{24} - \frac{9}{24} = \frac{1}{24}$$

What is the difference? Use a table to find the lowest common multiple.

1. $\frac{5}{6} - \frac{1}{3} =$ ______ $\frac{3}{6}$ or $\frac{1}{2}$
2. $\frac{11}{12} - \frac{3}{4} =$ ______ $\frac{2}{12}$ or $\frac{1}{6}$
3. $\frac{6}{7} - \frac{2}{3} =$ ______ $\frac{4}{21}$
4. $\frac{5}{6} - \frac{5}{9} =$ ______ $\frac{5}{18}$
5. $\frac{2}{3} - \frac{5}{12} =$ ______ $\frac{4}{8}$ or $\frac{1}{4}$
6. $\frac{4}{5} - \frac{1}{2} =$ ______ $\frac{3}{10}$
7. $\frac{9}{10} - \frac{1}{6} =$ ______ $\frac{22}{30}$ or $\frac{11}{15}$
8. $\frac{5}{8} - \frac{2}{11} =$ ______ $\frac{39}{88}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Subtraction Tic Tac Toe

Students practice subtracting fractions.

GO ONLINE

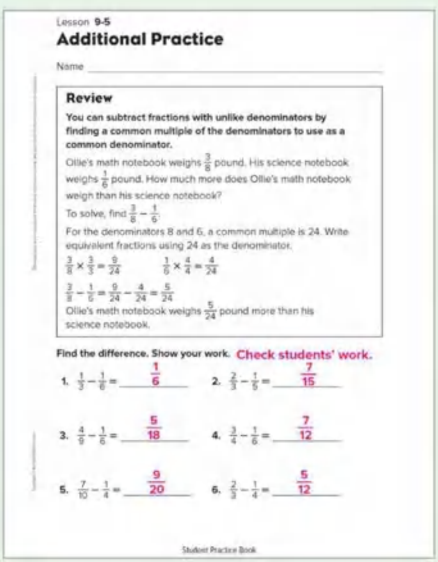
Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 97–98



Lesson 9-5

Additional Practice

Name ____________

Review

You can subtract fractions with unlike denominators by finding a common multiple of the denominators to use as a common denominator.

Ollie's math notebook weighs $\frac{3}{8}$ pound. His science notebook weighs $\frac{1}{6}$ pound. How much more does Ollie's math notebook weigh than his science notebook?

To solve, find $\frac{3}{8} - \frac{1}{6}$.

For the denominators 8 and 6, a common multiple is 24. Write equivalent fractions using 24 as the denominator.

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$ $\frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$

$\frac{3}{8} - \frac{1}{6} = \frac{9}{24} - \frac{4}{24} = \frac{5}{24}$

Ollie's math notebook weighs $\frac{5}{24}$ pound more than his science notebook.

Find the difference. Show your work. Check students' work.

1. $\frac{1}{3} - \frac{1}{6} =$ $\frac{1}{6}$
2. $\frac{2}{3} - \frac{1}{5} =$ $\frac{7}{15}$
3. $\frac{4}{9} - \frac{1}{6} =$ $\frac{5}{18}$
4. $\frac{3}{4} - \frac{1}{6} =$ $\frac{7}{12}$
5. $\frac{7}{10} - \frac{1}{4} =$ $\frac{9}{20}$
6. $\frac{2}{3} - \frac{1}{4} =$ $\frac{5}{12}$

Student Practice Book

Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Assign

Use It! Application Station

Get Moving Students design a car and measure the distance it travels.

The content of this card has concepts covered later in Lesson 9-8. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 97–98

Solve the problem. Show your work. Check students' work.

7. Raymundo walks $\frac{9}{10}$ mile. Mica walks $\frac{3}{5}$ mile. How much farther does Raymundo walk than Mica? $\frac{3}{10}$ mile

8. Yesterday, Deborah worked for $\frac{5}{6}$ hour on her homework. Today she worked $\frac{3}{4}$ hour on her homework. How much longer did Deborah spend on her homework yesterday than today? $\frac{1}{12}$ hour

9. A piece of string is $\frac{9}{10}$ foot long. Jody cuts the string so that one piece is $\frac{1}{2}$ foot long. How long is the other piece of string? $\frac{4}{10}$ or $\frac{2}{5}$ foot

10. The waterfall is $\frac{3}{4}$ kilometer from the nature center. Gary has hiked $\frac{2}{5}$ kilometer so far on the way to the waterfall. How much farther does Gary have to hike to get to the waterfall? $\frac{7}{20}$ kilometer

Math @Home Activity

Student Practice Book

Differentiation Resource Book, p. 98

Lesson 9-5 • Extend Thinking

Subtract Fractions with Unlike Denominators

Name ______________________

Fill in the missing values to complete each equation. The first one is done as an example for you. Show your work.

1. $\frac{7}{9} - \frac{2}{?} = \frac{?}{45}$
$\frac{7}{9} - \frac{2}{?} = \frac{?}{9 \times 5}$
$\frac{7}{9} - \frac{2}{5} = \frac{?}{9 \times 5}$
$\frac{7 \times 5}{9 \times 5} - \frac{2 \times 9}{5 \times 9} = \frac{?}{9 \times 5}$
$\frac{35}{45} - \frac{18}{45} = \frac{17}{45}$

2. $\frac{5}{\square} - \frac{1}{3} = \frac{\square}{33}$
$\frac{5}{?} - \frac{1}{3} = \frac{?}{11 \times 3}$
$\frac{5}{11} - \frac{1}{3} = \frac{?}{11 \times 3}$
$\frac{5 \times 3}{11 \times 3} - \frac{1 \times 11}{3 \times 11} = \frac{?}{5 \times 7}$
$\frac{15}{33} - \frac{11}{33} = \frac{4}{33}$
$\frac{5}{11} - \frac{1}{3} = \frac{4}{33}$ is the equation.

3. $\frac{3}{\square} - \frac{2}{5} = \frac{\square}{35}$
$\frac{3}{?} - \frac{2}{5} = \frac{?}{7 \times 5}$
$\frac{3}{7} - \frac{2}{5} = \frac{?}{7 \times 5}$
$\frac{3 \times 5}{7 \times 5} - \frac{2 \times 7}{5 \times 7} = \frac{?}{7 \times 5}$
$\frac{15}{35} - \frac{14}{35} = \frac{1}{35}$
$\frac{3}{7} - \frac{2}{5} = \frac{1}{35}$ is the equation.

4. $\frac{9}{5} - \frac{\square}{3} = \frac{17}{\square}$
$\frac{9}{5} - \frac{?}{3} = \frac{17}{5 \times 3}$
$\frac{9 \times 3}{5 \times 3} - \frac{? \times 5}{3 \times 5} = \frac{17}{5 \times 3}$
$\frac{27}{15} - \frac{? \times 5}{15} = \frac{17}{15}$
$\frac{27}{15} - \frac{10 \text{ or } 2 \times 5}{15} = \frac{17}{15}$
$\frac{9}{5} - \frac{2}{3} = \frac{17}{15}$ is the equation.

5. $\frac{7}{9} - \frac{\square}{8} = \frac{11}{\square}$
$\frac{7}{9} - \frac{?}{8} = \frac{11}{9 \times 8}$
$\frac{7 \times 8}{9 \times 8} - \frac{? \times 9}{8 \times 9} = \frac{11}{9 \times 8}$
$\frac{56}{72} - \frac{? \times 9}{72} = \frac{11}{72}$
$\frac{56}{72} - \frac{45 \text{ or } 5 \times 9}{72} = \frac{11}{72}$
$\frac{7}{9} - \frac{5}{8} = \frac{11}{72}$ is the equation.

6. $\frac{\square}{4} - \frac{\square}{6} = \frac{7}{12}$
$\frac{?}{4} - \frac{?}{6} = \frac{7}{4 \times 3}$
$\frac{? \times 3}{4 \times 3} - \frac{? \times 2}{6 \times 2} = \frac{7}{4 \times 3}$
$\frac{3 \times 3}{4 \times 3} - \frac{1 \times 2}{6 \times 2} = \frac{7}{4 \times 3}$
$\frac{9}{12} - \frac{2}{12} = \frac{7}{12}$
$\frac{3}{4} - \frac{1}{6} = \frac{7}{12}$ is the equation.

Differentiation Resource Book

LESSON **9-6**

Add Mixed Numbers with Unlike Denominators

Learning Targets

- I can add mixed numbers with unlike denominators.
- I can explain how to add mixed numbers with unlike denominators.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. $\left(\text{In general, } \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}\right)$.

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
equivalent fractions	establish
mixed number	relevant

Materials

The materials may be for any part of the lesson.

- fraction tiles
- index cards

Focus

Content Objectives	Language Objectives	SEL Objective
• Students add mixed numbers with unlike denominators. • Students explain how to add mixed numbers with unlike denominators.	• Students talk about adding mixed numbers with unlike denominators using *can* and *use*. • To support maximizing linguistic and cognitive meta-awareness, ELs participate in MLR5: Co-Craft Questions and Problems.	• Students identify multiple possible solutions for a given math problem.

Coherence

Previous	Now	Next
• Students understood, recognized, and generated equivalent fractions (Grade 4). • Students used equivalent fractions to subtract fractions having unlike denominators (Unit 9).	• Students decompose addends to add mixed numbers.	• Students decompose mixed numbers and use fractions greater than one to subtract mixed numbers (Unit 9). • Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding of fraction concepts and addition of fractions and mixed numbers with unlike denominators.	• Students develop proficiency adding mixed numbers with unlike denominators and develop skills to handle a range of cases.	• Students solve problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
What's Another Way to Write It? 5–7 min

Build Fluency Students build number sense as they write three different subtraction expressions to represent the number 27.75.

As solutions are given, record them for students to evaluate and compare. These prompts encourage students to talk about their reasoning:

- What type of subtraction expression might you think about first?
- What strategy could you use to find a minuend (or first number in the subtraction expression) for the expression?
- What strategy could you use to find a subtrahend (or second number in the subtraction expression) for the expression?
- Do you notice any patterns? Explain.

Launch 5–7 min

Sense-Making Routine

Purpose Students begin thinking about decomposing a mixed number, or how they can count the number of wholes, then parts of the whole.

Notice & Wonder

- What question could you ask?

Teaching Tip You may want to have students write down their questions on their own before discussing as a whole class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of decomposing addends to add mixed numbers and are based on possible comments and questions that students may make during the share out.

- How do quarters relate to dollars?
- How can you determine the value of the money shown?

Math is... Mindset

- What helps you know when there is a problem?

SEL Responsible Decision-Making – Identify Problems

Elicit from students that a key step in problem solving is analyzing information to identify the task. As students work through the Notice & Wonder routine, provide specific, constructive feedback that can help guide each student toward identifying any problems. As students work with adding mixed numbers with unlike denominators throughout the lesson, encourage them to connect and use their prior knowledge of adding fractions with unlike denominators. Encouraging use of prior knowledge can help students feel more competent and promote effective problem identification.

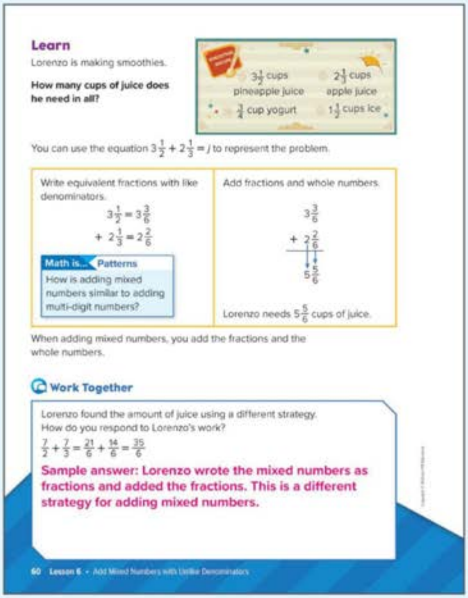
Transition to Explore & Develop

Ask questions that get students thinking about decomposing addends to add mixed numbers.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can decompose addends to add mixed numbers.

Explore & Develop
20 min



Learn

Lorenzo is making smoothies.

How many cups of juice does he need in all?

$3\frac{1}{2}$ cups pineapple juice

$2\frac{1}{3}$ cups apple juice

$\frac{3}{4}$ cup yogurt

$1\frac{1}{2}$ cups ice

You can use the equation $3\frac{1}{2} + 2\frac{1}{3} = j$ to represent the problem.

Write equivalent fractions with like denominators.	Add fractions and whole numbers.
$3\frac{1}{2} = 3\frac{3}{6}$ $+\ 2\frac{1}{3} = 2\frac{2}{6}$	$3\frac{3}{6}$ $+\ 2\frac{2}{6}$ $5\frac{5}{6}$
Math is... Patterns How is adding mixed numbers similar to adding multi-digit numbers?	Lorenzo needs $5\frac{5}{6}$ cups of juice.

When adding mixed numbers, you add the fractions and the whole numbers.

Work Together

Lorenzo found the amount of juice using a different strategy. How do you respond to Lorenzo's work?

$\frac{7}{2} + \frac{7}{3} = \frac{21}{6} + \frac{14}{6} = \frac{35}{6}$

Sample answer: Lorenzo wrote the mixed numbers as fractions and added the fractions. This is a different strategy for adding mixed numbers.

60 Lesson 6 • Add Mixed Numbers with Unlike Denominators

1 Pose the Problem

ETP Pose Purposeful Questions

- What information do you need to solve the problem? Where can you find it?
- What kinds of numbers are you working with?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Co-Craft Questions and Problems

Pair students and have them co-create a problem similar to the one on the Learn page. Have them work together to solve their problem and then trade their problem with another pair. After each pair solves the other pair's problem, have them form a group of four to check solutions and correct any mistakes that may have been made. Revisit the routine throughout the lesson for reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you decompose a mixed number?
- Explain how you can add mixed numbers that have unlike denominators.

Key Takeaway

- The sum of mixed numbers with unlike denominators can be found by decomposing the mixed number into whole-number parts and fraction parts.

Work Together

Students explore a different strategy used to solve the same problem and respond to it.

Common Error: Students may see that the numerators are the same in both fractions and think that Lorenzo does not need to determine equivalent fractions with like denominators. Remind students that the denominators must always be the same in order to add fractions.

LOM Language of Math

Remind students that, in math, to *decompose* a number means to break it down into simpler parts. The prefix *de-* means "off" or "from" and *compose* means "to make up a whole." Anything that decomposes breaks down into simpler parts that make up the whole, such as plants.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore extending the partial sums strategy to add mixed numbers.

Directions: Discuss with students how they used the partial sums strategy to add whole numbers and to add decimals.

- Do you think you can use a similar strategy to add mixed numbers?

Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- What operation is needed to solve the problem?
- Is there a way you can decompose the mixed numbers?
- Why can you change the order of the addends?
- Is your answer reasonable? How do you know?

Activity Debrief: Discuss with students that decomposing mixed numbers is one strategy for adding mixed numbers.

Math is... Patterns

- How is decomposing mixed numbers similar to the partial sums strategy?

Students step back for an overview and shift perspective, connecting a strategy for adding mixed numbers to a strategy for adding whole numbers.

Guided Exploration

Students add mixed numbers by decomposing them into a whole number and fraction.

ETP Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the sum. Ask:

- Will you use whole numbers or mixed numbers to estimate the sum? Why?
- If you use mixed numbers, what benchmark fractions will you use? Why?
- **Think About It:** What are some strategies you used to add whole numbers?
- Why can you change the order of the addends?
- How can you add $\frac{1}{2}$ and $\frac{1}{3}$?

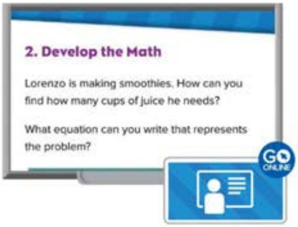
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

Math is... Patterns

- How is decomposing mixed numbers similar to the partial sums strategy?

Students step back for an overview and shift perspective, connecting a strategy for adding mixed numbers to a strategy for adding whole numbers.



EL English Learner Scaffolds

Entering/Emerging

Explain *in all*. Put 10 red chips on the desk. Say, *I have 10 red chips.* Put 8 blue chips on the desk. Say, *I have 8 blue chips.* Then count all the chips. Say, *I have 18 chips in all.* Repeat again with new chips. Then, repeat once more, placing 9 red chips and 6 blue chips on the desk. Ask, *Do I have 6, 9, or 15 chips in all?*

Developing/Expanding

Explain *in all*. Put 10 red chips on the desk. Say, *I have 10 red chips.* Put 8 blue chips on the desk. Say, *I have 8 blue chips.* Then count all the chips. Say, *I have 18 chips in all.* Repeat again with new chips. Then, repeat once more, placing 9 red chips and 6 blue chips on the desk. Ask, *How many chips do I have in all?*

Bridging/Reaching

Guide students to the problem at the top of the Learn page. Ask them to focus on the phrase *in all*. Instruct them to think of similar words they've used in the past that mean the same (*altogether, total, etc.*). Allow students to use a dictionary or thesaurus if desired. Then, ask students to use *in all* in a sentence, demonstrating with manipulatives. Provide validation and correction as needed.

Practice & Reflect 10 min

On My Own

Name ______________________

What is the sum? Choose the correct answer.

1. $3\frac{3}{10} + 4\frac{2}{5} = ?$
   A. $7\frac{5}{10}$
   B. $8\frac{7}{10}$
   C. $8\frac{5}{10}$
   D. $7\frac{7}{10}$

2. $1\frac{3}{4} + 5\frac{1}{6} = ?$
   A. $6\frac{4}{6}$
   B. $7\frac{4}{12}$
   C. $6\frac{11}{12}$
   D. $6\frac{5}{6}$

What is the sum?

3. $2\frac{2}{3} + 3\frac{1}{4} =$ $5\frac{11}{12}$
4. $4\frac{1}{2} + 5\frac{1}{3} =$ $9\frac{5}{6}$
5. $6\frac{3}{8} + 2\frac{1}{6} =$ $8\frac{13}{24}$
6. $3\frac{2}{9} + 1\frac{3}{4} =$ $4\frac{35}{36}$
7. $2\frac{1}{5} + 3\frac{1}{2} =$ $5\frac{7}{10}$
8. $5\frac{1}{3} + 4\frac{2}{5} =$ $9\frac{11}{15}$

9. Jill bought the strawberries and blueberries shown at a farmer's market. How many pounds of fruit did Jill buy?

   $3\frac{7}{8}$ pounds

10. Timothy rides his bike $1\frac{1}{2}$ miles to school. After school, he rides $2\frac{2}{5}$ miles to his piano lesson, then 2 miles back home. How many miles does Timothy ride in all?

    $5\frac{9}{10}$ miles

11. Marcus builds the body of this snowman. He then builds the head. How tall is Marcus's snowman?

    $4\frac{19}{24}$ feet tall

12. Solve the equation.

    $4\frac{7}{10} + 2\frac{3}{4} = ?$

    What do you notice about the sum of the two fractions? How can you rewrite the sum?

    $6\frac{29}{20}$; Sample answer: The sum is greater than 1; $7\frac{9}{20}$

13. **Extend Your Thinking** The chef at the restaurant uses $6\frac{2}{3}$ pounds of mushrooms on Saturday. On Sunday, she uses $1\frac{3}{8}$ more pounds of mushrooms than she did on Saturday. How many pounds of mushrooms did the chef use in all over the weekend?

    $14\frac{17}{24}$ pounds

Reflect

How can you add mixed numbers with unlike denominators?

Answers may vary.

Math is... Mindset

What helped you know when there was a problem?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–8 Students may be focused on determining like denominators and adding the fractions, and forget to add the whole numbers and include them as part of the sum. Remind students to write out the equation with the decomposed numbers and to determine the sum of the whole numbers.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–12	2	Application
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you add mixed numbers with unlike denominators?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you know when there was a problem?

Students reflect on how they practiced responsible decision-making.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can add mixed numbers with unlike denominators.
- I can explain how to add mixed numbers with unlike denominators.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Add mixed numbers with unlike denominators	5.NF.A.1
2	1	Add mixed numbers with unlike denominators	5.NF.A.1
3	2	Add mixed numbers with unlike denominators	5.NF.A.1
4	2	Add mixed numbers with unlike denominators	5.NF.A.1

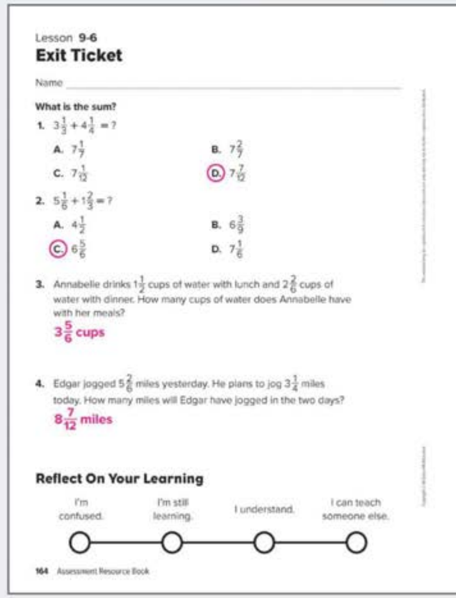
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 9-6

Exit Ticket

Name ______________________________

What is the sum?

1. $3\frac{1}{3} + 4\frac{1}{4} = ?$

   A. $7\frac{1}{7}$ B. $7\frac{2}{7}$

   C. $7\frac{1}{12}$ D. $7\frac{7}{12}$

2. $5\frac{1}{6} + 1\frac{2}{3} = ?$

   A. $4\frac{1}{2}$ B. $6\frac{3}{9}$

   C. $6\frac{5}{6}$ D. $7\frac{1}{6}$

3. Annabelle drinks $1\frac{1}{2}$ cups of water with lunch and $2\frac{2}{6}$ cups of water with dinner. How many cups of water does Annabelle have with her meals?

   $3\frac{5}{6}$ cups

4. Edgar jogged $5\frac{2}{6}$ miles yesterday. He plans to jog $3\frac{1}{4}$ miles today. How many miles will Edgar have jogged in the two days?

   $8\frac{7}{12}$ miles

Reflect On Your Learning

I'm confused. I'm still learning. I understand. I can teach someone else.

164 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Splitting Sticky Notes

Work with students in groups. Each student chooses three number cards. The greatest number will be the numerator, and the other two numbers will be the denominator and whole number. Students write the whole number and fraction on separate sticky notes. Students decompose the fraction into a mixed number, then add the mixed number and the whole number.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Add Unlike Mixed Numbers

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 99

Lesson **9-6 • Reinforce Understanding**

Add Mixed Numbers with Unlike Denominators

Name ______________________

Review

You can add mixed numbers by decomposing the addends into whole numbers and fractions. Consider the equation $\frac{5}{4} + \frac{2}{9} = ?$

$$\begin{array}{r} 5\frac{1}{4} \\ +2\frac{2}{9} \\ \hline \end{array} \rightarrow \begin{array}{r} 5\frac{1\times 9}{4\times 9} \\ +2\frac{2\times 4}{9\times 4} \\ \hline \end{array} \rightarrow \begin{array}{r} 5\frac{9}{36} \\ +2\frac{8}{36} \\ \hline 7\frac{17}{36} \end{array}$$

So, $5\frac{1}{4} + 2\frac{2}{9} = 7\frac{15}{36}$

What is the sum?

1. $2\frac{3}{4} + 1\frac{1}{8}$ = $3\frac{7}{8}$
2. $6\frac{2}{5} + 3\frac{3}{10}$ = $9\frac{7}{10}$
3. $3\frac{1}{9} + 4\frac{5}{6}$ = $7\frac{17}{18}$
4. $1\frac{1}{2} + 5\frac{3}{7}$ = $6\frac{13}{14}$
5. $3\frac{2}{9} + 2\frac{2}{3}$ = $5\frac{8}{9}$
6. $1\frac{1}{4} + 3\frac{5}{8}$ = $4\frac{7}{8}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Mixed Number Addition Concentration

Students practice adding mixed numbers.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 99–100

Lesson **9-6**

Additional Practice

Name ______________________

Review

You can add mixed numbers by adding the whole number parts and the fractional parts.

Myra walks $3\frac{1}{3}$ miles on Saturday and $4\frac{1}{4}$ miles on Sunday. How many miles does Myra walk on those two days?

To solve, find $3\frac{1}{3} + 4\frac{1}{4}$.

Decompose the addends: $3\frac{1}{3} = 3 + \frac{1}{3}$ and $4\frac{1}{4} = 4 + \frac{1}{4}$

Rewrite the sum: $3\frac{1}{3} + 4\frac{1}{4} = 3 + \frac{1}{3} + 4 + \frac{1}{4}$

Change the order of the addends so that the whole numbers are together and the fractions are together:

$3 + \frac{1}{3} + 4 + \frac{1}{4} = 3 + 4 + \frac{1}{3} + \frac{1}{4}$

Add the whole numbers: $3 + 4 = 7$

Add the fractions: $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

Add the whole numbers and the fractions: $7 + \frac{7}{12} = 7\frac{7}{12}$

Myra walked $7\frac{7}{12}$ miles on the two days.

What is the sum? Show your work. Check students' work.

1. $2\frac{1}{6} + 5\frac{2}{3} =$ $7\frac{5}{6}$
2. $8\frac{3}{4} + 3\frac{1}{10} =$ $11\frac{17}{20}$
3. $6\frac{3}{5} + 4\frac{1}{3} =$ $10\frac{14}{15}$
4. $5\frac{1}{4} + 3\frac{2}{3} =$ $8\frac{11}{12}$

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

WORKSTATIONS

Use It! Application Station

Create and Solve Students create a multi-step problem that adds and subtracts mixed numbers to solve. Then they use a digital tool to present the problem.

The content of this card has concepts covered later in Lesson 9-9. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

INDEPENDENT WORK

Student Practice Book, pp. 99–100

Solve the problem. Show your work.

5. Darcie runs $4\frac{1}{2}$ miles in the morning and $3\frac{3}{8}$ miles in the evening. How many miles does Darcie run in all?
   $7\frac{7}{8}$ miles

6. A frog jumps $2\frac{1}{4}$ feet. Then the frog jumps $2\frac{1}{6}$ feet. What is the total distance the frog jumped? $4\frac{5}{12}$ feet

7. A plant is $6\frac{1}{2}$ inches tall. After one week, the plant grows $2\frac{2}{5}$ inches. Now how tall is the plant? $8\frac{9}{10}$ inches

8. Clara buys two watermelons for a family picnic. One watermelon weighs $6\frac{1}{3}$ pounds. The second watermelon weighs $5\frac{5}{12}$ pounds. How much do the two watermelons weigh together?
   $11\frac{9}{12}$ or $11\frac{3}{4}$ pounds

Math @ Home Activity With your child, find two objects around your home that can be placed next to each other. Measure the length of each, using a foot-long ruler, and write the measurement as a mixed number of feet. Have your child add the two measurements, and then place the two objects next to each other and measure the total length to verify the sum. Find two other objects, and repeat as time permits.

Student Practice Book

Differentiation Resource Book, p. 100

Lesson 9-6 • Extend Thinking

Add Mixed Numbers with Unlike Denominators

Name ______________________

Fill in the missing values to complete each equation. The first one is done as an example. Show your work.

1. $1\frac{?}{3} + ?\frac{?}{?} = 5\frac{11}{12}$

   $1\frac{?}{3} + 4\frac{?}{4} = 5\frac{11}{3 \times 4}$

   $1\frac{? \times 4}{3 \times 4} + 4\frac{? \times 3}{4 \times 3} = 5\frac{8+3}{3 \times 4}$

   $1\frac{2 \times 4}{3 \times 4} + 4\frac{1 \times 3}{4 \times 3} = 5\frac{8+3}{3 \times 4}$

   $1\frac{8}{12} + 4\frac{3}{12} = 5\frac{11}{12}$

   $1\frac{2}{3} + 4\frac{1}{4} = 5\frac{11}{12}$

2. $?\frac{?}{3} + 4\frac{?}{?} = 7\frac{13}{15}$

   $3\frac{?}{3} + 4\frac{?}{5} = 7\frac{13}{3 \times 5}$

   $3\frac{2 \times 5}{3 \times 5} + 4\frac{1 \times 3}{5 \times 3} = 7\frac{10+3}{3 \times 5}$

   $3\frac{10}{15} + 4\frac{3}{15} = 7\frac{13}{15}$

   $3\frac{2}{3} + 4\frac{1}{5} = 7\frac{13}{15}$

3. $5\frac{?}{2} + ?\frac{?}{?} = 7\frac{9}{10}$

   $5\frac{?}{2} + 2\frac{?}{5} = 7\frac{9}{2 \times 5}$

   $5\frac{1 \times 5}{2 \times 5} + 2\frac{2 \times 2}{5 \times 2} = 7\frac{5+4}{2 \times 5}$

   $5\frac{5}{10} + 2\frac{4}{10} = 7\frac{9}{10}$

   $5\frac{1}{2} + 2\frac{2}{5} = 7\frac{9}{10}$

4. $3\frac{?}{?} + ?\frac{?}{7} = 4\frac{13}{14}$

   $3\frac{?}{2} + 1\frac{?}{7} = 4\frac{13}{2 \times 7}$

   $3\frac{1 \times 7}{2 \times 7} + 1\frac{3 \times 2}{7 \times 2} = 4\frac{7+6}{2 \times 7}$

   $3\frac{7}{14} + 1\frac{6}{14} = 4\frac{13}{14}$

   $3\frac{1}{2} + 1\frac{3}{7} = 4\frac{13}{14}$

5. $1\frac{?}{4} + ?\frac{?}{13} = 2\frac{47}{52}$

   $1\frac{?}{4} + 1\frac{?}{13} = 2\frac{47}{4 \times 13}$

   $1\frac{3 \times 13}{4 \times 13} + 1\frac{2 \times 4}{13 \times 4} = 2\frac{8+39}{4 \times 13}$

   $1\frac{39}{52} + 1\frac{8}{52} = 2\frac{47}{52}$

   $1\frac{3}{4} + 1\frac{2}{13} = 2\frac{47}{52}$

6. $3\frac{?}{?} + ?\frac{?}{9} = 5\frac{38}{45}$

   $3\frac{?}{?} + ?\frac{?}{9} = 5\frac{38}{45}$

   $3\frac{2 \times 9}{5 \times 9} + 2\frac{4 \times 5}{9 \times 5} = 5\frac{18+20}{45}$

   $3\frac{18}{45} + 2\frac{20}{45} = 5\frac{38}{45}$

   $3\frac{2}{5} + 2\frac{4}{9} = 5\frac{38}{45}$

Differentiation Resource Book

LESSON 9-7

Subtract Mixed Numbers with Unlike Denominators

Learning Targets

- I can subtract mixed numbers with unlike denominators.
- I can explain how to subtract mixed numbers with unlike denominators.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. $\left(\text{In general, } \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}\right)$.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

Vocabulary

Math Terms	Academic Terms
equivalent fractions	accurate
mixed number	assert

Materials

The materials may be for any part of the lesson.

- blank spinner
- fraction tiles

Focus

Content Objectives	Language Objectives	SEL Objective
• Students subtract mixed numbers with unlike denominators. • Students explain how to subtract mixed numbers with unlike denominators.	• Students talk about subtracting mixed numbers with unlike denominators using *can*, *should*, *same*, and *different*. • To support optimizing output, ELs participate in MLR7: Compare and Connect.	• Students practice segmenting a complex mathematical task into smaller achievable tasks.

Coherence

Previous	Now	Next
• Students understood, recognized, and generated equivalent fractions (Grade 4). • Students decomposed addends to add mixed numbers (Unit 9).	• Students decompose mixed numbers and use fractions greater than one to subtract mixed numbers.	• Students use regrouping to add and subtract mixed numbers (Unit 9). • Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students interpret and use representations to develop their understanding of subtracting mixed numbers with unlike denominators.	• Students build proficiency through repeated use of representations, such as pictures, tools, and equations.	• Students solve problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Would You Rather?

5–7 min

Build Fluency Students build number sense and understanding of fractions as they compare the sums of two fractions.

These prompts encourage students to talk about their reasoning:

- What strategy did you use to compare the amounts?
- Did you use equivalent fractions to compare the amounts? Why or why not?

Purpose Students begin to think about the difference between two mixed numbers having unlike denominators.

Notice & Wonder

- What could the question be?

Teaching Tip You may want to have students represent the image using their own fraction tiles to allow them to more closely examine the representation of the numbers.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' noticing and wondering about decomposing mixed numbers and using fractions greater than one to subtract mixed numbers, and are based on possible comments and questions that students may make during the share out.

- What numbers do the fraction tiles represent? How do you know?
- What kind of numbers do the fraction tiles represent? How do you know?

Math is... Mindset

- What helps you work well in a team?

SEL Relationship Skills: Teamwork

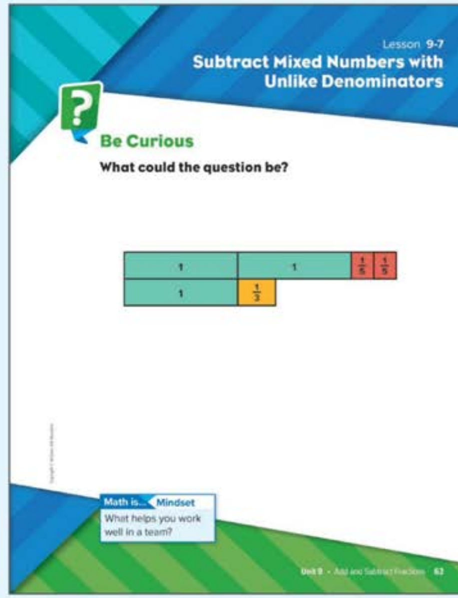
Establish a classroom culture in which students work effectively as a team, establishing a stronger learning community. Have students work as teams on the Notice & Wonder routine with each member listing as many questions as they can. As a group, have students compare lists and discuss questions that group members have in common.

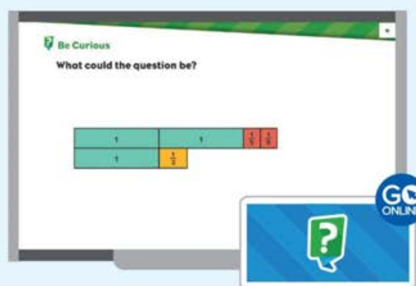
Transition to Explore & Develop

Ask questions that get students thinking about how they can decompose mixed numbers and use fractions greater than one to subtract mixed numbers.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about decomposing mixed numbers and using fractions greater than one to subtract mixed numbers.





Explore & Develop

20 min

Learn

How much longer is Jorge's leash than Magda's leash?

You can use the equation $2\frac{2}{3} - 1\frac{1}{4} = d$ to represent the problem.

One Way Rewrite the fractions as equivalent fractions with like denominators. Then, subtract.

$$2\frac{2}{3} = 2\frac{8}{12}$$
$$-\ 1\frac{1}{4} = 1\frac{3}{12}$$

$$\begin{array}{r} 2\frac{8}{12} \\ -\ 1\frac{3}{12} \\ \hline 1\frac{5}{12} \end{array}$$

Subtract the fractions, then the whole numbers.

Jorge's leash is $1\frac{5}{12}$ yards longer than Magda's leash.

Another Way Rewrite the mixed numbers as equivalent fractions with like denominators.

$$2\frac{2}{3} = \frac{8}{3} = \frac{32}{12}$$
$$-1\frac{1}{4} = \frac{5}{4} = \frac{15}{12}$$
$$\frac{32}{12} - \frac{15}{12} = \frac{17}{12}$$

Math is... Structure
How can you show that the differences found are the same?

Jorge's leash is $\frac{17}{12}$ yards longer than Magda's leash.

You can use different strategies to subtract mixed numbers with unlike denominators.

Work Together

Marcella walks $2\frac{2}{3}$ miles from her house to the bookstore.
Jacques walks $3\frac{5}{6}$ miles from his house to the bookstore.
How much farther does Jacques walk?

$1\frac{1}{6}$ **miles**

64 Lesson 7 • Subtract Mixed Numbers with Unlike Denominators

❶ Pose the Problem

ETP Pose Purposeful Questions

- What are some strategies you already know for subtracting fractions?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and give them a problem similar to the Work Together problem on the Learn page. Ask each to work individually to solve the problem, and then have them compare their strategies. Revisit this routine throughout the lesson to help students build proficiency.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you subtract mixed numbers by decomposing the mixed numbers?
- How can you subtract mixed numbers by using equivalent fractions that are greater than one?

Key Takeaway

- The difference of mixed numbers with unlike denominators can be found using strategies such as using equivalent fractions and decomposing the mixed numbers into whole-number parts and fraction parts.

Work Together

Students solve a subtraction word problem that involves fractions with unlike denominators.

Common Error: Students may mistakenly write the subtraction equation $2\frac{2}{3} - 3\frac{5}{6} = ?$ because $2\frac{2}{3}$ is the first number given in the problem. Encourage students to reread the problem make sure they are writing an equation that accurately represents the problem.

LOM Language of Math

Explain to students that the *mixed* in *mixed number* comes from the Latin word *miscere*, which means to mingle. Knowing this word can help students determine other words that come from the same root such as *miscellaneous*, which means a group of things from different sources.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore strategies to subtract mixed numbers.

Directions: Discuss with students how they decomposed by place value to subtract whole numbers and to subtract decimals.

- Do you think you can use a similar strategy to subtract mixed numbers?

Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- What operation is needed to solve this problem?
- How can you decompose $1\frac{1}{4}$?
- Is your answer reasonable? How do you know?

After students have solved by decomposing, challenge students to find another method to solve.

ETP Support Productive Struggle

- How can you represent $2\frac{2}{3}$ using only thirds? How can you write this as a fraction?
- How can you represent $1\frac{1}{4}$ using only fourths? How can you write this as a fraction?
- How can you use these equivalent representation to find the difference?

Math is... Exploring

- What is the same about these methods? What is different?

Students identify correspondences between different approaches to solving complex problems.

Activity Debrief: Discuss with students that decomposing is one strategy for adding mixed numbers. Another strategy is to write each mixed number as an equivalent fraction and then subtract the fractions.

Guided Exploration

Students subtract mixed numbers having unlike denominators by decomposing the mixed numbers and by writing equivalent fractions that are greater than one.

ETP Use and Connect Mathematical Representations

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the difference. Ask:

- Will you use whole numbers or mixed numbers to estimate the difference? Why?
- If you use mixed numbers, what benchmark fractions will you use? Why?
- How does this decomposing strategy relate to decomposing whole numbers to subtract?

Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?
- What steps are involved in rewriting the mixed numbers as equivalent fractions?
- How can you subtract once you have written the mixed numbers as fractions?
- How does the solution you found using equivalent fractions greater than one compare to the solution you found by decomposing the mixed numbers?

Math is... Exploring

- What is the same about these methods? What is different?

Students identify correspondences between different approaches to solving complex problems.

EL English Learner Scaffolds

Entering/Emerging

Explain *You can use ___ to ___*. Write an addition problem on the board. Say, *You can use a place value chart to solve the problem.* Solve the problem with a place value chart. Repeat with a new mathematical problem, using *You can use ___ to ___* to explain how to solve it. Then, repeat once more, asking students to answer the question: *Can I use [addition] or [division] to solve the problem?*

Developing/Expanding

Explain *You can use ___ to ___*. Write an addition problem on the board. Say, *You can use a place value chart to solve the problem.* Solve the problem with a place value chart. Repeat with a new mathematical problem, using *You can use ___ to ___* to explain how to solve it. Then, repeat once more, this time asking students to complete the sentence: *I can ___ to ___*.

Bridging/Reaching

Guide students to the Learn page and ask them to review the *You can use...* sentence below the table. Ask students to come up with a new sentence explaining how to do something, using *You can use ___ to ___*. Allow students to interject, correcting as needed. For example, *No, I don't think that's right. You can use...*

On My Own

Name ______________________

What is the difference? Choose the correct answer.

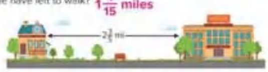
1. $3\frac{2}{3} - 1\frac{1}{5} = ?$
   A. $2\frac{7}{15}$
   B. $2\frac{1}{5}$
   C. $2\frac{1}{15}$
   D. $2\frac{1}{3}$

2. $6\frac{7}{8} - 5\frac{5}{6} = ?$
   A. $1\frac{5}{24}$
   B. $1\frac{1}{24}$
   C. $1\frac{4}{24}$
   D. $1\frac{2}{24}$

What is the difference?

3. $4\frac{3}{4} - 1\frac{1}{3} =$ $3\frac{5}{12}$
4. $2\frac{3}{5} - 1\frac{1}{2} =$ $1\frac{1}{10}$
5. $5\frac{5}{9} - 3\frac{1}{6} =$ $2\frac{7}{18}$
6. $3\frac{7}{10} - 1\frac{3}{8} =$ $2\frac{13}{40}$
7. $6\frac{1}{2} - 3\frac{1}{3} =$ $3\frac{1}{6}$
8. $4\frac{5}{8} - 3\frac{1}{5} =$ $1\frac{17}{40}$

9. The distance from Martin's house to school is shown. After 20 minutes, Martin walked $1\frac{1}{3}$ miles. What distance does he have left to walk? $1\frac{4}{15}$ miles

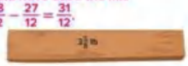


Unit 9 • Add and Subtract Fractions 65

10. Mrs. Williams bought $5\frac{1}{2}$ gallons of apple juice for the classroom party. She used $3\frac{1}{3}$ gallons during the party. How many gallons of apple juice does Mrs. Williams have left?
   $2\frac{1}{6}$ gallons

11. **Error Analysis** Brian solved this subtraction problem:
   $4\frac{5}{8} - 2\frac{1}{4} = \frac{29}{6} - \frac{9}{4} = \frac{20}{6}$
   Is Brian correct? Explain why or why not.
   No. Sample answer: He did not find like denominators when subtracting. He should have used the like denominator 12 to have $\frac{58}{12} - \frac{27}{12} = \frac{31}{12}$.

12. The combined weight of two wood planks is $6\frac{3}{4}$ pounds. The weight of one wood plank is shown. How many pounds is the second wood plank?



   $3\frac{1}{8}$ pounds

13. **Extend Your Thinking** Write and solve a real-world problem involving subtraction of two mixed numbers whose difference is less than $2\frac{1}{2}$.
   Check students' work.

Reflect

How can you subtract mixed numbers with unlike denominators?
Answers may vary.

Math is... Mindset
What helped you work well in a team?

66 Lesson 7 • Subtract Mixed Numbers with Unlike Denominators

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–8 If students choose to subtract by writing the mixed numbers as fractions greater than 1, they may convert mixed numbers to fractions greater than 1 incorrectly. Review the process if needed.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–10	2	Application
11	3	Conceptual Understanding
12	2	Application
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you subtract mixed numbers with unlike denominators?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you work well in a team?

Students reflect on how they practiced stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can subtract mixed numbers with unlike denominators.
- I can explain how to subtract mixed numbers with unlike denominators.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Subtract mixed numbers with unlike denominators	5.NF.A.1
2	1	Subtract mixed numbers with unlike denominators	5.NF.A.1
3	2	Subtract mixed numbers with unlike denominators	5.NF.A.1
4	2	Subtract mixed numbers with unlike denominators	5.NF.A.1

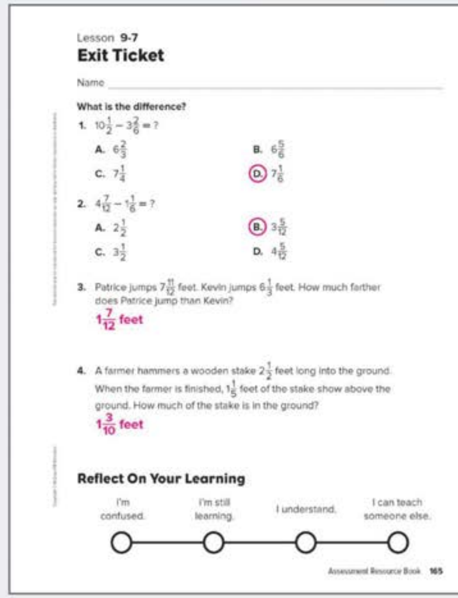
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking



Lesson 9-7
Exit Ticket

Name

What is the difference?

1. $10\frac{1}{2} - 3\frac{2}{6} = ?$

A. $6\frac{2}{3}$ B. $6\frac{5}{6}$
C. $7\frac{1}{4}$ D. $7\frac{1}{6}$

2. $4\frac{7}{12} - 1\frac{1}{6} = ?$

A. $2\frac{1}{2}$ B. $3\frac{5}{12}$
C. $3\frac{1}{2}$ D. $4\frac{5}{12}$

3. Patrice jumps $7\frac{11}{12}$ feet. Kevin jumps $6\frac{1}{3}$ feet. How much farther does Patrice jump than Kevin?

$1\frac{7}{12}$ feet

4. A farmer hammers a wooden stake $2\frac{1}{2}$ feet long into the ground. When the farmer is finished, $1\frac{1}{5}$ feet of the stake show above the ground. How much of the stake is in the ground?

$1\frac{3}{10}$ feet

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 165

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Subtract the Fractions

Work with students in groups. Students write mixed numbers on a blank spinner using the numbers 2, 3, and 4 as denominators. Each student spins the spinner once. Students subtract the lesser mixed number from the greater mixed number. If students have difficulty, help them to find like denominators for the fractions and represent the subtraction using fraction tiles.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Subtract Unlike Mixed Numbers

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 101

Lesson 9-7 • Reinforce Understanding

Subtract Mixed Numbers with Unlike Denominators

Name ______________________

Review

You can subtract mixed numbers by decomposing them into whole numbers and fractions.

Consider the equation: $5\frac{1}{4} - 2\frac{2}{9} = ?$

$$\begin{array}{r} 5\frac{1}{4} \\ -\ 2\frac{2}{9} \\ \hline \end{array} \longrightarrow \begin{array}{r} 5\frac{1\times 9}{4\times 9} \\ -\ 2\frac{2\times 4}{9\times 4} \\ \hline \end{array} \longrightarrow \begin{array}{r} 5\frac{9}{36} \\ -\ 2\frac{8}{36} \\ \hline 3\frac{1}{36} \end{array}$$

So, $5\frac{1}{4} - 2\frac{2}{9} = 3\frac{1}{36}$.

What is the difference?

1. $2\frac{3}{4} - 1\frac{1}{8} = 1\frac{5}{8}$
2. $6\frac{2}{5} - 3\frac{3}{10} = 3\frac{1}{10}$
3. $5\frac{5}{6} - 3\frac{1}{9} = 2\frac{13}{18}$
4. $5\frac{5}{6} - 1\frac{1}{4} = 4\frac{7}{12}$
5. $4\frac{7}{9} - 2\frac{2}{3} = 2\frac{1}{9}$
6. $3\frac{5}{6} - 2\frac{5}{8} = 1\frac{5}{24}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Subtracting Mixed Numbers Task Cards

Students practice subtracting mixed numbers.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 101–102

Lesson 9-7

Additional Practice

Name ______________________

Review

You can subtract mixed numbers by subtracting the whole number parts and the fractional parts.

Sheila jogs $4\frac{1}{4}$ miles on Wednesday and $6\frac{2}{3}$ miles on Saturday. How many miles more does Sheila jog on Saturday?

To solve, find $6\frac{2}{3} - 4\frac{1}{4}$.

Subtract the whole number: $6\frac{2}{3} - 4 = 2\frac{2}{3}$

Subtract the fraction: $2\frac{2}{3} - \frac{1}{4} = 2\frac{8}{12} - \frac{3}{12} = 2\frac{5}{12}$

Sheila jogged $2\frac{5}{12}$ miles more on Saturday.

What is the difference? Show your work. Check students' work.

1. $5\frac{2}{3} - 1\frac{1}{2} =$ $4\frac{1}{6}$
2. $8\frac{3}{4} - 3\frac{1}{10} =$ $5\frac{13}{20}$
3. $6\frac{3}{8} - 5\frac{1}{4} =$ $1\frac{1}{8}$
4. $3\frac{7}{10} - 1\frac{2}{5} =$ $2\frac{3}{10}$
5. $10\frac{5}{6} - 7\frac{1}{4} =$ $3\frac{7}{12}$
6. $7\frac{7}{9} - 3\frac{1}{3} =$ $4\frac{4}{9}$

Student Practice Book

Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Assign

Use It! Application Station

How Do You Say—Fractions? Students research to find how to say words related to adding and subtracting fractions in other languages.

GO ONLINE

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 101–102

Solve the problem. Show your work.

7. Edison has $5\frac{5}{6}$ gallons of paint. After painting his room, he has $2\frac{1}{4}$ gallons left. How much paint does Edison use to paint his room? $3\frac{7}{12}$ gallons

8. Conrad has two boards. One board is $4\frac{3}{5}$ feet long. The second board is $2\frac{1}{3}$ feet long. How much longer is the first board than the second board? $2\frac{4}{15}$ feet

9. A hiking trail is $6\frac{5}{8}$ kilometers long. Karen has hiked $4\frac{1}{3}$ kilometers so far. How much farther does Karen have to go to reach the end of the trail? $2\frac{7}{24}$ kilometers

10. Paul jumps $6\frac{1}{4}$ feet. Petra jumps $8\frac{3}{5}$ feet. How much farther does Petra jump than Paul? $2\frac{7}{20}$ feet

Math @Home Activity With your child, find two objects around your home whose lengths can be compared. Measure the length of each, using a foot-long ruler, and write the measurements as a mixed number of feet. Have your child subtract the two measurements to see how much longer one object is than the other, and then place the two objects next to each other and measure the difference in lengths to verify the result. Find two other objects, and repeat as time permits.

Student Practice Book

Differentiation Resource Book, p. 102

Lesson 9-7 • Extend Thinking

Subtract Mixed Numbers with Unlike Denominators

Name ______________________________

Fill in the missing values to complete each equation. The first one is done as an example. Show your work.

1. $?\frac{?}{?} - 1\frac{?}{3} = 5\frac{5}{12}$

$6\frac{?}{4} - 1\frac{?}{3} = 5\frac{5}{4 \times 3}$

$6\frac{? \times 3}{4 \times 3} - 1\frac{? \times 4}{3 \times 4} = 5\frac{9-4}{4 \times 3}$

$6\frac{3 \times 3}{4 \times 3} - 1\frac{1 \times 4}{3 \times 4} = 5\frac{9-4}{3 \times 4}$

$6\frac{9}{12} - 1\frac{4}{12} = 5\frac{5}{12}$

$6\frac{3}{4} - 1\frac{1}{3} = 5\frac{5}{12}$

2. $5\frac{?}{2} - ?\frac{?}{?} = 2\frac{5}{22}$

$5\frac{?}{2} - 3\frac{?}{11} = 2\frac{5}{2 \times 11}$

$5\frac{1 \times 11}{2 \times 11} - 3\frac{3 \times 2}{11 \times 2} = 2\frac{11-6}{2 \times 11}$

$5\frac{11}{22} - 3\frac{6}{22} = 2\frac{5}{22}$

$5\frac{1}{2} - 3\frac{3}{11} = 2\frac{5}{22}$

3. $?\frac{?}{5} - 3\frac{?}{?} = 1\frac{4}{15}$

$4\frac{?}{5} - 3\frac{?}{3} = 1\frac{4}{5 \times 3}$

$4\frac{3 \times 3}{5 \times 3} - 3\frac{1 \times 5}{3 \times 5} = 1\frac{9-5}{5 \times 3}$

$4\frac{9}{15} - 3\frac{5}{15} = 1\frac{4}{15}$

$4\frac{3}{5} - 3\frac{1}{3} = 1\frac{4}{15}$

4. $3\frac{?}{7} - 1\frac{?}{2} = 2\frac{5}{14}$

$3\frac{?}{7} - 1\frac{?}{2} = 2\frac{5}{7 \times 2}$

$3\frac{6 \times 2}{7 \times 2} - 1\frac{1 \times 7}{2 \times 7} = 2\frac{12-7}{7 \times 2}$

$3\frac{12}{14} - 1\frac{7}{14} = 2\frac{5}{14}$

$3\frac{6}{7} - 1\frac{1}{2} = 2\frac{5}{14}$

5. $?\frac{5}{?} - 1\frac{?}{2} = 2\frac{1}{18}$

$3\frac{?}{9} - 1\frac{?}{2} = 2\frac{1}{9 \times 2}$

$3\frac{5 \times 2}{9 \times 2} - 1\frac{1 \times 9}{2 \times 9} = 2\frac{10-9}{9 \times 2}$

$3\frac{10}{18} - 1\frac{9}{18} = 2\frac{1}{18}$

$3\frac{5}{9} - 1\frac{1}{2} = 2\frac{1}{18}$

6. $5\frac{?}{?} - ?\frac{?}{4} = 3\frac{7}{20}$

$5\frac{?}{5} - 2\frac{?}{4} = 3\frac{7}{5 \times 4}$

$5\frac{3 \times 4}{5 \times 4} - 2\frac{1 \times 5}{4 \times 5} = 3\frac{12-5}{5 \times 4}$

$5\frac{12}{20} - 2\frac{5}{20} = 3\frac{7}{20}$

$5\frac{3}{5} - 2\frac{1}{4} = 3\frac{7}{20}$

Differentiation Resource Book

LESSON **9-8**

Add and Subtract Mixed Numbers with Regrouping

Learning Target

- I can add and subtract mixed numbers with regrouping.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. $\left(\text{In general, } \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}\right)$.

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
equivalent fractions	debate
mixed number	eliminate

Materials

The materials may be for any part of the lesson.

- *Explain and Show Your Strategies* Teaching Resource
- fraction tiles
- index cards
- rulers

Focus

Content Objective	Language Objectives	SEL Objective
• Students add and subtract mixed numbers with regrouping.	• Students talk about adding and subtracting mixed numbers with regrouping using *rearrange* and *rename*. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students work toward completing a mathematical task independently using prior knowledge or understanding of mathematical concepts.

Coherence

Previous	Now	Next
• Students understood, recognized, and generated equivalent fractions (Grade 4). • Students decomposed mixed numbers and used fractions greater than one to subtract mixed numbers (Unit 9).	• Students use regrouping to add and subtract mixed numbers.	• Students choose and use known strategies to solve word problems that involve addition or subtraction of mixed numbers having unlike denominators (Unit 9). • Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding of fraction concepts to add and subtract mixed numbers with unlike denominators.	• Students develop proficiency in adding and subtracting mixed numbers for an increased range of cases.	• Students solve problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Would You Rather?

5–7 min

Build Fluency Students build number sense and understanding of fractions as they compare the difference of two fractions to another fraction.

These prompts encourage students to talk about their reasoning:

- What strategy did you use to compare the amounts?
- Did you use equivalent fractions to compare the amounts? Why or why not?
- How could inverse operations help with the comparison?

Launch 5–7 min

Sense-Making Routine

Purpose Students explore parts of a whole that, when combined, create wholes.

Notice & Wonder

- What do you notice?
- What do you wonder?

Teaching Tip You may want to have students model what is shown in the images using their own manipulatives to deepen their understanding of how parts of a whole make up a whole.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of using regrouping to add and subtract mixed numbers and are based on possible comments and questions that students may make during the share out.

- What is missing from each part?
- How can you make wholes?

Math is... Mindset

- Why is it useful to consider different possible solutions to a problem?

SEL **Responsible Decision-Making: Solve Problems**

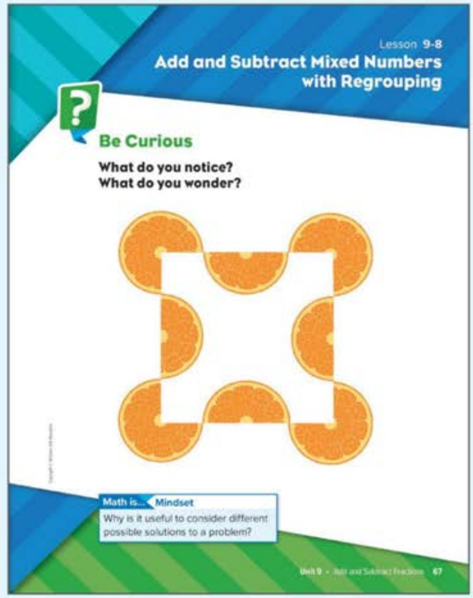
As you begin the Notice & Wonder routine, encourage students to become efficient problem solvers who can make informed decisions that lead to solutions. Discuss strategies students can use to help them focus on identifying information that is helpful in solving problems. Have students work in pairs to separate this information and information that does not help solve a problem.

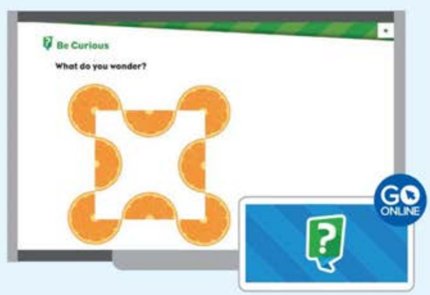
Transition to Explore & Develop

Ask questions that get students thinking about using regrouping to subtract mixed numbers.

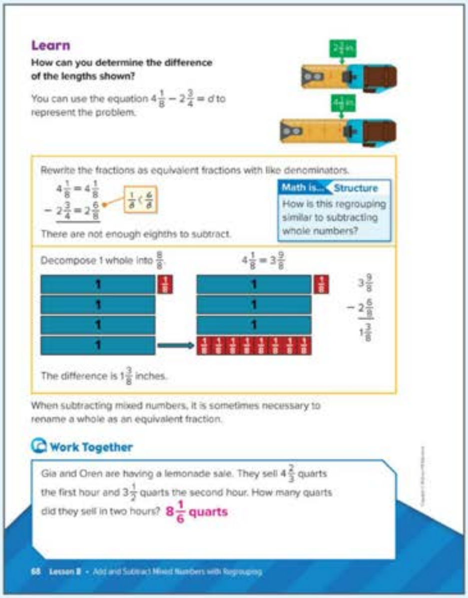
ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about when we need to, and how we can, use regrouping to subtract mixed numbers.





Explore & Develop 20 min



Learn

How can you determine the difference of the lengths shown?

You can use the equation $4\frac{1}{8} - 2\frac{3}{4} = d$ to represent the problem.

Rewrite the fractions as equivalent fractions with like denominators.

$$\begin{array}{r} 4\frac{1}{8} = 4\frac{1}{8} \\ -\ 2\frac{3}{4} = 2\frac{6}{8} \\ \hline \end{array}$$

$\frac{1}{8} < \frac{6}{8}$

There are not enough eighths to subtract.

Math is... Structure How is this regrouping similar to subtracting whole numbers?

Decompose 1 whole into $\frac{8}{8}$.

$4\frac{1}{8} = 3\frac{9}{8}$

$$\begin{array}{r} 3\frac{9}{8} \\ -\ 2\frac{6}{8} \\ \hline 1\frac{3}{8} \end{array}$$

The difference is $1\frac{3}{8}$ inches.

When subtracting mixed numbers, it is sometimes necessary to rename a whole as an equivalent fraction.

Work Together

Gia and Oren are having a lemonade sale. They sell $4\frac{2}{3}$ quarts the first hour and $3\frac{1}{2}$ quarts the second hour. How many quarts did they sell in two hours? $8\frac{1}{6}$ quarts

68 Lesson 8 • Add and Subtract Mixed Numbers with Regrouping

1 Pose the Problem

ETP Pose Purposeful Questions

- What strategies do you know that might be helpful in solving the problem?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about the steps they need to take to solve it Then have them share their work with their partner and compare. Revisit the routine throughout the lesson for reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you rename a whole as an equivalent fraction in a subtraction equation?
- When do you need to rename a whole as an equivalent fraction to help you subtract?
- Explain a strategy for subtracting mixed numbers that does not involve renaming a whole as an equivalent fraction.
- When might you need to rename a whole as an equivalent fraction to help you add?

Key Takeaway

- When adding and subtracting mixed numbers, it is sometimes necessary to rename a whole as an equivalent fraction.

Work Together

Students solve a word problem involving the addition of mixed numbers and regrouping. Use this to explain the "adding" part of the Key Takeaway.

Common Error: Students may be confused as they are now regrouping with addition instead of subtraction. Encourage them to start by adding the decomposed numbers, and to pay attention to whether the fractions are greater or less than 1. When they have a sum that is greater than 1, they can regroup the fraction that is equal to 1 as a whole.

LOM Language of Math

Tell students that *regrouping* means rearranging groups while still keeping them equal. People may also "regroup" after being unsuccessful at a task. The decomposition shown in the last lesson may be unsuccessful and force you to regroup.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore different strategies to subtract mixed numbers that involve regrouping.

Materials: *Explain and Show Your Strategies* Teaching Resource

Directions: Provide copies of the *Explain and Show Your Strategies* Teaching Resource. Have students solve the Pose the Problem using two different strategies.

ETP Support Productive Struggle

- What strategies have you learned for subtracting mixed numbers?
- Which strategy did you use first? Why did you decide to start with that strategy?
- What happened when you decomposed to subtract? How did you determine the next steps to continue using this strategy?
- What is the same about your two strategies? What is different?

Math is... Structure

- How is this regrouping similar to subtracting whole numbers?

Students recognize the structure in the number system that allows them to connect regrouping whole numbers when subtracting and regrouping mixed numbers when subtracting.

Activity Debrief: Discuss with students that problems can be solved using any known strategy. Some addition and subtraction strategies may be more efficient that others due to the quantities within the problem.

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students solve a subtraction equation involving mixed numbers and regrouping.

ETP Facilitate Meaningful Mathematical Discourse

Have the students estimate the difference. Ask:

- Will you use whole numbers or mixed numbers to estimate the difference? Why?
- If you use mixed numbers, what benchmark fractions will you use? Why?
- Why should you regroup 1 whole into eighths?
- Why are you able to subtract using $1\frac{9}{8}$ instead of $2\frac{1}{8}$?

Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

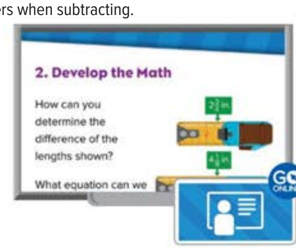
Have students solve the equation $4\frac{1}{8} - 2\frac{3}{4} = d$ in small groups by writing each mixed number as a fraction greater than 1. Ask:

- What steps do you need to take to write the mixed numbers as fractions greater than 1?
- How does the difference compare to the difference you calculated using regrouping?

Math is... Structure

- How is this regrouping similar to subtracting whole numbers?

Students recognize the structure in the number system that allows them to connect regrouping whole numbers when subtracting and regrouping mixed numbers when subtracting.



EL English Learner Scaffolds

Entering/Emerging

Explain *since*. Draw a triangle. Say, *This is a triangle since it has three sides.* Point to the three sides. Then draw a pentagon. Say, *This is a pentagon since it has five sides.* Point to the five sides. Finally, draw a square and a rectangle. Say, *This shape is a square since it has four equal sides.* Ask students to point or say the correct shape.

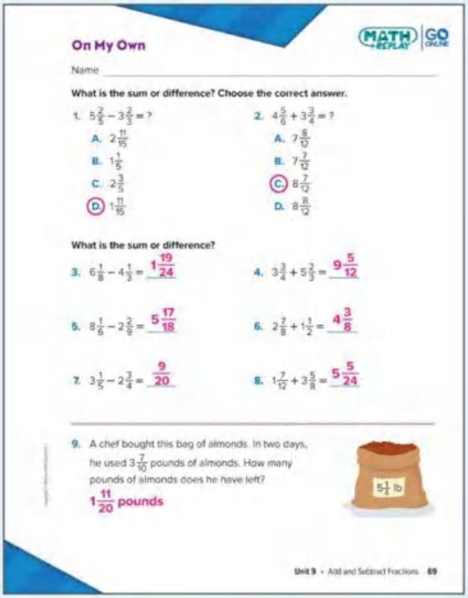
Developing/Expanding

Explain *since*. Draw a triangle. Say, *This is a triangle since it has three sides.* Point to the three sides. Then draw a pentagon. Say, *This is a pentagon since it has five sides.* Point. Draw a square and ask students to complete the following: *This shape is a square __ (since) it has four equal sides.* Finally, ask students to use since in their own sentence. Provide sentence frames if needed.

Bridging/Reaching

Guide students to the Learn page and have them focus on how *since* is used in the table. Ask students if they can think of other words that mean the same (*because, given that, etc.*). Ask students to use *since* in a sentence of their own. Validate and correct as necessary. Then ask students to think of another way *since* can be used (to say when something started, e.g., *I have gone to school here since the second grade.*

Practice & Reflect 10 min



On My Own

Name ______

What is the sum or difference? Choose the correct answer.

1. $5\frac{2}{5} - 3\frac{2}{3} = ?$
   A. $2\frac{11}{15}$
   B. $1\frac{1}{5}$
   C. $2\frac{3}{5}$
   D. $1\frac{11}{15}$

2. $4\frac{5}{6} + 3\frac{3}{4} = ?$
   A. $7\frac{8}{12}$
   B. $7\frac{7}{12}$
   C. $8\frac{7}{12}$
   D. $8\frac{8}{12}$

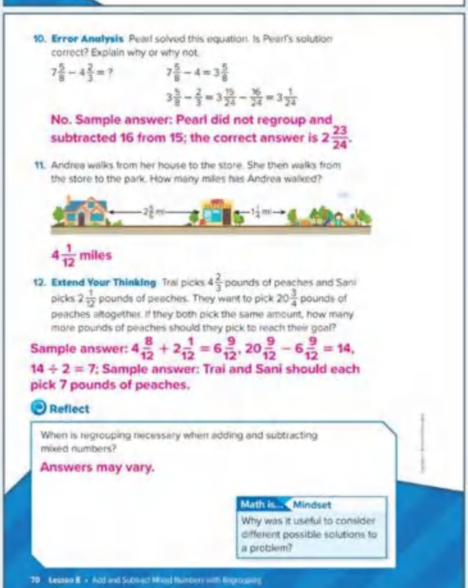
What is the sum or difference?

3. $6\frac{1}{8} - 4\frac{1}{3} =$ $1\frac{19}{24}$
4. $3\frac{3}{4} + 5\frac{2}{3} =$ $9\frac{5}{12}$
5. $8\frac{1}{6} - 2\frac{2}{9} =$ $5\frac{17}{18}$
6. $2\frac{7}{8} + 1\frac{1}{2} =$ $4\frac{3}{8}$
7. $3\frac{1}{5} - 2\frac{3}{4} =$ $\frac{9}{20}$
8. $1\frac{7}{12} + 3\frac{5}{8} =$ $5\frac{5}{24}$

9. A chef bought this bag of almonds. In two days, he used $3\frac{7}{10}$ pounds of almonds. How many pounds of almonds does he have left?
$1\frac{11}{20}$ pounds

$5\frac{1}{4}$ lb

Unit 9 • Add and Subtract Fractions 69



10. **Error Analysis** Pearl solved this equation. Is Pearl's solution correct? Explain why or why not.
$7\frac{5}{8} - 4\frac{2}{3} = ?$
$7\frac{5}{8} - 4 = 3\frac{5}{8}$
$3\frac{5}{8} - \frac{2}{3} = 3\frac{15}{24} - \frac{16}{24} = 3\frac{1}{24}$
No. Sample answer: Pearl did not regroup and subtracted 16 from 15; the correct answer is $2\frac{23}{24}$.

11. Andrea walks from her house to the store. She then walks from the store to the park. How many miles has Andrea walked?
$2\frac{5}{6}$ mi $1\frac{1}{4}$ mi
$4\frac{1}{12}$ miles

12. **Extend Your Thinking** Trai picks $4\frac{2}{3}$ pounds of peaches and Sani picks $2\frac{1}{12}$ pounds of peaches. They want to pick $20\frac{3}{4}$ pounds of peaches altogether. If they both pick the same amount, how many more pounds of peaches should they pick to reach their goal?
Sample answer: $4\frac{8}{12} + 2\frac{1}{12} = 6\frac{9}{12}$, $20\frac{9}{12} - 6\frac{9}{12} = 14$, $14 \div 2 = 7$; Sample answer: Trai and Sani should each pick 7 pounds of peaches.

Reflect

When is regrouping necessary when adding and subtracting mixed numbers?
Answers may vary.

Math is... Mindset
Why was it useful to consider different possible solutions to a problem?

70 Lesson 8 • Add and Subtract Mixed Numbers with Regrouping

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–8 Students may automatically subtract the smaller fraction from the larger fraction without regrouping. Remind students to look at the equation as a whole as they subtract, and to pay attention to the order of the numbers in the subtraction equations.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9	2	Application
10	3	Conceptual Understanding
11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- When is regrouping necessary when adding and subtracting mixed numbers?

Ask students to share their reflections with their classmates.

Math is... Mindset

- Why was it useful to consider different possible solutions to a problem?

Students reflect on how they practiced responsible decision-making.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can add and subtract mixed numbers with regrouping.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Add mixed numbers with regrouping	5.NF.A.1
2	1	Subtract mixed numbers with regrouping	5.NF.A.1
3	2	Subtract mixed numbers with regrouping	5.NF.A.1
4	2	Add mixed numbers with regrouping	5.NF.A.1

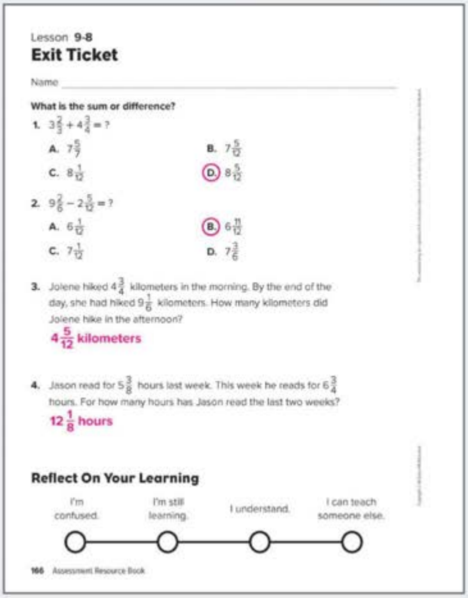
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 9-8

Exit Ticket

Name ______________________

What is the sum or difference?

1. $3\frac{2}{3} + 4\frac{3}{4} = ?$

   A. $7\frac{5}{7}$ B. $7\frac{5}{12}$

   C. $8\frac{1}{12}$ D. $8\frac{5}{12}$

2. $9\frac{2}{6} - 2\frac{5}{12} = ?$

   A. $6\frac{1}{12}$ B. $6\frac{11}{12}$

   C. $7\frac{1}{12}$ D. $7\frac{3}{6}$

3. Jolene hiked $4\frac{3}{4}$ kilometers in the morning. By the end of the day, she had hiked $9\frac{1}{6}$ kilometers. How many kilometers did Jolene hike in the afternoon?

   $4\frac{5}{12}$ kilometers

4. Jason read for $5\frac{3}{8}$ hours last week. This week he reads for $6\frac{3}{4}$ hours. For how many hours has Jason read the last two weeks?

   $12\frac{1}{8}$ hours

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

166 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Calculation Race

Work with students in pairs. Each student writes a mixed number on a card. When both cards are turned over, students add the mixed numbers and subtract the mixed numbers. Together, students check their work. If students have difficulty, help them to write fractions with like denominators and, if necessary, rewrite mixed numbers by regrouping 1 as an equivalent fraction before subtracting.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Add Unlike Mixed Numbers (Regroup)
- Subtract Mixed Numbers (Regroup)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 103

Lesson 9-8 • Reinforce Understanding

Add and Subtract Mixed Numbers with Regrouping

Name ____________________

Review

When adding and subtracting mixed numbers, it is sometimes necessary to rename whole numbers.

Consider the equation $5\frac{1}{6} - 2\frac{4}{5} = ?$

Step 1	Step 2	Step 3	Step 4
$5\frac{5}{30}$	$4 + 1 + \frac{5}{30}$	$4 + \frac{30}{30} + \frac{5}{30}$	$4\frac{35}{30}$
$-2\frac{24}{30}$	$-2\frac{24}{30}$	$-2\frac{24}{30}$	$-2\frac{24}{30}$
			$2\frac{11}{30}$

So, $5\frac{1}{6} - 2\frac{4}{5} = 2\frac{11}{30}$

Add or subtract.

1. $3\frac{5}{6} + 1\frac{1}{2} =$ ________ **$5\frac{2}{6}$ or $5\frac{1}{3}$**
2. $4\frac{2}{7} - 1\frac{1}{2} =$ ________ **$2\frac{11}{14}$**
3. $2\frac{4}{5} + 5\frac{1}{3} =$ ________ **$8\frac{2}{15}$**
4. $7\frac{1}{8} - 2\frac{3}{4} =$ ________ **$4\frac{3}{8}$**
5. $2\frac{5}{12} + 2\frac{7}{8} =$ ________ **$5\frac{7}{24}$**
6. $5\frac{1}{6} - 2\frac{7}{9} =$ ________ **$2\frac{7}{18}$**

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Mixed Number Addition Concentration

Students practice adding mixed numbers.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 103–104

Lesson 9-8

Additional Practice

Name ____________________

Review

You can add and subtract mixed numbers with regrouping.

Ken walked $2\frac{3}{4}$ miles yesterday. He walked $4\frac{2}{3}$ miles today.

How far did Ken walk both days?	How much farther did Ken walk today than yesterday?
To solve, find $2\frac{3}{4} + 4\frac{2}{3}$.	To solve, find $4\frac{2}{3} - 2\frac{3}{4}$.
Add the whole numbers: $2 + 4 = 6$	Write the fractions using a common denominator: $4\frac{8}{12} - 2\frac{9}{12}$
Add the fractions: $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$	Regroup 1 whole as $\frac{12}{12}$. $3\frac{20}{12} - 2\frac{9}{12}$
Add the whole number and the fraction: $6 + 1\frac{5}{12} = 7\frac{5}{12}$	Subtract the whole numbers and the fractions: $3\frac{20}{12} - 2\frac{9}{12} = 1\frac{11}{12}$
Ken walked $7\frac{5}{12}$ miles both days.	Ken walked $1\frac{11}{12}$ miles farther today than yesterday.

What is the sum or difference? Show your work. Check students' work.

1. $5\frac{1}{2} - 1\frac{2}{3} =$ **$3\frac{5}{6}$**
2. $3\frac{1}{4} + 3\frac{9}{10} =$ **$7\frac{3}{20}$**
3. $8\frac{1}{4} - 3\frac{7}{10} =$ **$4\frac{11}{20}$**
4. $2\frac{7}{8} + 1\frac{5}{6} =$ **$4\frac{17}{24}$**

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Use It! Application Station

Get Moving Students design a car and measure the distance it travels.

GO ONLINE

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

INDEPENDENT WORK

Student Practice Book, pp. 103–104

Solve the problem. Show your work.

6. Maren lives $3\frac{1}{4}$ miles from school. Stormy lives $2\frac{4}{5}$ miles from school. How much farther away from the school does Maren live than Stormy? $\frac{9}{20}$ mile(s)

7. Mark hikes $2\frac{3}{5}$ kilometers to the scenic overlook. He then hikes $1\frac{9}{10}$ kilometers further to the nature center. How many kilometers does Mark hike? $4\frac{5}{10}$ or $4\frac{1}{2}$ kilometers

8. A length of ribbon is $4\frac{2}{9}$ feet long. Millie cuts a piece of ribbon that is $1\frac{5}{6}$ feet long. How long is the remaining piece of ribbon? $2\frac{7}{18}$ feet

9. Martin buys $2\frac{5}{6}$ pounds of peanuts and $4\frac{3}{4}$ pounds of almonds. How many pounds of nuts does Martin buy? $7\frac{7}{12}$ pounds

Math @ Home Activity With your child, find two objects around your home whose lengths can be either added or compared. Measure the length of each, using a foot-long ruler, and write the measurements as a mixed number of feet. Have your child add or subtract the two measurements, and then measure the sum or to verify the result. Find two other objects, and repeat as time permits.

Student Practice Book

Differentiation Resource Book, p. 104

Lesson 9-8 • Extend Thinking

Add and Subtract Mixed Numbers with Regrouping

Name ____________________

Find the answer to Problem A. Write the answer in the answer column. Then complete Problem B so it uses regrouping and has the same answer as Problem A. The first one has been done for you as an example. Show your work.

Problem A	Answer	Problem B
$5\frac{1}{8} - 2\frac{5}{6}$ $5\frac{3}{24} - 2\frac{20}{24}$ $4\frac{27}{24} - 2\frac{20}{24}$	$2\frac{7}{24}$	$4\frac{1}{4} - ?\frac{7}{24}$ $4\frac{6}{24} - ?\frac{7}{24}$ Since 6 is less than 7, we will use regrouping. $3\frac{30}{24} - 1\frac{23}{24}$
$3\frac{2}{9} - 1\frac{2}{3}$ $3\frac{2}{9} - 1\frac{6}{9}$ $2\frac{11}{9} - 1\frac{6}{9}$	$1\frac{5}{9}$	$5\frac{1}{6} - \square\frac{\square}{18}$ $4\frac{21}{18} - 3\frac{11}{18}$ $5\frac{1}{6} - 3\frac{11}{18}$
$4\frac{2}{3} + 3\frac{3}{4}$ $4\frac{8}{12} + 3\frac{9}{12}$ $7\frac{17}{12}$ or $8\frac{5}{12}$	$8\frac{5}{12}$	$10\frac{1}{6} - \square\frac{\square}{4}$ $9\frac{14}{12} - 1\frac{9}{12}$ $10\frac{1}{6} - 1\frac{3}{4}$
$7\frac{1}{2} - 2\frac{6}{7}$ $7\frac{7}{14} - 2\frac{12}{14}$ $6\frac{21}{14} - 2\frac{12}{14}$	$4\frac{9}{14}$	$1\frac{5}{21} + \square\frac{\square}{42}$ $1\frac{10}{42} + 3\frac{17}{42}$ $1\frac{5}{21} + 3\frac{17}{42}$

Differentiation Resource Book

LESSON **9-9**

Solve Problems Involving Fractions and Mixed Numbers

Learning Target

- I can solve word problems involving fractions.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.A** Use equivalent fractions as a strategy to add and subtract fractions.

◇ **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Math Practices and Processes

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	Academic Terms
equivalent fractions	reflect
mixed number	suggest

Materials

The materials may be for any part of the lesson.

- fraction circles
- fraction tiles
- number cubes
- *Problem-Solving Tool* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students solve word problems involving fractions.	• Students explain how to solve word problems with fractions using *can*, *should*, *reasonable*, and *estimate*. • To support sense-making, ELs participate in MLR3: Three Reads.	• Students identify a problem and execute the steps necessary to solve the problem.

Coherence

Previous	Now	Next
• Students compared two fractions having different numerators and different denominators (Grade 4). • Students used regrouping to add and subtract mixed numbers (Unit 9).	• Students choose and use known strategies to solve word problems that involve addition or subtraction of mixed numbers having unlike denominators.	• Students multiply fractions (Unit 10).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of adding and subtracting mixed numbers. *Conceptual Understanding is not a targeted element of rigor for this standard.*	• Students develop proficiency in adding and subtracting mixed numbers. *Procedural Skill & Fluency is not a targeted element of rigor for this standard.*	• Students add and subtract mixed numbers involving unlike denominators to solve problems with real-world contexts.

Number Routine

GO ONLINE

Would You Rather?

5–7 min

Build Fluency Students build number sense and understanding of fractions as they compare the sums and differences of fractions.

These prompts encourage students to talk about their reasoning:

- What strategy did you use to make your choice?
- How did you compare the amounts?
- Did you use the same strategy for addition expressions as for subtraction expressions? Explain.

Launch 5–7 min

Sense-Making Routine

Purpose Students explore how they can solve problems by knowing how mixed numbers relate to one another.

Notice & Wonder

- What question could you ask?

Teaching Tip You may want to have students discuss a real-world scenario that the image represents as a class before discussing the questions they can ask about it.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' understanding of choosing and using the strategies they know for adding and subtracting mixed numbers having unlike denominators, and are based on possible comments and questions that students may make during the share out.

- What kinds of numbers does answering the question you asked involve? How do you know?
- What operation does answering the question you asked involve? How do you know?
- What kind of denominators does answering the question you asked involve? How do you know?

Math is... Mindset

- How can you use your abilities and skills to be successful today?

SEL **Self-Awareness: Self-Efficacy**

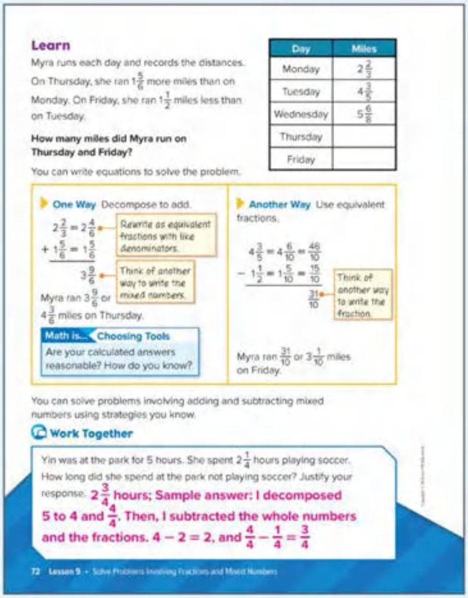
Help students develop skills that would help them become more likely to persevere to complete a challenging task. As students begin the Notice & Wonder routine, encourage them to first identify the problem, then think critically about what they will do to solve the problem. As you come together to collaboratively discuss the Notice & Wonder routine, you can invite students to share how their self-efficacy helped them if they ran into roadblocks when they solved the routine.

Transition to Explore & Develop

Ask students questions that get them thinking about choosing and using known strategies to solve real-world problems that involve fractions and mixed numbers.

ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about how we can choose and use the strategies we know to solve real-world problems that involve addition and subtraction of fractions and mixed numbers.



Learn

Myra runs each day and records the distances. On Thursday, she ran $1\frac{5}{6}$ more miles than on Monday. On Friday, she ran $1\frac{1}{2}$ miles less than on Tuesday.

Day	Miles
Monday	$2\frac{2}{3}$
Tuesday	$4\frac{3}{5}$
Wednesday	$5\frac{6}{8}$
Thursday	
Friday	

How many miles did Myra run on Thursday and Friday?

You can write equations to solve the problem.

One Way Decompose to add.

$$\begin{array}{r} 2\frac{2}{3} = 2\frac{4}{6} \\ +\ 1\frac{5}{6} = 1\frac{5}{6} \\ \hline 3\frac{9}{6} \end{array}$$

Rewrite as equivalent fractions with like denominators.

Think of another way to write the mixed numbers.

Myra ran $3\frac{9}{6}$ or $4\frac{3}{6}$ miles on Thursday.

Math is... Choosing Tools

Are your calculated answers reasonable? How do you know?

Another Way Use equivalent fractions.

$$\begin{array}{r} 4\frac{3}{5} = 4\frac{6}{10} = \frac{46}{10} \\ -\ 1\frac{1}{2} = 1\frac{5}{10} = \frac{15}{10} \\ \hline \frac{31}{10} \end{array}$$

Think of another way to write the fraction.

Myra ran $\frac{31}{10}$ or $3\frac{1}{10}$ miles on Friday.

You can solve problems involving adding and subtracting mixed numbers using strategies you know.

Work Together

Yin was at the park for 5 hours. She spent $2\frac{1}{4}$ hours playing soccer. How long did she spend at the park not playing soccer? Justify your response. **$2\frac{3}{4}$ hours; Sample answer: I decomposed 5 to 4 and $\frac{4}{4}$. Then, I subtracted the whole numbers and the fractions. $4 - 2 = 2$, and $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$**

72 Lesson 9 • Solve Problems Involving Fractions and Mixed Numbers

❶ Pose the Problem

ETP Pose Purposeful Questions

- How does the table help you understand the problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Three Reads

1st read: Instruct students to look at the problem at the top of the Learn page. Ensure students understand the situation and key words: records, distances, more miles than, miles less than.

2nd read: Focus students' attention on the How can... question.

3rd read: Instruct students to brainstorm ways to solve the problem.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you choose which strategy to use to solve problems involving mixed numbers and fractions?

Key Takeaway

- Problems involving fractions can be solved using known strategies for addition and subtraction of fractions or mixed numbers.

Work Together

Students work together to solve a problem that involves subtraction of a whole number and a mixed number.

Common Error: Students may be confused because one of the numbers in the problem is a whole number instead of a mixed number or fraction. Remind students that whole numbers can be written as fractions greater than one or "decomposed" with no fraction part.

LOM Language of Math

Explain to students that the word *fraction* comes from the Latin word *frangere* which means "to break." Knowing the roots of words can help us understand their meaning. Other words that come from *frangere* include *fragment*, which is a small or broken off part of something, and *fractious*, which means "breaking the rules."

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore known strategies to solve a real-world word problem involving mixed numbers.

Materials: *Problem-Solving Tool* Teaching Resource

Directions: Provide copies of the *Problem-Solving Tool* Teaching Resource. Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- How do you know which operation to use?
- What equations can you write?
- What are some strategies you know for adding mixed numbers? Subtracting mixed numbers?
- How does your solution method compare to others?

Math is... Choosing Tools

- Are your calculated answers reasonable? How do you know?

Students detect possible errors by strategically using estimation and other mathematical knowledge.

Activity Debrief: Have students share their solutions and strategies for solving the problem. Encourage students to find similarities and differences among the solution methods.

A PDF of the Teaching Resource is in the Digital Teacher Center.

Guided Exploration

Students solve a word problem involving fractions and mixed numbers by choosing and using known strategies.

ETP Facilitate Meaningful Mathematical Discourse

- **Think About It:** What is an estimate for the distance Maya ran on Thursday?
- **Think About It:** What is an estimate for the distance Maya ran on Friday?

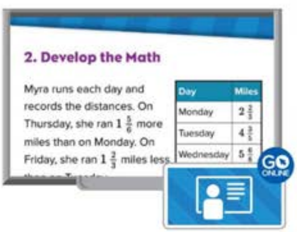
Have the students solve the equation $4\frac{3}{5} - 1\frac{1}{2} = f$ by decomposing. Ask:

- How can you decompose the mixed numbers?
- How can you find like denominators and make equivalent fractions?
- Do you need to regroup any wholes as fractions to solve?

Math is... Choosing Tools

- Are your calculated answers reasonable? How do you know?

Students detect possible errors by strategically using estimation and other mathematical knowledge.



EL English Learner Scaffolds

Entering/Emerging

Explain *spend [time] doing something.* Use a timer while you do a task, such as coloring. Stop the timer and say, *I spent [45 seconds] coloring.* Repeat with another task. Then write your name on the board. Have a student time you. When you're done, ask students, *How long did I spend writing my name?* For students who need more help, prompt students to refer to the timer.

Developing/Expanding

Explain *spend [time] doing something.* Use a timer while you do a task, such as coloring. Stop the timer and say, *I spent [45 seconds] coloring.* Repeat with another task. Then write your name on the board. Have a student time you. Ask students to complete the sentence: *You ___ (spent) 20 seconds ___ your name.* Then ask students to provide their own demonstration using *spent time doing something.* Provide sentence frames if needed.

Bridging/Reaching

Check student understanding of *spend [time] doing something* by guiding them to the Learn page and having them focus on the Work Together problem. Ask them to use *spent [time] doing something* in their own sentence, demonstrating with a timer. Then ask students to talk about how long they usually spend working on certain tasks *(i.e. homework, cleaning).*

On My Own

Name ____________________

1. Jonah walks $2\frac{7}{8}$ miles on Monday. On Tuesday, he walks $1\frac{2}{3}$ miles. How many miles does Jonah walk on Monday and Tuesday?
   - A. $3\frac{9}{24}$ mi
   - B. $3\frac{13}{24}$ mi
   - C. $4\frac{13}{24}$ mi
   - D. $4\frac{15}{24}$ mi

2. Kai has $4\frac{2}{5}$ ounces of juice in his cup. Martha pours $5\frac{7}{10}$ more ounces into his cup. How many ounces of juice are in Kai's cup?
   - A. $9\frac{1}{10}$ oz
   - B. $9\frac{9}{10}$ oz
   - C. $10\frac{1}{10}$ oz
   - D. $10\frac{9}{10}$ oz

3. Alyana buys $4\frac{3}{10}$ pounds of potatoes. She uses $2\frac{3}{4}$ pounds in a recipe. How many pounds does she have left?
   $1\frac{11}{20}$ pounds

4. Mark has a sheet of wrapping paper that is $1\frac{1}{3}$ yards long. He uses $\frac{3}{5}$ yard of the wrapping paper. How much of the sheet does Mark have left?
   $\frac{11}{15}$ yard

5. Ben and Gina go apple picking. The weights of the apples they pick are shown. How many pounds do they pick altogether?
   $16\frac{3}{10}$ pounds

6. Caleb walks $2\frac{1}{4}$ miles from his home to the park. Andre walks $1\frac{2}{3}$ miles from his home to the park. Who lives closer to the park? By how much?
   Andre, by $\frac{7}{12}$ mile

7. At the beginning of summer, Rick was $54\frac{5}{6}$ inches tall. He grew $1\frac{1}{4}$ inches over the summer. How tall is he now?
   $56\frac{1}{12}$ inches

8. Andy walks his dog for $2\frac{2}{3}$ miles on Saturday. On Sunday, he walks his dog for $3\frac{1}{2}$ miles. How many miles does he walk his dog on Saturday and Sunday?
   $6\frac{1}{6}$ miles

9. **STEM Connection** Poppy is helping to clean up a park. The trash bag she is using can hold up to 15 pounds. There are $10\frac{5}{8}$ pounds in the bag now. How many more pounds of trash can Poppy collect with the same bag? **$4\frac{3}{8}$ pounds**

10. **Extend Your Thinking** Write a word problem that involves addition or subtraction of fractions and mixed numbers to solve. The solution to the problem should equal a whole number. **Answers may vary.**

Reflect

How does knowing how to add and subtract mixed numbers and fractions help you solve problems?

Answers may vary.

Math is... Mindset

How did you use your abilities and skills to be successful today?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 4 Students may be confused by this problem as one of the numbers is a fraction instead of a mixed number. Remind students that they can use the same strategies they have learned when subtracting a fraction from a mixed number.

Item Analysis

Item	DOK	Rigor
1–9	2	Application
10	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How does knowing how to add and subtract mixed numbers and fractions help you solve problems?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you use your abilities and skills to be successful today?

Students reflect on how they practiced self-management.

Learning Targets

Ask students to reflect on the Learning Target of the lesson.

- I can solve word problems involving fractions.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Solve problems involving mixed numbers	5.NF.A.2
2	2	Solve problems involving mixed numbers	5.NF.A.2
3	2	Solve problems involving mixed numbers	5.NF.A.2
4	2	Solve problems involving mixed numbers	5.NF.A.2

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

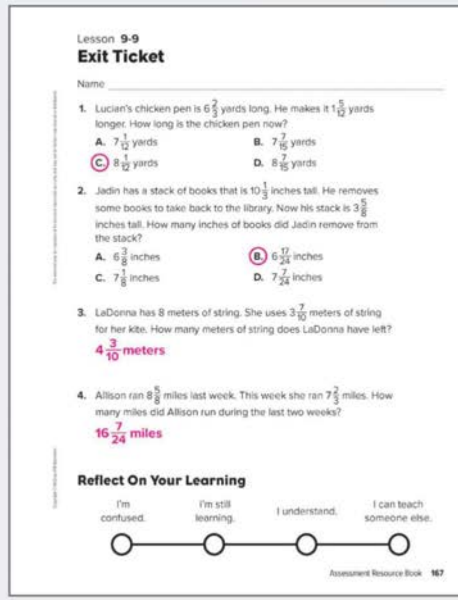
If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 9-9

Exit Ticket

Name ____________________

1. Lucian's chicken pen is $6\frac{2}{3}$ yards long. He makes it $1\frac{5}{12}$ yards longer. How long is the chicken pen now?

   A. $7\frac{1}{12}$ yards B. $7\frac{7}{15}$ yards

   C. $8\frac{1}{12}$ yards D. $8\frac{7}{15}$ yards

2. Jadin has a stack of books that is $10\frac{1}{3}$ inches tall. He removes some books to take back to the library. Now his stack is $3\frac{5}{8}$ inches tall. How many inches of books did Jadin remove from the stack?

   A. $6\frac{3}{8}$ inches B. $6\frac{17}{24}$ inches

   C. $7\frac{1}{8}$ inches D. $7\frac{7}{24}$ inches

3. LaDonna has 8 meters of string. She uses $3\frac{7}{10}$ meters of string for her kite. How many meters of string does LaDonna have left?

   $4\frac{3}{10}$ meters

4. Allison ran $8\frac{5}{8}$ miles last week. This week she ran $7\frac{2}{3}$ miles. How many miles did Allison run during the last two weeks?

   $16\frac{7}{24}$ miles

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 167

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Is the Difference the Same?

Work with students in pairs. Provide fraction tiles and circles. One partner rolls three number cubes. The other partner uses the numbers rolled to make a mixed number with: a 1-digit whole number, numerator, and denominator. Students add that number to or subtract from $7\frac{1}{8}$Remind students that they may need to regroup the mixed number before subtracting.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Add & Subtract Fractions (Model)
- Add & Subtract Fractions (Equations)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 105

Lesson 9-9 • Reinforce Understanding

Solve Problems Involving Mixed Numbers

Name ______________________

Review

Problems involving mixed numbers can be solved using known strategies for addition and subtraction of mixed numbers.

Problem	Work and Solution
Nancy has $3\frac{2}{5}$ cords of firewood. David has $4\frac{9}{10}$ cords of firewood. How many cords of firewood do they have in total?	$3\frac{2}{5} + 4\frac{9}{10}$ $= 3\frac{4}{10} + 4\frac{9}{10}$ $= 7\frac{13}{10}$ $= 8\frac{3}{10}$ cords

1. Sheridan and Natalie are shucking corn. Sheridan shucks $6\frac{5}{8}$ pounds of corn. Natalie shucks $2\frac{3}{4}$ pounds of corn less than Sheridan. How many pounds of corn did Natalie shuck?
   $3\frac{7}{8}$ pounds of corn
2. Corben runs $4\frac{1}{2}$ miles on Monday, $6\frac{3}{10}$ miles on Tuesday, and $5\frac{4}{5}$ miles on Wednesday. How many total miles did Corben run?
   $16\frac{6}{10}$ or $16\frac{3}{5}$ miles
3. Rico is baking bread. He has $7\frac{7}{9}$ cups of flour and he uses $3\frac{5}{6}$ cups. How many cups of flour does he have left?
   $3\frac{17}{18}$ cups of flour

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction and Mixed Number Addition Race

Students practice adding fractions and mixed numbers.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 105–106

Lesson 9-9

Additional Practice

Name ______________________

Review

You can solve problems by adding and subtracting mixed numbers with regrouping.

Carla bought $2\frac{7}{8}$ pounds of cashews and $4\frac{1}{6}$ pounds of raisins for a mixture.

How many pounds is Carla's mixture? To solve, find $2\frac{7}{8} + 4\frac{1}{6}$. Write the fractions using a common denominator: $2\frac{7}{8} + 4\frac{1}{6} = 2\frac{21}{24} + 4\frac{4}{24}$ Add the mixed numbers: $2\frac{21}{24} + 4\frac{4}{24} = 6\frac{25}{24}$ Regroup the fraction: $6 + 1\frac{1}{24} = 7\frac{1}{24}$ Carla's mixture weighed $7\frac{1}{24}$ pounds.	How many more pounds of raisins did Carla buy than cashews? To solve, find $4\frac{1}{6} - 2\frac{7}{8}$. Write the fractions using a common denominator: $4\frac{4}{24} - 2\frac{21}{24}$ Regroup 1 whole as $\frac{24}{24}$: $3\frac{28}{24} - 2\frac{21}{24}$ Subtract the mixed numbers: $3\frac{28}{24} - 2\frac{21}{24} = 1\frac{7}{24}$ Carla bought $1\frac{7}{24}$ pounds more raisins.

Solve the problem. Show your work. Check students' work.

1. Allison buys $4\frac{1}{3}$ pounds of tomatoes. She uses $2\frac{4}{5}$ pounds of tomatoes to make a salad. How many pounds of tomatoes does Allison have left? $1\frac{8}{15}$ pounds

Student Practice Book

Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division of multi-digit numbers.

Use It! Application Station

Create and Solve Students create a multi-step problem that adds and subtracts mixed numbers to solve. Then they use a digital tool to present the problem.

WORKSTATIONS

Spiral Review Practice

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

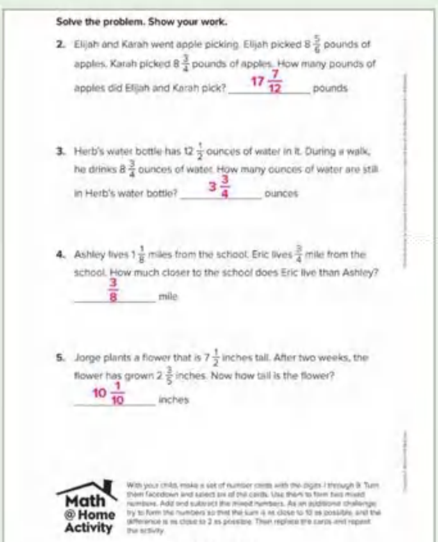
STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

GO ONLINE

INDEPENDENT WORK

Student Practice Book, pp. 105–106



Solve the problem. Show your work.

2. Elijah and Karah went apple picking. Elijah picked $8\frac{5}{6}$ pounds of apples. Karah picked $8\frac{3}{4}$ pounds of apples. How many pounds of apples did Elijah and Karah pick? $17\frac{7}{12}$ pounds

3. Herb's water bottle has $12\frac{1}{2}$ ounces of water in it. During a walk, he drinks $8\frac{3}{4}$ ounces of water. How many ounces of water are still in Herb's water bottle? $3\frac{3}{4}$ ounces

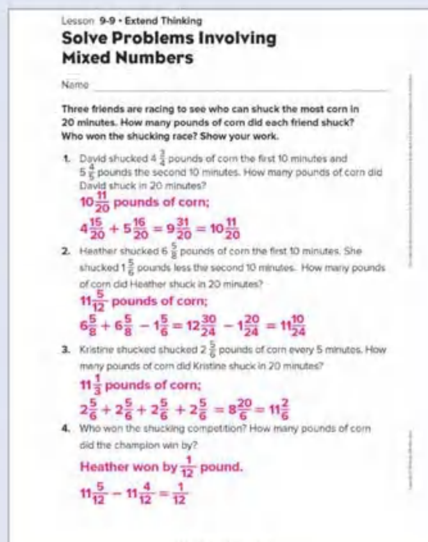
4. Ashley lives $1\frac{1}{8}$ miles from the school. Eric lives $\frac{3}{4}$ mile from the school. How much closer to the school does Eric live than Ashley? $\frac{3}{8}$ mile

5. Jorge plants a flower that is $7\frac{1}{2}$ inches tall. After two weeks, the flower has grown $2\frac{3}{5}$ inches. Now how tall is the flower? $10\frac{1}{10}$ inches

Math @ Home Activity

Student Practice Book

Differentiation Resource Book, p. 106



Lesson 9-9 • Extend Thinking

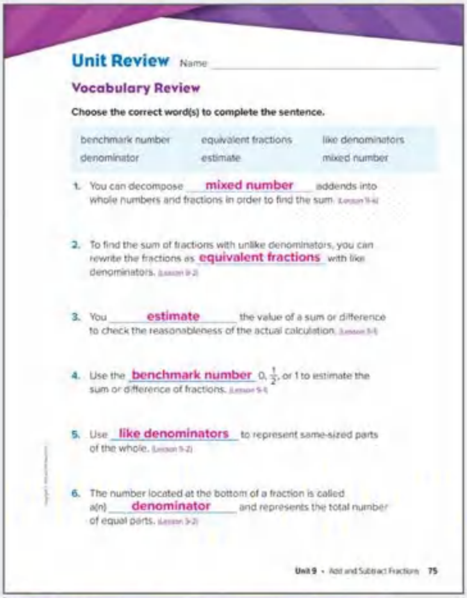
Solve Problems Involving Mixed Numbers

Name ______________________

Three friends are racing to see who can shuck the most corn in 20 minutes. How many pounds of corn did each friend shuck? Who won the shucking race? Show your work.

1. David shucked $4\frac{3}{4}$ pounds of corn the first 10 minutes and $5\frac{4}{5}$ pounds the second 10 minutes. How many pounds of corn did David shuck in 20 minutes?
   $10\frac{11}{20}$ pounds of corn;
   $4\frac{15}{20} + 5\frac{16}{20} = 9\frac{31}{20} = 10\frac{11}{20}$
2. Heather shucked $6\frac{5}{8}$ pounds of corn the first 10 minutes. She shucked $1\frac{5}{6}$ pounds less the second 10 minutes. How many pounds of corn did Heather shuck in 20 minutes?
   $11\frac{5}{12}$ pounds of corn;
   $6\frac{5}{8} + 6\frac{5}{8} - 1\frac{5}{6} = 12\frac{30}{24} - 1\frac{20}{24} = 11\frac{10}{24}$
3. Kristine shucked shucked $2\frac{5}{6}$ pounds of corn every 5 minutes. How many pounds of corn did Kristine shuck in 20 minutes?
   $11\frac{1}{3}$ pounds of corn;
   $2\frac{5}{6} + 2\frac{5}{6} + 2\frac{5}{6} + 2\frac{5}{6} = 8\frac{20}{6} = 11\frac{2}{6}$
4. Who won the shucking competition? How many pounds of corn did the champion win by?
   Heather won by $\frac{1}{12}$ pound.
   $11\frac{5}{12} - 11\frac{4}{12} = \frac{1}{12}$

Differentiation Resource Book



Unit Review

Name ______

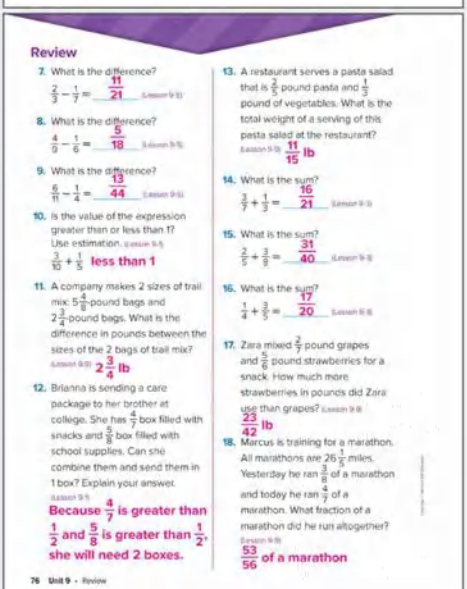
Vocabulary Review

Choose the correct word(s) to complete the sentence.

benchmark number	equivalent fractions	like denominators
denominator	estimate	mixed number

1. You can decompose **mixed number** addends into whole numbers and fractions in order to find the sum. (Lesson 9-6)
2. To find the sum of fractions with unlike denominators, you can rewrite the fractions as **equivalent fractions** with like denominators. (Lesson 9-2)
3. You **estimate** the value of a sum or difference to check the reasonableness of the actual calculation. (Lesson 9-1)
4. Use the **benchmark number** 0, $\frac{1}{2}$, or 1 to estimate the sum or difference of fractions. (Lesson 9-1)
5. Use **like denominators** to represent same-sized parts of the whole. (Lesson 9-2)
6. The number located at the bottom of a fraction is called a(n) **denominator** and represents the total number of equal parts. (Lesson 9-2)

Unit 9 • Add and Subtract Fractions 75



Review

7. What is the difference?
$\frac{2}{3} - \frac{1}{7} =$ **$\frac{11}{21}$** (Lesson 9-5)

8. What is the difference?
$\frac{4}{9} - \frac{1}{6} =$ **$\frac{5}{18}$** (Lesson 9-5)

9. What is the difference?
$\frac{6}{11} - \frac{1}{4} =$ **$\frac{13}{44}$** (Lesson 9-5)

10. Is the value of the expression greater than or less than 1? Use estimation. (Lesson 9-1)
$\frac{3}{10} + \frac{1}{5}$ **less than 1**

11. A company makes 2 sizes of trail mix: $5\frac{4}{8}$-pound bags and $2\frac{3}{4}$-pound bags. What is the difference in pounds between the sizes of the 2 bags of trail mix? (Lesson 9-9) **$2\frac{3}{4}$ lb**

12. Brianna is sending a care package to her brother at college. She has $\frac{4}{7}$ box filled with snacks and $\frac{5}{8}$ box filled with school supplies. Can she combine them and send them in 1 box? Explain your answer. (Lesson 9-1)
Because $\frac{4}{7}$ is greater than $\frac{1}{2}$ and $\frac{5}{8}$ is greater than $\frac{1}{2}$, she will need 2 boxes.

13. A restaurant serves a pasta salad that is $\frac{2}{5}$ pound pasta and $\frac{1}{3}$ pound of vegetables. What is the total weight of a serving of this pasta salad at the restaurant? (Lesson 9-9) **$\frac{11}{15}$ lb**

14. What is the sum?
$\frac{3}{7} + \frac{1}{3} =$ **$\frac{16}{21}$** (Lesson 9-3)

15. What is the sum?
$\frac{2}{5} + \frac{3}{8} =$ **$\frac{31}{40}$** (Lesson 9-3)

16. What is the sum?
$\frac{1}{4} + \frac{3}{5} =$ **$\frac{17}{20}$** (Lesson 9-3)

17. Zara mixed $\frac{2}{7}$ pound grapes and $\frac{5}{6}$ pound strawberries for a snack. How much more strawberries in pounds did Zara use than grapes? (Lesson 9-9)
$\frac{23}{42}$ lb

18. Marcus is training for a marathon. All marathons are $26\frac{1}{5}$ miles. Yesterday he ran $\frac{3}{8}$ of a marathon and today he ran $\frac{4}{7}$ of a marathon. What fraction of a marathon did he run altogether? (Lesson 9-9)
$\frac{53}{56}$ of a marathon

76 Unit 9 • Review

Students can complete the **Unit Review** to prepare for the **Unit Assessment.** Students may complete the Review in their Interactive eBook in the Digital Student Center.

Vocabulary Review

Item Analysis

Item	Lesson
1	9-6
2	9-2
3	9-1
4	9-1
5	9-2
6	9-2

Review

Item Analysis

Item	DOK	Lesson	Standard
7	1	9-5	5.NF.A.1
8	1	9-5	5.NF.A.1
9	1	9-5	5.NF.A.1
10	1	9-1	5.NF.A.2
11	2	9-9	5.NF.A.2
12	3	9-1	5.NF.A.2
13	2	9-9	5.NF.A.2
14	1	9-3	5.NF.A.1
15	1	9-3	5.NF.A.1
16	1	9-3	5.NF.A.1
17	2	9-9	5.NF.A.2
18	2	9-9	5.NF.A.2

To review the lessons in this unit, have students watch the Math Replay video in their Digital Student Center.

Assign the Unit Review practice to students from the Digital Teacher Center.

Item Analysis (continued)

Item	DOK	Lesson	Standard
19	1	9-4	5.NF.A.1
20	1	9-4	5.NF.A.1
21	1	9-2	5.NF.A.1
22	2	9-9	5.NF.A.2
23	1	9-2	5.NF.A.1
24	1	9-1	5.NF.A.2
25	1	9-6	5.NF.A.1
26	1	9-7	5.NF.A.1
27	1	9-6	5.NF.A.1
28	1	9-7	5.NF.A.1
29	2	9-9	5.NF.A.2
30	2	9-8	5.NF.A.1

Performance Task

Standards: 5.NF.A.1, 5.NF.A.2

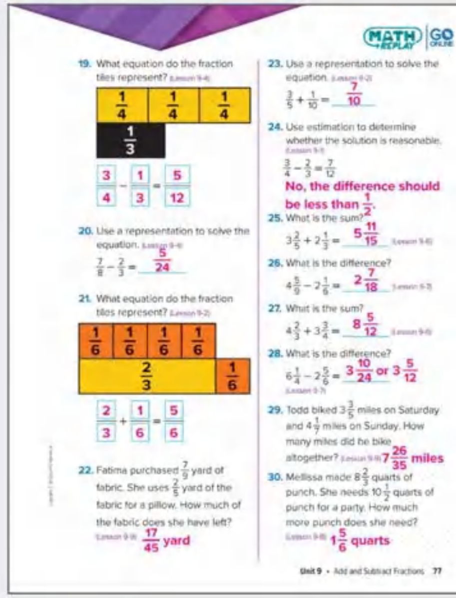
Rubric (4 points)

Part A (DOK 3) – 2 points	
2 POINTS	Student's work reflects proficiency with adding mixed numbers.
1 POINT	Student's work reflects developing proficiency with adding mixed numbers.
0 POINTS	Student's work shows weak proficiency with adding mixed numbers.

Part B (DOK 3) – 2 points	
2 POINTS	Student's work reflects proficiency with adding and subtracting mixed numbers.
1 POINT	Student's work reflects developing proficiency with adding and subtracting mixed numbers.
0 POINTS	Student's work shows weak proficiency with adding and subtracting mixed numbers.

Reflect

The Reflect question provides an opportunity for students to express their understanding of the unit level focus question.



MATH REPLAY | GO ONLINE

19. What equation do the fraction tiles represent?

$\frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

20. Use a representation to solve the equation.

$\frac{7}{8} - \frac{2}{3} = \frac{5}{24}$

21. What equation do the fraction tiles represent?

$\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$

22. Fatima purchased $\frac{7}{9}$ yard of fabric. She uses $\frac{2}{5}$ yard of the fabric for a pillow. How much of the fabric does she have left? $\frac{17}{45}$ yard

23. Use a representation to solve the equation.

$\frac{3}{5} + \frac{1}{10} = \frac{7}{10}$

24. Use estimation to determine whether the solution is reasonable.

$\frac{3}{4} - \frac{2}{3} = \frac{7}{12}$

No, the difference should be less than $\frac{1}{2}$.

25. What is the sum?

$3\frac{2}{5} + 2\frac{1}{3} = 5\frac{11}{15}$

26. What is the difference?

$4\frac{5}{9} - 2\frac{1}{6} = 2\frac{7}{18}$

27. What is the sum?

$4\frac{2}{3} + 3\frac{3}{4} = 8\frac{5}{12}$

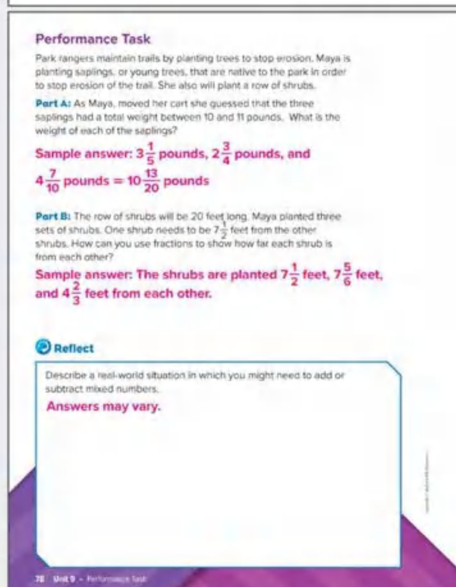
28. What is the difference?

$6\frac{1}{4} - 2\frac{5}{6} = 3\frac{10}{24}$ or $3\frac{5}{12}$

29. Todd biked $3\frac{3}{5}$ miles on Saturday and $4\frac{1}{7}$ miles on Sunday. How many miles did he bike altogether? $7\frac{26}{35}$ miles

30. Melissa made $8\frac{2}{3}$ quarts of punch. She needs $10\frac{1}{2}$ quarts of punch for a party. How much more punch does she need? $1\frac{5}{6}$ quarts

Unit 9 • Add and Subtract Fractions 77



Performance Task

Park rangers maintain trails by planting trees to stop erosion. Maya is planting saplings, or young trees, that are native to the park in order to stop erosion of the trail. She also will plant a row of shrubs.

Part A: As Maya, moved her cart she guessed that the three saplings had a total weight between 10 and 11 pounds. What is the weight of each of the saplings?

Sample answer: $3\frac{1}{5}$ pounds, $2\frac{3}{4}$ pounds, and $4\frac{7}{10}$ pounds $= 10\frac{13}{20}$ pounds

Part B: The row of shrubs will be 20 feet long. Maya planted three sets of shrubs. One shrub needs to be $7\frac{1}{2}$ feet from the other shrubs. How can you use fractions to show how far each shrub is from each other?

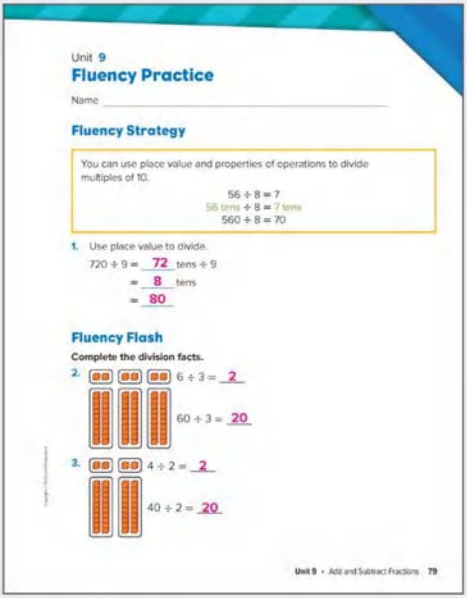
Sample answer: The shrubs are planted $7\frac{1}{2}$ feet, $7\frac{5}{6}$ feet, and $4\frac{2}{3}$ feet from each other.

Reflect

Describe a real-world situation in which you might need to add or subtract mixed numbers.

Answers may vary.

78 Unit 9 • Performance Task


Unit 9

Fluency Practice

Name ______________________

Fluency Strategy

You can use place value and properties of operations to divide multiples of 10.

$56 \div 8 = 7$

56 tens $\div 8 =$ 7 tens

$560 \div 8 = 70$

1. Use place value to divide.

$720 \div 9 =$ 72 tens $\div 9$

= 8 tens

= 80

Fluency Flash

Complete the division facts.

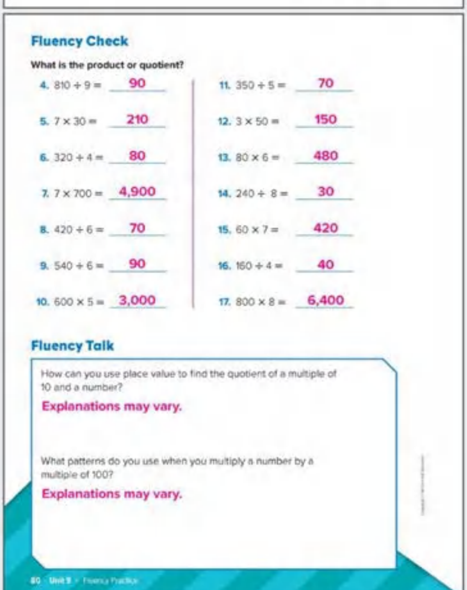
2. $6 \div 3 =$ 2

$60 \div 3 =$ 20

3. $4 \div 2 =$ 2

$40 \div 2 =$ 20

Unit 9 • Add and Subtract Fractions 79


Fluency Check

What is the product or quotient?

4. $810 \div 9 =$ 90

5. $7 \times 30 =$ 210

6. $320 \div 4 =$ 80

7. $7 \times 700 =$ 4,900

8. $420 \div 6 =$ 70

9. $540 \div 6 =$ 90

10. $600 \times 5 =$ 3,000

11. $350 \div 5 =$ 70

12. $3 \times 50 =$ 150

13. $80 \times 6 =$ 480

14. $240 \div 8 =$ 30

15. $60 \times 7 =$ 420

16. $160 \div 4 =$ 40

17. $800 \times 8 =$ 6,400

Fluency Talk

How can you use place value to find the quotient of a multiple of 10 and a number?

Explanations may vary.

What patterns do you use when you multiply a number by a multiple of 100?

Explanations may vary.

80 Unit 9 • Fluency Practice

Fluency practice helps students develop procedural fluency, that is, the “ability to apply procedures accurately, efficiently, and flexibly.” Because there is no expectation of speed, students should not be timed when completing the practice activity.

Build Fluency **Objective** Students practice dividing multiples of 10.

Fluency Progression

Unit	Skill	Standard
1	Use Partial Sums to Add	4.NBT.B.4
2	Decompose by Place Value to Subtract	4.NBT.B.4
3	Use an Algorithm to Add	4.NBT.B.4
4	Use an Algorithm to Subtract	4.NBT.B.4
5	Choose a Strategy to Add	4.NBT.B.4
6	Choose a Strategy to Subtract	4.NBT.B.4
7	Multiply by Multiples of 10	5.NBT.B.5
8	Multiply by Multiples of 100	5.NBT.B.5
9	**Divide Multiples of 10**	**5.NBT.B.6**
10	Divide Multiples of 100	5.NBT.B.6
11	Use an Algorithm to Multiply (2- and 3-Digit Numbers by 1-Digit Numbers)	5.NBT.B.5
12	Use an Algorithm to Multiply (2-Digit Numbers by 2-Digit Numbers)	5.NBT.B.5
13	Choose a Strategy to Multiply	5.NBT.B.5
14	Choose a Strategy to Multiply	5.NBT.B.5

Fluency Expectations

Grade 4

- Add and subtract within 1,000,000.

Grade 5

- Multiply multi-digit whole numbers.

Grade 6

- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Performance Task

Valentina's Celebration

Students draw on their understanding of addition and subtraction of fractions and mixed numbers with unlike denominators. Use the rubric shown to evaluate students' work.

Standards: 5.NF.A.1, 5.NF.A.2

Rubric (8 points)

Parts A and B (DOK 2) – 4 points	
4 POINTS	Student's work shows proficiency with adding/subtracting mixed numbers with unlike denominators. The student's answers and work are correct.
2 POINTS	Student's work shows developing proficiency with adding/subtracting mixed numbers with unlike denominators. The student's answers or work is incorrect.
0 POINTS	Student's work shows weak proficiency with adding/subtracting mixed numbers with unlike denominators. The student's answers and work are incorrect.

Part C (DOK 2) – 2 points	
2 POINTS	Student's work shows proficiency with adding mixed numbers with unlike denominators. The student's response and explanation are reasonable.
1 POINT	Student's work shows developing proficiency with adding mixed numbers with unlike denominators. The student's response or explanation is not reasonable.
0 POINTS	Student's work shows weak proficiency with adding mixed numbers with unlike denominators. The student's response and explanation are not reasonable.

Part D (DOK 2) – 2 points	
2 POINTS	Student's work shows proficiency with subtracting mixed numbers from whole numbers. The student's answer and work are correct.
1 POINT	Student's work shows developing proficiency with subtracting mixed numbers from whole numbers. The student's answer or work is incorrect.
0 POINTS	Student's work shows weak proficiency with subtracting mixed numbers from whole numbers. The student's answer and work are incorrect.

Unit 9

Performance Task

Name ____________________

Valentina's Celebration!

Valentina has several things to do before her party.

Part A

Valentina drives $4\frac{2}{3}$ miles to the bakery to pick up muffins. Valentina then drives $5\frac{1}{2}$ miles to the grocery store to pick up some snacks for the party. How many miles did Valentina drive? Show two different ways to add.

$10\frac{1}{6}$ miles; Sample Answer:

$4\frac{2}{3} + 5\frac{1}{2} = \frac{14}{3} + \frac{11}{2} = \frac{28}{6} + \frac{33}{6} = \frac{61}{6} = 10\frac{1}{6};$

$4\frac{2}{3} + 5\frac{1}{2} = 4\frac{4}{6} + 5\frac{3}{6} = 9\frac{7}{6} = 10\frac{1}{6}$

Part B

Valentina still has 3 more places to go before the party.

Location	Distance (mi)
The mall for favors	$4\frac{5}{6}$
The restaurant to pick up food	$3\frac{1}{12}$
Home to decorate	$8\frac{7}{12}$

The restaurant is $1\frac{3}{4}$ miles closer than the mall from where she is. Home is $5\frac{1}{2}$ miles farther than the restaurant. Fill in the distances to the restaurant and to home. Show your work.

Restaurant: $3\frac{1}{12}$ miles; Home: $8\frac{7}{12}$ miles; Sample answer:

$4\frac{5}{6} - 1\frac{3}{4} = 4\frac{10}{12} - 1\frac{9}{12} = 3\frac{1}{12}; 3\frac{1}{12} + 5\frac{1}{2} = 3\frac{1}{12} + 5\frac{6}{12} = 8\frac{7}{12}$

Part C

Valentina makes punch to serve at her party. During the first hour, the guests drink $4\frac{1}{3}$ quarts of punch and $3\frac{1}{2}$ quarts the second hour. To figure out how much total punch the guests have drank so far, Valentina uses the following strategies:

$4\frac{1}{3} + 3\frac{1}{2} = p$	$4\frac{1}{3} + 3\frac{1}{2} = p$
$\frac{13}{3} + \frac{7}{2} = p$	$4\frac{1}{3} + 3 = 7\frac{1}{3}$
$\frac{39}{6} + \frac{21}{6} = p$	$7\frac{1}{3} + \frac{1}{2} = p$
$\frac{60}{6} = p$	$7\frac{2}{6} + \frac{3}{6} = p$
$10 = p$	$7\frac{5}{6} = p$

How do you respond to her?

Sample Answer: The work on the left is incorrect. She mutliplied 13 times 3 instead of 13 times 2. The work on the right is correct.

How is adding numbers with unlike denominators similar to subtracting fractions with unlike denominators? Explain.

Sample Answer: The process is exactly the same. The denominators must be the same before adding or subtracting.

Part D

During the 3-hour party, Valentina and her friends spend $1\frac{1}{4}$ hours playing basketball. How long did they spend not playing basketball? Show your work.

$1\frac{3}{4}$ hours; Sample Answer: $3 - 1\frac{1}{4} = 2\frac{4}{4} - 1\frac{1}{4} = 1\frac{3}{4}$

Unit Assessments

Two forms of the Unit Assessment, Form A and Form B, are available for either print or digital administration. The items on the two assessments are parallel items, assessing the same concept and standard. The table below provides the item analysis for both forms.

Both Unit Assessments are available in the Assessment Resource Book or as downloadable files from the Digital Teacher Center.

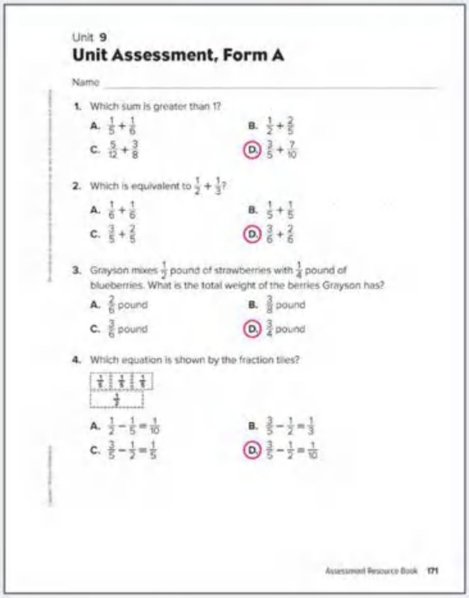
Data When students complete the Unit Assessment in the Digital Student Center, their responses are auto-scored.

Item Analysis

Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	1	9-1	Estimate & Check (Benchmark Fractions)	5.NF.A.2
2	1	9-2	Add Fractions-Rename Both (Models)	5.NF.A.1
3	2	9-2, 9-3	Add Fractions (Rename Both)	5.NF.A.1
4	2	9-4	Subtract Fractions (Model and Rename)	5.NF.A.1
5	2	9-4, 9-5	Subtract Fractions (Rename Both)	5.NF.A.1
6	2	9-6	Add Unlike Mixed Numbers	5.NF.A.1
7	2	9-7	Subtract Unlike Mixed Numbers	5.NF.A.1
8	2	9-8	Subtract Mixed Numbers (Regroup)	5.NF.A.1
9	2	9-1	Estimate & Check (Benchmark Fractions)	5.NF.A.2
10	2	9-3	Add Fractions (Rename Both)	5.NF.A.1
11	2	9-6	Add Unlike Mixed Numbers	5.NF.A.1
12	2	9-9	Add & Subtract Fractions (Model)	5.NF.A.2
13	2	9-1	Estimate & Check (Benchmark Fractions)	5.NF.A.2
14	3	9-9	Add & Subtract Fractions (Equations)	5.NF.A.2

GO ONLINE

Assign the digital Unit Assessment (Form A or B) to students or download and print PDFs from the Digital Teacher Center.



Unit 9

Unit Assessment, Form A

Name ______________________

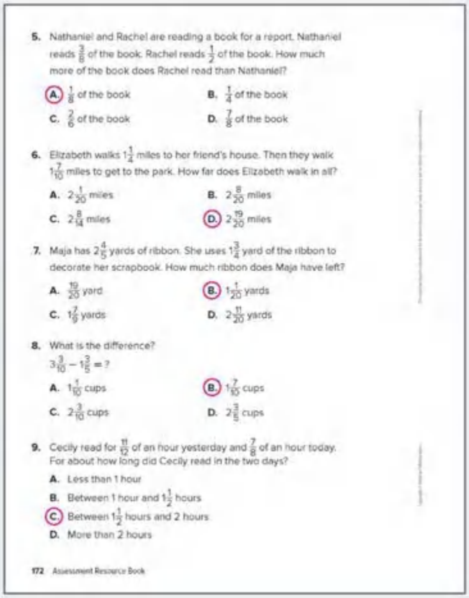
1. Which sum is greater than 1?
   A. $\frac{1}{5}+\frac{1}{6}$ B. $\frac{1}{2}+\frac{2}{5}$
   C. $\frac{5}{12}+\frac{3}{8}$ D. $\frac{3}{5}+\frac{7}{10}$

2. Which is equivalent to $\frac{1}{2}+\frac{1}{3}$?
   A. $\frac{1}{6}+\frac{1}{6}$ B. $\frac{1}{5}+\frac{1}{5}$
   C. $\frac{3}{5}+\frac{2}{5}$ D. $\frac{3}{6}+\frac{2}{6}$

3. Grayson mixes $\frac{1}{2}$ pound of strawberries with $\frac{1}{4}$ pound of blueberries. What is the total weight of the berries Grayson has?
   A. $\frac{2}{6}$ pound B. $\frac{3}{8}$ pound
   C. $\frac{3}{6}$ pound D. $\frac{3}{4}$ pound

4. Which equation is shown by the fraction tiles?
   A. $\frac{1}{2}-\frac{1}{5}=\frac{1}{10}$ B. $\frac{3}{5}-\frac{1}{2}=\frac{1}{3}$
   C. $\frac{3}{5}-\frac{1}{2}=\frac{1}{5}$ D. $\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$

Assessment Resource Book 171



5. Nathaniel and Rachel are reading a book for a report. Nathaniel reads $\frac{3}{8}$ of the book. Rachel reads $\frac{1}{2}$ of the book. How much more of the book does Rachel read than Nathaniel?
   A. $\frac{1}{8}$ of the book B. $\frac{1}{4}$ of the book
   C. $\frac{2}{6}$ of the book D. $\frac{7}{8}$ of the book

6. Elizabeth walks $1\frac{1}{4}$ miles to her friend's house. Then they walk $1\frac{7}{10}$ miles to get to the park. How far does Elizabeth walk in all?
   A. $2\frac{1}{20}$ miles B. $2\frac{8}{20}$ miles
   C. $2\frac{8}{14}$ miles D. $2\frac{19}{20}$ miles

7. Maja has $2\frac{4}{5}$ yards of ribbon. She uses $1\frac{3}{4}$ yard of the ribbon to decorate her scrapbook. How much ribbon does Maja have left?
   A. $\frac{19}{20}$ yard B. $1\frac{1}{20}$ yards
   C. $1\frac{7}{9}$ yards D. $2\frac{11}{20}$ yards

8. What is the difference?
   $3\frac{3}{10}-1\frac{3}{5}=?$
   A. $1\frac{1}{10}$ cups B. $1\frac{7}{10}$ cups
   C. $2\frac{3}{10}$ cups D. $2\frac{3}{5}$ cups

9. Cecily read for $\frac{11}{12}$ of an hour yesterday and $\frac{7}{8}$ of an hour today. For about how long did Cecily read in the two days?
   A. Less than 1 hour
   B. Between 1 hour and $1\frac{1}{2}$ hours
   C. Between $1\frac{1}{2}$ hours and 2 hours
   D. More than 2 hours

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Unit **9**

Unit Assessment, Form A (continued)

Name ______________________________

10. To find $\frac{5}{8} + \frac{5}{6}$, which number may be used as a common denominator?

A. 8 **B.** 14

C. 24 **D.** 36

11. Bradley uses $1\frac{2}{3}$ cups of chopped nuts and $2\frac{1}{4}$ cups of fresh berries in a granola mix. How many cups of chopped nuts and fresh berries does Bradley use in all?

A. $3\frac{1}{4}$ cups **B.** $3\frac{3}{7}$ cups

C. $3\frac{11}{12}$ cups **D.** $4\frac{1}{12}$ cups

12. Cynthia practices the piano for $2\frac{3}{8}$ hours on Friday and for $3\frac{1}{4}$ hours on Saturday. How much longer does Cynthia practice the piano on Saturday than on Friday?

$\frac{7}{8}$ hour

13. Jackie's thermos contains $\frac{11}{12}$ cup of soup. After a walk, she eats $\frac{5}{8}$ cup of the soup. Which best describes how much soup is left in the thermos?

A. almost none

B. about $\frac{1}{2}$ cup

C. about 1 cup

D. about $1\frac{1}{2}$ cups

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14. Don jumped a distance of $7\frac{1}{3}$ feet. His younger brother jumped a distance of $4\frac{1}{2}$ feet and then jumped $3\frac{11}{12}$ feet. Who jumped farther? How much farther? Explain how you found your answer.

Don's younger brother; $1\frac{1}{12}$ feet; Sample answer: I added the lengths of the brother's two jumps: $4\frac{1}{2} + 3\frac{11}{12} = 8\frac{5}{12}$. Since $8\frac{5}{12}$ is greater than $7\frac{1}{3}$, Don's brother jumped farther. To find how much farther, I subtracted $7\frac{1}{3}$ from $8\frac{5}{12}$ to get $1\frac{1}{12}$.

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Form B

Unit 9

Unit Assessment, Form B

Name ______________________________

1. Which sum is less than 1?

2. Which is equivalent to $\frac{1}{4} + \frac{1}{5}$?

3. Vincente mixes [illegible] liter of orange juice with [illegible] liter of pineapple juice. How much juice does Vincente have?

4. Which equation is shown by the fraction tiles?

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A. 1 pound **B.** $1\frac{5}{24}$ pounds

C. [illegible] pounds **D.** $2\frac{11}{24}$ pounds

8. What is the difference?

A. $\frac{5}{8}$ pound **B.** [illegible] pound

C. [illegible] pounds **D.** [illegible] pounds

9. Angela walked for [illegible] hour yesterday and [illegible] hour today. For about how long did Angela walk in the two days?

A. Less than 1 hour

B. Between 1 hour and $1\frac{1}{2}$ hours

C. Between $1\frac{1}{2}$ hours and 2 hours

D. More than 2 hours

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12. Catherine practices the flute for [illegible] hours on Friday and for [illegible] hours on Saturday. How much longer does Catherine practice the flute on Saturday than on Friday?

$\frac{7}{8}$ hour

13. Julia's thermos contains [illegible] cup of soup. After a walk, she eats [illegible] cup of the soup. Which best describes how much soup is left in the thermos?

A. almost none **B.** about $\frac{1}{2}$ cup

C. about 1 cup **D.** about $1\frac{1}{2}$ cups

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UNIT 10 PLANNER

Multiply Fractions

PACING: 13 days

LESSON		MATH OBJECTIVE	LANGUAGE OBJECTIVE	SOCIAL AND EMOTIONAL LEARNING OBJECTIVE
Unit Opener	Folding Fractions on a Strip	Students explore how much is represented when folding a strip.		
10-1	Represent Multiplication of a Whole Number by a Fraction	Students use a representation to multiply a whole number by a fraction.	Students discuss using representations to multiply a whole number by a fraction using *other ways* and *different ways*.	Students identify personal traits that make them good students, peers, and math learners.
10-2	Multiply a Whole Number by a Fraction	Students multiply a whole number by a fraction.	Students explain multiplying a whole number by a fraction using the verbs *notice* and *apply*, and the phrase *make a shortcut*.	Students demonstrate thoughtful reflection through identifying the causes of challenges and successes while completing a mathematical task.
Math Probe	Which is Greater?	Students identify the quantity that is greater.		
10-3	Represent Multiplication of a Fraction by a Fraction	Students use a representation to multiply a fraction by a fraction.	Students explain how to represent multiplication of a fraction by a fraction using the verbs *partition* and *show* and the nouns *patterns* and *shortcuts*.	Students offer constructive feedback to the mathematical ideas posed by others.
10-4	Multiply a Fraction by a Fraction	Students multiply a fraction by a fraction by multiplying the numerators and multiplying the denominators.	Students talk about multiplying a fraction by a fraction by multiplying the numerators and denominators using *relate*.	Students analyze the components of a problem to make informed decisions when engaging in mathematical practices.
10-5	Determine the Area of Rectangles with Fractional Side Lengths	Students find the area of a rectangle with fractional side lengths by tiling. Students find the area of a rectangle with fractional side lengths by multiplying the side lengths.	Students explain how to find the area of a rectangle with fractional side lengths using the verb *tile*.	Students discuss how a mathematical rule or routine can help develop mathematical skills and knowledge.
10-6	Represent Multiplication of Mixed Numbers	Students use an area model to represent multiplication of mixed numbers. Students find partial products using an area model.	Students talk about using an area model to represent multiplication of mixed numbers using the terms *similar to* and *different from*.	Students engage in respectful discourse with peers about various perspectives for approaching a mathematical challenge.
10-7	Multiply Mixed Numbers	Students use partial products to multiply mixed numbers. Students write mixed numbers as fractions to find the product.	Students discuss multiplying mixed numbers using the verb *find*.	Students exchange ideas for completing a mathematical task with a peer and reflect on the value of their similarities and differences.
10-8	Multiplication as Scaling	Students explain how the size of the factors impacts the size of the product without performing the multiplication.	Students explain why the product of a given number and a fraction greater than 1 results in a product greater than the given number.	Students discover and discuss personal interests related to mathematics and share these interests with peers.
10-9	Solve Word Problems Involving Fractions	Students solve word problems involving fractions.	Students talk about solving word problems involving fractions using the verb *determine*.	Students develop and execute a plan for mathematical problem solving.

Unit Review
Fluency Practice

Performance Task
Unit Assessment

FOCUS QUESTION:
How can I multiply fractions?

LESSON	KEY VOCABULARY: Math Terms	KEY VOCABULARY: Academic Terms	MATERIALS TO GATHER	RIGOR FOCUS	STANDARD
10-1	fraction model multiplication partition	reflect suggest	• counters • fraction circles • fraction tiles	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.4, 5.NF.B.4.a
10-2	denominator numerator	citation complex	• fraction circles • fraction tiles	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.4, 5.NF.B.4.a
10-3	fraction model multiplication	procedure speculate	• fraction circles • fraction tiles • grid paper	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.4, 5.NF.B.4.a
10-4	denominator numerator	arguably speculate	• grid paper • index cards	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.4, 5.NF.B.4.a
10-5	area square unit	expand reflect	• blank spinners • grid paper • rulers	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.4.b
10-6	area model decompose mixed number partial products	accurate establish	• blank spinners • fraction tiles • grid paper	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.4, 5.NF.B.4.a
10-7	decompose partial products	accurate transition	• grid paper	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.4, 5.NF.B.4.a
10-8	scaling	complex infer	• index cards	Conceptual Understanding	5.NF.B.5.a, 5.NF.B.5.b
10-9	equation unknown variable	assert reflect	• grid paper • *Problem-Solving Tool* Teaching Resource	Application	5.NF.B.6

Unit Overview

Focus

Multiplying Fractions and Multiplication as Scaling

In this unit, students review and build on their work from Grade 4, when they multiplied fractions and mixed numbers by whole numbers using models. That work is extended to give students a generalized understanding of multiplication with whole numbers, fractions, and mixed numbers in any combination.

Throughout the unit, students continue the practice of estimating to check the reasonableness of answers. They use tools such as drawings, fraction tiles, and area models to make connections to multiplication of whole numbers and to make sense of what it means to multiply fractions. Students use that understanding to develop efficient strategies.

To start, students build their understanding of multiplying a fraction by a whole number and develop two processes for approaching the concept.

Scaling is essentially the resizing of a number. Understanding this concept enables students to reason about the size of a product without having to multiply. Multiplying a number by a factor greater than 1 generates a product that is greater than the number, and multiplying a number by a factor less than 1 generates a product that is less than the number. Multiplying two (positive) fractions that are both less than 1 generates a product less than either of the two factors. The idea that it is possible for a product to be less than one or both factors is a difficult concept.

Coherence

What Students Have Learned

- Students applied and extended previous understandings of multiplication to multiply a fraction by a whole number. (Grade 4)
- Students represented a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and used this understanding to multiply a fraction by a whole number. In general, $n \times \left(\frac{a}{b}\right) = \frac{(n \times a)}{b}$. (Grade 4)
- Students solved word problems involving multiplication of a fraction by a whole number. (Grade 4)

What Students Are Learning

- Students multiply a fraction by a whole number or another fraction.
- Students find the area of a rectangle with fractional side lengths.
- Students compare the size of the product to the size of a factor based on the size of the other factor.

What Students Will Learn

- Students interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. (Grade 6)
- Students solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (Grade 6)

Rigor

Conceptual Understanding

Students develop understanding of
- using representations for multiplying fractions and whole numbers;
- using representations for multiplying fractions and fractions;
- using representations for multiplying mixed numbers;
- how the size of one factor impacts the size of the product relative to the other factor.

Procedural Skill & Fluency

Students build proficiency with
- multiplying fractions and whole numbers;
- multiplying fractions and fractions;
- multiplying mixed numbers;
- using area models and partial products to multiply fractions and mixed numbers.

Application

Students apply their knowledge of
- interpreting multiplication with fractions and mixed numbers in real-world contexts;
- multiplication strategies to solve and write fraction and mixed number multiplication problems with real-world contexts.

Effective Teaching Practices

Use and Connect Mathematical Representations

Mathematics is by nature an abstract subject, so to teach, learn, or discuss it, we depend on representations to make it understandable. A representation can be visual, physical, contextual, verbal, or symbolic.

A visual representation can be created individually or in small groups, and because of its recorded nature, it can then be easily shared among groups or with the whole class for discussion. These visuals often come from the use of hands-on learning materials as part of the Concrete-Representational-Abstract (C-R-A) methodology. This instructional approach involves frequent opportunities for students to create and sketch their physical setups and other graphic constructions on the way toward understanding and mastering the related symbolic manipulations.

Students will have a deep understanding of a concept when they can switch and make connections between different perspectives.

For example, to represent multiplication with fractions and mixed numbers, students will use fraction tiles, area models, and expressions and equations, and they will generate and explain problem situations in real-world contexts. As representations are suggested, used, and encountered, spend some time having students discuss them. For example—

- Have them identify representations that possess similar characteristics, such as number lines, fraction tiles, and fraction models, or arrays and area models, and have them describe the similar characteristics and how they are useful.
- Have them create a symbolic representation when given a visual and vice versa—for example, have them write an equation to represent the decomposition of a product into partial products given the area model or draw the area model given the equation.

Math Practices and Processes

Reason Abstractly and Quantitatively

Mathematics is a combination of abstract concepts and real-world applications, and the goal in teaching it is to help students work flexibly in and between both realms. All meaning in mathematics is tied to its application, so as we help students develop their understanding of concepts and their fluency with procedures, we must keep application on their minds.

To do this, we introduce concepts and procedures by way of problem situations. We expose students to problems with content that is familiar and realistic and require them to understand the quantities in a problem and how the quantities relate to the situation. With realistic content, students should then be able to use representations to describe the situation, manipulate the representations as the situation changes and understand the meaning behind the representations.

To help students build proficiency with reasoning abstractly and quantitatively, give them opportunities to interact with a variety of problems. Make it a habit to promote and display flexible thinking.

- Encourage students to make sense of the quantities and relationships in problem situations by creating a variety of representations. For multiplying fractions and mixed numbers, students will make frequent use of fraction tiles and fraction models, area models, and expressions and equations based on fraction concepts and the properties of multiplication.
- Have students explain their representations—for example, how a fraction model represents quantities or how the labeling of an area model connects to the situation.

Social and Emotional Learning

- **Self-Awareness – Self-Confidence** (Lesson 10-1): Self-confident students are more willing to take risks, allowing them to learn from mistakes.
- **Responsible Decision-Making – Reflect** (Lesson 10-2): When students reflect, they can make connections between effort and achievement.
- **Relationship Skills – Social Engagement** (Lesson 10-3): Engaging with others allows students to develop relationships and establish a sense of security and belonging in the classroom community.
- **Responsible Decision-Making – Solve Problems** (Lesson 10-4): Efficient problem solvers can make informed decisions that lead to solutions.
- **Self-Management – Organizational Skills** (Lesson 10-5): Organizing information and work can help students work through challenging mathematical tasks.
- **Social Awareness – Develop Perspective** (Lesson 10-6): Developing perspective can help students understand different ways of thinking.
- **Relationship Skills – Build Relationships** (Lesson 10-7): Building positive relationships can help establish a strong classroom community.
- **Self-Awareness – Identify Emotions** (Lesson 10-8): Students who can identify and understand their own feelings and emotions can better manage the reactions to those feelings and emotions.
- **Self-Management – Goal Setting** (Lesson 10-9): Setting goals can help motivate students to take initiative and stay focused.

Language of Math

Mathematical Nouns

Students will be using these key terms in this unit:

- **Decompose** (Lesson 10-5): Students were introduced to this term in the context of addition and subtraction with whole numbers in Grade 2.
- **Mixed number** (Lesson 10-5): Students were also introduced to this term in the context of adding and subtracting mixed numbers in Grade 4.
- **Partial Products** (Lesson 10-7): Students were also introduced to this term in the context of multiplication of multi-digit whole numbers in Grade 4. They know *product* from Grade 3, and they learned that *partial products* are the products they generate by using the Distributive Property to multiply.
- **Partition** (Lesson 10-1): Students were also introduced to this term in the context of dividing shapes into equal shares in Grade 2. In this unit, the concept is the same but is applied to multiplication by fractions.
- **Scaling*** (Lesson 10-8): Students might know *scale* as a device for measuring weight. In this lesson, *scaling* refers to the multiplicative relationship between a product and its factors. Students use the concept to determine how the size of a product will compare to the sizes of the factors without having to multiply.

*This is a new term.

Math Language Development

A Focus on Writing

Writing in mathematics entails both writing mathematics and writing *about* mathematics—that is, writing can be used to form and execute our thoughts, reasoning, and procedures and to explain them to others.

Writing in mathematics can be a challenge. It is difficult for many students to understand the language of math and to translate math language to everyday language or everyday language to math language. Nonetheless, writing is beneficial and necessary, because it is a tool for students to organize and focus their thoughts and to preserve and share their work.

When students are writing, encourage them to include all the written tools they know of and are learning about—words, expressions, equations, and pictures. Expressions, equations, and pictures can say a lot, but make a point of having students sometimes write out the meanings of these representations and describe why they are being used. This will promote deeper learning and give you insight into students' thinking.

In this unit, students can write to:

- Describe how to estimate the quantities in a multiplication problem to generate an approximation of the solution and explain why they would do that.
- Explain the use of unit fractions as building blocks for finding products.
- Describe an algorithm for multiplying fractions and explain why it works.
- Describe strategies for translating the quantities in a problem into equations with one or more unknowns.
- Explain how to partition an area to represent multiplication of a fraction by a fraction.
- Explain what they know about multiplication with whole numbers that helps them multiply with fractions and mixed numbers.
- Explain the use of the Distributive Property in connection to an area model.

English Language Learner

In this unit, students are provided with a number of scaffolds to support their comprehension of the language used to present and explain strategies related to multiplying fractions. Because many of the words (*shade, shortcut, whether, yield, involving*) and phrases *(that area, whole piece, similar to, different from, you can either ______ or ______*) used in this unit could prove challenging to ELs, they are supported in understanding and using them so that the instruction is more accessible.

Lesson 10-1 – *shade*
Lesson 10-2 – *shortcut*
Lesson 10-3 – *that area*
Lesson 10-4 – *whole piece*
Lesson 10-5 – *whether*
Lesson 10-6 – *similar to, different from*
Lesson 10-7 – *you can either ______ or ______*
Lesson 10-8 – *yield*
Lesson 10-9 – *involving*

Unit Routines

Number Routines

Build Fluency The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the thinking habits of mind that are important for proficient doers of math.

Decompose It

Purpose: Build flexibility with numbers.

Overview: Students generate multiple (at least 3) ways to decompose given numbers and share their thinking for each decomposition. The teacher records decompositions and then facilitates a discussion of patterns in the decompositions.

Which Benchmark Is It Closest To?

Purpose: Enhance rounding and reasoning skills.

Overview: Students determine to which benchmark the given number is closest and explain their reasoning.

Find the Pattern, Make a Pattern

Purpose: Build efficiency with recognizing and building patterns.

Overview: Students determine the rule(s) for a given pattern, then use the rule(s) to create a new pattern. The teacher records students' new patterns and facilitates a discussion to validate the pattern and its rules.

Greater Than or Less Than

Purpose: Build proficiency with number and place value sense; estimating and comparing skills.

Overview: Students use mental math to estimate or evaluate the value of given expressions and then compare the value of the expressions to a target benchmark number. Students share their solutions and thinking.

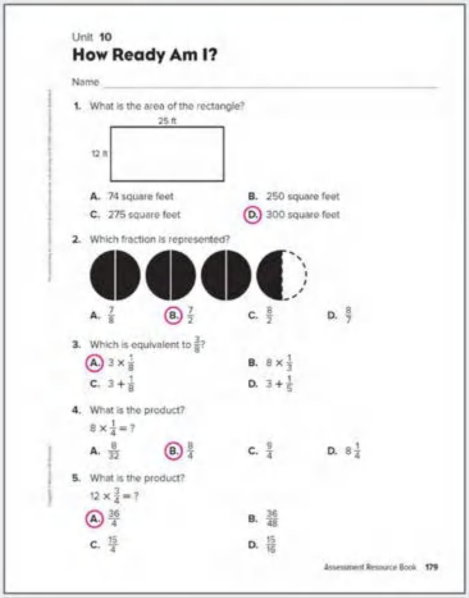
Sense-Making Routines

- **Notice & Wonder: How are they the same? How are they different?** (Lesson 10-9) Students connect the context of real-world problems with different types of models.
- **Notice & Wonder: Tell me everything you can.** (Lesson 10-8) Students comment on how the size of the factors impacts the size of the product.
- **Notice & Wonder: What do you notice? What do you wonder?** (Lessons 10-1, 10-3, 10-6) In Lesson 10-1, students make observations about a model that represents the product of a fraction and a whole number. In Lesson 10-3, they represent a familiar problem using fractions instead of decimals. In Lesson 10-6, students see the decomposition of a mixed number multiplication problem.
- **Notice & Wonder: What could the question be?** (Lesson 10-4) Students consider a situation for multiplying a fraction by a fraction.
- **Notice & Wonder: What do you see?** (Lesson 10-5) Students notice that the area of a rectangle sometimes includes partial tiles.
- **Numberless Word Problem: What math do you see?** (Lesson 10-2) Students discuss a mathematical situation without numbers, including what strategies would be used to solve it.
- **Numberless Word Problem: What is the question?** (Lesson 10-7) Students discuss a mathematical situation without numbers, including what strategies would be used to solve it.

MLR Math Language Routines

The Mathematical Language Routines used in this unit give teachers a structured, yet adaptable format for amplifying and developing students' social and academic language. These routines can also be used as formative assessment opportunities as students develop proficiency in English and mathematical language. They can be used in ways that support real-time-, peer-, and self-assessment. For more information on the Math Language Routines, see the Appendix.

- Lesson 10-1 – Students participate in MLR8: Discussion Supports.
- Lesson 10-2 – Students participate in MLR1: Stronger and Clearer Each Time.
- Lesson 10-3 – Students participate in MLR6: Three Reads.
- Lesson 10-4 – Students participate in MLR3: Critique, Correct, and Clarify.
- Lesson 10-5 – Students participate in MLR2: Collect and Display.
- Lesson 10-6 – Students participate in MLR4: Info Gap.
- Lesson 10-7 – Students participate in MLR7: Compare and Connect.
- Lesson 10-8 – Students participate in MLR1: Stronger and Clearer Each Time.
- Lesson 10-9 – Students participate in MLR5: Co-Craft Questions and Problems.



Unit 10

How Ready Am I?

Name ______________________

1. What is the area of the rectangle?

25 ft

12 ft

A. 74 square feet B. 250 square feet

C. 275 square feet D. 300 square feet

2. Which fraction is represented?

A. $\frac{7}{8}$ B. $\frac{7}{2}$ C. $\frac{8}{2}$ D. $\frac{8}{7}$

3. Which is equivalent to $\frac{3}{8}$?

A. $3 \times \frac{1}{8}$ B. $8 \times \frac{1}{3}$

C. $3 + \frac{1}{8}$ D. $3 + \frac{1}{5}$

4. What is the product?

$8 \times \frac{1}{4} = ?$

A. $\frac{8}{32}$ B. $\frac{8}{4}$ C. $\frac{9}{4}$ D. $8\frac{1}{4}$

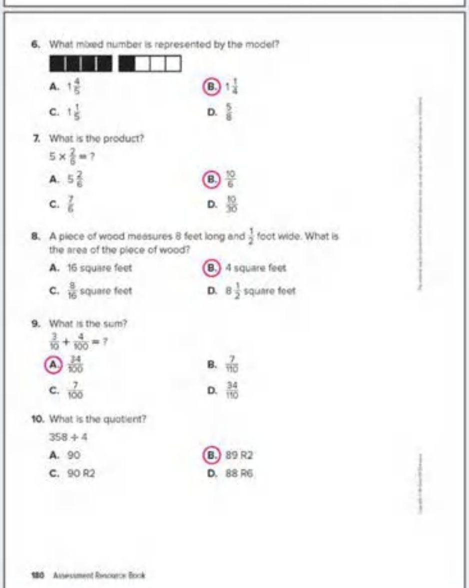
5. What is the product?

$12 \times \frac{3}{4} = ?$

A. $\frac{36}{4}$ B. $\frac{36}{48}$

C. $\frac{15}{4}$ D. $\frac{15}{16}$

Assessment Resource Book 179



6. What mixed number is represented by the model?

A. $1\frac{4}{5}$ B. $1\frac{1}{4}$

C. $1\frac{1}{5}$ D. $\frac{5}{8}$

7. What is the product?

$5 \times \frac{2}{6} = ?$

A. $5\frac{2}{6}$ B. $\frac{10}{6}$

C. $\frac{7}{6}$ D. $\frac{10}{30}$

8. A piece of wood measures 8 feet long and $\frac{1}{2}$ foot wide. What is the area of the piece of wood?

A. 16 square feet B. 4 square feet

C. $\frac{8}{16}$ square feet D. $8\frac{1}{2}$ square feet

9. What is the sum?

$\frac{3}{10} + \frac{4}{100} = ?$

A. $\frac{34}{100}$ B. $\frac{7}{110}$

C. $\frac{7}{100}$ D. $\frac{34}{110}$

10. What is the quotient?

$358 \div 4$

A. 90 B. 89 R2

C. 90 R2 D. 88 R6

180 Assessment Resource Book

Administer the Readiness Diagnostic to determine your students' readiness for this unit.

Targeted Intervention

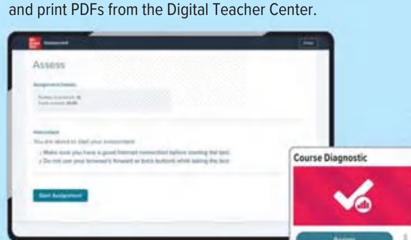
Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	1	Multiply whole numbers to find area	Area of Rectangles and Squares	4.MD.A.3
2	1	Identify improper fractions from models	Build Fractions from Unit Fractions	4.NF.B.3
3	1	Multiply to find equivalent fractions	Equivalent Fractions with Multiplication	4.NF.A.1
4	1	Multiply fractions by whole numbers	Unit Fractions by Whole Number (Models)	4.NF.B.4
5	1	Multiply fractions by whole numbers	Fraction by Whole Number (Equations)	4.NF.B.4
6	1	Identify mixed numbers from models	Build Fractions from Unit Fractions	4.NF.B.3
7	1	Multiply fractions by whole numbers	Fraction by Whole Number (Equations)	4.NF.B.4
8	2	Multiply fractions by whole numbers in word problems	Multiplication in Fraction Word Problems	4.NF.B.4.c
9	1	Add fractions with denominators 10 and 100	Add Fractions in 10ths and 100ths	4.NF.C.5
10	1	Divide 3-digit by 1-digit with remainder	Three-Digit Dividends (Partial Quotients)	4.NBT.B.6

GO ONLINE

Assign the digital Readiness Diagnostic to students or download and print PDFs from the Digital Teacher Center.



Focus Question

Introduce the Focus Question: *How can I multiply fractions?*

Ask students to think about what they know about multiplying fractions.

- What do you know about multiplying numbers?
- How is multiplication related to addition?
- What do you know about adding fractions?

Remind students that at the end of the unit, they will reflect back on what they learned.

Family Letter

Each letter presents an overview of the math in the unit and home activities to support student learning.

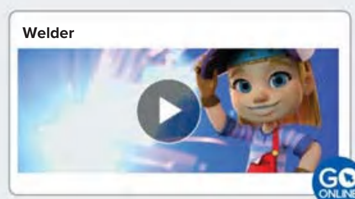
STEM in Action

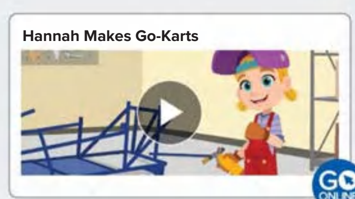
Videos

Students can watch the two STEM videos.

STEM Career: Welder Hannah discusses her aspirations to be a welder.

Hannah Makes Go-Karts Hannah uses multiplication of fractions to find out how much work she can get done in a day.



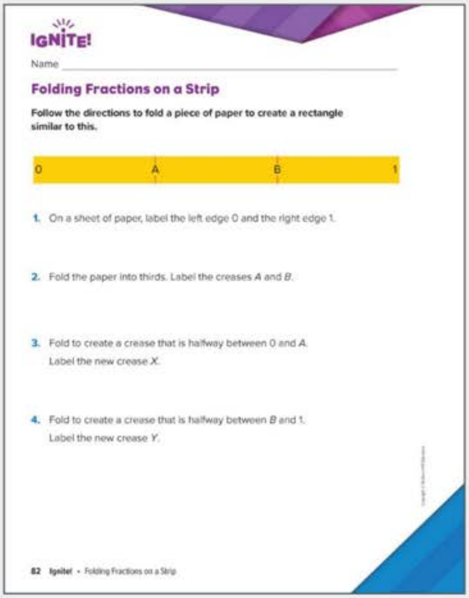


STEM Project Card

Students can complete the STEM Project Card during their workstation time.

STEM Adventure

Students can complete the STEM Adventure during their workstation time.



IGNITE!

Name

Folding Fractions on a Strip

Follow the directions to fold a piece of paper to create a rectangle similar to this.

1. On a sheet of paper, label the left edge 0 and the right edge 1.
2. Fold the paper into thirds. Label the creases *A* and *B*.
3. Fold to create a crease that is halfway between 0 and *A*. Label the new crease *X*.
4. Fold to create a crease that is halfway between *B* and 1. Label the new crease *Y*.

82 Ignite! • Folding Fractions on a Strip

Ignite!

Folding Fractions on a Strip

Students explore how much is represented when folding a strip.

1. Direct attention to the rectangle on the student page.
   - What do you notice about the rectangle?

2. Have students label a piece of paper so that the left edge is labeled as 0 and the right edge is labeled as 1. Show students how to fold a piece of paper “like a letter” into three parts, label the creases A and B.
   - What can you say about the three parts of the strip?

3. Have students fold the left edge to crease A, creating a new crease that is halfway between 0 and A. Have them mark the new crease with a line and label it X.
   - What fraction of the entire strip is the part from 0 to X? How do you know?

4. Have students fold the right edge to crease B. Have them mark the crease with a line and label it Y.
   - What fraction of the entire strip is from Y to 1? Explain.
   - Think of the strip as being a number line from 0 to 1. What fraction would point Y be on the number line? Explain.

Unit Resources At-A-Glance

Workstations

Reveal Math offers rich and varied resources that teachers can use to differentiate and enrich students' instructional experiences with the unit content. The table presents an overview of the resources available for the unit with recommendations for when to use.

Station	Activity	Description	Use After Lesson
Game Station	Game Station	Students build proficiency with multiplication of fractions and mixed numbers.	
		• **Fraction Multiplication Tic Tac Toe**	10-1
		• **Representing Fraction Multiplication Task Cards**	10-2
		• **Fraction Multiplication Task Cards**	10-3
		• **Fraction Multiplication Showdown**	10-4
		• **Mixed Number Task Cards**	10-5
		• **Mixed Number Concentration**	10-6
		• **Area with Fractions Task Cards**	10-7
		• **Product Size Sort**	10-8
		• **Fraction Problem Race**	10-9
Digital Station	Digital Game	**Mad Lab Mix-Up** Students practice multiplying multi-digit numbers.	10-1
Application Station	**Have students complete at least one of the Use It! activities for this unit.**		
	STEM Project Card	**This or That** Students follow instructions to measure and cut materials.	10-8
	Connection Card	**Fraction of a Fraction** Students use fractions to follow and change a recipe.	10-3
	Real World Card	**If, Then** Students use if/then statements to write a problem in which they make a true or false statement that uses multiplication of fractions.	10-7

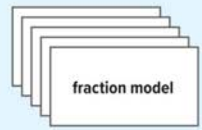
Additional Resources

Use the resources below to provide additional support for this unit.

GO ONLINE

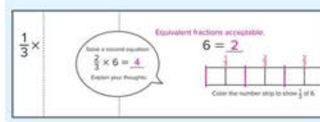
Vocabulary

Use the vocabulary cards to help students learn the vocabulary in this unit. Encourage students to write their own definitions of the new term on the front side of the card.



Foldables

Use the unit foldables with Lessons 10-1 and 10-3.



Spiral Review

Students can complete the Spiral Review at any point during the unit as either a paper-and-pencil or digital activity.

Lesson	Standard
10-1	5.NBT.A.1
10-2	5.MD.C.3
10-3	5.NBT.A.2
10-4	5.NF.A.2
10-5	5.NF.A.1
10-6	5.NBT.B.7
10-7	5.MD.C.4
10-8	5.NF.B.4
10-9	5.NF.B.5

LESSON **10-1**

Represent Multiplication of a Whole Number by a Fraction

Learning Targets

- I can use a representation to multiply a whole number by a fraction.
- I can explain how to use a representation to multiply a whole number by a fraction.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

◇ **5.NF.B.4.a** Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$).

Math Practices and Processes

MPP Look for and express regularity in repeated reasoning.

Vocabulary

Math Terms	Academic Terms
fraction model	reflect
multiplication	suggest
partition	

Materials

The materials may be for any part of the lesson.

- counters
- fraction circles
- fraction tiles

Focus

Content Objective	Language Objectives	SEL Objective
• Students use a representation to multiply a whole number by a fraction.	• Students discuss using representations to multiply a whole number by a fraction using *other ways* and *different ways*. • To support maximizing cognitive and linguistic meta-awareness, ELs participate in MLR8: Discussion Supports.	• Students identify personal traits that make them good students, peers, and math learners.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students multiplied decimals (Unit 6).	• Students use a representation to multiply a whole number by a fraction.	• Students multiply a whole number by a fraction (Unit 10). • Students interpret and compute quotients of fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students interpret different representations used when multiplying fractions by whole numbers.	• Students find a fraction of a whole number.	• Students assess solutions to word problems. *Application is not a targeted element of rigor for this standard.*

Number Routine
Find the Pattern, Make a Pattern 5–7 min

Build Fluency Students build number sense with fractions and mixed numbers as they identify the pattern rule and use the rule to find the missing numbers. Students then create a new pattern that follows the same rule.

These prompts encourage students to talk about their reasoning:

- What pattern do you notice?
- What terms helped you determine the pattern? Explain.
- How did you determine what is missing?
- Create a new pattern. How did you begin your new pattern?
- How do you know your new pattern follows the same rule?

Launch 5–7 min

Sense-Making Routine

Purpose Students consider a model of $\frac{3}{4} \times 5 = \frac{3 \times 5}{4}$ by showing 3 parts of 5 objects each divided into 4 equal parts.

Notice & Wonder™

- What do you notice?
- What do you wonder?

Teaching Tip Have students work in small groups to discuss what they think they know about the image, and share ideas about what mathematics they might be able to do to solve a problem?

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of how to represent multiplication of whole numbers by fractions and are based on possible comments and questions that students may make during the share out.

- How are the watermelons divided?
- How much of each watermelon is outlined?

Math is... Mindset

- Why is it important to have confidence in your work?

SEL Self-Awareness: Self-Confidence

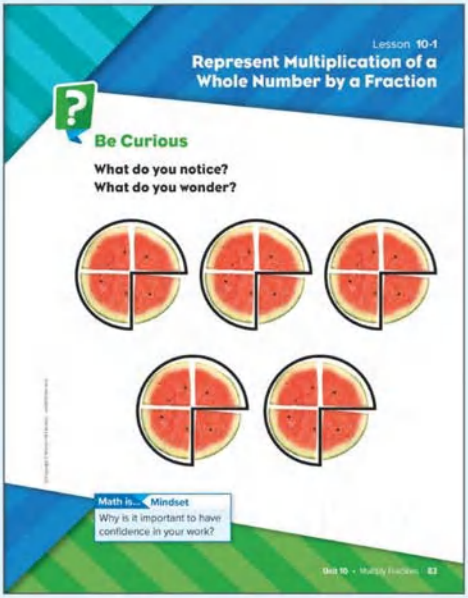
Throughout the Notice & Wonder routine, provide opportunities for students to feel confident in themselves. Model and encourage giving positive feedback for sharing ideas, good effort, or creative thinking. Make sure students understand that being good students can also include being helpful peers and active members of the classroom community. Remind students that some tasks are more challenging than others, and they can demonstrate self-confidence by speaking up and asking for help if they need it. Throughout their work with multiplying fractions and whole numbers, they continue to find opportunities to allow students to give positive feedback to their classmates.

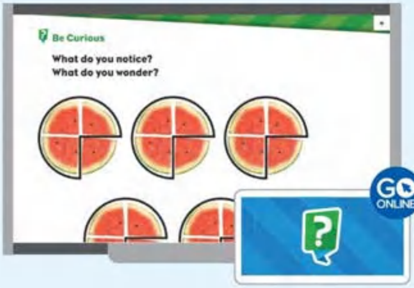
Transition to Explore & Develop

Ask questions that get students thinking about strategies for representing fractions.

ETP Establish Mathematics Goals to Focus Learning

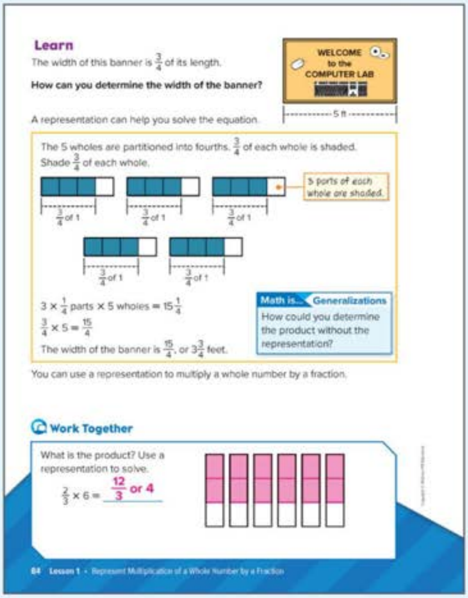
- Let's think about using a representation to multiply a whole number by a fraction.





Explore & Develop

20 min



Learn

The width of this banner is $\frac{3}{4}$ of its length.

How can you determine the width of the banner?

WELCOME to the COMPUTER LAB

5 ft

A representation can help you solve the equation.

The 5 wholes are partitioned into fourths. $\frac{3}{4}$ of each whole is shaded. Shade $\frac{3}{4}$ of each whole.

$\frac{3}{4}$ of 1 $\frac{3}{4}$ of 1 $\frac{3}{4}$ of 1 $\frac{3}{4}$ of 1 $\frac{3}{4}$ of 1

3 parts of each whole are shaded.

$3 \times \frac{1}{4}$ parts $\times$ 5 wholes $= 15\frac{1}{4}$

$\frac{3}{4} \times 5 = \frac{15}{4}$

The width of the banner is $\frac{15}{4}$, or $3\frac{3}{4}$ feet.

Math is... Generalizations
How could you determine the product without the representation?

You can use a representation to multiply a whole number by a fraction.

Work Together

What is the product? Use a representation to solve.

$\frac{2}{3} \times 6 =$ $\frac{12}{3}$ or 4

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❶ Pose the Problem

MLR Discussion Supports

As students talk about what they know, have them pay attention to others' understandings in order to increase their ability to work through multiplication of a whole number by a fraction. Restate statements they make as a question to seek clarification and provide vocabulary or grammar prompts for students who need more guidance.

ETP Pose Purposeful Questions

- Have you seen problems like this before? How were they similar? How were they different?

❷ Develop the Math

Choose the option that best meets your instructional goals.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you use partitioning to multiply a whole number by a fraction?
- How can you represent multiplying a whole number by a fraction?

Key Takeaways

- Multiplying a whole number by a fraction involves partitioning.
- Representations can be used to show the partitioning.

Work Together

The Work Together activity can be used as a formative assessment opportunity to check students' understanding of representing multiplication of fractions by whole numbers.

Common Misconception: Students can easily confuse which numbers determine the number of wholes, partions, and shaded partions. Emphasize the "fraction of" idea. They are representing two thirds of 6 wholes here.

LOM Language of Math

Tell students that another way we use the word *partition* is to describe a divider like ones you might see at a bank that separates or divides the customers from the employees.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how to represent multiplication of a whole number by a fraction.

Directions: Present students with expressions, such as: $\frac{1}{3} \times 6, \frac{1}{4} \times 2, \frac{1}{4} \times 5$. In pairs or small groups, have students determine strategies for solving each expression.

ETP Support Productive Struggle

- Is the product greater than the whole number? How do you know?
- What tools can you use to help you find each product?
- How is finding $\frac{1}{3} \times 6$ similar to finding $\frac{1}{4} \times 2$? How is it different?
- How is finding $\frac{1}{4} \times 2$ similar to finding $\frac{1}{4} \times 5$? How is it different?

Math is... Generalizations

- How could you determine the product without the representation?

Students look for and use repeated reasoning or calculations to help them solve problems.

Activity Debrief: Facilitate a discussion to ensure students understand multiplication of a whole number by a fraction can be represented by taking a part of the whole number. Sometimes it is necessary to partition the whole number into equal parts.

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the width of the banner?

Guided Exploration

Students extend their understanding of multiplication and fractions to represent multiplication of a whole number by a fraction.

ETP Use and Connect Mathematical Representations

- **Think About It:** What are some different ways you have represented fractions?
- Why do you think you represent the 5 wholes first?
- **Think About It:** What other ways could you represent 5 wholes?

Have students represent three-fourths of each whole. Ask:

- How can you partition each whole into fourths?
- Why does that partition make fourths?
- How would you shade the partitions to show three-fourths?
- How can you determine how many fourths are shaded?
- **Think About It:** How can you write $\frac{15}{4}$ in another way?

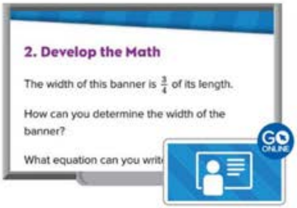
Have students interpret the equation $\frac{3}{4} \times 5$. Ask:

- How many wholes did you represent? What part of the multiplication equation shows that?
- How many equal parts did you partition each whole into? What part of the multiplication equation shows that?
- How many partitions of each whole did you shade? What part of the multiplication equation shows that?

Math is... Generalizations

- How could you determine the product without the representation?

Students look for and use repeated reasoning or calculations to help them solve problems.



EL English Learner Scaffolds

Entering/Emerging Ensure comprehension of *shade*. Draw a circle and divide it into four equal parts. Say *I'm going to shade two parts.* After demonstrating, draw another shape, and repeat. Finally, draw a rectangle and split it into five equal parts. Shade three, but don't provide verbal cues. After you shade the parts, ask students *How many parts did I shade? 2, 3, or 5?*

Developing/Expanding Ensure comprehension of *shade*. Draw a circle and divide it into four equal parts. Say *I'm going to shade two parts*. After demonstrating, draw another shape, and repeat. Finally, draw a rectangle and split it into five equal parts. Shade three, but don't provide verbal cues. After you shade the parts, ask students, *How many parts did I shade?*

Bridging/Reaching Ask students to look at the table on the Learn page, focusing on *Shade $\frac{3}{4}$ of each whole*. Then draw a circle and divide it into four equal parts, shading two. Ask, *What did I do? (You shaded [two parts] of the circle).* Then ask students to discuss similar words to *shade (darken, color in, fill in, etc.)*. Allow them to use a dictionary or thesaurus if desired.

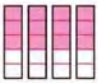
Practice & Reflect 10 min

On My Own

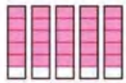
Name ____________________

What is the product? Use a representation to solve.

1. $\frac{3}{5} \times 4 =$ $\frac{12}{5}$ or $2\frac{2}{5}$



2. $\frac{5}{6} \times 5 =$ $\frac{25}{6}$ or $4\frac{1}{6}$



3. $\frac{2}{5} \times 8 =$ $\frac{16}{5}$ or $3\frac{1}{5}$

4. $\frac{3}{8} \times 7 =$ $\frac{21}{8}$ or $2\frac{5}{8}$

5. Anna walked $\frac{5}{8}$ of this trail. How many miles did she walk?
$\frac{45}{8}$ or $5\frac{5}{8}$ mi

6. George is making 6 loaves of zucchini bread. Each loaf uses $\frac{3}{4}$ cup of grated zucchini. How many cups of grated zucchini will George use?

A. $\frac{18}{24}$ cups
B. $\frac{9}{4}$ cups
C. $\frac{18}{4}$ cups
D. $\frac{6}{4}$ cups

Unit 10 • Multiply Fractions 85

7. Deena is making a bracelet that is the length shown. White beads cover $\frac{7}{10}$ of its length. What is the length of the part of the bracelet that is strung with white beads?

$\frac{63}{10}$ cm or $6\frac{3}{10}$ cm

8. **Error Analysis** Louise multiplied $\frac{4}{5} \times 6$ and found the product $\frac{24}{30}$. Explain the error that Louise made. Then, find the correct product.
Sample answer: Louise did not first divide the 6 wholes into fifths. The product should be $\frac{24}{5}$.

9. The Johnson family ordered 6 pizzas. The family ate $\frac{7}{8}$ of each pizza. How much pizza did the Johnson family eat in all?
$\frac{42}{8}$ pizzas or $5\frac{2}{8}$ pizzas

10. **Extend Your Thinking** Julio and Rafael share a package of 12 markers. Julio takes $\frac{2}{3}$ of the markers. Rafael takes $\frac{1}{4}$ of the markers. How many markers will each have? How many markers are left in the package?
Julio takes $\frac{24}{3} = 8$ markers and Rafael takes $\frac{12}{4} = 3$ markers. There is 1 marker left in the package.

Reflect

Explain how you can use a representation to multiply a whole number by a fraction.
Explanations may vary.

Math is... Mindset
Why was it important to have confidence in your work?

86 Lesson 1 • Represent Multiplication of a Whole Number by a Fraction

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 10 When finding the number of markers that Rafael takes, students may incorrectly find one-fourth of Julio's markers instead of one-fourth of the whole package. Make sure they read problems carefully.

Practice Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill & Fluency
5–7	2	Application
8	3	Conceptual Understanding
9	2	Application
10	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- Explain how you can use a representation to multiply a whole number by a fraction.

Ask students to share their reflections with their classmates.

Math is... Mindset

- Why was it important to have confidence in your work?

Students reflect on how they practiced self-awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can use a representation to multiply a whole number by a fraction.
- I can explain how to use a representation to multiply a whole number by a fraction.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Multiply a whole number by a fraction using representations	5.NF.B.4.a
2	1	Multiply a whole number by a fraction using representations	5.NF.B.4.a
3	2	Multiply a whole number by a fraction using representations	5.NF.B.4.a

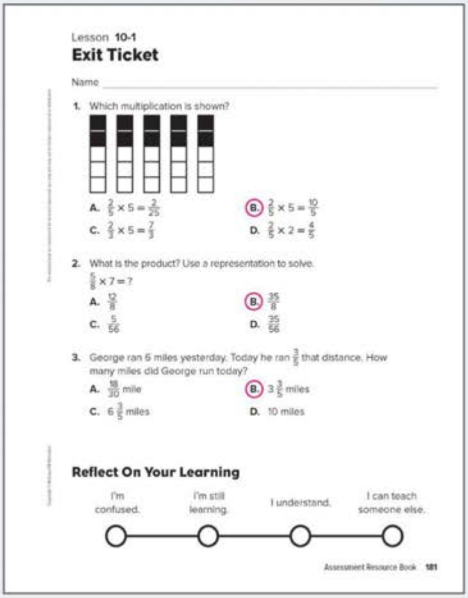
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking



Lesson 10-1

Exit Ticket

Name

1. Which multiplication is shown?

A. $\frac{2}{5} \times 5 = \frac{2}{25}$ B. $\frac{2}{5} \times 5 = \frac{10}{5}$
C. $\frac{2}{3} \times 5 = \frac{7}{3}$ D. $\frac{2}{5} \times 2 = \frac{4}{5}$

2. What is the product? Use a representation to solve.
$\frac{5}{8} \times 7 = ?$
A. $\frac{12}{8}$ B. $\frac{35}{8}$
C. $\frac{5}{56}$ D. $\frac{35}{56}$

3. George ran 6 miles yesterday. Today he ran $\frac{3}{5}$ that distance. How many miles did George run today?
A. $\frac{18}{30}$ mile B. $3\frac{3}{5}$ miles
C. $6\frac{3}{5}$ miles D. 10 miles

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 181

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Make It, Write it

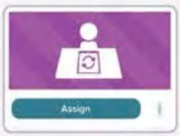
Have each student separate a handful of counters to make 2 unequal groups. Students write an equation showing Fraction × Whole = Part of whole. So, if students have 4 counters and separate 1, they write $\frac{1}{4} \times 4 = 1$. Make sure students understand that the whole is the total number of counters, the part of the whole is the number of counters they separated, and the fraction is the portion of the whole the separated counters represent.

GO ONLINE

Take Another Look Lesson

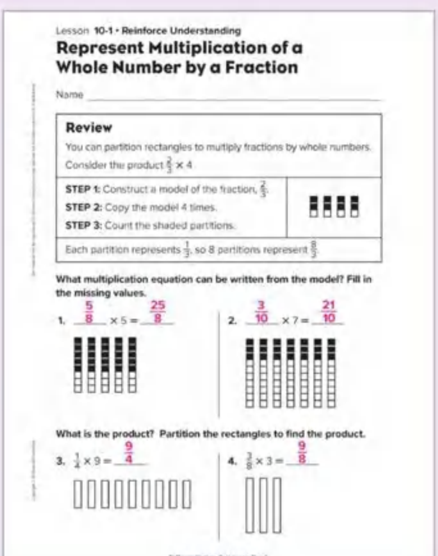
Assign the interactive lesson to reinforce targeted skills.

- Multiply Whole Number by Fraction-Models



INDEPENDENT WORK

Differentiation Resource Book, p. 107



Lesson 10-1 • Reinforce Understanding

Represent Multiplication of a Whole Number by a Fraction

Name

Review

You can partition rectangles to multiply fractions by whole numbers.

Consider the product $\frac{2}{5} \times 4$

STEP 1: Construct a model of the fraction, $\frac{2}{5}$.

STEP 2: Copy the model 4 times.

STEP 3: Count the shaded partitions.

Each partition represents $\frac{1}{5}$, so 8 partitions represent $\frac{8}{5}$.

What multiplication equation can be written from the model? Fill in the missing values.

1. $\frac{5}{8} \times 5 = \frac{25}{8}$

2. $\frac{3}{10} \times 7 = \frac{21}{10}$

What is the product? Partition the rectangles to find the product.

3. $\frac{1}{4} \times 9 = \frac{9}{4}$

4. $\frac{3}{8} \times 3 = \frac{9}{8}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Multiplication Tic Tac Toe

Students practice multiplying fractions by whole numbers.

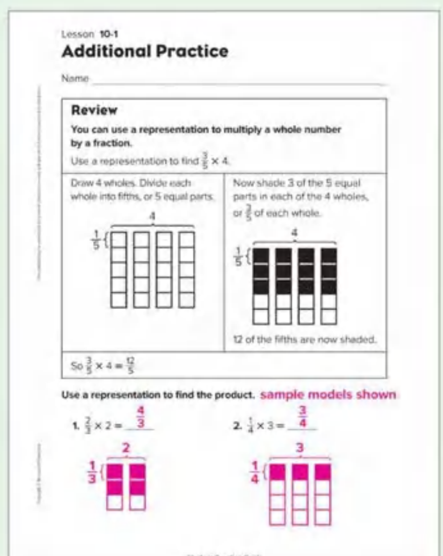
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 107–108



Lesson 10-1

Additional Practice

Name

Review

You can use a representation to multiply a whole number by a fraction.

Use a representation to find $\frac{3}{5} \times 4$.

Draw 4 wholes. Divide each whole into fifths, or 5 equal parts.

Now shade 3 of the 5 equal parts in each of the 4 wholes, or $\frac{3}{5}$ of each whole.

12 of the fifths are now shaded.

So $\frac{3}{5} \times 4 = \frac{12}{5}$

Use a representation to find the product. sample models shown

1. $\frac{2}{3} \times 2 = \frac{4}{3}$

2. $\frac{1}{4} \times 3 = \frac{3}{4}$

Student Practice Book

Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Assign

Use It! Application Station

If, Then Students use if/then statements to write a problem in which they make a true or false statement that uses multiplication of fractions.

The content of this card has concepts covered later in Lesson 10-7. You may want to assign this card to students ready to explore content covered later in this unit.

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

GO ONLINE

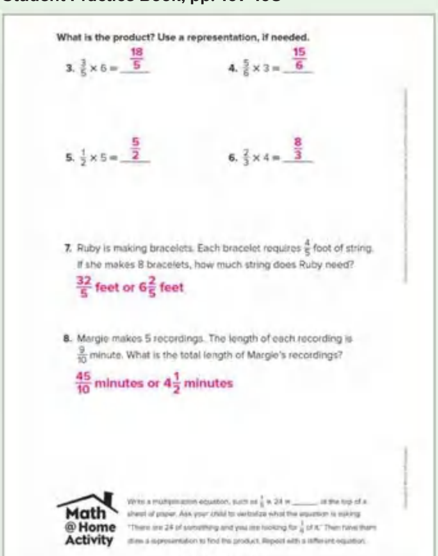
STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 107-108



What is the product? Use a representation, if needed.

3. $\frac{3}{5} \times 6 = \underline{\frac{18}{5}}$

4. $\frac{5}{6} \times 3 = \underline{\frac{15}{6}}$

5. $\frac{1}{2} \times 5 = \underline{\frac{5}{2}}$

6. $\frac{2}{3} \times 4 = \underline{\frac{8}{3}}$

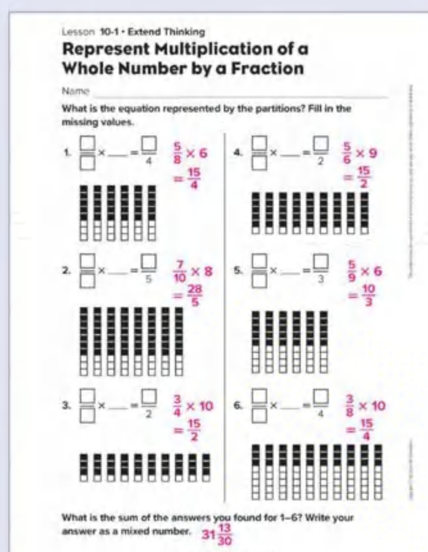
7. Ruby is making bracelets. Each bracelet requires $\frac{4}{5}$ foot of string. If she makes 8 bracelets, how much string does Ruby need?
$\frac{32}{5}$ feet or $6\frac{2}{5}$ feet

8. Margie makes 5 recordings. The length of each recording is $\frac{9}{10}$ minute. What is the total length of Margie's recordings?
$\frac{45}{10}$ minutes or $4\frac{1}{2}$ minutes

Math @ Home Activity Write a multiplication equation, such as $\frac{1}{6} \times 24 =$ _____, at the top of a sheet of paper. Ask your child to verbalize what the equation is asking: "There are 24 of something and you are looking for $\frac{1}{6}$ of it." Then have them draw a representation to find the product. Repeat with a different equation.

Student Practice Book

Differentiation Resource Book, p. 108



Lesson 10-1 • Extend Thinking

Represent Multiplication of a Whole Number by a Fraction

Name __________

What is the equation represented by the partitions? Fill in the missing values.

1. $\frac{\square}{\square} \times$ ___ $= \frac{\square}{4}$ $\frac{5}{8} \times 6 = \frac{15}{4}$

2. $\frac{\square}{\square} \times$ ___ $= \frac{\square}{5}$ $\frac{7}{10} \times 8 = \frac{28}{5}$

3. $\frac{\square}{\square} \times$ ___ $= \frac{\square}{2}$ $\frac{3}{4} \times 10 = \frac{15}{2}$

4. $\frac{\square}{\square} \times$ ___ $= \frac{\square}{2}$ $\frac{5}{6} \times 9 = \frac{15}{2}$

5. $\frac{\square}{\square} \times$ ___ $= \frac{\square}{3}$ $\frac{5}{9} \times 6 = \frac{10}{3}$

6. $\frac{\square}{\square} \times$ ___ $= \frac{\square}{4}$ $\frac{3}{8} \times 10 = \frac{15}{4}$

What is the sum of the answers you found for 1–6? Write your answer as a mixed number. $31\frac{13}{30}$

Differentiation Resource Book

LESSON **10-2**

Multiply a Whole Number by a Fraction

Learning Target

- I can multiply a whole number by a fraction.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

◇ **5.NF.B.4.a** Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$).

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
denominator	citation
numerator	complex

Material

The materials may be for any part of the lesson.

- fraction circles
- fraction tiles

Focus

Content Objective	Language Objectives	SEL Objective
• Students multiply a whole number by a fraction by multiplying the numerator times the whole number, and using that as the numerator in the product and the denominator of the fraction as the denominator.	• Students explain multiplying a whole number by a fraction using the verbs *notice* and *apply*, and the phrase *make a shortcut.* • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students demonstrate thoughtful reflection through identifying the causes of challenges and successes while completing a mathematical task.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students used a representation to multiply a whole number by a fraction (Unit 10).	• Students multiply a whole number by a fraction using properties of operations.	• Students represent multiplication of a fraction by a fraction (Unit 10). • Students interpret and compute quotients of fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students expand their understanding of multiplying fractions by discovering how to multiply the numerator by the whole number to find the product.	• Students apply strategies to gain proficiency with multiplying fractions by whole numbers.	• Students solve word problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

Number Routine

Find the Pattern, Make a Pattern

GO ONLINE

5–7 min

Build Fluency Students build number sense with fractions and mixed numbers as they identify the pattern rule and use the rule to find the missing numbers. Students then create a new pattern that follows the same rule.

These prompts encourage students to talk about their reasoning:

- What pattern do you notice?
- What terms helped you determine the pattern? Explain.
- How did you determine what is missing?
- What is another way to think about the pattern?
- How do you know your new pattern follows the same rule?

Purpose Students prepare for story contexts that involve multiplying a whole number by a fraction.

Numberless Word Problem

- What is the question?

Teaching Tip Have students work in pairs to share their thoughts about the numberless word problem. They should share what operations the word problem suggests and how they could solve it.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of multiplying a whole number by a fraction and are based on possible comments and questions that students may make during the share out.

- Do you think the judges ate part or all of each pie? Why?
- How many pies were entered in the contest?

Math is... Mindset

- What helps you make decisions?

SEL Responsible Decision-Making: Reflect

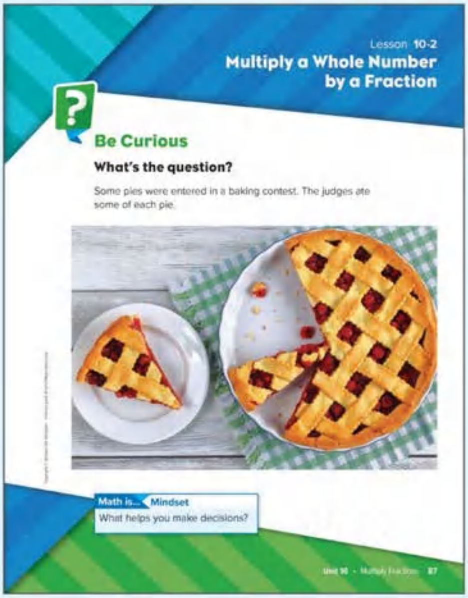
When students reflect, they can make connections between effort and achievement. After working through the Numberless Word Problem routine, allow students time to thoughtfully reflect on their work. Invite them to think about what may have been challenging as well as the ways in which they were successful and why. Encourage students to also consider what parts of the Numberless Word Problem routine that they enjoyed and why.

Transition to Explore & Develop

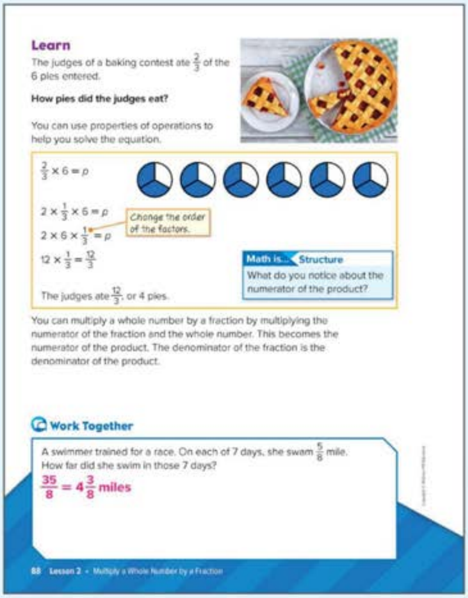
Ask questions that get students thinking about using properties of operations to multiply a whole number by a fraction.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how to multiply a whole number by a fraction using properties of operations.



Explore & Develop
20 min



Learn

The judges of a baking contest ate $\frac{2}{3}$ of the 6 pies entered.

How pies did the judges eat?

You can use properties of operations to help you solve the equation.

$\frac{2}{3} \times 6 = p$

$2 \times \frac{1}{3} \times 6 = p$

$2 \times 6 \times \frac{1}{3} = p$ Change the order of the factors.

$12 \times \frac{1}{3} = \frac{12}{3}$

The judges ate $\frac{12}{3}$, or 4 pies.

Math is... Structure
What do you notice about the numerator of the product?

You can multiply a whole number by a fraction by multiplying the numerator of the fraction and the whole number. This becomes the numerator of the product. The denominator of the fraction is the denominator of the product.

Work Together

A swimmer trained for a race. On each of 7 days, she swam $\frac{5}{8}$ mile. How far did she swim in those 7 days?

$\frac{35}{8} = 4\frac{3}{8}$ miles

88 Lesson 2 • Multiply a Whole Number by a Fraction

❶ Pose the Problem

ETP Pose Purposeful Questions

- How are the quantities in this problem related?
- What models might help you make sense of these quantities? How would they help you?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about how to solve the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How could you explain to a friend how to multiply a whole number by a fraction?

Key Takeaway

- One way to multiply a whole number by a fraction is to multiply the numerator times the whole number and use that as the numerator in the product and use the denominator of the fraction as the denominator.

Work Together

The Work Together activity focuses on students' understanding of solving word problems involving the multiplication of fractions by whole numbers.

Common Error: Some students may think that the problem is asking for how much she swam each day, instead of how far she swam in all. Make sure the students read word problems carefully.

LOM Language of Math

Properties of operations are qualities that belong to the operation, as in, "a property of addition is that it is commutative." Similary, *property* is something, like land or a home, that belongs to someone, as in "my grandparents own property in the country."

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore efficient strategies for multiplying a whole number by a fraction.

Directions: Display 4 or 5 multiplication expressions involving a fraction times a whole number. Tell students that they will find each product and look for patterns.

ETP **Support Productive Struggle**

- What strategy did you use to find each product?
- Are your products reasonable? How do you know?
- Can you find any shortcuts to help you solve more efficiently? If so, describe them.
- What patterns do you notice when multiplying a fraction times a whole number? Why do those patterns exist?

Math is... Structure

- How do you know the Associative Property of Multiplication works for fractions?

Students understand the nature of the properties they have used to multiply a whole number by a fraction.

Activity Debrief: Students should notice that one way to multiply a whole number by a fraction is to multiply the numerator times the whole number, and use that as the numerator in the product, and use the denominator of the fraction as the denominator. They can use properties of operations to explain why this occurs.

Have students revisit the Pose the Problem question and discuss answers.

- How much pie did the judges eat?

Guided Exploration

Students extend their understanding of multiplication with fractions using equations.

ETP **Facilitate Meaningful Mathematical Discourse**

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have students represent the equation $\frac{2}{3} \times 11$. Ask:

- How many wholes should you represent? Why? What part of the multiplication equation shows that?
- How many equal parts should you partition each whole into? Why? What part of the multiplication equation shows that?
- How many partitions of each whole should you shade? Why? What part of the multiplication equation shows that?
- You know that $\frac{2}{3} = 2 \times \frac{1}{3}$. How do you know that $\frac{2}{3} = \frac{1}{3} \times 2$?
- Why can you rewrite $(\frac{1}{3} \times 2) \times 11$ as $\frac{1}{3} \times (2 \times 11)$? What Property do you use?
- You know that $2 \times 11 = 22$. Why can you rewrite $\frac{1}{3} \times (2 \times 11)$ as $22 \times \frac{1}{3}$? What Property do you use?
- How do you know that $22 \times \frac{1}{3} = \frac{22}{3}$?

Have the students look for a pattern or shortcut. Ask:

- What do you notice about the numerator of the product?
- What do you notice about the denominator of the product?
- What shortcut could you use when multiplying a whole number by a fraction?

Math is... Structure

- How do you know the Associative Property of Multiplication works for fractions?

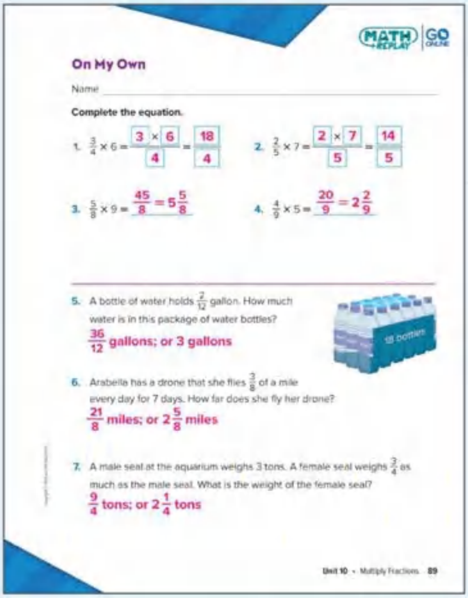
Students understand the nature of the properties they have used to multiply a whole number by a fraction.

EL **English Learner Scaffolds**

Entering/Emerging Ensure understanding of *shortcut*. Demonstrate with 5 tens rods. Say *Let's count the cubes*. Start counting one at a time, but then stop and say, *Wait. I know a shortcut*. Count them instead by 10s. Say *That was faster*. Finally, write a math problem and solve it without using shortcuts. Ask *Is this a shortcut?* Then solve it again using a shortcut and ask *Is this a shortcut?*

Developing/Expanding Ensure understanding of *shortcut*. Demonstrate with 5 tens rods. Say *Let's count the cubes*. Start counting one at a time, but then stop and say, *Wait. I know a shortcut.* Count them instead by 10s. Say *That was faster*. Finally, guide students to the Learn page and point to the multiplication problem in the table. Ask *Is this a shortcut?*

Bridging/Reaching Ask students to review the Math Is...Generalizations question on the Learn page. Ensure comprehension by asking for the meaning of *shortcut* (a shorter or faster way of doing something). Allow them to use a dictionary if desired. Then ask students to identify other math shortcuts they know and explain why the shortcut works; for example, skip counting by 5, doubles/near doubles, etc.


MATH REPLAY | GO ONLINE

On My Own

Name ______

Complete the equation.

1. $\frac{3}{4} \times 6 = \frac{3 \times 6}{4} = \frac{18}{4}$

2. $\frac{2}{5} \times 7 = \frac{2 \times 7}{5} = \frac{14}{5}$

3. $\frac{5}{8} \times 9 = \frac{45}{8} = 5\frac{5}{8}$

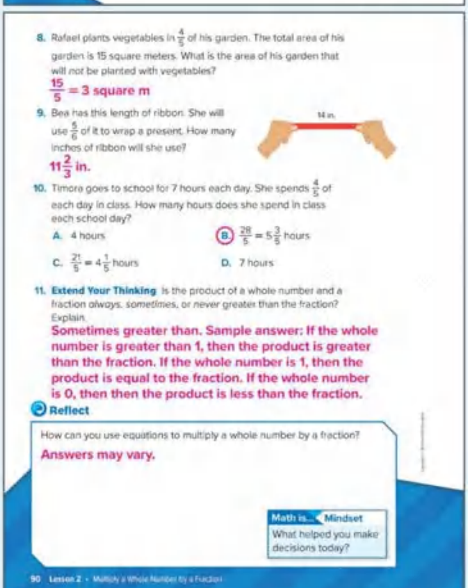
4. $\frac{4}{9} \times 5 = \frac{20}{9} = 2\frac{2}{9}$

5. A bottle of water holds $\frac{2}{12}$ gallon. How much water is in this package of water bottles?
$\frac{36}{12}$ gallons; or 3 gallons

6. Arabella has a drone that she flies $\frac{3}{8}$ of a mile every day for 7 days. How far does she fly her drone?
$\frac{21}{8}$ miles; or $2\frac{5}{8}$ miles

7. A male seal at the aquarium weighs 3 tons. A female seal weighs $\frac{3}{4}$ as much as the male seal. What is the weight of the female seal?
$\frac{9}{4}$ tons; or $2\frac{1}{4}$ tons

Unit 10 • Multiply Fractions 89


8. Rafael plants vegetables in $\frac{4}{5}$ of his garden. The total area of his garden is 15 square meters. What is the area of his garden that will *not* be planted with vegetables?
$\frac{15}{5} = 3$ square m

9. Bea has this length of ribbon. She will use $\frac{5}{6}$ of it to wrap a present. How many inches of ribbon will she use?
14 in.
$11\frac{2}{3}$ in.

10. Timora goes to school for 7 hours each day. She spends $\frac{4}{5}$ of each day in class. How many hours does she spend in class each school day?
A. 4 hours
B. $\frac{28}{5} = 5\frac{3}{5}$ hours
C. $\frac{21}{5} = 4\frac{1}{5}$ hours
D. 7 hours

11. **Extend Your Thinking** Is the product of a whole number and a fraction *always*, *sometimes*, or *never* greater than the fraction? Explain.
Sometimes greater than. Sample answer: If the whole number is greater than 1, then the product is greater than the fraction. If the whole number is 1, then the product is equal to the fraction. If the whole number is 0, then then the product is less than the fraction.

Reflect

How can you use equations to multiply a whole number by a fraction?
Answers may vary.

Math is... Mindset
What helped you make decisions today?

90 Lesson 2 • Multiply a Whole Number by a Fraction

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–4 As students find the product of fractions and whole numbers, make sure that they do not multiply the denominator of the fraction by the whole number. The denominator of the product should be equal to the denominator of the fraction.

Practice Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill & Fluency
5–10	2	Application
11	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you use equations to multiply a whole number by a fraction?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you make decisions today?

Students reflect on how they practiced responsible decision-making

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can multiply a whole number by a fraction.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Multiply a whole number by a fraction	5.NF.B.4.a
2	1	Multiply a whole number by a fraction	5.NF.B.4.a
3	2	Multiply a whole number by a fraction	5.NF.B.4.a
4	2	Multiply a whole number by a fraction	5.NF.B.4.a

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

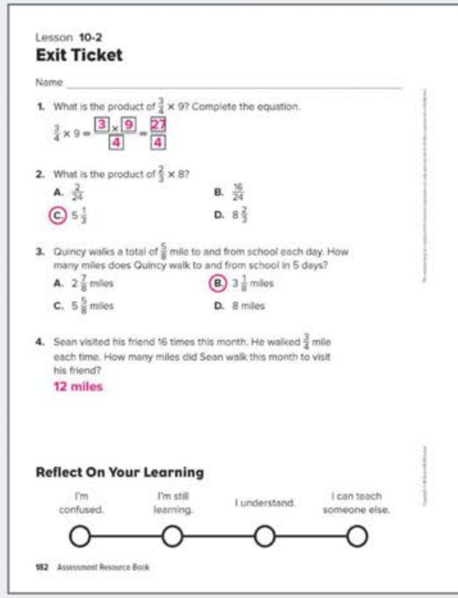
If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 10-2

Exit Ticket

Name ______

1. What is the product of $\frac{3}{4} \times 9$? Complete the equation.

$\frac{3}{4} \times 9 = \frac{\boxed{3} \times \boxed{9}}{\boxed{4}} = \frac{\boxed{27}}{\boxed{4}}$

2. What is the product of $\frac{2}{3} \times 8$?

A. $\frac{2}{24}$ B. $\frac{16}{24}$

C. $5\frac{1}{3}$ D. $8\frac{2}{3}$

3. Quincy walks a total of $\frac{5}{8}$ mile to and from school each day. How many miles does Quincy walk to and from school in 5 days?

A. $2\frac{7}{8}$ miles B. $3\frac{1}{8}$ miles

C. $5\frac{5}{8}$ miles D. 8 miles

4. Sean visited his friend 16 times this month. He walked $\frac{3}{4}$ mile each time. How many miles did Sean walk this month to visit his friend?

12 miles

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

182 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Multiply It

Work with students in pairs. Have students solve $\frac{1}{5} \times 20$.

Then each student multiplies 20 by $\frac{2}{5}$, by $\frac{3}{5}$, by $\frac{4}{5}$, and by $\frac{5}{5}$.

If students have difficulty, help them recognize that once they have the first product, they can multiply it by the numerators to find the other products. Students share their solutions and discuss any patterns they see.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Multiply Whole Number by Fraction

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 109

Lesson 10-2 • Reinforce Understanding

Multiply a Whole Number by a Fraction

Name ____________

Review

To find the product of whole number and a fraction, multiply the numerator of the fraction by the whole number.

Consider the product $\frac{3}{4} \times 5$.

There are 5 groups of 3 shaded regions. So there are 3×5, or 15 shaded regions.

Each shaded region is one-fourth of a bar, so the shaded regions show 15 fourths, or $\frac{15}{4}$.

From the representation, we see $\frac{3}{4} \times 5 = \frac{3 \times 5}{4} = \frac{15}{4}$.

What is the product? Fill in the blanks to show your work.

1. $\frac{5}{6} \times 3 = \frac{15}{6}$ or $\frac{5}{2}$

   $\frac{\square \times \square}{6}$ $\frac{5 \times 3}{6}$

2. $\frac{5}{8} \times 5 = \frac{25}{8}$

   $\frac{\square \times \square}{8}$ $\frac{5 \times 5}{8}$

3. $\frac{2}{9} \times 7 = \frac{14}{9}$

   $\frac{\square \times \square}{9}$ $\frac{2 \times 7}{9}$

4. $\frac{3}{8} \times 6 = \frac{18}{8}$ or $\frac{9}{4}$

   $\frac{\square \times \square}{\square}$ $\frac{3 \times 6}{8}$

5. $\frac{3}{5} \times 4 = \frac{12}{5}$

   $\frac{\square \times \square}{\square}$ $\frac{3 \times 4}{5}$

6. $\frac{4}{7} \times 8 = \frac{32}{7}$

   $\frac{\square \times \square}{\square}$ $\frac{4 \times 8}{7}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Representing Fraction Multiplication Task Cards

Students practice representing the multiplication of whole numbers by fractions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 109-110

Lesson 10-2

Additional Practice

Name ____________

Review

You can multiply a fraction and a whole number by multiplying the numerator of the fraction by the whole number, and keeping the denominator the same.

Johnny walks $\frac{3}{4}$ mile to and from school each day. How many miles did he walk this week if school met all 5 days?

To solve, find $\frac{3}{4} \times 5$.

Multiply the numerator and the whole number: $3 \times 5 = 15$.

This is the numerator. Keep the denominator the same: $\frac{3}{4} \times 5 = \frac{15}{4}$.

Write the answer as a mixed number: $\frac{15}{4} = 3\frac{3}{4}$

Johnny walked $3\frac{3}{4}$ miles to and from school this week.

What is the product?

1. $\frac{5}{6} \times 5 = \frac{25}{6}$ or $4\frac{1}{6}$
2. $\frac{1}{4} \times 9 = \frac{9}{4}$ or $2\frac{1}{4}$
3. $\frac{3}{8} \times 6 = \frac{18}{8}$ or $2\frac{1}{4}$
4. $\frac{2}{3} \times 8 = \frac{16}{3}$ or $5\frac{1}{3}$

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Assign

Use It! Application Station

This or That Students follow instructions to measure and cut materials.

The content of this card has concepts covered later in Lesson 10-8. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 109-110

5. A bottle of water contains $\frac{3}{4}$ liter of water. How much water do you get when you buy a package of 15 bottles of water?

$\frac{45}{4}$ liters or $11\frac{1}{4}$ liters

6. One lap around the track covers $\frac{3}{5}$ mile. Mary walked 8 laps. How many miles did Mary walk?

$\frac{24}{5}$ miles or $4\frac{4}{5}$ miles

7. An elephant weighs about 4 tons. A younger elephant weighs about $\frac{5}{8}$ that much. About how many tons does the younger elephant weigh?

$\frac{20}{8}$ tons or $2\frac{1}{2}$ tons

8. Margaret buys 20 pounds of flour to use for making treats for the bake sale. She used $\frac{5}{6}$ of the available flour. How many pounds of flour did Margaret use?

$\frac{100}{6}$ pounds or $16\frac{2}{3}$ pounds

Math @ Home Activity With your child, make two stacks of index cards. On the cards in the first stack, write different fractions. On the cards in the second stack, write different whole numbers. Have your child randomly choose a card from each stack. Then have them show you how to find the product of the numbers on the cards. Continue the activity with different pairs of cards.

Student Practice Book

Differentiation Resource Book, p. 110

Lesson 10-2 • Extend Thinking

Multiply a Whole Number by a Fraction

Name ____________________

What value completes the equation? Show your work. The first one is done as an example.

1. $\frac{?}{9} \times 15 = 3\frac{1}{3}$

$\frac{?}{9} \times \frac{15}{1} = \frac{10}{3}$

$\frac{?}{9} \times \frac{15}{1} = \frac{30}{9}$

$\frac{2}{9} \times 15 = 3\frac{1}{3}$

2. $\frac{\square}{8} \times 12 = 10\frac{1}{2}$

$\frac{?}{8} \times \frac{12}{1} = \frac{21}{2}$

$\frac{7}{8} \times \frac{12}{1} = \frac{84}{8}$

$\frac{7}{8} \times 12 = 10\frac{1}{2}$

3. $\frac{\square}{15} \times 10 = 7\frac{1}{3}$

$\frac{?}{15} \times \frac{10}{1} = \frac{22}{3}$

$\frac{11}{15} \times \frac{10}{1} = \frac{110}{15}$

$\frac{11}{15} \times 10 = 7\frac{1}{3}$

4. $\frac{\square}{6} \times 14 = 11\frac{2}{3}$

$\frac{?}{6} \times \frac{14}{1} = \frac{35}{3}$

$\frac{5}{6} \times \frac{14}{1} = \frac{70}{6}$

$\frac{5}{6} \times 14 = 11\frac{2}{3}$

5. $\frac{\square}{22} \times 33 = 4\frac{1}{2}$

$\frac{?}{22} \times 33 = \frac{9}{2}$

$\frac{3}{22} \times \frac{33}{1} = \frac{99}{22}$

$\frac{3}{22} \times 33 = 4\frac{1}{2}$

6. $\frac{\square}{16} \times 20 = 16\frac{1}{4}$

$\frac{?}{16} \times 20 = \frac{65}{4}$

$\frac{13}{16} \times \frac{20}{1} = \frac{260}{16}$

$\frac{13}{16} \times 20 = 16\frac{1}{4}$

Differentiation Resource Book

Unit 10

Fraction Problems

Name ____________________

Choose the best estimate for each problem. Do not actually solve the problems.

1. Ms. Garcia is making bows from ribbon. She uses $\frac{5}{8}$ yard of ribbon for each bow. How much ribbon does she need to make 9 bows?

   Circle the best estimate.

   a. 3 yards
   b. 5 yards
   c. 7 yards
   d. 8 yards

   Explain or show your thinking.
   Explanations may vary.

2. The price of a sweatshirt is \$32. The price of a T-shirt is $\frac{3}{10}$ the price of the sweatshirt. What is the price of the T-shirt?

   Circle the best estimate.

   a. \$10
   b. \$16
   c. \$24
   d. \$31

   Explain or show your thinking.
   Explanations may vary.

Unit 10 • Multiply Fractions 91

Choose the best estimate for the problem. Do not actually solve the problems.

3. Chantal drinks $\frac{2}{3}$ cup of orange juice every morning. How many cups does she drink in 20 days?

   Circle the best estimate.

   a. 6 cups
   b. 10 cups
   c. 14 cups
   d. 19 cups

   Explain or show your thinking.
   Explanations may vary.

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

92 Math Probe • Fraction Problems

Analyze the Probe ✔ Formative Assessment

Targeted Concept Determine the product of a whole number and a fraction by reasoning about the magnitude of numbers and the meaning of multiplication within a word-problem context.

Targeted Misconceptions Some students choose an incorrect operation when solving problems. They may misinterpret the problem context or focus on "key words" that may suggest an incorrect operation. Some students misinterpret an expression for an equation.

Some students have difficulty determining an estimated magnitude of a product when working with fractions. They may overgeneralize about the size of fractions in general or misjudge the magnitude of a particular fraction. Others may simply look for a match between the numbers in the problem and the numbers in the choices without considering the effect of the operation and magnitude of the numbers involved in solving the problem.

Authentic Student Work

Below are examples of students' explanations.

Sample A

1. Ms. Garcia is making bows from ribbon. She uses $\frac{5}{8}$ yard of ribbon for each bow. How much ribbon does she need to make 9 bows?

   Circle the best estimate.

   A. 3 yards
   B. 5 yards
   C. 7 yards
   D. 8 yards

Explain or show your thinking.

$\frac{5}{8}$ is about $\frac{1}{2}$
$\frac{1}{2}$ of 9 is $4\frac{1}{2}$
But really it is a bit more so 5 just makes alot of sense!

Sample B

3. Chantal drinks $\frac{2}{3}$ cup of orange juice every morning. How many cups does she drink in 20 days?

   Circle the best estimate.

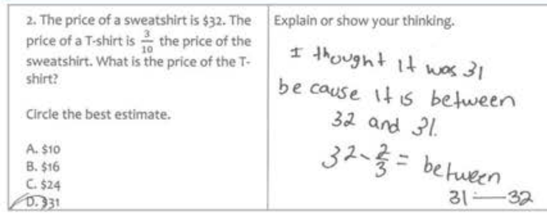
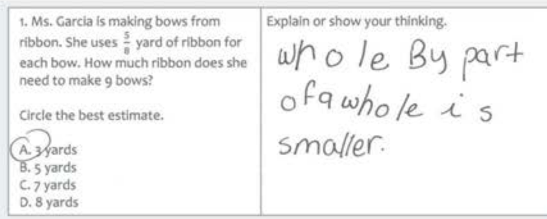
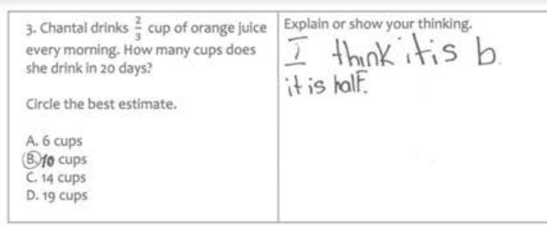
   A. 6 cups
   B. 10 cups
   C. 14 cups
   D. 19 cups

Explain or show your thinking.

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ estimating
6 6 6 + 2 xtra
12 more
round up to 14

Collect and Assess Student Work

Collect and review student responses to determine possible misconceptions. See examples in If-Then chart.

IF incorrect...	THEN the student likely...	Sample Misconceptions
1. d 2. d 3. d	chooses the number closest to the whole number in the problem without considering the context provided. The student may have subtracted rather than multiplied.	 2. The price of a sweatshirt is \$32. The price of a T-shirt is $\frac{3}{10}$ the price of the sweatshirt. What is the price of the T-shirt? Circle the best estimate. A. \$10 B. \$16 C. \$24 D. \$31 Explain or show your thinking. I thought it was 31 because it is between 32 and 31. $32-\frac{2}{3}=$ between 31—32
1. a 2. a 3. a	chooses the least value, having overgeneralized that "finding a fraction of a number makes smaller." Note this reasoning results in a correct response for Exercise 2 (choice a).	 1. Ms. Garcia is making bows from ribbon. She uses $\frac{5}{8}$ yard of ribbon for each bow. How much ribbon does she need to make 9 bows? Circle the best estimate. A. 3 yards B. 5 yards C. 7 yards D. 8 yards Explain or show your thinking. whole By part of a whole is smaller.
1. b 2. b 3. b	uses $\frac{1}{2}$ as a benchmark, not recognizing that there is a more precise estimate. For example, in Exercise 3, half of 20 is 10 (choice b). But the problem calls for $\frac{2}{3}$ of 20, so 14 is a better estimate (choice c). Note this reasoning results in a correct response for Exercise 1.	 3. Chantal drinks $\frac{2}{3}$ cup of orange juice every morning. How many cups does she drink in 20 days? Circle the best estimate. A. 6 cups B. 10 cups C. 14 cups D. 19 cups Explain or show your thinking. I think it is b. it is half.

Many of the above difficulties result in a combination of correct and incorrect responses.
For correct responses, be sure to check for sound reasoning.

Take Action

Choose from the following resources or suggestions:

- Revisit finding products in Lessons 10-1 and 10-2.
- Build skills in interpreting word problems through strategies such as acting out word problems, removing numbers from problems, or leaving the question out in order for students to predict what might be asked or what values might be given.
- Use approaches that include asking students to estimate an answer before computing, make a drawing to represent the situation, and comparing their final answer to their estimate. Include problems where only an estimated answer is needed so that students do not rely only on exact answers.
- Have students create their own word problems to share and discuss.

Revisit the Probe after additional instruction. Have students review their initial answers to the probe. Use these questions for discussion:

- Are there any answers you would like to change? Explain why you might want to change them.
- Are there any questions you still have about any of the Exercises on this probe?

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

LESSON **10-3**

Represent Multiplication of a Fraction by a Fraction

Learning Targets

- I can use a representation to multiply a fraction by a fraction.
- I can explain how to use a representation to multiply a fraction by a fraction.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

◇ **5.NF.B.4.a** Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$).

Math Practices and Processes

MPP Reason abstractly and quantitatively.

Vocabulary

Math Terms	Academic Terms
fraction model	procedure
multiplication	speculate

Materials

The materials may be for any part of the lesson.

- fraction circles
- fraction tiles
- grid paper

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use a representation to multiply a fraction by a fraction. • Students explain how to use a representation to multiply a fraction by a fraction.	• Students explain how to represent multiplication of a fraction by a fraction using the verbs *partition* and *show* and the nouns *patterns* and *shortcuts*. • To support sense-making, ELs participate in MLR3: Three Reads.	• Students offer constructive feedback to the mathematical ideas posed by others.

Coherence

Previous	Now	Next
• Students mulitplied a fraction by a whole number (Grade 4). • Students multiplied a whole number by a fraction (Unit 10).	• Students represent multiplication of a fraction by a fraction.	• Students multiply a fraction by a fraction (Unit 10). • Students interpret and compute quotients of fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of multiplication as they use a representation to multiply two fractions.	• Students build proficiency with fractions and strategies for multiplying fractions.	• Students multiply fractions to solve word problems. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Decompose It 5–7 min

Build Fluency Students build number sense as they decompose a whole number in at least three ways.

These prompts encourage students to talk about their reasoning:

- Retell the problem in your own words. What decomposed parts are related? What patterns do you see?
- How can a pattern help you find new decomposed parts?
- What is the importance of using the word "and" when decomposing whole numbers?
- How did you use place value to help you decompose whole numbers?

Purpose Students refresh the "part of a part" idea that they were introduced to when they represented multiplying decimals by decimals.

Notice & Wonder

- What do you notice?
- What do you wonder?

Teaching Tip Have students work in pairs to discuss what they think they notice and wonder about the image, and share ideas about what mathematics they might be able to apply to it.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of how to represent multiplication of fractions by fractions and are based on possible comments and questions that students may make during the share out.

- What fraction of the shelves have towels in them? How do you know?
- What fraction of the towels are red? How do you know?
- What fraction of the shelves have red towels in them? How do you know?

Math is... Mindset

- How can you work well with a classmate even when you might disagree?

SEL Relationship Skills: Social Engagement

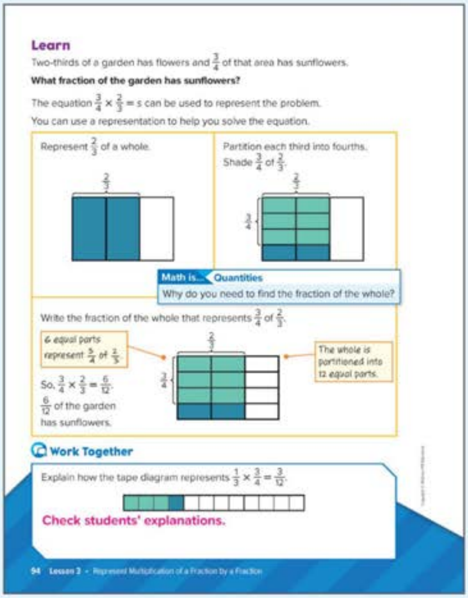
As students engage in collaborative discourse around the Notice & Wonder routine, invite them to give constructive or helpful feedback to their peers. As students engage and discuss what they noticed and wondered, they are strengthening their relationship skills. Remind students that active listening and building on the ideas of others can help them connect with one another and work toward achieving shared goals.

Transition to Explore & Develop

Ask questions that get students thinking about partitioning wholes. Guide the discussion to have the student think about how to represent fractions with different partitions. If students bring up partitions of partitions organically, bring that into the discussion, but if students do not introduce the concept during this part of the lesson, they will be reminded of it in the Explore & Develop.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about using a representation to multiply a fraction by a fraction.



Learn

Two-thirds of a garden has flowers and $\frac{3}{4}$ of that area has sunflowers.

What fraction of the garden has sunflowers?

The equation $\frac{3}{4} \times \frac{2}{3} = s$ can be used to represent the problem.

You can use a representation to help you solve the equation.

Represent $\frac{2}{3}$ of a whole.

Partition each third into fourths. Shade $\frac{3}{4}$ of $\frac{2}{3}$.

Math is... Quantities

Why do you need to find the fraction of the whole?

Write the fraction of the whole that represents $\frac{3}{4}$ of $\frac{2}{3}$.

6 equal parts represent $\frac{3}{4}$ of $\frac{2}{3}$.

The whole is partitioned into 12 equal parts.

So, $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$.

$\frac{6}{12}$ of the garden has sunflowers.

Work Together

Explain how the tape diagram represents $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$.

Check students' explanations.

94 Lesson 2 • Represent Multiplication of a Fraction by a Fraction

❶ Pose the Problem

MLR Three Reads

1st read: Ensure students understand what *that area* refers to, and also ensure comprehension of key words: *garden, plant,* and *sunflowers.*

2nd read: Focus students' attention on the *What...* question.

3rd read: Instruct students to brainstorm ways to solve the problem.

ETP Pose Purposeful Questions

- Have you seen problems like this before? How were they similar? How were they different?

❷ Develop the Math

Choose the option that best meets your instructional goals.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- Explain why multiplying fractions is like finding a fraction of a fraction.

Key Takeaway

- A representation showing a fraction of a fraction can be used to visualize multiplication of a fraction by a fraction.

Work Together

Students practice their understanding of representing multiplication of fractions, this time using a tape diagram.

Common Error: Students who think "one-third of three-fourths" are correct, but may be confused by this representation. Suggest that they think "three-fourths of one-third."

LOM Language of Math

To *represent* can also mean to act or speak for, like a representative in government or an athlete representing their country, or can mean "amount to," as in "the cows represent half the animals in the barn," or can mean "to be a symbol," as in "the blue in their flag represents justice."

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how to represent the multiplication of two fractions.

Materials: grid paper

Directions: Have students work together to solve the Pose the Problem. Students may use any strategy to solve.

Implement Tasks That Promote Reasoning and Problem Solving

- Can you draw a picture or make a representation to show the problem?
- How does your representation show $\frac{2}{3}$?
- How does your representation show $\frac{3}{4}$ of $\frac{2}{3}$?
- How does your represenation show the answer?
- How is your representation the same as other's representations? How is it different?

Math is... Quantities

- Why do you need to find the fraction of the whole when determining the product?

Students make sense of quantities and their relationships in problem situations.

Activity Debrief: Have students share their representations of products of fractions. Ensure students understanding that when representing $\frac{2}{3}$, they are showing $\frac{2}{3}$ of the whole garden. When representing $\frac{3}{4}$ of $\frac{2}{3}$, they are showing a part of a part of the garden. Finally, when determining the product, they are showing the fraction of the whole garden.

Guided Exploration

Students extend their understanding of multiplication with fractions to represent multiplication of a fraction by a fraction.

Use and Connect Mathematical Representations

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students determine the fraction of the whole that represents $\frac{3}{4}$ of $\frac{2}{3}$. Ask:

- How many equal parts is the whole partitioned into? How do you know?
- How many equal parts represent $\frac{3}{4}$ of $\frac{2}{3}$? How do you know?
- What fraction of the whole represents $\frac{3}{4}$ of $\frac{2}{3}$? Explain why.
- **Think About It:** Explain why you need to find the fraction of the whole represented.

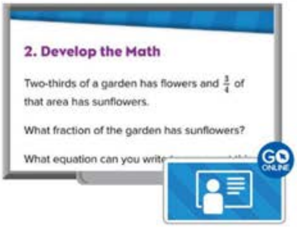
Have the students look for a pattern or shortcut. Ask:

- What do you notice about the numerator of the product?
- What do you notice about the denominator of the product?

Math is... Quantities

- Why do you need to find the fraction of the whole?

Students make sense of quantities and their relationships in problem situations.



English Learner Scaffolds

Entering/Emerging Support students in understanding the term *that area*. Draw a square and divide it into three equal parts. Shade one of the parts. Point to it and say *That area is shaded*. Repeat again with a new drawing, and then send students to the Learn page. Point to the unshaded part. Ask *Is this area shaded?* Then point to the shaded parts. Ask *Is that area shaded?*

Developing/Expanding Support students in understanding the term *that area*. Draw a square and divide it into three equal parts. Shade one of the parts. Point to it and say *That area is shaded*. Repeat again with a new drawing, and then send students to the Learn page. Point to the shaded part and ask students to complete the sentence: ______ (That area) *is shaded*.

Bridging/Reaching Ask students to read the word problem at the top of the Learn page. Ask them to tell you what *that area* refers to (the school garden). Then ask students to brainstorm similar words to *area* and share with the class (*section, space, part, etc.*). Allow students to use a dictionary or thesaurus if desired.

On My Own

Name

What is the product? Use a representation to solve.

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. $\frac{5}{6} \times \frac{3}{5} = \frac{15}{30} = \frac{1}{2}$

3. $\frac{5}{8} \times \frac{2}{3} = \frac{10}{24}$

4. $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

5. $\frac{4}{5} \times \frac{5}{6} = \frac{20}{30}$

6. $\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$

7. Matias prepared $\frac{2}{3}$ of the garden for vegetables. He is planting $\frac{3}{8}$ of the vegetable garden with potatoes. What fraction of the whole garden will be the potato garden?
$\frac{3}{8} \times \frac{2}{3} = \frac{6}{24}$

8. Hazel travels $\frac{5}{8}$ mile. She ran $\frac{2}{5}$ of that distance. How far did Hazel run?
$\frac{5}{8} \times \frac{2}{5} = \frac{10}{40} = \frac{1}{4}$ mi

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9. Jordan saved two-thirds of his earnings last month from babysitting. He spent $\frac{3}{5}$ of that savings to buy new sneakers. How much of his earnings did he spend on sneakers?
$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$

10. Kevin wants to make half of the recipe. How many cups of walnuts pieces does he use?

A. $\frac{1}{8}$ cup
B. $\frac{1}{4}$ cup
C. $\frac{3}{8}$ cup
D. $\frac{4}{6}$ cup

RECIPE
Ingredients
1 egg
$\frac{3}{4}$ c of walnuts
$\frac{3}{4}$ tsp of baking soda

11. Using the same recipe, how much baking soda would Kevin need if he makes a recipe that is $\frac{3}{4}$ of the original recipe?
$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ tsp

12. **Extend Your Thinking** Will the product of $\frac{3}{9} \times \frac{4}{7}$ be the same as the product of $\frac{1}{3} \times \frac{4}{7}$? Explain.
Yes. Sample answer: $\frac{3}{9}$ is equivalent to $\frac{1}{3}$.

Reflect

When you multiply two fractions, is the product greater than, less than, or the same as the two fractions?
Answers may vary.

Math is... Mindset
What helped you work well with a classmate even when you might disagree?

96 Lesson 2 • Represent Multiplication of a Fraction by a Fraction

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–6 Be sure students are shading to show a part of a part to show multiplication rather than shading to show addition. Have students use two different colors such as red and yellow so they can be sure they are making a new color to show the answer (orange).

Practice Item Analysis

Item	DOK	Rigor
1–6	1	Procedural Skill & Fluency
7–11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.
- How can you represent the multiplication of two fractions? Why is the product less than the two fractions?

Ask students to share their reflections with their classmates.

Math is... Mindset
- What helped you work well with a classmate even when you might disagree?

Students reflect on how they developed stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.
- I can use a representation to multiply a fraction by a fraction.
- I can explain how to use a representation to multiply a fraction by a fraction.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Multiply a fraction by a fraction using representations	5.NF.B.4.a
2	2	Multiply a fraction by a fraction using representations	5.NF.B.4.a
3	2	Multiply a fraction by a fraction using representations	5.NF.B.4.a

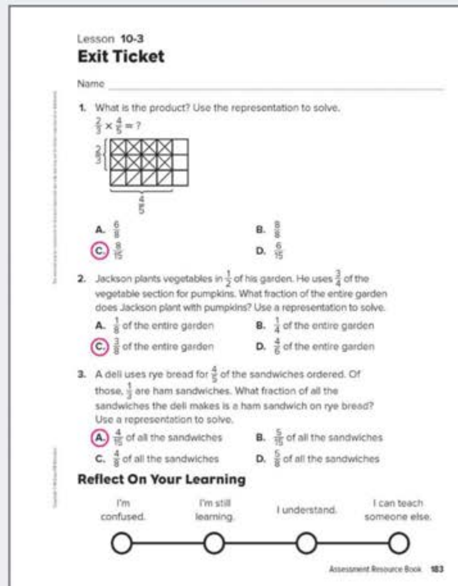
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 10-3

Exit Ticket

Name ______

1. What is the product? Use the representation to solve.
$\frac{2}{3} \times \frac{4}{5} = ?$
A. $\frac{6}{8}$ B. $\frac{8}{8}$
C. $\frac{8}{15}$ D. $\frac{6}{15}$

2. Jackson plants vegetables in $\frac{1}{2}$ of his garden. He uses $\frac{3}{4}$ of the vegetable section for pumpkins. What fraction of the entire garden does Jackson plant with pumpkins? Use a representation to solve.
A. $\frac{1}{8}$ of the entire garden B. $\frac{1}{4}$ of the entire garden
C. $\frac{3}{8}$ of the entire garden D. $\frac{4}{6}$ of the entire garden

3. A deli uses rye bread for $\frac{4}{5}$ of the sandwiches ordered. Of those, $\frac{1}{3}$ are ham sandwiches. What fraction of all the sandwiches the deli makes is a ham sandwich on rye bread? Use a representation to solve.
A. $\frac{4}{15}$ of all the sandwiches B. $\frac{5}{15}$ of all the sandwiches
C. $\frac{4}{8}$ of all the sandwiches D. $\frac{5}{8}$ of all the sandwiches

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 183

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Fraction Windows

Work with students in pairs. Students divide their paper into equal parts, and color some of the parts, and write the fraction represented by the colored parts. Then they combine their papers into a single image using both colors and work together to write a fraction. If students have difficulty, help them recognize that the numerator is the number of parts of each color and the denominator is the number of parts on each paper.

GO ONLINE

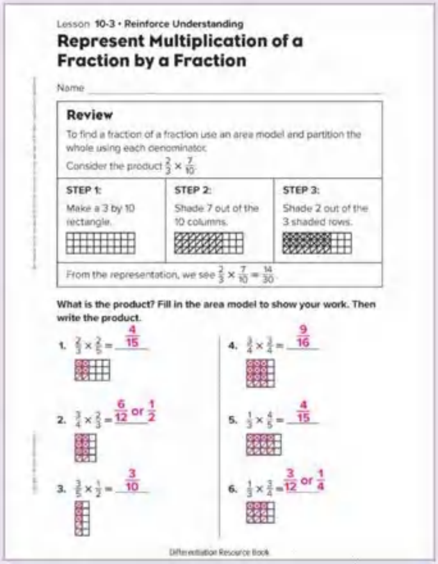
Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Multiply Two Fractions-Models

INDEPENDENT WORK

Differentiation Resource Book, p. 111


Lesson 10-3 • Reinforce Understanding

Represent Multiplication of a Fraction by a Fraction

Name

Review

To find a fraction of a fraction use an area model and partition the whole using each denominator.

Consider the product $\frac{2}{3} \times \frac{7}{10}$.

STEP 1:	STEP 2:	STEP 3:
Make a 3 by 10 rectangle.	Shade 7 out of the 10 columns.	Shade 2 out of the 3 shaded rows.

From the representation, we see $\frac{2}{3} \times \frac{7}{10} = \frac{14}{30}$.

What is the product? Fill in the area model to show your work. Then write the product.

1. $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$
2. $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$ or $\frac{1}{2}$
3. $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$
4. $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$
5. $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$
6. $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$ or $\frac{1}{4}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Multiplication Task Cards

Students practice representing the multiplication of fractions.

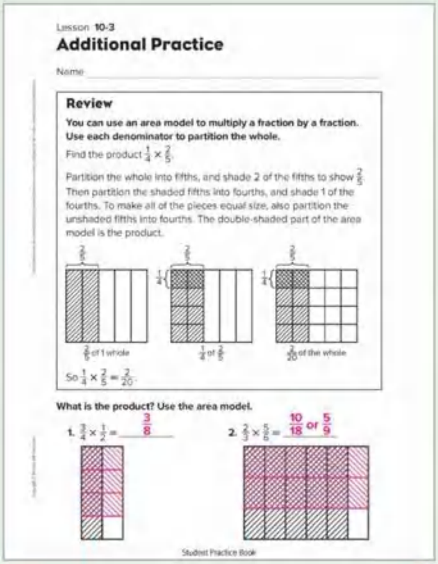
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 111–112


Lesson 10-3

Additional Practice

Name

Review

You can use an area model to multiply a fraction by a fraction. Use each denominator to partition the whole.

Find the product $\frac{1}{4} \times \frac{2}{5}$.

Partition the whole into fifths, and shade 2 of the fifths to show $\frac{2}{5}$. Then partition the shaded fifths into fourths, and shade 1 of the fourths. To make all of the pieces equal size, also partition the unshaded fifths into fourths. The double-shaded part of the area model is the product.

$\frac{2}{5}$ of 1 whole $\quad \frac{1}{4}$ of $\frac{2}{5} \quad \frac{2}{20}$ of the whole

So $\frac{1}{4} \times \frac{2}{5} = \frac{2}{20}$.

What is the product? Use the area model.

1. $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
2. $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18}$ or $\frac{5}{9}$

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Assign

Use It! Application Station

Fraction of a Fraction Students use fractions to follow and change a recipe.

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

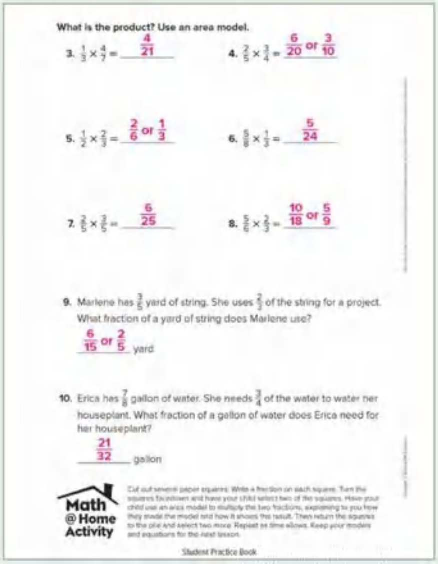
STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

GO ONLINE

Student Practice Book, pp. 111–112



What is the product? Use an area model.

3. $\frac{1}{3} \times \frac{4}{7} =$ $\frac{4}{21}$

4. $\frac{2}{5} \times \frac{3}{4} =$ $\frac{6}{20}$ or $\frac{3}{10}$

5. $\frac{1}{2} \times \frac{2}{3} =$ $\frac{2}{6}$ or $\frac{1}{3}$

6. $\frac{5}{8} \times \frac{1}{3} =$ $\frac{5}{24}$

7. $\frac{2}{5} \times \frac{3}{5} =$ $\frac{6}{25}$

8. $\frac{5}{6} \times \frac{2}{3} =$ $\frac{10}{18}$ or $\frac{5}{9}$

9. Marlene has $\frac{3}{5}$ yard of string. She uses $\frac{2}{3}$ of the string for a project. What fraction of a yard of string does Marlene use?

$\frac{6}{15}$ or $\frac{2}{5}$ yard

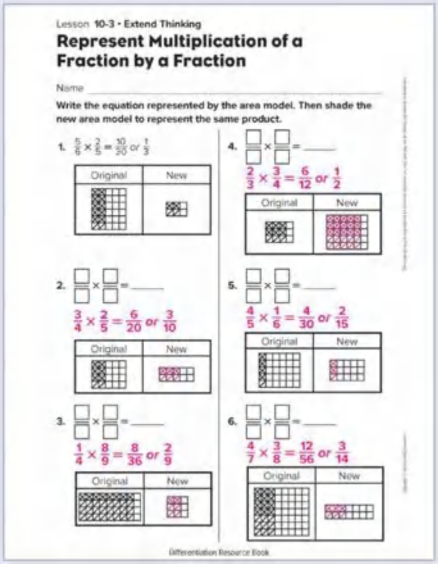
10. Erica has $\frac{7}{8}$ gallon of water. She needs $\frac{3}{4}$ of the water to water her houseplant. What fraction of a gallon of water does Erica need for her houseplant?

$\frac{21}{32}$ gallon

Math @ Home Activity Cut out several paper squares. Write a fraction on each square. Turn the squares facedown and have your child select two of the squares. Have your child use an area model to multiply the two fractions, explaining to you how they made the model and how it shows the result. Then return the squares to the pile and select two more. Repeat as time allows. Keep your models and equations for the next lesson.

Student Practice Book

Differentiation Resource Book, p. 112



Lesson 10-3 • Extend Thinking

Represent Multiplication of a Fraction by a Fraction

Name ______

Write the equation represented by the area model. Then shade the new area model to represent the same product.

1. $\frac{5}{6} \times \frac{2}{5} = \frac{10}{30}$ or $\frac{1}{3}$

Original | New

2. $\frac{\square}{\square} \times \frac{\square}{\square} =$ ______

$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$ or $\frac{3}{10}$

Original | New

3. $\frac{\square}{\square} \times \frac{\square}{\square} =$ ______

$\frac{1}{4} \times \frac{8}{9} = \frac{8}{36}$ or $\frac{2}{9}$

Original | New

4. $\frac{\square}{\square} \times \frac{\square}{\square} =$ ______

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$ or $\frac{1}{2}$

Original | New

5. $\frac{\square}{\square} \times \frac{\square}{\square} =$ ______

$\frac{4}{5} \times \frac{1}{6} = \frac{4}{30}$ or $\frac{2}{15}$

Original | New

6. $\frac{\square}{\square} \times \frac{\square}{\square} =$ ______

$\frac{4}{7} \times \frac{3}{8} = \frac{12}{56}$ or $\frac{3}{14}$

Original | New

Differentiation Resource Book

INDEPENDENT WORK

LESSON **10-4**

Multiply a Fraction by a Fraction

Learning Target

- I can multiply a fraction by a fraction.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

◇ **5.NF.B.4.a** Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$).

Math Practices and Processes

MPP Look for and express regularity in repeated reasoning.

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
denominator	arguably
numerator	speculate

Materials

The materials may be for any part of the lesson.

- grid paper
- index cards

Focus

Content Objective	Language Objectives	SEL Objective
• Students multiply a fraction by a fraction by multiplying the numerators and multiplying the denominators.	• Students talk about multiplying a fraction by a fraction by multiplying the numerators and denominators using *relate*. • To support cultivating conversation, ELs participate in MLR3: Critique, Correct, and Clarify.	• Students analyze the components of a problem to make informed decisions when engaging in mathematical practices.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students represented multiplication of a fraction by a fraction (Unit 10).	• Students multiply a fraction by a fraction.	• Students determine the area of a rectangle with fractional side lengths (Unit 10). • Students interpret and compute quotients of fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students notice and generalize a pattern that connects the area model to an equation.	• Students build proficiency with representations and multiplication involving fractions.	• Students solve word problems. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine

Decompose It 5–7 min

Build Fluency Students build number sense as they decompose a fraction in at least two ways. These prompts encourage students to talk about their reasoning:

- What are you being asked to do?
- What do you notice about the fraction?
- What do you think you should do first?
- What strategy can you use to decompose the fraction?
- What patterns do you see?
- How can a pattern help you decompose a fraction in a different way?

Purpose Students consider story contexts that involve multiplying a fraction by a fraction.

Notice & Wonder

- What could the question be?

Teaching Tip Have students work in pairs to discuss prior knowledge about similar problems.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of multiplying a fraction by a fraction and are based on possible comments and questions that students may make during the share out.

- What operations could you do with the numbers given?
- What quantities could you find that make sense in this problem?

Math is... Mindset

- How does a plan help you solve a problem?

SEL Responsible Decision-Making: Solve Problems

As students work through the Notice & Wonder routine, encourage them to analyze or think about the problem critically before they take steps toward solving. Having them consider what information they have, what question is being asked, and what tools they may use can help students make sense of the problem. This analysis can help students make more informed mathematical decisions.

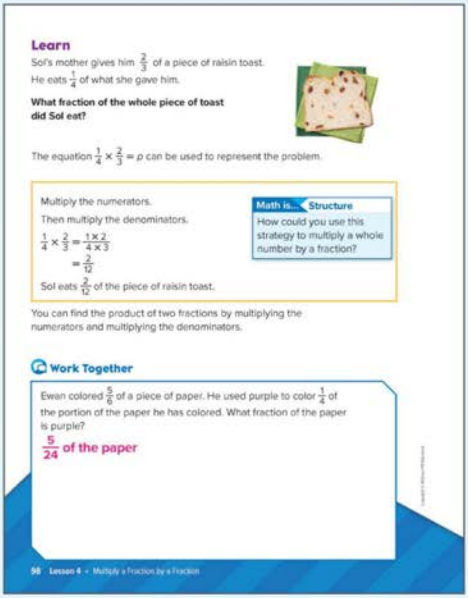
Transition to Explore & Develop

Ask questions that get students thinking about multiplying fractions. Guide the discussion to have the students think about how to find the product of fractions. If students bring up multiplying numerators and multiplying denominators organically, bring that into the discussion, but if students do not introduce the concept during this part of the lesson, they will be reminded of it in the Explore & Develop.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about a strategy for multiplying fractions.

Explore & Develop
20 min



Learn

Sol's mother gives him $\frac{2}{3}$ of a piece of raisin toast. He eats $\frac{1}{4}$ of what she gave him.

What fraction of the whole piece of toast did Sol eat?

The equation $\frac{1}{4} \times \frac{2}{3} = p$ can be used to represent the problem.

Multiply the numerators.

Then multiply the denominators.

$\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3}$

$= \frac{2}{12}$

Sol eats $\frac{2}{12}$ of the piece of raisin toast.

Math is... Structure

How could you use this strategy to multiply a whole number by a fraction?

You can find the product of two fractions by multiplying the numerators and multiplying the denominators.

Work Together

Ewan colored $\frac{5}{6}$ of a piece of paper. He used purple to color $\frac{1}{4}$ of the portion of the paper he has colored. What fraction of the paper is purple?

$\frac{5}{24}$ of the paper

98 Lesson 4 • Multiply a Fraction by a Fraction

❶ Pose the Problem

ETP Pose Purposeful Questions

- How can you use the picture to help you understand this problem?
- How can you represent this problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Critique, Correct, and Clarify

Make a false claim for students to critique. Write the following on the board: $\frac{3}{4} \times \frac{2}{3} = \frac{3+2}{4+3} = \frac{5}{7}$. *Am I correct?* Ask students to correct the statement, explaining how they know it's incorrect. Revisit this routine throughout the lesson to provide reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- What shortcut or rule have you found for multiplying a fraction by a fraction?

Key Takeaway

- To multiply a fraction by a fraction, multiply the numerators and multiply the denominators.

Work Together

Students use equations to multply a fraction by a fraction.

Common Error: When multiplying fractions, students may only multiply numerators and keep the denominator of the first factor as they did when multiplying a fraction and a whole number. Remind them that both ned to be multiplied.

LOM Language of Math

Denominator is a Latin word meaning "that which names." The denominator names the partition the fraction is representing.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore efficient strategies for multiplying two fractions.

Directions: Display 4 or 5 multiplication expressions involving two fractions. Tell students that they will find each product and look for patterns.

ETP Support Productive Struggle

- What strategy did you use to find each product?
- Are your products reasonable? How do you know?
- Can you find any shortcuts to help you solve more efficiently? If so, describe them.
- What patterns do you notice when multiplying two fractions? Why do those patterns exist?

Math is... Structure

- How could you use this strategy to multiply a whole number by a fraction?

Students step back for an overview and shift perspective, allowing them to apply the strategy they have just learned to a scenario they investigated earlier.

Activity Debrief: Students should notice that one way to multiply two fractions is to multiply the numerators and multiply the denominators. These products are the numerator and denominator of the product. They can use properties of operations to explain why this occurs.

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine what fraction of the whole piece of toast Sol ate?

Guided Exploration

Students extend their understanding of multiplying fractions to using equations to find the product.

ETP Use and Connect Mathematical Representations

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

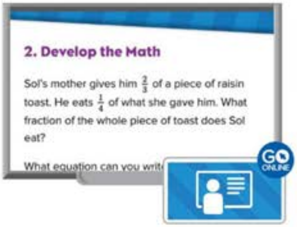
Have students represent the equation $\frac{1}{4} \times \frac{2}{3}$. Ask:

- What fraction of a whole should you represent? Why?
- How should you partition that fraction? Why?
- What should you shade next? Why?
- How many equal parts is the whole partitioned into?
- **Think About It:** Does Sol eat more or less than $\frac{2}{3}$ of the piece of raisin toast? How do you know?

Math is... Structure

- How could you use this strategy to multiply a whole number by a fraction?

Students step back for an overview and shift perspective, allowing them to apply the strategy they have just learned to a scenario they investigated earlier.



EL English Learner Scaffolds

Entering/Emerging Explain *whole piece of toast,* focusing on *whole* and *piece*. Show students a piece of paper. Say *This is a piece of paper*. Then cut it in two. Hold one half up and say *This is half a piece of paper*. Show students another piece of paper. Say *This is a whole piece of paper.* Then guide students to the Learn page. Point to the picture and ask *Is this a whole piece of toast?*

Developing/Expanding Explain *whole piece of toast,* using a piece of paper. Say *This is a piece of paper.* Then cut it in two. Hold one half up and say *This is half a piece of paper.* Show students another piece of paper. Say *This is a whole piece of paper.* Then send students to the Learn page and point to the picture. Have them complete the sentence: *That's a _____ (whole) piece of toast.*

Bridging/Reaching Ask students to read the word problem at the top of the Learn page, focusing on the phrase *whole piece of toast.* Ask them to come up with other phrases that can have both *whole* and *piece* in them (*whole piece of paper, whole piece of bread, whole piece of cake, etc.*). For students who may need more help, provide visual cues or references.

On My Own

Name ____________

Complete the equation.

1. $\frac{1}{5} \times \frac{1}{5} = \frac{1 \times 1}{5 \times 5} = \frac{1}{25}$
2. $\frac{2}{3} \times \frac{7}{8} = \frac{2 \times 7}{3 \times 8} = \frac{14}{24}$
3. $\frac{2}{3} \times \frac{4}{9} = \frac{8}{27}$
4. $\frac{3}{7} \times \frac{4}{5} = \frac{12}{35}$
5. On Sunday, Aisha used $\frac{3}{4}$ of a bag of oranges to make fresh orange juice. On Monday, she used $\frac{4}{5}$ as many oranges as on Sunday. How many bags of oranges did she use on Monday?
   $\frac{3}{4} \times \frac{4}{5} = \frac{12}{20}$; $\frac{12}{20}$ bag
6. Tabitha and Ally are putting together a puzzle. They have $\frac{3}{5}$ of the puzzle completed. If Tabitha put $\frac{1}{2}$ of the partly-finished puzzle together, what fraction of the puzzle did she put together?
   
   $\frac{3}{10}$ of the puzzle
7. Christina and her friends shared $\frac{2}{3}$ of a bag of snacks. Her friends ate $\frac{4}{5}$ of what was shared. How much of the bag of snacks did they eat?
   $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$; $\frac{8}{15}$ of the bag

Unit 10 • Multiply Fractions 99

8. **Error Analysis** Joelle thinks that the product of $\frac{7}{8} \times \frac{3}{10}$ is greater than the product of $\frac{3}{8} \times \frac{7}{10}$. How do you respond to Joelle's thinking?
   $\frac{7}{8} \times \frac{3}{10} = \frac{21}{80}$; $\frac{3}{8} \times \frac{7}{10} = \frac{21}{80}$; The product of each pair of fractions is equal.
9. Complete the equation.
   $\frac{1}{8} \times \frac{1}{3} = \frac{1}{24}$
10. **STEM Connection** Saffron is baking a sweet potato pie. Her recipe calls for $\frac{2}{3}$ cup of sugar. If she wants to make $\frac{1}{2}$ of the recipe, how much sugar will she need?
    
    $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$
11. **Extend Your Thinking** When you add fractions, the denominators stay the same. But, when you multiply fractions they do not. Explain why.
    When you add fractions you are counting parts of the whole, so parts stay the same size. When you multiply, the parts may change size because you are finding part of a part.

Reflect

How can you multiply fractions without using a drawing?
Answers may vary.

Math is... Mindset
How has a plan helped you solve a problem?

100 Lesson 4 • Multiply a Fraction by a Fraction

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 8 Students may think that because the first factor is greater in the first multiplication, the product is greater, but the products of the numerators, and the products of the denominators, are equal in both expressions.

Practice Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill & Fluency
5–7	2	Application
8	3	Conceptual Understanding
9	1	Procedural Skill & Fluency
10	2	Application
11	3	Conceptual Understanding

Reflect

Students complete the Reflect question.
- How can you multiply fractions without using a drawing?

Ask students to share their reflections with their classmates.

Math is... Mindset
- How has a plan helped you solve a problem?

Students reflect on how they practiced responsible decision-making.

Learning Targets

Ask students to reflect on the Learning Target of the lesson.
- I can multiply a fraction by a fraction.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Multiply a fraction by a fraction	5.NF.B.4.a
2	1	Multiply a fraction by a fraction	5.NF.B.4.a
3	2	Multiply a fraction by a fraction	5.NF.B.4.a

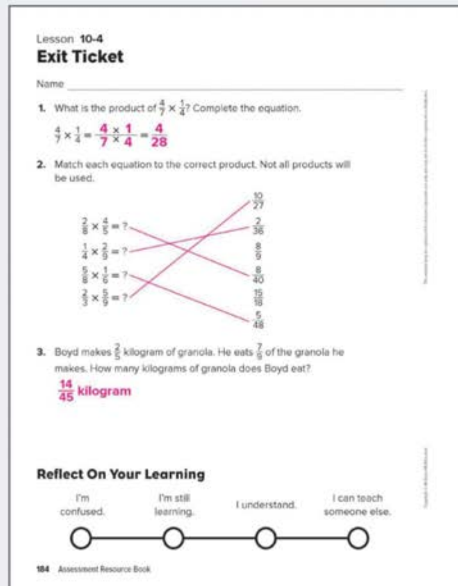
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 10-4

Exit Ticket

Name ____________________

1. What is the product of $\frac{4}{7} \times \frac{1}{4}$? Complete the equation.

$\frac{4}{7} \times \frac{1}{4} = \frac{4 \times 1}{7 \times 4} = \frac{4}{28}$

2. Match each equation to the correct product. Not all products will be used.

$\frac{2}{8} \times \frac{4}{5} = ?$ — $\frac{8}{40}$

$\frac{1}{4} \times \frac{2}{9} = ?$ — $\frac{2}{36}$

$\frac{5}{8} \times \frac{1}{6} = ?$ — $\frac{5}{48}$

$\frac{2}{3} \times \frac{5}{9} = ?$ — $\frac{10}{27}$

Products: $\frac{10}{27}$, $\frac{2}{36}$, $\frac{8}{9}$, $\frac{8}{40}$, $\frac{15}{18}$, $\frac{5}{48}$

3. Boyd makes $\frac{2}{5}$ kilogram of granola. He eats $\frac{7}{9}$ of the granola he makes. How many kilograms of granola does Boyd eat?

$\frac{14}{45}$ kilogram

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

184 Assessment Resource Book

 10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Flip It, Solve It!

Work with students in pairs. Write 6 fractions (for example: $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$) on the board. Provide notecards and multiplication charts, and have them copy one fraction onto each notecard. Students will take turns flipping over one card and multiplying it by $\frac{1}{2}$. Students discuss if they agree on the product.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Multiply Two Fractions
- Multiply Two Fractions-Word Problems

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 113

Lesson 10-4 • Reinforce Understanding

Multiply a Fraction by a Fraction

Name ______________________

Review

We can multiply the numerators and multiply the denominators to find the product of two fractions.

Consider $\frac{3}{5} \times \frac{4}{7}$.

Multiply 3 times 4 to find the numerator.

Multiply 5 times 7 to find the denominator.

$\boxed{\frac{3}{5} \times \frac{4}{7} = \frac{3 \times 4}{5 \times 7}}$

As a result, $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$.

What is the product?

1. $\frac{2}{9} \times \frac{5}{11} = \frac{10}{99}$
2. $\frac{4}{7} \times \frac{4}{9} = \frac{16}{63}$
3. $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$ or $\frac{3}{10}$
4. $\frac{2}{11} \times \frac{1}{3} = \frac{2}{33}$
5. $\frac{8}{9} \times \frac{3}{4} = \frac{24}{36}$ or $\frac{2}{3}$
6. $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$
7. $\frac{3}{5} \times \frac{7}{10} = \frac{21}{50}$
8. $\frac{5}{6} \times \frac{3}{8} = \frac{15}{48}$ or $\frac{5}{16}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Multiplication Showdown

Students practice multiplying fractions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 113–114

Lesson 10-4

Additional Practice

Name ______________________

Review

You can multiply a fraction by a fraction by multiplying the numerators and multiplying the denominators.

Allen lives $\frac{2}{5}$ mile from the park. He ran $\frac{3}{4}$ of the way to the park, then walked. How far did Allen run?

To solve, find $\frac{3}{4} \times \frac{2}{5}$.

Multiply the denominators of the factors to get the denominator of the product. Multiply the numerators of the factors to get the numerator of the product.

$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20}$

Allen ran $\frac{6}{20}$ mile on the way to the park.

What is the product?

1. $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$
2. $\frac{2}{4} \times \frac{5}{7} = \frac{10}{28}$
3. $\frac{4}{5} \times \frac{3}{7} = \frac{12}{35}$
4. $\frac{6}{11} \times \frac{5}{7} = \frac{30}{77}$

Student Practice Book

Own It! Digital Station

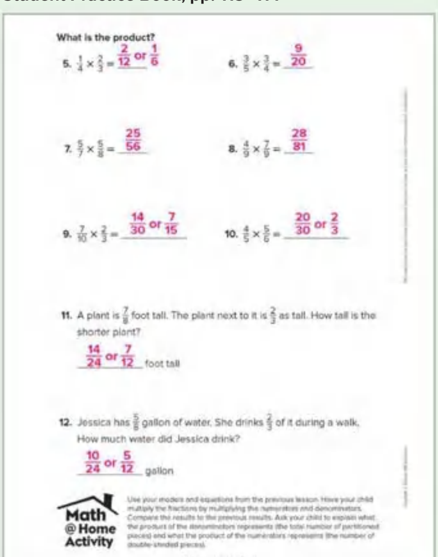
Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Student Practice Book, pp. 113–114



What is the product?

5. $\frac{1}{4} \times \frac{2}{3} =$ $\frac{2}{12}$ or $\frac{1}{6}$

6. $\frac{3}{5} \times \frac{3}{4} =$ $\frac{9}{20}$

7. $\frac{5}{7} \times \frac{5}{8} =$ $\frac{25}{56}$

8. $\frac{4}{9} \times \frac{7}{9} =$ $\frac{28}{81}$

9. $\frac{7}{10} \times \frac{2}{3} =$ $\frac{14}{30}$ or $\frac{7}{15}$

10. $\frac{4}{5} \times \frac{5}{6} =$ $\frac{20}{30}$ or $\frac{2}{3}$

11. A plant is $\frac{7}{8}$ foot tall. The plant next to it is $\frac{2}{3}$ as tall. How tall is the shorter plant?

$\frac{14}{24}$ or $\frac{7}{12}$ foot tall

12. Jessica has $\frac{5}{8}$ gallon of water. She drinks $\frac{2}{3}$ of it during a walk. How much water did Jessica drink?

$\frac{10}{24}$ or $\frac{5}{12}$ gallon

Math @ Home Activity Use your models and equations from the previous lesson. Have your child multiply the fractions by multiplying the numerators and denominators. Compare the results to the previous results. Ask your child to explain what the product of the denominators represents (the total number of partitioned pieces) and what the product of the numerators represents (the number of double-shaded pieces).

Student Practice Book

WORKSTATIONS

GO ONLINE

INDEPENDENT WORK

E Extend Thinking

Use It! Application Station

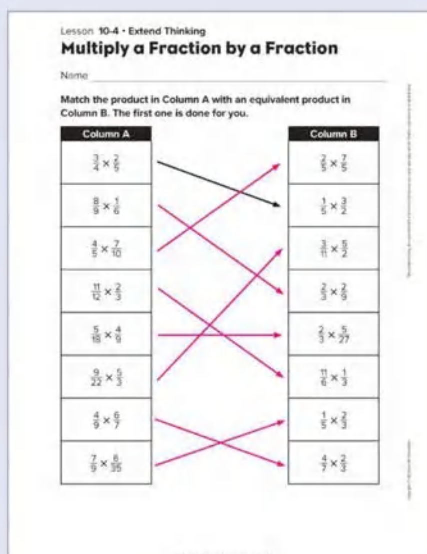
If, Then Students use if/then statements to write a problem in which they make a true or false statement that uses multiplication of fractions.

The content of this card has concepts covered later in Lesson 10-7. You may want to assign this card to students ready to explore content covered later in this unit.

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Differentiation Resource Book, p. 114



Lesson 10-4 • Extend Thinking

Multiply a Fraction by a Fraction

Name ______________________

Match the product in Column A with an equivalent product in Column B. The first one is done for you.

Column A	Column B
$\frac{3}{4} \times \frac{2}{5}$	$\frac{2}{5} \times \frac{7}{5}$
$\frac{8}{9} \times \frac{1}{6}$	$\frac{1}{5} \times \frac{3}{2}$
$\frac{4}{5} \times \frac{7}{10}$	$\frac{3}{11} \times \frac{5}{2}$
$\frac{11}{12} \times \frac{2}{3}$	$\frac{2}{3} \times \frac{2}{9}$
$\frac{5}{18} \times \frac{4}{9}$	$\frac{2}{3} \times \frac{5}{27}$
$\frac{9}{22} \times \frac{5}{3}$	$\frac{11}{6} \times \frac{1}{3}$
$\frac{4}{9} \times \frac{6}{7}$	$\frac{1}{5} \times \frac{2}{3}$
$\frac{7}{9} \times \frac{6}{35}$	$\frac{4}{7} \times \frac{2}{3}$

Differentiation Resource Book

LESSON **10-5**

Determine the Area of Rectangles with Fractional Side Lengths

Learning Targets

- I can find the area of a rectangle with fractional side lengths by tiling it with unit squares with unit fraction side lengths.
- I can find the area of a rectangle with fractional side lengths by multiplying the side lengths.

Vocabulary

Math Terms	Academic Terms
area	expand
square unit	reflect

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

◇ **5.NF.B.4.b** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Math Practices and Processes

MPP Look for and express regularity in repeated reasoning.

Materials

The materials may be for any part of the lesson.

- blank spinners
- grid paper
- rulers

Focus

Content Objectives	Language Objectives	SEL Objective
• Students find the area of a rectangle with fractional side lengths by tiling it with unit squares. • Students find the area of a rectangle with fractional side lengths by multiplying the side lengths.	• Students explain how to find the area of a rectangle with fractional side lengths using the verb *tile*. • To support sense-making, ELs participate in MLR2: Collect and Display.	• Students discuss how a mathematical rule or routine can help develop mathematical skills and knowledge.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students multiplied a fraction by a fraction (Unit 10).	• Students determine the area of a rectangle with fractional side lengths.	• Students multiply mixed numbers using area models and partial products (Unit 10). • Students interpret and compute quotients of fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding about multiplying fractions using the concept of area.	• Students build proficiency multiplying fractions.	• Students solve word problems. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Which Benchmark Is It Closest To? 5–7 min

Build Fluency Students build understanding of fractions as they compare fractions to benchmarks. Students decide which benchmark each fraction is closest to.

Remind students that exact positioning is not necessary, but they should decide between which benchmarks the fractions belong and approximately how close to the benchmarks each fraction would be.

These prompts encourage students to talk about their reasoning:

- What are you being asked to do?
- How did you know where to place each fraction along the number line?
- How can benchmarks help you compare fractions?

Launch 5–7 min

Sense-Making Routine

Purpose Students reflect on tiling a region with unit squares to find area to prepare them for tiling rectangles with fractional side lengths to find area.

Notice & Wonder

- What do you see?

Teaching Tip Have students work in pairs to discuss what they think they notice and wonder about the image, and share ideas about what mathematics they might be able to do to solve a problem.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of find the area of a rectangle with fractional side lengths by tiling it and are based on possible comments and questions that students may make during the share out.

- How does finding area use muitiplication?
- What do you notice about the tiling of this floor that is similar to the tiling you have used to find the area of a rectangle? What is different?

Math is... Mindset

- What strategies help you work more efficiently?

SEL Self-Management: Organizational Skills

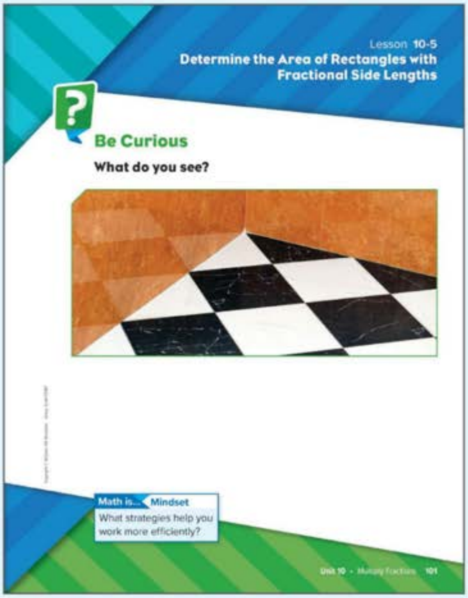
Organizing information and work can help students work through challenging mathematical tasks. Invite students to organize the rules or routines they will follow while working through the Notice & Wonder routine. Have them list criteria they used to organize, and have groups of students compare these criteria. Ask them to discuss the pros and cons of each method, and then use the organizational pros to develop an organizational method they think will be useful in finding areas of rectangles.

Transition to Explore & Develop

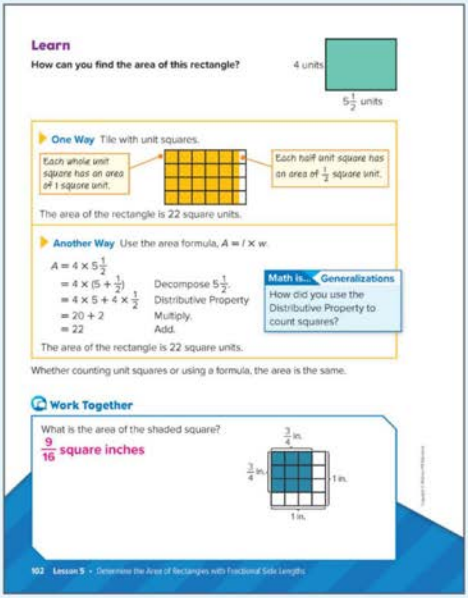
Ask questions to get students thinking about how they would find the area of the floor shown.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about a strategy for finding the area of rectangles where at least one side has a fractional length.



Explore & Develop 20 min



Learn

How can you find the area of this rectangle?

4 units

$5\frac{1}{2}$ units

One Way Tile with unit squares.

Each whole unit square has an area of 1 square unit.

Each half unit square has an area of $\frac{1}{2}$ square unit.

The area of the rectangle is 22 square units.

Another Way Use the area formula, $A = l \times w$.

$A = 4 \times 5\frac{1}{2}$	
$= 4 \times (5 + \frac{1}{2})$	Decompose $5\frac{1}{2}$.
$= 4 \times 5 + 4 \times \frac{1}{2}$	Distributive Property
$= 20 + 2$	Multiply.
$= 22$	Add.

Math is... Generalizations
How did you use the Distributive Property to count squares?

The area of the rectangle is 22 square units.

Whether counting unit squares or using a formula, the area is the same.

Work Together

What is the area of the shaded square?

$\frac{9}{16}$ square inches

$\frac{3}{4}$ in.

$\frac{3}{4}$ in.

1 in.

1 in.

102 Lesson 5 • Determine the Area of Rectangles with Fractional Side Lengths

❶ Pose the Problem

MLR Collect and Display

As students discuss the questions, record relevant words and phrases they may use such as *determine, tiling, unit squares,* and *distributive property*. Display the words for student reference. Use the student-generated expressions to help students make connections between student language and math vocabulary.

ETP Pose Purposeful Questions

- How did you tile a rectangle with unit squares previously to find area?
- What formula did you use to find area previously?
- What is different about this problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How is finding the area of a rectangle with whole number side lengths similar to finding the area of one with fractional side lengths? How is it different?

Key Takeaway

- Strategies for finding area of rectangles with whole number side lengths extend to finding the area of rectangles with fractional side lengths.

Work Together

Students find the area of a region whose dimensions are both fractions.

Common Misconception: Students may not realize they have the strategies needed to solve this problem. Point out they can find the area without tiling, but by using the area formula and multiplying the length by the width.

LOM Language of Math

Numerator comes from the Latin word *numero*, meaing "count." The numerator counts the number of partitions represented by the fraction.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how to find the area of a rectangle that has fractional side lengths.

Materials: grid paper, fraction tiles

Directions: Have students work together to solve the Pose the Problem. Students may use any strategy or tool to solve.

ETP Implement Tasks that Promote Reasoning and Problem Solving

- Can you draw a picture or representation to help you solve?
- How does your work show $5\frac{1}{2}$ units?
- How does your work show 4 units?
- How does your work show the area of the rectangle?
- Is there another way to explain how to find the area of the rectangle?

Activity Debrief: Have students share their strategies for finding the area. Encourage students to look for similarties and differences among using grid paper to count unit squares and using fraction tiles. Compare both of these representations to using the area formula, focusing attention on using the Distributive Property to solve.

Math is... Generalizations

- How did you use the Distributive Property to count squares?

Students notice if calculations are repeated. In this case that the whole and the half unit squares in the counting strategy appear in the area forrmula as the (4×5) and $\left(4 \times \frac{1}{2}\right)$ terms.

Guided Exploration

Students extend their understanding of multiplication with fractions to multiply with a mixed number to find the area of a rectangle.

ETP Use and Connect Mathematical Representations

Have the students determine how many whole unit squares they would use. Ask:

- What are the length and width of the rectangle?
- How many rows and columns of unit squares can you use?
- How many unit squares are in that tiling or array of unit squares?
- **Think About It:** How do you know you can use half unit squares to tile the remaining area?

Have the students determine how many half unit squares they would use. Ask:

- How many half unit squares can you use to fill the remaining area? How do you know?

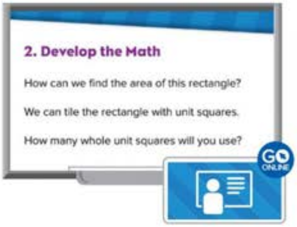
Have the students determine the number of whole unit squares they used in all. Ask:

- How many whole unit squares did you use?
- How many whole unit squares are 4 half unit squares equal to?

Math is... Generalizations

- How did you use the Distributive Property to count squares?

Students notice if calculations are repeated. In this case that the whole and half unit squares in the counting strategy appear in the area formula as the (4×5) and $\left(4 \times \frac{1}{2}\right)$ terms.

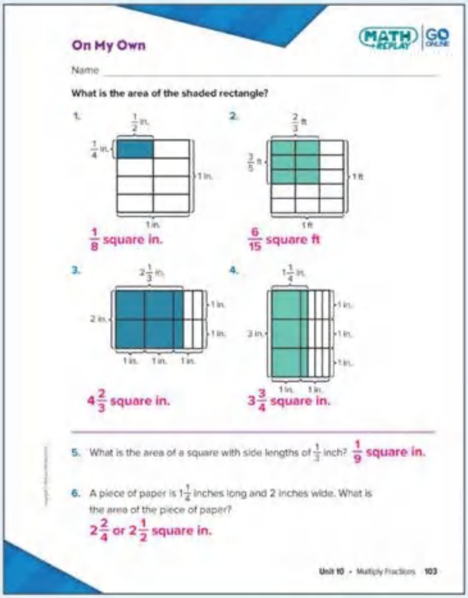


EL English Learner Scaffolds

Entering/Emerging Explain *whether*. Put 20 cm cubes on the desk and count them aloud. Then sort them into groups of 2 and count by 2. Say *If I sort them into groups of 2, the total is still 20.* Repeat, sorting them into groups of 5. Then say *Whether I sort these into groups of 2 or 5, the total is still 20.* Ask *Whether grouping by 2 or 5, is the total the same?*

Developing/Expanding Explain *whether*. Put cubes on the desk and count them aloud. Then sort them into groups of 2 and count by 2. Say *If I sort them into groups of 2, the total is still 20.* Repeat, sorting them into groups of 5. Then say *Whether I sort these into groups of 2 or 5, the total is still 20.* Then present the students with 10 counters and the sentence frame: ___ *I sort these into groups of* ___ *or* ___*, the total is the same.*

Bridging/Reaching Guide students to the Learn page. Show them the sentence using *whether*. To ensure comprehension, ask what they think *whether* means (*either way, no matter what,* etc.). Then ask students to use *whether* in a sentence. Validate and make corrections to vocabulary, grammar, and meaning as needed.



On My Own

Name

What is the area of the shaded rectangle?

1. $\frac{1}{2}$ in., $\frac{1}{4}$ in., 1 in., 1 in.
$\frac{1}{8}$ square in.

2. $\frac{2}{3}$ ft, $\frac{3}{5}$ ft, 1 ft, 1 ft
$\frac{6}{15}$ square ft

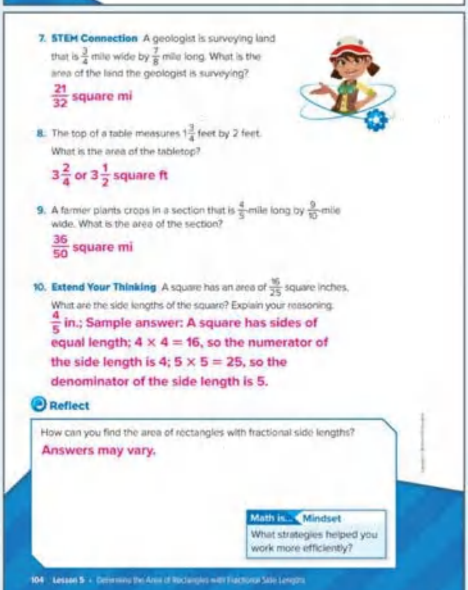
3. $2\frac{1}{3}$ in., 2 in., 1 in., 1 in., 1 in., 1 in., 1 in.
$4\frac{2}{3}$ square in.

4. $1\frac{1}{4}$ in., 3 in., 1 in., 1 in., 1 in., 1 in., 1 in.
$3\frac{3}{4}$ square in.

5. What is the area of a square with side lengths of $\frac{1}{3}$ inch? $\frac{1}{9}$ square in.

6. A piece of paper is $1\frac{1}{4}$ inches long and 2 inches wide. What is the area of the piece of paper?
$2\frac{2}{4}$ or $2\frac{1}{2}$ square in.

Unit 10 • Multiply Fractions 103



7. **STEM Connection** A geologist is surveying land that is $\frac{3}{4}$ mile wide by $\frac{7}{8}$ mile long. What is the area of the land the geologist is surveying?
$\frac{21}{32}$ square mi

8. The top of a table measures $1\frac{3}{4}$ feet by 2 feet. What is the area of the tabletop?
$3\frac{2}{4}$ or $3\frac{1}{2}$ square ft

9. A farmer plants crops in a section that is $\frac{4}{5}$ mile long by $\frac{9}{10}$ mile wide. What is the area of the section?
$\frac{36}{50}$ square mi

10. **Extend Your Thinking** A square has an area of $\frac{16}{25}$ square inches. What are the side lengths of the square? Explain your reasoning.
$\frac{4}{5}$ in.; Sample answer: A square has sides of equal length; $4 \times 4 = 16$, so the numerator of the side length is 4; $5 \times 5 = 25$, so the denominator of the side length is 5.

Reflect

How can you find the area of rectangles with fractional side lengths?
Answers may vary.

Math is... Mindset
What strategies helped you work more efficiently?

104 Lesson 5 • Determine the Area of Rectangles with Fractional Side Lengths

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 5 Students may have forgotten some geometry and stumble over this question. Remind them that: 1) squares are rectangles so the strategies they have learned apply to them; and 2) the side length of one side of a square is the same as all its other side lengths.

Practice Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill & Fluency
5–9	2	Application
10	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you find the area of rectangles with fractional side lengths?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What strategies helped you work more efficiently?

Students reflect on how they practiced self-management.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can find the area of a rectangle with fractional side lengths by tiling it with unit squares with unit fraction side lengths.
- I can find the area of a rectangle with fractional side lengths by multiplying the side lengths.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Find area of rectangles with fractional side lengths	5.NF.B.4.b
2	1	Find area of rectangles with fractional side lengths	5.NF.B.4.b
3	2	Find area of rectangles with fractional side lengths	5.NF.B.4.b

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

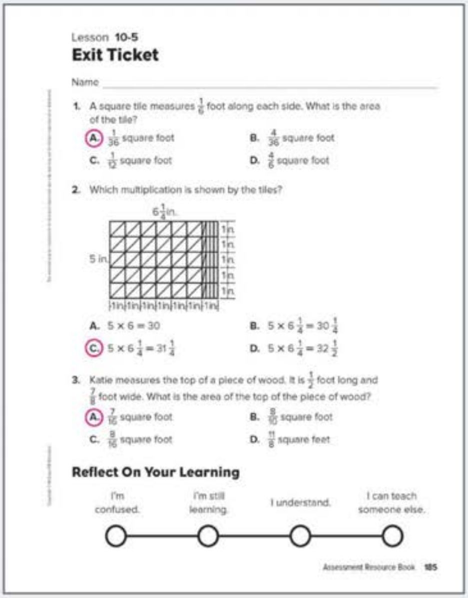
If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 10-5

Exit Ticket

Name ____________________

1. A square tile measures $\frac{1}{6}$ foot along each side. What is the area of the tile?

   A. $\frac{1}{36}$ square foot B. $\frac{4}{36}$ square foot

   C. $\frac{1}{12}$ square foot D. $\frac{4}{6}$ square foot

2. Which multiplication is shown by the tiles?

   $6\frac{1}{4}$ in.; 5 in.; 1 in. 1 in. 1 in. 1 in. 1 in. 1 in. 1 in.; 1 in. 1 in. 1 in. 1 in. 1 in.

   A. $5 \times 6 = 30$ B. $5 \times 6\frac{1}{4} = 30\frac{1}{4}$

   C. $5 \times 6\frac{1}{4} = 31\frac{1}{4}$ D. $5 \times 6\frac{1}{4} = 32\frac{1}{2}$

3. Katie measures the top of a piece of wood. It is $\frac{1}{2}$ foot long and $\frac{7}{8}$ foot wide. What is the area of the top of the piece of wood?

   A. $\frac{7}{16}$ square foot B. $\frac{8}{10}$ square foot

   C. $\frac{8}{16}$ square foot D. $\frac{11}{8}$ square feet

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 185

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Areas with Mixed Numbers

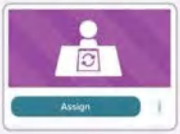
Work with students in pairs. Have students fill in a blank spinner with mixed numbers with denominators of 2, 3, or 4. Each student spins the spinner once. One student draws the length of a rectangle and labels it with number spun. The other student draws and labels the width of the rectangle. The students work together to find the area of the rectangle. Encourage students to separate the rectangle into parts and find the area o f each part.

GO ONLINE

Take Another Look Lesson

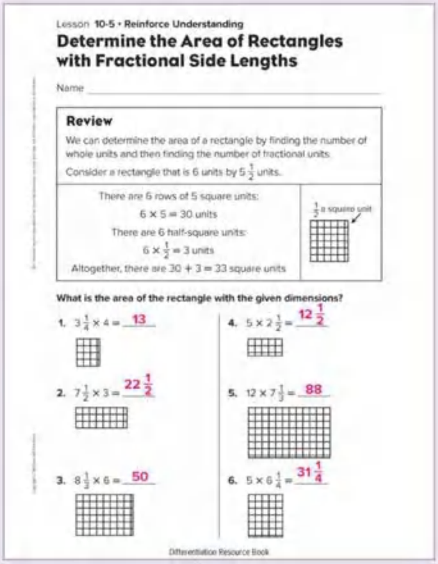
Assign the interactive lesson to reinforce targeted skills.

- Multiply Two Fractions-Models



INDEPENDENT WORK

Differentiation Resource Book, p. 115



Lesson 10-5 • Reinforce Understanding

Determine the Area of Rectangles with Fractional Side Lengths

Name

Review

We can determine the area of a rectangle by finding the number of whole units and then finding the number of fractional units.

Consider a rectangle that is 6 units by $5\frac{1}{2}$ units.

There are 6 rows of 5 square units:

$6 \times 5 = 30$ units

There are 6 half-square units:

$6 \times \frac{1}{2} = 3$ units

Altogether, there are $30 + 3 = 33$ square units

$\frac{1}{2}$ a square unit

What is the area of the rectangle with the given dimensions?

1. $3\frac{1}{4} \times 4 =$ 13
2. $7\frac{1}{2} \times 3 =$ $22\frac{1}{2}$
3. $8\frac{1}{3} \times 6 =$ 50
4. $5 \times 2\frac{1}{2} =$ $12\frac{1}{2}$
5. $12 \times 7\frac{1}{3} =$ 88
6. $5 \times 6\frac{1}{4} =$ $31\frac{1}{4}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Mixed Number Task Cards Students practice representing the multiplication of mixed numbers.

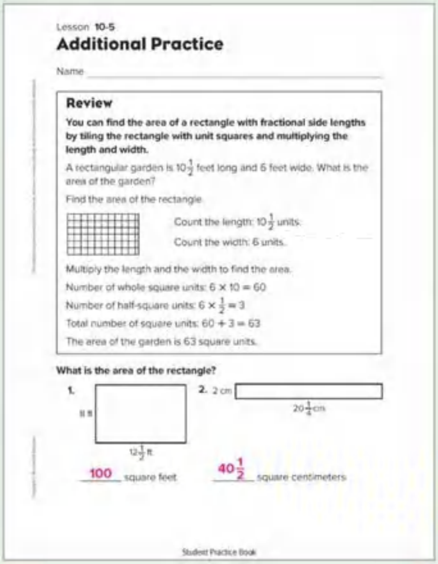
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 115–116



Lesson 10-5

Additional Practice

Name

Review

You can find the area of a rectangle with fractional side lengths by tiling the rectangle with unit squares and multiplying the length and width.

A rectangular garden is $10\frac{1}{2}$ feet long and 6 feet wide. What is the area of the garden?

Find the area of the rectangle.

Count the length: $10\frac{1}{2}$ units.

Count the width: 6 units.

Multiply the length and the width to find the area.

Number of whole square units: $6 \times 10 = 60$

Number of half-square units: $6 \times \frac{1}{2} = 3$

Total number of square units: $60 + 3 = 63$

The area of the garden is 63 square units.

What is the area of the rectangle?

1. 8 ft; $12\frac{1}{2}$ ft — 100 square feet
2. 2 cm; $20\frac{1}{4}$ cm — $40\frac{1}{2}$ square centimeters

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers

Assign

Use It! Application Station

This or That Students follow instructions to measure and cut materials.

The content of this card has concepts covered later in Lesson 10-8. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 115–116

What is the area of a rectangle with the given dimensions?

3. 5 inches by $3\frac{1}{2}$ inches

   $17\frac{1}{2}$ square inches

4. 8 feet by $4\frac{2}{3}$ feet

   $37\frac{1}{3}$ square feet

5. 10 yards by $2\frac{1}{5}$ yards

   22 square yards

6. 5 meters by $2\frac{3}{4}$ meters

   $13\frac{3}{4}$ square meters

7. A ceramic tile is $\frac{3}{4}$ foot wide and $\frac{3}{4}$ foot long. What is the area of the tile?

   $\frac{9}{16}$ square foot

8. Jill's rectangular bedroom is 11 feet long and $9\frac{1}{2}$ feet wide. What is the area of the floor in Jill's bedroom?

   $104\frac{1}{2}$ square feet

9. Jake's vegetable garden is in the shape of a rectangle. The garden is 24 feet long and $5\frac{5}{6}$ feet wide. What is the area of Jake's vegetable garden?

   140 square feet

Math @ Home Activity With your child, look for some rectangles or rectangular objects around your home. Use a ruler or tape measure to measure the dimensions, using fractions of a foot instead of inches. If neither dimension is a whole number, round one of the dimensions to the nearest whole foot. Have your child find the area of the object.

Student Practice Book

Differentiation Resource Book, p. 116

Lesson 10-5 • Extend Thinking

Determine the Area of Rectangles with Fractional Side Lengths

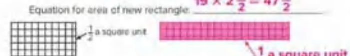
Name ______________________

Fill in the requested information for the rectangle. Then draw a different rectangle with the same area. Write the multiplication equation that represents the area of the new rectangle.

Answers will vary for 2nd rectangle.

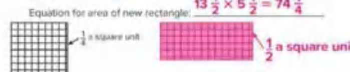
1. The rectangle has the dimensions: $9\frac{1}{2}$ units by 5 units and an area of $47\frac{1}{2}$ square units.

   Equation for area of new rectangle: $19 \times 2\frac{1}{2} = 47\frac{1}{2}$



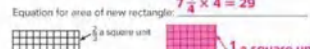
2. The rectangle has the dimensions: $8\frac{1}{4}$ units by 9 units and an area of $74\frac{1}{4}$ square units.

   Equation for area of new rectangle: $13\frac{1}{2} \times 5\frac{1}{2} = 74\frac{1}{4}$



3. The rectangle has the dimensions: $9\frac{2}{3}$ units by 3 units and an area of 29 square units.

   Equation for area of new rectangle: $7\frac{1}{4} \times 4 = 29$



Differentiation Resource Book

LESSON 10-6
Represent Multiplication of Mixed Numbers

Learning Targets

- I can use an area model to represent multiplication of mixed numbers.
- I can use an area model to find partial products when multiplying mixed numbers.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

◇ **5.NF.B.4.a** Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$).

Math Practices and Processes

MPP Model with mathematics.

Vocabulary

Math Terms	Academic Term
area model	accurate
decompose	establish
mixed number	
partial products	

Materials

The materials may be for any part of the lesson.

- blank spinners
- fraction tiles
- grid paper

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use an area model to represent multiplication of mixed numbers. • Students find partial products using an area model.	• Students talk about using an area model to represent multiplication of mixed numbers using the terms *similar to* and *different from.* • To support optimizing output, students participate in MLR3 Info Gap.	• Students engage in respectful discourse with peers about various perspectives for approaching a mathematical challenge.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students determined the area of a rectangle with fractional side lengths (Unit 10).	• Students multiply mixed numbers using area models and partial products.	• Students multiply mixed numbers using equations and partial products (Unit 10). • Students interpret and compute quotients of fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding of multiplying mixed numbers using representations.	• Students build proficiency for adding fractions by using multiple strategies.	• Students solve problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

Number Routine
Decompose It
5–7 min

Build Fluency Students build number sense as they decompose each decimal in at least two ways.

Record decompositions as students share with classmates.

These prompts encourage students to talk about their reasoning:

- What are you being asked to do?
- What do you notice about the numbers? What do you think you should do first?
- What strategy can you use to decompose the numbers?
- How can a pattern help you decompose a decimal in a different way?

Purpose Students are given entry into using area models to multiply mixed numbers by comparing a mixed number area model or a whole number area model.

Notice & Wonder

- How are they the same?
- How are they different?

Teaching Tip As students notice and wonder, you may want to have them copy down the area models on a piece of paper so they can examine and compare them more easily.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of using an area model to represent multiplication of mixed numbers and are based on possible comments and questions that students may make during the share out.

- When have you used an area model before?
- What do you notice about the numbers represented in the area models?

Math is... Mindset

- How does a different perspective help you with your work?

SEL Social Awareness: Develop Perspective

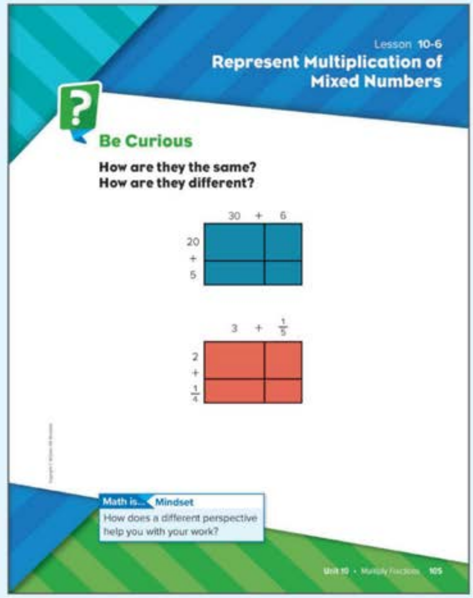
Encourage students to think about different ways to consider the Notice & Wonder routine. With a partner, have them share different tools/strategies/representations/methods they can use to explore how the area models are related. Invite students to consider and build off their partner's ideas.

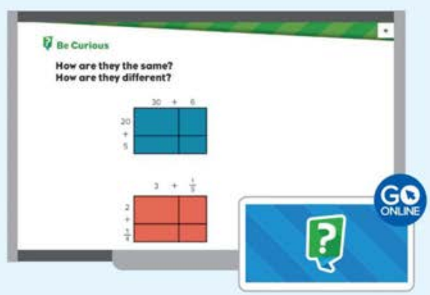
Transition to Explore & Develop

Ask questions that get students thinking about how decomposing multi-digit numbers is similar to decomposing mixed numbers.

ETP Establish Mathematics Goals to Focus Learning

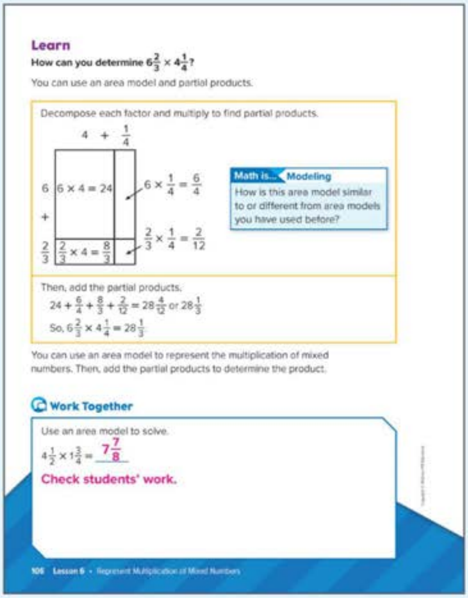
- Let's think about how we can use area models and partial products to multiply mixed numbers.





Explore & Develop

20 min



Learn

How can you determine $6\frac{2}{3} \times 4\frac{1}{4}$?

You can use an area model and partial products.

Decompose each factor and multiply to find partial products.

Math is... Modeling
How is this area model similar to or different from area models you have used before?

Then, add the partial products.

$24 + \frac{6}{4} + \frac{8}{3} + \frac{2}{12} = 28\frac{4}{12}$ or $28\frac{1}{3}$

So, $6\frac{2}{3} \times 4\frac{1}{4} = 28\frac{1}{3}$

You can use an area model to represent the multiplication of mixed numbers. Then, add the partial products to determine the product.

Work Together

Use an area model to solve.

$4\frac{1}{2} \times 1\frac{3}{4} =$ $7\frac{7}{8}$

Check students' work.

106 Lesson 6 • Represent Multiplication of Mixed Numbers

1 Pose the Problem

ETP Pose Purposeful Questions

- Based on what you know about mathematics, can you make a conjecture about how to represent the product?
- Do you think there will only be one way to determine the product? Why or why not?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Info Gap

Pair students. Provide Partner A with a problem like the one on the Learn page. Provide Partner B with the information to solve the problem. Instruct Partner B to ask A what information they need, and for A to respond, explaining why they need it. Have students continue until the problem is completed.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you represent multiplication of mixed numbers using an area model?
- How can you determine the partial products using an area model that represents mixed numbers?
- What are some ways you can add the partial products when they are fractions?

Key Takeaways

- An area model can be used to represent multiplication of mixed numbers.
- The partial products strategy can be extended to mixed numbers.

Work Together

Students solve a multiplication equation involving mixed numbers using an area model and partial products. Encourage students to use any known strategy to add the partial products.

Common Error: Remind students to first decompose each factor before creating their area model.

LOM Language of Math

Explain to students that a *mixed number* is called that because it is composed of more than one type of number—a whole number as well as a fraction. Similarly, mixed berries or mixed nuts are composed of more than one type of berry or nut. Encourage students to use the term *mixed number* throughout the lesson so that they become more familiar with it.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore area models for multiplication to extend to multiplying two mixed numbers.

Directions: Ask students to write a multiplication problem involving two 2-digit numbers and draw an area model to represent the product. Have students record as many ways as possible to decompose the factors. Invite students to share ways they decomposed the factors, focus attention on similar methods of decomposing, such as by place value.

- Do you think these methods of decomposing will work for multiplying two mixed numbers?

Have students explore different ways to decompose the factors $6\frac{2}{3}$ and $4\frac{1}{4}$ to find their product.

ETP Support Productive Struggle

- How can you decompose the factors?
- How can you determine the partial products?
- How can you determine the product?
- Is your answer reasonable? How do you know?
- How is using an area model helpful when multiplying mixed numbers?

Math is... Modeling

- How is this area model similar to or different from area models you have used before?

Students are connecting the models they have used to understand multiplication.

Activity Debrief: Discuss with students that an area model is one method they can use to multiply mixed numbers. Using this method, they can decompose each factor, find partial products, and add the partial products to calculate the product.

Guided Exploration

Students represent a multiplication equation involving mixed numbers using an area model and partial products.

ETP Facilitate Meaningful Mathematical Discourse

- **Think About It:** What representations did you use to multiply whole numbers?

Have the students decompose the factors. Ask:

- What can you decompose a mixed number into?

Have the students determine the partial products. Ask:

- What multiplication expression can you use to represent the area of each region of the area model?
- What is the partial product for each region?

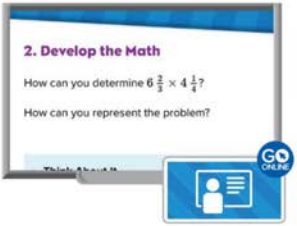
Have the students add the partial products. Ask:

- What like denominator will you use?
- How will you rewrite each fraction as an equivalent fraction?

Math is... Modeling

- How is this area model similar to or different from area models you have used before?

Students are connecting the models they have used to understand multiplication.



EL English Learner Scaffolds

Entering/Emerging Explain *similar to* and *different from*. Show students similar objects. Point and say *This one is similar to that one.* Name similarities. Show two objects that are different. Point and say *This one is different from that one.* Name differences. Choose two new pairs of objects, one pair being similar, one not. Point and ask *Are they similar to each other? Are they different from each other?*

Developing/Expanding Explain *similar to* and *different from.* Show students similar objects. Point and say *This one is similar to that one.* Name similarities. Show two objects that are different. Point and say *This one is different from that one.* Name differences. Then have students repeat the activity with new objects, using *similar to* and *different from.* Provide sentence frames for students who need help.

Bridging/Reaching Ask students to explain the phrases *similar to* and *different from,* using classroom manipulatives to support their explanations. Allow students to interject, pointing out any mistakes that they may make in meaning or understanding. For example, *No, those items are not similar to/different from each other because... or No, that's not correct because...*

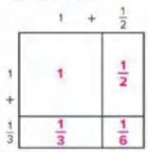
Practice & Reflect
10 min

On My Own

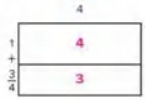
Name ______________________

Complete the area model. What is the product?

1. $1\frac{1}{3} \times 1\frac{1}{2} =$ **2**



2. $1\frac{3}{4} \times 4 =$ **7**



What is the product? Use an area model to solve.

3. $1\frac{1}{4} \times 1\frac{1}{5} =$ $1\frac{1}{2}$

4. $\frac{3}{5} \times 4\frac{1}{2} =$ $2\frac{7}{10}$

5. $3\frac{1}{3} \times 1\frac{1}{2} =$ **5**

6. $2\frac{1}{4} \times 2\frac{2}{3} =$ **6**

7. Aiden made $3\frac{2}{3}$ boxes of pasta for the baseball team's dinner. They ate only $\frac{1}{3}$ of that amount. How many boxes of pasta did the team eat?

   $1\frac{2}{9}$ boxes

Unit 10 • Multiply Fractions 107

8. **STEM Connection** Saffron used $2\frac{1}{2}$ times more flour than sugar while baking. She used $3\frac{1}{4}$ cups of sugar. How much flour did she use?

   $8\frac{1}{8}$ cups

9. Kayla fills her flowerpots with $3\frac{1}{2}$ quarts of potting soil. Leon has $2\frac{1}{3}$ times as much soil as Kayla. How much potting soil does Leon have?

   $8\frac{1}{6}$ quarts

10. **Extend Your Thinking** How is decomposing mixed numbers different from decomposing numbers that contain decimals?

    Sample answer: You decompose a mixed number so it is a whole number and then a fraction, while you decompose a number that contains a decimal by place value.

Reflect

How can area models help you represent multiplication of mixed numbers?

Answers may vary.

Math is... Mindset
How has a different perspective helped you with your work today?

108 Lesson 6 • Represent Multiplication of Mixed Numbers

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 3–6 Students may forget to include all partial products when adding to determine the final product. As they add fractions and mixed numbers, encourage them to use their area models to ensure they have included each partial product.

Practice Item Analysis

Item	DOK	Rigor
1–6	1	Procedural Skill & Fluency
7–9	2	Application
10	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can area models help you represent multiplication of mixed numbers?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How has a different perspective helped you with your work today?

Students reflect on how they practiced social awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can use an area model to represent multiplication of mixed numbers.
- I can use an area model to find partial products when multiplying mixed numbers.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Represent multiplication of mixed numbers	5.NF.B.4.a
2	2	Represent multiplication of mixed numbers	5.NF.B.4.a

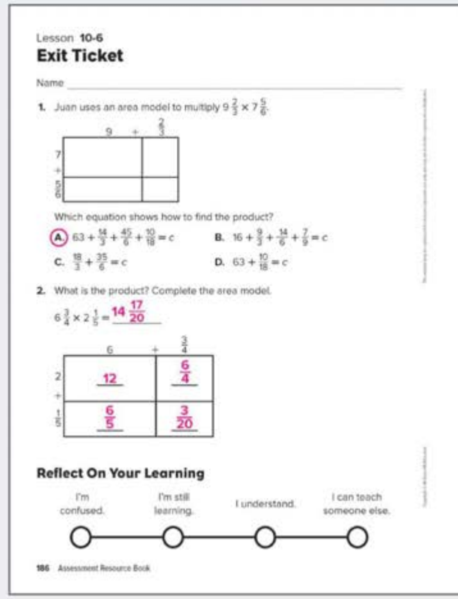
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
2 of 2	Additional Practice or any of the B or E activities
1 of 2	*Take Another Look* or any of the B activities
0 of 2	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 10-6

Exit Ticket

Name ____________

1. Juan uses an area model to multiply $9\frac{2}{3}\times 7\frac{5}{6}$.

Which equation shows how to find the product?

A. $63+\frac{14}{3}+\frac{45}{6}+\frac{10}{18}=c$ B. $16+\frac{9}{3}+\frac{14}{6}+\frac{7}{9}=c$

C. $\frac{18}{3}+\frac{35}{6}=c$ D. $63+\frac{10}{18}=c$

2. What is the product? Complete the area model.

$6\frac{3}{4}\times 2\frac{1}{5}=14\frac{17}{20}$

	6	$\frac{3}{4}$
2	12	$\frac{6}{4}$
$\frac{1}{5}$	$\frac{6}{5}$	$\frac{3}{20}$

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

186 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Multiply Mixed Numbers

Work with students in pairs. Students fill in a blank spinner with mixed numbers with denominators of 2, 3, or 4. Each student spins the spinner once and multiplies the two mixed numbers using an area model and partial products. Students compare their results. If students have difficulty, help them break each mixed number into a whole number and a fraction, then multiply fractions and whole numbers to find the partial products.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Multiply Fractions/Mixed Numbers (Model)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 117

Lesson 10-6 • Reinforce Understanding

Represent Multiplication of Mixed Numbers

Name ______________________

Review

You can use an area model and partial products to represent the multiplication of mixed numbers.

Consider the product $2\frac{1}{3} \times 2\frac{1}{4}$ units.

	2	$\frac{1}{4}$
2	$2 \times 2 = 4$	$2 \times \frac{1}{4} = \frac{2}{4}$
$\frac{1}{3}$	$\frac{1}{3} \times 2 = \frac{2}{3}$	$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

The product of $2\frac{1}{3} \times 2\frac{1}{4}$ is $5\frac{1}{4}$.

Add the partial products to determine the product:

$4 + \frac{2}{4} + \frac{2}{3} + \frac{1}{12}$

$= 4 + \frac{6}{12} + \frac{8}{12} + \frac{1}{12}$

$= 4 + \frac{15}{12}$

$= 4 + 1\frac{3}{12}$

$= 5\frac{3}{12}$ or $5\frac{1}{4}$

What is the product? Complete the area model to show your work.

1. $1\frac{1}{4} \times 4\frac{3}{4} = 5\frac{15}{16}$

	4	$\frac{3}{4}$
1	4	$\frac{3}{4} = \frac{12}{16}$
$\frac{1}{4}$	1	$\frac{3}{16}$

2. $2\frac{1}{6} \times 3\frac{1}{2} = 7\frac{7}{12}$

	3	$\frac{1}{2}$
2	6	1
$\frac{1}{6}$	$\frac{1}{2} = \frac{6}{12}$	$\frac{1}{12}$

3. $6 \times 5\frac{1}{4} = 31\frac{1}{2}$

	5	$\frac{1}{4}$
6	30	$\frac{6}{4} = 1\frac{1}{2}$

4. $2 \times 3\frac{4}{5} = 7\frac{3}{5}$

	3	$\frac{4}{5}$
2	6	$\frac{8}{5} = 1\frac{3}{5}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Mixed Number Concentration

Students practice multiplying mixed numbers and fractions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 117-118

Lesson 10-6

Additional Practice

Name ______________________

Review

You can use an area model to multiply mixed numbers.

Find the product $2\frac{1}{3} \times 3\frac{3}{4}$.

Use an area model.

	3	+ $\frac{3}{4}$
2	6	$\frac{6}{4}$
+ $\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{12}$

Write each mixed number as a sum. Write the product, or area, in each smaller rectangle.

Add the four partial products: $6 + 1 + \frac{6}{4} + \frac{3}{12}$

Add the whole numbers: $6 + 1 = 7$

Add the fractions: $\frac{6}{4} + \frac{3}{12} = \frac{18}{12} + \frac{3}{12} = \frac{21}{12} = 1\frac{9}{12}$ or $1\frac{3}{4}$

Add the whole numbers and fractions: $7 + 1\frac{3}{4} = 8\frac{3}{4}$

So, $2\frac{1}{3} \times 3\frac{3}{4} = 8\frac{3}{4}$.

What is the product? Complete the area model.

1. $1\frac{2}{3} \times 1\frac{4}{5} = 3$

	1	+ $\frac{4}{5}$
1	1	$\frac{4}{5}$
+ $\frac{2}{3}$	$\frac{2}{3}$	$\frac{8}{15}$

2. $2\frac{5}{6} \times 3\frac{1}{5} = 9\frac{1}{15}$

	3	+ $\frac{1}{5}$
2	6	$\frac{2}{5}$
+ $\frac{5}{6}$	$\frac{15}{6}$	$\frac{5}{30}$

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Assign

Use It! Application Station

Fraction of a Fraction Students use fractions to follow and change a recipe.

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

GO ONLINE

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 117-118

What is the product?

3. $2\frac{5}{6} \times 3\frac{1}{4} =$ $9\frac{5}{24}$

4. $1\frac{3}{5} \times 8\frac{1}{2} =$ $13\frac{3}{5}$

5. $6\frac{1}{3} \times 5\frac{5}{8} =$ $35\frac{5}{8}$

6. $2\frac{1}{10} \times 4\frac{3}{4} =$ $9\frac{39}{40}$

7. Greg walked $2\frac{2}{5}$ miles yesterday. Today he walked $2\frac{1}{2}$ times as far. How many miles did Greg walk today?

6 miles

8. Giselle's bedroom wall is $10\frac{3}{4}$ feet long and $8\frac{1}{4}$ feet tall. What is the area of the wall?

$88\frac{11}{16}$ square feet

Math @ Home Activity With your child, create a set of 9 number cards, each with one of the digits 1 through 9. Turn them facedown and have your child select 6 of the cards. Use the digits to form two mixed numbers. Use an area model to multiply the mixed numbers. Try to get a product that is as great as possible and then rearrange the numbers to get a product that is as low as possible. Then replace the cards and repeat the activity.

Student Practice Book

Differentiation Resource Book, p. 118

Lesson 10-6 • Extend Thinking

Represent Multiplication of Mixed Numbers

Name ____________________

What equation can be written from the area model? Complete the area model to write the equation.

1. ____ × ____ = ____

$2\frac{1}{3} \times 3\frac{5}{6} = 8\frac{17}{18}$

	3	$\frac{5}{6}$
2	6	$\frac{10}{6}$
$\frac{1}{3}$	1	$\frac{5}{18}$

2. ____ × ____ = ____

$5\frac{3}{4} \times 1\frac{1}{2} = 8\frac{5}{8}$

	1	$\frac{1}{2}$
5	5	$\frac{5}{2}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{8}$

3. ____ × ____ = ____

$6 \times 3\frac{5}{8} = 21\frac{3}{4}$

	3	$\frac{5}{8}$
6	18	$\frac{30}{8}$

4. ____ × ____ = ____

$1\frac{4}{5} \times 5\frac{3}{7} = 9\frac{27}{35}$

	5	$\frac{3}{7}$
1	5	$\frac{3}{7}$
$\frac{4}{5}$	4	$\frac{12}{35}$

5. ____ × ____ = ____

$2\frac{1}{2} \times 4\frac{2}{7} = 10\frac{5}{7}$

	4	$\frac{2}{7}$
2	8	$\frac{4}{7}$
$\frac{1}{2}$	2	$\frac{2}{14}$

6. ____ × ____ = ____

$2 \times 3\frac{7}{9} = 7\frac{5}{9}$

	3	$\frac{7}{9}$
2	6	$\frac{14}{9}$

Differentiation Resource Book

LESSON **10-7**

Multiply Mixed Numbers

Learning Targets

- I can use partial products to multiply mixed numbers.
- I can multiply mixed numbers by writing the mixed numbers as fractions and then multiplying fractions.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

◇ **5.NF.B.4.a** Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$).

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
decompose	accurate
partial products	transition

Material

The materials may be for any part of the lesson.

- grid paper

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use partial products to multiply mixed numbers by decomposing the mixed number into a whole number part and a fractional part. • Students write mixed numbers as fractions, then multiply two fractions to find the product.	• Students discuss multiplying mixed numbers using the verb *find*. • To support optimizing output, ELs participate in MLR7: Compare and Connect.	• Students exchange ideas for completing a mathematical task with a peer and reflect on the value of their similarities and differences.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students multiplied mixed numbers using area models and partial products (Unit 10).	• Students multiply mixed numbers using equations and partial products.	• Students interpret multiplication as scaling (Unit 10). • Students interpret and compute quotients of fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding of multiplying mixed numbers as they relate visual representations to equations.	• Students build proficiency for multiplying mixed numbers by using multiple strategies.	• Students solve word problems. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Which Benchmark Is It Closest To? 5–7 min

Build Fluency Students build number sense as they decide which benchmarks fractions are closest to.

Remind students that exact positioning is not necessary, but that they should decide between which benchmarks the fractions belong and approximately how close to the benchmarks each fraction would be.

These prompts encourage students to talk about their reasoning:

- How can benchmarks help you compare fractions?
- How did you know where to place the dot along the number line?

Purpose Students consider story contexts that involve multiplying a mixed number by a mixed number.

Numberless Word Problem

- What is the question?

Teaching Tip You may want to have students write down their own answers and thoughts about what questions could be asked using the information given before having them share with the class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of choosing strategies to solve word problems involving multiplying a mixed number by a mixed number and are based on possible comments and questions that students may make during the share out.

- How would the question be different if you knew the amounts of soil?
- What kinds on numbers could be used to show the amount of soil they shoveled?

Math is... Mindset

- What are some ways to build a positive relationship with classmates?

SEL Relationship Skills: Build Relationships

As students engage in collaborative discourse around the Numberless Word Problem routine, remind them that building relationships with others is an important part of being an effective and respectful communicator. Explain that one way to do this is by listening attentively when others are sharing their ideas about what question the numberless word problem is asking.

Transition to Explore & Develop

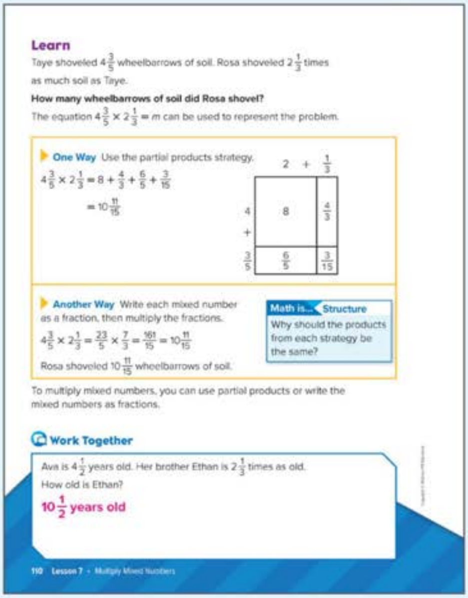
Ask questions that get students thinking about how they would answer the question they posed if the problem involved amounts of soil that were mixed numbers.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can multiply mixed numbers.

Explore & Develop

20 min



Learn

Taye shoveled $4\frac{3}{5}$ wheelbarrows of soil. Rosa shoveled $2\frac{1}{3}$ times as much soil as Taye.

How many wheelbarrows of soil did Rosa shovel?

The equation $4\frac{3}{5} \times 2\frac{1}{3} = m$ can be used to represent the problem.

One Way Use the partial products strategy.

$4\frac{3}{5} \times 2\frac{1}{3} = 8 + \frac{4}{3} + \frac{6}{5} + \frac{3}{15}$

$= 10\frac{11}{15}$

	2	$\frac{1}{3}$
4	8	$\frac{4}{3}$
$\frac{3}{5}$	$\frac{6}{5}$	$\frac{3}{15}$

Another Way Write each mixed number as a fraction, then multiply the fractions.

$4\frac{3}{5} \times 2\frac{1}{3} = \frac{23}{5} \times \frac{7}{3} = \frac{161}{15} = 10\frac{11}{15}$

Rosa shoveled $10\frac{11}{15}$ wheelbarrows of soil.

Math is... Structure
Why should the products from each strategy be the same?

To multiply mixed numbers, you can use partial products or write the mixed numbers as fractions.

Work Together

Ava is $4\frac{1}{2}$ years old. Her brother Ethan is $2\frac{1}{3}$ times as old. How old is Ethan?

$10\frac{1}{2}$ years old

110 Lesson 7 • Multiply Mixed Numbers

1 Pose the Problem

ETP Pose Purposeful Questions

- What are the important quantities in this problem?
- What repesentations do you know that might help you make sense of those quantities?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and assign a problem like the one on the Learn page. Instruct one student to use an area model and partial products to multiply and the other to write the mixed numbers as fractions. Then have them compare their strategies. Revisit this routine throughout the lesson as needed.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- Which strategy for multiplying mixed numbers do you find more efficient? Why?

Key Takeaways

- Partial products can be used to multiply mixed numbers by decomposing the mixed number into a whole number part and a fractional part.
- Mixed numbers can also be written as fractions, then multiply two fractions to find the product.

Work Together

Students solve a word problem involving the multiplication of mixed numbers. Encourage students to use both an area model as well as writing the mixed numbers as fractions to solve.

Common Error: Students may mistakenly multiply the mixed numbers before renaming them as fractions, for example multiplying 4×2 and $\frac{1}{2} \times \frac{1}{3}$ to get $8\frac{1}{6}$. Make sure students understand that neither strategy they have studied in this lesson works that way.

LOM Language of Math

Make sure that students are precise as they discuss mixed numbers and fractions. A mixed number is not a fraction. A mixed number is wrtitten using a whole number part and a fraction part. Similarly, a fraction greater than 1, if it is not written using a whole number part, is called a fraction, not a mixed number.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore different strategies for multiplying two mixed numbers.

Directions: Present students with these equations:

$1\frac{1}{2} \times 2\frac{3}{4} = 4\frac{1}{8}$ $\quad \frac{3}{2} \times \frac{11}{4} = \frac{33}{8}$

Display these questions for students to consider as they analyze the equations:

- Is the product for each equation correct? How do I know?
- What is the same about these equations?
- What is different about these equations?
- How do the quantities in these equations relate?

ETP Support Productive Struggle

- How can you multiply two mixed numbers?
- How can you write a fraction that is equivalent to a mixed number?
- How can you multiply two fractions?

Math is... Structure

- Why should the products from each strategy be the same?

Students step back for an overview and understand that, if strategies work, they must all yield the same results.

Activity Debrief: Have groups share what they discovered when analyzing these equations. Facilitate a discussion that there are two strategies that students can use to multiply two mixed numbers. One is using an area model to find partial products. Another is writing each mixed number as an equivalent fraction and multiplying the two fractions.

Have students revisit the Pose the Problem question and discuss answers.

- Taye shoveled $4\frac{3}{5}$ wheelbarrows of soil. Rosa shoveled $2\frac{1}{3}$ times as much soil as Taye. How many wheelbarrows of soil did Rosa shovel?

Guided Exploration

Students transition from using area models to multiply mixed numbers to writing mixed numbers as fractions and then multiplying.

ETP Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students determine the partial products. Ask:

- How will you decompose the factors? Why?
- What multiplication expression can you use to represent the area of each region of the area model?
- What is the partial product for each region?
- How will you add the partial products?
- **Think About It:** Why will writing each mixed number as a fraction let you solve this equation?

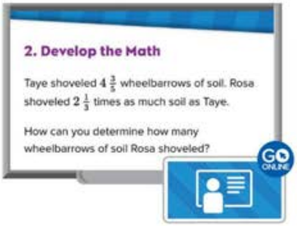
Have the students rewrite the mixed numbers as fractions. Ask:

- How will you rewrite the mixed numbers as fractions? Why does your method work?
- **Think About It:** What is a shortcut for multiplying fractions?

Math is... Structure

- Why should the products from each strategy be the same?

Students step back for an overview and understand that, if strategies work, they must all yield the same results.



EL English Learner Scaffolds

Entering/Emerging Write a multiplication problem with mixed numbers on the board. Then say *I can either use an area model and partial products* (demonstrate this) *or write the mixed number as fractions* (demonstrate this too) *to multiply mixed numbers* (point back to the multiplication problem). Guide students to this same sentence on the Learn page, pointing to what each part refers to on the page.

Developing/Expanding Write a multiplication problem with mixed numbers on the board. Then say *I can either use an area model and partial products* (demonstrate this) *or write the mixed number as fractions* (demonstrate this too) *to multiply mixed numbers* (point back to the multiplication problem). Guide students to this same sentence on the Learn page and have them point to what each part refers to on the page.

Bridging/Reaching Guide students to the Learn page and point them to the sentence *To multiply mixed numbers, you* can either *use...* Ask them to restate the meaning of the sentence (*You have a choice of doing it this one way or this other way*.). Validate or correct as necessary and then ask students to present two ways of solving a math problem using the structure *You can either* _____ *or* _____.

Practice & Reflect 10 min

On My Own

Name ______________________

What is the product?

1. $2\frac{3}{5} \times 1\frac{1}{3} =$ $3\frac{7}{15}$
2. $1\frac{3}{4} \times 3\frac{1}{3} =$ $5\frac{5}{6}$
3. $2\frac{2}{4} \times 3\frac{1}{2} =$ $8\frac{3}{4}$
4. $1\frac{1}{3} \times 4\frac{2}{3} =$ $6\frac{2}{9}$
5. $3\frac{3}{4} \times 2\frac{1}{2} =$ $9\frac{3}{8}$
6. $2\frac{2}{5} \times 4\frac{1}{4} =$ $10\frac{1}{5}$
7. $5\frac{1}{8} \times 2\frac{3}{4} =$ $14\frac{3}{32}$
8. $2\frac{1}{4} \times 3\frac{3}{5} =$ $8\frac{1}{10}$

9. The weight of Natalie's backpack is shown. Her brother's backpack weighs $2\frac{1}{4}$ times that much. How much does Natalie's brother's backpack weigh?
15 pounds

10. The street Michelle lives on is $1\frac{1}{2}$ miles long. The street Lucas lives on is $1\frac{2}{5}$ times as long as Michelle's street. How long is the street Lucas lives on?
$2\frac{1}{10}$ miles

11. Benson bought this much dog food last week. This week he bought $2\frac{1}{3}$ times as much as last week. How many pounds of dog food did Benson buy this week?
$8\frac{1}{6}$ pounds

12. A rectangle has a length of $1\frac{1}{3}$ yards and a width of $5\frac{1}{4}$ yards. What is the area of the rectangle?
7 square yards

13. **Error Analysis** Bernardo solved the following problem. Did Bernardo multiply correctly? Explain why or why not.
$5\frac{1}{2} \times \frac{2}{3} = 5\frac{2}{6}$
No; he multiplied only the fractions and did not multiply the whole number and $\frac{2}{3}$; the correct answer is $3\frac{2}{3}$.

14. **Extend Your Thinking** Will the product of mixed numbers always be greater than the factors? How do you know?
Yes. Sample answer: mixed numbers include whole numbers that are greater than 1, so when factors that are greater than 1 are multiplied by each other, the product will be greater than either factor.

Reflect

How can you multiply mixed numbers using fractions?
Answers may vary.

Math is... Mindset
What helped you build a positive relationship with classmates?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–8 If students multiply the mixed numbers by rewriting them as fractions, make sure students remain adept at converting mixed numbers into fractions greater than 1. The conversion can be a source of "cascading" calculation errors.

Practice Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–12	2	Application
13–14	3	Conceptual Understanding

Reflect

Students complete the Reflect question.
- How can you multiply mixed numbers using fractions?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you build a positive relationship with classmates?

Students reflect on how they developed stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.
- I can use partial products to multiply mixed numbers.
- I can multiply mixed numbers by writing the mixed numbers as fractions and then multiplying fractions.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Multiply mixed numbers	5.NF.B.4.a
2	1	Multiply mixed numbers	5.NF.B.4.a
3	2	Multiply mixed numbers	5.NF.B.4.a

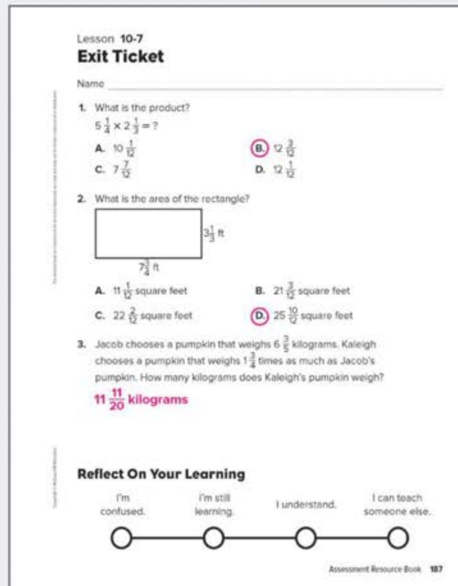
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking



Lesson 10-7
Exit Ticket

Name

1. What is the product?
$5\frac{1}{4} \times 2\frac{1}{3} = ?$
A. $10\frac{1}{12}$
B. $12\frac{3}{12}$
C. $7\frac{7}{12}$
D. $12\frac{1}{12}$

2. What is the area of the rectangle?
$3\frac{1}{3}$ ft
$7\frac{3}{4}$ ft
A. $11\frac{1}{12}$ square feet
B. $21\frac{3}{12}$ square feet
C. $22\frac{2}{12}$ square feet
D. $25\frac{10}{12}$ square feet

3. Jacob chooses a pumpkin that weighs $6\frac{3}{5}$ kilograms. Kaleigh chooses a pumpkin that weighs $1\frac{3}{4}$ times as much as Jacob's pumpkin. How many kilograms does Kaleigh's pumpkin weigh?
$11\frac{11}{20}$ kilograms

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 187

R Reinforce Understanding

SMALL GROUP

Mixed Numbers Multiplication

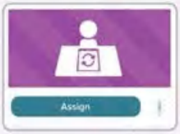
Work with students in pairs. Each partner should write a mixed number, then both partners write a multiplication equation with the mixed numbers. Partners should solve using area models and partial products, and then should solve by writing the mixed numbers as equivalent fractions. Encourage students to discuss which strategy they prefer.

GO ONLINE

Take Another Look Lessons

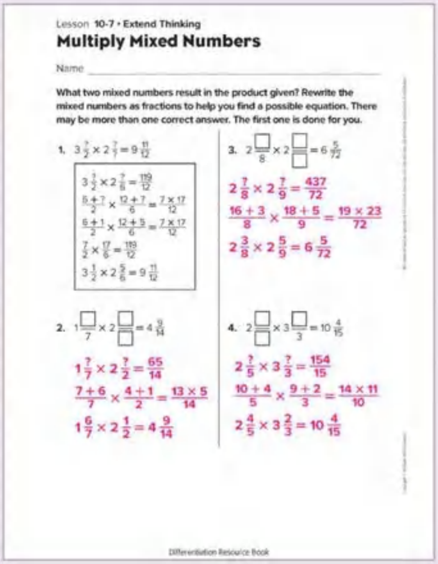
Assign the interactive lessons to reinforce targeted skills.

- Multiplication of Mixed Numbers
- Multiply Mixed Numbers
- Multiply Fractions/Mixed Numbers



INDEPENDENT WORK

Differentiation Resource Book, p. 119



Lesson 10-7 • Extend Thinking

Multiply Mixed Numbers

Name ____________________

What two mixed numbers result in the product given? Rewrite the mixed numbers as fractions to help you find a possible equation. There may be more than one correct answer. The first one is done for you.

1. $3\frac{?}{2}\times 2\frac{?}{?}=9\frac{11}{12}$

$3\frac{?}{2}\times 2\frac{?}{6}=\frac{119}{12}$
$\frac{6+?}{2}\times\frac{12+?}{6}=\frac{7\times 17}{12}$
$\frac{6+1}{2}\times\frac{12+5}{6}=\frac{7\times 17}{12}$
$\frac{7}{2}\times\frac{17}{6}=\frac{119}{12}$
$3\frac{1}{2}\times 2\frac{5}{6}=9\frac{11}{12}$

2. $1\frac{\square}{7}\times 2\frac{\square}{\square}=4\frac{9}{14}$

$1\frac{?}{7}\times 2\frac{?}{2}=\frac{65}{14}$
$\frac{7+6}{7}\times\frac{4+1}{2}=\frac{13\times 5}{14}$
$1\frac{6}{7}\times 2\frac{1}{2}=4\frac{9}{14}$

3. $2\frac{\square}{8}\times 2\frac{\square}{\square}=6\frac{5}{72}$

$2\frac{?}{8}\times 2\frac{?}{9}=\frac{437}{72}$
$\frac{16+3}{8}\times\frac{18+5}{9}=\frac{19\times 23}{72}$
$2\frac{3}{8}\times 2\frac{5}{9}=6\frac{5}{72}$

4. $2\frac{\square}{\square}\times 3\frac{\square}{3}=10\frac{4}{15}$

$2\frac{?}{5}\times 3\frac{?}{3}=\frac{154}{15}$
$\frac{10+4}{5}\times\frac{9+2}{3}=\frac{14\times 11}{10}$
$2\frac{4}{5}\times 3\frac{2}{3}=10\frac{4}{15}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Area with Fractions Task Cards

Students practice multiplying mixed numbers to find area.

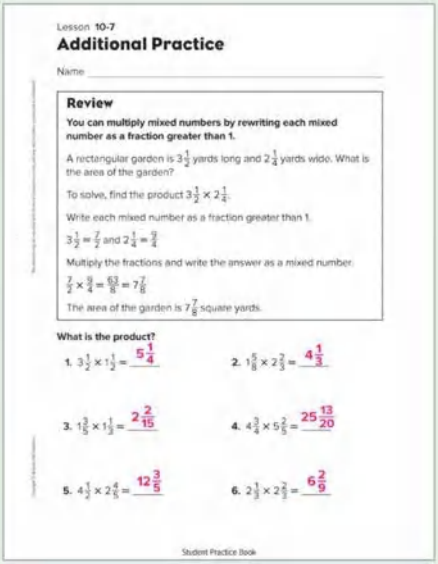
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 119-120



Lesson 10-7

Additional Practice

Name ____________________

Review

You can multiply mixed numbers by rewriting each mixed number as a fraction greater than 1.

A rectangular garden is $3\frac{1}{2}$ yards long and $2\frac{1}{4}$ yards wide. What is the area of the garden?

To solve, find the product $3\frac{1}{2}\times 2\frac{1}{4}$.

Write each mixed number as a fraction greater than 1.

$3\frac{1}{2}=\frac{7}{2}$ and $2\frac{1}{4}=\frac{9}{4}$

Multiply the fractions and write the answer as a mixed number.

$\frac{7}{2}\times\frac{9}{4}=\frac{63}{8}=7\frac{7}{8}$

The area of the garden is $7\frac{7}{8}$ square yards.

What is the product?

1. $3\frac{1}{2}\times 1\frac{1}{2}=$ $5\frac{1}{4}$
2. $1\frac{5}{8}\times 2\frac{2}{3}=$ $4\frac{1}{3}$
3. $1\frac{3}{5}\times 1\frac{1}{3}=$ $2\frac{2}{15}$
4. $4\frac{3}{4}\times 5\frac{2}{5}=$ $25\frac{13}{20}$
5. $4\frac{1}{2}\times 2\frac{4}{5}=$ $12\frac{3}{5}$
6. $2\frac{1}{3}\times 2\frac{2}{3}=$ $6\frac{2}{9}$

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Assign

Use It! Application Station

If, Then Students use if/then statements to write a problem in which they make a true or false statement that uses multiplication of fractions.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 119–120

7. Walter ran $3\frac{2}{5}$ miles yesterday. Today he ran $1\frac{1}{2}$ times as far. How many miles did Walt run today?

$5\frac{1}{10}$ miles

8. Wanda bought a plant that was $1\frac{1}{4}$ inches tall. After two weeks, the plant was $2\frac{1}{2}$ times as tall. How tall was the plant after two weeks?

$3\frac{1}{8}$ inches

9. Jodie's backpack weighs $2\frac{3}{8}$ pounds. Jeff's backpack weighs $1\frac{3}{4}$ times as much as Jodie's backpack. How much does Jeff's backpack weigh?

$4\frac{5}{32}$ pounds

10. Kyla lives $1\frac{2}{3}$ miles from the park. The library is $2\frac{1}{3}$ times as far from Kyla's house. How far is the library from Kyla's house?

$3\frac{8}{9}$ miles

Math @Home Activity With your child, create a set of 9 number cards, each with one of the digits 1 through 9. Turn them facedown and have your child select 6 of the cards. Use the digits to form two mixed numbers. Write the mixed numbers as fractions greater than 1 and multiply. Try to get a product that is as great as possible and then rearrange the numbers to get a product that is as low as possible. Then replace the cards and repeat the activity.

Student Practice Book

Differentiation Resource Book, p. 120

Lesson 10-7 • Extend Thinking

Multiply Mixed Numbers

Name ______________________

What two mixed numbers result in the product given? Rewrite the mixed numbers as fractions to help you find a possible equation. There may be more than one correct answer. The first one is done for you.

1. $3\frac{?}{2} \times 2\frac{?}{?} = 9\frac{11}{12}$

$3\frac{?}{2} \times 2\frac{?}{6} = \frac{119}{12}$

$\frac{6+?}{2} \times \frac{12+?}{6} = \frac{7 \times 17}{12}$

$\frac{6+1}{2} \times \frac{12+5}{6} = \frac{7 \times 17}{12}$

$\frac{7}{2} \times \frac{17}{6} = \frac{119}{12}$

$3\frac{1}{2} \times 2\frac{5}{6} = 9\frac{11}{12}$

2. $1\frac{\square}{7} \times 2\frac{\square}{\square} = 4\frac{9}{14}$

$1\frac{?}{7} \times 2\frac{?}{2} = \frac{65}{14}$

$\frac{7+6}{7} \times \frac{4+1}{2} = \frac{13 \times 5}{14}$

$1\frac{6}{7} \times 2\frac{1}{2} = 4\frac{9}{14}$

3. $2\frac{\square}{8} \times 2\frac{\square}{\square} = 6\frac{5}{72}$

$2\frac{?}{8} \times 2\frac{?}{9} = \frac{437}{72}$

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$2\frac{?}{5} \times 3\frac{?}{3} = \frac{154}{15}$

$\frac{10+4}{5} \times \frac{9+2}{3} = \frac{14 \times 11}{10}$

$2\frac{4}{5} \times 3\frac{2}{3} = 10\frac{4}{15}$

Differentiation Resource Book

LESSON **10-8**

Multiplication as Scaling

Learning Targets

- I can explain how the size of the factors impacts the size of the product without performing the multiplication.
- I can explain why the product of a given number and a fraction greater than 1 is greater than the given number and why the product of a given number and a fraction less than 1 is less than the given number.

Vocabulary

Math Term	Academic Terms
scaling	complex
	infer

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.5** Interpret multiplication as scaling (resizing), by:

◇ **5.NF.B.5.a** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

◇ **5.NF.B.5.b** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

Math Practices and Processes

MPP Model with mathematics.

Materials

The materials may be for any part of the lesson.

- index cards

Focus

Content Objectives	Language Objectives	SEL Objective
• Students explain how the size of the factors impacts the size of the product without performing the multiplication. • Students explain how the product of a given number and a fraction is related to the size of the fraction.	• Students explain why the product of a given number and a fraction is related to the size of the fraction. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students discover and discuss personal interests related to mathematics and share these interests with peers.

Coherence

Previous	Now	Next
• Students interpreted multiplication as a comparison (Grade 4). • Students multiplied mixed numbers using equations and partial products (Unit 10).	• Students interpret multiplication as scaling.	• Students solve problems involving fractions (Unit 10). • Students understand ratio concepts and language (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding of multiplication as they determine how the size of one factor impacts the size of the product relative to the other factor.	• Students build proficiency interpreting multiplication. *Procedural Skill & Fluency is not a targeted element of rigor for this standard.*	• Students interpret multiplication with fractions and mixed numbers in real-world contexts. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Greater Than or Less Than

5–7 min

Build Fluency Students build number sense as they determine whether an addition expression is greater than or less than the benchmark of 10,000.

Remind students that this is a mental activity, and that exact answers are not needed.

These prompts encourage students to talk about their reasoning:

- How did you determine the value of each expression?
- How did you know that your estimates were reasonable?
- Who thought about it in a different way? What is another way to think about the estimate? What is the value of each expression?

Sense-Making Routine

Purpose Students see that the size of the factors impacts the size of their product, in preparation for justifying and understanding why this is so.

Notice & Wonder

- How are they the same?
- How are they different?

Teaching Tip You may want to have students discuss what they notice and wonder about the equation in pairs before discussing as a whole class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of how the size of the factors impacts the size of the product in a multiplication equation and are based on possible comments and questions that students may make during the share out.

- What changes in the equations? What stays the same?
- Why do you think the equations on the right have products that are greater than the equations on the left?

Math is... Mindset

- What makes you feel excited when doing math?

SEL Self-Awareness: Identify Emotions

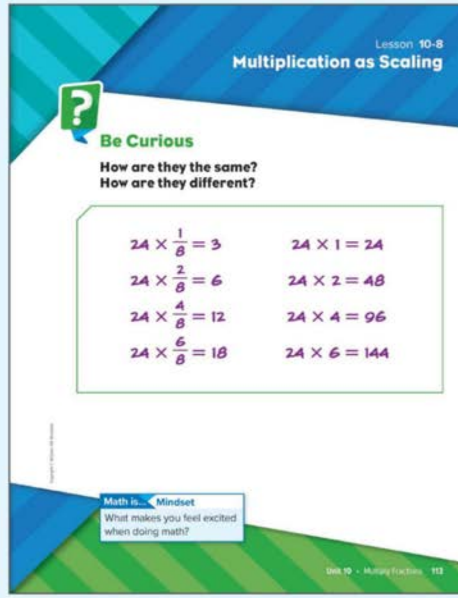
Give students opportunities to share about themselves to reinforce their sense of identity and their emotions. As students work collaboratively to complete the Notice & Wonder routine, invite them to share an emotion they feel related to math. Encourage them to think about how that emotion can help them with their work as they understand multiplication as scaling.

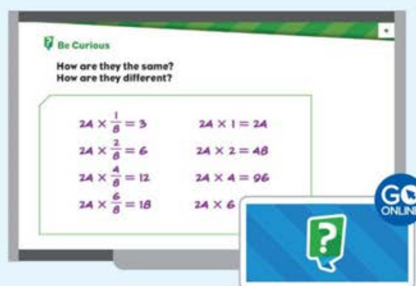
Transition to Explore & Develop

Ask questions that get students thinking about multiplying fractions that are less than 1 and multiplying fractions that are greater than 1 by whole numbers.

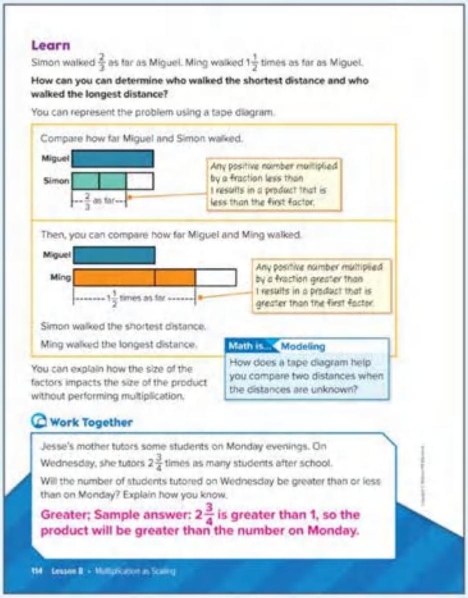
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how our knowledge of multiplying fractions can help us explain how the size of factors impacts the size of their product.





Explore & Develop 20 min



Learn

Simon walked $\frac{2}{3}$ as far as Miguel. Ming walked $1\frac{1}{2}$ times as far as Miguel.

How can you can determine who walked the shortest distance and who walked the longest distance?

You can represent the problem using a tape diagram.

Compare how far Miguel and Simon walked.

Then, you can compare how far Miguel and Ming walked.

Simon walked the shortest distance.

Ming walked the longest distance.

You can explain how the size of the factors impacts the size of the product without performing multiplication.

Math is... Modeling
How does a tape diagram help you compare two distances when the distances are unknown?

Work Together

Jesse's mother tutors some students on Monday evenings. On Wednesday, she tutors $2\frac{3}{4}$ times as many students after school.

Will the number of students tutored on Wednesday be greater than or less than on Monday? Explain how you know.

Greater; Sample answer: $2\frac{3}{4}$ is greater than 1, so the product will be greater than the number on Monday.

114 Lesson 8 • Multiplication as Scaling

1 Pose the Problem

ETP Pose Purposeful Questions

- What are you trying to determine?
- Do you know how far Miguel walked?
- How can the information given help you solve the problem?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about how to solve the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you use what you know about fractions to explain what the size of a product will be?

Key Takeaways

- How the size of the factors impacts the size of the product can be described without performing the multiplication.
- The product of a given number and a fraction greater than than 1 results in a product greater than the given number, and the product of a given number and a fraction less than 1 results in a product less than the given number.

Work Together

Students solve a word problem involving scaling. Encourage students to use a tape diagram to represent the problem.

Common Misconception: Students may think that they need to know how many students Jesse's mother tutors on Monday in order to know if there are more students on Monday or Wednesday. Remind students that they can determine the answer using only one factor, by thinking about if that factor is greater than or less than 1.

LOM Language of Math

Students may be familiar with scale models. A scale model has a scale. For example, some scale model cars are $\frac{1}{18}$th scale. If the real car is 18 feet long, the scale model car will be $18 \times \frac{1}{18} = 1$ foot long. Models of small objects will have a scale greater than 1 to make their small details easily seen.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how the size of factors impacts the size of a product in a multiplication equation.

Directions: Have students work together to solve the Pose the Problem.

ETP **Implement Tasks that Promote Reasoning and Problem Solving**

- What do you know about the problem?
- What questions can you ask about the problem before looking to solve it?
- How do the values and quantities in the problem relate?
- How are you approaching the problem?
- What assumptions can you make to help you solve the problem?

Math is... Modeling

- How can a representation help you compare two quantities when the quantities are unknown?

Students are considering if the representation has or has not served its purpose.

Activity Debrief: Have groups share how they worked through the problem and what they discovered about how the size of the factors impacts the size of the products in a multiplication equation.

Guided Exploration

Students learn how to explain the size of a product using the size of factors in a multiplication equation.

ETP **Facilitate Meaningful Mathematical Discourse**

Ask students to share their generalizations about the comparison of Miguel's and Simon's distances. Make sure they justify their conclusions, communicate them to others, and respond to the arguments of others. Most importantly make sure they ask useful questions to clarify or improve each other's arguments. Ask:

- How can you check if your generalization is true?

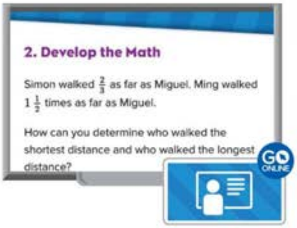
Ask students to share their generalizations about the comparison of Miguel's and Ming's distances. Make sure they justify their conclusions, communicate them to others, and respond to the arguments of others. Most importantly make sure they ask useful questions to clarify or improve each other's arguments. Ask:

- How can you check if your generalization is true?
- **Think About It:** Why were you able to solve the problem without multiplying?

Math is... Modeling

- How does a tape diagram help you compare two distances when the distances are unknown?

Students are considering if the representation has or has not served its purpose.



EL **English Learner Scaffolds**

Entering/Emerging Ensure understanding of *yield*. Write a multiplication problem involving a number multiplied by a fraction less than 1. While pointing, say *Any number multiplied by a fraction less than 1 yields a product that is less than the first factor*. Repeat with a number multiplied by a fraction greater than 1. Then instruct students to go to the Learn page and find the 2 instances of *yields*.

Developing/Expanding Ensure understanding of *yield*. Write a multiplication problem involving a number multiplied by a fraction less than 1. While pointing, say *Any number multiplied by a fraction less than 1 yields a product that is less than the first factor*. Repeat with a number multiplied by a fraction greater than 1. Then instruct students to go to the Learn page and read the sentences with *yields*, pointing to what each part is referring to on the page.

Bridging/Reaching Guide students to the Learn page and have them focus on the sentences that use *yields*. Ask them to tell you what they think it means (*produces, results in, etc*.). Then ask them to use *yield* in their own sentence. Allow students to interject, agreeing or disagreeing and providing correction when necessary. For example, *I don't think you used* yield *correctly*. or *Are you sure that results in...*.

Practice & Reflect 10 min

On My Own

Name ______________________

1. Which fraction will result in a product that is greater than $\frac{3}{4}$?
   $\frac{3}{4} \times$ ____
   A. $\frac{1}{3}$
   B. $\frac{1}{2}$
   C. $\frac{5}{8}$
   D. $\frac{5}{4}$

2. Which fraction will result in a product that is less than $\frac{8}{7}$?
   $\frac{8}{7} \times$ ____
   A. $\frac{5}{7}$
   B. $\frac{12}{6}$
   C. $\frac{10}{7}$
   D. $\frac{8}{6}$

3. Which expression has a product that is less than the first factor? Select all that apply.
   A. $42 \times \frac{1}{2}$
   B. $35 \times \frac{2}{1}$
   C. $78 \times \frac{1}{5}$
   D. $26 \times \frac{4}{5}$

4. Which expression has a product that is greater than the second factor? Select all that apply.
   A. $\frac{3}{4} \times \frac{2}{1}$
   B. $\frac{2}{1} \times 75$
   C. $26 \times \frac{3}{2}$
   D. $\frac{9}{10} \times 5$

5. In a cross-country race, Duante took $\frac{8}{9}$ as long as James to finish. Kofi took $\frac{7}{8}$ as long as James to finish. Order their race times from least to greatest. **Kofi, Duante, James**

6. Jamie is $\frac{4}{5}$ as tall as Harriet. Jenny is $1\frac{2}{5}$ as tall as Harriet. Who is the tallest? How do you know?
   Jenny; Sample answer: Harriet is shorter than Jenny. Jamie is the shortest.

7. **STEM Connection** Hannah is measuring three sheets of metal. Sheet A is $1\frac{1}{2}$ times as long as Sheet B. Sheet C is $2\frac{2}{3}$ times as long as Sheet A. How should Hannah order the sheets from least length to greatest length?
   Sheet B, Sheet A, Sheet C

8. Tyler has three dogs of different sizes: Max, Daisy, and Charlie. Daisy weighs $\frac{1}{3}$ as much as Max. Charlie weighs $\frac{3}{8}$ as much as Daisy. Who weighs the least? How do you know?
   Charlie; Daisy weighs more than Charlie. Max weighs the most.

9. Hugo is organizing his books from shortest to tallest. His math book is $2\frac{1}{4}$ times as tall as his science book. His reading book is $\frac{7}{9}$ as tall as his science book. In what order should Hugo organize his books?
   reading book, science book, math book

10. **Extend Your Thinking** What will happen to the product if a whole number is multiplied by $\frac{8}{8}$?
    The product will have the same value as the other factor because the fraction is equal to 1.

Reflect

How can you explain what the size of a product will be if you know the sizes of the factors?
Answers may vary.

Math is... Mindset
What made you feel excited when doing math?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–4 Students may think that numbers greater than 1 are actually less than 1 because they are written as fractions instead of mixed numbers. Encourage students to notice which fractions could be written as mixed numbers to help them determine whether or not the numbers are greater than 1.

Practice Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill & Fluency
5–9	2	Application
10	3	Conceptual Understanding

Reflect

Students complete the Reflect question.
- How can you explain what the size of a product will be if you know the sizes of the factors?

Ask students to share their reflections with their classmates.

Math is... Mindset
- What made you feel excited when doing math?

Students reflect on how they practiced self-awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.
- I can explain how the size of the factors impacts the size of the product without performing the multiplication.
- I can explain why the product of a given number and a fraction greater than 1 is greater than the given number and why the product of a given number and a fraction less than 1 is less than the given number.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Multiplication as scaling	5.NF.B.5.b
2	1	Multiplication as scaling	5.NF.B.5.b
3	2	Multiplication as scaling	5.NF.B.5.b

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

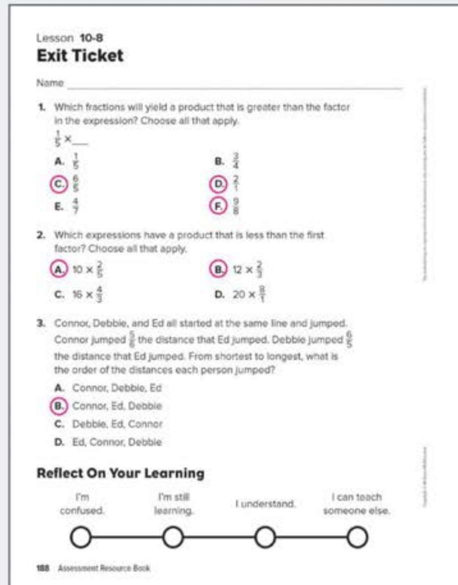
If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 10-8

Exit Ticket

Name ____________________

1. Which fractions will yield a product that is greater than the factor in the expression? Choose all that apply.

   $\frac{1}{5} \times$ ___

   A. $\frac{1}{5}$ B. $\frac{3}{4}$

   C. $\frac{6}{5}$ D. $\frac{2}{1}$

   E. $\frac{4}{7}$ F. $\frac{9}{8}$

2. Which expressions have a product that is less than the first factor? Choose all that apply.

   A. $10 \times \frac{2}{5}$ B. $12 \times \frac{2}{3}$

   C. $16 \times \frac{4}{3}$ D. $20 \times \frac{8}{1}$

3. Connor, Debbie, and Ed all started at the same line and jumped. Connor jumped $\frac{5}{6}$ the distance that Ed jumped. Debbie jumped $\frac{6}{5}$ the distance that Ed jumped. From shortest to longest, what is the order of the distances each person jumped?

   A. Connor, Debbie, Ed

   B. Connor, Ed, Debbie

   C. Debbie, Ed, Connor

   D. Ed, Connor, Debbie

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

188 Assessment Resource Book

R Reinforce Understanding

SMALL GROUP

Factor Up 1

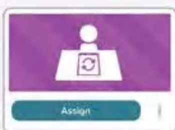
Students each create 2 cards with a fraction, 2 with a mixed number, and 2 with a whole number from 1 to 10. Shuffle the fraction and mixed number cards and place them face down. Do the same with the whole number cards. Draw 1 card from each pile and predict the relative size of the product compared to each factor. If students have difficulty, encourage them to think about whether they are multiplying by a number less than 1 or greater than 1.

GO ONLINE

Take Another Look Lesson

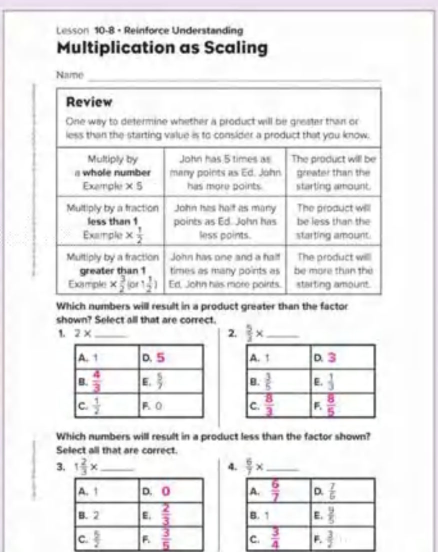
Assign the interactive lesson to reinforce targeted skills.

- Multiplication as Scaling



INDEPENDENT WORK

Differentiation Resource Book, p. 121



Lesson 10-8 • Reinforce Understanding

Multiplication as Scaling

Name ______

Review

One way to determine whether a product will be greater than or less than the starting value is to consider a product that you know.

Multiply by a **whole number** Example × 5	John has 5 times as many points as Ed. John has more points.	The product will be greater than the starting amount.
Multiply by a fraction **less than 1** Example × $\frac{1}{2}$	John has half as many points as Ed. John has less points.	The product will be less than the starting amount.
Multiply by a fraction **greater than 1** Example: × $\frac{3}{2}$ (or $1\frac{1}{2}$)	John has one and a half times as many points as Ed. John has more points.	The product will be more than the starting amount.

Which numbers will result in a product greater than the factor shown? Select all that are correct.

1. $2 \times$ ______

A. 1	D. 5
B. $\frac{4}{3}$	E. $\frac{5}{7}$
C. $\frac{1}{2}$	F. 0

2. $\frac{5}{3} \times$ ______

A. 1	D. 3
B. $\frac{3}{5}$	E. $\frac{1}{3}$
C. $\frac{8}{3}$	F. $\frac{8}{5}$

Which numbers will result in a product less than the factor shown? Select all that are correct.

3. $1\frac{2}{3} \times$ ______

A. 1	D. 0
B. 2	E. $\frac{2}{3}$
C. [illegible]	F. $\frac{3}{5}$

4. $\frac{6}{7} \times$ ______

A. $\frac{6}{7}$	D. $\frac{7}{6}$
B. 1	E. [illegible]
C. $\frac{3}{4}$	F. [illegible]

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Product Size Sort

Students practice determining whether products involving fractions are greater than or less than factors.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 121-122

Lesson 10-8

Additional Practice

Name ______

Review

You can predict whether a product will be greater or less than one of the factors without performing the multiplication.

The rosebush is 2 feet tall. The sunflower is $1\frac{1}{2}$ times as tall as the rosebush. The tulip is $\frac{7}{8}$ the height of the rosebush. What is the order of the flowers from shortest to tallest?

Flower	Height	
Rosebush	2 feet	
Sunflower	$2 \times 1\frac{1}{2}$; taller than rosebush	When multiplying by a factor greater than 1, such as $1\frac{1}{2}$, the answer will be greater than the factor.
Tulip	$2 \times \frac{7}{8}$; shorter than rosebush	When multiplying by a factor less than 1, such as $\frac{7}{8}$, the answer will be less than the factor.

From shortest to tallest, the order of the flowers is tulip, rosebush, and sunflower.

1. Which expressions have a value greater than 38? Choose all that apply.

A. $38 \times \frac{3}{2}$

B. $38 \times \frac{4}{3}$

C. $38 \times \frac{5}{8}$

D. $38 \times \frac{5}{2}$

E. $38 \times \frac{10}{6}$

F. $38 \times \frac{3}{7}$

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Assign

Use It! Application Station

This or That Students follow instructions to measure and cut materials.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 121-122

Circle the lesser number.

2. $\boxed{7 \times \frac{9}{10}}$ or $7 \times \frac{5}{2}$

3. $15 \times \frac{6}{5}$ or $\boxed{15 \times \frac{5}{6}}$

4. $\boxed{\frac{1}{3} \times 2}$ or $\frac{7}{5} \times 2$

5. $\frac{15}{4} \times 20$ or $\boxed{\frac{12}{15} \times 20}$

Write a fraction that makes each sentence true.

6. $\frac{1}{2} \times$ ____ $< \frac{1}{2}$
any fraction less than 1

7. $\frac{8}{13} \times$ ____ $> \frac{8}{13}$
any fraction greater than 1

8. $\frac{15}{6} \times$ ____ $> \frac{15}{6}$
any fraction greater than 1

9. $\frac{7}{9} \times$ ____ $< \frac{7}{9}$
any fraction less than 1

10. On Monday, Willie ran $2\frac{1}{2}$ miles. On Wednesday, he ran $\frac{3}{5}$ that distance. On Friday, he ran $1\frac{3}{5}$ times that distance. What is the order of the days from shortest run to longest run?
Wednesday, Monday, Friday

Math @Home Activity With your child, practice predicting whether a product will be greater than or less than a certain factor. For example, if the distance from your home to school is 2 miles, and your child's friend lives $\frac{1}{2}$ the distance from the school, have your child determine who lives farther from the school.

Student Practice Book

Differentiation Resource Book, p. 122

Lesson **10-8 • Extend Thinking**

Multiplication as Scaling

Name ____________________

1. Elsa is $\frac{7}{8}$ times as old as Dylan. Abigail is $\frac{1}{2}$ times as old as Bryce. Felipe is $1\frac{1}{3}$ times as old as Elsa. Dylan is $1\frac{1}{4}$ times as old as Colin. Bryce is $\frac{5}{6}$ times as old as Colin. Create tape diagrams to show all of their ages and list the people in order from youngest to oldest.

Abigail
Bryce
Colin
Dylan
Elsa
Felipe

Abigail, Bryce, Colin, Elsa, Dylan, Felipe; diagrams may vary

2. Suppose today is Colins' birthday, and he is exactly 18 years old. Find the ages of the other people and record them in the table. Show your work.

Abigail	Bryce	Colin	Dylan	Elsa	Felipe
$7\frac{1}{2}$	15	18	$22\frac{1}{2}$	$19\frac{11}{16}$	$26\frac{1}{4}$

Bryce is $\frac{5}{6} \times 18 = 15$ years old. Abigail is $\frac{1}{2} \times 15 = 7\frac{1}{2}$ years old. Dylan is $18 \times 1\frac{1}{4} = 22\frac{1}{2}$ years old. Elsa is $\frac{7}{8} \times 22\frac{1}{2} = 19\frac{11}{16}$ years old. Felipe is $19\frac{11}{16} \times 1\frac{1}{3} = 26\frac{1}{4}$ years old.

3. Tony is $\frac{3}{4}$ times as old as Frances. Sheila is $\frac{5}{4}$ times as old as Leonard. Leonard is $\frac{4}{5}$ times as old as Tony. Use tape diagrams to solve. What do you notice?

Frances
Tony
Leonard
Sheila

Tony and Sheila are the same age.

Differentiation Resource Book

LESSON **10-9**

Solve Problems Involving Fractions

Learning Target

- I can solve word problems involving fractions.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B** Apply and extend previous understandings of multiplication and division.

◇ **5.NF.B.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

Vocabulary

Math Terms	Academic Term
equation	assert
unknown	reflect
variable	

Materials

The materials may be for any part of the lesson.

- grid paper
- *Problem-Solving Tool* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students solve word problems involving fractions.	• Students talk about solving word problems involving fractions using the verb *determine*. • To support maximizing linguistic and cognitive meta-awareness, ELs participate in MLR5: Co-Craft Questions and Problems.	• Students develop and execute a plan for mathematical problem solving.

Coherence

Previous	Now	Next
• Students multiplied or divided to solve word problems involving multiplicative comparison (Grade 4). • Students interpreted multiplication as scaling (Unit 10).	• Students choose and use known methods to solve problems involving fractions.	• Students relate fractions to division and divide fractions (Unit 11). • Students solve word problems involving division of fractions by fractions (Grade 6),

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of fraction and mixed number multiplication by solving real-world problems. *Conceptual Understanding is not a targeted element of rigor for this standard.*	• Students build proficiency with strategies for multiplying fractions and mixed numbers. *Procedural Skill & Fluency is not a targeted element of rigor for this standard.*	• Students apply their understanding of multiplication strategies to solve and write fraction and mixed number multiplication problems with real-world contexts.

GO ONLINE

Number Routine
Greater Than or Less Than

5–7 min

Build Fluency Students build number sense as they determine whether addition expressions are greater than or less than the benchmarks $\frac{1}{2}$ and 1. Remind students that this is a mental activity, and that exact answers are not needed.

These prompts encourage students to talk about their reasoning:

- How did you determine the value of each expression?
- How did you know that your estimates were reasonable?
- Who thought about it in a different way?
- What is another way to think about the expressions?

Launch 5–7 min

Sense-Making Routine

Purpose Students begin to recognize that real-world problems involving multiplication of fractions demand different models and strategies based on context.

Notice & Wonder

- How are they the same?
- How are they different?

Teaching Tip You may want to have students discuss their responses in pairs or small groups before discussing with the class as a whole.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of how they can solve real-world word problems involving fractions and are based on possible comments and questions that students may make during the share out.

- How would you model each of the three scenarios?
- What would be the same about each model? What would be different?

Math is... Mindset

- What goal do you want to accomplish today?

SEL Self-Regulation: Goal Setting

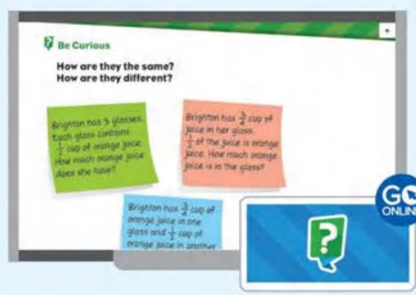
As students begin the Notice & Wonder routine, have them think about or develop goals to solve the problem. Have them consider what they will do first to help them work through exploring how to model different multiplication equations with fractions. You can invite students to write or draw out goals to help them organize their work. Throughout the lesson, students may also find it helpful to develop goals for their problem-solving work with solving problems involving fractions.

Transition to Explore & Develop

Ask students to think about the strategies they know for multiplying fractions and how they can be applied to real-world problems.

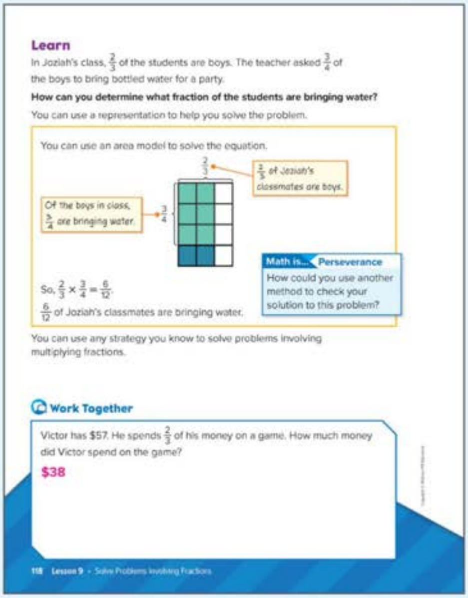
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can solve real-world problems involving multiplication of fractions.



Explore & Develop

20 min



Learn

In Joziah's class, $\frac{2}{3}$ of the students are boys. The teacher asked $\frac{3}{4}$ of the boys to bring bottled water for a party.

How can you determine what fraction of the students are bringing water?

You can use a representation to help you solve the problem.

You can use an area model to solve the equation.

$\frac{2}{3}$ of Joziah's classmates are boys.

Of the boys in class, $\frac{3}{4}$ are bringing water.

So, $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$.

$\frac{6}{12}$ of Joziah's classmates are bringing water.

Math is... Perseverance

How could you use another method to check your solution to this problem?

You can use any strategy you know to solve problems involving multiplying fractions.

Work Together

Victor has \$57. He spends $\frac{2}{3}$ of his money on a game. How much money did Victor spend on the game?

\$38

118 Lesson 9 • Solve Problems Involving Fractions

1 Pose the Problem

ETP Pose Purposeful Questions

- Do you need to know how many students are in Joziah's class? Why or why not?
- Do you need to find out what fraction of the boys are bringing water? Why or why not?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Co-Craft Questions and Problems

Pair students and have them co-create a problem similar to the one on the Learn page. Have them work together to solve their problem and then trade their problem with another pair. After each pair solves the other pair's problems, have them form a group of four to check solutions and correct any mistakes that may have been made. Revisit the routine throughout the lesson for reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How would you choose a multiplication strategy you already know to solve a problem?
- How would you use a multiplication strategy you already know to solve a problem?

Key Takeaway

- Problems involving fractions can be solved using known strategies for multiplication of fractions.

Work Together

Students solve a word problem involving the multiplication of fractions. Have students choose a known strategy.

Common Misconception: Students may be tempted to use an area model since the example used one. Have students describe the type of numbers involved in the problem, and recall strategies that they know for problems involving those types of numbers.

LOM Language of Math

Remind students that the word *strategy* means "a plan designed to achieve an overall aim." They can choose a strategy to solve math problems just as they would choose a strategy when playing a game, writing an essay, or cleaning a room.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore solving word problems involving fractions.

Materials: *Problem-Solving Tool* Teaching Resource

Directions: Provide copies of the *Problem-Solving Tool* Teaching Resource. Have students work together to solve the Pose the Problem.

ETP **Support Productive Struggle**

- How do you know what equation to write?
- What strategies do you know of that you can use to solve the problem?
- Which strategy do you prefer to use? Explain.
- How can you represent each factor?
- How can you determine the product?

Math is... Perseverance

- How could you use another method to check your solution to this problem?

Students check their answers to problems using a different method.

Activity Debrief: Have students share their solutions and strategies for solving the problem. Encourage students to find similarities and differences among the solution methods.

A PDF of the Teaching Resource is available in the Digital Teacher Center.


Problem-Solving Tool

Guided Exploration

Students use multiplication strategies that are familiar to them to solve problems involving the multiplication of fractions.

ETP **Use and Connect Mathematical Representations**

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have students represent the equation $\frac{2}{3} \times \frac{3}{4}$. Ask:

- How could you represent the whole class?
- What fraction of Joziah's whole class are boys? What fraction of a whole should you represent? Why?
- What fraction of those boys are bringing bottled water? How can you represent that?

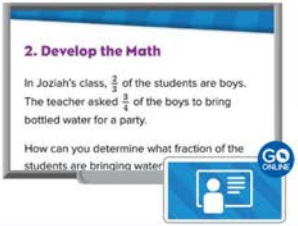
Have students find the fraction of the whole class bringing bottled water. Ask:

- Why do you have to partition the whole?
- How can you partition the whole to find out what fraction of the whole class is bringing bottled water?
- How many equal parts is the whole partitioned into?
- **Think About It:** Is there another way you could write the calculated product?

Math is... Perseverance

- How could you use another method to check your solution to this problem?

Students check their answers to problems using a different method.



EL **English Learner Scaffolds**

Entering/Emerging Support students in understanding *involving*. Say *Let's look at equations involving adding*. Show or write different addition equations. Then say *Let's look at equations involving subtracting*. Show or write different subtraction problems. Then write one equation using division and one equation using multiplication. Point to each and ask *Is this an equation involving dividing or multiplying?*

Developing/Expanding Support students in understanding *involving*. Say *Let's look at equations involving adding*. Show or write different addition equations. Then say *Let's look at equations involving subtracting*. Show or write different subtraction problems. Then show or write division equations. Ask students to complete the sentence: *These are equations* _____ (involving) *dividing*.

Bridging/Reaching Ensure understanding of *involving* by asking students to review the sentence on the Learn page and restate the meaning of it in their own words. Then have them write different equations and talk about them using *involving*.

Practice & Reflect 10 min

On My Own

Name ______________________

1. Sasha's family had $\frac{3}{4}$ of a pizza left over. Sasha ate $\frac{1}{2}$ of what was left. How much of the pizza did Sasha eat?
   - A. $\frac{5}{8}$
   - B. $\frac{3}{8}$
   - C. $\frac{4}{8}$
   - D. $\frac{4}{6}$

2. Jessica ran $\frac{7}{8}$ of a mile. Sheila ran $1\frac{1}{3}$ times as far as Jessica. How far did Sheila run?
   - A. $1\frac{1}{2}$ mi
   - B. $2\frac{1}{3}$ mi
   - C. $1\frac{1}{6}$ mi
   - D. $1\frac{3}{4}$ mi

3. The first chapter of a book has 40 pages. Connor read $\frac{7}{8}$ of the pages in the first chapter. Simon read $\frac{1}{5}$ of the pages that Connor read. How many pages have Connor and Simon each read?
   Connor read 35 pages; Simon read 7 pages

4. What is the area of the piece of rug?
   $4\frac{2}{3}$ square feet

5. In a fifth grade class, $\frac{3}{5}$ of the children play an instrument. Of those, $\frac{5}{8}$ are girls. What fraction of girls in the fifth-grade class plays an instrument?
   $\frac{3}{8}$

6. **STEM Connection** Saffron has a bag of flour that contains 8 cups of flour. The bag is $\frac{6}{7}$ full. Saffron uses $\frac{1}{3}$ of the bag to make a batch of muffins. How many cups of flour does Saffron use?
   $2\frac{2}{7}$ cups of flour

7. Maya has a flower garden. In her garden, $\frac{2}{3}$ of the flowers are roses. Of the roses, $\frac{5}{6}$ are pink. How many of the flowers in Maya's garden are pink roses?
   $\frac{5}{9}$ are pink roses

8. One fifth-grade class donated $1\frac{3}{4}$ boxes of canned goods to the food pantry. Another fifth-grade class donated $2\frac{1}{2}$ times as many boxes of canned goods. How many boxes did the second fifth-grade class donate?
   $4\frac{3}{8}$ boxes

9. Daniel has a collection of stickers. Of his collection, $\frac{4}{7}$ of the stickers are round. Of the round stickers, $\frac{1}{4}$ are red. How many of Daniel's sticker collection is red and round? **$\frac{1}{7}$ of the collection**

10. **Extend Your Thinking** Write and solve a word problem involving multiplying two fractions that are both less than 1.
    Answers may vary.

Reflect

How can you solve real world problems involving multiplication of fractions?

Answers may vary.

Math is... Mindset
What goal did you accomplish today?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 3 Students may find $\frac{1}{5}$ of the total pages rather than $\frac{1}{5}$ of the pages that Connor read. Make sure they read the problem carefully.

Practice Item Analysis

Item	DOK	Rigor
1–9	2	Application
10	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you solve real world problems involving multiplication of fractions?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What goal did you accomplish today?

Students reflect on how they practiced self-regulation.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can solve word problems involving fractions.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Solve problems involving fractions	5.NF.B.6
2	2	Solve problems involving fractions	5.NF.B.6
3	2	Solve problems involving fractions	5.NF.B.6
4	2	Solve problems involving fractions	5.NF.B.6

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking

Lesson 10-9

Exit Ticket

Name ______________________

1. Benjamin makes a quilt that is $4\frac{1}{8}$ feet wide and $3\frac{2}{3}$ feet long. What is the area of Benjamin's quilt?
   $15\frac{3}{24}$ square feet

2. Rufus lives $\frac{4}{5}$ mile from school. Leah lives $\frac{3}{4}$ the distance from school as Rufus. How far does Leah live from the school?
   $\frac{12}{20}$ mile

3. At the football game, $\frac{11}{12}$ of the people are cheering for the home team. Of those people, $\frac{2}{3}$ of them are wearing team shirts. What fraction of the people at the game are cheering for the home team wearing a team shirt?
   $\frac{22}{36}$ of the people

4. Dora's vegetable garden is in the shape of a rectangle. It is $12\frac{1}{2}$ yards long and 8 yards wide. What is the area of Dora's vegetable garden?
   100 square yards

Reflect On Your Learning

I'm confused.	I'm still learning.	I understand.	I can teach someone else.

Assessment Resource Book 189

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Fraction Word Problem

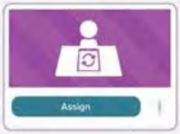
Work with students in pairs. Each student in a pair writes a word problem that requires fraction multiplication to solve. Students then solve each other's problems. If students struggle, help them to draw area models to show what they are multiplying.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Multiply Fractions/Mixed Numbers (Model)
- Multiply Fractions/Mixed Numbers



INDEPENDENT WORK

Differentiation Resource Book, p. 123

Lesson **10-9 • Reinforce Understanding**

Solve Problems Involving Fractions

Name ______________________

Review

Use one of the strategies learned to solve word problems involving fractions. One option is to write mixed numbers as fractions.

Mark is $1\frac{2}{5}$ times as tall as Sara, who is $3\frac{3}{4}$ feet tall. How tall is Mark?

$1\frac{2}{5} \times 3\frac{3}{4} = \frac{5+2}{5} \times \frac{12+3}{4} = \frac{7}{5} \times \frac{15}{4} = \frac{7 \times 15}{5 \times 4} = \frac{105}{20} = 5\frac{5}{20} = 5\frac{1}{4}$

Mark is $5\frac{1}{4}$ feet tall.

Solve each problem using one of the strategies you learned.

1. In a local animal shelter, $\frac{2}{3}$ of the animals are dogs. Of the dogs, $\frac{1}{4}$ are puppies. What fraction of the animals at the shelter are puppies?
   $\frac{1}{6}$ of the animals are puppies
2. A painting is $2\frac{4}{5}$ feet long and $1\frac{1}{2}$ wide. What is the area of the painting?
   $4\frac{1}{5}$ square feet
3. Esther has a bag of rice that holds 10 cups of rice when full. The bag is $\frac{3}{4}$ full. Esther uses $\frac{3}{8}$ of the rice in the bag to make a batch of paella. How many cups of rice does Esther use?
   $2\frac{13}{16}$ cups of rice
4. Nico is $4\frac{1}{2}$ feet tall. Noah is $1\frac{1}{7}$ times as tall as Nico. How tall is Noah?
   $5\frac{1}{7}$ feet tall
5. $\frac{4}{5}$ of the students in a class have siblings. Of the students who have siblings, $\frac{1}{3}$ have an older brother. What fraction of the students in the class have an older brother?
   $\frac{4}{15}$ of the students

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Problem Race Students multiply fractions to solve problems.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 123-124

Lesson **10-9**

Additional Practice

Name ______________________

Review

You can solve problems involving multiplication of fractions and mixed numbers.

Mrs. Adler has a poster that is $14\frac{1}{2}$ inches long and $8\frac{1}{2}$ inches wide. What is the area of the poster?

Use a representation.

	14	+ $\frac{1}{2}$
8	112	4
+ $\frac{1}{2}$	7	$\frac{1}{4}$

$112 + 7 + 4 + \frac{1}{4} = 123\frac{1}{4}$

Rewrite mixed numbers as fractions greater than 1.

$14\frac{1}{2} \times 8\frac{1}{2} =$

$\frac{29}{2} \times \frac{17}{2} =$

$\frac{493}{4} = 123\frac{1}{4}$

The area of the poster is $123\frac{1}{4}$ square inches.

1. Bella is painting a sign that is $4\frac{3}{4}$ feet long and $3\frac{1}{2}$ feet wide. What is the area of the sign?
   $16\frac{5}{8}$ square feet
2. Ariana lives $\frac{5}{8}$ mile from school. Fred lives $\frac{3}{4}$ the distance from school as Ariana. How far does Fred live from the school?
   $\frac{15}{32}$ mile

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying multi-digit numbers.

Assign

Use It! Application Station

Fraction of a Fraction Students use fractions to follow and change a recipe.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 123–124

3. There are 42 flowers in a vase. Of the flowers, $\frac{1}{2}$ are roses. Of the roses, $\frac{3}{7}$ are white. How many white roses are in the vase?

   9 white roses

4. A photograph of a skyline is enlarged to be $2\frac{1}{4}$ feet tall and 6 feet long. What is the area of the enlarged photograph?

   $13\frac{1}{2}$ square feet

5. At the basketball game, $\frac{9}{10}$ of the people are cheering for the home team. Of those people, $\frac{3}{4}$ of them are wearing team shirts. What fraction of the people at the game are cheering for the home team wearing a team shirt?

   $\frac{27}{40}$ of the people

6. A family is building a rectangular patio in the backyard. The patio is to be $5\frac{3}{4}$ yards long and $3\frac{1}{4}$ yards wide. What is the area of the patio?

   $18\frac{11}{16}$ square yards

7. In a fifth-grade class, $\frac{3}{4}$ of the students play an instrument. Of those students, $\frac{2}{5}$ of them play the piano. What fraction of the students in the class play the piano?

   $\frac{6}{20}$ or $\frac{3}{10}$ of the students

Math @Home Activity With your child, identify situations around your home where multiplication of fractions and mixed numbers would be necessary. For example, if the length of a piece of fabric is [illegible] foot long and the width of the fabric is [illegible] feet long, students would multiply to find the area of the piece of fabric. Ask your child to show how to solve the problem in two different ways. Have them explain how each strategy yields the same product.

Student Practice Book

Differentiation Resource Book, p. 124

Lesson 10-9 • Extend Thinking

Solve Problems Involving Fractions

Name ____________________

Solve each problem. Show your work.

1. In Library A, $\frac{3}{5}$ of the books are fiction. Of the fiction books, $\frac{7}{18}$ are from the mystery genre. Of the books in the mystery genre, $\frac{2}{9}$ are from the whodunit subgenre. What fraction of the books in the library are from the whodunit subgenre?

   $\frac{3}{5} \times \frac{7}{18} = \frac{7}{30}$ and $\frac{7}{30} \times \frac{2}{9} = \frac{7}{135}$; $\frac{7}{135}$ of the library books

2. If there are 27,000 books in Library A, and $\frac{2}{7}$ of the whodunit books are checked out, How many whodunit books are available to check out?

   $27{,}000 \times \frac{7}{135} = 1{,}400$ whodunit books, and $\frac{2}{7} \times 1{,}400 = 400$ books are checked out. So there are $1{,}400 - 400 = 1{,}000$ whodunit books left that can be checked out.

3. $\frac{3}{10}$ of the non-fiction books in Library A are biographies and $\frac{1}{9}$ of the biographies are about a U.S. President. How many books in Library A are biographies about a U.S. President?

   $\frac{2}{5} \times \frac{3}{10} = \frac{3}{25}$ and $\frac{3}{25} \times \frac{1}{9} = \frac{1}{75}$. So there are $\frac{1}{75} \times 27{,}000 = 360$ books about U.S. Presidents.

4. Library B has $1\frac{2}{3}$ more biographies about a U.S. President than Library A, and half of those books are checked out. How many biographies about a U.S. President are checked out from Library B?

   $360 \times 1\frac{2}{3} = 600$ and $\frac{1}{2} \times 600 = 300$. 300 biographies about a U.S. President are checked out from Library B.

Differentiation Resource Book

Unit Review

Name ____________________

Vocabulary Review

Choose the correct word(s) to complete the sentence.

area	numerator	partition
area model	partial products	scaling
denominator		

1. The **denominator** is the bottom number of a fraction representing the total. (Lessons 10-2, 10-4)
2. The product of the dimensions of a rectangle gives the **area** of its flat surface. (Lesson 10-5)
3. The **numerator** is the top number of a fraction indicating how many parts of the total. (Lessons 10-2, 10-4)
4. To find the product of a whole number and a fraction, **partition** each whole representing the model by the value of the denominator. (Lesson 10-1)
5. Use a(n) **area model** to find the partial products of a product. (Lesson 10-6)
6. The **partial products** are added to find the product. (Lessons 10-6, 10-7)
7. **Scaling** compares the size of a product to the size of one of the factors. (Lesson 10-8)

Unit 10 • Multiply Fractions 121

Review

8. What is the product? Show your work. (Lesson 10-2)
$\frac{3}{7} \times 9 =$ **$\frac{27}{7}$ or $3\frac{6}{7}$**

9. A mother rabbit weighs 5 pounds. The baby rabbit weighs $\frac{2}{9}$ of the mother rabbit's weight. What is the weight of the baby rabbit? (Lesson 10-9)
$\frac{10}{9}$ or $1\frac{1}{9}$ pounds

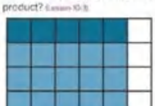
10. What is the product? (Lesson 10-2)
$\frac{3}{5} \times 8 =$ **$4\frac{4}{5}$ or $\frac{24}{5}$**

11. Sarah rides her bike to and from school. A round trip is $\frac{3}{4}$ mile. How many miles does she ride her bike to and from school in 5 days? (Lesson 10-9)
$3\frac{3}{4}$ or $\frac{15}{4}$ miles

12. Solve by drawing an area model. (Lesson 10-3)
$\frac{7}{9} \times \frac{3}{4} =$ **$\frac{21}{36}$**

13. What is the product? (Lesson 10-4)
$\frac{7}{12} \times \frac{7}{8} =$ **$\frac{49}{96}$**

14. The area model represents what product? (Lesson 10-3)



A. $\frac{1}{4} \times \frac{3}{5}$
B. $\frac{1}{6} \times \frac{3}{4}$
C. $\frac{1}{4} \times \frac{5}{6}$
D. $\frac{4}{5} \times \frac{5}{6}$

15. Jackson shares $\frac{1}{2}$ of a bag of trail mix with his friends. They eat $\frac{2}{3}$ of the trail mix that was in the bag. How much of the bag of trail mix did they eat? (Lesson 10-9)
$\frac{1}{3}$ of the bag

122 Unit 10 • Review

Students can complete the **Unit Review** to prepare for the **Unit Assessment.** Students may complete the Review in their Interactive eBook in the Digital Student Center.

Vocabulary Review

Item Analysis

Item	Lesson
1	10-2
2	10-5
3	10-2
4	10-1
5	10-6
6	10-7
7	10-8

Review

Item Analysis

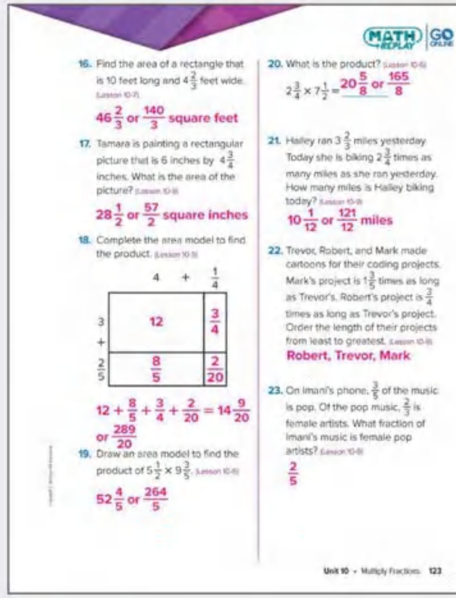
Item	DOK	Lesson	Standard
8	1	10-2	5.NF.B.4.a
9	2	10-9	5.NF.B.6
10	1	10-2	5.NF.B.4.a
11	2	10-9	5.NF.B.6
12	1	10-3	5.NF.B.4.a
13	1	10-4	5.NF.B.4.a
14	1	10-3	5.NF.B.4.a
15	2	10-9	5.NF.B.6

To review the lessons in this unit, have students watch the Math Replay video in their Digital Student Center.

Assign the Unit Review practice to students from the Digital Teacher Center.

Item Analysis (continued)

Item	DOK	Lesson	Standard
16	1	10-7	5.NF.B.4.a
17	2	10-9	5.NF.B.6
18	1	10-5	5.NF.B.4.b
19	1	10-5	5.NF.B.4.b
20	1	10-6	5.NF.B.4.a
21	2	10-9	5.NF.B.6
22	2	10-9	5.NF.B.6
23	2	10-9	5.NF.B.6



16. Find the area of a rectangle that is 10 feet long and $4\frac{2}{3}$ feet wide. (Lesson 10-7)

$46\frac{2}{3}$ or $\frac{140}{3}$ square feet

17. Tamara is painting a rectangular picture that is 6 inches by $4\frac{3}{4}$ inches. What is the area of the picture?

$28\frac{1}{2}$ or $\frac{57}{2}$ square inches

18. Complete the area model to find the product.

	4	$\frac{1}{4}$
3	12	$\frac{3}{4}$
$\frac{2}{5}$	$\frac{8}{5}$	$\frac{2}{20}$

$12 + \frac{8}{5} + \frac{3}{4} + \frac{2}{20} = 14\frac{9}{20}$ or $\frac{289}{20}$

19. Draw an area model to find the product of $5\frac{1}{2} \times 9\frac{3}{5}$.

$52\frac{4}{5}$ or $\frac{264}{5}$

MATH REPLAY | GO ONLINE

20. What is the product?

$2\frac{3}{4} \times 7\frac{1}{2} =$ $20\frac{5}{8}$ or $\frac{165}{8}$

21. Hailey ran $3\frac{2}{3}$ miles yesterday. Today she is biking $2\frac{3}{4}$ times as many miles as she ran yesterday. How many miles is Hailey biking today?

$10\frac{1}{12}$ or $\frac{121}{12}$ miles

22. Trevor, Robert, and Mark made cartoons for their coding projects. Mark's project is $1\frac{3}{5}$ times as long as Trevor's. Robert's project is $\frac{3}{4}$ times as long as Trevor's project. Order the length of their projects from least to greatest.

Robert, Trevor, Mark

23. On Imani's phone, $\frac{3}{5}$ of the music is pop. Of the pop music, $\frac{2}{3}$ is female artists. What fraction of Imani's music is female pop artists?

$\frac{2}{5}$

Unit 10 • Multiply Fractions 123

Performance Task

Standards: 5.NF.B.4, 5.NF.B.4.a, 5.NF.B.4.b, 5.NF.B.5.a, 5.NF.B.5.b, 5.NF.B.6

Rubric (8 points)

Parts A, B, C (DOK 2) – 2 points each	
2 POINTS	Student's work reflects a proficiency with multiplying fractions.
1 POINT	Student's work reflects developing proficiency with multiplying fractions.
0 POINTS	Student's work reflects weak proficiency with multiplying fractions.

Part D (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with multiplying and ordering fractions.
1 POINT	Student's work reflects developing proficiency with multiplying and ordering fractions.
0 POINTS	Student's work reflects weak proficiency with multiplying and ordering fractions.

Performance Task

Hannah is welding three different types of rectangular shaped tables.

Part A: The coffee table has a length of $4\frac{1}{2}$ feet and a width of $3\frac{3}{4}$ feet. The side table is $\frac{4}{5}$ the area of the coffee table. What is the area of the side table?

$10\frac{1}{8}$ square feet

Part B: The area of the patio table is $2\frac{1}{5}$ times the area of the side table. What is the area of the patio table?

$22\frac{11}{40}$ square feet

Part C: It took her $1\frac{1}{9}$ times as long to weld the coffee table as it did to weld the side table. It took her $\frac{7}{12}$ times as long to weld the patio table as it did to weld the side table. Place in order from least to greatest the amounts of time it took Hannah to weld the tables.

patio table, side table, coffee table

Reflect

How does the size of the factors indicate whether the product will be less than both factors, less than one of the factors, or less than neither of the factors?

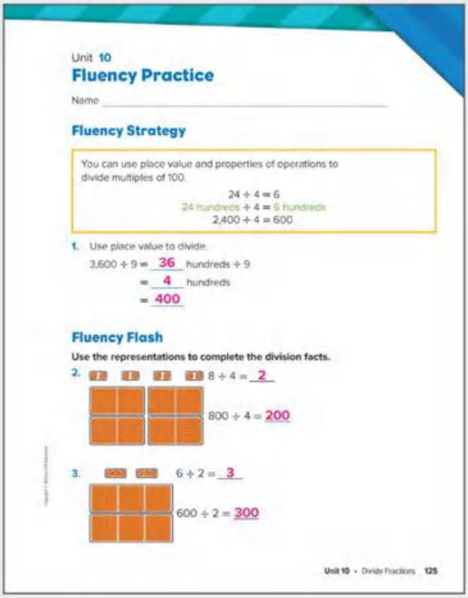
Answers may vary.

124 Unit 10 • Performance Task

Reflect

The Reflect question provides an opportunity for students to express their understanding of the unit level focus question.

Fluency Practice



Unit 10

Fluency Practice

Name ______

Fluency Strategy

You can use place value and properties of operations to divide multiples of 100.

$24 \div 4 = 6$

24 hundreds ÷ 4 = 6 hundreds

$2{,}400 \div 4 = 600$

1. Use place value to divide.

$3{,}600 \div 9 =$ 36 hundreds ÷ 9

= 4 hundreds

= 400

Fluency Flash

Use the representations to complete the division facts.

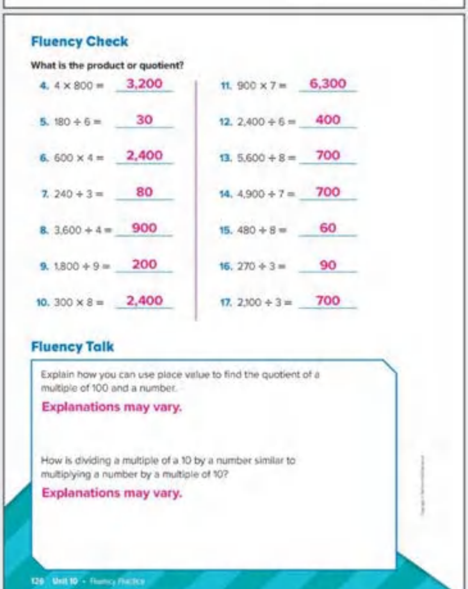
2. $8 \div 4 =$ 2

$800 \div 4 =$ 200

3. $6 \div 2 =$ 3

$600 \div 2 =$ 300

Unit 10 • Divide Fractions 125



Fluency Check

What is the product or quotient?

4. $4 \times 800 =$ 3,200
5. $180 \div 6 =$ 30
6. $600 \times 4 =$ 2,400
7. $240 \div 3 =$ 80
8. $3{,}600 \div 4 =$ 900
9. $1{,}800 \div 9 =$ 200
10. $300 \times 8 =$ 2,400
11. $900 \times 7 =$ 6,300
12. $2{,}400 \div 6 =$ 400
13. $5{,}600 \div 8 =$ 700
14. $4{,}900 \div 7 =$ 700
15. $480 \div 8 =$ 60
16. $270 \div 3 =$ 90
17. $2{,}100 \div 3 =$ 700

Fluency Talk

Explain how you can use place value to find the quotient of a multiple of 100 and a number.

Explanations may vary.

How is dividing a multiple of a 10 by a number similar to multiplying a number by a multiple of 10?

Explanations may vary.

126 Unit 10 • Fluency Practice

Fluency practice helps students develop procedural fluency, that is, the "ability to apply procedures accurately, efficiently, and flexibly." Because there is no expectation of speed, students should not be timed when completing the practice activity.

Build Fluency **Objective** Students practice dividing multiples of 100.

Fluency Progression

Unit	Skill	Standard
1	Use Partial Sums to Add	4.NBT.B.4
2	Decompose by Place Value to Subtract	4.NBT.B.4
3	Use an Algorithm to Add	4.NBT.B.4
4	Use an Algorithm to Subtract	4.NBT.B.4
5	Choose a Strategy to Add	4.NBT.B.4
6	Choose a Strategy to Subtract	4.NBT.B.4
7	Multiply by Multiples of 10	5.NBT.B.5
8	Multiply by Multiples of 100	5.NBT.B.5
9	Divide Multiples of 10	5.NBT.B.6
10	**Divide Multiples of 100**	**5.NBT.B.6**
11	Use an Algorithm to Multiply (2- and 3-Digit Numbers by 1-Digit Numbers)	5.NBT.B.5
12	Use an Algorithm to Multiply (2-Digit Numbers by 2-Digit Numbers)	5.NBT.B.5
13	Choose a Strategy to Multiply	5.NBT.B.5
14	Choose a Strategy to Multiply	5.NBT.B.5

Fluency Expectations

Grade 4

- Add and subtract within 1,000,000.

Grade 5

- Multiply multi-digit whole numbers.

Grade 6

- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Animal Rescue

Students draw on their understanding of multiplying with fractions. Use the rubric shown to evaluate students' work.

Standards: 5.NF.B.4, 5.NF.B.4.a, 5.NF.B.4.b, 5.NF.B.6

Rubric (12 points)

Part A (DOK 2) – 4 points	
4 POINTS	Student's work reflects a proficiency with multiplying a fraction by a fraction and by a whole number. The student's answers and work are correct.
2 POINTS	Student's work reflects a developing proficiency with multiplying a fraction by a fraction and by a whole number. Some of student's answers and work are incorrect.
0 POINTS	Student's work reflects a weak proficiency with multiplying a fraction by a fraction and by a whole number. The student's answers and work are incorrect.

Parts B and E (DOK 2) – 4 points	
4 POINTS	Student's work reflects a proficiency with multiplying mixed numbers. The student's answer and work are correct.
2 POINTS	Student's work reflects a developing proficiency with multiplying mixed numbers. The student's answer or work is incorrect.
0 POINTS	Student's work reflects a weak proficiency with multiplying mixed numbers. The student's answer and work are incorrect.

Part C (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with using multiplication for scaling. The student accurately lists the adoption fees from least to greatest.
1 POINT	Student's work reflects a developing proficiency with using multiplication for scaling. The student has minor issues accurately listing the adoption fees from least to greatest.
0 POINTS	Student's work reflects a weak proficiency with using multiplication for scaling. The student was unable to accurately list the adoption fees from least to greatest.

Part D (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with multiplying a fraction by a whole number. The student's answers and work are correct.
1 POINT	Student's work reflects a developing proficiency with multiplying a fraction by a whole number. Some of student's answers and work are incorrect.
0 POINTS	Student's work reflects a weak proficiency with multiplying a fraction by a whole number. The student's answers and work are incorrect.

Unit 10

Performance Task

Name ______________________

Animal Rescue

At the local animal rescue, there are currently 150 animals available for adoption. The rescue categorizes the animals as shown in the table.

Age	Canine	Feline
Less than 6 months	Puppy	Kitten
6 months or older	Dog	Cat

Part A

Of the animals available for adoption, $\frac{2}{3}$ are canines. Of the canines available for adoption, $\frac{1}{4}$ are puppies. What fraction of all the animals available for adoption are puppies? How many puppies are there at the animal rescue? What fraction of all the animals available for adoptions are dogs? How many dogs are there at the animal rescue? Show your work.

$\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$, so $\frac{1}{6}$ of the animals are puppies. $\frac{1}{6} \times 150 = 25$, so there are 25 puppies at the animal rescue. $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$, so $\frac{1}{2}$ of the animals are dogs. $\frac{1}{2} \times 150 = 75$, so there are 75 dogs.

Part B

For the canines that are dogs, it is recommended that the kennel for the dogs is at least $6\frac{3}{4}$ feet wide by $9\frac{1}{2}$ feet deep. Determine the minimum area of the floor for each kennel in ft². Show your work.

$64\frac{1}{8}$ square feet; $6\frac{3}{4} \times 9\frac{1}{2} = (6 + \frac{3}{4}) \times (9 + \frac{1}{2}) =$
$54 + 3 + 6\frac{3}{4} + \frac{3}{8} = 57 + 6\frac{6}{8} + \frac{3}{8} = 57 + 6\frac{9}{8} = 57 + 7\frac{1}{8}$

Assessment Resource Book 191

Part C

The adoption fee for kittens is $\frac{1}{2}$ as much as the adoption fee for puppies. The adoption fee for cats is $\frac{3}{10}$ the cost of kittens. The adoption fee for dogs is 2 times the adoption fee of cats. Order the adoption fees from least to greatest. Use a diagram to represent the problem.

cats, dogs, kittens, puppies

Part D

If the adoption price for a kitten is \$100, what are the adoption prices of cats, dogs, and puppies? Show your work.

The price for puppies will be $100 \times 2 = \$200$. The price for cats will be $100 \times \frac{3}{10} = \$30$. The price for dogs will be $30 \times 2 = \$60$.

Part E

The table shows the minimum recommended daily food amount for dogs based on their weight. How many cups of dog food will an 80-pound dog require in a year?

Weight (lbs)	5	10	20	40	60	80	100
Food Per Day (cups)	$\frac{1}{2}$	$\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	3	$3\frac{2}{3}$	$4\frac{1}{4}$

$3\frac{2}{3} \times 365 = 1{,}320$ cups

192 Assessment Resource Book

Unit Assessments

Two forms of the Unit Assessment, Form A and Form B, are available for either print or digital administration. The items on the two assessments are parallel items, assessing the same concept and standard. The table below provides the item analysis for both forms.

Both Unit Assessments are available in the Assessment Resource Book or as downloadable files from the Digital Teacher Center.

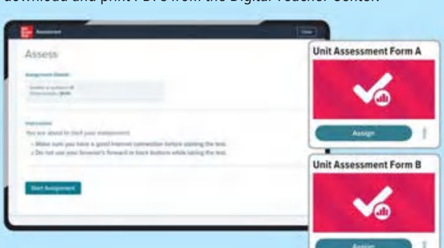
Data When students complete the Unit Assessment in the Digital Student Center, their responses are auto-scored.

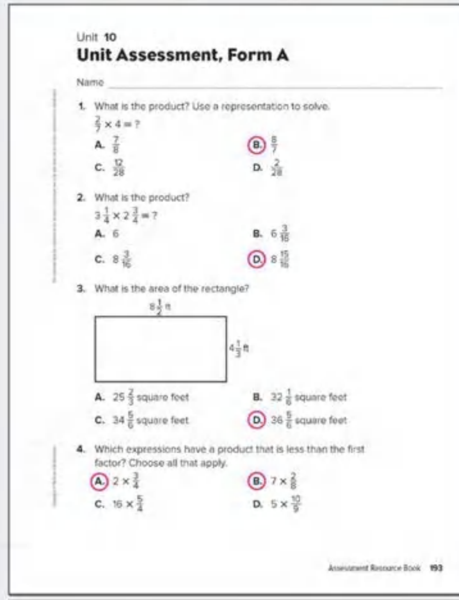
Item Analysis

Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	1	10-1	Multiply Whole Number by Fraction-Models	5.NF.B.4.a
2	1	10-7	Multiplication of Mixed Numbers	5.NF.B.4.a
3	1	10-5, 10-7	Multiply Two Fractions-Models	5.NF.B.4.a, 5.NF.B.4.b
4	1	10-8	Multiplication as Scaling	5.NF.B.5.b
5	1	10-4	Multiply Two Fractions	5.NF.B.4.a
6	2	10-4, 10-9	Multiply Two Fractions-Word Problems	5.NF.B.4.a, 5.NF.B.6
7	2	10-2	Multiply Whole Number by Fraction	5.NF.B.4.a
8	1	10-1	Multiply Whole Number by Fraction-Models	5.NF.B.4.a
9	1	10-7	Multiplication of Mixed Numbers	5.NF.B.4.a
10	1	10-5	Multiply Two Fractions-Models	5.NF.B.4.b
11	1	10-3	Multiply Two Fractions-Models	5.NF.B.4.a
12	2	10-7	Multiply Fractions/Mixed Numbers	5.NF.B.4.a
13	2	10-9	Multiply Whole Number by Fraction	5.NF.B.6
14	2	10-9	Multiply Two Fractions-Models	5.NF.B.6
15	2	10-9	Multiply Fractions/Mixed Numbers	5.NF.B.6

GO ONLINE

Assign the digital Unit Assessment (Form A or B) to students or download and print PDFs from the Digital Teacher Center.





Unit 10

Unit Assessment, Form A

Name ______________________

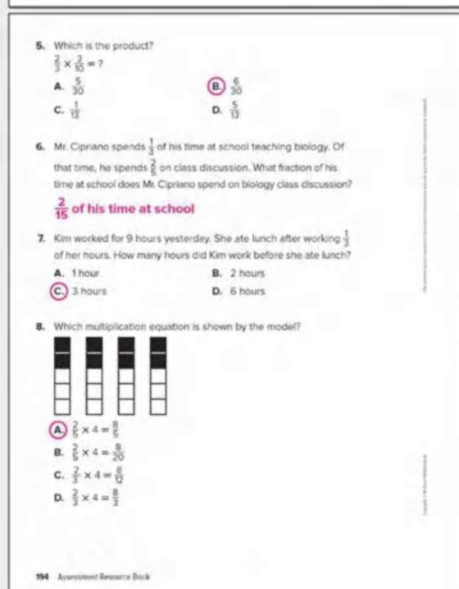
1. What is the product? Use a representation to solve.
$\frac{2}{7} \times 4 = ?$
A. $\frac{7}{8}$ B. $\frac{8}{7}$
C. $\frac{12}{28}$ D. $\frac{2}{28}$

2. What is the product?
$3\frac{1}{4} \times 2\frac{3}{4} = ?$
A. 6 B. $6\frac{3}{16}$
C. $8\frac{3}{16}$ D. $8\frac{15}{16}$

3. What is the area of the rectangle?
$8\frac{1}{2}$ ft
$4\frac{1}{3}$ ft
A. $25\frac{2}{3}$ square feet B. $32\frac{1}{6}$ square feet
C. $34\frac{5}{6}$ square feet D. $36\frac{5}{6}$ square feet

4. Which expressions have a product that is less than the first factor? Choose all that apply.
A. $2 \times \frac{3}{4}$ B. $7 \times \frac{2}{8}$
C. $16 \times \frac{5}{4}$ D. $5 \times \frac{10}{9}$

Assessment Resource Book 193



5. Which is the product?
$\frac{2}{3} \times \frac{3}{10} = ?$
A. $\frac{5}{30}$ B. $\frac{6}{30}$
C. $\frac{1}{13}$ D. $\frac{5}{13}$

6. Mr. Cipriano spends $\frac{1}{3}$ of his time at school teaching biology. Of that time, he spends $\frac{2}{5}$ on class discussion. What fraction of his time at school does Mr. Cipriano spend on biology class discussion?
$\frac{2}{15}$ **of his time at school**

7. Kim worked for 9 hours yesterday. She ate lunch after working $\frac{1}{3}$ of her hours. How many hours did Kim work before she ate lunch?
A. 1 hour B. 2 hours
C. 3 hours D. 6 hours

8. Which multiplication equation is shown by the model?
A. $\frac{2}{5} \times 4 = \frac{8}{5}$
B. $\frac{2}{5} \times 4 = \frac{8}{20}$
C. $\frac{2}{3} \times 4 = \frac{8}{12}$
D. $\frac{2}{3} \times 4 = \frac{8}{3}$

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Unit 10
Unit Assessment, Form A (continued)

Name ______________________

9. What is the product?
$6\frac{1}{3} \times 2\frac{2}{3} = ?$
A. 18　B. $16\frac{8}{9}$　C. $12\frac{3}{8}$　D. $12\frac{2}{9}$

10. What is the area of the rectangle?
$5\frac{1}{4}$ ft
3 ft
$15\frac{3}{4}$ square feet

11. What is the product? Use the representation to solve.
$\frac{2}{3} \times \frac{3}{4} = ?$
$\frac{2}{3}$
$\frac{3}{4}$
A. $\frac{6}{12}$　B. $\frac{9}{12}$　C. $\frac{5}{7}$　D. $\frac{6}{7}$

12. Skye picks $3\frac{1}{2}$ pints of blueberries. Kate picks $2\frac{1}{4}$ times as many pints as Skye. How many pints of blueberries does Kate pick?
A. $5\frac{3}{4}$ pints　B. $6\frac{1}{8}$ pints
C. $7\frac{7}{8}$ pints　D. 8 pints

Assessment Resource Book 195

13. There are 4 people on the middle-school relay team. Each of them runs $\frac{1}{3}$ mile. What is the total distance that the team runs? Explain how you found your answer.
$\frac{4}{3}$ miles or $1\frac{1}{3}$ miles; Sample answer: I multiplied 4 times the numerator 1 from $\frac{1}{3}$ and that equals 4 for the numerator of the answer. Then I used the denominator from $\frac{1}{3}$ as the denominator for the answer.

14. Rory uses buttons for a project. Of the buttons, $\frac{1}{2}$ of them are round. Of the round buttons, $\frac{2}{3}$ have holes. What fraction of the buttons that Rory uses are round with holes? Explain your answer using the area model.
$\frac{1}{2}$
$\frac{2}{3}$
$\frac{2}{6}$ or $\frac{1}{3}$ of the buttons; Sample answer: $\frac{2}{3}$ is the right two columns of the whole. $\frac{1}{2}$ is the bottom row of the whole. So the whole has 6 equal parts. The overlap shows $\frac{2}{3}$ of $\frac{1}{2}$. This section is 2 parts out of the 6 parts in the whole rectangle that represents all the buttons.

15. Mrs. Mishler uses carpet squares to cover the floor. One carpet square measures $1\frac{1}{2}$ feet by $1\frac{1}{2}$ feet. If Mrs. Mishler uses 15 carpet squares, what is the total area of the floor? Show your work.
$33\frac{3}{4}$ square feet; Sample answer: Area of carpet square: $1\frac{1}{2} \times 1\frac{1}{2} = A$; $\frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$; Total area of the floor: $2\frac{1}{4} \times 15 = A$; $\frac{9}{4} \times 15 = \frac{135}{4} = 33\frac{3}{4}$

196 Assessment Resource Book

Form B

Unit 10
Unit Assessment, Form B

Name ______________________

1. Which is the product?

2. Which is the product?

3. Which is the area of the rectangle?

4. Which expressions have a product that is greater than the first factor? Choose all that apply.

A. 2 miles　B. 3 miles

6. Which multiplication is shown by the model?

11. What is the product? Use the representation to solve.

$\frac{7}{40}$ of the bread; Sample answer: $\frac{7}{8}$ is the right seven columns of the whole. $\frac{1}{5}$ is the top row of the whole. So the whole has 40 equal parts. The overlap shows $\frac{7}{8}$ of $\frac{1}{5}$. This section is 7 parts out of the 40 parts in the whole rectangle that represents all the bread loaves.

15. Ahmed makes a mosaic using square tiles. Each tile measures $1\frac{1}{3}$ centimeters by $1\frac{1}{3}$ centimeters. If Ahmed uses 15 tiles, what is the total area of his mosaic? Show your work.
$26\frac{6}{9}$ or $26\frac{2}{3}$ square centimeters; Sample answer: Area of square tile: $1\frac{1}{3} \times 1\frac{1}{3} = A$; $\frac{4}{3} \times \frac{4}{3} = \frac{16}{9} = 1\frac{7}{9}$. Total area of the mosaic: $1\frac{7}{9} \times 15 = A$; $\frac{16}{9} \times 15 = \frac{240}{9} = 26\frac{6}{9}$

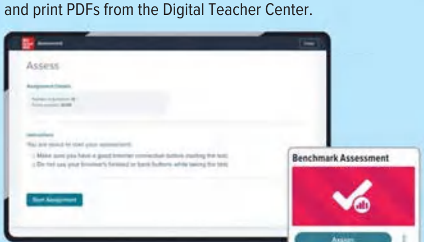
The Benchmark Assessment 3 is available in both print and digital.

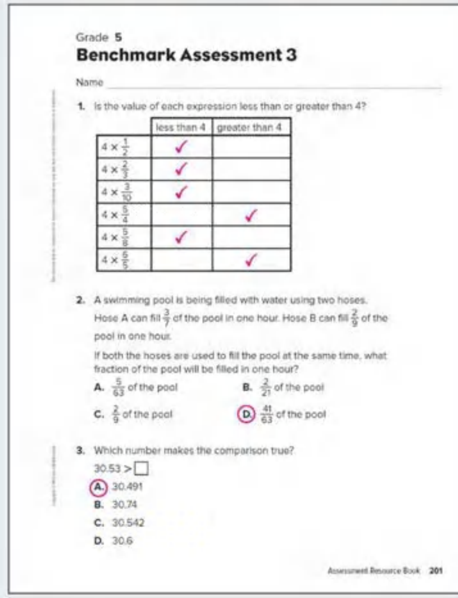
Data When students complete the Benchmark Assessment in the Digital Student Center, their responses are auto-scored.

Item Analysis

Item	DOK	Skill	Standard
1	3	Interpret multiplication of fractions as scaling	5.NF.B.5
2	2	Solve addition word problems involving fractions	5.NF.A.2
3	2	Compare decimals	5.NBT.A.3.b
4	2	Divide multi-digit numbers	5.NBT.B.6
5	2	Represent multiplication of fractions and whole numbers	5.NF.B.4
6	2	Divide decimals	5.NBT.B.7
7	2	Subtract fractions	5.NF.A.1
8	2	Multiply decimals	5.NBT.B.7
9	2	Add decimals	5.NBT.B.7
10	2	Use area models to multiply	5.NBT.B.5
11	2	Solve volume word problems	5.MD.C.5
12	3	Interpret multiplication of fractions as scaling	5.NF.B.5
13	2	Solve addition word problems involving fractions	5.NF.A.2
14	2	Solve division word problems involving decimals	5.NBT.B.7
15	2	Round decimals	5.NBT.A.4
16a	2	Use partial products to multiply	5.NBT.B.5
16b	2	Multiply multi-digit numbers	5.NBT.B.5
17	2	Use place-value patterns to divide decimals	5.NBT.B.7
18	2	Represent subtraction of decimals	5.NBT.B.7
19	2	Solve division word problems	5.NBT.B.6
20	2	Solve multiplication word problems involving fractions	5.NF.B.6
21a	2	Represent subtraction of fractions	5.NF.A.1
21b	2	Subtract fractions	5.NF.A.1

Assign the digital Benchmark Assessment to students or download and print PDFs from the Digital Teacher Center.





Grade 5

Benchmark Assessment 3

Name ______________________

1. Is the value of each expression less than or greater than 4?

	less than 4	greater than 4
$4 \times \frac{1}{2}$	✓	
$4 \times \frac{2}{3}$	✓	
$4 \times \frac{3}{10}$	✓	
$4 \times \frac{5}{4}$		✓
$4 \times \frac{5}{8}$	✓	
$4 \times \frac{6}{5}$		✓

2. A swimming pool is being filled with water using two hoses. Hose A can fill $\frac{3}{7}$ of the pool in one hour. Hose B can fill $\frac{2}{9}$ of the pool in one hour.

If both the hoses are used to fill the pool at the same time, what fraction of the pool will be filled in one hour?

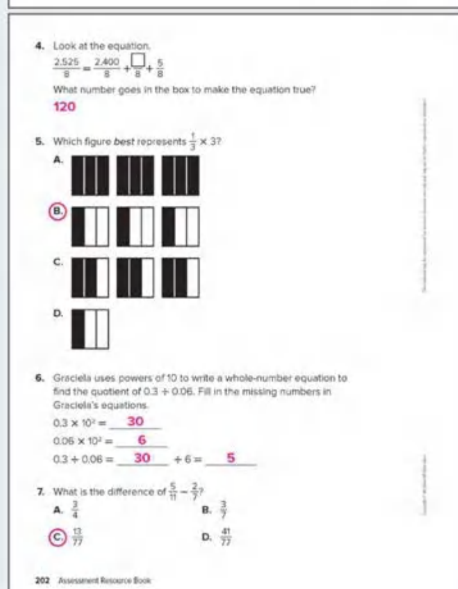
A. $\frac{5}{63}$ of the pool
B. $\frac{2}{21}$ of the pool
C. $\frac{2}{9}$ of the pool
D. $\frac{41}{63}$ of the pool

3. Which number makes the comparison true?

$30.53 > \square$

A. 30.491
B. 30.74
C. 30.542
D. 30.6

Assessment Resource Book 201



4. Look at the equation.

$\frac{2{,}525}{8} = \frac{2{,}400}{8} + \frac{\square}{8} + \frac{5}{8}$

What number goes in the box to make the equation true?

120

5. Which figure *best* represents $\frac{1}{3} \times 3$?

A.

B.

C.

D.

6. Graciela uses powers of 10 to write a whole-number equation to find the quotient of $0.3 \div 0.06$. Fill in the missing numbers in Graciela's equations.

$0.3 \times 10^2 =$ 30

$0.06 \times 10^2 =$ 6

$0.3 \div 0.06 =$ 30 $\div 6 =$ 5

7. What is the difference of $\frac{5}{11} - \frac{2}{7}$?

A. $\frac{3}{4}$
B. $\frac{3}{7}$
C. $\frac{13}{77}$
D. $\frac{41}{77}$

202 Assessment Resource Book

Grade 5
Benchmark Assessment 3 (continued)

Name ______________________

8. What is the product?

6.5×0.04

A. 0.026 **B.** 0.26 **C.** 2.60 **D.** 26.0

9. Which expression can be used to find the sum of $7.03 + 0.54$?

A. $1.2 + 0.07$ **B.** $0.12 + 0.04$ **C.** $7 + 0.8 + 0.04$ **D.** $7 + 0.5 + 0.07$

10. Humberto uses an area model to solve a multiplication problem.

	80 +	?
40 +	3200	120
?	720	27

What product does the area model represent?

49 × **83** = **4,067**

11. Jana buys a rectangular storage crate that has a volume of 12 cubic feet.

Which dimensions could Jana's storage crate be? Choose all that apply.

A. 2 feet × 2 feet × 8 feet
B. 3 feet × 2 feet × 2 feet
C. 4 feet × 3 feet × 1 foot
D. 6 feet × 2 feet × 4 feet
E. 1 foot × 6 feet × 2 feet

12. Read the following statement.

12 × b is greater than 12, but less than 24.

Which value of *b* makes this statement true?

A. $\frac{1}{2}$ **B.** $\frac{2}{3}$ **C.** $1\frac{1}{2}$ **D.** $2\frac{1}{3}$

13. Rita walks $\frac{2}{5}$ mile to the store. Then she walks $\frac{3}{7}$ mile to the community center.

How far does Rita walk altogether?

A. $\frac{6}{35}$ mile **B.** $\frac{29}{35}$ mile **C.** $\frac{5}{12}$ mile **D.** $\frac{17}{12}$ mile

14. Stu has 2.19 pounds of ground beef to make 6 burger patties. He uses the same amount of ground beef for each patty.

How much ground beef does Stu use to make each burger patty?

0.365 pound

15. What is 223.754 rounded to the nearest tenth?

223.8

16a. What are the partial products of 45×34? Choose all that apply.

A. 20 **B.** 120 **C.** 150 **D.** 160 **E.** 200 **F.** 1,200

16b. What is the product?

$45 \times 34 =$ **1,530**

Grade 5
Benchmark Assessment 3 (continued)

Name ______________________

17. Write the quotient for each division problem.

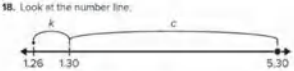
$0.053 \div 0.01 =$ **5.3**

$1.8 \div 0.01 =$ **180**

$0.002 \div 0.1 =$ **0.02**

$4 \div 0.1 =$ **40**

18. Look at the number line.



What are the values of *k* and *c*?

$k =$ **0.04**

$c =$ **4**

19. There are 4,572 people at a stadium. The same number of people are sitting in each of 12 sections in the stadium.

How many people are sitting in each section?

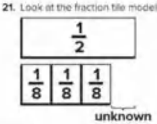
A. 4,584 people
B. 4,560 people
C. 381 people
D. 306 people

20. John travels $7\frac{5}{6}$ miles to reach school. Albert travels $\frac{1}{3}$ the number of miles John travels to reach the same school.

How far does Albert travel to school?

A. $2\frac{11}{18}$ miles
B. $7\frac{5}{18}$ miles
C. $7\frac{1}{2}$ miles
D. $8\frac{1}{6}$ miles

21. Look at the fraction tile model.



a. Which expression does the model represent?

A. $\frac{1}{2} + \frac{3}{8}$ **B.** $\frac{1}{2} - \frac{3}{8}$ **C.** $\frac{1}{2} + \frac{3}{24}$ **D.** $\frac{1}{2} - \frac{3}{24}$

b. What is the value of the unknown in the model?

$\frac{1}{8}$

UNIT 11 PLANNER

Divide Fractions

PACING: 10 days

LESSON		MATH OBJECTIVE	LANGUAGE OBJECTIVE	SOCIAL AND EMOTIONAL LEARNING OBJECTIVE
Unit Opener	IGNITE! Number Strings	Students use patterns to divide with fractions.		
11-1	Relate Fractions to Division	Students represent the quotient to a division equation as a fraction or mixed number.	Students talk about relating fractions to division with the gerund *using*.	Students discuss and practice positive strategies for managing emotional reactions to stressful situations.
11-2	Solve Problems Involving Division	Students determine whether a quotient should be written with a remainder or as a mixed number.	Students discuss whether a quotient should be written with a remainder or as a mixed number using *apply*.	Students exercise creativity by solving a problem using more than one approach.
11-3	Represent Division of Whole Numbers by Unit Fractions	Students use representations to divide whole numbers by unit fractions.	Students talk about using representations to divide whole numbers by unit fractions using *can* and *should*.	Students collaborate with peers to solve a mathematical problem.
11-4	Divide Whole Numbers by Unit Fractions	Students use the meaning of multiplication as equal groups to divide whole numbers by unit fractions.	Students discuss if a calculated quotient is correct using a related multiplication equation using *should*, *might*, and *could*.	Students identify and use mathematical tools to organize work.
11-5	Represent Division of Unit Fractions by Non-Zero Whole Numbers	Students use representations to divide unit fractions by non-zero whole numbers.	Students explain how to use representations to divide unit fractions by non-zero whole numbers using *similar* and *related*.	Students determine the representations and analyses necessary to make informed decisions when engaging in mathematical practices.
11-6	Divide Unit Fractions by Non-Zero Whole Numbers	Students extend their understanding that dividing by a whole is the same as multiplying by a unit fraction to divide unit fractions by whole numbers.	Students explain if a calculated quotient is correct using *different* and *related*.	Students demonstrate self-awareness of personal strengths and areas of challenge in mathematics.
Math Probe Which Expressions Represent the Situation?		Students choose expressions that can be used to solve problems involving division and fractions.		
11-7	Solve Problems Involving Fractions	Students solve word problems involving division of fractions using strategies such as using fraction models.	Students discuss solving word problems involving division of fractions using different strategies, using *another way*.	Students advocate for their mathematical problem solving and adjust their understanding based on constructive feedback.
Unit Review **Fluency Practice**				
Unit Assessment **Performance Task**				

FOCUS QUESTION:
How can I divide fractions?

LESSON	KEY VOCABULARY		MATERIALS TO GATHER	RIGOR FOCUS	STANDARD
	Math Terms	Academic Terms			
11-1	denominator dividend divisor numerator quotient	prove reflect	• fraction circles • number cubes	Conceptual Understanding, Application	5.NF.B.3
11-2	mixed number quotient remainder	analyze reflect	• number cards • *Problem-Solving Tool* Teaching Resource	Application	5.NF.B.3
11-3	division fraction model unit fraction	evaluate reflect	• fraction circles • fraction tiles • number cube	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.7, 5.NF.B.7.b
11-4	dividend division divisor unit fraction	arguably speculate	• spinners • *Unit Fractions & Whole Numbers* Teaching Resource	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.7, 5.NF.B.7.b
11-5	division fraction model unit fraction	analyze suggest	• *Dividing Fractions Puzzle Pieces* Teaching Resource	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.7, 5.NF.B.7.a
11-6	dividend division divisor unit fraction	accurate evaluate	• fraction circles • *Unit Fractions and Whole Numbers* Teaching Resource	Conceptual Understanding, Procedural Skill & Fluency	5.NF.B.7, 5.NF.B.7.a
11-7	equation unknown variable	establish relevant	• *Problem-Solving Tool* Teaching Resource	Conceptual Understanding, Procedural Skill & Fluency, Application	5.NF.B.7, 5.NF.B.7.c

Unit Overview

Focus

Divide Fractions

This unit builds on earlier work with division and fractions to establish that a fraction describes an indicated division. Students divide fractions, limited to division of a whole number by a unit fraction and division of a unit fraction by a non-zero whole number. They explore situations involving *equal sharing* division and *equal grouping* division. Students use models to help determine quotients. Formal procedures for dividing fractions and mixed numbers are developed in Grade 6. In making connections between division and fractions, students write equations with fractions to describe division situations.

Students use different representations when they find the quotient of a unit fraction divided by a whole number in equal-sharing situations.

They observe patterns, but some still have difficulty comprehending how a quotient can be greater than the dividend. The use of simple models illustrates how dividing a whole number (the dividend) by a unit fraction (the divisor) involves finding the number of small pieces that fit into a larger piece (the whole). In such situations, the number of little pieces that fit in is greater than the dividend. A common misconception is that students may interpret a problem such as $8 \div \frac{1}{4}$ as 8 *being equally divided* into 4 parts. To verify their results, students can use the relationship between multiplication and division to check their answers.

Coherence

What Students Have Learned

- Students used the four operations to solve word problems. (Grade 4)
- Students interpreted a multiplication equation as a comparison. (Grade 4)
- Students multiplied or divided to solve word problems. (Grade 4)
- Students multiplied a fraction by a whole number. (Grade 4)

What Students Are Learning

- Students interpret a fraction as division of the numerator by the denominator.
- Students divide unit fractions by whole numbers and whole numbers by unit fractions.
- Students divide a unit fraction by a non-zero whole number.
- Students divide a whole number by a unit fraction.
- Students solve real-world problems involving division with unit fractions and whole numbers.

What Students Will Learn

- Students use ratio language to describe a ratio relationship between two quantities. (Grade 6)
- Students understand the concept of a unit rate and describe a ratio relationship. (Grade 6)
- Students compute quotients of fractions. (Grade 6)
- Students solve word problems involving division of fractions. (Grade 6)

Rigor

Conceptual Understanding

Students develop understanding of
- fractions as division of the numerator by the denominator;
- dividing unit fractions by non-zero whole numbers;
- dividing whole numbers by unit fractions.

Procedural Skill & Fluency

Students build proficiency with
- dividing unit fractions by non-zero whole numbers;
- dividing whole numbers by unit fractions.

Application

Students apply their knowledge of
- dividing unit fractions by non-zero whole numbers and whole number by unit fractions to solve real-world problems.

Effective Teaching Practices

Implement Tasks That Promote Reasoning and Problem Solving

This unit presents opportunities for students to use reasoning to make sense of quantities and operations. The lessons present division with unit fractions in a concrete, visual way, encouraging analysis and deep understanding.

As you progress through the lessons, make sure that students look closely at the visual models and fully understand what they represent. When dividing, ask students to explain how the visual representation shows the equal-sharing process throughout each step.

This in-depth and repeated analysis gives students the best chance of taking meaning from the step-by-step representations as they are developed, and to transfer their understanding as they make their own visual models.

To promote reasoning, provide problems that students have already completed. When a problem presents itself where the solution strategy is dividing a whole number by a fraction, ask students to predict how the quotient will be affected by dividing a larger or smaller whole number by the same unit fraction. Similarly, have them predict how the quotient will be affected if the same whole number is divided by a larger or smaller unit fraction.

When students develop their own representations, ask them to explain the reasoning behind each step. Have students discuss representations and solutions with partners, small groups, and the full class to provide varied levels of independence and style of explanation. Explaining their own reasoning helps solidify students' awareness of their own strategies and helps them discover any gaps in their understanding of concepts and procedures.

As students solve problems and explain their reasoning, encourage them to share differences in how they perceive problems and representations.

Math Practices and Processes

Look for and Express Regularity in Repeated Reasoning

Encourage students to take note of how their solution to one problem relates to the next problem. Students should notice, for example, that each time they divide a unit fraction by a non-zero whole number, they can predict how the visual representation will look and that the quotient will be less than the dividend. On the other hand, each time they divide a whole number by a unit fraction (other than by $\frac{1}{1}$), the quotient will be greater than the dividend.

Throughout the unit, students express regularity in repeated reasoning when they apply to fractions the do-undo relationship between division and multiplication that they have used with whole numbers. They write a related multiplication equation for a division equation, knowing that the related equation expresses the same relationship by using the numbers from the same *fact family*.

Provide consistent opportunities for students to build their reasoning skills. Some suggestions include:

- Students in pairs examine two problems that both require dividing a fraction by a non-zero whole number. Each partner represents the problem visually and writes an equation. Then, partners explain their reasoning to one another.
- Students in pairs examine these two problems: one dividing a unit fraction by a non-zero whole number; the other dividing the same whole number by the same unit fraction. Each partner represents the problem visually and with an equation, and then partners discuss any patterns that they notice.
- Students show division with a fraction using fraction tiles. Then, individual students represent the same division using another representation. Students explain how their representations are the same and how they are different.

Social and Emotional Learning

Self-Management – Manage Stress (Lesson 11-1): Students who can regulate their stress are resilient and better prepared for academic success.

Self-Awareness – Recognize Strengths (Lesson 11-2): When students recognize their own strengths, they can see themselves as resourceful and may be more willing to attempt to problem solve and help others.

Social Awareness – Respect Others (Lesson 11-3): When students are respectful of one another, they strengthen their class community.

Self-Management – Organizational Skills (Lesson 11-4): Organizing information and work can help students work through challenging mathematical tasks.

Responsible Decision-Making – Evaluate (Lesson 11-5): When students evaluate their own logic and reasoning, they can develop understanding that helps them make informed decisions.

Self-Awareness – Accurate Self-Perception (Lesson 11-6): Having accurate self-perception allows students to determine areas of strength as well as areas in which they need to focus and practice.

Relationship Skills – Communication (Lesson 11-7): Students who can communicate effectively are more likely to build strong relationships and contribute to a positive classroom culture.

Unit Overview

Language of Math

Vocabulary

Students will be using these key terms in this unit:

- **Denominator** (Lesson 11-1): Students were introduced to this term in Grade 3. Emphasize that the denominator tells the number by which the whole has been divided into equal parts.
- **Dividend** (Lesson 11-1): Students were also introduced to this term with of division of whole numbers. Encourage them to use division terminology consistently and discuss the meaning of each word.
- **Divisor** (Lesson 11-1): Students were also introduced to this term with division. Encourage them to notice the action meaning of *divisor*; the *-or* ending indicates that this word applies to the active part of an equation, the number *doing the dividing*.
- **Numerator** (Lesson 11-1): Students were also introduced to this term in Grade 3. Emphasize that the numerator tells how many fractional parts are under consideration.
- **Quotient** (Lesson 11-1): Students were also introduced to this term with division. Help students remember which term is the quotient– mention that the dividend and divisor, the two "D words," are on one side of the equation; the quotient is on the other side.

Math Language Development

A Focus on Fraction and Division Vocabulary

Throughout this unit, students will use many mathematical terms with which they should already be familiar.

Emphasize terminology in the context of division with fractions. Although students are familiar with terms such as *quotient* and *numerator*, encourage them to use those terms consistently as they work with division and fractions.

The idea that the quotient for a division problem can be a fraction may be new to many students.

Encourage students to use language such as $\frac{3}{4}$ means *3 divided by 4* or *3 divided into 4 equal groups* to promote the connection.

Many students believe that *division* means "to make smaller." Although dividing a unit fraction by a whole number (greater than 1) "makes smaller" because the unit fraction is split into equal groups, emphasize that dividing a whole number (greater than 1) by a unit fraction (less than 1) results in a quotient that is greater than the whole number because we are finding how many copies of the unit fraction are needed to make the whole number.

Some suggestions include:

- Have pairs write a division equation and make a drawing showing objects represented by the division. Have students label the equation and drawing with the terms *dividend, divisor,* and *quotient*. Have them write a related multiplication equation, labeling the *factors* and the *product*.
- Have students write a division problem such as $3 \div 4$ (where the dividend is less than the divisor). Then have them draw a representation to show the quotient as the fraction. Have students make connections between the dividend 3 and the numerator 3, and between the divisor 4 and the denominator 4. Repeat with a division problem where the dividend is greater than the divisor.

English Language Learner

In this unit, students are provided with a number of scaffolds to support their comprehension of the language used to present and explain strategies related to dividing fractions. Because many of the words (*pour, faces*) and phrases (*full sheets, 5-foot board, serving size, share among, can be rewritten*) used in this unit could prove challenging to ELs, they are supported in understanding and using them so that the instruction is more accessible.

Lesson 11-1 – *pour*

Lesson 11-2 – *full sheets*

Lesson 11-3 – *5-foot board*

Lesson 11-4 – *serving size*

Lesson 11-5 – *share among equally*

Lesson 11-6 – *can be rewritten*

Lesson 11-7 – *faces*

Unit Routines

Number Routines

Build Fluency The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the thinking habits of mind that are important for proficient doers of math.

Greater Than or Less Than

Purpose: Build proficiency with number and place value sense.

Overview: Students use mental math to estimate or evaluate the value of given expressions and then compare the value of the expressions to a target benchmark number. Students share their solutions and thinking.

Where Does It Go?

Purpose: Build estimating skills using benchmarks.

Overview: Students place a target number on number lines with different endpoints and justify their placement.

About How Much?

Purpose: Build estimating skills.

Overview: Students estimate the value of expressions (with operations) shown, explaining their strategies and thinking. The teacher records students' estimates, then reveals the value of the expression. Students analyze the estimates and discuss which are closest to the actual value of the expression.

What's Another Way to Write It?

Purpose: Build flexibility with number sense and mental math operations.

Overview: Given a number, students generate expressions using operations that, when evaluated, have the same value as the number. The teacher records expressions as students share. Students then look for relationships amongst the expressions.

Sense-Making Routines

- **Is it always true?** (Lesson 11-6) Students think about the meanings of multiplication and division and determine whether the relationship is unique or holds for every situation.
- **Notice & Wonder: How are they the same? How are they different?** (Lesson 11-7) Students think about whether the context of the problem can be represented by a whole number divided by a whole number, whole number divided by a fraction, or a fraction divided by a non-zero whole number.
- **Notice & Wonder: What do you notice? What do you wonder?** (Lessons 11-1, 11-3, 11-4) In Lesson 11-1, students notice a whole being broken into parts to make equal shares. In Lesson 11-3, students connect division of whole numbers by unit fractions to real-world situations. In Lesson 11-4, students think about grouping fractional parts to make wholes.
- **Numberless Word Problem** (Lesson 11-2) Students are presented with a problem that would involve division if it had numbers. Students then consider a solution strategy and what numbers could be in the problem.
- **Which doesn't belong?** (Lesson 11-5) Students explore unit fractions being partitioned into equal parts.

Math Language Routines

The Mathematical Language Routines used in this unit give teachers a structured, yet adaptable format for amplifying and developing students' social and academic language. For more information on the Math Language Routines, see the Appendix.

- Lesson 11-1: Students participate in MLR7: Compare and Connect so that students' oral and written output can be fostered as they compare different ways to solve and represent division problems.
- Lesson 11-2: Students participate in MLR1: Stronger and Clearer Each Time so that they have a structured and interactive opportunity to revise and refine their ideas while representing and solving division problems.
- Lesson 11-3: Students participate in MLR2: Collect and Display so that students' oral words and phrases can be captured into a stable, collective reference as they discuss dividing whole numbers by unit fractions.
- Lesson 11-4: Students participate in MLR8: Discussion Supports so that students' meta-awareness can be fostered as they discuss what they know in order to divide whole numbers by unit fractions.
- Lesson 11-5: Students participate in MLR3: Critique, Correct, and Clarify so that they have an opportunity to analyze, reflect on, and develop a piece of mathematical writing pertaining to dividing unit fractions by non-zero whole numbers that is not their own.
- Lesson 11-7: Students participate in MLR5: Co-Craft Questions and Problems so that they can produce the language of mathematical questions related to dividing fractions.

Readiness Diagnostic

Unit 11

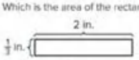
How Ready Am I?

Name ____________________

1. Which best describes $\frac{3}{5}$?
   - A. 3 groups of 5 objects
   - B. 5 groups of 3 objects
   - C. 3 wholes divided into 5 equal shares
   - D. 5 wholes divided into 3 equal shares
2. Which fraction represents 5?
   - A. $\frac{5}{5}$
   - B. $\frac{10}{2}$
   - C. $\frac{10}{5}$
   - D. $\frac{1}{5}$
3. Mary has 48 cards. She puts them into 5 piles. How many cards are left over?
   - A. 9 cards
   - B. 6 cards
   - C. 3 cards
   - D. 5 cards
4. Which mixed number is equivalent to $\frac{13}{4}$?
   - A. $2\frac{3}{4}$
   - B. $3\frac{1}{13}$
   - C. $3\frac{1}{4}$
   - D. $3\frac{3}{4}$
5. Which fraction is equivalent to $6\frac{3}{5}$?
   - A. $\frac{14}{5}$
   - B. $\frac{23}{5}$
   - C. $\frac{23}{3}$
   - D. $\frac{33}{5}$

Assessment Resource Book 207

6. Which is the product $\frac{1}{3} \times 4$?
   - A. $\frac{1}{12}$
   - B. $\frac{4}{12}$
   - C. $\frac{4}{3}$
   - D. $4\frac{1}{3}$
7. Which is the product $4 \times \frac{2}{5}$?
   - A. $4\frac{2}{5}$
   - B. $\frac{8}{5}$
   - C. $\frac{8}{20}$
   - D. $\frac{1}{10}$
8. Which is the area of the rectangle?



   - A. $\frac{1}{3}$ square inch
   - B. $\frac{2}{3}$ square inch
   - C. $2\frac{1}{3}$ square inches
   - D. $4\frac{2}{3}$ square inches
9. Which is the product of $\frac{3}{5} \times \frac{3}{5}$?
   - A. $\frac{9}{25}$
   - B. $\frac{9}{5}$
   - C. $\frac{6}{25}$
   - D. $\frac{6}{5}$
10. Which is the product of $\frac{3}{8} \times \frac{3}{4}$?
   - A. $\frac{6}{12}$
   - B. $\frac{9}{8}$
   - C. $\frac{12}{24}$
   - D. $\frac{9}{32}$

208 Assessment Resource Book

Administer the Readiness Diagnostic to determine your students' readiness for this unit.

Targeted Intervention

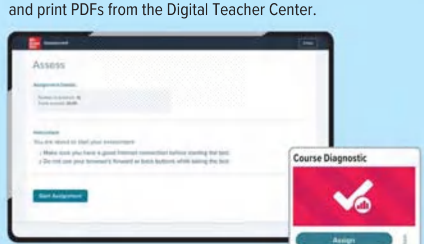
Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	1	Understand fractions as equal groups	Understand Unit Fractions	3.NF.A.1
2	1	Represent whole numbers as fractions	Fractions with Numerators Greater Than 1	3.NF.A.3.c
3	2	Interpret remainders	Interpret Remainders in Word Problems	4.OA.A.3
4	1	Change fractions to mixed numbers	Decompose Fractions into Sums	4.NF.B.3
5	1	Change mixed numbers to fractions	Decompose Fractions into Sums	4.NF.B.3
6	1	Multiply fractions by whole numbers	Unit Fraction By Whole Number (Models)	4.NF.B.4.b
7	1	Multiply fractions by whole numbers	Fraction by Whole Number (Models)	4.NF.B.4.b
8	1	Multiply fractions by whole numbers	Fraction by Whole Number (Models)	4.NF.B.4.b
9	1	Multiply fractions by fractions	Multiply Two Fractions	5.NF.B.4
10	1	Multiply fractions by fractions	Multiply Two Fractions	5.NF.B.4

GO ONLINE

Assign the digital Readiness Diagnostic to students or download and print PDFs from the Digital Teacher Center.



Focus Question

Introduce the Focus Question: *How can I divide fractions?*
Ask students to think about what they know about dividing fractions.

- What does it mean to divide?
- What do you know about fractions?

Remind students that at the end of the unit, they will reflect back on what they learned.

Family Letter

Each letter presents an overview of the math in the unit and home activities to support student learning.

STEM in Action

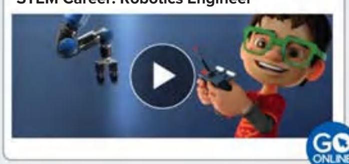
Videos

Students can watch the two STEM videos.

STEM Career: Robotics Engineer Antonio about his aspirations to be robotics engineer.

Antonio Divides Fractions Students watch to see how Antonio uses division of a whole number by a unit fraction to determine how long a robot's batteries will last.

STEM Career: Robotics Engineer



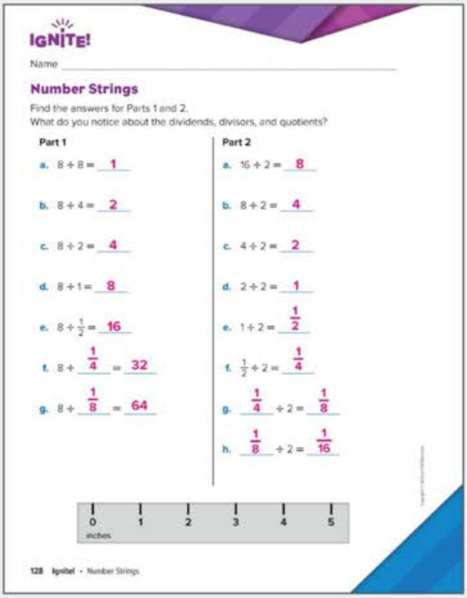
Antonio Divides Fractions

STEM Project Card

Students can complete the STEM project during their workstation time.

STEM Adventure

Students can complete the STEM adventure during their workstation time.



IGNITE!

Name ____________

Number Strings

Find the answers for Parts 1 and 2.
What do you notice about the dividends, divisors, and quotients?

Part 1	Part 2
a. $8 \div 8 = $ 1	a. $16 \div 2 = $ 8
b. $8 \div 4 = $ 2	b. $8 \div 2 = $ 4
c. $8 \div 2 = $ 4	c. $4 \div 2 = $ 2
d. $8 \div 1 = $ 8	d. $2 \div 2 = $ 1
e. $8 \div \frac{1}{2} = $ 16	e. $1 \div 2 = $ $\frac{1}{2}$
f. $8 \div $ $\frac{1}{4}$ $=$ 32	f. $\frac{1}{2} \div 2 = $ $\frac{1}{4}$
g. $8 \div $ $\frac{1}{8}$ $=$ 64	g. $\frac{1}{4}$ $\div 2 = $ $\frac{1}{8}$
	h. $\frac{1}{8}$ $\div 2 = $ $\frac{1}{16}$

0 1 2 3 4 5
inches

128 Ignite! • Number Strings

Ignite!

Number Strings

Students analyze number strings to set the stage for learning how to divide with unit fractions.

1. Mention that a number string is a set of related math problems. Direct students to Part 1 on the *Student Edition* page.
2. Have students study the answers for patterns.
   - What do you notice about the dividends, divisors, and quotients in problems a–d?
3. Now ask students to examine problem e in Part 1.
   - Why is $\frac{1}{2}$ the divisor in problem e?
   - Describe a strategy to solve problem e.
4. Have students use the patterns they discovered in the number string to solve problems f and g. As needed, assist students.
5. Explore a common misconception about division.
   - When you divide, do you think that the quotient will always be less than or equal to the dividend? Explain.
   - How do the quotients in problems e–g of Part 1 compare to their dividends?
   - Why do you suppose the quotients in problems e–g are each greater than their dividend, 8?
6. Have students use the ruler at the bottom of the page to consider the problem $5 \div \frac{1}{2}$
   - Do you think the quotient for $5 \div \frac{1}{2}$ will be greater than 5? Use the ruler to help you find how many $\frac{1}{2}$-inch segments fit into 5.
7. Have students complete Part 2, problems a–d. Then ask students to look for patterns.
   - What do you notice about the divisors, dividends, and quotients in problems a–d?
8. Ask students to complete the rest of Part 2.
   - Use the patterns you have discovered to solve problems e–h. How is problem e different from the problems a–d?

Workstations

Reveal Math offers rich and varied resources that teachers can use to differentiate and enrich students' instructional experiences with the unit content. The table presents an overview of the resources available for the unit with recommendations for when to use.

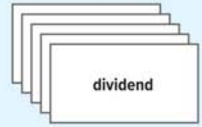
Station	Activity	Description	Use After Lesson
Game Station	Game Station	Students build proficiency with the division of fractions and whole numbers.	
		• **Fractions as Division Four in a Row**	11-1
		• **Fractions as Division Four in a Row**	11-2
		• **Represent Division of Whole Numbers by Unit Fractions Concentration**	11-3
		• **Dividing Whole Numbers by Unit Fractions Bingo**	11-4
		• **Fraction Division Match, Concentration, and Showdown**	11-5
		• **Fraction Division Bingo**	11-6
		• **Dividing Fractions Race**	11-7
Digital Station	Digital Game	**Dino Dig** Students practice multiplying using area models.	11-1
Application Station	**Have students complete at least one of the Use It! activities for this unit.**		
	STEM Project Card	**How Fast Is Your Robot?** Students create a robot and measure the distance it travels. They experiment with techniques to increase the distance and speed.	11-4
	Connection Card	**Potluck with a Twist** Students use whole numbers, fractions, and division to create a 14-dish menu for a potluck.	11-7
	Real World Card	**Can You Hear Me?** Students compare and contrast face-to-face and online communication.	11-1

Additional Resources

Use the resources below to provide additional support for this unit.

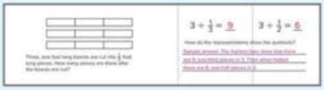
Vocabulary

Use the vocabulary cards to help students learn the vocabulary in this unit. Encourage students to write their own definitions of the key terms on the front side of the card.



Foldables

Use the unit foldables with Lessons 11-3 and 11-5.



Spiral Review

Students can complete the Spiral Review at any point during the unit as either a paper-and-pencil or digital activity.

Lesson	Standard
11-1	5.NF.B.6
11-2	5.NF.B.4
11-3	5.NF.B.5
11-4	5.NF.B.3
11-5	5.MD.C.5
11-6	5.NF.B.7
11-7	5.NF.A.1

LESSON **11-1**

Relate Fractions to Division

Learning Targets

- I can represent the quotient of a division equation as a fraction or mixed number.
- I can explain why the quotient of a division equation can always be expressed as a fraction.
- I can explain why division of whole numbers can be written as a multiplication expression.

Standards ◇ Major △ Supporting ○ Additional

Content

◇**5.NF.B.3** Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b} = a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Math Practices and Processes

MPP Model with mathematics.

Vocabulary

Math Terms	Academic Terms
denominator	prove
dividend	reflect
divisor	
numerator	
quotient	

Materials

The materials may be for any part of the lesson.

- fraction circles
- number cubes

Focus

Content Objectives	Language Objectives	SEL Objective
• Students represent the quotient to a division equation as a fraction or mixed number. • Students explain why the quotient of a division equation can be expressed as a fraction. • Students explain why division of whole numbers can be written as a multiplication expression.	• Students discuss relating fractions to division with the gerund *using*. • To support sense-making, ELs participate in MLR7: Compare and Connect.	• Students discuss and practice positive strategies for managing emotional reactions to stressful situations.

Coherence

Previous	Now	Next
• Students interpreted a multiplication expression as a comparison (Grade 4). • Students divided decimals to hundredths (Unit 8).	• Students extend their understanding of fractions to understand fractions as division.	• Students solve word problems and determine how the quotient should be written (Unit 11). • Students understand the concept of a unit rate (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of fractions as another way to write a division expression.	• Students build proficiency with division of whole numbers that result by dividing the numerator by the denominator. *Procedural Skill & Fluency is not a specific element of rigor for this standard.*	• Students solve problems involving division of whole numbers that result in a quotient that is a fraction or mixed number.

GO ONLINE

Number Routine
Greater Than or Less Than 5–7 min

Build Fluency Students build number sense as they determine whether estimated sums of fractions are greater than or less than a benchmark.

Remind students that this is a mental activity, and that exact sums are not needed.

These prompts encourage students to talk about their reasoning:

- What do you notice about the fractions in each expression?
- What is the same and different about the two expressions?
- What strategy did you use to estimate the size of each fraction and of each sum?
- How do you know if your estimate is reasonable?

Launch 5–7 min

Purpose Students explore how they can divide wholes into parts to make equal shares.

Notice & Wonder™

- What do you notice?
- What do you wonder?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students discuss with partners if they have ever experienced a situation like the one shown in the image.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of relating fractions to division and are based on possible comments and questions that students may make during the share out.

- Can the people share the sandwiches now? Why or why not?
- How do you think the people could share the sandwiches?

Math is... Mindset

- What are some ways you can avoid stress?

SEL Self-Management: Manage Stress

After students have completed the Notice & Wonder routine, invite them to share what may have caused them stress. For example, students may have experienced stress if they did not understand a peer's reasoning for making equal parts or struggled with thinking about how to make equal parts. Discuss ways students can avoid that stress in the future as well as how they can manage or relieve it now. Strategies such as developing a manageable plan, getting organized, taking breaks, and asking for help can help students manage emotional reactions to stress.

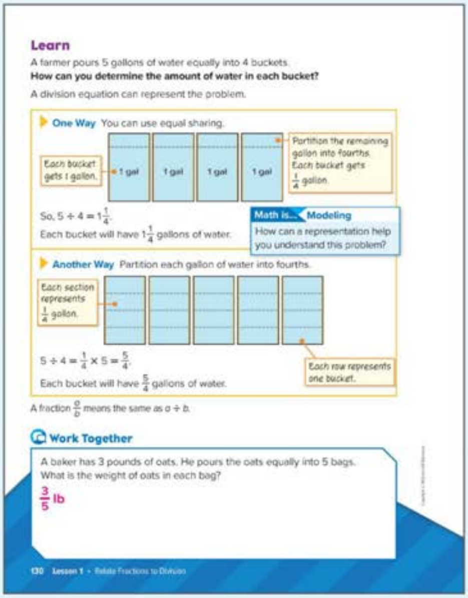
Transition to Explore & Develop

Ask questions that get students thinking about the connections among making equal shares, fractions, and division.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can relate division and fractions.

Explore & Develop 20 min



Learn

A farmer pours 5 gallons of water equally into 4 buckets.

How can you determine the amount of water in each bucket?

A division equation can represent the problem.

One Way You can use equal sharing.

So, $5 \div 4 = 1\frac{1}{4}$.

Each bucket will have $1\frac{1}{4}$ gallons of water.

Math is... Modeling
How can a representation help you understand this problem?

Another Way Partition each gallon of water into fourths.

$5 \div 4 = \frac{1}{4} \times 5 = \frac{5}{4}$.

Each bucket will have $\frac{5}{4}$ gallons of water.

A fraction $\frac{a}{b}$ means the same as $a \div b$.

Work Together

A baker has 3 pounds of oats. He pours the oats equally into 5 bags. What is the weight of oats in each bag?

$\frac{3}{5}$ lb

130 Lesson 1 • Relate Fractions to Division

1 Pose the Problem

ETP Pose Purposeful Questions

- What are you trying to determine?
- Do you need to know the capacity of the buckets? Why or why not?
- How can you figure out what operation to use?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and give them an equation to solve similar to the one on the Learn page. Instruct one student to represent by equal sharing and the other by partitioning. Then have them compare their strategies. Revisit this activity throughout the lesson to help students build proficiency.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you determine the quotient of whole numbers using a fraction?
- How can you determine the quotient of whole numbers using multiplication?

Key Takeaways

- A fraction $\frac{a}{b}$ can be interpreted as meaning $a \div b$.
- The quotient of a division equation can be represented as a fraction or mixed number.
- Division of whole numbers is the same as multiplying by a unit fraction; the denominator of the unit fraction is the same as the divisor.

Work Together

Students solve a problem involving division that can be written as a fraction by interpreting division as a fraction, by partitioning, or by equal shares. Have students attempt all 3 approaches.

Common Error: Students may initially be confused as the amount they are dividing is less than the amount they are dividing by. Remind them that they can still write a division equation or fraction that is equal to a number less than 1.

LOM Language of Math

Explain to students that *division* comes from the Latin word *dividere*, which means “to force apart.” Students can use their knowledge of the word’s origin to determine the meaning of other words that come from *dividere*, such as *divisive* or *individual*.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how to express a division equation involving whole numbers using fraction circles and equations.

Materials: fraction tiles or fraction circles

Directions: Distribute a set of fraction tiles or circles to each pair of students. Have students solve the Pose the Problem using any strategy they choose.

Math is... Modeling

- How can a representation help you understand this problem?

Students consider what a model tells them and assess how it will or will not help them.

ETP Support Productive Struggle

- How do you know which number to represent as the numerator?
- How do you know which number to represent as the denominator?
- What does the fraction tell you about how many gallons of water are in each bucket?
- Are there other ways you could write the fraction?

Activity Debrief: Invite students to share the steps they took to explore the problem and solution. Have students discuss and share how their work is similar and different from each other's work. Facilitate a discussion to ensure students understand that any division of whole numbers can be written as a fraction and vice versa. In addition, they understand dividing by a whole number is the same as multiplying by a unit fraction with the whole number as the denominator.

Guided Exploration

Students explore how division of whole numbers can be related to fractions.

ETP Use and Connect Mathematical Representations

Have the students perform the equal sharing. Ask:

- If you put 1 gallon into each bucket, what do you notice?
- How can you share the remaining water?
- What fraction can you use to partition the remaining water?
- What is the total amount of water in each bucket?

Have the students perform the partitioning. Ask:

- Into how many equal parts should you partition each gallon of water? How do you know?
- How can you represent the number of gallons in each bucket?
- How many one-fourths will be in each bucket?
- **Think About It:** How can you show 5 one-fourths using a multiplication expression?
- **Think About It:** How is partitioning similar to equal sharing? How is it different?

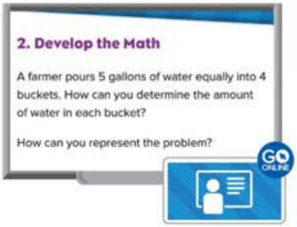
Have students think about the two representations of $5 \div 4$. Ask:

- What equation was the result of the first representation?
- What equation was the result of the second representation?
- How are the equations the same? How are they different? What conclusions can you draw?

Math is... Modeling

- How can a representation help you understand this problem?

Students consider what a model tells them and assess how it will or will not help them.



EL English Learner Scaffolds

Entering/Emerging Ensure comprehension of *pour*. Draw a picture of a person pouring water into a glass and say, *[She's] pouring water*. Next, mime holding a cup and ask, *Am I pouring?* Then mime pouring and ask, again *Am I pouring?*

Developing/Expanding Ensure comprehension of *pour*. Draw a picture of a person pouring water into a glass and say, *[She's] pouring water*. Next, mime pouring and ask, *What am I doing?*

Bridging/Reaching Instruct students to name things that can be poured; for example, *water, milk, paint, sand, sugar*, etc. Then, ask them to categorize their items by liquid or solid; for example, solids: *sand, sugar*, etc., and liquids: *water, milk, paint*, etc. Allow students to look items up in a dictionary, if desired.

On My Own

Name ______

1. Marie equally divides 6 bags of soil into these flowerpots. How many bags of soil are in each pot?

$\frac{6}{4}$ bags or $1\frac{2}{4}$ bags

Complete the equation.

2. $\underline{5} \div \underline{9} = \frac{5}{9}$
3. $\underline{13} \div \underline{4} = \frac{13}{4}$
4. $3 \div 8 = \underline{\frac{3}{8}}$
5. $7 \div 9 = \underline{\frac{7}{9}}$
6. $\underline{\frac{1}{3}} \times 7 = 7 \div 3$
7. $\frac{1}{4} \times 5 = 5 \div \underline{4}$

8. A farmer pours 3 pounds of chicken feed equally into 4 bags. What is the weight of the chicken feed in each bag?
   - **A.** $\frac{3}{4}$ pound
   - B. $1\frac{3}{4}$ pounds
   - C. $\frac{4}{3}$ pounds
   - D. $1\frac{1}{4}$ pounds

9. An artist divides 4 pounds of clay equally into 3 containers. What is the weight of the clay in each container? Circle all correct answers.
   - A. $1\frac{1}{4}$ pounds
   - **B.** $1\frac{1}{3}$ pounds
   - C. $\frac{3}{4}$ pound
   - **D.** $\frac{4}{3}$ pounds

Unit 11 • Divide Fractions 131

10. Aki pours the same amount of aquarium pebbles from this bag into each of 3 aquariums. What is the weight of the pebbles in each aquarium?

$\frac{7}{3}$ pounds or $2\frac{1}{3}$ pounds

11. What is the unknown divisor? Explain how you know.

$2 \div \_\_\_\_ = \frac{2}{3}$

3; Sample answer: In a fraction, the divisor is the denominator.

12. **Error Analysis** Spencer divides 6 pounds of food from the food drive into 3 boxes. He says each box has $\frac{3}{6}$ pounds of food. Is he right? How do you know? No. Sample answer: Spencer divided 3 pounds of food into 6 boxes instead of 6 pounds into 3 boxes. The correct answer is $\frac{6}{3}$ or 2.

13. **Extend Your Thinking** Write a word problem involving division in which the quotient is $\frac{8}{5}$. Answers will vary but should involve groups of 8 that need to be divided into 5 parts.

Reflect

How is a fraction another way to write a division expression?

Answers may vary.

Math is... Mindset

What did you do to avoid stress today?

132 Lesson 1 • Relate Fractions to Division

Practice

Build Prodcedural Fluency from Conceptual Understanding

Common Error: Exercises 2–7 Students may not pay attention to the operation symbols in each equation and miss that some of the numbers are being multiplied, not divided. Encourage them to pay attention to the symbols and to think about how multiplication relates to division of whole numbers.

Item Analysis

Item	DOK	Rigor
1	2	Application
2–7	2	Procedural Skill and Fluency
8–10	2	Application
11–13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How is a fraction another way to write a division expression?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What did you do to avoid stress today?

Students reflect on how they practiced self-management.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can represent the quotient of a division equation as a fraction or mixed number.
- I can explain why the quotient of a division equation can always be expressed as a fraction.
- I can explain why division of whole numbers can be written as a multiplication expression.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Match division to fractions	5.NF.B.3
2	2	Relate fractions to division	5.NF.B.3
3	2	Relate fractions to division	5.NF.B.3

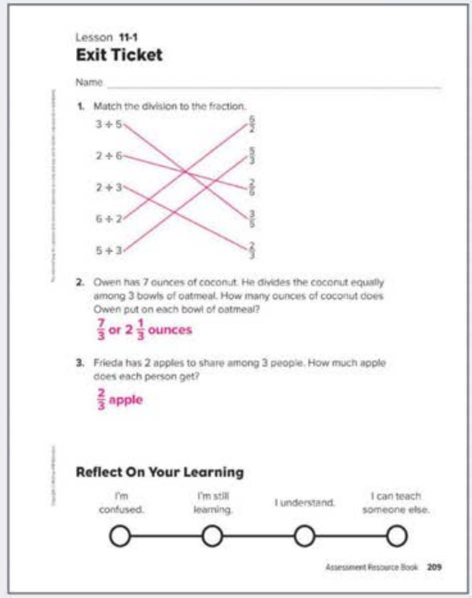
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 11-1

Exit Ticket

Name ______________________

1. Match the division to the fraction.

$3 \div 5$	$\frac{6}{2}$
$2 \div 6$	$\frac{5}{3}$
$2 \div 3$	$\frac{2}{6}$
$6 \div 2$	$\frac{3}{5}$
$5 \div 3$	$\frac{2}{3}$

2. Owen has 7 ounces of coconut. He divides the coconut equally among 3 bowls of oatmeal. How many ounces of coconut does Owen put on each bowl of oatmeal?

$\frac{7}{3}$ or $2\frac{1}{3}$ ounces

3. Frieda has 2 apples to share among 3 people. How much apple does each person get?

$\frac{2}{3}$ apple

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 209

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Roll It, Write It, Show It!

Give students one number cube and a partially completed (1 filled) equation frame. Students roll once to determine the unknown divisor and the denominator to complete the equation using fraction circles, if needed. If students have difficulty, encourage them to think about the division sentences as corresponding multiplication sentences. Repeat with additional examples with 2 filled.

GO ONLINE

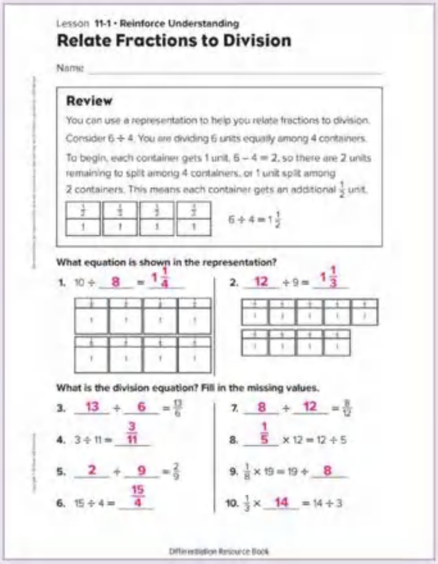
Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Quotient Fractions (Models)

INDEPENDENT WORK

Differentiation Resource Book, p. 125


Lesson 11-1 • Reinforce Understanding

Relate Fractions to Division

Name

Review

You can use a representation to help you relate fractions to division.

Consider $6 \div 4$. You are dividing 6 units equally among 4 containers.

To begin, each container gets 1 unit. $6 - 4 = 2$, so there are 2 units remaining to split among 4 containers, or 1 unit split among 2 containers. This means each container gets an additional $\frac{1}{2}$ unit.

$6 \div 4 = 1\frac{1}{2}$

What equation is shown in the representation?

1. $10 \div$ 8 $=$ $1\frac{1}{4}$
2. 12 $\div 9 =$ $1\frac{1}{3}$

What is the division equation? Fill in the missing values.

3. 13 $\div$ 6 $= \frac{13}{6}$
4. $3 \div 11 =$ $\frac{3}{11}$
5. 2 $\div$ 9 $= \frac{2}{9}$
6. $15 \div 4 =$ $\frac{15}{4}$
7. 8 $\div$ 12 $= \frac{8}{12}$
8. $\frac{1}{5}$ $\times 12 = 12 \div 5$
9. $\frac{1}{8} \times 19 = 19 \div$ 8
10. $\frac{1}{3} \times$ 14 $= 14 \div 3$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fractions as Division Four in a Row

Students practice solving fraction word problems using division.

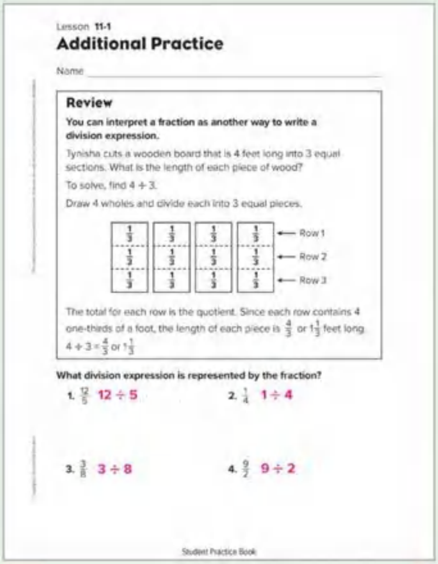
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 125–126


Lesson 11-1

Additional Practice

Name

Review

You can interpret a fraction as another way to write a division expression.

Tynisha cuts a wooden board that is 4 feet long into 3 equal sections. What is the length of each piece of wood?

To solve, find $4 \div 3$.

Draw 4 wholes and divide each into 3 equal pieces.

The total for each row is the quotient. Since each row contains 4 one-thirds of a foot, the length of each piece is $\frac{4}{3}$ or $1\frac{1}{3}$ feet long.

$4 \div 3 = \frac{4}{3}$ or $1\frac{1}{3}$

What division expression is represented by the fraction?

1. $\frac{12}{5}$ $12 \div 5$
2. $\frac{1}{4}$ $1 \div 4$
3. $\frac{3}{8}$ $3 \div 8$
4. $\frac{9}{2}$ $9 \div 2$

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying using area models.

Assign

Use It! Application Station

Can You Hear Me? Students compare and contrast face-to-face and online communication.

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

GO ONLINE

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 125–126

What fraction is represented by the division expression?

5. $7 \div 6 = \frac{7}{6}$
6. $12 \div 3 = \frac{12}{3}$
7. $2 \div 5 = \frac{2}{5}$
8. $6 \div 8 = \frac{6}{8}$
9. $10 \div 10 = \frac{10}{10}$
10. $1 \div 4 = \frac{1}{4}$
11. Giselle has 3 pounds of peanuts. She shares the peanuts by putting an equal amount into each of 5 bags. What is the weight of the peanuts in each bag?
 $\frac{3}{5}$ pound
12. Juan walks 8 miles. He divides the walk into 3 equal parts so he knows when to stop for water. How far does Juan walk between stops?
 $\frac{8}{3}$ miles or $2\frac{2}{3}$ miles
13. Aubrey draws a line that is 34 centimeters long. She divides the line into 6 equal parts. How long is each part of the line?
 $\frac{34}{6}$ centimeters or $5\frac{2}{3}$ centimeters

Math @Home Activity Provide opportunities for your child to explore how fractions and division are related. For example, ask him or her to determine how much each person would receive if a given amount, such as 2 pounds of granola, was divided equally among each person in your family. Have your child write a division expression and a fraction for the situation.

Student Practice Book

Differentiation Resource Book, p. 126

Lesson 11-1 • Extend Thinking

Relate Fractions to Division

Name ______________________

Donna created some perfume and divided it up equally among her 3 sisters and some of her friends. The representation shows how Donna divided up her perfume. Use the representation to answer the following problems.

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
1	1	1	1	1	1	1	1

1. Donna split her perfume up among **8** people.
2. Donna shared her perfume with **5** of her friends.
3. Donna made **12** ounces of perfume to share.
4. How much perfume would Donna's sisters get if she only shared her perfume with them? **4 ounces**
5. How much perfume would Donna's friends get if she only shared her perfume with them? $2\frac{2}{5}$ **ounces**
6. How much perfume would Donna need to make so that each person got $2\frac{5}{8}$ ounces of perfume each? **21 ounces**

Differentiation Resource Book

LESSON 11-2

Solve Problems Involving Division

Learning Target

- I can determine whether a quotient should be written with a remainder or as a mixed number.

Standards ◇ Major △ Supporting ○ Additional

Content

◇**5.NF.B.3** Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b} = a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

Vocabulary

Math Terms	Academic Terms
mixed number	analyze
quotient	reflect
remainder	

Materials

The materials may be for any part of the lesson.

- number cards
- *Problem-Solving Tool* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students determine whether a quotient should be written with a remainder or as a mixed number.	• Students discuss whether a quotient should be written with a remainder or as a mixed number using the verb *apply*. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students exercise creativity by solving a problem using more than one approach.

Coherence

Previous	Now	Next
• Students found whole-number quotients and remainders with up to four-digit dividends and one-digit divisors (Grade 4). • Students extended their understanding of fractions to understand fractions as division (Unit 11).	• Students solve word problems and determine if the quotient should be written with a remainder or as a mixed number based on the context of the problem.	• Students represent division of a whole numbers by unit fractions (Unit 11). • Students understand the concept of a unit rate associated with a ratio and use rate language in the context of a ratio relationship (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of division and mixed numbers by determining how they should write a quotient for division problems.	• Students build their fluency with division as they practice strategies and skills for dividing whole numbers.	• Students apply their understanding of division to solve problems with real-world contexts. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine

What's Another Way to Write It? 5–7 min

Build Fluency Students build number sense as they are given a number and asked to write addition expressions that are equivalent to the target fraction. Students' suggestions are recorded for discussion.

These prompts encourage students to talk about their reasoning:

- Explain your first step in writing a sum.
- Explain how two of your expressions are related. What patterns do you notice?
- How can we use ______'s expression to create a new expression?
- If you wrote an expression with more than two addends, what was your thinking?

Launch 5–7 min

Sense-Making Routine

Purpose Students focus on the context of a problem to make sense of it.

Numberless Word Problem

- What math do you see in this problem?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students write down their own thoughts about the numberless word problem before beginning a class discussion.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of how the context of a word problem can impact the answer and are based on possible comments and questions that students may make during the share out.

- What do you need to know to be able to solve this problem?
- What are some possible results of how the sheets are handed out?

Math is... Mindset

- How can your strengths in other areas help you in math?

SEL Self-Awareness: Recognize Strengths

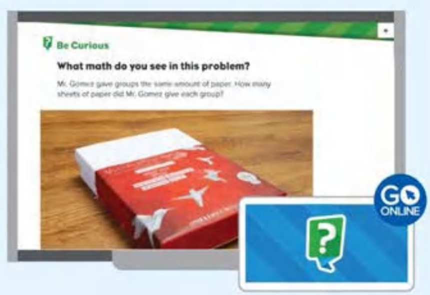
As students work through the Numberless Word Problem routine, encourage students to recognize their own strengths. As a result, they can see themselves as resourceful and may be more willing to attempt to problem solve and help others. As a class, discuss some student strengths and how they helped work through the routine.

Transition to Explore & Develop

Ask questions that get students thinking about how the answer to a division problem can depend on when things can be divided equally and when they cannot.

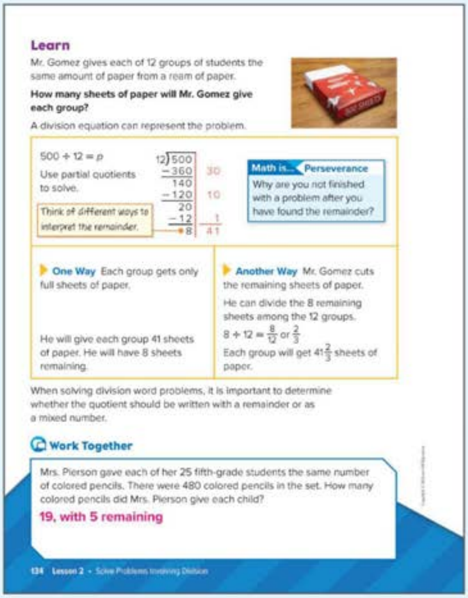
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we need to consider the context of a division problem to find the answer.



Explore & Develop

20 min



Learn

Mr. Gomez gives each of 12 groups of students the same amount of paper from a ream of paper.

How many sheets of paper will Mr. Gomez give each group?

A division equation can represent the problem.

$500 \div 12 = p$

Use partial quotients to solve.

$$\begin{array}{r|l} 12\overline{)500} & \\ -360 & 30 \\ \hline 140 & \\ -120 & 10 \\ \hline 20 & \\ -12 & 1 \\ \hline 8 & 41 \end{array}$$

Think of different ways to interpret the remainder.

Math is... Perseverance
Why are you not finished with a problem after you have found the remainder?

One Way Each group gets only full sheets of paper.

He will give each group 41 sheets of paper. He will have 8 sheets remaining.

Another Way Mr. Gomez cuts the remaining sheets of paper.

He can divide the 8 remaining sheets among the 12 groups.

$8 \div 12 = \frac{8}{12}$ or $\frac{2}{3}$

Each group will get $41\frac{2}{3}$ sheets of paper.

When solving division word problems, it is important to determine whether the quotient should be written with a remainder or as a mixed number.

Work Together

Mrs. Pierson gave each of her 25 fifth-grade students the same number of colored pencils. There were 480 colored pencils in the set. How many colored pencils did Mrs. Pierson give each child?

19, with 5 remaining

134 Lesson 2 • Solve Problems Involving Division

❶ Pose the Problem

ETP Pose Purposeful Questions

- What are you trying to find?
- What are some strategies you know to divide whole numbers?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about how to solve the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- When would you write a quotient with a remainder?
- When would you write a quotient as a mixed number?

Key Takeaway

- When solving division word problems, it is important to consider the context and/or unit to determine whether the quotient should be written with a remainder or as a mixed number.

Work Together

Students solve a division word problem and determine if they need to write the quotient with a remainder or as a mixed number.

Common Error: Students may think that they should write the quotient both ways, as they saw in the Pose the Problem. Remind students to pay attention to the context of the problem and provide the answer that is appropriate for the context.

LOM Language of Math

Remind students that the remainder is the number that is “remaining” after they have shared equally.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how the context of a word problem impacts the quotient.

Materials: *Problem-Solving Tool* Teaching Resource

Directions: Provide copies of the *Problem-Solving Tool* Teaching Resource. Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- How do you know which operation to use?
- What strategies do you know for solving a division problem?
- If there is a remainder, how does that remainder apply to the context of the problem?
- What happens if Mr. Gomez only wants to give out full sheets of paper?
- What happens if Mr. Gomez does not want any leftover paper and gives out partial sheets of paper to the students?

Math is... Perseverance

- Why are you not finished with a problem after you have found the remainder?

Students explain that they are not finished until they interpret the remainder in the context of the problem.


Problem-Solving Tool
Start
Solve
Reflect

Activity Debrief: Invite students to share their findings with others. Encourage students to think about how their solution applies to the context of the problem.

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students explore how they can use the context of a word problem to know whether to write the quotient with a remainder or as a mixed number.

ETP Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

- **Think About It:** What strategy would you use to solve $500 \div 12 = p$? Explain why.

Have the students estimate the solution. Ask:

- What compatible numbers will you use to estimate the solution? Why?
- How will basic facts and place-value patterns help you estimate the solution?

Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

- **Think About It:** Why can't Mr. Gomez share the remaining 8 full sheets?

Have the students discuss their understanding of how the context of a division problem affects how to write the quotient. Have students share and compare their ideas. Make sure they ask useful questions that improve each other's ideas. Ask:

- How does the context help you determine whether the quotient should be written with a remainder or whether the quotient should be written as a mixed number?

Math is... Perseverance

- Why are you not finished with a problem after you have found the remainder?

Students explain that they are not finished until they interpret the remainder in the context of the problem.

EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *full sheets*. Hand out two full sheets of lined paper or white printer paper. Say, *These are full sheets*. Then, ask students to cut one of their sheets. Using one student's half sheets, hold both halves up. Say, *These are half sheets*. Show students two sheets of construction paper, one full and one cut in half. Ask, *Which is a full sheet?* Allow pointing.

Developing/Expanding Ensure understanding of *full sheets*. Hand out two full sheets of lined paper or white printer paper. Say, *These are full sheets*. Then, ask students to cut one of their sheets. Using one student's half sheets, hold both halves up. Say, *These are half sheets*. Pick up a sheet of construction paper. Hold it up and ask, *What is this?* Expect at least *a full sheet* as the answer.

Bridging/Reaching Ask students to show you full sheets of paper that they have in their desks (i.e. notebook paper, loose-leaf, etc.). Then, have them concentrate on both *full* and *sheet* and think of other words they may know that mean the same; for example, *full: entire, whole, uncut; sheet: page, piece of paper*. Allow students to use a dictionary, if desired.

On My Own

Name ______

Solve each problem. If there is a remainder, decide how to represent and interpret the remainder.

1. Grace walked the number of miles shown over the course of 7 days. She walked the same number of miles each day. How many miles did she walk each day?

$2\frac{6}{7}$ miles

2. There were 210 balloons at a fair. Each of the 50 children that attended the fair got the same number of balloons. How many balloons did each child get?
4 balloons; there were 10 left over

3. Dawn made 50 bracelets. She gave each of her 12 friends the same number of bracelets. How many bracelets did Dawn give to each of her friends? She gave 4 bracelets to each of her friends; 2 were left over.

Would you write the quotient for the problem with a remainder or as a mixed number?

4. Equal amounts of juice are poured into different glasses.
A. remainder
B. mixed number

5. The same number of books must be put on each shelf.
A. remainder
B. mixed number

6. A dog is fed the same amount of food every day.
A. remainder
B. mixed number

7. Someone gives out the same number of flowers to each of 5 friends.
A. remainder
B. mixed number

Unit 11 • Divide Fractions 135

Solve each problem. If there is a remainder, decide how to represent and interpret the remainder.

8. A water cooler holds 80 cups of water. If 30 people each get an equal amount of water, how many cups of water does each person get? $2\frac{2}{3}$ cups

9. A baker has this bag of flour. He puts equal amounts of flour in 4 canisters. How many pounds of flour are in each canister? $6\frac{1}{4}$ pounds

10. Ryan has 320 pencils. He gives an equal number of pencils to each of 15 friends. How many pencils does he give each friend? 21, with 5 remaining

11. Rose has a piece of ribbon that is 150 inches long. She is cutting the ribbon into 20 equal pieces. How long will each piece be?
$7\frac{1}{2}$ inches

12. **Extend Your Thinking** Drew has 169 toy cars that he is organizing into boxes. Each box can hold 30 cars. How many boxes does he need? 6 boxes; The answer to this problem needs to be greater than the quotient with a remainder or the mixed number because the remainder or fraction requires another box.

Reflect

How do you know if a quotient should be written with a remainder or as a mixed number?
Answers may vary.

Math is... Mindset
How have your strengths in other areas helped you in math?

136 Lesson 2 • Solve Problems Involving Division

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 4–7 Students may think that they cannot answer the question without knowing the exact amounts being divided. Remind students to think only about the context, and whether something can be divided into fractional units or not.

Item Analysis

Item	DOK	Rigor
1–3	2	Application
4–7	2	Conceptual Understanding
8–11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How do you know if a quotient should be written with a remainder or as a mixed number?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How have your strengths in other areas helped you in math?

Students reflect on how they practiced self-awareness.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can determine whether a quotient should be written with a remainder or as a mixed number.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Interpret remainders in division	5.NF.B.7
2	2	Interpret remainders in division	5.NF.B.7
3	2	Interpret remainders in division	5.NF.B.7
4	2	Interpret remainders in division	5.NF.B.7

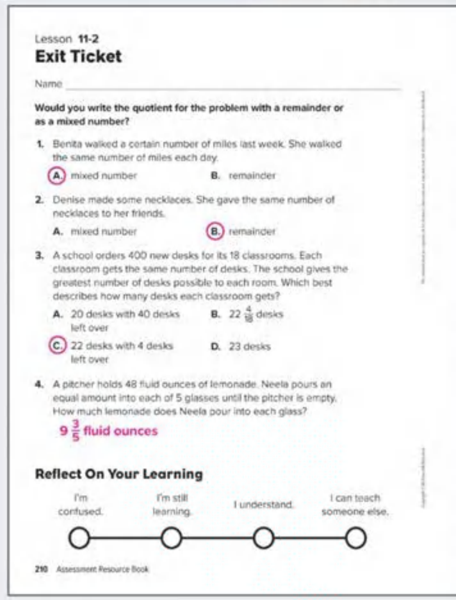
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking


Lesson 11-2
Exit Ticket

Name ______

Would you write the quotient for the problem with a remainder or as a mixed number?

1. Benita walked a certain number of miles last week. She walked the same number of miles each day.
   A. mixed number B. remainder
2. Denise made some necklaces. She gave the same number of necklaces to her friends.
   A. mixed number B. remainder
3. A school orders 400 new desks for its 18 classrooms. Each classroom gets the same number of desks. The school gives the greatest number of desks possible to each room. Which best describes how many desks each classroom gets?
   A. 20 desks with 40 desks left over B. $22\frac{4}{18}$ desks
   C. 22 desks with 4 desks left over D. 23 desks
4. A pitcher holds 48 fluid ounces of lemonade. Neela pours an equal amount into each of 5 glasses until the pitcher is empty. How much lemonade does Neela pour into each glass?
   $9\frac{3}{5}$ fluid ounces

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

210 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

It's a Problem

Work with students in pairs. Students pick two number cards and write a division equation with the greater number as the dividend and the lesser number as the divisor. One student comes up with a context for the problem, and the other student should solve the problem. If students have difficulty determining the context, ask the questions "What unit will this be measured in?" and "Would that unit make sense as a mixed number?"

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Quotient Fractions (Word Problems)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 127

Lesson **11-2 • Reinforce Understanding**

Solve Problems Involving Division

Name ____________________

Review

You can use context to determine whether to write a quotient with a remainder or as a mixed number.

Fiona has 30 paperback books. She gives each of her 4 friends the same number of books. How many books did she give each friend? $30 \div 4 = 7 \text{ R } 2$ Fiona gives each friend 7 books and has 2 books remaining	Andy is dividing 30 pounds of flour equally among 4 bags. How many pounds of flour does he put in each bag? $30 \div 4 = 7\frac{2}{4} = 7\frac{1}{2}$ Andy puts $7\frac{1}{2}$ pounds of flour in each bag.

What is the quotient? Determine whether the answer should be written with a remainder or as a mixed number.

1. Gavin ran a total of 49 miles the last two weeks. He ran the same number of miles each day. How many miles did Gavin run each day?
   Gavin ran $3\frac{1}{2}$ miles each day.
2. Angela has 100 eggs to sell at market. She packages the eggs in cartons which hold 12 eggs each. How many cartons does she have to sell at market?
   Angela has 8 cartons to sell and 4 eggs left over.
3. Chris has 50 pounds of deer sausage. He puts an equal amount into 18 bags. How much sausage will be in each bag?
   Each bag has $2\frac{7}{9}$ pounds of deer sausage.
4. Elyse made 72 bookmarks for her book club. She gave each of the 15 members the same number of bookmarks. How many bookmarks did each club member get?
   Each club member received 4 bookmarks and Elyse had 12 left over.

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fractions as Division Four In a Row

Students practice solving fraction word problems using division.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 127–128

Lesson **11-2**

Additional Practice

Name ____________________

Review

You can determine whether the quotient should be written with a remainder or as a mixed number.

A pitcher holds 42 fluid ounces of lemonade. Helene pours an equal amount into each of 5 glasses until the pitcher is empty. How much lemonade does Helene pour into each glass?

To solve, find $42 \div 5$.

With a remainder, $42 \div 5 = 8 \text{ R}2$.

As a mixed number, $42 \div 5 = 8\frac{2}{5}$

Since fractional parts of fluid ounces can be poured, write the answer as a mixed number.

Helene poured $8\frac{2}{5}$ fluid ounces into each glass.

How would you write the quotient for the problem?

1. Callie walked a certain number of miles last week. She walked the same number of miles each day. How many miles did she walk each day?
   A. as a mixed number
   B. with a remainder
   C. either way is appropriate
2. Debbie made some bracelets. She gave the same number of bracelets to her friends. How many bracelets did she give to each friend?
   A. as a mixed number
   B. with a remainder
   C. either way is appropriate

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying using area models.

Assign

WORKSTATIONS

Use It! Application Station

How Fast Is Your Robot? Students create a robot and measure the distance it travels. They experiment with techniques to increase the distance and speed. *The content of this card has concepts covered later in Lesson 11-4. You may want to assign this card to students ready to explore content covered later in this unit.*

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 127–128

3. A 10-kilometer race is divided into 3 equal sections. How long is each section of the race?
   $3\frac{1}{3}$ **kilometers**

4. A teacher orders a box of 100 pencils to give to the students. Each of the 18 students receives the same number of pencils. How many pencils does each student get?
   5 pencils, with 10 pencils remaining

5. A fence is 40 yards long. Fence posts are placed so that there are 6 equal sections. How far apart are the fence posts?
   $6\frac{4}{6}$ **or** $6\frac{2}{3}$ **yards apart**

6. A grocer has 50 peaches to sell. He packages them in groups of 3. How many packages does the grocer make?
   16 packages with 2 peaches left over

Math @ Home Activity Provide opportunities for your child to explore how division might be represented with a remainder or with a mixed number. For example, if you make a 64-fluid ounce pitcher of a drink, and you have 10 plastic cups, how many fluid ounces will be in each cup if all cups have the same amount of the drink?

Student Practice Book

Differentiation Resource Book, p. 128

Lesson 11-2 • Extend Thinking

Solve Problems Involving Division

Name ____________________

Fran has 275 pens. $\frac{3}{5}$ of the pens are black ink. The remaining pens are blue ink. Altogether, these 275 pens weigh 54 ounces.

1. If Fran gives an equal number of the black pens to each of her 12 friends, how many black pens will each friend receive?
   13; $275 \times \frac{3}{5} = 165$ black pens. $165 \div 12 = 13$ R 9.

2. If Fran gives an equal number of the blue pens to each of her six family members, how many blue pens will Fran have left?
   2; $275 - 165 = 110$ blue pens. $110 \div 6 = 18$ R2

3. How many pens will Fran have left over if she divides the pens up evenly, without regard to color, between her friends and family?
   5; $275 \div 18 = 15$ R 5.

4. Fran is thinking about mailing her best friend some pens. If Fran divides up all of the pens evenly between her 12 friends, how much will her best friend's package of pens weigh?
   $4\frac{1}{2}$ **ounces; $54 \div 12 = 4\frac{1}{2}$**

5. Fran is thinking about mailing an equal number of the 275 pens to each of her 4 siblings. How much will each package weigh?
   $13\frac{1}{2}$ **ounces; $54 \div 4 = 13\frac{1}{2}$**

6. How much does one pen weigh?
   $\frac{54}{275}$ **ounces**

Differentiation Resource Book

LESSON 11-3

Represent Division of Whole Numbers by Unit Fractions

Learning Target

- I can use a representation to divide whole numbers by unit fractions.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

◇ **5.NF.B.7.b** Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div \frac{1}{5}$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{5} = 20$ because $20 \times \frac{1}{5} = 4$.

Math Practices and Processes

MPP Model with mathematics.

Vocabulary

Math Terms	Academic Terms
division	evaluate
fraction model	reflect
unit fraction	

Materials

The materials may be for any part of the lesson.

- fraction circles
- fraction tiles
- number cube

Focus

Content Objective	Language Objectives	SEL Objective
• Students use representations to divide whole numbers by unit fractions.	• Students talk about using representation to divide whole numbers by unit fractions using *can* and *should*. • To support sense-making, ELs participate in MLR2: Collect and Display.	• Students collaborate with peers to solve a mathematical problem.

Coherence

Previous	Now	Next
• Students multiplied fractions by whole numbers (Grade 4). • Students solved word problems and determined how to write the quotient (Unit 11).	• Students represent division of whole numbers by unit fractions.	• Students use the relationship of multiplication and division to divide whole numbers by unit fractions (Unit 11). • Students divide fractions by fractions and understand rate and ratio concepts (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build their understanding of division of whole numbers by unit fractions as they relate the concept to different representations.	• Students build their fluency with multiplication and division as they develop strategies and skills for dividing whole numbers by unit fractions.	• Students apply their understanding of division to solve problems with real-world contexts. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
What's Another Way to Write It? 5–7 min

Build Fluency Students build number sense as they write subtraction expressions to represent a given difference expressed as a fraction.

These prompts encourage students to talk about their **reasoning:**

- How can you use your understanding of inverse relationships to find two expressions within the same fact family?
- What is another way to write an expression thought about it in a different way?
- Is it necessary for all of the numbers used to have the same denominator? Explain.

Launch 5–7 min

Purpose Students connect division of whole numbers by unit fractions to real-world situations.

Notice & Wonder

- What do you notice?
- What do you wonder?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students share their ideas and questions with a partner before discussing with the class as a whole.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of how they can represent dividing a whole number by a unit fraction and are based on possible comments and questions that students may make during the share out.

- What wholes do you see? Explain.
- What parts do you see? Explain.

Math is... Mindset

- What behaviors show respect towards someone?

SEL Social Awareness: Respect Others

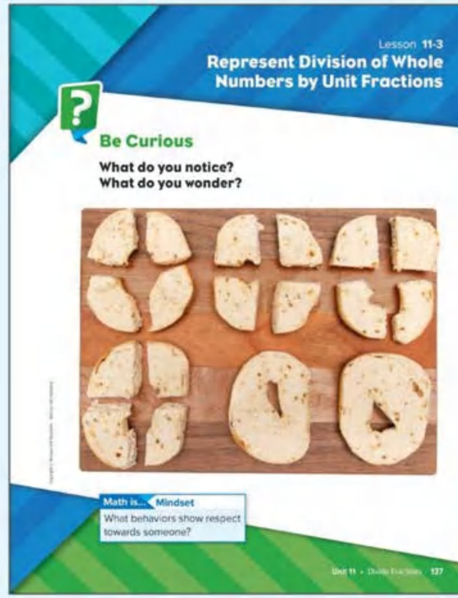
As students work through the Notice & Wonder routine, invite them to be conscious of being respectful of their peers by listening to them, accepting the value of their ideas, and being willing to be either a leader or allowing others to lead. When students are respectful of one another, they strengthen their class community.

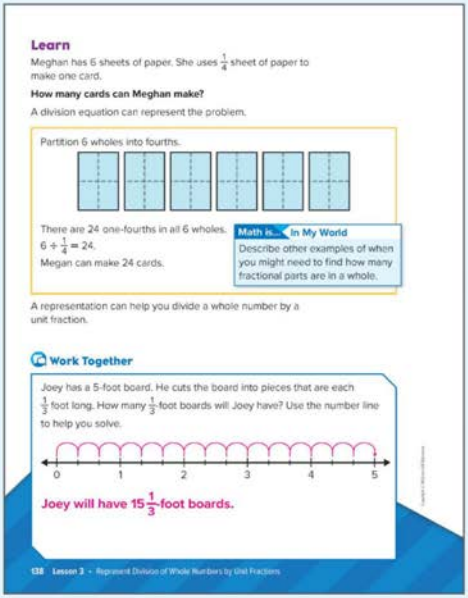
Transition to Explore & Develop

Ask questions that get students thinking about how they could represent dividing a whole number by a unit fraction.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can use representations to divide whole numbers by unit fractions.





Learn

Meghan has 6 sheets of paper. She uses $\frac{1}{4}$ sheet of paper to make one card.

How many cards can Meghan make?

A division equation can represent the problem.

Partition 6 wholes into fourths.

There are 24 one-fourths in all 6 wholes.
$6 \div \frac{1}{4} = 24$.
Megan can make 24 cards.

Math is... In My World
Describe other examples of when you might need to find how many fractional parts are in a whole.

A representation can help you divide a whole number by a unit fraction.

Work Together

Joey has a 5-foot board. He cuts the board into pieces that are each $\frac{1}{3}$ foot long. How many $\frac{1}{3}$-foot boards will Joey have? Use the number line to help you solve.

Joey will have 15 $\frac{1}{3}$-foot boards.

138 Lesson 3 • Represent Division of Whole Numbers by Unit Fractions

1 Pose the Problem

MLR Collect and Display

As students discuss the questions, record relevant words and phrases they may use such as *fraction model, represent*, and *partition*. Display the words for student reference. Use the student-generated expressions to help students make connections between student language and math vocabulary.

ETP Pose Purposeful Questions

- What are some ways you can explain what $6 \div 2$ means?
- What strategies have you used in the past to represent division of whole numbers?

2 Develop the Math

Choose the option that best meets your instructional goals.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you use a fraction model to represent division of whole numbers by unit fractions?
- How do you use the representation help you determine the quotient?

Key Takeaway

- Fraction models can be used to represent division of whole numbers by unit fractions.

Work Together

Students solve a word problem involving division of a whole number by a fraction using a number line instead of partitioning.

Common Error: Students may think they need to partition the whole 5 feet into 3 shares to get $5 \div 3 = \frac{5}{3}$. Point out that this question asks "How many one-thirds are in 5?" which can be written $5 \div \frac{1}{3}$.

LOM Language of Math

Explain to students that *partition* is a verb that means to break something up into parts. It is also a noun. A *partition* is a light movable wall or screen that can divide a room into smaller rooms. *Partition* comes from the Latin word *partiri,* meaning "divide into parts."

Activity-Based Exploration

Students explore extending the meaning of division to divide a whole number by a unit fraction.

Directions: Present a division expression, such as 1,288 ÷ 23. Have students discuss with a partner the meaning of division, such as finding how many groups of 23 can be made from 1,288. Present the expression $3 \div \frac{1}{2}$. Encourage students to think about how they can use the meaning of division to help them divide a whole number by a unit fraction.

ETP Support Productive Struggle

- What tools can you use to represent the division?
- How can you represent the dividend?
- How can you represent the divisor?
- How does your representation show the quotient?

Math is... In My World

- Describe other examples of when you might need to find how many fractional parts are in a whole.

Students apply the mathematics they know to solve problems arising in everyday life.

Activity Debrief: Invite students to discuss their findings and the representations they explored. Facilitate a discussion to ensure students understand that the meaning of division is the same when dividing whole numbers by unit fractions.

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the number of cards Meghan can make?

Guided Exploration

Students represent division of a whole numbers by unit fractions.

ETP Use and Connect Mathematical Representations

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

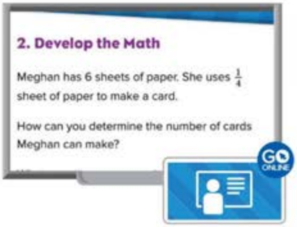
- Should the number of one-fourths in 6 be less than or greater than 6? Why?
- How does that help you estimate the solution?

- What do the 6 wholes in the fraction model represent? Explain why.
- Why should you partition the wholes?
- **Think About It:** How does the number of $\frac{1}{4}$s compare to the number of wholes? Explain why.

Math is... In My World

- Describe other examples of when you might need to find how many fractional parts are in a whole.

Students apply the mathematics they know to solve problems arising in everyday life.



EL English Learner Scaffolds

Entering/Emerging Ensure students' comprehension of *5-foot board*. Measure your classroom door. Say, *This is a [7-foot] door. It's [seven] feet tall*. Then, measure another object, such as a bulletin board. Say, *This is an [8-foot] board. It's [eight] feet long*. Guide students to the Work Together problem on the Learn page. Ask, *How long is the board? (five feet).*

Developing/Expanding Ensure comprehension of *5-foot board*. Measure your classroom door. Say, *This is a [7-foot] door. It's [seven] feet tall*. Write "7-foot door." Then, measure another object such as a bulletin board. Say, *This is an [8-foot] board. It's [eight] feet long*. Finally, ask students to measure an object and use *x-foot (object)* to describe it.

Bridging/Reaching Instruct students to review the Work Together problem on the Learn page. Have them focus on *5-foot board*. Then, have students measure classroom objects, using *x-foot (object)* in complete sentences to describe the objects. Finally, ask students to explore other similar terms that require a hyphen, such as *6-hr school day* and *2-hour drive*.

Practice & Reflect 10 min

On My Own

Name ____________________

What is the quotient? Use a representation to solve. Check students' drawings.

1. $6 \div \frac{1}{3} =$ 18
2. $9 \div \frac{1}{4} =$ 36
3. $7 \div \frac{1}{8} =$ 56
4. $5 \div \frac{1}{5} =$ 25
5. $4 \div \frac{1}{2} =$ 8
6. $2 \div \frac{1}{9} =$ 18
7. $4 \div \frac{1}{6} =$ 24
8. $3 \div \frac{1}{10} =$ 30

9. Jamal cuts the board into pieces that are each $\frac{1}{2}$ foot long. How many pieces does he have? 14 pieces

10. **Error Analysis** Kevin has a 5 feet length of wrapping paper. He uses a $\frac{1}{3}$ foot length of wrapping paper for each present. He writes an equation to help him determine how many presents he can wrap using all of the paper. Is Kevin correct? Explain your thinking. No, Kevin multiplied instead of divided. He can wrap 15 presents.

$5 \div \frac{1}{3} = \frac{5}{3}$

11. Mrs. Lopez has 2 large pizzas for her class to share. Each slice is $\frac{1}{8}$ of the pizza. How many slices of pizza does Mrs. Lopez have? 16 slices; Check students' work

12. A house painter pours the paint from this 5-gallon can into smaller cans that each hold $\frac{1}{2}$ gallon. How many small cans will he fill? Use a fraction model to justify your answer. 10 small cans; Check students' work

13. A baker has 4 pounds of flour. She divides it evenly into bags that hold $\frac{1}{3}$ pound each. Show how many bags the baker can fill using a fraction model. 12 bags; Check students' work

14. **Extend Your Thinking** Find a whole number and unit fraction whose quotient is 24.
Sample answer: $8 \div \frac{1}{3} = 24$

Reflect

How does using representations help you understand division of a whole number by a unit fraction?

Answers may vary.

Math is... Mindset
How did you use behaviors that show respect towards someone?

Practice

ETP **Build Procedural Fluency from Conceptual Understanding**

Common Error: Exercises 1–8 Students may lose track of how many partitions they have made in the whole as they count, especially with greater numbers like in exercise 3. Encourage students to write how many partitions are in each whole above the representation, and add them when they have counted each one.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9	2	Application
10	3	Conceptual Understanding
11–13	2	Application
14	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How does using representations help you understand division of a whole number by a unit fraction?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you use behaviors that show respect towards someone?

Students reflect on how they practiced social awareness.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can use a representation to divide whole numbers by unit fractions.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Divide whole number by unit fraction with representation	5.NF.B.7.c
2	2	Divide whole number by unit fraction with representation	5.NF.B.7.c
3	2	Divide whole number by unit fraction with representation	5.NF.B.7.c
4	2	Divide whole number by unit fraction with representation	5.NF.B.7.c

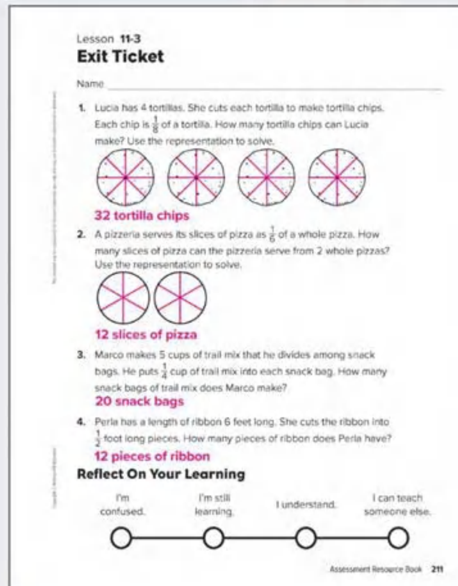
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 11-3

Exit Ticket

Name ____________________

1. Lucia has 4 tortillas. She cuts each tortilla to make tortilla chips. Each chip is $\frac{1}{8}$ of a tortilla. How many tortilla chips can Lucia make? Use the representation to solve.
   32 tortilla chips
2. A pizzeria serves its slices of pizza as $\frac{1}{6}$ of a whole pizza. How many slices of pizza can the pizzeria serve from 2 whole pizzas? Use the representation to solve.
   12 slices of pizza
3. Marco makes 5 cups of trail mix that he divides among snack bags. He puts $\frac{1}{4}$ cup of trail mix into each snack bag. How many snack bags of trail mix does Marco make?
   20 snack bags
4. Perla has a length of ribbon 6 feet long. She cuts the ribbon into $\frac{1}{2}$ foot long pieces. How many pieces of ribbon does Perla have?
   12 pieces of ribbon

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 211

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Roll It, Draw It, Solve It!

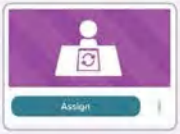
Work with students in pairs. Provide a number cube and fraction tiles or fraction circles. One student rolls for a dividend to divide by $\frac{1}{2}$. Both players use the manipulatives to show the problem. Then they write and solve. If students have difficulty, help them recognize that the number of halves will be two times the dividend.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Divide Whole Numbers by Unit Fractions



INDEPENDENT WORK

Differentiation Resource Book, p. 129

Lesson 11-3 • Reinforce Understanding

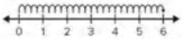
Represent Division of Whole Numbers by Unit Fractions

Name ____________________

Review

You can use a number line to help you solve division problems of whole numbers by unit fractions.

Consider $6 \div \frac{1}{4}$. Make a number line with tick marks from 0 to 6. Since you are dividing by $\frac{1}{4}$, start at 0 and draw four jumps from 0 to 1, four jumps from 1 to 2, and so on. Count the total number of jumps, which should be 24.



$6 \div \frac{1}{4} = 24$

What is the quotient? Use the number line representation to help you solve.

1. $8 \div \frac{1}{3} =$ 24
2. $5 \div \frac{1}{2} =$ 10
3. $2 \div \frac{1}{7} =$ 14
4. $3 \div \frac{1}{4} =$ 12
5. $6 \div \frac{1}{3} =$ 18
6. $4 \div \frac{1}{5} =$ 20
7. $7 \div \frac{1}{3} =$ 21
8. $9 \div \frac{1}{2} =$ 18

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Represent Division of Whole Numbers by Unit Fractions Concentration

Students practice solving fraction division problems by matching pictures, expressions, and answers.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 129–130

Lesson 11-3

Additional Practice

Name ____________________

Review

You can use a representation to find the quotient of a whole number divided by a unit fraction.

Rosanna has 5 large pieces of fabric. To make a quilt, she needs to cut each large piece of fabric into 5 pieces or $\frac{1}{5}$s. How many smaller pieces of fabric will she have?

To solve, find $5 \div \frac{1}{5}$.

Use a representation to find the quotient. Draw 5 wholes. Divide each whole into $\frac{1}{5}$s.

There are 25 pieces that are $\frac{1}{5}$ of a whole.

Rosanna will have 25 smaller pieces of fabric.

What is the quotient? Use a representation to solve.

Check students' work.

1. $3 \div \frac{1}{8} =$ 24
2. $8 \div \frac{1}{5} =$ 40
3. $4 \div \frac{1}{4} =$ 16
4. $2 \div \frac{1}{6} =$ 12

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying using area models.

Assign

Use It! Application Station

Potluck with a Twist Students use whole numbers, fractions, and division to create a 14-dish menu for a potluck. *The content of this card has concepts covered later in Lesson 11-7. You may want to assign this card to students ready to make sense of content covered later in this unit.*

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 129–130

5. Carl has a board that is 4 feet long. He makes shelves that are $\frac{1}{2}$ foot long. How many shelves can he cut from the board?

   8 shelves

6. A baker has 8 pounds of flour. Each cake needs $\frac{1}{3}$ pound of flour. How many cakes can be made with the available flour?

   24 cakes

7. A medium pizza is cut so that each slice is $\frac{1}{6}$ of the pizza. How many slices are there in 3 medium pizzas?

   18 slices

8. A caterer makes 6 pans of fruit salad. Each serving is to be $\frac{1}{10}$ the size of the pan. How many cups of fruit salad can be served?

   60 cups

Math @ Home Activity Give your child several sheets of paper. First, have your child fold one sheet of paper in half. Ask them how many sections were created. Then have your child calculate how many sections there would be if a given number, such as 5, 6, or 7 sheets of paper, was folded the same way. Repeat the activity with different numbers of sections.

Student Practice Book

Differentiation Resource Book, p. 130

Lesson 11-3 • Extend Thinking

Represent Division of Whole Numbers by Unit Fractions

Name ______________________

What is the division problem and quotient shown in the number line representation? What is a second division problem that results in the same quotient? The first one is done for you. Answers may vary.

	Number Line & Problem	1st poblem	Quotient	2nd Problem
1.	0 1 2 3 4 5 6	$6 \div \frac{1}{2}$	12	$3 \div \frac{1}{4}$
2.	0 1 2 3 4 5 6 7	$7 \div \frac{1}{3}$	21	$3 \div \frac{1}{7}$
3.	0 1 2 3 4 5	$5 \div \frac{1}{4}$	20	$10 \div \frac{1}{2}$
4.	0 1 2 3 4	$4 \div \frac{1}{6}$	24	$2 \div \frac{1}{12}$
5.	0 1 2 3 4 5 6	$6 \div \frac{1}{6}$	36	$4 \div \frac{1}{9}$
6.	0 1 2	$2 \div \frac{1}{4}$	8	$1 \div \frac{1}{8}$
7.	0 1 2 3	$3 \div \frac{1}{5}$	15	$5 \div \frac{1}{3}$
8.	0 1 2 3 4 5 6 7 8	$8 \div \frac{1}{2}$	16	$4 \div \frac{1}{4}$
9.	0 1 2 3 4 5	$5 \div \frac{1}{6}$	30	$10 \div \frac{1}{3}$

Differentiation Resource Book

LESSON **11-4**

Divide Whole Numbers by Unit Fractions

Learning Targets

- I can use the meaning of multiplication as equal groups to divide whole numbers by unit fractions.
- I can check if a calculated quotient is correct using a related multiplication equation.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

◇ **5.NF.B.7.b** Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div \frac{1}{5}$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{5} = 20$ because $20 \times \frac{1}{5} = 4$.

Math Practices and Processes

MPP Attend to precision.

Vocabulary

Math Terms	Academic Terms
dividend	arguably
division	speculate
divisor	
unit fraction	

Materials

The materials may be for any part of the lesson.

- spinners
- *Unit Fractions & Whole Numbers* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use the meaning of multiplication as equal groups to divide whole numbers by unit fractions. • Students check the quotient using a related multiplication equation.	• Students discuss if a calculated quotient is correct using a related multiplication equation using *should, might*, and *could*. • To support maximizing cognitive and linguistic meta-awareness, ELs participate in MLR8: Discussion Supports.	• Students identify and use mathematical tools to organize work.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students represented division of a whole numbers by unit fractions (Unit 11).	• Students use the relationship between multiplication and division to divide whole numbers by unit fractions.	• Students represent division of unit fractions by non-zero whole numbers (Unit 11). • Students divide fractions by fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of the relationship between multiplication and division as they justify the quotient of a whole number divided by a unit fraction.	• Students build proficiency by solving problems involving a whole number divided by a unit fraction using pictures, words, and numbers.	• Students apply their understanding of dividing a whole number by a unit fraction in solving problems in real-life contexts. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
About How Much?

5–7 min

Build Fluency Students build number sense as they estimate the sum of two fractions and the sum of two mixed numbers.

Students complete a think-pair-share, and the teacher records the estimates. These prompts encourage students to talk about their **reasoning:**

- What are you being asked to find? What do you notice about the numbers?
- What strategy can you use to solve the problems? How did you determine your estimate?
- What did you do first? What did you do next? How could this have been done another way?

Launch 5–7 min

Sense-Making Routine

Purpose Students begin to think about grouping fractional parts to make wholes.

Notice & Wonder

- What do you notice?
- What do you wonder?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students write down the fractions they associate with each image before beginning class discussion.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of using the relationship between multiplication and division to divide whole numbers by unit fractions and are based on possible comments and questions that students may make during the share out.

- How can you find how many parts make up each whole?
- How do you know which fraction each image represents?

Math is... Mindset

- How do you organize your work to be successful?

SEL **Self-Management: Organizational Skills**

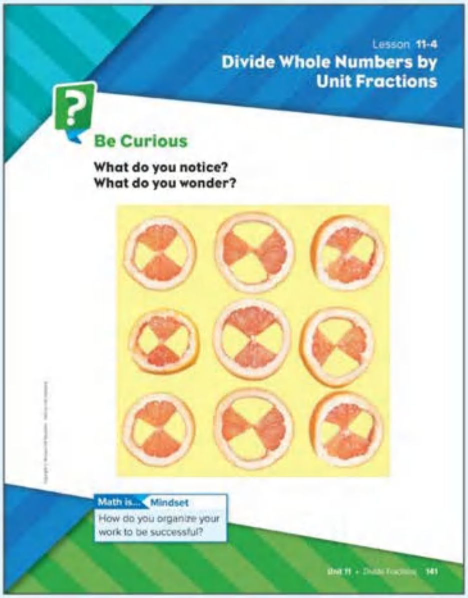
As students complete the Notice & Wonder routine, invite them to share how they organized their work or what they noticed about how you organized your work. You may have written down what students noticed and wondered as two organized lists. Invite students to discuss the tools they may use to organize their work while dividing wholes by fractions, such as number lines or fraction tiles. Encourage them to think about why each tool may be helpful for their work with dividing wholes by fractions.

Transition to Explore & Develop

Ask questions that get students thinking about how they can use the relationship between multiplication and division to divide whole numbers by unit fractions.

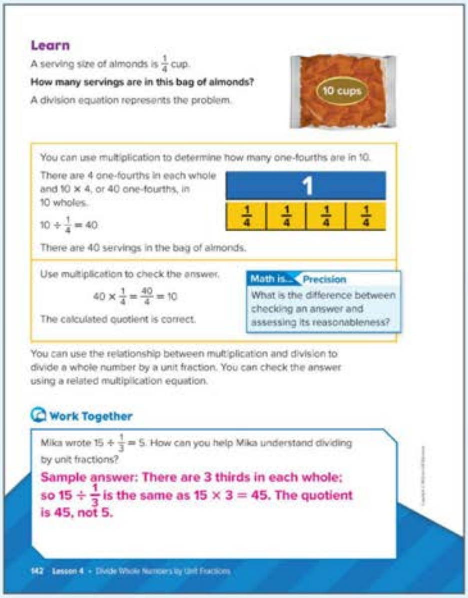
ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about how we can use the relationship between multiplication and division to divide whole numbers by unit fractions.



Explore & Develop

20 min



Learn

A serving size of almonds is $\frac{1}{4}$ cup.

How many servings are in this bag of almonds?

A division equation represents the problem.

10 cups

You can use multiplication to determine how many one-fourths are in 10.

There are 4 one-fourths in each whole and 10 × 4, or 40 one-fourths, in 10 wholes.

$10 \div \frac{1}{4} = 40$

1

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

There are 40 servings in the bag of almonds.

Use multiplication to check the answer.

$$40 \times \frac{1}{4} = \frac{40}{4} = 10$$

The calculated quotient is correct.

Math is... Precision

What is the difference between checking an answer and assessing its reasonableness?

You can use the relationship between multiplication and division to divide a whole number by a unit fraction. You can check the answer using a related multiplication equation.

Work Together

Mika wrote $15 \div \frac{1}{3} = 5$. How can you help Mika understand dividing by unit fractions?

Sample answer: There are 3 thirds in each whole; so $15 \div \frac{1}{3}$ is the same as $15 \times 3 = 45$. The quotient is 45, not 5.

142 Lesson 4 • Divide Whole Numbers by Unit Fractions

1 Pose the Problem

MLR **Discussion Supports**

As students talk about what they know, have them pay attention to others' understandings in order to increase their ability to work through multiplication equations with a calculated quotient. Restate statements they make as a question to seek clarification and provide vocabulary or grammar prompts for students who need more guidance.

ETP **Pose Purposeful Questions**

- How can you explain the relationship between multiplication and division?
- How can you represent division of whole numbers by unit fractions?

2 Develop the Math

Choose the option that best meets your instructional goals.

3 Bring It Together

ETP **Elicit and Use Evidence of Student Thinking**

- How does the relationship between multiplication and division help you divide whole numbers by unit fractions?
- How can you use multiplication to check that a calculated quotient is correct?

Key Takeaway

- The relationship between multiplication and division can be used to divide whole numbers by unit fractions.

Work Together

Students work together to correct the quotient of a division equation and explain dividing whole numbers by unit fractions.

Common Error: If students think that Mika's work is correct, ask them to think about how dividing by a unit fraction is different from multiplying by a unit fraction.

LOM **Language of Math**

Explain to students that the word *dividend* comes from the Latin word *dividendum,* meaning "thing to be divided."

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore efficient strategies for dividing a whole number by a unit fraction.

Materials: *Unit Fractions and Whole Numbers* Teaching Resource

Directions: Provide copies of *Unit Fractions and Whole Numbers*. Students will write a division equation using a whole number as the dividend and a unit fraction as the divisor. Tell students that they will find each quotient and look for patterns.

ETP Support Productive Struggle

- How many groups of the unit fraction are needed to make 1 whole? 2 wholes?
- What patterns do you notice when dividing a whole number by a unit fraction?
- How does thinking about *groups of* help you divide whole numbers by unit fractions?

Activity Debrief: Students should notice that the quotient is the product of the dividend and the denominator of the divisor. Facilitate a discussion about using a related multiplication equation to check their work.

Math is... Precision

- What is the difference between checking an answer and assessing its reasonableness?

Students discuss the difference between a check, which determines if a calculated result is correct, and assessing the reasonableness of a calculated answer (often done by rounding or compatible numbers),


Unit Fractions and Whole Numbers

5	$\frac{1}{2}$
$\frac{1}{5}$	8
6	$\frac{1}{4}$
$\frac{1}{8}$	3
10	$\frac{1}{6}$
$\frac{1}{3}$	4
7	$\frac{1}{10}$
$\frac{1}{9}$	2

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine how many servings are in this bag of almonds?

The PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students use the relationship between multiplication and division to divide whole numbers by unit fractions.

ETP Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- Should the number of $\frac{1}{4}$s in 10 be less than or greater than 10? Why?
- How does that help you estimate the solution?
- How does the bar diagram help you understand how to solve the problem?

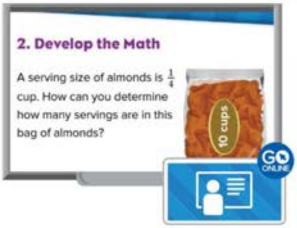
Have the students draw bar diagrams to help and assist them solve the problem. Ask:

- How many bars should you use? Why?
- How should the bars be partitioned? Why?
- How can these bar diagrams help you solve the problem?
- **Think About It:** How are division by fractions and division by whole numbers similar?

Math is... Precision

- What is the difference between checking an answer and assessing its reasonableness?

Students discuss the difference between a check, which determines if a calculated result is correct, and assessing the reasonableness of a calculated answer (often done by rounding or compatible numbers).



EL English Learner Scaffolds

Entering/Emerging Explain *serving* (*size*). Draw a bag of nuts and a bottle of juice. Label each as having 8 servings. Draw serving sizes next to them: $\frac{1}{2}$ cup for the nuts and $\frac{3}{4}$ cup for the juice. Point to the bag of nuts. Say, *This bag has eight servings*. Point to the $\frac{1}{2}$ cup. Say, *Each serving is $\frac{1}{2}$ a cup*. Repeat with the juice. Show students the Learn page and ask, *What is the serving size: 10 cups or $\frac{1}{4}$ cup?*

Developing/Expanding Explain *serving* (*size*). Draw a bag of nuts and a bottle of juice. Label each as having 8 servings. Draw serving sizes next to them: $\frac{1}{2}$ cup for the nuts and $\frac{3}{4}$ cup for the juice. Point to the bag of nuts. Say, *This bag has eight servings*. Point to the $\frac{1}{2}$ cup. Say, *Each serving is $\frac{1}{2}$ a cup*. Repeat with the juice. Show students the Learn page and ask, *What is the serving size of the almonds?*

Bridging/Reaching Ask students to read the problem on the Learn page. Ask them to explain the meaning of *serving size*. To help aid them in their explanation, prompt them to use a dictionary or thesaurus to look up similar words for serving size (*portion, amount, quantity*).

Practice & Reflect
10 min

On My Own

MATH REPLAY | GO ONLINE

Name ______________________

What is the quotient?

1. $3 \div \frac{1}{5} =$ **15**
2. $6 \div \frac{1}{3} =$ **18**
3. $4 \div \frac{1}{4} =$ **16**
4. $7 \div \frac{1}{2} =$ **14**
5. $12 \div \frac{1}{3} =$ **36**
6. $9 \div \frac{1}{5} =$ **45**
7. $6 \div \frac{1}{6} =$ **36**
8. $10 \div \frac{1}{10} =$ **100**
9. $8 \div \frac{1}{7} =$ **56**

10. Keri is making trail mix that contains $\frac{1}{3}$ cup of sunflower seeds per serving. How many servings can she make with this bag?
6 servings

11. A clock chimes every $\frac{1}{4}$ hour. How many times will the clock chime in 6 hours?
24 times

12. Mia hiked 4 miles. There were trail markers every $\frac{1}{10}$ mile. How many trail markers did Mia see during her hike?
40 markers

13. **STEM Connection** Poppy is visiting a park that is 15 acres. The park is divided into sections that are each $\frac{1}{3}$ acre. How many sections does the park have?
$15 \div \frac{1}{3} = 45$

14. Jaxon has 10 gallons of punch. He pours the punch into pitchers that each hold $\frac{1}{2}$ gallon. How many pitchers does Jaxon use?
20 pitchers

15. **Extend Your Thinking** When a whole number is divided by a fraction that is less than 1, will the quotient always be greater than the whole number? Explain why or why not.
Yes. Sample answer: The whole number is being divided into more parts, so the quotient will be greater.

Reflect

How does using the relationship between multiplication and division help you divide whole numbers by fractions?
Answers may vary.

Math is... Mindset
How have you organized your work to be successful?

Practice

ETP **Build Procedural Fluency from Conceptual Understanding**

Common Error: Exercises 1–9 When the dividend is a multiple of the denominator of the unit fraction, some students may divide the dividend by that denominator. For example, in exercise 2, they may divide 6 by 3 to obtain 2. Encourage students to use a representation to show how many one-thirds are in 6.

Item Analysis

Item	DOK	Rigor
1–9	1	Procedural Skill & Fluency
10–14	2	Application
15	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How does using the relationship between multiplication and division help you divide whole numbers by fractions?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How have you organized your work to be successful?

Students reflect on how they practiced self-management.

Learning Targets

Ask students to reflect on the Learning Targets and of the lesson.

- I can use the meaning of multiplication as equal groups to divide whole numbers by unit fractions.
- I can check if a calculated quotient is correct using a related multiplication equation.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

GO ONLINE | MATH REPLAY

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Divide whole numbers by unit fractions	5.NF.B.7.b
2	1	Divide whole numbers by unit fractions	5.NF.B.7.b
3	2	Divide whole numbers by unit fractions	5.NF.B.7.b
4	2	Divide whole numbers by unit fractions	5.NF.B.7.b

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

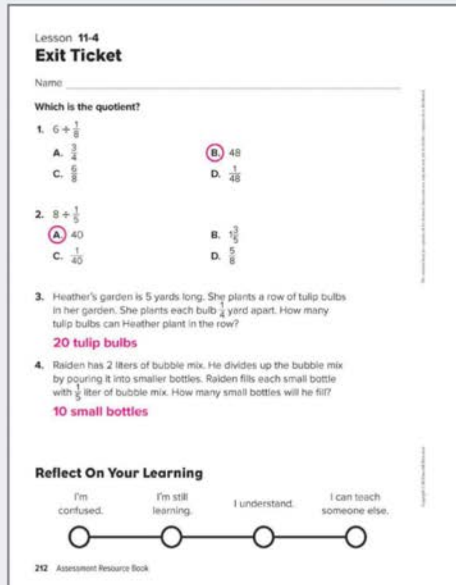
If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking


Lesson 11-4

Exit Ticket

Name

Which is the quotient?

1. $6 \div \frac{1}{8}$

A. $\frac{3}{4}$ B. 48 C. $\frac{6}{8}$ D. $\frac{1}{48}$

2. $8 \div \frac{1}{5}$

A. 40 B. $1\frac{3}{5}$ C. $\frac{1}{40}$ D. $\frac{5}{8}$

3. Heather's garden is 5 yards long. She plants a row of tulip bulbs in her garden. She plants each bulb $\frac{1}{4}$ yard apart. How many tulip bulbs can Heather plant in the row?

20 tulip bulbs

4. Raiden has 2 liters of bubble mix. He divides up the bubble mix by pouring it into smaller bottles. Raiden fills each small bottle with $\frac{1}{5}$ liter of bubble mix. How many small bottles will he fill?

10 small bottles

Reflect On Your Learning

I'm confused. I'm still learning. I understand. I can teach someone else.

212 Assessment Resource Book

 10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Divide the Whole

Write the numbers 2, 3, 4, and 8 in the spaces on a spinner. Each student spins the spinner twice. The first spin represents the whole number; the second spin represents the denominator of a unit fraction. Each student writes a division equation for the numbers. Students switch equations, find the quotient, and share their strategy. If students have difficulty, help them represent the problem by using drawings, number lines, or fraction tiles.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Divide Whole Numbers by Unit Fractions

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 131

Lesson 11-4 • Reinforce Understanding

Divide Whole Numbers by Unit Fractions

Name ______________________

Review

To divide whole numbers by unit fractions, you can check your work using a related multiplication equation.

Consider $13 \div \frac{1}{3}$.

There are 3 thirds in 1 unit.

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

This means there are $13 \times 3 = 39$ thirds in 13.

To check your work, use the equation $39 \times \frac{1}{3}$.

$39 \times \frac{1}{3} = 13$

Therefore, $13 \div \frac{1}{3} = 39$.

What is the quotient? Use a related multiplication equation to check your answer. Show your work.

1. $10 \div \frac{1}{8} =$ 80
   Check: $80 \times \frac{1}{8} = 10$.
2. $5 \div \frac{1}{3} =$ 15
   Check: $15 \times \frac{1}{3} = 5$.
3. $7 \div \frac{1}{4} =$ 28
   Check: $28 \times \frac{1}{4} = 7$.
4. $9 \div \frac{1}{10} =$ 90
   Check: $90 \times \frac{1}{10} = 9$.
5. $15 \div \frac{1}{5} =$ 75
   Check: $75 \times \frac{1}{5} = 15$.
6. $12 \div \frac{1}{7} =$ 84
   Check: $84 \times \frac{1}{7} = 12$.
7. How many quarter-cups are in 1 cup of flour? 4
8. How many slices are in 3 pies, if each slice is $\frac{1}{8}$ of a pie? 24

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Dividing Whole Numbers by Unit Fractions Bingo Students practice solving expressions involving fractions by playing bingo.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 131–132

Lesson 11-4

Additional Practice

Name ______________________

Review

You can use multiplication to find the quotient of a whole number divided by a unit fraction.

A baker has a 4-pound bag of flour. A recipe uses $\frac{1}{5}$ pound of flour. How many times can the baker make the recipe?

To solve, find $4 \div \frac{1}{5}$.

There are 5 $\frac{1}{5}$-pounds in each pound of flour.

1	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
2	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
3	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
4	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

So there are $4 \times 5 = 20$ $\frac{1}{5}$-pounds in 4 pounds.

The baker can make the recipe 20 times.

What is the quotient?

1. $9 \div \frac{1}{8} =$ 72
2. $11 \div \frac{1}{5} =$ 55
3. $6 \div \frac{1}{4} =$ 24
4. $5 \div \frac{1}{6} =$ 30
5. $3 \div \frac{1}{2} =$ 6
6. $8 \div \frac{1}{3} =$ 24

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying using area models.

Assign

Use It! Application Station

How Fast Is Your Robot? Students create a robot and measure the distance it travels. They experiment with techniques to increase the distance and speed.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 131–132

7. A large bag of granola weighs 6 pounds. If smaller bags are made that each contain $\frac{1}{8}$ pound of granola, how many smaller bags can be filled?

48 smaller bags

8. A watermelon weighs 7 pounds. Slices are cut so that each piece weighs $\frac{1}{4}$ pound. How many slices are cut from the watermelon?

28 slices

9. Rosita's garden is 8 yards long. She plants a row of flowers in her garden. She plants each flower $\frac{1}{3}$ yard apart. How many flowers does Rosita plant?

24 flowers

10. Winnie has 9 apples. She cuts each apple so that each slice is $\frac{1}{5}$ of the apple. How many apple slices does Winnie have?

45 slices

Math @ Home Activity Write the numbers 2 through 12 on separate index cards. Next write the unit fractions with denominators 2 through 10 on separate index cards. Place the whole-number cards and the fraction cards in two separate piles, and have your child randomly select a card from each pile. Then have them divide the whole number by the fraction. Repeat the activity, putting the cards back into their respective piles each time.

Student Practice Book

Differentiation Resource Book, p. 132

Lesson 11-4 • Extend Thinking

Divide Whole Numbers by Unit Fractions

Name ______________________

Match the division statement in Column A with the answer statement in Column B and fill in the divisor. The first one is done for you. Answers may vary.

Column A	Column B
$14 \div \frac{1}{2}$	There are 120 sixths in 20 wholes.
$12 \div \frac{1}{5}$	There are 144 eighths in 18 wholes.
$9 \div \frac{1}{7}$	There are 36 thirds in 12 wholes.
$20 \div \frac{1}{6}$	There are 28 halves in 14 wholes.
$5 \div \frac{1}{4}$	There are 20 fourths in 5 wholes.
$18 \div \frac{1}{8}$	There are 45 thirds in 15 wholes.
$15 \div \frac{1}{3}$	There are 60 fifths in 12 wholes.
$12 \div \frac{1}{3}$	There are 63 sevenths in 9 wholes.

Differentiation Resource Book

LESSON **11-5**

Represent Division of Unit Fractions by Non-Zero Whole Numbers

Learning Target

- I can use a representation to divide unit fractions by non-zero whole numbers.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

◇ **5.NF.B.7.a** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$.

Math Practices and Processes

MPP Reason abstractly and quantitatively.

Vocabulary

Math Terms	Academic Terms
division	analyze
fraction model	suggest
unit fraction	

Material

The materials may be for any part of the lesson.

- *Dividing Fractions Puzzle Pieces* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students use representations to divide unit fractions by non-zero whole numbers.	• Students explain how to use representations to divide unit fractions by non-zero whole numbers using *similar* and *related*. • To support cultivating conversation, ELs participate in MLR3: Critique, Correct, and Clarify.	• Students determine the representations and analyses necessary to make informed decisions when engaging in mathematical practices.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students used the relationship between multiplication and division to divide whole numbers by unit fractions (Unit 11).	• Students represent division of unit fractions by non-zero whole numbers.	• Students use the relationship between multiplication and division to divide unit fractions by whole numbers (Unit 11). • Students divide fractions by fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students extend their understanding of division with fractions by representing division of unit fractions by non-zero whole numbers.	• Students evaluate representations used to divide fractions by non-zero whole numbers.	• Students apply their understanding of dividing fractions by non-zero whole numbers to solve problems. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
About How Much?

5–7 min

Build Fluency Students build number sense and estimation skills as they estimate differences involving fractions and mixed numbers.

Students share their estimates with partners and reason with classmates. The teacher records estimates and discloses the actual differences. These prompts encourage students to talk about their reasoning:

- What are you asked to find?
- What do you notice about the numbers? What do you think you should do first?
- What strategies did you use to determine your estimates?

Purpose Students explore unit fractions being partitioned into equal parts.

Which Doesn't Belong?

- Which doesn't belong?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students model the images by themselves with fraction tiles to give them a more hands-on experience with the activity.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of using representations to divide unit fractions by non-zero whole numbers and are based on possible comments and questions that students may make during the share out.

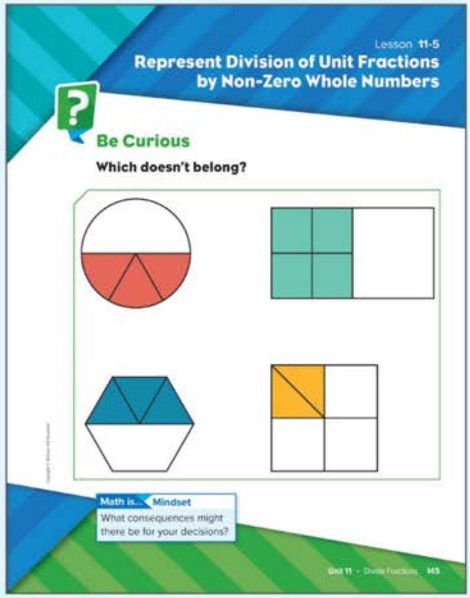
- How can you know what the shaded parts represent?
- How can you know how the shaded parts are divided?

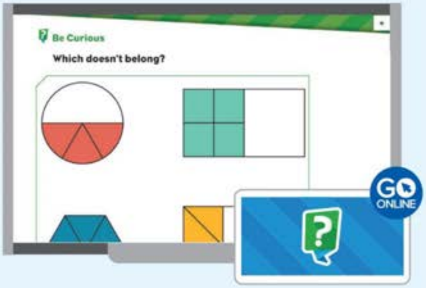
Math is... Mindset

- What consequences might there be for your decisions?

SEL Responsible Decision-Making: Evaluate

As students begin the Which doesn't belong? routine, have them think about different ways to evaluate the image. As they evaluate the image, encourage them to think about different attributes or characteristics so that they can develop understanding that helps them make informed decisions.





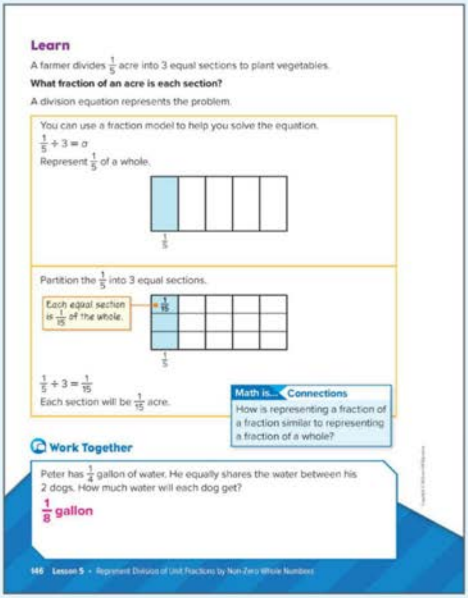
Transition to Explore & Develop

Ask questions that get students thinking about using representations to divide unit fractions by non-zero whole numbers.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can use representations to divide unit fractions by non-zero whole numbers.

Explore & Develop 20 min



Learn

A farmer divides $\frac{1}{5}$ acre into 3 equal sections to plant vegetables.

What fraction of an acre is each section?

A division equation represents the problem.

You can use a fraction model to help you solve the equation.

$\frac{1}{5} \div 3 = a$

Represent $\frac{1}{5}$ of a whole.

$\frac{1}{5}$

Partition the $\frac{1}{5}$ into 3 equal sections.

Each equal section is $\frac{1}{15}$ of the whole.

$\frac{1}{15}$

$\frac{1}{5}$

$\frac{1}{5} \div 3 = \frac{1}{15}$

Each section will be $\frac{1}{15}$ acre.

Math is... Connections

How is representing a fraction of a fraction similar to representing a fraction of a whole?

Work Together

Peter has $\frac{1}{4}$ gallon of water. He equally shares the water between his 2 dogs. How much water will each dog get?

$\frac{1}{8}$ gallon

146 Lesson 5 • Represent Division of Unit Fractions by Non-Zero Whole Numbers

1 Pose the Problem

ETP Pose Purposeful Questions

- What strategies in the past have you used to represent division involving fractions?
- Do you think the sections will be greater than $\frac{1}{5}$ acre? Why or why not?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Critique, Correct, and Clarify

Make a false claim for students to critique. Write the following on the board: $\frac{1}{5} \div 4 = \frac{4}{5}$. *Am I correct?* Ask students to correct the statement, explaining how they know it's incorrect. Revisit this routine throughout the lesson to provide reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you use a fraction model to represent division of unit fractions by non-zero whole numbers?
- How can you use the representation to help you determine the quotient?

Key Takeaway

- Fraction models can be used to divide unit fractions by non-zero whole numbers.

Work Together

Students solve a word problem involving division of a unit fraction by a non-zero whole number.

Common Error: Make sure students understand they cannot start by just representing $\frac{1}{4}$. They need to show a whole, and then partition the whole into $\frac{1}{4}$s. They will partition each $\frac{1}{4}$ of that whole into 2 sections to solve the problem, which is why this step is important.

LOM Language of Math

Explain to students that *quotient* comes from the Latin *quotiens*, which means "how many times." Finding the quotient involves finding how many times the divisor goes into the dividend.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using representations to divide unit fractions by non-zero whole numbers.

Materials: *Dividing Fractions Puzzle Pieces* Teaching Resource

Directions: Provide copies of the *Dividing Fractions Puzzle Pieces* Teaching Resource. In pairs or small groups, invite students to find matches with the puzzle pieces. Encourage students to use representations to help them make matches.

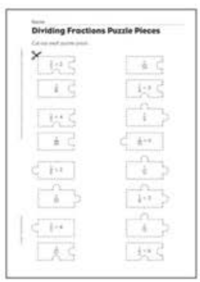
ETP Support Productive Struggle

- How can you use the meaning of division to help you create a representation?
- How does your representation show the dividend?
- How does your representation show the divisor?
- How does your representation show the quotient?

Math is... Connections

- How is representing a fraction of a fraction similar to representing a fraction of a whole?

Students connect their understanding of representing part of a whole to part of a fraction.



Activity Debrief: Have students share their representations to explore the concept of division of unit fractions by non-zero whole numbers.

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine the fraction of an acre for each section?

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students represent division of unit fractions by non-zero whole numbers.

ETP Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- Should the each of the 3 equal shares of $\frac{1}{5}$ be less than or greater than $\frac{1}{5}$? Why?
- How does that help you estimate the solution?
- **Think About It:** Why are 3 equal shares of the whole, and not just the $\frac{1}{5}$, shown in the model?
- How can you know that each share is $\frac{1}{15}$ of the whole?

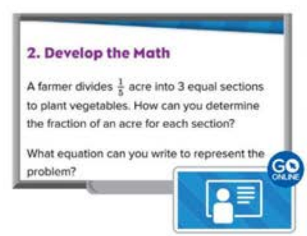
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

Math is... Connections

- How is representing a fraction of a fraction similar to representing a fraction of a whole?

Students connect their understanding of representing part of a whole to part of a fraction.



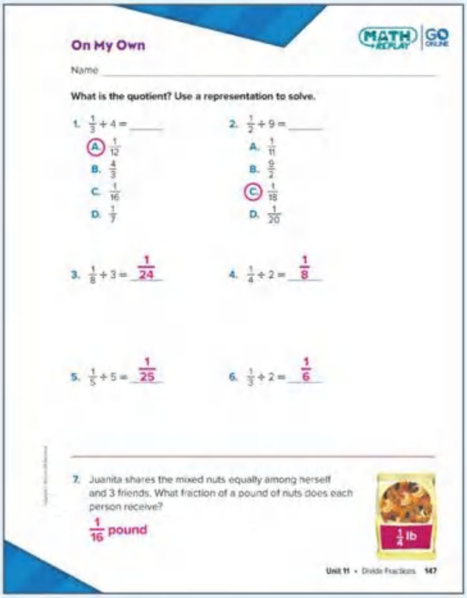
EL English Learner Scaffolds

Entering/Emerging Ensure students' comprehension of *share among equally*. Put enough counters in a group so that you can pass out 2-3 to each student. Say, *I'm going to share the counters among us equally*. Pass them out so that each student has the same number. Repeat twice, once sharing among everyone equally, and once not. Ask, each time, *Did I share among us equally?*

Developing/Expanding Ensure students' comprehension of *share among equally*. Put enough counters in a group so that you can pass out 2-3 to each student. Say, *I'm going to share the counters among us equally*. Pass them out so that each student has the same number. Repeat with new items to share among the class equally. Ask, *What did I do?* Provide sentence frames for students who need more guidance.

Bridging/Reaching Have students read the Work Together problem on the Learn page, focusing on the phrase *share among equally*. Then give students a number of counters. Tell them to share them among the students equally, saying what they're doing. Have them restate in their own words; for example, *I'm distributing them evenly*.

Practice & Reflect 10 min

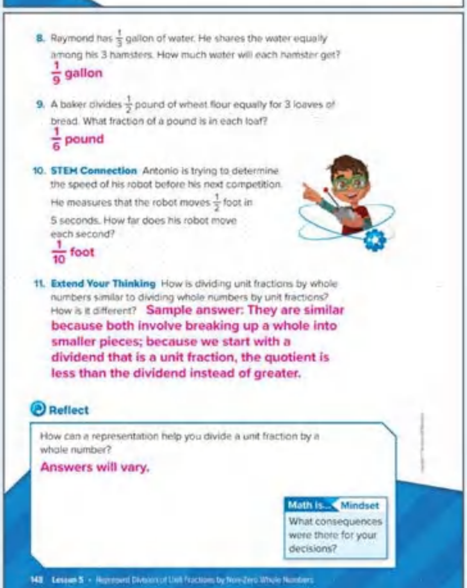

On My Own

Name

What is the quotient? Use a representation to solve.

1. $\frac{1}{3} \div 4 =$ ____
   - A. $\frac{1}{12}$
   - B. $\frac{4}{3}$
   - C. $\frac{1}{16}$
   - D. $\frac{1}{7}$
2. $\frac{1}{2} \div 9 =$ ____
   - A. $\frac{1}{11}$
   - B. $\frac{9}{2}$
   - C. $\frac{1}{18}$
   - D. $\frac{1}{20}$
3. $\frac{1}{8} \div 3 = \frac{1}{24}$
4. $\frac{1}{4} \div 2 = \frac{1}{8}$
5. $\frac{1}{5} \div 5 = \frac{1}{25}$
6. $\frac{1}{3} \div 2 = \frac{1}{6}$
7. Juanita shares the mixed nuts equally among herself and 3 friends. What fraction of a pound of nuts does each person receive?
   $\frac{1}{16}$ pound

Unit 11 • Divide Fractions 147


8. Raymond has $\frac{1}{3}$ gallon of water. He shares the water equally among his 3 hamsters. How much water will each hamster get?
   $\frac{1}{9}$ gallon
9. A baker divides $\frac{1}{2}$ pound of wheat flour equally for 3 loaves of bread. What fraction of a pound is in each loaf?
   $\frac{1}{6}$ pound
10. **STEM Connection** Antonio is trying to determine the speed of his robot before his next competition. He measures that the robot moves $\frac{1}{2}$ foot in 5 seconds. How far does his robot move each second?
    $\frac{1}{10}$ foot
11. **Extend Your Thinking** How is dividing unit fractions by whole numbers similar to dividing whole numbers by unit fractions? How is it different? **Sample answer: They are similar because both involve breaking up a whole into smaller pieces; because we start with a dividend that is a unit fraction, the quotient is less than the dividend instead of greater.**

Reflect

How can a representation help you divide a unit fraction by a whole number?

Answers will vary.

Math is... Mindset

What consequences were there for your decisions?

148 Lesson 5 • Represent Division of Unit Fractions by Non-Zero Whole Numbers

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 9 Students may divide by 3 instead of 4 if they do not realize that Juanita is also getting a share of the nuts. Make sure students read each word problem carefully before they begin to solve it.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–12	2	Application
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can a representation help you divide a unit fraction by a whole number?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What consequences were there for your decisions?

Students reflect on how they practiced responsible decision-making.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can use a representation to divide unit fractions by non-zero whole numbers.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standards
1	2	Divide unit fractions by whole numbers with representation	5.NF.B.7.a
2	2	Divide unit fractions by whole numbers with representation	5.NF.B.7.c
3	2	Divide unit fractions by whole numbers with representation	5.NF.B.7.c

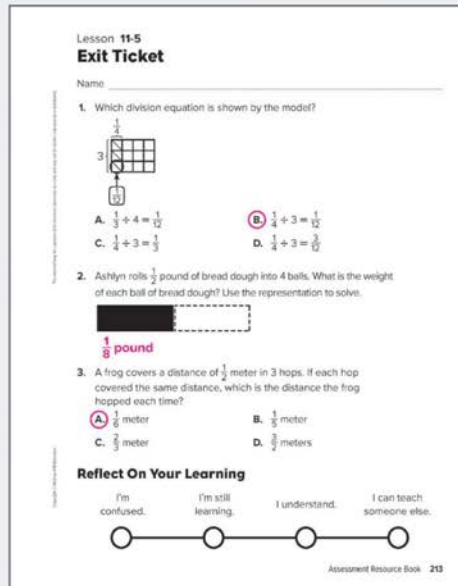
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 11-5

Exit Ticket

Name ______________________

1. Which division equation is shown by the model?

A. $\frac{1}{3} \div 4 = \frac{1}{12}$ B. $\frac{1}{4} \div 3 = \frac{1}{12}$

C. $\frac{1}{4} \div 3 = \frac{1}{3}$ D. $\frac{1}{4} \div 3 = \frac{3}{12}$

2. Ashlyn rolls $\frac{1}{2}$ pound of bread dough into 4 balls. What is the weight of each ball of bread dough? Use the representation to solve.

$\frac{1}{8}$ pound

3. A frog covers a distance of $\frac{1}{2}$ meter in 3 hops. If each hop covered the same distance, which is the distance the frog hopped each time?

A. $\frac{1}{6}$ meter   B. $\frac{1}{5}$ meter

C. $\frac{2}{3}$ meter   D. $\frac{3}{2}$ meters

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 213

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

What's the Story?

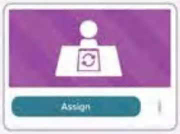
Work with students in pairs. Have students write a word problem to match the expression $\frac{1}{6} \div 3$. Then have students trade word problems with a partner and solve. Have them compare and discuss their word problems and representations. Encourage students to check their work by multiplying the quotient by the divisor. Remind students that since they are dividing a number into smaller parts, the quotient must be less than the dividend.

GO ONLINE

Take Another Look Lesson

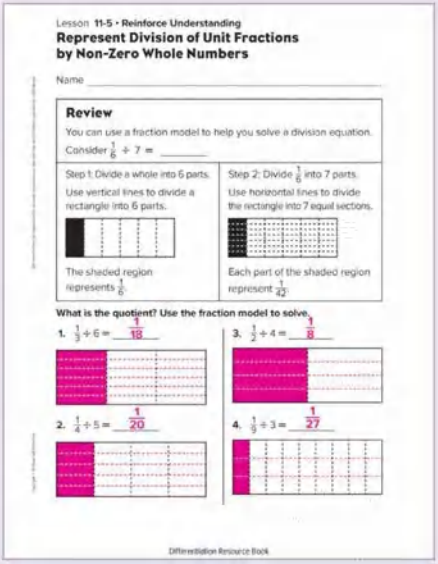
Assign the interactive lesson to reinforce targeted skills.

- Divide Unit Fractions by Whole Numbers



INDEPENDENT WORK

Differentiation Resource Book, p. 133



Lesson 11-5 • Reinforce Understanding

Represent Division of Unit Fractions by Non-Zero Whole Numbers

Name ______

Review

You can use a fraction model to help you solve a division equation.

Consider $\frac{1}{6} \div 7 =$ ______

Step 1: Divide a whole into 6 parts.

Use vertical lines to divide a rectangle into 6 parts.

The shaded region represents $\frac{1}{6}$.

Step 2: Divide $\frac{1}{6}$ into 7 parts.

Use horizontal lines to divide the rectangle into 7 equal sections.

Each part of the shaded region represent $\frac{1}{42}$.

What is the quotient? Use the fraction model to solve.

1. $\frac{1}{3} \div 6 =$ $\frac{1}{18}$
2. $\frac{1}{4} \div 5 =$ $\frac{1}{20}$
3. $\frac{1}{2} \div 4 =$ $\frac{1}{8}$
4. $\frac{1}{9} \div 3 =$ $\frac{1}{27}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Division Match, Concentration, and Showdown Students practice matching expressions, word problems, representations, and answers.

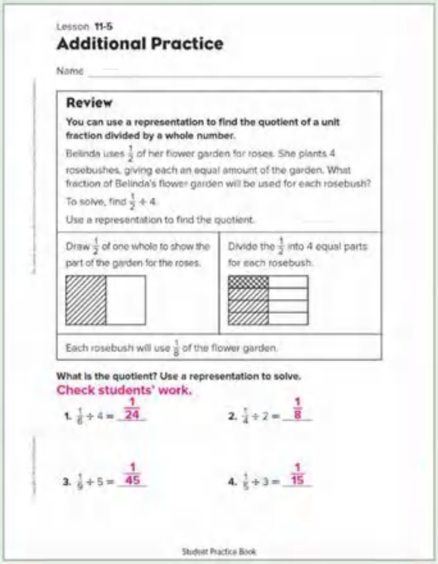
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 133–134



Lesson 11-5

Additional Practice

Name ______

Review

You can use a representation to find the quotient of a unit fraction divided by a whole number.

Belinda uses $\frac{1}{2}$ of her flower garden for roses. She plants 4 rosebushes, giving each an equal amount of the garden. What fraction of Belinda's flower garden will be used for each rosebush?

To solve, find $\frac{1}{2} \div 4$.

Use a representation to find the quotient.

Draw $\frac{1}{2}$ of one whole to show the part of the garden for the roses.

Divide the $\frac{1}{2}$ into 4 equal parts for each rosebush.

Each rosebush will use $\frac{1}{8}$ of the flower garden.

What is the quotient? Use a representation to solve.

Check students' work.

1. $\frac{1}{6} \div 4 =$ $\frac{1}{24}$
2. $\frac{1}{4} \div 2 =$ $\frac{1}{8}$
3. $\frac{1}{9} \div 5 =$ $\frac{1}{45}$
4. $\frac{1}{5} \div 3 =$ $\frac{1}{15}$

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying using area models.

Assign

WORKSTATIONS

Use It! Application Station

Can You Hear Me? Students compare and contrast face-to-face and online communication.

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

GO ONLINE

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 133–134

What is the quotient?

5. $\frac{1}{3} \div 8 =$ $\frac{1}{24}$

6. $\frac{1}{7} \div 6 =$ $\frac{1}{42}$

7. $\frac{1}{10} \div 2 =$ $\frac{1}{20}$

8. $\frac{1}{9} \div 4 =$ $\frac{1}{36}$

9. In 3 minutes, Javier can walk $\frac{1}{6}$ mile. How far does Javier walk in 1 minute?

$\frac{1}{18}$ mile

10. A baker has $\frac{1}{2}$ pound of flour. From this amount, the baker can make 5 cakes. How much flour does the baker use to make each cake?

$\frac{1}{10}$ pound

11. A swimmer swims 5 lengths of the pool to swim $\frac{1}{4}$ kilometer. What fraction of a kilometer is each length of the pool?

$\frac{1}{20}$ kilometer

Math @Home Activity Set out measuring cups and measuring spoons that represent unit fractions, $\frac{1}{3}$ cup or $\frac{1}{4}$ teaspoon. Have your child practice dividing each unit fraction into 2, 3, or 4 smaller, equal amounts. Use other measuring cups or spoons to verify the results, if possible.

Student Practice Book

Differentiation Resource Book, p. 134

Lesson **11-5 • Extend Thinking**

Represent Division of Unit Fractions by Non-Zero Whole Numbers

Name ____________________

What division statements give the quotient in the Answer column? Complete the fraction models to represent the quotient in the Answer column and write the division statement for each model. The first one is done for you.

	Model and Problem	Answer	Model and Problem
1.	$\frac{1}{5} \div 4$	$\frac{1}{20}$	$\frac{1}{2} \div 10$
2.	$\frac{1}{8} \div 2$	$\frac{1}{16}$	$\frac{1}{4} \div 4$
3.	$\frac{1}{9} \div 4$	$\frac{1}{36}$	$\frac{1}{6} \div 6$
4.	$\frac{1}{3} \div 6$	$\frac{1}{18}$	$\frac{1}{9} \div 2$
5.	$\frac{1}{8} \div 7$	$\frac{1}{56}$	$\frac{1}{14} \div 4$

Differentiation Resource Book

LESSON **11-6**

Divide Unit Fractions by Non-Zero Whole Numbers

Learning Targets

- I can divide unit fractions by non-zero whole numbers.
- I can check if a calculated quotient is correct using a related multiplication equation.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

◇ **5.NF.B.7.a** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$.

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
dividend	accurate
division	evaluate
divisor	
unit fraction	

Materials

The materials may be for any part of the lesson.

- number cubes
- *Unit Fractions and Whole Numbers* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students extend their understanding that dividing by a whole is the same as multiplying by a unit fraction to divide unit fractions by whole numbers. • Students check if a calculated quotient is correct using a related multiplication equation.	• Students explain if a calculated quotient is correct using *different* and *related*. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students demonstrate self-awareness of personal strengths and areas of challenge in mathematics.

Coherence

Previous	Now	Next
• Students multiplied a fraction by a whole number (Grade 4). • Students represented division of unit fractions by non-zero whole numbers (Unit 11).	• Students use the relationship between multiplication and division to divide unit fractions by non-zero whole numbers.	• Students use strategies to solve division word problems involving fractions and whole numbers (Unit 11). • Students divide fractions by fractions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build their understanding of dividing unit fractions by non-zero whole numbers by using multiplication to justify their solutions.	• Students interpret multiplication equations to solve related division equations.	• Students apply their understanding of division of unit fractions by non-zero whole numbers to solve problems. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
Where Does It Go?

5–7 min

Build Fluency Students build number sense and fraction understanding as they draw a point for the same mixed number on two different number lines—one number line with marked endpoints 0 and 10; the other with marked endpoints 0 and 8.

These prompts encourage students to talk about their reasoning:

- How are the number lines similar? How are they different?
- How did you determine where to place the point on each number line?

Purpose Students think about the relationship between multiplication and division to divide unit fractions by non-zero whole numbers.

Is It Always True?

- Is the statement always true?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students work in small groups to discuss whether the statement is always true before discussing as a class. Remind them they only need to find one instance where it is not true to say that it is not always true.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of how to use the relationship between multiplication and division to divide unit fractions by non-zero whole numbers and are based on possible comments and questions that students may make during the share out.

- What could $3 \div 4$ represent?
- What could $\frac{1}{4} \times 3$ represent?
- How can you tell if something is *always* true?

Math is... Mindset

- What strengths will you rely on to be successful today?

SEL Self-Awareness: Accurate Self-Perception

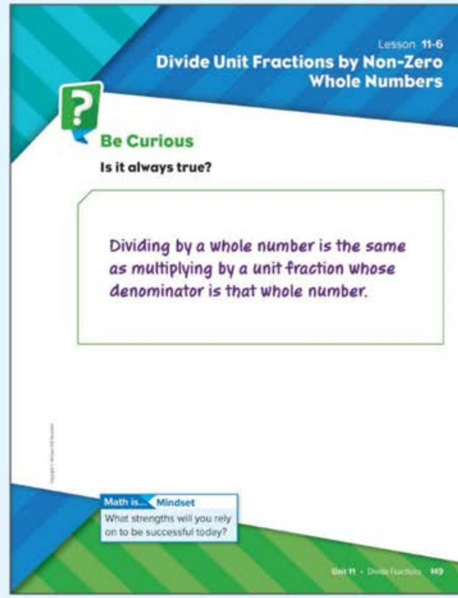
As students begin to think about the relationship between multiplication and division in the Is it always true? routine, encourage them to make connections to concepts they are more familiar or comfortable with, such as dividing and multiplying whole numbers. They can also use more familiar strategies to check their answers. As students continue to divide unit fractions by whole numbers, differentiate instruction to provide opportunities for students to experience success and gratification as well as encounter appropriate amounts of productive struggle.

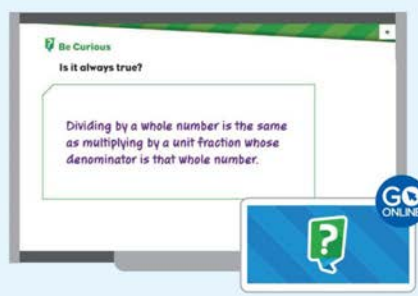
Transition to Explore & Develop

Ask questions that get students thinking about how the relationship between multiplication and division can help them divide unit fractions by non-zero whole numbers.

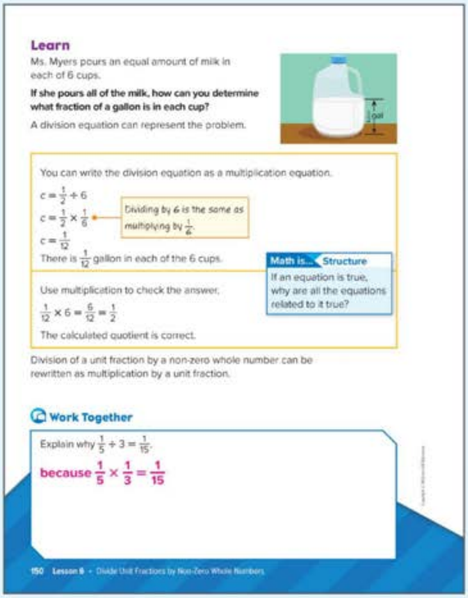
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can use the relationship between multiplication and division to divide unit fractions by non-zero whole numbers.





Explore & Develop 20 min



Learn

Ms. Myers pours an equal amount of milk in each of 6 cups.

If she pours all of the milk, how can you determine what fraction of a gallon is in each cup?

A division equation can represent the problem.

You can write the division equation as a multiplication equation.

$c = \frac{1}{2} \div 6$

$c = \frac{1}{2} \times \frac{1}{6}$ — Dividing by 6 is the same as multiplying by $\frac{1}{6}$.

$c = \frac{1}{12}$

There is $\frac{1}{12}$ gallon in each of the 6 cups.

Use multiplication to check the answer.

$\frac{1}{12} \times 6 = \frac{6}{12} = \frac{1}{2}$

The calculated quotient is correct.

Math is... Structure
If an equation is true, why are all the equations related to it true?

Division of a unit fraction by a non-zero whole number can be rewritten as multiplication by a unit fraction.

Work Together

Explain why $\frac{1}{5} \div 3 = \frac{1}{15}$.

because $\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$

150 Lesson 6 • Divide Unit Fractions by Non-Zero Whole Numbers

1 Pose the Problem

ETP Pose Purposeful Questions

- How can you explain the relationship between multiplication and division?
- How can you represent division of unit fractions by non-zero whole numbers?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about how to solve the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How does the relationship between multiplication and division help you divide unit fractions by non-zero whole numbers?
- How can you use multiplication to check that a calculated quotient is correct?

Key Takeaway

- The relationship between multiplication and division can be used to divide unit fractions by non-zero whole numbers.

Work Together

Students explain why the quotient of a division equation is accurate by using multiplication. Encourage students to solve the problem by rewriting the equation as multiplication of a unit fraction, then checking the answer by multiplying the quotient by 3.

Common Error: Students may explain the equation using a representation. Ask them to also use a multiplication equation to demonstrate why the equation is true.

LOM Language of Math

Explain to students that the multiplication and division equations are *related* when they are part of the same fact family, just as people are related when they are part of the same family.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore efficient strategies for dividing a unit fraction by a non-zero whole number.

Materials: *Unit Fractions and Whole Numbers* Teaching Resource

Directions: Provide copies of *Unit Fractions and Whole Numbers*. Students will write a division equation using a unit fraction as the dividend and a whole number as the divisor. Tell students that they will find each quotient and look for patterns.

ETP Support Productive Struggle

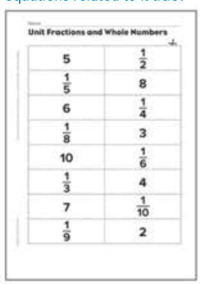
- What patterns do you notice when dividing a unit fraction by a whole number?
- How does dividing by a whole number compare to multiplying by a unit fraction whose denominator is that whole number?

Activity Debrief: Students should notice that the quotient is the product of the dividend and a unit fraction whose denominator is the whole number. Facilitate a discussion about using a related multiplication equation to check their work.

Math is... Structure

- If an equation is true, why are all the equations related to it true?

Students understand the structure of mathematics that allows them to use the truth of an equation to establish the truth of a related equation and to check an answer.


Unit Fractions and Whole Numbers

5	$\frac{1}{2}$
$\frac{1}{5}$	8
6	$\frac{1}{4}$
$\frac{1}{8}$	3
10	$\frac{1}{6}$
$\frac{1}{3}$	4
7	$\frac{1}{10}$
$\frac{1}{9}$	2

Have students revisit the Pose the Problem question and discuss answers.

- How can you determine what fraction of a gallon is in each cup?

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students use the relationship between multiplication and division to divide unit fractions by non-zero whole numbers.

ETP Facilitate Meaningful Mathematical Discourse

Have the students create the equation. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

Have the students estimate the solution. Ask:

- Should the each of the 6 equal shares of $\frac{1}{2}$ be less than or greater than $\frac{1}{2}$? Why?
- How does that help you estimate the solution?

Make sure students understand the division expression and the multiplication expression represent the same quantity.

- What does $\frac{1}{2} \div 6$ represent?
- What does $\frac{1}{6} \times \frac{1}{2}$ represent?
- **Think About It:** How do you know that $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$?

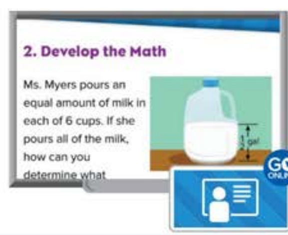
Have the students use their estimate to assess the reasonableness of the calculated solution. Ask:

- Is the calculated solution reasonable? Why or why not?

Math is... Structure

- If an equation is true, why are all the equations related to it true?

Students understand the structure of mathematics that allows them to use the truth of an equation to establish the truth of a related equation and to check an answer.



EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *can be*. Use a pencil to write something. Write it messily. Say, *This can be rewritten to be neater*. Rewrite it more neatly. Stack a pile of books on the table. Say, *These can be restacked*. Stack them differently. Finally, group 12 counters into 3 groups of 4. Ask, *Can these be regrouped*? Then place just one counter on the table and ask, *Can this be regrouped*?

Developing/Expanding Ensure understanding of *can be*. Use a pencil to write something. Write it messily. Say, *This can be rewritten to be neater*. Rewrite it more neatly. Stack a pile of books on the table. Say, *These can be restacked*. Stack them differently. Finally, group 12 counters into 3 groups of 4. Ask, *What can be done with these?* Provide the frame *They* ____ ____ *regrouped,* if needed.

Bridging/Reaching Guide students to the Learn page and ask them to focus on the sentence containing *can be rewritten*. Have them restate the meaning in their own words using *again* instead of *re-*. Provide the sentence starter *You can ...* Have students share other words they know with the prefix *re-* where the meaning is to do something again (*redo, regroup,* etc.).

On My Own

Name ______________________

Is the quotient correct or incorrect? How do you know?

1. $\frac{1}{2} \div 3 = \frac{1}{6}$
Correct. Sample answer: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

2. $\frac{1}{4} \div 2 = \frac{1}{2}$
Incorrect. Sample answer: $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

3. $\frac{1}{3} \div 6 = \frac{1}{9}$
Incorrect. Sample answer: $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

4. $\frac{1}{6} \div 4 = \frac{1}{24}$
Correct. Sample answer: $\frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

What is the quotient?

5. $\frac{1}{5} \div 5 = \underline{\frac{1}{25}}$

6. $\frac{1}{7} \div 2 = \underline{\frac{1}{14}}$

7. $\frac{1}{8} \div 10 = \underline{\frac{1}{80}}$

8. $\frac{1}{9} \div 3 = \underline{\frac{1}{27}}$

9. A garden has an area of $\frac{1}{10}$ acre. What fraction of an acre is each of the 4 sections?
$\frac{1}{40}$ acre

10. **Error Analysis** Peter buys $\frac{1}{4}$ pound of ham. Peter says that if he makes 2 ham sandwiches, each will have $\frac{1}{2}$ pound of ham. Is Peter correct? Explain why or why not.
No; Sample answer: Peter is not correct. He divided 4 by 2 instead of multiplying $\frac{1}{2} \times \frac{1}{4}$, which is $\frac{1}{8}$.

11. Theo cuts this board into 4 equal sections. What is the length of each section?
$\frac{1}{16}$ ft

12. Sasha spends $\frac{1}{2}$ of each school day in math class, science class, and history class. If the time spent in each class is the same, what fraction of the school day does Sasha spend in math class?
$\frac{1}{6}$ of the school day

13. **Extend Your Thinking** When a unit fraction is divided by a non-zero whole number, will the quotient always be less than the unit fraction? Explain why or why not.
Yes, the fraction is being divided into smaller pieces, so the quotient will be less than the unit fraction.

Reflect

How did you think like a mathematician today?
Answers may vary.

Math is... Mindset
What strengths did you rely on to be successful today?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1–8 Students may mistakenly divide the whole number by the fraction. Remind students to pay attention to the order in which each equation is written, and that the placement of the dividend and divisor are not interchangeable.

Item Analysis

Item	DOK	Rigor
1–4	1	Conceptual Understanding
5–8	1	Procedural Skill & Fluency
9	2	Application
10	3	Conceptual Understanding
11–12	3	Application
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How did you think like a mathematician today?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What strengths did you rely on to be successful today?

Students reflect on how they practiced self-awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can divide unit fractions by non-zero whole numbers.
- I can check if a calculated quotient is correct using a related multiplication equation.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket
Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Divide unit fractions by whole numbers	5.NF.B.7.a
2	1	Divide unit fractions by whole numbers	5.NF.B.7.a
3	2	Divide unit fractions by whole numbers	5.NF.B.7.c
4	2	Divide unit fractions by whole numbers	5.NF.B.7.c

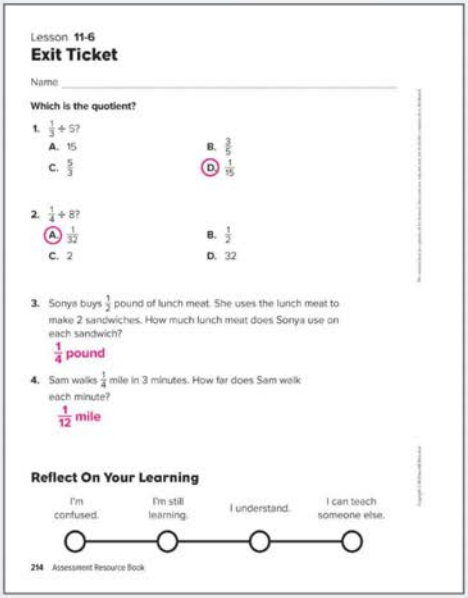
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking



Lesson 11-6
Exit Ticket

Name

Which is the quotient?

1. $\frac{1}{3} \div 5$?
A. 15
B. $\frac{3}{5}$
C. $\frac{5}{3}$
D. $\frac{1}{15}$

2. $\frac{1}{4} \div 8$?
A. $\frac{1}{32}$
B. $\frac{1}{2}$
C. 2
D. 32

3. Sonya buys $\frac{1}{2}$ pound of lunch meat. She uses the lunch meat to make 2 sandwiches. How much lunch meat does Sonya use on each sandwich?
$\frac{1}{4}$ pound

4. Sam walks $\frac{1}{4}$ mile in 3 minutes. How far does Sam walk each minute?
$\frac{1}{12}$ mile

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

214 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Divide It Up!

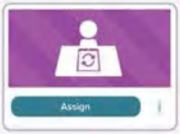
Work with students in groups. Students roll a number cube for the denominator and roll again for the whole number. Students then solve by rewriting the division problem as a multiplication problem. If students have difficulty, encourage them to model the problem using area to see that the number of unit fractions that it takes to fill the whole number is equal to the product of the whole number and the denominator of the fraction.

GO ONLINE

Take Another Look Lesson

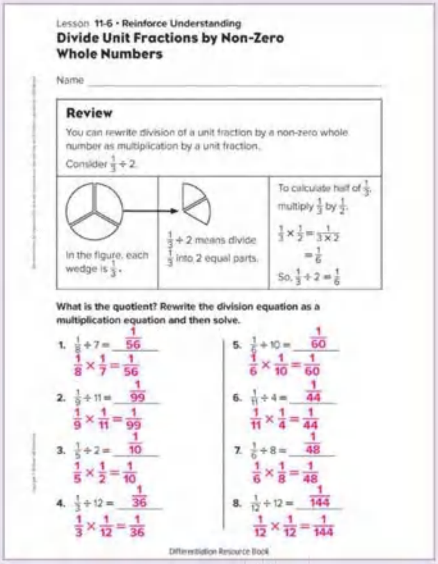
Assign the interactive lesson to reinforce targeted skills.

- Divide Unit Fractions in Word Problems



INDEPENDENT WORK

Differentiation Resource Book, p. 135



Lesson 11-6 • Reinforce Understanding

Divide Unit Fractions by Non-Zero Whole Numbers

Name ______

Review

You can rewrite division of a unit fraction by a non-zero whole number as multiplication by a unit fraction.

Consider $\frac{1}{3} \div 2$.

In the figure, each wedge is $\frac{1}{3}$.

$\frac{1}{3} \div 2$ means divide $\frac{1}{3}$ into 2 equal parts.

To calculate half of $\frac{1}{3}$, multiply $\frac{1}{3}$ by $\frac{1}{2}$.

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{3 \times 2}$
$= \frac{1}{6}$

So, $\frac{1}{3} \div 2 = \frac{1}{6}$

What is the quotient? Rewrite the division equation as a multiplication equation and then solve.

1. $\frac{1}{8} \div 7 = \frac{1}{56}$; $\frac{1}{8} \times \frac{1}{7} = \frac{1}{56}$
2. $\frac{1}{9} \div 11 = \frac{1}{99}$; $\frac{1}{9} \times \frac{1}{11} = \frac{1}{99}$
3. $\frac{1}{5} \div 2 = \frac{1}{10}$; $\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$
4. $\frac{1}{3} \div 12 = \frac{1}{36}$; $\frac{1}{3} \times \frac{1}{12} = \frac{1}{36}$
5. $\frac{1}{6} \div 10 = \frac{1}{60}$; $\frac{1}{6} \times \frac{1}{10} = \frac{1}{60}$
6. $\frac{1}{11} \div 4 = \frac{1}{44}$; $\frac{1}{11} \times \frac{1}{4} = \frac{1}{44}$
7. $\frac{1}{6} \div 8 = \frac{1}{48}$; $\frac{1}{6} \times \frac{1}{8} = \frac{1}{48}$
8. $\frac{1}{12} \div 12 = \frac{1}{144}$; $\frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Fraction Division Bingo Students practice solving expressions involving fractions by playing bingo.

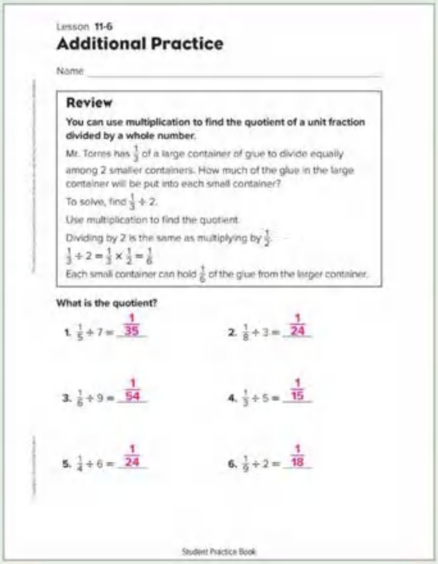
GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 135–136



Lesson 11-6

Additional Practice

Name ______

Review

You can use multiplication to find the quotient of a unit fraction divided by a whole number.

Mr. Torres has $\frac{1}{3}$ of a large container of glue to divide equally among 2 smaller containers. How much of the glue in the large container will be put into each small container?

To solve, find $\frac{1}{3} \div 2$.

Use multiplication to find the quotient.

Dividing by 2 is the same as multiplying by $\frac{1}{2}$.

$\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Each small container can hold $\frac{1}{6}$ of the glue from the larger container.

What is the quotient?

1. $\frac{1}{5} \div 7 = \frac{1}{35}$
2. $\frac{1}{8} \div 3 = \frac{1}{24}$
3. $\frac{1}{6} \div 9 = \frac{1}{54}$
4. $\frac{1}{3} \div 5 = \frac{1}{15}$
5. $\frac{1}{4} \div 6 = \frac{1}{24}$
6. $\frac{1}{9} \div 2 = \frac{1}{18}$

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying using area models.

Use It! Application Station

How Fast Is Your Robot? Students create a robot and measure the distance it travels. They experiment with techniques to increase the distance and speed.

GO ONLINE

Spiral Review

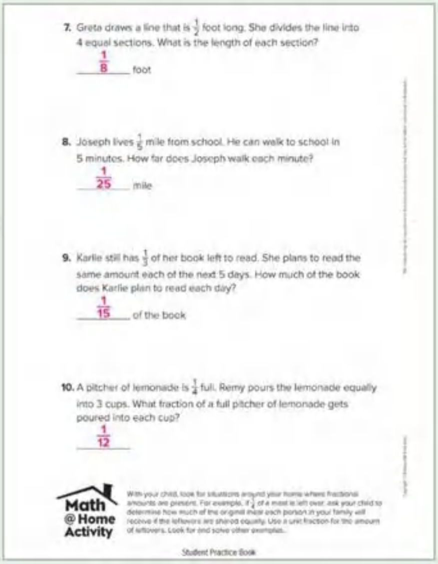
Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

INDEPENDENT WORK

Student Practice Book, pp. 135–136



7. Greta draws a line that is $\frac{1}{2}$ foot long. She divides the line into 4 equal sections. What is the length of each section?
   $\frac{1}{8}$ foot

8. Joseph lives $\frac{1}{5}$ mile from school. He can walk to school in 5 minutes. How far does Joseph walk each minute?
   $\frac{1}{25}$ mile

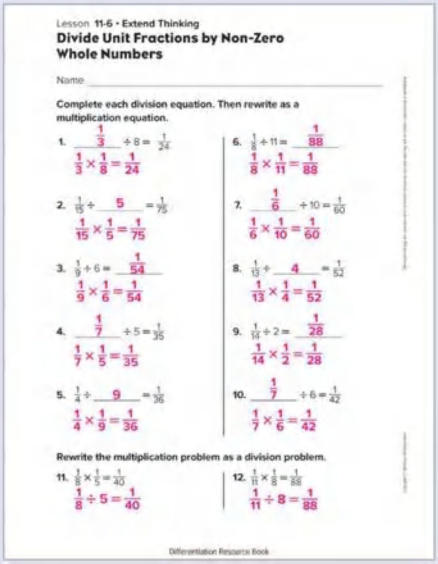
9. Karlie still has $\frac{1}{3}$ of her book left to read. She plans to read the same amount each of the next 5 days. How much of the book does Karlie plan to read each day?
   $\frac{1}{15}$ of the book

10. A pitcher of lemonade is $\frac{1}{4}$ full. Remy pours the lemonade equally into 3 cups. What fraction of a full pitcher of lemonade gets poured into each cup?
    $\frac{1}{12}$

Math @ Home Activity With your child, look for situations around your home where fractional amounts are present. For example, if $\frac{1}{4}$ of a meal is left over, ask your child to determine how much of the original meal each person in your family will receive if the leftovers are shared equally. Use a unit fraction for the amount of leftovers. Look for and solve other examples.

Student Practice Book

Differentiation Resource Book, p. 136



Lesson 11-6 • Extend Thinking

Divide Unit Fractions by Non-Zero Whole Numbers

Name ______________________

Complete each division equation. Then rewrite as a multiplication equation.

1. $\frac{1}{3} \div 8 = \frac{1}{24}$
   $\frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$

2. $\frac{1}{15} \div 5 = \frac{1}{75}$
   $\frac{1}{15} \times \frac{1}{5} = \frac{1}{75}$

3. $\frac{1}{9} \div 6 = \frac{1}{54}$
   $\frac{1}{9} \times \frac{1}{6} = \frac{1}{54}$

4. $\frac{1}{7} \div 5 = \frac{1}{35}$
   $\frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$

5. $\frac{1}{4} \div 9 = \frac{1}{36}$
   $\frac{1}{4} \times \frac{1}{9} = \frac{1}{36}$

6. $\frac{1}{8} \div 11 = \frac{1}{88}$
   $\frac{1}{8} \times \frac{1}{11} = \frac{1}{88}$

7. $\frac{1}{6} \div 10 = \frac{1}{60}$
   $\frac{1}{6} \times \frac{1}{10} = \frac{1}{60}$

8. $\frac{1}{13} \div 4 = \frac{1}{52}$
   $\frac{1}{13} \times \frac{1}{4} = \frac{1}{52}$

9. $\frac{1}{14} \div 2 = \frac{1}{28}$
   $\frac{1}{14} \times \frac{1}{2} = \frac{1}{28}$

10. $\frac{1}{7} \div 6 = \frac{1}{42}$
    $\frac{1}{7} \times \frac{1}{6} = \frac{1}{42}$

Rewrite the multiplication problem as a division problem.

11. $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$
    $\frac{1}{8} \div 5 = \frac{1}{40}$

12. $\frac{1}{11} \times \frac{1}{8} = \frac{1}{88}$
    $\frac{1}{11} \div 8 = \frac{1}{88}$

Differentiation Resource Book

Unit 11

Which Expressions Represent the Situation?

Name ______________________

Read each problem. Circle the expression or expressions that could be used to solve the problem. Do not actually solve the problem.

1. Mia has $\frac{1}{2}$ pound of cheese to be split equally into 6 small boxes. Which expression or expressions could be used to find the weight of the cheese in each box?

   Circle *all* correct choices.

   a. $\frac{1}{2} \times 6$
   b. $\frac{1}{2} \div 6$
   c. $\frac{1}{2} + \frac{1}{6}$
   d. $\frac{1}{2} \div \frac{1}{6}$
   e. $6 \div \frac{1}{2}$
   f. $\frac{1}{6} \times \frac{1}{2}$

   Explain your choice(s).

   Explanations may vary.

2. The price of a scarf is $\frac{1}{3}$ the price of a jacket. The price of the jacket is \$45. Which expression or expressions can be used to find the price of a scarf?

   Circle *all* correct choices.

   a. $45 \div 3$
   b. $\frac{1}{3} \times 45$
   c. $\frac{1}{3} \div 45$
   d. $\frac{1}{3} + 45$
   e. $45 \div \frac{1}{3}$
   f. $45 - \frac{1}{3}$

   Explain your choice(s).

   Explanations may vary.

Unit 11 • Divide Fractions 153

Read the problem. Circle the expression or expressions that could be used to solve the problem. Do not actually solve the problem.

3. Ari shares 4 pies among 16 friends. Which expression or expressions can be used to determine how much pie each friend receives?

   Circle *all* correct choices.

   a. $16 - 4$
   b. $16 \div 4$
   c. $4 \div 16$
   d. $\frac{1}{4} \times 16$
   e. $\frac{1}{16} \times 4$
   f. $16 \div \frac{1}{4}$

   Explain your choice(s).

   Explanations may vary.

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

154 Math Probe • Which Expressions Represent the Situation?

Analyze the Probe

Formative Assessment

Targeted Concept Identify expressions that represent a problem situation involving multiplication or division with a whole number and a unit fraction. Understand that more than one expression can be used to describe a problem situation.

Targeted Misconceptions Some students apply a "keyword" strategy to determine the operation—which can lead them off track. Some students think that the sequence of terms within an expression must match the sequence in which the values appear in the given word problem. Other students may not recognize equivalent expressions or think that a given problem can only be represented with one expression.

Authentic Student Work

Below are examples of students' explanations.

Sample A

2. Circle *all* correct choices.

   a. $45 \div 3$
   b. $\frac{1}{3} \times 45$
   c. $\frac{1}{3} \div 45$
   d. $\frac{1}{3} \div 45$
   e. $45 \div \frac{1}{3}$
   f. $45 - \frac{1}{3}$

Explain your choice(s).

the jacket would be 3 times the $ from the scarf. I would See how many times 3 goes in to 45 which would be 15. $45 \div 3 = 15$
$1/3$ of $45 = 15$.

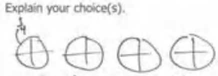
Sample B

3. Circle *all* correct choices.

   a. $16 - 4$
   b. $16 \div 4$
   c. $4 \div 16$ $\frac{1}{4}$
   d. $\frac{1}{4} \times 16$
   e. $\frac{1}{16} \times 4 = \frac{1}{4}$
   f. $16 \div \frac{1}{4}$

Explain your choice(s).

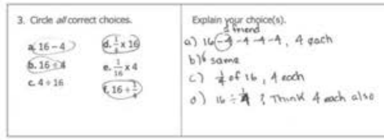
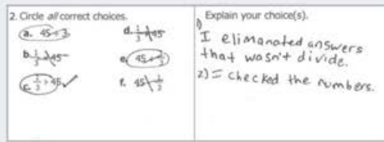
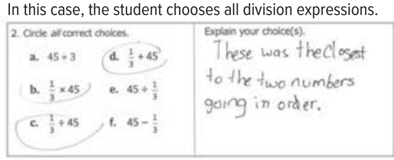
$\frac{1}{4}$



(c) 4 pies with 4 slices will work
(e) $\frac{1}{4}$ pie each

Collect and Assess Student Work

Collect and review student responses to determine possible misconceptions. See examples in If-Then chart.

IF incorrect...	THEN the student likely...	Sample Misconceptions
1. c 2. d, f 3. a	does not interpret the situation as a relationship that can be represented with multiplication or division, but instead identifies the operation as addition or subtraction.	In this case, the student understands the correct answer but has difficulty choosing the expressions that represent the situation. 
1. a, d 2. c	thinks that the sequence of terms in an expression must match the sequence in which the values appear in the word problem. Note that this misconception could lead to a correct answer for Exercise 1 (choice b) and a correct answer for Exercise 2 (choice b).	In this case, the student looked for the numbers in the order shown in the problem. 
1. d, e 3. b, f	identifies a division relationship, but chooses an expression that contains the reciprocal of the divisor (in Exercise 1, choice d; in Exercise 3, choice f); OR transposes the divisor and the dividend (in Exercise 1, choice e; in Exercise 3, choice b).	In this case, the student chooses all division expressions. 
Chooses only one expression for each Exercise	does not identify equivalent expressions or thinks there is only one expression that can represent a given situation.	

Many of the above difficulties result in a combination of correct and incorrect responses.
For correct responses, be sure to check for sound reasoning.

Take Action

Choose from the following resources or suggestions:

- Revisit the activities in Lessons 11-2 through 11-5 that relate to writing expressions to describe division situations.
- Provide problems where the quantities have been removed to promote reasoning and estimation about the problem context.
- Use concrete and visual representations to build understanding of equivalent expressions that model a given problem.
- Focus on language that promotes conceptual understanding of division and multiplication. A division problem such $3 \div \frac{1}{8}$ may be thought of as *How many* $\frac{1}{8}$*s fit into 3?* Multiplication can be thought of as *groups of*.
- Build understanding of the structure of word problems by having students sort a collection of problems based on common structures such as *equal groups* or *compare* situations.

Revisit the Probe After additional instruction, have students review their initial answers to the probe. Use these questions for discussion:

- Are there any answers you would like to change? Explain.
- Are there any questions that you still have about any of the exercises on this probe?

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

LESSON **11-7**

Solve Problems Involving Fractions

Learning Target

- I can solve word problems involving division of fractions.

Standards ◇ Major △ Supporting ○ Additional

Content

◇ **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

◇ **5.NF.B.7.c** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

Vocabulary

Math Terms	Academic Terms
equation	establish
unknown	relevant
variable	

Material

The materials may be for any part of the lesson.

- *Problem-Solving Tool* Teaching Resource

Focus

Content Objective

- Students solve word problems involving division of fractions using strategies such as using fraction models or the relationship between multiplication and division.

Language Objectives

- Students discuss solving word problems involving division of fractions using *another way*.
- To support maximizing linguistic and cognitive meta-awareness, ELs participate in MLR5: Co-Craft Questions and Problems.

SEL Objective

- Students advocate for their mathematical problem solving and adjust their understanding based on constructive feedback.

Coherence

Previous

- Students applied previous understandings of multiplication to multiply a fraction by a whole number (Grade 4).
- Students used the relationship between multiplication and division to divide unit fractions by whole numbers (Unit 11).

Now

- Students choose and use strategies to solve division word problems that involve fractions and whole numbers.

Next

- Students convert measurement units and represent data (Unit 12).
- Students interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions (Grade 6).

Rigor

Conceptual Understanding

- Students extend their understanding of operations with fractions by solving word problems.

Procedural Skill & Fluency

- Students build fluency in interpreting word problems and finding the solutions using operations involving fractions and whole numbers.

Application

- Students apply their understanding of operations with fractions and whole numbers to solve problems with real-world contexts.

GO ONLINE

Number Routine
Where Does It Go?

5–7 min

Build Fluency Students build number sense as they draw a point for a decimal number 23.54 on two different number lines marked with different end points.

These prompts encourage students to talk about their reasoning:

- What did you notice about each number line?
- How did you determine where to place the point for 23.54 on each number line?
- How did you think about each number line before placing each point?
- Why do the points appear to be located in about the same position on each number line? How can that be correct?

Launch 5–7 min

Sense-Making Routine

Purpose Students think about the context of word problems involving division and fractions and how that affects their solution strategy.

Notice & Wonder

- How are they the same?
- How are they different?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students write down on their own what they notice is the same and different before beginning discussion as a class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' ability to choose and use strategies to solve division word problems that involve fractions and whole numbers and are based on possible comments and questions that students may make during the share out.

- How do you know which operation to use to solve each problem?
- What types of numbers does each situation involve?

Math is... Mindset

- How can you show that you are listening attentively?

SEL Relationship Skills: Communication

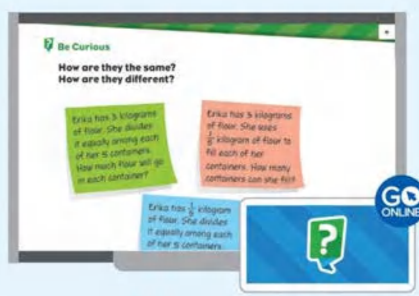
After students work through the Notice & Wonder routine independently, have them share their reasoning with a partner and communicate their chosen strategy. If students have used different strategies to determine how the problems are similar and different, invite them to work together to understand one another's reasoning. Remind students that strong learners are willing to learn from not only communication by their teachers but also their peers.

Transition to Explore & Develop

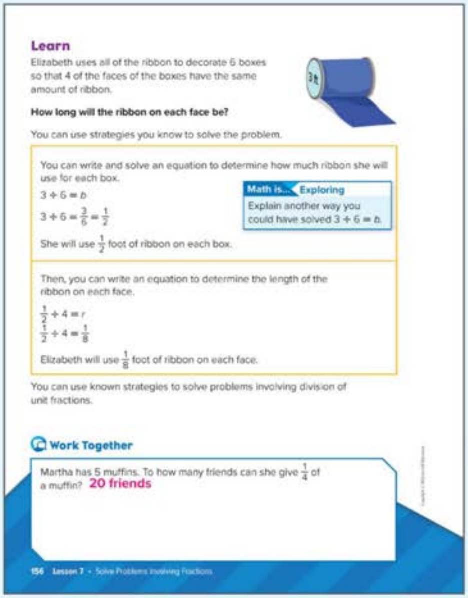
Ask questions that get students thinking about choosing and using strategies to solve division word problems that involve fractions and whole numbers.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about choosing and using strategies to solve division word problems that involve fractions and whole numbers.



Explore & Develop 20 min



Learn

Elizabeth uses all of the ribbon to decorate 6 boxes so that 4 of the faces of the boxes have the same amount of ribbon.

How long will the ribbon on each face be?

You can use strategies you know to solve the problem.

You can write and solve an equation to determine how much ribbon she will use for each box.

$3 \div 6 = b$

$3 \div 6 = \frac{3}{6} = \frac{1}{2}$

She will use $\frac{1}{2}$ foot of ribbon on each box.

Math is... Exploring
Explain another way you could have solved $3 \div 6 = b$.

Then, you can write an equation to determine the length of the ribbon on each face.

$\frac{1}{2} \div 4 = r$

$\frac{1}{2} \div 4 = \frac{1}{8}$

Elizabeth will use $\frac{1}{8}$ foot of ribbon on each face.

You can use known strategies to solve problems involving division of unit fractions.

Work Together

Martha has 5 muffins. To how many friends can she give $\frac{1}{4}$ of a muffin? **20 friends**

156 Lesson 7 • Solve Problems Involving Fractions

❶ Pose the Problem

ETP Pose Purposeful Questions

- How can you make sense of this problem?
- What do think a plan for solving this problem would be?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Co-Craft Questions and Problems

Pair students and have them co-create a problem similar to the one on the Learn page. Have them work together to solve their problem and then trade their problem with another pair. After each pair solves the other pair's problems, have them form a group of four to check solutions and correct any mistakes that may have been made. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you choose a strategy to solve a division word problem that involves fractions and whole numbers?
- What are some strategies you know that you can use to solve division word problems that involve fractions and whole numbers?

Key Takeaway

- Problems involving division of unit fractions can be solved using known strategies, such as using fraction models or the relationship between multiplication and division.

Work Together

Students work together to solve a word problem involving division of a whole number by a unit fraction.

Common Error: Because students divided a unit fraction by a whole number in the Pose the Problem, they may automatically begin to do the same in the Work Together. Remind students to read the problem carefully, using representations such as fraction models to help them understand the problem, and only then determine how they will solve the problem.

LOM Language of Math

People can buy a small part of ownership in a company; this small part is called a *share*. Often, some of the company's profits is divided equally among the people who own shares. That amount that are divided equally is called a *dividend*.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore solving problems involving division of fractions.

Materials: *Problem-Solving Tool* Teaching Resource

Directions: Distribute copies of the *Problem-Solving Tool* Teaching Resource. Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- How can you represent the problem?
- What strategies do you know for dividing whole numbers?
- How does the context of the problem help you know how to solve it?
- How does your solution method compare to others?

Math is... Exploring

- Explain another way you could have solved $4 \div 8 = b$.

Students understand other approaches to solving problems.

Activity Debrief: Have students share their solutions and strategies for solving the problem. Encourage students to find similarities and differences among the solution methods.

A PDF of the Activity-Based Exploration is available in the Digital Teacher Center.

Guided Exploration

Students choose and use strategies to solve a multi-step word problem that involves fractions and whole numbers.

ETP Facilitate Meaningful Mathematical Discourse

Have the students solve the equation for how much ribbon will be used on each box. Ask:

- What kind of division equation is $4 \div 8$?
- How can you solve the equation?
- Should the quotient be less than or greater than 4? How do you know?
- **Think About It:** How can you check if $4 \div 8 = \frac{1}{2}$ is correct?

Have the students create the equation for the length of ribbon on each face. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?

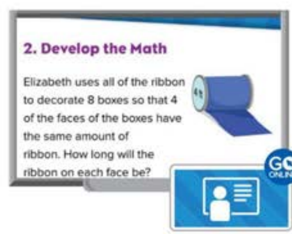
Have the students solve the equation for how much ribbon will be used on each box. Ask:

- What kind of division equation is $\frac{1}{2} \div 4$?
- How can you solve the equation?
- Should the quotient be less than or greater than $\frac{1}{2}$? How do you know?

Math is... Exploring

- Explain another way you could have solved $4 \div 8 = b$.

Students understand other approaches to solving problems.

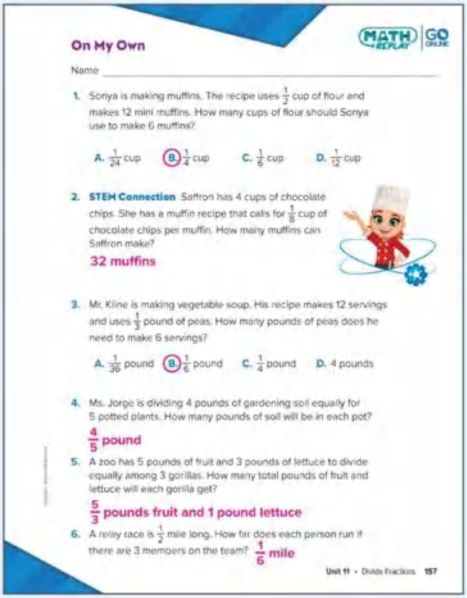


EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *faces* as in "faces of a box." Demonstrate using a manipulative with faces, such as a cube or box. Count the faces. Say, *This has [six] faces*. Then, point to three of its faces. Say, *I pointed to three of its faces*. Point again to two of its faces. Ask, *How many faces did I point to? [6] or [2]?*

Developing/Expanding Ensure understanding of *faces* as in "faces of a box." Demonstrate using a manipulative with faces, such as a cube or box. Count the faces. Say, *This has [six] faces.* Then point to three of its faces. Say, *I pointed to three of its faces*. Point again to two of its faces. Ask, *How many faces did I point to?* Expect the use of faces in students' responses.

Bridging/Reaching Ask students to review the problem at the top of the Learn page. Have them focus on the word *face*. Instruct students to look for objects in the class that have six faces (*box, cube, etc*.). Finally, ask them if they can think of other words that can be used in its place (*surface, side, etc.*). Allow them to use a dictionary if desired.



On My Own

Name ______

1. Sonya is making muffins. The recipe uses $\frac{1}{2}$ cup of flour and makes 12 mini muffins. How many cups of flour should Sonya use to make 6 muffins?

A. $\frac{1}{24}$ cup B. $\frac{1}{4}$ cup C. $\frac{1}{6}$ cup D. $\frac{1}{12}$ cup

2. **STEM Connection** Saffron has 4 cups of chocolate chips. She has a muffin recipe that calls for $\frac{1}{8}$ cup of chocolate chips per muffin. How many muffins can Saffron make?

32 muffins

3. Mr. Kline is making vegetable soup. His recipe makes 12 servings and uses $\frac{1}{3}$ pound of peas. How many pounds of peas does he need to make 6 servings?

A. $\frac{1}{36}$ pound B. $\frac{1}{6}$ pound C. $\frac{1}{4}$ pound D. 4 pounds

4. Ms. Jorge is dividing 4 pounds of gardening soil equally for 5 potted plants. How many pounds of soil will be in each pot?

$\frac{4}{5}$ pound

5. A zoo has 5 pounds of fruit and 3 pounds of lettuce to divide equally among 3 gorillas. How many total pounds of fruit and lettuce will each gorilla get?

$\frac{5}{3}$ pounds fruit and 1 pound lettuce

6. A relay race is $\frac{1}{2}$ mile long. How far does each person run if there are 3 members on the team? **$\frac{1}{6}$ mile**

Unit 11 • Divide Fractions 157

7. Shaun is making 3 bags of trail mix. He has $\frac{1}{5}$ pound of dried cranberries to divide equally among the bags. How many pounds of dried cranberries will be in each bag?

A. $\frac{1}{15}$ pound B. $\frac{3}{5}$ pound C. $\frac{1}{3}$ pound D. 15 pounds

8. Lucy brings 4 cakes to the bake sale. Each piece of cake is $\frac{1}{6}$ of the whole. How many pieces of cake does she have? Write and solve the equation.

$4 \div \frac{1}{6} = 24$ pieces

9. Mike made 60 cookies. He divided the cookies equally among his 8 friends and kept the rest for himself. How many cookies did Mike give his friends, and how many did he keep?

He gave 7 cookies to each friend and kept 4 to himself.

10. Ingrid buys this piece of cheese. She uses equal amounts of it to make 3 sandwiches. How much cheese is on each sandwich?

$\frac{1}{4}$ lb

$\frac{1}{12}$ pound

11. **Extend Your Thinking** Write a division word problem that involves a unit fraction. Then, solve it. **Check students' work.**

Reflect

What strategy do you like to use to solve real-world problems involving the division of fractions? Explain your answer.

Answers may vary.

Math is... Mindset
How have you shown that you were listening attentively?

158 Lesson 7 • Solve Problems Involving Fractions

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 1 Students may see that the recipe is being cut in half and try to divide by $\frac{1}{2}$. Encourage students to think about how they divide when splitting something in 2 equally.

Item Analysis

Item	DOK	Rigor
1–10	2	Application
11	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- What strategy do you like to use to solve real-world problems involving the division of fractions? Explain your answer.

Ask students to share their reflections with their classmates.

Math is... Mindset

- How have you shown that you were listening attentively?

Students reflect on how they developed stronger relationship skills.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can solve word problems involving division of fractions.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Solve problems involving fractions	5.NF.B.7.c
2	2	Solve problems involving fractions	5.NF.B.7.c
3	2	Solve problems involving fractions	5.NF.B.7.c
4	2	Solve problems involving fractions	5.NF.B.7.c

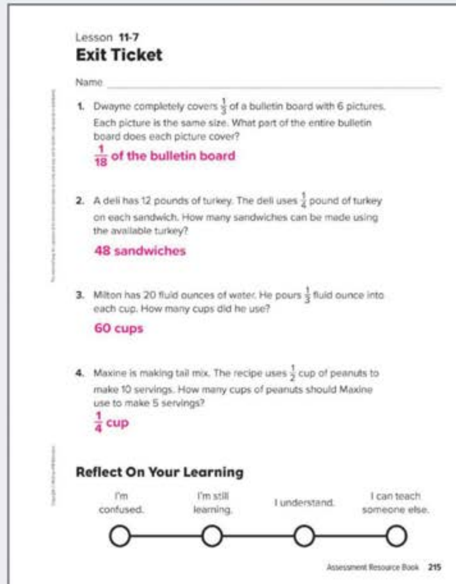
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking


Lesson 11-7
Exit Ticket

Name ____________

1. Dwayne completely covers $\frac{1}{3}$ of a bulletin board with 6 pictures. Each picture is the same size. What part of the entire bulletin board does each picture cover?
$\frac{1}{18}$ of the bulletin board

2. A deli has 12 pounds of turkey. The deli uses $\frac{1}{4}$ pound of turkey on each sandwich. How many sandwiches can be made using the available turkey?
48 sandwiches

3. Milton has 20 fluid ounces of water. He pours $\frac{1}{3}$ fluid ounce into each cup. How many cups did he use?
60 cups

4. Maxine is making trail mix. The recipe uses $\frac{1}{2}$ cup of peanuts to make 10 servings. How many cups of peanuts should Maxine use to make 5 servings?
$\frac{1}{4}$ cup

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 215

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Write 'n' Solve!

Work with students in pairs. Have each student write a word problem involving division with a whole number and a fraction. Then have the students trade problems and solve. If students have difficulty, help them to draw bar diagrams and write equations. Have students check their solutions using multiplication.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills

- Divide Unit Fractions in Word Problems

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 137

Lesson 11-7 • Reinforce Understanding

Solve Problems Involving Fractions

Name ____________________

Review

Be careful when solving problems involving division of unit fractions.

Dividing a Whole Number by a Whole Number	6 foot of rope cut into 10 equal pieces. How long is each piece?	$6 \div 10 = \frac{6}{10}$ or $\frac{3}{5}$
Dividing a Whole Number by a Unit Fraction	One dime is $\frac{1}{10}$ of a dollar. How many dimes in \$6.00?	$6 \div \frac{1}{10} = 6 \times 10 = 60$
Dividing a Unit Fraction by a Whole Number	A $\frac{1}{6}$ acre garden plot is divided into 10 equal size flower beds. How big is each flower bed?	$\frac{1}{6} \div 10 = \frac{1}{6} \times \frac{1}{10} = \frac{1}{60}$

Solve each problem. Show your work.

1. A chicken noodle soup recipe calls for $\frac{1}{4}$ cup of chopped parsley and makes 6 servings. How much chopped parsley is in each serving?
   $\frac{1}{4} \div 6 = \frac{1}{24}$ cup of chopped parsley per serving.
2. Walter is dividing 6 pounds of flour equally among 8 containers. How many pounds of flour will be in each container?
   $6 \div 8 = \frac{6}{8} = \frac{3}{4}$ pounds of flour in each container.
3. Mary has 4 pounds of pulled pork and 9 pounds of brisket to divide equally among five customers. How many total pounds of each type of meat will each customer get?
   $4 \div 5 = \frac{4}{5}$ pounds of pulled pork and $9 \div 5 = \frac{9}{5}$ pounds of brisket for each customer.
4. Soo has 5 cups of orange juice. She has a smoothie recipe which calls for $\frac{1}{3}$ cup of orange juice per smoothie. How many smoothies can Soo make? $5 \div \frac{1}{3} = 5 \times 3 = 15$ smoothies can be made from the orange juice Soo has available.

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Dividing Fractions Race Students practice dividing fractions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 137–138

Lesson 11-7

Additional Practice

Name ____________________

Review

You can use strategies you know to help you solve problems involving division.

A sandwich shop uses $\frac{1}{4}$ pound of lunch meat on its sandwiches. Yesterday, the sandwich shop used 20 pounds of lunch meat. How many sandwiches were served yesterday?

To solve, find $20 \div \frac{1}{4}$.

There are four $\frac{1}{4}$s in each whole.

So, $20 \times 4 = 80$.

The sandwich shop served 80 sandwiches yesterday.

1. Deanne covers $\frac{1}{3}$ of her notebook cover with 5 stickers. Each sticker is the same size. What part of the entire notebook cover does each sticker cover?
   $\frac{1}{15}$
2. Marvin uses a mix and some water to make 54 fluid ounces of fruit punch. He pours an equal amount into 8 glasses for himself and seven friends. How much fruit punch does each person get?
   $6\frac{3}{4}$ fluid ounces

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with multiplying using area models.

Assign

Use It! Application Station

Potluck with a Twist Students use whole numbers, fractions, and division to create a 14-dish menu for a potluck.

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

GO ONLINE

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 137–138

3. A baker has 10 pounds of flour on hand. Each batch of cookies needs $\frac{1}{2}$ pound of flour. How many batches of cookies can the baker make using the available flour?

   20 batches

4. Maxine has 2 pounds of raisins. She places an equal amount into each of 15 snack bags. How many pounds of raisins are in each snack bag?

   $\frac{2}{15}$ pound

5. Andrea has 50 perennials to plant. She plants the flowers in 6 equal rows, using as many flowers as possible. How many perennials are in each row? How many are left unplanted?

   8 perennials in each row;

   2 perennials left unplanted

6. Matthew has $\frac{1}{3}$ pound of trail mix. He eats all of it in 4 equal servings during his hike. How much trail mix does Matthew eat in one serving?

   $\frac{1}{12}$ pound

Math @ Home Activity With your child, look for situations around your home where your child can practice solving problems involving division. For example, if there are 3 apples left, and 5 people each want some, how much does each person get if they share equally? Look for and solve other examples that have been studied in this unit.

Student Practice Book

Differentiation Resource Book, p. 138

Lesson 11-7 • Extend Thinking

Solve Problems Involving Fractions

Name ______

1. Stephanie has a recipe to make muffins. The recipe uses $\frac{1}{4}$ cup of melted butter and makes 12 muffins. How many cups of melted butter should Stephanie use to make 4 muffins?

   $\frac{1}{4} \div 3 = \frac{1}{12}$ cup of melted butter

2. A stick of butter is $\frac{1}{2}$ cup. What fraction of a stick of butter would Stephanie need to make 4 muffins?

   $\frac{1}{2} \div \frac{1}{12} = 6$, meaning she needs $\frac{1}{6}$ of a stick of butter

3. Ari has four sticks of butter. How many muffins can Ari make with four sticks of butter?

   Ari has $4 \times \frac{1}{2} = 2$ cups of butter. One muffin takes $\frac{1}{4} \div 12 = \frac{1}{48}$ cup of butter. Ari can make $2 \div \frac{1}{48} = 96$ muffins.

4. Ari made 8 batches of 12 muffins and wants to give each of his 14 friends the same number of muffins. How many muffins does each of his friends receive?

   Ari made $8 \times 12 = 96$ muffins. $96 \div 14 = 6$ R 12. Each friend receives 6 muffins and Ari has 12 muffins left over.

5. A variation for the muffin recipe calls for adding $\frac{1}{3}$ cup of raisins to make 12 muffins. How many cups of raisins will each muffin contain?

   $\frac{1}{3} \times \frac{1}{12} = \frac{1}{36}$ cup of raisins per muffin.

Differentiation Resource Book

Unit Review

Name ______________

Vocabulary Review

Choose the correct word(s) to complete each sentence.

denominator	numerator	unit fraction
dividend	quotient	unknown
divisor	remainder	

1. A **dividend** represents the amount to be shared equally. (Lesson 11-2)
2. A **unit fraction** has a 1 as the numerator. (Lesson 11-3)
3. In a fraction that represents a quotient, the denominator is the same as the **divisor**. (Lesson 11-1)
4. A **quotient** of a division equation can be represented as a fraction or mixed number. (Lesson 11-1)
5. In a fraction that represents a quotient, the **numerator** is the same as the dividend. (Lesson 11-1)
6. The number that you are looking for when you solve an equation is called the **unknown**. (Lesson 11-7)
7. The **denominator** of a mixed number can represent a divisor. (Lesson 11-1)
8. When an improper fraction is rewritten as a mixed number the **remainder** is the numerator in the fraction. (Lesson 11-2)

Unit 11 • Divide Fractions 159

Review

9. Shaylin has 3 pounds of yogurt. She shares the yogurt equally with 3 of her friends. What is the weight of the yogurt Shaylin and each of her friends will receive? Write a division equation that represents the problem. Then, write the answer. (Lesson 11-2)

   $3 \div 4 = \frac{3}{4}$; $\frac{3}{4}$ pound

10. A rancher has 7 pounds of hay. He divides it equally into 3 troughs. How many pounds of hay does the rancher pour in each trough? Use a drawing to justify your solution. (Lesson 11-1)

    $\frac{7}{3}$ pounds; Check students' drawings.

11. The guitar teacher received a box of 200 guitar picks. She wants to share the guitar picks equally among her 16 students. How many guitar picks does each student receive? (Lesson 11-2)

    Each student will receive 12 guitar picks.

12. Which equation can match the model? (Lesson 11-3)

    A. $5 \div 3 = n$
    B. $3 \div \frac{1}{5} = n$
    C. $5 \div \frac{1}{3} = n$
    D. $3 \div 5 = n$

13. Jason has 9 yards of wire to make necklaces. He uses $\frac{1}{3}$ yard to make each necklace. How many necklaces can Jason make? (Lesson 11-4)

    27 necklaces

14. Ciera made 12 pints of fruit punch. She is going to put $\frac{1}{2}$ pint in each glass. How many glasses can Ciera fill? Write a division equation that represents the problem. Then, write the answer. (Lesson 11-4)

    $12 \div \frac{1}{2} = 24$; 24 glasses

160 Unit 11 • Review

Students can complete the **Unit Review** to prepare for the **Unit Assessment**. Students may complete the Review in their Interactive eBook in the Digital Student Center.

Vocabulary Review

Item Analysis

Item	Lesson
1	11-2
2	11-3
3	11-1
4	11-1
5	11-1
6	11-7
7	11-1
8	11-2

Review

Item Analysis

Item	DOK	Lesson	Standard
9	2	11-2	5.NF.B.7.c
10	2	11-1	5.NF.B.7
11	2	11-2	5.NF.B.7.c
12	1	11-3	5.NF.B.7, 5.NF.B.7.b
13	2	11-4	5.NF.B.7, 5.NF.B.7.b
14	2	11-4	5.NF.B.7, 5.NF.B.7.b

To review the lessons in this unit, have students watch the Math Replay video in their Digital Student Center.

Assign the Unit Review practice to students from the Digital Teacher Center.

GO ONLINE

MATH REPLAY

Item Analysis (continued)

Item	DOK	Lesson	Standard
15	1	11-4	5.NF.B.7, 5.NF.B.7.b
16	1	11-4	5.NF.B.7, 5.NF.B.7.b
17	1	11-4	5.NF.B.7, 5.NF.B.7.b
18	2	11-6	5.NF.B.7, 5.NF.B.7.a
19	1	11-2	5.NF.B.7
20	1	11-5	5.NF.B.7, 5.NF.B.7.a
21	1	11-5	5.NF.B.7, 5.NF.B.7.a

Performance Task

Standard: 5.NF.B.3, 5.NF.B.7, 5.NF.B.7.a, 5.NF.B.7.b, 5.NF.B.7.c

Rubric (6 points)

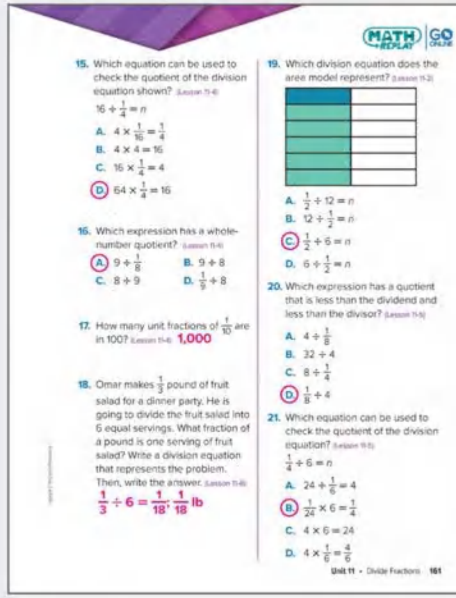
Part A (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with dividing a whole number by a fraction.
1 POINT	Student's work reflects developing proficiency with dividing a whole number by a fraction.
0 POINTS	Student's work shows weak proficiency with dividing a whole number by a fraction.

Part B (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with dividing a fraction by a fraction.
1 POINT	Student's work reflects developing proficiency with dividing a fraction by a fraction.
0 POINTS	Student's work shows weak proficiency with dividing a fraction by a fraction.

Part C (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with dividing an improper fraction by a fraction.
1 POINT	Student's work reflects developing proficiency with dividing an improper fraction by a fraction.
0 POINTS	Student's work shows weak proficiency with dividing an improper fraction by a fraction.

Reflect

The Reflect question provides an opportunity for students to express their understanding of the unit level focus question.



MATH REPLAY | GO ONLINE

15. Which equation can be used to check the quotient of the division equation shown?

$16 \div \frac{1}{4} = n$

A. $4 \times \frac{1}{16} = \frac{1}{4}$
B. $4 \times 4 = 16$
C. $16 \times \frac{1}{4} = 4$
D. $64 \times \frac{1}{4} = 16$

16. Which expression has a whole-number quotient?

A. $9 \div \frac{1}{8}$
B. $9 \div 8$
C. $8 \div 9$
D. $\frac{1}{9} \div 8$

17. How many unit fractions of $\frac{1}{10}$ are in 100? **1,000**

18. Omar makes $\frac{1}{3}$ pound of fruit salad for a dinner party. He is going to divide the fruit salad into 6 equal servings. What fraction of a pound is one serving of fruit salad? Write a division equation that represents the problem. Then, write the answer.

$\frac{1}{3} \div 6 = \frac{1}{18}$; $\frac{1}{18}$ lb

19. Which division equation does the area model represent?

A. $\frac{1}{2} \div 12 = n$
B. $12 \div \frac{1}{2} = n$
C. $\frac{1}{2} \div 6 = n$
D. $6 \div \frac{1}{2} = n$

20. Which expression has a quotient that is less than the dividend and less than the divisor?

A. $4 \div \frac{1}{8}$
B. $32 \div 4$
C. $8 \div \frac{1}{4}$
D. $\frac{1}{8} \div 4$

21. Which equation can be used to check the quotient of the division equation?

$\frac{1}{4} \div 6 = n$

A. $24 \div \frac{1}{6} = 4$
B. $\frac{1}{24} \times 6 = \frac{1}{4}$
C. $4 \times 6 = 24$
D. $4 \times \frac{1}{6} = \frac{4}{6}$

Unit 11 • Divide Fractions 161

Performance Task

Antonio is programming a robot to pick fresh fruit and place it into separate containers by weight for customers to take home.

Part A: The robot picks 2 pounds of strawberries and puts $\frac{1}{5}$ pound in each container. How many containers of strawberries will the robot fill?

10 containers

Part B: The robot picks $\frac{1}{3}$ pound of blueberries and puts $\frac{1}{9}$ pound in each container. How many containers of blueberries will the robot fill?

3 containers

Part C: The robot picks $\frac{11}{4}$ pounds of raspberries and puts $\frac{1}{6}$ pound in each container. How many containers of raspberries will the robot fill? Will the robot be able to put the same amount of raspberries in each container? Explain your answer.

No; Sample answer: The robot will fill 16 full containers and then 1 container with half of the amount or $\frac{1}{12}$ pound.

Reflect

How might you divide fractions by fractions in the real world?

Answers may vary.

162 Unit 11 • Performance Task

Fluency Practice

Unit 11

Fluency Practice

Name ______________________

Fluency Strategy

You can multiply using an algorithm.

Step 1 Multiply the ones.
8 × 4 = 32
Regroup 32 as 3 tens and 2 ones.

$$\begin{array}{r} 314 \\ \times\ 8 \\ \hline 2{,}512 \end{array}$$

Step 2 Multiply the tens.
8 × 10 = 80
Add the 3 tens.
80 + 30 = 110
Regroup 110 tens as 1 hundred and 1 ten.

Step 3 Multiply the hundreds.
8 × 300 = 2,400
Add the 1 hundred.
2,400 + 100 = 2,500

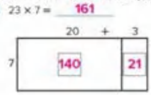
1. Use the algorithm to multiply.

$$\begin{array}{r} 456 \\ \times\ 9 \\ \hline 4{,}104 \end{array}$$

Fluency Flash

Use the area model to find the product.

2. 23 × 7 = 161



Fluency Check

What is the product or quotient?

3. 4,500 ÷ 5 = 900
4. 2,800 ÷ 4 = 700
5. 480 ÷ 6 = 80
6. 160 ÷ 4 = 40
7. 35 × 8 = 280
8. 456 × 8 = 3,648
9. 350 ÷ 7 = 50
10. 240 ÷ 4 = 60
11. 3,200 ÷ 8 = 400
12. 180 ÷ 9 = 20
13. 652 × 7 = 4,564
14. 289 × 6 = 1,734

Fluency Talk

How can you multiply a 3-digit number by a 1-digit number?
Explanations may vary.

How can you divide a multiple of 100?
Explanations may vary.

Fluency practice helps students develop procedural fluency; that is, the "ability to apply procedures accurately, efficiently, and flexibly." Because there is no expectation of speed, students should not be timed when completing the practice activity.

Build Fluency **Objective** Students practice using an algorithm to multiply.

Fluency Progression

Unit	Focus	Standard
1	Use Partial Sums to Add	4.NBT.B.4
2	Decompose by Place Value to Subtract	4.NBT.B.4
3	Use an Algorithm to Add	4.NBT.B.4
4	Use an Algorithm to Subtract	4.NBT.B.4
5	Choose a Strategy to Add	4.NBT.B.4
6	Choose a Strategy to Subtract	4.NBT.B.4
7	Multiply by Multiples of 10	5.NBT.B.5
8	Multiply by Multiples of 100	5.NBT.B.5
9	Divide Multiples of 10	5.NBT.B.6
10	Divide Multiples of 100	5.NBT.B.6
11	**Use an Algorithm to Multiply (2- and 3-Digit Numbers by 1-Digit Numbers)**	**5.NBT.B.5**
12	Use an Algorithm to Multiply (2-Digit Numbers by 2-Digit Numbers)	5.NBT.B.5
13	Choose a Strategy to Multiply	5.NBT.B.5
14	Choose a Strategy to Multiply	5.NBT.B.5

Fluency Expectations

Grade 4

- Add and subtract within 1,000,000.

Grade 5

- Multiply multi-digit whole numbers.

Grade 6

- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Field Day Fun!

Students draw on their understanding of solving problems involving division with fractions. Use the rubric shown to evaluate students' work.

Standard: 5.NF.B.3, 5.NF.B.7

Rubric (10 points)

Part A (DOK 2) – 2 points	
2 POINTS	Student's work shows proficiency representing quotients as a fraction. The student's answer and explanation are accurate.
1 POINT	Student's work shows developing proficiency representing quotients as a fraction. The student's answer or explanation is not accurate.
0 POINTS	Student's work shows weak proficiency representing quotients as a fraction. The student's answer and explanation are not accurate.
Parts B, C, E (DOK 2) – 6 points	
6 POINTS	Student's work shows proficiency with solving division word problems involving fractions. The student's answer and work are correct.
3 POINTS	Student's work shows developing proficiency with solving division word problems involving fractions. The student's answer or work is incorrect.
0 POINTS	Student's work shows weak proficiency with solving division word problems involving fractions. The student's answer and work are incorrect.
Part D (DOK 3) – 2 points	
2 POINTS	Student's work shows proficiency with determining if a quotient should be written as a remainder. The student's answer is correct and the explanations are reasonable.
1 POINT	Student's work shows developing proficiency with determining if a quotient should be written as a remainder. The student's answer is incorrect or the explanations are not reasonable.
0 POINTS	Student's work shows weak proficiency with determining if a quotient should be written as a remainder. The student's answer is incorrect and the explanations are not reasonable.

Unit 11

Performance Task

Name ________________

Field Day Fun!

The 5th grade is having their yearly field day competition.

Part A

For the first event, each of the 8 teams will need a bucket of sand. If there are 6 pounds of sand, what weight of sand will each team get? Write an equation and solve. Explain your work.

$\frac{3}{4}$ pound; Sample Answer: $p = 6 \div 8$, $p = \frac{3}{4}$. Divide the weight of the sand by the number of teams.

Part B

For the second event, a team of students runs 2 times around the track. If each student runs $\frac{1}{10}$ of the track, how many students do they need on each team? Use a representation to solve.

20 students; Check students' drawings.

Part C

For the third event, student teams will bunny hop $\frac{1}{4}$ of the way around the track. If there are 6 students on a team, how far will each person go? Show your work.

$\frac{1}{24}$ of the way around the track; Sample Answer: $\frac{1}{4} \div 6 = \frac{1}{24}$

Part D

For showing great sportsmanship, Mrs. Garcia rewarded her students by letting them pick from a prize box. The prize box has 81 prizes that include special pencils, notepads, stickers, and small toys. If there are 24 students in the class, what is the greatest number of prizes each student can pick if everyone is to get the same amount? Explain your answer.

3 prizes; $81 \div 24 = 3$ R9; The remainder isn't used since not all the students will get another prize.

Should the quotient be written as a mixed number or with a remainder? Explain how to know.

Sample answer: The quotient shouldn't be written as a mixed number because each prize cannot be divided into parts.

Part E

Mr. Johnson brought healthy snacks for his class to enjoy after the field day fun. Mr. Johnson has 50 large granola bars. He wants to give $\frac{1}{2}$ of a granola bar to each student. How many students will get $\frac{1}{2}$ of a granola bar? Write an equation and solve.

100 students; Sample answer: $50 \div \frac{1}{2} = s$; $s = 100$

Unit Assessments

Two forms of the Unit Assessment, Form A and Form B, are available for either print or digital administration. The items on the two assessments are parallel items, assessing the same concept and standard. The table below provides the item analysis for both forms.

Both Unit Assessments are available in the Assessment Resource Book or as downloadable files from the Digital Teacher Center.

Data When students complete the Unit Assessment in the Digital Student Center, their responses are auto-scored.

Item Analysis

Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	2	11-1	Quotient Fractions (Models)	5.NF.B.3
2	2	11-1	Quotient Fractions (Models)	5.NF.B.3
3	2	11-2	Quotient Fractions (Word Problems)	5.NF.B.7
4	2	11-2	Quotient Fractions (Word Problems)	5.NF.B.7
5	2	11-3, 11-7	Divide Whole Numbers by Unit Fractions	5.NF.B.7.b, 5.NF.B.7.c
6	2	11-4, 11-7	Divide Whole Numbers by Unit Fractions	5.NF.B.7.b, 5.NF.B.7.c
7	1	11-4	Divide Whole Numbers by Unit Fractions	5.NF.B.7.b
8	2	11-6, 11-7	Divide Unit Fractions in Word Problems	5.NF.B.7.a, 5.NF.B.7.c
9	1	11-5	Divide Unit Fractions by Whole Numbers	5.NF.B.7.a
10	1	11-6	Divide Unit Fractions in Word Problems	5.NF.B.7.a
11	1	11-1	Quotient Fractions (Models)	5.NF.B.3
12	2	11-2	Quotient Fractions (Word Problems)	5.NF.B.7
13	2	11-6, 11-7	Divide Unit Fractions in Word Problems	5.NF.B.7.a, 5.NF.B.7.c
14	3	11-4, 11-7	Divide Whole Numbers by Unit Fractions	5.NF.B.7.b, 5.NF.B.7.c

GO ONLINE

Assign the digital Unit Assessment (Form A or B) to students or download and print PDFs from the Digital Teacher Center.

Unit 11

Unit Assessment, Form A

Name ____________________

1. A vet feeds 8 dogs every week using 3 pounds of dog food. If each dog gets the same amount, how many pounds of dog food does each dog get every week?
   A. $\frac{3}{8}$ pound B. $\frac{8}{3}$ pounds
   C. 5 pounds D. 24 pounds

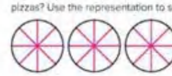
2. There are 7 children in Uriah's swim class. They have 9 cups of grapes to divide equally among them. Which fraction shows how many cups of grapes each child receives?
   A. $\frac{2}{9}$ cup B. $\frac{7}{9}$ cup
   C. $\frac{9}{2}$ cups D. $\frac{9}{7}$ cups

3. A teacher orders a set of 50 new pencils for her 15 students. Each student gets the same number of pencils. The teacher gives the greatest number of pencils possible to each student. Which best describes how many pencils each student gets?
   A. 2 pencils with 20 pencils left over
   B. 3 pencils with 5 pencils left over
   C. $3\frac{5}{15}$ pencils
   D. 4 pencils

4. A pitcher holds 64 fluid ounces of juice. Akeem pours an equal amount into each of 6 glasses until the pitcher is empty. How much juice does Akeem pour into each glass?
   A. $\frac{6}{64}$ fluid ounces
   B. 10 fluid ounces with 4 fluid ounces left over
   C. $10\frac{4}{6}$ fluid ounces
   D. 14 fluid ounces

Assessment Resource Book 219

5. A restaurant serves its slices of pizza as $\frac{1}{8}$ of a whole pizza. How many slices of pizza can the restaurant serve from 3 whole pizzas? Use the representation to solve.



24 slices of pizza

6. A local radio station gives weather updates every $\frac{1}{5}$ hour. How many weather updates does the radio announcer give during a 4-hour shift?
   A. $\frac{4}{5}$ weather update B. $\frac{5}{4}$ weather updates
   C. 9 weather updates D. 20 weather updates

7. What is the quotient?
   $4 \div \frac{1}{7}$
   A. 28 B. $\frac{4}{7}$
   C. $\frac{7}{4}$ D. $\frac{1}{28}$

8. In Ferdinand's flower garden, $\frac{1}{4}$ of the flowers are roses. He plants 3 different colors of roses. What fraction of Ferdinand's flower garden is each color of the roses?
   A. $\frac{1}{12}$ B. $\frac{3}{4}$
   C. $\frac{3}{12}$ D. $\frac{12}{1}$

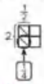
220 Assessment Resource Book

Unit **11**

Unit Assessment, Form A (continued)

Name ____________________

9. Which division equation is represented by the model?



A. $\frac{1}{2} \div 2 = \frac{1}{4}$ **B.** $\frac{1}{2} \div 2 = \frac{1}{2}$

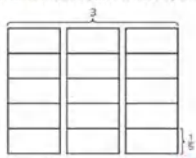
C. $2 \div \frac{1}{2} = 4$ **D.** $2 \div \frac{1}{2} = \frac{1}{4}$

10. What is the quotient?

$\frac{1}{4} \div 6$

$\frac{1}{24}$

11. Which division equation is shown by the model?



A. $15 \div \frac{1}{5} = 3$ **B.** $3 \div 15 = \frac{1}{5}$

C. $\frac{1}{5} \div 3 = \frac{1}{15}$ **D.** $15 \div 3 = \frac{1}{5}$

12. Carlita made 80 necklaces to sell at a craft show. She sold the same number of necklaces each of the 6 days of the show. Did Carlita sell all of her necklaces? Explain.

No; Sample answer: $80 \div 6 = 13$ R2, so she could have sold at most 13 necklaces each day, but then she would have 2 necklaces left over. To sell all of her necklaces, she would have had to sell $13\frac{2}{6}$ or $13\frac{1}{3}$ necklaces each day, but it is not possible to sell $\frac{1}{3}$ of a necklace.

13. Heidi has $\frac{1}{5}$ of a novel left to read. She wants to finish the novel by reading the same amount each day for the next 2 days. How much of the novel should Heidi read each day? Explain.

$\frac{1}{10}$ of the novel; Sample answer: Over 2 days, she should read $\frac{1}{5} \div 2 = \frac{1}{10}$ of the novel each day.

14. The middle school student council has 10 cups of confetti to decorate the tables for the banquet. They plan to use $\frac{1}{3}$ cup of confetti on each table. If they use $\frac{1}{4}$ cup of confetti on each table instead of $\frac{1}{3}$ cup of confetti, how many more tables can they decorate? Explain.

10 more tables; Sample answer: Using $\frac{1}{3}$ cup allows $10 \div \frac{1}{3} = 30$ tables to be decorated. Using $\frac{1}{4}$ cup allows $10 \div \frac{1}{4} = 40$ tables to be decorated. Then $40 - 30 = 10$, so 10 more tables can be decorated.

Form B

Unit **11**

Unit Assessment, Form B

Name ____________________

1. A group of 8 hikers has 7 pounds of trail mix to share equally. How many pounds of trail mix does each hiker get?

A. [illegible] pound **B.** [illegible] pounds

C. 1 pound **D.** 56 pounds

2. A baker makes 6 loaves of bread using 7 pounds of dough. If each loaf is the same weight, how many pounds is one loaf of bread?

A. [illegible] pound **B.** 1 pound

C. [illegible] pounds **D.** 13 pounds

3. A basketball league orders 50 new basketballs for the 8 teams in the league. Each team gets the same number of basketballs. The league gives the greatest number of basketballs possible to each team. Which best describes how many basketballs each team gets?

A. 5 basketballs with 10 basketballs left over

B. 6 basketballs with 2 basketballs left over

C. $6\frac{2}{8}$ basketballs

D. 8 basketballs

4. A pitcher holds 56 fluid ounces of lemonade. Alice pours an equal amount into each of 6 glasses until the pitcher is empty. How much lemonade does Alice pour into each glass?

A. [illegible] fluid ounces

B. 9 fluid ounces with 2 fluid ounces left over

C. $9\frac{1}{3}$ fluid ounces

D. 10 fluid ounces

7. What is the quotient?

$8 \div \frac{1}{3}$

A. [illegible] **B.** 24

C. [illegible] **D.** $\frac{1}{24}$

8. In Felicia's flower garden, $\frac{1}{3}$ of the flowers are tulips. She plants 3 different colors of tulips. What fraction of Felicia's flower garden is each color of the tulips?

A. [illegible] **B.** [illegible]

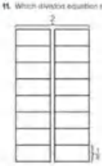
C. [illegible] **D.** [illegible]

10. What is the quotient?

$\frac{1}{3} \div 6$

$\frac{1}{18}$

11. Which division equation is shown by the model?



A. $2 \div 16 = \frac{1}{8}$

B. $16 \div \frac{1}{8} = 2$

C. $\frac{1}{8} \div 2 = \frac{1}{16}$

D. $16 \div 2 = \frac{1}{8}$

14. A caterer provides 10 pounds of coleslaw for an event. They plan to serve $\frac{1}{6}$ pound in each serving bowl. If they use $\frac{1}{8}$ pound of coleslaw in each bowl instead of $\frac{1}{6}$ pound, how many more bowls of coleslaw can they serve? Explain.

20 more bowls; Sample answer: Using $\frac{1}{6}$ pound allows $10 \div \frac{1}{6} = 60$ bowls to be served. Using $\frac{1}{8}$ pound allows $10 \div \frac{1}{8} = 80$ bowls to be served. Then $80 - 60 = 20$, so 20 more bowls can be served.

UNIT 12 PLANNER

Measurement and Data

PACING: 9 days

LESSON		MATH OBJECTIVE	LANGUAGE OBJECTIVE	SOCIAL AND EMOTIONAL LEARNING OBJECTIVE
Unit Opener IGNITE! Which Sums Occur Least and Most? Students roll a pair of number cubes and explore how often each sum occurs. The data can be visualized with a line plot later in the module.				
12-1	Convert Customary Units	Students use the relationship between customary units of measurement to convert measurements. Students use the relationship between units of time to convert measurements.	Students discuss the relationship between customary units of measurement and time to convert measurements using the verb *decide*.	Students foster personal curiosity about mathematics by relating a mathematical concept to their own lives and interests.
12-2	Convert Metric Units	Students use the relationship between metric units of measurement to convert measurements.	Students discuss the relationship between metric units of measurements to convert measurements using the verb *help*.	Students explain their thinking for how they solved a mathematical problem, including how a correct solution was found or what caused confusion and why.
12-3	Solve Multi-Step Problems Involving Measurement Units	Students solve multi-step problems by identifying and answering a hidden question and using that answer to solve the initial problem.	Students discuss solving multi-step problems by identifying and answering a hidden question to solve the initial problem using *make sense of* and *determine*.	Students describe the logic and reasoning used to make a mathematical decision.
12-4	Represent Measurement Data on a Line Plot	Students create a line plot to display a data set involving measurement. Students interpret line plots.	Students discuss line plots using the modals *might, can,* and *could*.	Students recognize and build upon personal mathematical strengths of self and others within the classroom math community.
12-5	Solve Problems Involving Measurement Data on Line Plots	Students solve problems using data in a line plot and performing operations on the data.	Students discuss solving problems using operations and line plot data using *amount* and the superlatives *greatest* and *least*.	Students acknowledge different representations that can be used to complete a mathematical task, and reflect on the value of the similarities and differences.
Math Probe Line Plots Students interpret a line plot with fractional data.				
Unit Review **Fluency Practice**				
Unit Assessment **Performance Task**				

FOCUS QUESTION:
How can I convert measurement units and represent measurement data?

LESSON	KEY VOCABULARY: Math Terms	KEY VOCABULARY: Academic Terms	MATERIALS TO GATHER	RIGOR FOCUS	STANDARD
12-1	capacity convert customary system length weight	accurate infer	• *Customary Conversion Tables* Teaching Resource • *Customary Measurement Cards* Teaching Resource	Procedural Skill & Fluency, Application	5.MD.A.1
12-2	capacity convert length mass metric system	emphasize note	• base-ten blocks (ones and tens only) • *Metric Conversion Tables* Teaching Resource • number cubes	Procedural Skill & Fluency, Application	5.MD.A.1
12-3	convert	analyze procedure	• *Customary Conversion Tables* Teaching Resource • index cards • *Metric Conversation Tables* Teaching Resource • *Problem-Solving Tool* Teaching Resource	Procedural Skill & Fluency, Application	5.MD.A.1
12-4	data line plot outlier	accurate reflect	• dry spaghetti noodles • *Water Remaining Line Plot* Teaching Resource	Procedural Skill & Fluency, Application	5.MD.B.2
12-5	data line plot	emphasize suggest	• blank number cubes • index cards • *Problem-Solving Tool* Teaching Resource	Procedural Skill & Fluency, Application	5.MD.B.2

Unit Overview

Focus

Data and Line Plots

In Kindergarten and Grade 1, students become familiar with categorical data and ways to display it, such as picture graphs and bar graphs. In Grade 2, they are introduced to measurement data. They generate measurement data by measuring and recording lengths to the nearest whole unit and represent the data on a line plot. In Grade 3, they work with measurements in fractions of a unit, and in Grades 4 and 5, they create line plots for a variety of data sets and solve problems based on the data using operations appropriate for the grade. A line plot uses an appropriately scaled number line to present the values of the measurements in a data set. Each measurement is represented by an X or dot placed above the number line, directly over the location of its value.

The line plot is an efficient way to display, compare, and interpret the data. When students are proficient at constructing line plots, they can be presented with line plots for a variety of measurement contexts and asked to interpret them. The work in this unit has connections to the fraction expectations for Grade 5, which also call for solving problems using computations with fractions and mixed numbers.

Students also learn that the same measure can be expressed in different units. Students learn to convert between units within a measurement system using their previously-learned skills in multiplication and division.

Coherence

What Students Have Learned

- Students have learned the measurement units and expressed the same measure in different units. (Grade 4)
- Students have used the four operations to solve problems involving measurement. (Grade 4)
- Students have made a line plot to display fractional units of measures. (Grade 4)
- Students have solved problems using data from line plots. (Grade 4)

What Students Are Learning

- Students convert measures within a given system.
- Students solve problems using measurement conversions.
- Students make a line plot of measurement data expressed in fractions of units.
- Students operate with fractions to solve problems involving information presented in a line plot.

What Students Will Learn

- Students identify statistical questions. (Grade 6)
- Students describe data by its center, spread, and overall shape. (Grade 6)
- Students display numerical data in plots on a number line. (Grade 6)

Rigor

Conceptual Understanding

Students develop understanding of
- converting among customary units of measure;
- converting among metric units of measure;
- using statistical representations to display fractional data.

Procedural Skill & Fluency

Students build proficiency with
- multiplying and dividing to convert units of measure;
- solving multi-step problems involving unit conversions;
- representing data on a number line using a line plot;
- interpreting data represented in a line plot.

Application

Students apply their knowledge of
- multiplying and dividing with fractions to convert among units of measure;
- relative sizes of units of measure to solve problems with real-world contexts;
- data to represent data from a real-world context on a number line;
- fractional operations to solve problems involving data on line plots in real-world contexts.

Effective Teaching Practices

Support Productive Struggle in Learning Mathematics

Struggle is part of learning. This is especially true in mathematics. It is a well-known truth, and in response, teachers might believe they do students a favor by taking out the struggle. However, learning without some struggle is probably not deep learning.

Productive struggle promotes the deep learning we seek for students, because it forces a more thorough, active kind of engagement. A major role of the teacher is to understand this and make sure his or her students understand it too. Too often students believe their struggle means they are just not good at math. In fact, they might have already been influenced by adults to believe this. Teachers can correct this by encouraging students to reject the idea that they are just not good at math and to see struggle as an opportunity to learn. Students must believe it is true, and teachers can show them that it is true by engaging them correctly.

As a facilitator, help students accept and engage themselves in productive struggle.

For example—

- Anticipate what students might struggle with and be ready to help them know the struggle is normal. Remind them of their prior work when they create scales for line plots and when they encounter unlike denominators.
- If students struggle unproductively, provide tools such as fraction circles.
- Give students the sense that questions indicate hard work and participation.
- Encourage students to talk with each other and recognize each other's good efforts. Create a culture in which students are advocates for each other.
- Students started working with line plots in Grade 2, so they have a good foundation for handling struggles now. Allow them some time to do so. Issues are likely to be associated with the introduction of fraction data.

Math Practices and Processes

Make Sense of Problems and Persevere in Solving Them

This practice goes hand in hand with the teaching practice of supporting productive struggle. The connection is the idea that perseverance is the key to success.

Students need to thoroughly consider a problem and understand what is being asked. Proficient students identify the givens and how they relate to each other and what the problem is asking for. They form an idea of what the solution may look like and seek representations that might help them find it. They recognize that there could be more than one way to solve a problem, so they consider alternative approaches when possible.

Proficient students know how to use problem-solving strategies such as drawing pictures, solving analogous or simpler problems, and looking for patterns. They recognize that it might be necessary to change course along the way. They check their solution against an estimate or alternative approach when possible, and they ask themselves whether their solution makes sense.

Solving problems involving data on a line plot requires students to both interpret the plot and determine the operations to use. This might slow students in finding a starting point. If necessary, focus them on the mechanics of the line plot first, making sure they can describe what it means. Then have them address the problem and what it asks for.

Students will need to handle unlike denominators. Note whether they wait to convert to like fractions when needed or if they convert the tick labels on the line plot. In some cases, one might be more efficient than the other.

When students divide, have them revisit the concept of fractions as division if necessary.

Social and Emotional Learning

Self-Awareness – Self-Efficacy (Lesson 12-1): Students with high self-efficacy are more likely to persevere to complete a challenging task.

Self-Management – Self-Motivation (Lesson 12-2): Students who self-motivate can take initiative and persevere through challenging tasks.

Responsible Decision-Making – Analyze Situations (Lesson 12-3): Students make sense through analysis, which helps them make informed decisions.

Relationship Skills – Social Engagement (Lesson 12-4): Engaging with others allows students to develop relationships and establish a sense of security and belonging in the classroom community.

Social Awareness – Appreciate Diversity (Lesson 12-5): When students appreciate diversity, they create a stronger, more inclusive classroom community.

Language of Math

Mathematical Nouns

- **Convert*** (Lesson 12-1) Students are introduced to the term as they learn to change a measure from one unit to another within a measurement system. They do not convert between the metric and customary measurement systems.
- **Data*** (Lesson 12-4) Students are familiar with the concept of data from their earliest work with it in Kindergarten but have not been formally introduced to the term until now. Data are numbers or symbols, sometimes collected from a survey or experiment, to show information. *Data* is plural.
- **Line plot** (Lesson 12-1) Students were introduced to this term in the context of displaying measurement data in Grade 2, and they encountered it again in Grades 3 and 4. A line plot is a method of displaying a set of measurement data. It uses a number line to present the values of the measurements, each measurement being depicted as an X or dot placed directly over the corresponding location for that value on the number line. When the plot uses dots to mark the measurements, it can be called a *dot plot*.
- **Outlier*** (Lesson 12-4) An outlier is a piece of data that doesn't fitwithin the pattern of the rest of the data. It is usually an extreme value that can skew the interpretation or summary of the data.

*This is a new term.

Math Language Development

A Focus on Reading

In many respects, reading in math can be similar to reading in any academic discipline. In any discipline, students read for comprehension. They seek to learn new ideas, and they must learn and incorporate new vocabulary.
In some ways, reading in math is different and requires different strategies. Consider these unique characteristics of mathematics text.

- Math text is conceptually dense.
- A single sentence might communicate multiple layers of content.
- Math text looks different. It includes prose, equations, graphs, tables, symbols, and other means for communicating ideas.
- Math ideas are developed logically, with the conclusion at the end.
- Math is a language that uses common words but with different meanings.

As a facilitator, interact with students before, while, and after they read a passage or problem.

Before reading—
- If the passage or problem has a title or other telling features, ask students to inspect them briefly and predict what the text passage or problem is about.

While reading—
- Have students restate the content in their own words and address possible comprehension issues.
- Have students notice the ways that new ideas are built on familiar ones–for example, have them recall prior work placing and labeling tick marks for displays of whole number tasks.

After reading—
- Check with students that the problem or passage makes sense to them.
- Help students make connections to similar problems they have solved before.

English Language Learner

In this unit, students are provided with a number of scaffolds to support their comprehension of the language used to present and explain strategies related to measurement and data. Because many of the words (*mixture, so, enough*) and phrases (*make sense of, whether*) used in this unit could prove challenging to ELs, they are supported in understanding and using them so that the instruction is more accessible.

Lesson 12-1 – *mixture*
Lesson 12-2 – *make sense of*
Lesson 12-3 – *enough*
Lesson 12-4 – *whether*
Lesson 12-5 – *so (that)*

Number Routines

Build Fluency The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the habits of mind that are important for proficient doers of math.

Find the Pattern, Make a Pattern

Purpose: Build efficiency with recognizing and building patterns.

Overview: Students determine the rule(s) for a given pattern, then use the rule(s) to create a new pattern. The teacher records students' new patterns and facilitates a discussion to validate the pattern and its rules.

Decompose It

Purpose: Build flexibility with numbers.

Overview: Students generate multiple (at least 3) ways to decompose given numbers and share their thinking for each decomposition. The teacher records decompositions and then facilitates a discussion of patterns in the decompositions.

Which Benchmark Is It Closest To?

Purpose: Enhance rounding and reasoning skills.

Overview: Students determine to which benchmark the given number is closest and explain their reasoning.

Sense-Making Routines

Notice & Wonder

- **Notice & Wonder: Tell me everything you can.** (Lesson 12-1) Students observe different-sized containers.
- **Notice & Wonder: What do you see?** (Lesson 12-2) Students see a balanced scale with a large object on one side and lots of smaller objects on the other side.
- **Which doesn't belong?** (Lesson 12-3) Students see several unit conversions and compare them to see how they can group them to find one that doesn't belong.
- **Numberless Graph: What's the question?** (Lesson 12-4) Students see a line plot without labels and discuss what they can know about the data and what the data could represent.
- **Notice & Wonder** (Lesson 12-5) Students see many bowls on a table with various numbers of tomatoes in each bowl.

Math Language Routines

The Mathematical Language Routines used in this unit give teachers a structured, yet adaptable format for amplifying and developing students' social and academic language. For more information on the Math Language Routines, see the Appendix.

- Lesson 12-1 – Students participate in MLR8: Discussion Supports so that students' meta-awareness can be fostered as they discuss what they know in order to solve a problem that involves converting customary units.
- Lesson 12-2 – Students participate in MLR7: Compare and Connect so that students' oral and written output can be fostered as they compare and contrast different ways to convert metric units.
- Lesson 12-3 – Students participate in MLR2: Collect and Display so that students' oral words and phrases can be captured into a stable, collective reference when relating units of measure.
- Lesson 12-4 – Students participate in MLR1: Stronger and Clearer Each Time so that they have a structured and interactive opportunity to revise and refine both their ideas and their verbal and written output while representing data on a line plot.
- Lesson 12-5 – Students participate in MLR5: Co-Craft Questions and Problems so that they can produce the language of mathematical questions related to solving problems involving data on line plots.

Unit 12

How Ready Am I?

Name ______________________

1. Barb fills a bottle with 1 liter of water. How many milliliters of water are in the bottle?
   A. 10 milliliters B. 100 milliliters
   C. 1,000 milliliters D. 10,000 milliliters
2. The height of a room is 2.8 meters. How many centimeters is this?
   A. 0.028 centimeter B. 0.28 centimeter
   C. 280 centimeters D. 2,800 centimeters
3. Which statement is true?
   A. There are 100 millimeters in 1 meter.
   B. There are 1,000 centimeters in 1 kilometer.
   C. There are 1,000 meters in 1 kilometer.
   D. There are 10 centimeters in 1 meter.
4. A football player gained 9 yards. How many feet is this?
   A. 3 feet B. 12 feet
   C. 18 feet D. 27 feet
5. Sonya bought 2 gallons of bottled water. How many quarts of water did Sonya buy?
   A. 1 quart B. 4 quarts
   C. 6 quarts D. 8 quarts

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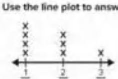
6. The running time of a movie is 3 hours. How many minutes is the running time of the movie?
   A. 3 minutes B. 60 minutes
   C. 180 minutes D. 310 minutes
7. Which is the quotient 74 ÷ 1,000?
   A. 0.074 B. 0.74
   C. 74 D. 74,000
8. Which is the quotient 6.5 ÷ 10?
   A. 0.065 B. 0.65
   C. 65 D. 650

Use the line plot to answer questions 9 and 10.



Apple Weights (pounds)

9. How many apples were weighed?
   A. 3 apples B. 6 apples
   C. 8 apples D. 10 apples
10. Which weight occurs most often?
   A. $\frac{1}{4}$ pound B. $\frac{1}{2}$ pound C. $\frac{3}{4}$ pound

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Administer the Readiness Diagnostic to determine your students' readiness for this unit.

Targeted Intervention

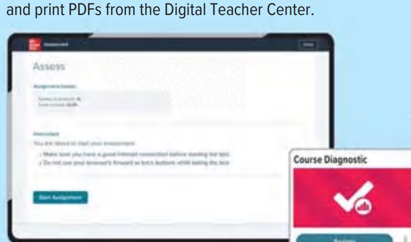
Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	2	Convert metric units	Metric Units of Liquid Volume	4.MD.A.1
2	2	Convert metric units	Metric Units of Length	4.MD.A.1
3	1	Identify metric conversions	Metric Units of Length	4.MD.A.1
4	2	Convert customary units	Customary Units of Length	4.MD.A.1
5	2	Convert customary units	Customary Units of Liquid Volume	4.MD.A.1
6	2	Convert customary units	Time	4.MD.A.1
7	1	Divide whole numbers by powers of 10	Divide by Powers of 10 (Decimal Point)	5.NBT.A.2
8	1	Divide decimals by powers of 10	Divide by Powers of 10 (Decimal Point)	5.NBT.A.2
9	2	Interpret a line plot	Line Plots (Quarters)	4.MD.B.4
10	2	Interpret a line plot	Line Plots (Quarters)	4.MD.B.4

GO ONLINE

Assign the digital Readiness Diagnostic to students or download and print PDFs from the Digital Teacher Center.



Focus Question

Introduce the Focus Question: *How can I convert measurement units and represent measurement data?*

Ask students to think about what they know about measurement data.

- What types of measurements have you made before?
- What units have you used to measure?
- What kinds of data have you used?
- What graphs have you used to represent data?

Remind students that at the end of the unit, they will reflect back on what they learned.

Family Letter

Each letter presents an overview of the math in the unit and home activities to support student learning.

STEM in Action

Videos

Students can watch the two STEM videos.

STEM Career: Construction Manager Finn describes his aspirations to be a construction manager.

Finn Buys Drywall Students see how Finn uses a line plot to keep track of drywall thickness.

STEM Project

Students can complete the STEM Project Card during their workstation time.

STEM Adventure

Students can complete the STEM Adventure during their workstation time.


IGNITE!

Name

Which Sums Occur Least and Most?

Listen for directions.

A Sum	B Tallies for Your Group	C Totals for Your Group	D Combined Results
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

166 Ignite! • Which Sums Occur Least and Most?

Ignite!

Which Sums Occur Least and Most?

Students roll a pair of number cubes and explore how often each sum occurs. The data can be visualized with a line plot later in the unit.

Materials: number cubes marked 1–6

Pose the following experiment.

- Suppose you roll two number cubes marked 1–6 many times. After each roll, you find the sum of the two numbers that appear. What are the possible sums that can be rolled?
- Do you think each sum will occur about the same number of times?

1. Give each pair of students two number cubes marked 1–6.
   Roll a pair of number cubes 50 times and record a tally mark in Column B for each sum that is rolled. After the 50 rolls, record the number of times each sum occurred in Column C.
2. Ask students to analyze their results.
   - What do you notice about the results?
   - Are you surprised by your results?
3. Have students combine their data with other groups of students.
   - Combine your data with two other groups, and record the combined results in Column D. What do you notice about your combined results?
4. Have students conjecture about the data.
   - Explain why sums of 2 and 12 occur least often.
   - Explain why sums of 6, 7, and 8 occur more often than the other sums.

You may want to combine the data from the entire class to see if there are any changes in the sums that appeared most and least frequently.

Unit Resources At-A-Glance

Workstations

Reveal Math offers rich and varied resources that teachers can use to differentiate and enrich students' instructional experiences with the unit content. The table presents an overview of the resources available for the unit with recommendations for when to use them.

	Activity	Description	Use After Lesson
Game Station	Game Station	Students build proficiency with converting measurement units and creating line plots.	
		• **Product Size Sort**	12-1
		• **Convert Metric Units Race**	12-2
		• **Metric Units of Measurement Race**	12-3
		• **Create a Line Plot Task Cards**	12-4
		• **Line Plot Task Cards**	12-5
Digital Station	Digital Game	**Space Race** Students practice finding volume.	12-1
Application Station	**Have students complete at least one of the Use It! activities for this unit.**		
	STEM Project Card	**Environmentally Friendly** Students use measurements to create 5 environmentally friendly home improvements.	12-3
	Connection Card	**City of Trees** Students create a line plot for plant growth data.	12-5
	Real World Card	**Find a Pattern and Repeat** Students use a repeat function to loop computer code.	12-1

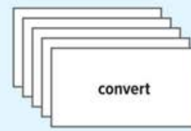
Additional Resources

Use the resources below to provide additional support for this unit.

GO ONLINE

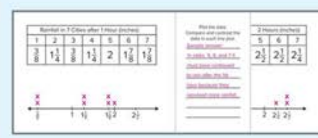
Vocabulary

Use the vocabulary cards to help students learn the vocabulary in this unit. Encourage students to write their own definitions of the new terms on the front side of the card.



Foldables

Use the unit foldables with Lessons 12-4 and 12-5.



Spiral Review

Students can complete the Spiral Review at any point during the unit as either a paper-and-pencil or digital activity.

Lesson	Standard
12-1	5.NF.B.3
12-2	5.NF.B.7
12-3	5.NF.B.6
12-4	5.NF.B.5
12-5	5.NF.B.4

LESSON **12-1**

Convert Customary Units

Learning Targets

- I can convert customary units of measure and time.
- I can explain which operation to use when converting.

Standards ◆ Major △ Supporting ● Additional

Content

△ **5.MD.A** Convert like measurement units within a given measurement system.

△ **5.MD.A.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	Academic Terms
capacity	accurate
convert	infer
customary system	
length	
weight	

Materials

The materials may be for any part of the lesson.

- *Customary Conversion Tables* Teaching Resource
- *Customary Measurement Cards* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use the relationship between customary units of measurement to convert measurements. • Students use the relationship between units of time to convert measurements.	• Students discuss the relationship between customary units of measurement and time to convert measurements using *decide*. • To support maximizing cognitive and linguistic meta-awareness, ELs participate in MLR8: Discussion Supports.	• Students foster personal curiosity about mathematics by relating a mathematical concept to their own lives and interests.

Coherence

Previous	Now	Next
• Students converted measurements within a single system of measurement (Grade 4).	• Students use the relationships between customary units of measurement and units of time to convert measurements.	• Students use the relationships between metric units of mass, length, or capacity to convert measurements (Unit 12).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students use understanding of multiplication and division with fractions to convert among customary units of measure. *Conceptual understanding is not a targeted element of rigor for this standard.*	• Students develop proficiency with multiplying and dividing to convert among customary units of measure.	• Students apply knowledge of multiplying and dividing with fractions to convert among customary units of measure.

GO ONLINE

Number Routine

Find the Pattern, Make a Pattern

5–7 min

Build Fluency Students build number sense as they determine the rule for a pattern and use it to find missing terms. Students then use the same rule to make a new number sequence with different numbers.

These prompts encourage students to talk about their reasoning:

- How did you think about the pattern?
- What did you notice about the numbers?
- How did you create your own sequence of numbers?
- How do you know that your sequence follows thesame rule?

Launch 5–7 min

Sense-Making Routine

Purpose Students begin thinking about the correspondences among containers with varying capacity.

Notice & Wonder™

- Tell me everything you can.

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students talk in small pairs about similarities and differences they notice about the containers before discussing as a class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' noticing and wondering about the relationships among the capacities of the containers and are based on possible comments and questions that students may make during the share out.

- What could help you understand how the capacities of these containers are related?
- What are some ways you can describe the relationship between the capacities of these containers?

Math is... Mindset

- What behaviors have helped you be successful in the past?

SEL Self-Awareness: Self-Efficacy

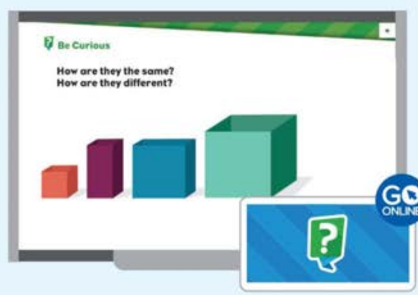
Students with high self-efficacy are more likely to persevere to complete a challenging task. As students participate in the Notice & Wonder routine, encourage them to relate units of measure to their own lives. Invite them to consider when they have had to test the efficacy of several different methods that could be used to solve a problem.

Transition to Explore & Develop

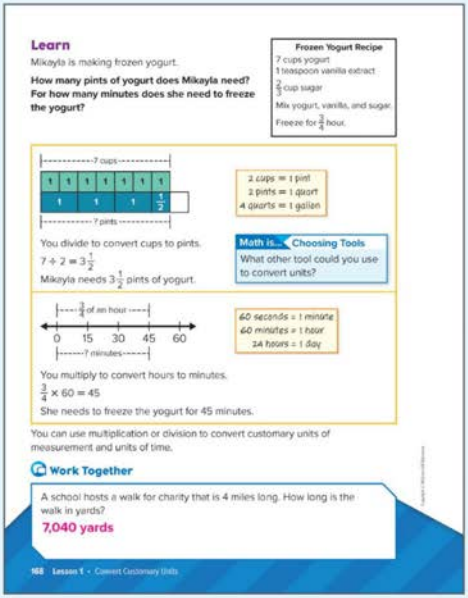
Ask questions that get students thinking about how they can convert smaller units of measure to larger units of measure, and vice versa.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can use multiplication and division to convert customary units of length, capacity, and weight, and units of time.



Explore & Develop 20 min



Learn

Mikayla is making frozen yogurt.

How many pints of yogurt does Mikayla need? For how many minutes does she need to freeze the yogurt?

Frozen Yogurt Recipe
7 cups yogurt
1 teaspoon vanilla extract
$\frac{2}{3}$ cup sugar
Mix yogurt, vanilla, and sugar.
Freeze for $\frac{3}{4}$ hour.

7 cups
1 1 1 1 1 1 1
1 1 1 $\frac{1}{2}$
? pints

2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

You divide to convert cups to pints.

$7 \div 2 = 3\frac{1}{2}$

Mikayla needs $3\frac{1}{2}$ pints of yogurt.

Math is... Choosing Tools
What other tool could you use to convert units?

$\frac{3}{4}$ of an hour
0 15 30 45 60
? minutes

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

You multiply to convert hours to minutes.

$\frac{3}{4} \times 60 = 45$

She needs to freeze the yogurt for 45 minutes.

You can use multiplication or division to convert customary units of measurement and units of time.

Work Together

A school hosts a walk for charity that is 4 miles long. How long is the walk in yards?

7,040 yards

168 Lesson 1 • Convert Customary Units

❶ Pose the Problem

MLR Discussion Supports

As students talk about what they know, have them pay attention to others' understandings in order to increase their ability to work through converting customary units. Restate statements they make as a question to seek clarification and provide vocabulary or grammar prompts for students who need more guidance.

ETP Pose Purposeful Questions

- Do you think the number of pints will be greater than or less than the number of cups? Why?
- How do you know if the number of minutes will be greater than or less than the minutes in an hour?

❷ Develop the Math

Choose the option that best meets your instructional goals.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you know when to use multiplication to convert customary units of measure?
- How do you know when to use division to convert customary units of measure?
- How does knowing the relationships between different customary units of measure help you convert among the units?

Key Takeaway

- Multiplication and division are used to convert among different customary units of length, capacity, weight, and units of time.

Work Together

Students use division to convert miles to yards using multiplication or division.

Common Error Students will often use the wrong operation. Make sure they keep in mind that converting from a longer distance unit like miles to a shorter one like yards should result in more of the shorter unit.

LOM Language of Math

Explain to students that *customary* means "according to the customs or practices of a particular place." The units of measure taught in this lesson are customary in the United States, while most other countries use metric units of measure.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore converting customary units of measure.

Materials: *Customary Conversion Tables* Teaching Resource, *Customary Measurement Cards* Teaching Resource

Directions: Have students match *Customary Measurement Cards* that represent the same quantity. After students have matched each measurement card, have them write a multiplication and division equation that represents each conversion.

ETP Implement Tasks That Promote Reasoning and Problem Solving

- How did you begin to think about this task? How did you organize your information/thinking?
- How did you determine the factors and product of your multiplication equations?
- How did you determine the dividend, divisor, and quotient of your division equations?
- When you convert a smaller unit to a larger unit, are more or fewer of the larger units needed? Explain why.

Math is... Choosing Tools

- Can you draw a picture or make a model that supports your equations?

Students use tools to deepen their understanding of mathematical relationships.

Activity Debrief: Have groups share and explain their matches and equations.

Have students revisit the Pose the Problem question and discuss answers.

- How can you find how many pints of yogurt Mikayla needs and how many minutes she needs to refrigerate the mixture?

PDFs of the Teaching Resources are available in the Digital Teacher Center.

Customary Measurement Cards

60 in.	5,280 ft	3 lb	32 oz
2 gal	5 c	10 pt	4 qt
8,000 lb	8 pt	9 ft	$\frac{1}{4}$ mi
20 c	108 in.	5 ft	40 fl oz
40 oz	1 mi	2 lb	1,320 ft
4 T	8 qt	$\frac{1}{2}$ T	4 in.
$\frac{1}{3}$ ft	1,000 lb	$\frac{1}{2}$ gal	2 qt

Guided Exploration

Students learn how to convert units of capacity and time using multiplication and division. You may wish to provide copies of the *Customary Conversion Tables* Teaching Resource for students to use.

ETP Use and Connect Mathematical Representations

Before creating a bar diagram that solves the problem, have the students create a bar diagram showing the relationship between a pint and cups. Ask:

- Are there a greater number of cups in a pint or pints in a cup? How do you know?
- How will this bar diagram help you create a bar diagram that heps you solve the problem?

Make sure students understand that when a conversion to a smaller unit is made, more of the smaller units are needed, and vice versa. Ask:

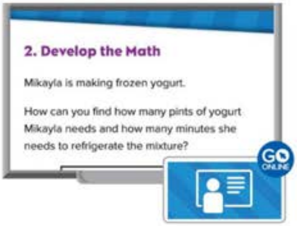
- When you convert a larger unit to a smaller unit, are more or fewer of the smaller units needed? Explain why.
- When you convert a smaller unit to a larger unit, are more or fewer of the larger units needed? Explain why.

- **Think About It:** Why should you use division to convert cups to pints?
- **Think About It:** Why should you use multiplication to convert hours to minutes?

Math is... Choosing Tools

- How can a bar diagram help you decide which operation to use?

Students recognize the insight to be gained from the tools they choose and use.



EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *mixture*. First, make sure students understand what yogurt is. Point to all of the ingredients and say, using a stirring gesture, *Yogurt is a mixture of all of these things*. Point to them again and say *These make yogurt. Yogurt is a mixture*. Check by pointing to either just one ingredient or all of them and asking *Is this a mixture?*

Developing/Expanding Ensure understanding of *mixture*. First, make sure students understand what yogurt is. Point to all of the ingredients and say, using a stirring gesture, *Yogurt is a mixture of all of these things*. Point to them again and say *These make yogurt. Yogurt is a mixture*. Check by asking *What is yogurt?* (a mixture)

Bridging/Reaching Ask students to talk about common mixtures they may make or have at home (*iced tea, ice cream*, etc.). Then have them brainstorm and list similar words to *mixture* and share their list with the class (*mix, blend*, etc.). Allow students to use a dictionary or thesaurus if desired.

Practice & Reflect 10 min

On My Own

Name

Which operation will you use for the conversion? Explain your reasoning.

1. cups to fluid ounces **multiplication; finding more of a smaller unit**
2. hours to days **division; finding fewer of a larger unit**

Complete the conversion.

3. 36 in = **3** ft
4. 2 T = **4,000** lb
5. 16 pt = **2** gal
6. 3 yr = **36** mo
7. 48 oz = **3** lb
8. 4 hr = **240** min
9. A basketball court is 84 feet long. How does 84 feet compare to 30 yards? Explain how you know. **84 feet is less than 30 yards. Sample answer: $30 \times 3 = 90$, so 30 yards is 90 feet, and 84 is less than 90.**
10. James needs this much ribbon for an art project. How many inches of ribbon does he need? **8 in.** $\frac{2}{3}$ ft

11. During a reading contest, Mike read for a total of 120 hours. How many days is equal to 120 hours? **5 days**
12. Amy's dog weighs 272 ounces. How many pounds does her dog weigh? **17 lb**
13. Lauren goes for a walk that is $\frac{7}{8}$ mile long. How many feet did she walk? **4,620 ft**
14. **STEM Connection** Finn needs to cut a piece of wood that is 144 inches long. He thinks it would be easier to measure the piece of wood in yards. What is the length in yards? Explain your answer. **4 yd; Sample answer: $144 \div 36 = 4$**

15. **Extend Your Thinking** A rope is 100 inches long. What is the length in feet and inches? Explain your reasoning. **8 ft 4 in.; Sample answer: $100 \div 12 = 8$ with a remainder of 4, so there are 8 feet and 4 inches remaining.**

Reflect

How can you use multiplication and division to convert among different customary units of measure?

Answers may vary.

Math is... Mindset
What behaviors have helped you be successful in the past?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 9 Students may simply compare 84 and 30 without considering the units used to determine each measurement quantity. Remind students that when comparing, the units must be the same.

Item Analysis

Item	DOK	Rigor
1–2	1	Conceptual Understanding
3–8	1	Procedural Skill & Fluency
9–14	2	Application
15	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you use multiplication and division to convert among different customary units of measure?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What behaviors have helped you be successful in the past?

Students reflect on how they practiced self-awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can convert customary units of measure and time.
- I can explain which operation to use when converting.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Understand how to convert customary units	5.MD.A.1
2	1	Convert customary units	5.MD.A.1
3	2	Convert customary units	5.MD.A.1
4	2	Convert customary units	5.MD.A.1

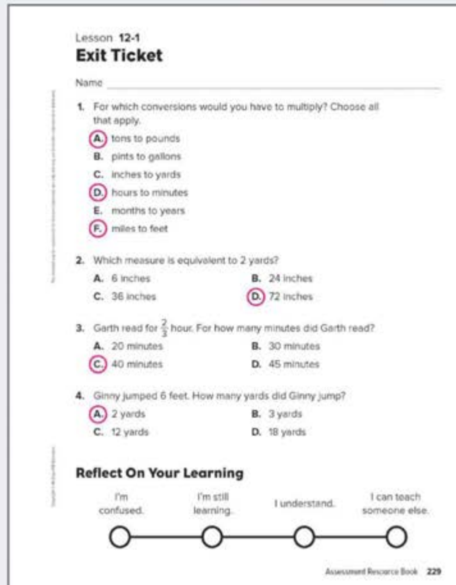
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking



Lesson 12-1
Exit Ticket

Name ______________________

1. For which conversions would you have to multiply? Choose all that apply.
   - A. tons to pounds
   - B. pints to gallons
   - C. inches to yards
   - D. hours to minutes
   - E. months to years
   - F. miles to feet

2. Which measure is equivalent to 2 yards?
   - A. 6 inches
   - B. 24 inches
   - C. 36 inches
   - D. 72 inches

3. Garth read for $\frac{2}{3}$ hour. For how many minutes did Garth read?
   - A. 20 minutes
   - B. 30 minutes
   - C. 40 minutes
   - D. 45 minutes

4. Ginny jumped 6 feet. How many yards did Ginny jump?
   - A. 2 yards
   - B. 3 yards
   - C. 12 yards
   - D. 18 yards

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 229

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Customary Conversion Tables

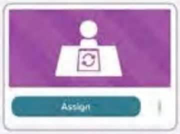
Work with students in pairs. Using the *Customary Conversion Tables* Teaching Resource, one student should present the other with a specific customary measurement. The other student should convert the measurement to a smaller or larger unit of measure. If students have difficulty, ask whether they are moving from smaller to larger or larger to smaller units, and whether they should use multiplication or division to make the conversion.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Length Conversions
- Weight and Mass Conversions
- Liquid Volume Conversions



INDEPENDENT WORK

Differentiation Resource Book, p. 139

Lesson 12-1 • Reinforce Understanding

Convert Customary Units

Name ______________________

Review

You can use multiplication or division to convert customary units of measurement and units of time.

Using Multiplication to Convert	Multiply to find the number of smaller units.	
	9 gallons to quarts	1 gal = 4 qt $9 \times 4 = 36$ qt
Using Division to Convert	Divide to find the number of larger units.	
	720 minutes to hours	60 min = 1 hr $720 \div 60 = 12$ hr

Which operation should you use for the conversion? Explain your answer.

1. days to minutes **multiplication; finding smaller units**
2. cups to quarts **division; finding larger units**

Match the measurement in Column A to its equivalent measurement in Column B.

	Column A	Column B
3.	4 pounds	3 hours
4.	180 minutes	40 quarts
5.	5 years	64 ounces
6.	12 quarts	480 minutes
7.	10 gallons	8 years
8.	8 hours	7 pounds
9.	112 ounces	60 months
10.	96 months	3 gallons

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Product Sort Size Students practice comparing products and factors.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 139–140

Lesson 12-1

Additional Practice

Name ______________________

Review

You can use multiplication or division to convert customary units of measurement and units of time.

To convert from a larger measure to a smaller measure, multiply because there will be more of the smaller unit.

To convert from a smaller measure to a larger measure, divide because there will be fewer of the larger unit.

Write equivalent measures in yards and in inches for 6 feet.

Use the equivalent measures:

1 yard = 3 feet Divide since the conversion is from feet to yards: $6 \div 3 = 2$	1 foot = 12 inches Multiply since the conversion is from feet to inches: $6 \times 12 = 72$

So 6 feet is equivalent to 2 yards and to 72 inches.

Which operation do you use for the conversion?

1. hours to minutes **multiplication**
2. gallons to quarts **multiplication**
3. ounces to pounds **division**
4. inches to feet **division**

Student Practice Book

Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with finding volume.

Assign

Use It! Application Station

Find a Pattern and Repeat Students use a repeat function to loop computer code.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 139–140

What is the equivalent measure?

5. 120 min = **2** h
6. 3 lb = **48** oz
7. 48 mo = **4** yr
8. 10 ft = **120** in.
9. 2 gal = **8** qt
10. $\frac{2}{3}$ hr = **40** min
11. A football team has to advance 10 yards to get a first down. How many feet is this? **30** feet
12. The running time for a movie is 180 minutes. How many hours long is the movie? **3** hours
13. Betty bought 2 gallons of milk. Jane bought 6 quarts of milk. Who bought more? How much more? **Betty; 2 quarts more**

Math @Home Activity Provide opportunities for your child to convert customary units. For example, measure the dimensions of a window in feet and have your child show how to convert the measures to inches. Or if your child has 60 minutes until bedtime, ask them to tell how many hours it is until bedtime.

Student Practice Book

Differentiation Resource Book, p. 140

Lesson 12-1 • Extend Thinking

Convert Customary Units

Name ______________________

How can you complete the conversions? Fill in the table to show how. The first one is done for you as an example. Show your work.

1.	**360** inches $30 \times 12 = 360$	30 feet	**10** yards $30 \div 3 = 10$
2.	______ ounces **16,000; $1{,}000 \times 16$**	1,000 pounds	______ tons **0.5; $1{,}000 \div 2{,}000$**
3.	2,160 minutes	______ hours **36; $2{,}160 \div 60$**	______ days **1.5; $36 \div 24$**
4.	______ pints **56; 28×2**	______ quarts **28; 7×4**	7 gallons
5.	______ days **18,250; 50×365**	50 years	______ decades **5; $50 \div 10$**
6.	______ seconds **50,400; 840×60**	______ minutes **840; 14×60**	14 hours
7.	______ inches **540; 45×12**	45 feet	______ yards **15; $45 \div 3$**
8.	______ ounces **68; 8.5×8**	8.5 cups	______ pints **4.25; $8.5 \div 2$**
9.	______ seconds **9,000; 150×60**	150 minutes	______ hours **2.5; $150 \div 60$**
10.	100 pints	______ quarts **50; $100 \div 2$**	______ gallons **12.5; $50 \div 4$**

Differentiation Resource Book

LESSON 12-2
Convert Metric Units

Learning Targets

- I can convert metric units of measure.
- I can explain which operation to use when converting.

Standards ◆ Major △ Supporting ● Additional

Content

△ **5.MD.A** Convert like measurement units within a given measurement system.

△ **5.MD.A.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

MPP Model with mathematics.

Vocabulary

Math Terms	Academic Terms
capacity	emphasize
convert	note
length	
mass	
metric system	

Materials

The materials may be for any part of the lesson.

- base-ten blocks (ones and tens only)
- *Metric Conversion Tables* Teaching Resource
- number cubes

Focus

Content Objective	Language Objectives	SEL Objective
• Students use the relationship between metric units of measurement to convert measurements.	• Students discuss the relationship between metric units of measurements to convert measurements using *help*. • To support optimizing output, ELs participate in MLR7: Compare and Connect.	• Students explain their thinking for how they solved a mathematical problem, including how a correct solution was found or what caused confusion and why.

Coherence

Previous	Now	Next
• Students converted measurements within a single system of measurement (Grade 4). • Students used the relationships between customary units of measurement and units of time to convert measurements (Unit 12).	• Students use the relationships between metric units of mass, length, or capacity to convert measurements.	• Students solve multi-step problems involving metric and customary unit conversions (Unit 12).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students use multiplication and division to convert among metric units of measure. *Conceptual understanding is not a targeted element of rigor for this standard.*	• Students develop proficiency with multiplying and dividing to convert among metric units of measure.	• Students apply knowledge of multiplying and dividing with fractions to convert among metric units of measure.

GO ONLINE

Number Routine
Find the Pattern, Make a Pattern
5–7 min

Build Fluency Students build number sense as they determine the rule for a pattern and use it to find missing terms. Students then use the same rule to make a new number sequence with different numbers.

These prompts encourage students to talk about their reasoning:

- How did you determine the rule that was used for the given sequence?
- How did you figure out the missing numbers in the sequence?
- What other sequence of numbers would follow the same rule?
- How do you know that your sequence follows the rule?

Launch 5–7 min

Sense-Making Routine

Purpose Students begin to understand how they can express equal amounts using different metric units of measure.

Notice & Wonder

- What do you see?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students write down their own thoughts and questions about the image before having them discuss as a class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' noticing and wondering about how to express equal amounts using different units and are based on possible comments and questions that students may make during the share out.

- What are some different units of measure you could use to express weight or mass?
- How can you express equal amounts in different ways?

Math is... Mindset

- What helps you be motivated to do your best work?

SEL Self-Management: Self-Motivation

After students work through the Notice & Wonder routine, have them share with a partner what motivated them to choose what they saw in the diagram. Encourage students to think about how they made sense of the image or what caused confusion as they worked to understand the image. As students move on to work with converting metric units of measure, encourage them to reflect on their work by asking themselves to compare how their motivations might differ from those of other students.

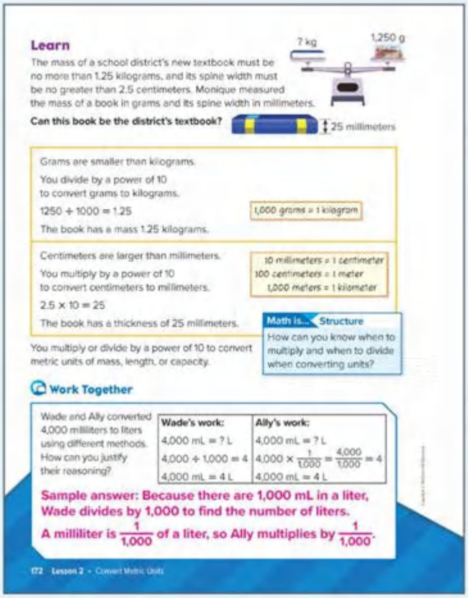
Transition to Explore & Develop

Ask questions that get students thinking about how they can convert larger units of mass to smaller units and vice versa.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can use multiplication and division to convert metric units of measurement.

Explore & Develop 20 min



Learn

The mass of a school district's new textbook must be no more than 1.25 kilograms, and its spine width must be no greater than 2.5 centimeters. Monique measured the mass of a book in grams and its spine width in millimeters.

Can this book be the district's textbook?

Grams are smaller than kilograms.

You divide by a power of 10 to convert grams to kilograms.

$1250 \div 1000 = 1.25$

The book has a mass 1.25 kilograms.

1,000 grams = 1 kilogram

Centimeters are larger than millimeters.

You multiply by a power of 10 to convert centimeters to millimeters.

$2.5 \times 10 = 25$

The book has a thickness of 25 millimeters.

10 millimeters = 1 centimeter
100 centimeters = 1 meter
1,000 meters = 1 kilometer

You multiply or divide by a power of 10 to convert metric units of mass, length, or capacity.

Math is... Structure
How can you know when to multiply and when to divide when converting units?

Work Together

Wade and Ally converted 4,000 milliliters to liters using different methods. How can you justify their reasoning?

Wade's work:	Ally's work:
4,000 mL = ? L	4,000 mL = ? L
$4{,}000 \div 1{,}000 = 4$	$4{,}000 \times \frac{1}{1{,}000} = \frac{4{,}000}{1{,}000} = 4$
4,000 mL = 4 L	4,000 mL = 4 L

Sample answer: Because there are 1,000 mL in a liter, Wade divides by 1,000 to find the number of liters. A milliliter is $\frac{1}{1{,}000}$ of a liter, so Ally multiplies by $\frac{1}{1{,}000}$.

172 Lesson 2 • Convert Metric Units

1 Pose the Problem

ETP Pose Purposeful Questions

- Is the scale is balanced? How do you know? What does that tell you?
- How can you determine if both sides of the scale have the same mass?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and have them both work on the same problem, similar to the one on the Learn page. Have one student use multiplication and one use division, and then have them compare their work with their partner. Revisit this activity throughout the lesson to help students build proficiency.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you know when to use division to convert metric units of measure?
- In what situations might you use multiplication to convert metric units of measure?
- How does knowing the relationship between metric units of measure help you convert among the units?

Key Takeaways

- Multiplication and division are used to convert among different metric units of length, capacity, and mass.
- When converting within the metric system, one can multiply or divide by powers of 10.

Work Together

Students explore how two students solved the same problem using different strategies.

Common Misconception Because the strategy that Ally used was not the one taught in the lesson, students might assume that her work was done incorrectly. Remind students that multiplying 4,000 by $\frac{1}{1{,}000}$ is the same as dividing 4,000 by 1,000.

LOM Language of Math

Explain to students that *convert* means "to cause to change in form." Just as they can convert metric units from one unit of measure to another, they can convert U.S. dollars to a different currency, convert a building for a different purpose, or convert a solid to a liquid in science class.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore measuring objects using base-ten blocks.

Materials: base-ten blocks (ones and tens only)

Directions: Have students measure the length of various objects using the ones cubes only. After students have measured lengths using the ones cubes, have students predict the length of their objects using the tens rods only to measure. Students can then test their predictions by measuring the objects using the tens rods.

ETP Implement Tasks That Promote Reasoning and Problem Solving

- Based on your measurement with the ones cubes, what can you predict about the measurement using the tens rods?
- How does the relationship between the ones cube and the tens rod help you make a prediction?
- How is this task similar to converting units of measure?

Math is... Modeling

- How can you use an equation to represent the relationship between the measurement with ones cubes and the measurement with tens rods?

Students use mathematics to decontextualize a problem.

Activity Debrief: Have groups of students share how they worked through the problem, including the representations they used and any solutions they found. Explain that the metric system is based on powers of 10 similar to the base-ten number system.

Have students revisit the Pose the Problem question and discuss answers. You may wish to provide copies of the *Metric Conversion Tables* Teaching Resource for students to use.

- How can you determine the mass of the bag in kilograms?

Guided Exploration

Students use division to convert metric units of measurement. You may wish to provide copies of the *Metric Conversion Tables* Teaching Resource for students to use.

ETP Facilitate Meaningful Mathematical Discourse

- **Think About It:** Why should you use division to convert grams to kilograms?

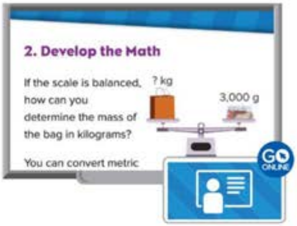
Discuss the nature of metric conversions and how they are similar to and different from customary conversions. Ask:

- What kind of number will you always multiply or divide by when converting metric units? How do you know?
- How is converting metric units similar to converting customary units? How is it different?

Math is... Modeling

- How does the bar diagram help you make sense of the problem?

Students are mapping relationships using diagrams and analyzing those relationships mathematically to draw conclusions.



EL English Learner Scaffolds

Entering/Emerging Ensure comprehension of *make sense of*. Write an equation on the board. Say *Let's make sense of this problem*. Use tens rods to help you solve the problem while saying *I'm making sense of this problem*. Repeat with another problem and another tool. Repeat once more, this time asking students *What can I use to make sense of this problem?* Give them two tools to choose from.

Developing/Expanding Ensure comprehension of *make sense of*. Write an equation on the board. Say *Let's make sense of this problem*. Use tens rods to help you solve the problem while saying *I'm making sense of this problem*. Repeat with another problem and another tool. Repeat once more, this time asking students *How can I make sense of this problem?* Provide a sentence from to those who need it.

Bridging/Reaching Ask students what they do to try to help make sense of a math problem they don't understand (*I use counters, I use a chart, I group items by...*, etc.). Allow students to interject, giving their own opinions and providing corrections when needed. For example, *That's not how I try to make sense of a problem. I....* or *When I'm trying to make sense of ...* .

On My Own

Name ______________________

Which operation should you use for the conversion? Explain your answer.

1. milligrams to grams **division; Sample answer: finding fewer of a larger unit**
2. meters to centimeters **multiplication; Sample answer: finding more of a smaller unit**

Complete the conversion.

3. 3 L = **3,000** mL
4. 100 mL = **0.1** L
5. 500 kg = **500,000** g
6. 6 km = **6,000** m
7. 70 mg = **0.07** g
8. 800 kL = **800,000** L

9. Andrew's height is given in centimeters. What is Andrew's height in meters? **1.42 m**

10. **Error Analysis** A cooler contains 50 liters of water. Emily calculated to determine how many milliliters of water are in the cooler. Check Emily's work. Did she make any mistakes? If so, how could she correct her work?

$50 \times 100 = 5{,}000$

There are 5,000 milliliters of water.

Sample answer: She should have multiplied 50 by 1,000 instead of 100.

Unit 12 • Measurement and Data 173

11. The maximum mass an elevator can hold is 450 kilograms. What is the maximum mass in grams? **450,000 g**

12. How many liters of water are in the pool? **375,000 L**

13. Ryan has a sheet of paper that is 0.75 meter long. What is the length in centimeters? **75 cm**

14. Ada's backpack has a mass of 9,080 grams. What is the mass in kilograms? **9.08 kg**

15. **Extend Your Thinking** Explain how you can determine how many millimeters are in a kilometer. **Sample answer: A meter is 1,000 millimeters, and a kilometer is 1,000 meters, so there are 1,000 × 1,000 = 1,000,000 millimeters in a kilometer.**

Reflect

How can you use multiplication and division to convert metric units of measure?

Answers may vary.

Math is... Mindset

What helped to motivate you to do your best work?

174 Lesson 2 • Convert Metric Units

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 13 Students may mistakenly answer 7,500 by multiplying 75× 100. Remind students to read the problem carefully and that when multiplying a decimal by a power of 10, the digits shift the same number of places as the exponent or the number of zeros.

Item Analysis

Item	DOK	Rigor
1–2	1	Conceptual Understanding
3–8	1	Procedural Skill & Fluency
9	2	Application
10	2	Conceptual Understanding
11–14	2	Application
15	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you use multiplication and division to convert metric units of measure?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped to motivate you to do your best work?

Students reflect on how they practiced self-management.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can convert metric units of measure.
- I can explain which operation to use when converting.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Understand how to convert metric units	5.MD.A.1
2	1	Convert metric units	5.MD.A.1
3	2	Convert metric units	5.MD.A.1
4	2	Convert metric units	5.MD.A.1

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking

Lesson 12-2
Exit Ticket

Name

1. For which conversions would you have to divide? Choose all that apply.
   - A. millimeters to meters
   - B. meters to kilometers
   - C. centimeters to millimeters
   - D. liters to milliliters
   - E. grams to kilograms
   - F. meters to centimeters

2. Which measure is equivalent to 3 kilograms?
   - A. 30 milligrams
   - B. 30 grams
   - C. 3,000 milligrams
   - D. 3,000 grams

3. A pencil is 78 centimeters long. How many meters is this?
   - A. 0.078 meter
   - B. 0.78 meter
   - C. 7.8 meters
   - D. 7,800 meters

4. A water cooler can hold 8.5 liters of water. How many milliliters of water is this?
   - A. 0.0085 milliliter
   - B. 0.085 milliliter
   - C. 8,500 milliliters
   - D. 85,000 milliliters

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

230 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

Reinforce Understanding

SMALL GROUP

Metric Conversions

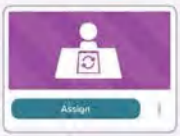
Work with students in pairs. One student rolls two number cubes and writes a measurement of grams, meters, or liters times a power of 10. That student gives the measurement to the other student, who converts it to a smaller or larger unit of measurement. If students have difficulty ask whether they are moving from smaller to larger or larger to smaller units, and whether they should use multiplication or division to make the conversion.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Length Conversions Word Problems
- Mass or Weight Conversions Word Problems
- Liquid Volume Conversions Word Problems



INDEPENDENT WORK

Differentiation Resource Book, p. 141

Lesson 12-2 • Reinforce Understanding

Convert Metric Units

Name ______

Review

You can use multiplication or division to convert metric units of measurement and units of time.

Using Multiplication to Convert	Multiply when converting to a smaller unit.	
	12 meters to centimeters	1 m = 100 cm 12 × 100 = 1,200 cm
Using Division to Convert	Divide when converting to a larger unit.	
	540 kilograms to grams	1 kg = 1,000 g 540 ÷ 1,000 = 0.54 g

Which operation should you use for the conversion? Explain your answer.

1. liters to milliliters **multiplication; I am finding a smaller unit.**
2. meters to kilometers **division; I am finding a larger unit.**

Match the measurement in Column A to its equivalent measurement in Column B.

	Column A	Column B
3.	5 kilograms	70,000 milliliters
4.	70 liters	0.008 grams
5.	14 meters	12 meters
6.	9,000 meters	9 kilometers
7.	8 milligrams	120 meters
8.	1,200 centimeters	5.4 liters
9.	540 centiliters	1,400 centimeters
10.	0.12 kilometers	5,000 grams

Differentiation Resource Book

Build Proficiency

WORKSTATIONS

Practice It! Game Station

Convert Metric Units Race Students practice using metric conversions to solve problems.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 141–142

Lesson 12-2

Additional Practice

Name ______

Review

You can use multiplication or division to convert metric units of mass, length, or capacity.

To convert from a larger measure to a smaller measure, multiply because there will be more of the smaller unit.

To convert from a smaller measure to a larger measure, divide because there will be fewer of the larger unit.

Write equivalent measures in meters and in millimeters for 45 centimeters.

Use the equivalent measures.

1 meter = 100 centimeters Divide since the conversion is from centimeters to meters: 45 ÷ 100 = 0.45	1 centimeter = 10 millimeters Multiply since the conversion is from centimeters to millimeters: 45 × 10 = 450

So 45 centimeters is equivalent to 0.45 meter and to 450 millimeters.

Which operation do you use for the conversion?

1. kilograms to grams **multiplication**
2. milliliters to liters **division**
3. meters to kilometers **division**
4. grams to milligrams **multiplication**

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with finding volume.

Assign

Use It! Application Station

City of Trees Students create a line plot for plant growth data.

The content of this card has concepts covered later in Lesson 12-5. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 141–142

What is the equivalent measure?

5. 3 m = 300 cm
6. 5.2 L = 5,200 mL
7. 240 g = 0.24 kg
8. 1,200 m = 1.2 km
9. 500 mg = 0.5 g
10. 40 mL = 0.04 L
11. Phyllis ran in a 5-kilometer race to help raise money for the school. How many meters long is the race? 5,000 meters
12. Jenny's water bottle holds 1.3 liters of water. How many milliliters of water does the water bottle hold? 1,300 milliliters
13. Joe's dog has a mass of 6.52 kilograms. How many grams is the mass of Joe's dog? 6,520 grams

Math @ Home Activity Provide opportunities for your child to convert metric units. For example, scales can measure in pounds but also in kilograms or grams. Have your child convert a number of kilograms to a number of grams, or grams to kilograms. Containers often show measures in fluid ounces, but also in liters or milliliters. Measurements of length can be made in meters or centimeters—have your child explain how to convert a measure to the other unit.

Student Practice Book

Differentiation Resource Book, p. 142

Lesson 12-2 • Extend Thinking

Convert Metric Units

Name ______________________

How can you complete the conversions? Fill in the table to show how. The first one is done for you as an example. Show your work.

1.	400 millimeters 40 × 10 = 400	40 centimeters	0.4 meters 40 ÷ 100 = 0.40
2.	____ centimeters 6,500; 65 × 100	65 meters	____ kilometers 0.065; 65 ÷ 1,000
3.	7,400 milliliters	____ liters 7.4; 7,400 ÷ 1,000	____ kiloliters 0.0074; 7.4 ÷ 1,000
4.	____ milligrams 3,100,000; 3,100 × 1,000	____ grams 3,100; 3.1 × 1,000	3.1 kilograms
5.	____ milliliters 520,000; 520 × 1,000	520 liters	____ kiloliters 0.52; 520 ÷ 1,000
6.	____ millimeters 8,100; 810 × 10	____ centimeters 810; 8.1 × 100	8.1 meters
7.	11,000 milliliters	____ liters 11; 11,000 ÷ 1,000	____ kiloliters 0.011; 11 ÷ 1,000
8.	____ milligrams 9,200,000; 9,200 × 1,000	____ grams 9,200; 9.2 × 1,000	9.2 kilograms

Differentiation Resource Book

LESSON **12-3**

Solve Multi-Step Problems Involving Measurement Units

Learning Target

- I can solve multi-step problems by identifying and answering a hidden question and using that answer to solve the initial problem.

Vocabulary

Math Term	Academic Terms
convert	analyze

Standards ◆ Major △ Supporting ● Additional

Content

△ **5.MD.A** Convert like measurement units within a given measurement system.

△ **5.MD.A.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Math Practices and Processes

MPP Make sense of problems and persevere in solving them.

MPP Look for and express regularity in repeated reasoning.

Materials

The materials may be for any part of the lesson.

- *Customary Conversion Tables* Teaching Resource
- index cards
- *Metric Conversation Tables* Teaching Resource
- *Problem-Solving Tool* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students solve multi-step problems by identifying and answering a hidden question and using that answer to solve the initial problem.	• Students discuss solving multi-step problems using *make sense* of and *determine*. • To support sense-making, ELs participate in MLR2: Collect and Display.	• Students describe the logic and reasoning used to make a mathematical decision.

Coherence

Previous	Now	Next
• Students used the four operations to solve word problems involving measurement units (Grade 4). • Students used the relationships between metric units of mass, length, or capacity to convert measurements (Unit 12).	• Students solve multi-step problems involving metric and customary unit conversions.	• Students represent and interpret measurement data to eighths of a unit on a line plot (Unit 12).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students deepen their understanding of multiplying fractions and converting units of measurement. *Conceptual Understanding is not a specific element of rigor for this standard.*	• Students build their proficiency with multiplication involving whole numbers and fractions, and in converting units of measurement.	• Students apply knowledge of relative size of unit of measurement and multiplication and division to solve problems with real-world contexts.

Number Routine
Decompose It

GO ONLINE

5–7 min

Build Fluency Students build fluency with fractions as they use equivalent fractions and addition and/ or subtraction concepts to decompose a mixed number in different ways.

These prompts encourage students to talk about their reasoning:

- What do you notice about the number? In what way is the number composed?
- What strategy did you use to solve the problem? What did you do first? Next?
- How did one way of decomposing the number help you think of other ways to decompose the number?
- How can you use a pattern to think of more ways to decompose the number?

Launch 5–7 min

Sense-Making Routine

Purpose Students begin thinking about multi-step problems involving conversions of units.

Which Doesn't Belong?

- Which doesn't belong?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students discuss their thinking with partners before sharing as a whole class.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of multi-step problems involving conversions of units and are based on possible comments and questions that students may make during the share out.

- Which conversions are between customary units? Metric units?
- What do the units in each conversion measure?
- What units are smaller than quarts? Larger than cups?

Math is... Mindset

- What helps you make sense of a situation?

SEL **Responsible Decision-Making: Analyze Situations**

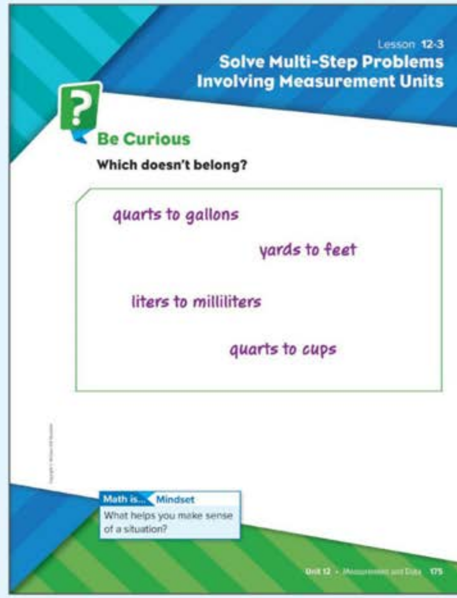
As students work through the Which doesn't belong? routine, have them write, draw, or speak to each of the steps of their logic and reasoning. This can help students analyze their thought processes as they analyze situations, choose appropriate strategies, and identify solutions.

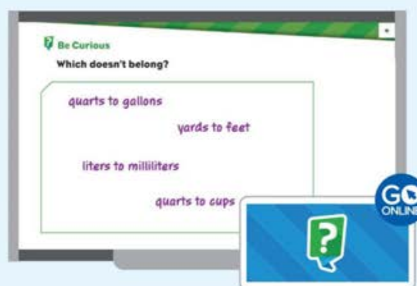
Transition to Explore & Develop

Ask questions that get students thinking about how they can solve multi-step word problems involving measurement conversions.

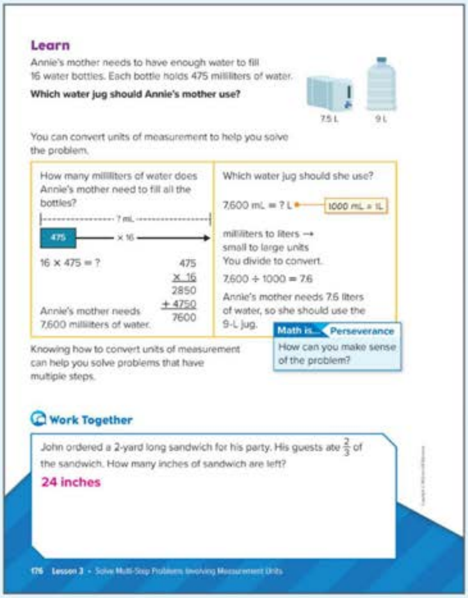
ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about how we can solve multi-step problems involving conversions of units of measure.





Explore & Develop
20 min


Learn

Annie's mother needs to have enough water to fill 16 water bottles. Each bottle holds 475 milliliters of water.

Which water jug should Annie's mother use?

You can convert units of measurement to help you solve the problem.

How many milliliters of water does Annie's mother need to fill all the bottles?

16 × 475 = ?

$$\begin{array}{r} 475 \\ \times\ 16 \\ \hline 2850 \\ +\ 4750 \\ \hline 7600 \end{array}$$

Annie's mother needs 7,600 milliliters of water.

Which water jug should she use?

7,600 mL = ? L — 1000 mL = 1L

milliliters to liters →
small to large units
You divide to convert.

7,600 ÷ 1000 = 7.6

Annie's mother needs 7.6 liters of water, so she should use the 9-L jug.

Knowing how to convert units of measurement can help you solve problems that have multiple steps.

Math is... Perseverance
How can you make sense of the problem?

Work Together

John ordered a 2-yard long sandwich for his party. His guests ate $\frac{2}{3}$ of the sandwich. How many inches of sandwich are left?

24 inches

176 Lesson 3 • Solve Multi-Step Problems Involving Measurement Units

❶ Pose the Problem

MLR Collect and Display

As students discuss the questions, record relevant words and phrases they may use such as *make sense of, determine,* and *convert*. Display the words for student reference. Use the student-generated expressions to help students make connections between student language and math vocabulary.

ETP Pose Purposeful Questions

- What are you trying to find?
- What information do you have that can help you solve the problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- Explain why some problems are called multi-step problems.
- How does planning ahead help you solve a problem?

Key Takeaway

- When solving multi-step problems, it is necessary to answer a hidden question first and use that answer to solve the initial problem.

Work Together

Students solve a multi-step word problem involving customary units of measurement.

Common Error Make sure students understand that they are trying to determine how many inches of the sandwich are left, not how many inches the guests ate. This will require multiple steps: for example, finding out how much sandwich is left, and then finding how long that remaining sandwich measures in inches.

LOM Language of Math

Explain to students that the prefix *centi-* indicates one-hundredth and *milli-* indicates one-thousandth. They can think of the words *century*, meaning a time period of 100 years, and *millennium*, a time period of 1,000 years, to remind them of these values.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore solving multi-step word problems involving units of measurement.

Materials: *Problem-Solving Tool* Teaching Resource

Directions: Distribute copies of the *Problem-Solving Tool* Teaching Resource to each student or pairs. Have students solve the Pose the Problem. You may wish to provide copies of the *Customary Conversion Tables* Teaching Resource and the *Metric Conversion Tables* Teaching Resource.

Math is... Perseverance

- How can you make sense of the problem?

Students understand the meaning of the problem and realize they will have to persevere through more than one step to solve it.

ETP Support Productive Struggle

- How can you represent the information given in the problem?
- What do you need to determine before solving the problem?
- How can you determine which operation you should use first?
- How can you use the relationship between milliliters and liters to solve the problem?

Activity Debrief: Have groups share the plans they made before solving the problem as well as the steps they took to solve the problem. Encourage students to explain their thinking behind each step they took to solve the problem.

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students use what they know about converting units of measurement to solve a multi-step word problem. You may wish to provide copies of the *Customary Conversion Tables* Teaching Resource and the *Metric Conversion Tables* Teaching Resource.

ETP Facilitate Meaningful Mathematical Discourse

Encourage students to make and share plans for solving the problem. Make sure they ask useful questions to improve each other's ideas. Ask:

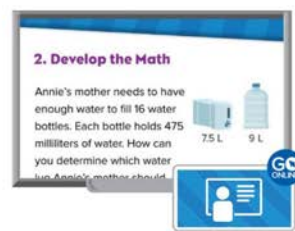
- What are the steps you would perform to solve this problem?
- Can you understand other students' plans?
- How are their plans similar to yours? How are they different?

- **Think About It:** Why should you use multiplication to find the total number of milliliters?
- **Think About It:** Why should you use division to convert milliliters to liters?
- **Think About It:** How can you use what you know about division patterns to find the quotient 7,600 ÷ 1,000?

Math is... Perseverance

- How can you make sense of the problem?

Students understand the meaning of a problem and realize they will need to persevere through more than one step to solve it.



EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *enough*. Show a container and more than enough manipulatives to fill it. Say *I think I have enough (cubes) to fill this container*. Fill it. Nod and say *I had enough cubes*. Then show a larger container. Ask *Do I have enough to fill this?* Put the (cubes) in. Shake your head and say *I didn't have enough (cubes).* Repeat again, asking *Do I have enough (cubes) to fill the container?*

Developing/Expanding Ensure understanding of *enough*. Show a container and more than enough manipulatives to fill it. Say *I think I have enough (cubes) to fill this container*. Fill it. Nod and say *I had enough cubes*. Then show a larger container. Ask *Do I have enough to fill this?* Put the (cubes) in. Shake your head and say *I didn't have enough (cubes).* Repeat again and say *Tell me about the (cubes) and container*. Expect a response with *enough*.

Bridging/Reaching Ensure comprehension of *enough*. Then have students come up with similar words and phrases for *have enough (plenty, sufficient, right amount)*. Then ask students to come up with other words that are often paired with *enough (good enough, long enough,* etc.) and to tell the class what they mean. Allow students to use a dictionary or thesaurus if desired.

Practice & Reflect 10 min

On My Own

Name ______

1. Adrien has a roll of wrapping paper that is 3 yards long. He uses $\frac{1}{3}$ of the wrapping paper to wrap a present. What is the length, in feet, of the paper left on the roll?
   A. 1 ft
   B. 3 ft
   C. 6 ft

2. Ruby's backpack has a mass of 4 kilograms. She removes a book that has a mass of 120 grams. What is the mass of Ruby's backpack after she removes the book?
   A. 2.8 kg
   B. 3.88 kg
   C. 38.8 kg

3. Amy's family has 2 gallons of milk in the refrigerator. At dinner, her family drinks $\frac{3}{8}$ of the milk in the refrigerator. How many cups of milk are left? **20 C**

4. A track at the school is 400 meters long. Jackson walks around the track $3\frac{1}{2}$ times. How many kilometers did Jackson walk? **1.4 km**

5. **STEM Connection** Finn knows that a cubic yard of concrete weighs about 4,050 pounds. A cement truck can hold 10 cubic yards of concrete. How many tons of concrete can the truck hold? **20.25 tons**

6. Robin is selling lemonade. She makes 3 liters of lemonade and sells glasses of 250 milliliters of lemonade each. In the first hour, she sells 6 glasses of lemonade. How many liters does she have left? **1.5 L**

Unit 12 • Measurement and Data 177

7. Brian is walking to his friend's house that is 2.6 kilometers away. He stops when he is $\frac{7}{8}$ of the way there. How many meters does he still have to walk?
   325 m

8. Nell is aiming to drink the amount of water shown per day. By 3 p.m., she is $\frac{3}{4}$ of the way to her goal. How many more fluid ounces does she need to drink to reach her goal?
   16 fl oz

8 cups per day

9. Tyler wants to send his cousin 5 books that are each 1,500 grams. He has a box that can hold up to 6 kilograms. Will Tyler be able to use the box he has? Explain.
   No. Sample answer: He will need a bigger box because the total mass of the books is 7.5 kilograms.

10. Gina is growing a houseplant. When she measures it at the beginning of the month, it is 3 feet tall. When she measures it at the end of the month, it is $1\frac{1}{4}$ the size it was at the beginning of the month. How many inches did the houseplant grow? **9 in.**

11. **Extend Your Thinking** Christa has 3 gallons of water. Jaylen has 36 pints of water. Who has more water? Explain your reasoning.
    Jaylen; Sample answer: 3 gallons is the same as 24 pints, and 36 > 24.

Reflect

How can you solve multi-step word problems involving units of measurement?

Answers may vary.

Math is... Mindset
What helped you make sense of a situation?

178 Lesson 3 • Solve Multi-Step Problems Involving Measurement Units

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 1-10 Make sure students understand which unit of measurement they should be using to express their answers. For example, in Exercise 1, the length is given in yards but students are asked to give their answer in feet.

Item Analysis

Item	DOK	Rigor
1–10	2	Application
11	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you solve multi-step word problems involving units of measurement?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you make sense of a situation?

Students reflect on how they practiced responsible decision-making.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can solve multi-step problems by identifying and answering a hidden question and using that answer to solve the initial problem.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Solve multi-step word problems with conversions	5.MD.A.1
2	2	Solve multi-step word problems with conversion	5.MD.A.1
3	2	Solve multi-step word problems with conversions	5.MD.A.1
4	2	Solve multi-step word problems with conversions	5.MD.A.1

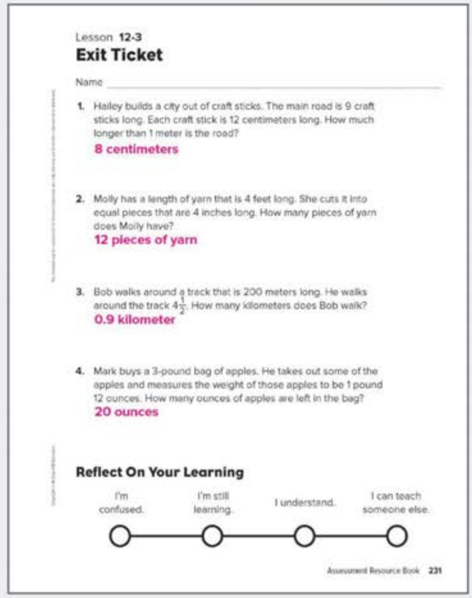
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 12-3

Exit Ticket

Name

1. Hailey builds a city out of craft sticks. The main road is 9 craft sticks long. Each craft stick is 12 centimeters long. How much longer than 1 meter is the road?
8 centimeters

2. Molly has a length of yarn that is 4 feet long. She cuts it into equal pieces that are 4 inches long. How many pieces of yarn does Molly have?
12 pieces of yarn

3. Bob walks around a track that is 200 meters long. He walks around the track $4\frac{1}{2}$. How many kilometers does Bob walk?
0.9 kilometer

4. Mark buys a 3-pound bag of apples. He takes out some of the apples and measures the weight of those apples to be 1 pound 12 ounces. How many ounces of apples are left in the bag?
20 ounces

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 231

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Find Your Match!

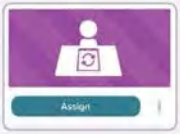
Show students the sentence "_______ L = _______ mL." Have each student fill in the blanks in 4 different ways. Then have students write each of their numbers on index cards. Mix up and then lay out the cards and have students take turns trying to make a match that shows a true conversion. If necessary, remind students that one liter is equivalent to 1,000 milliliters, so the number of milliliters must be 1,000 times the equivalent number of liters.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Multi-Step Problems (Measurement)



INDEPENDENT WORK

Differentiation Resource Book, p. 143

Lesson 12-3 • Reinforce Understanding

Solve Multi-Step Problems Involving Measurement Units

Name ____________________

Review

You can convert units at the beginning or the end of a problem.

The longest NFL field goal kick on record is 64 yards. Mitch is $\frac{3}{4}$ of the way to reaching the record. How far can Mitch kick, in feet?		
Conversion as 1st Step	$64 \times 3 = 192$ feet	$192 \times \frac{3}{4} = 144$ ft
Conversion as 2nd Step	$64 \times \frac{3}{4} = 48$ yards	$48 \times 3 = 144$ ft

Fill in each blank to solve the problem.

1. Griffin has completed $\frac{3}{4}$ of a 5-kilometer run. How many meters has he run?

   Griffin has run **3.75** kilometers, which is **3,750** meters.

2. A skein of yarn is 220 yards. Sharon's ball of yarn is $\frac{4}{9}$ of a skein. How many feet of yarn does Sharon have?

   Sharon has **$97\frac{7}{9}$** yards of yarn, which is **$293\frac{1}{3}$** feet.

3. The average person spends 78,000 hours watching television over their lifetime. $\frac{1}{4}$ of that time is spent watching commercials. How many days are spent watching commercials?

   The average person watches **19,500** hours of television in their lifetime, which is **812.5** days.

4. Sophie has 5 kilograms of potatoes. She peels 900 grams of potatoes for a dinner party. How many kilograms of potatoes does she have left?

   Sophie has peeled **0.9** kilgrams of potatoes. Sophie has **4.1** kilograms of potatoes left to peel.

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Metric Units of Measurement Race

Students practice using metric conversions to solve problems.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 143–144

Lesson 12-3

Additional Practice

Name ____________________

Review

You can convert units of measurement to help you solve problems that have multiple steps.

Kathryn has a new spool of ribbon that holds a total length of 2.5 meters of ribbon. She uses 225 centimeters of ribbon to wrap some gift boxes. How much ribbon does Kathryn have left?

To solve, find 2.5 meters − 225 centimeters.

First, convert 2.5 meters to an equivalent measure in centimeters: $2.5 \times 100 = 250$, so 2.5 m = 250 cm.

Then subtract: 250 cm − 225 cm = 25 cm.

Kathryn has 25 cm of ribbon left.

1. Zach has a pitcher that holds 1.5 L of lemonade. Each cup holds 280 mL of lemonade. He pours out 5 glasses for himself and 4 friends. How much lemonade will be left in the pitcher?

   Sample answer: 100 mL

2. A bag of apples weighs 3 pounds. Each apple weighs 6 ounces. How many apples are in the bag? **8** apples

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with finding volume.

Assign

Use It! Application Station

Environmentally Friendly Students use measurements to create five environmentally friendly home improvements.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 143–144

3. A hiking trail is 3.2 km long. From the start of the trail, the bridge is 1.4 km along the trail. Once at the bridge, a waterfall is 900 m farther. How far is it from the waterfall to the end of the trail?
Sample answers: 0.9 km or 900 m

4. A deli uses 4 ounces of meat on each of its sandwiches. How many sandwiches can be made from 5 pounds of meat?
20 sandwiches

5. A bush is 3 feet 4 inches tall. Herb trims the bush so that the bush is now $\frac{3}{4}$ the height it was. How tall is the bush now?
Sample answer: 2 feet 6 inches tall

Math @Home Activity Look for situations in which two of the same types of measurements occur such as millimeters and centimeters, or feet and inches. With your child create a problem situation that involves the two measures. Have your child explain how to solve the problem by converting one of the measures to an equivalent measure using the other unit.

Student Practice Book

Differentiation Resource Book, p. 144

Lesson **12-3 • Extend Thinking**

Solve Multi-Step Problems Involving Measurement Units

Name ____________________

Five friends are preparing for a hike. It is recommended that hikers carry a maximum of 30 pounds. Keri's backpack weighs 35 pounds. How can you use this information to solve the word problem? Show your work.

1. Justin has five 12-ounce water bottles in his backpack, and his gear weighs $\frac{9}{10}$ that of Keri's. How many water bottles must Justin take out so that his backpack is within the recommened weight?
2 bottles; Sample answer: $35 \times \frac{9}{10} = 31.5$. He is 1.5 lb or $1.5 \times 16 = 24$ oz over, which is 2 bottles.

2. Abigail's gear weighs $\frac{4}{5}$ that of Keri's. If she carries Keri's excess gear and an additional 12-ounce water bottle, how many ounces over or under the recommended weight will her gear be?
20 oz under; Sample answer: $35 \times \frac{4}{5} = 28$ lb; $28 \times 16 = 448$ oz. $448 + 12 = 460$; $30 \times 16 = 480$ oz; $480 - 460 = 20$ oz.

3. Finn's gear weighs $\frac{5}{8}$ that of Keri's. Finn is collecting rock samples during the hike. How many ounces of rock sample can Finn carry home and keep his backpack within the recommended weight?
130 oz; Sample answer: $35 \times \frac{5}{8} = 21.875$ lb; $30 - 21.875 = 8.125$ lb; $8.125 \times 16 = 130$ oz.

4. Floyd's gear weighs 72 ounces more Finn's. How many 0.84 ounce granola bars can Floyd add to his pack and keep his pack within the recommended weight?
69 bars; Sample answer: $21.875 \times 16 = 350$ oz; $350 + 72 = 422$; 30 lb $= 480$ oz; $480 - 422 = 58$ oz; $58 \div 0.84 = 69\frac{4}{84}$; so 69 whole bars.

Differentiation Resource Book

LESSON **12-4**

Represent Measurement Data on a Line Plot

Learning Targets

- I can create line plots of data sets involving measurement data.
- I can interpret line plots.

Standards ◇ Major △ Supporting ○ Additional

Content

△ **5.MD.B** Represent and interpret data.

△ **5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Use operations on fractions for this grade to solve problems involving information presented in line plots.

Math Practices and Processes

MPP Model with mathematics.

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	Academic Terms
data	accurate
line plot	reflect
outlier	

Materials

The materials may be for any part of the lesson.

- dry spaghetti noodles
- *Water Remaining Line Plot* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students create a line plot to display a data set involving measurement. • Students interpret line plots.	• Students discuss line plots using the modals *might, can*, and *could*. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students recognize and build upon personal mathematical strengths of self and others within the classroom math community.

Coherence

Previous	Now	Next
• Students displayed a data set of measurements in fractions of a unit on line plots and solved problems by using information presented in line plots (Grade 4). • Students solved multi-step problems involving unit conversions (Unit 12).	• Students represent and interpret measurement data to eighths of a unit on a line plot.	• Students solve real-world problems with data in fractional measurements on a line plot using operations (Unit 12). • Students develop understanding of statistical variability and summarize and describe distributions (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students use understanding of relationships between fractional values and how various statistical representations are used to better understand a data set. *Conceptual understanding is not a specific element or rigor for this standard.*	• Students represent measurement data by labeling a number line and placing an X to represent each data value above the number line.	• Students apply measurement concepts and number sense to organize, represent, and interpret data within real-world contexts.

Number Routine

Which Benchmark Is It Closest To? 5–7 min

GO ONLINE

Build Fluency Students build understanding of fractions as they identify the closest benchmark numbers to given fractions.

These prompts encourage students to talk about their reasoning:

- Retell the problem in your own words. What do you notice about the numbers?
- How can you tell which benchmark to choose?
- How do the benchmarks help you understand and compare the fractions?
- How could putting the numbers in order from greatest to least help you place them on the number line?

Launch 5–7 min

Sense-Making Routine

Purpose Students begin thinking about how measurements are represented in line plots and what conclusions can be drawn from them about the measurements.

Numberless Graph

- What is the question?

Teaching Tip You may want to have students take a few minutes to jot down various questions they have about the line plot before discussing as a class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of line plots and are based on possible comments and questions that students may make during the share out.

- What does the title tell you about the line plot?
- What do you think the Xs represent?

Math is... Mindset

- What helps you be part of the classroom community?

SEL Relationship Skills: Social Engagement

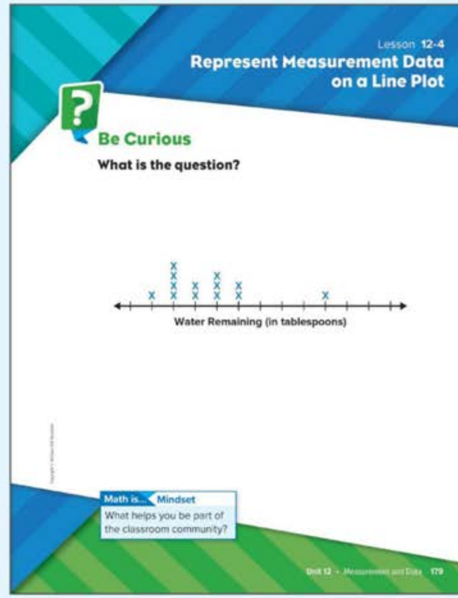
Help students identify and understand the value of socially engaging with other students within the class community. As students collaborate in small groups to complete the Numberless Word Problem routine, invite each student to acknowledge the value of each group member. As students think about what questions they could ask, have them identify their peers' as well as their own contributions to the collaborative group effort.

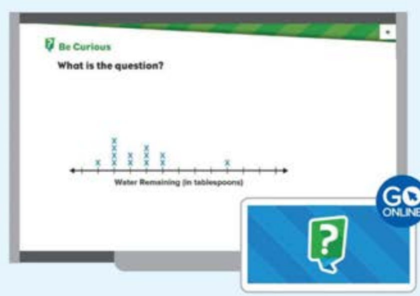
Transition to Explore & Develop

Ask questions that get students thinking about the information they can learn from a line plot.

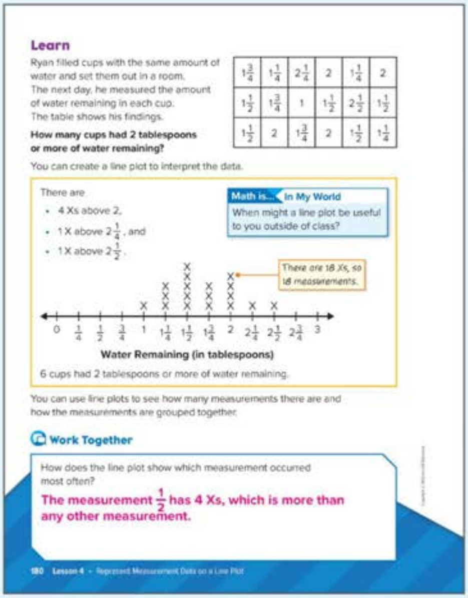
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can interpret information that is represented on a line plot.





Explore & Develop 20 min



Learn

Ryan filled cups with the same amount of water and set them out in a room. The next day, he measured the amount of water remaining in each cup. The table shows his findings.

How many cups had 2 tablespoons or more of water remaining?

$1\frac{3}{4}$	$1\frac{1}{4}$	$2\frac{1}{4}$	2	$1\frac{1}{4}$	2
$1\frac{1}{2}$	$1\frac{3}{4}$	1	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$
$1\frac{1}{2}$	2	$1\frac{3}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{4}$

You can create a line plot to interpret the data.

There are
- 4 Xs above 2,
- 1 X above $2\frac{1}{4}$, and
- 1 X above $2\frac{1}{2}$.

Math is... In My World
When might a line plot be useful to you outside of class?

6 cups had 2 tablespoons or more of water remaining.

You can use line plots to see how many measurements there are and how the measurements are grouped together.

Work Together

How does the line plot show which measurement occurred most often?

The measurement $\frac{1}{2}$ has 4 Xs, which is more than any other measurement.

180 Lesson 4 • Represent Measurement Data on a Line Plot

❶ Pose the Problem

ETP Pose Purposeful Questions
- Why would you represent data?
- What are some ways you represented data in the past?
- How is data represented in a line plot?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time
Pair students and have them work on a problem like the problem on the Learn page. Have them individually write how to solve the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking
- What information is shown in a line plot?
- What are some conclusions about data you can make by interpreting a line plot?

Key Takeaway
- Line plots can be used to display data sets that involve measurement.

Work Together

Students use the line plot to determine which measurement occurred most often.

Common Misconception Students often confuse the meanings of the labels on the number line in a line plot and the number of Xs above them. Many will conclude, in this example, that $2\frac{1}{4}$ occurred most often because it is the greatest number on the number line, or that 1 occurred most often because there is 1 X over the greatest number on the number line. Remind them that each X represents a time that the label on the number line appeared in the data set.

LOM Language of Math
Tell students that the correct pronunciation of *data* is “day-tah, “ not “dah-tah.” Also point out that a line plot contains data, which is plural. One piece of data is called a *datum*. More than one bacterium are bacteria. More than one datum are data.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore interpreting data from a line plot.

Materials: *Water Remaining Line Plot* Teaching Resource

Directions: Distribute copies of the *Water Remaining Line Plot* Teaching Resource to each student or pairs. Have students solve the Pose the Problem.

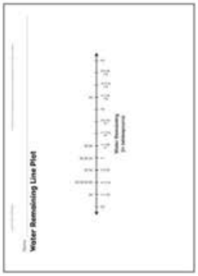
ETP Support Productive Struggle

- How do you know what the numbers on the line plot mean?
- How do you know what the Xs in the line plot represent?
- Why do some numbers not have any Xs above them?
- How can you know how many cups of water are represented in the line plot?
- What can you learn about how water evaporates from the line plot?

Math is... In My World

- When might a line plot be useful to you outside of class?

Students apply the mathematics they know to solve problems arising in everyday life.



Activity Debrief: Invite students to discuss as a class the statements they made about the line plot. Make sure students support their statements with information that can be found on the line plot.

A PDF of the Teaching Resource is available in the Digital Teacher Center.

Guided Exploration

Students explore how they can make statements about measurements that are represented in a line plot.

ETP Use and Connect Mathematical Representations

- How do you know what the Xs represent?
- Why might it be important to know how many cups are represented?
- **Think About It:** How is a line plot similar to other representations of measurements that you know?
- How could knowing where the measurements are grouped be helpful in analyzing the data?

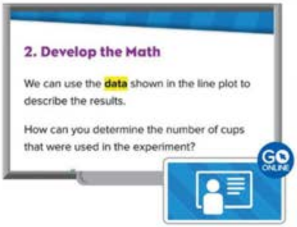
Have students discuss the nature of outliers in data.

- What might have caused that cup to be an outlier?
- Do you think you should ignore outliers when you interpret data? Why or why not?

Math is... In My World

- When might a line plot be useful to you outside of class?

Students apply the mathematics they know to solve problems arising in everyday life.



EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *whether*. Put 24 centimeter cubes on the desk. Say, *Let's see whether there are more than 20 cubes.* Count and confirm. Then say, *Let's see whether we have enough counters to fill this container*. Put them in a container too big to fill. Then, put 3 tens rods on the desk. Say, *Tell me whether we have more than 30 cubes*. (*yes/no*)

Developing/Expanding Ensure understanding of *whether*. Put 24 centimeter cubes on the desk. Say, *Let's see whether there are more than 20 cubes*. Count and confirm. Then say, *Let's see whether we have enough counters to fill this container*. Put them in a container too big to fill. Then, put 3 tens rods on the desk. Ask, *What can we do to know whether we have more than 30 cubes?*

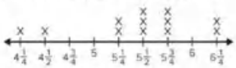
Bridging/Reaching Ask students to use *whether* in a sentence. For example, *I will go to the store whether it is raining or not*. Validate and make corrections to vocabulary, grammar, and meaning as needed.

Practice & Reflect
10 min

On My Own

Name ____________

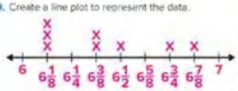
This line plot shows the lengths of various mice from nose to the tip of the tail. Use the line plot to answer the questions.



Mice Length (in inches)

1. How many mice are in the data set?
   12 mice
2. How long is the shortest mouse?
   $4\frac{1}{4}$ in.
3. How long is the longest mouse?
   $6\frac{1}{4}$ in.
4. Which measurement or measurements occurred the most often?
   $5\frac{1}{2}$ in. and $5\frac{3}{4}$ in.
5. Which measurement or measurements occur the least often?
   $4\frac{1}{4}$ in. and $4\frac{1}{2}$ in.
6. How many mice are longer than 5 inches?
   10 mice
7. How many mice are shorter than 5 inches?
   2 mice
8. What is the difference in inches between the longest and the shortest mice?
   2 in.

9. Create a line plot to represent the data.



Pencil Lengths (in inches)

Pencil Lengths (in.)			
$6\frac{3}{4}$	$6\frac{1}{8}$	$6\frac{1}{2}$	$6\frac{1}{8}$
$6\frac{7}{8}$	$6\frac{3}{8}$	$6\frac{1}{8}$	$6\frac{3}{8}$

10. How did you know how to label the measurements on the line plot?
    Sample answer: Some of the measurements are in eighths, so I labeled each tick mark counting by eighths.
11. How did you know how many Xs to place above each measurement?
    Sample answer: I counted how many of each measurement there are in the table and placed that many Xs.
12. Are there any measurements with no Xs above them? Explain.
    There are no Xs above 6, $6\frac{1}{4}$, $6\frac{5}{8}$, or 7; there are no pencils of those lengths.
13. **Extend Your Thinking** Another pencil was found. It has a length that is $1\frac{1}{2}$ inches shorter than the longest pencil in the table. What is the length of this new pencil?
    $5\frac{3}{8}$ in.

Reflect

How can you use line plots to interpret measurements?
Answers may vary.

Math is... Mindset
How were you part of the classroom community?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 9 Students may initially be confused by how to space the measurements in the line plot as the fractions have different denominators. Encourage students to first find a common denominator for all of the measurements before determining how to label and space the measurements on the line plot.

Item Analysis

Item	DOK	Rigor
1–8	1	Application
9–12	2	Procedural Skill & Fluency
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you use line plots to interpret measurements?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How were you part of the classroom community?

Students reflect on how they developed stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can create line plots of data sets involving measurement data.
- I can interpret line plots.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Interpret a line plot	5.MD.B.2
2	2	Interpret a line plot	5.MD.B.2
3	2	Interpret a line plot	5.MD.B.2

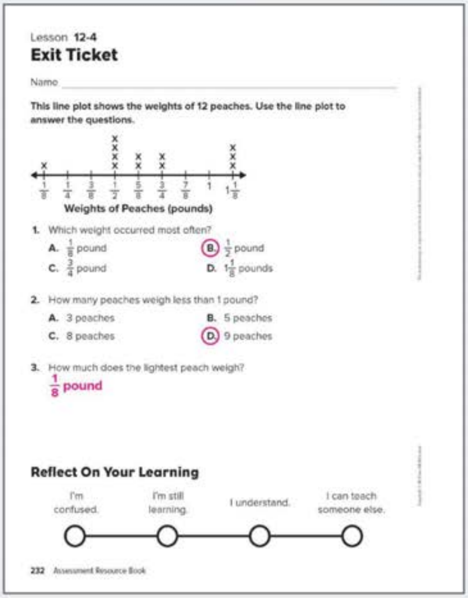
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking


Lesson 12-4

Exit Ticket

Name

This line plot shows the weights of 12 peaches. Use the line plot to answer the questions.

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$

Weights of Peaches (pounds)

1. Which weight occurred most often?
   A. $\frac{1}{8}$ pound B. $\frac{1}{2}$ pound
   C. $\frac{3}{4}$ pound D. $1\frac{1}{8}$ pounds

2. How many peaches weigh less than 1 pound?
   A. 3 peaches B. 5 peaches
   C. 8 peaches D. 9 peaches

3. How much does the lightest peach weigh?
   $\frac{1}{8}$ pound

Reflect On Your Learning

I'm confused. I'm still learning. I understand. I can teach someone else.

232 Assessment Resource Book

R Reinforce Understanding

SMALL GROUP

Spaghetti Fractions

Have students break 6–8 strands of uncooked spaghetti in 2 pieces, aiming for halves. Help the students measure the length of the pieces to the nearest eighth of an inch. Then work together to make a line plot with their combined data. Discuss with students how they can identify the most common spaghetti length. Make sure students understand that the most common length has the most marks shown on the line plot.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Line Plots with Operations (Halves)
- Line Plots with Operations (Quarters)
- Line Plots with Operations (Eighths)

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 145

Lesson 12-4 • Reinforce Understanding

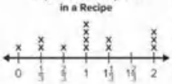
Represent Measurement Data on a Line Plot

Name ____________________

Review

Line plots show data as a mark above a value on a number line.

Cups of Flour Required in a Recipe



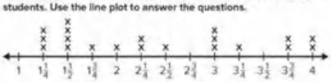
There are 13 Xs, so there are 13 recipes represented.

The most commonly used measure of flour is 1 cup.

1 of the recipes calls for 0 cups.

3 of the recipes call for 2 cups.

This line plot shows the hours a week spent reading by a group of students. Use the line plot to answer the questions.



1. How many students are represented on the line plot?
   20
2. What is the longest time spent reading per week?
   4 hours
3. How many students read more than 3 hours?
   5 students
4. What is the shortest time spent reading per week?
   $1\frac{1}{4}$ hours
5. What is the most common time spent reading?
   $1\frac{1}{2}$ hours
6. How many students read for $3\frac{1}{2}$ hours?
   none or 0 students

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Create a Line Plot Task Cards Students practice finding equivalent fractions on a number line.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 145–146

Lesson 12-4

Additional Practice

Name ____________________

Review

You can create a line plot from a set of data and use it to make observations about the data.

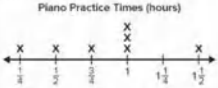
The times, in hours, that Kaylee practices the piano are shown. Which time or times occurs most often?

$\frac{1}{4}$, 1, $\frac{1}{2}$, 1, $1\frac{1}{2}$, $\frac{3}{4}$, 1

To solve, make a line plot of the data.

Make a number line showing all of the possible times. Use an X to mark one occurrence.

Piano Practice Times (hours)



Since there are 3 X only above 1, the time that occurs most often is 1 hour.

Use the line plot above for questions 1 and 2.

1. How many times did Kaylee practice for 1 hour or more?
   4 times
2. How many times did Kayle practice for exactly $1\frac{1}{4}$ hours? Explain.
   0 times; Sample answer: There are no X above $1\frac{1}{4}$ on the line plot.

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with finding volume.

Use It! Application Station

Find a Pattern and Repeat Students use a repeat function to loop computer code.

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review from the Digital Teacher Center.

GO ONLINE

STEM Adventure

Assign a digital simulation to apply skills and extend thinking.

INDEPENDENT WORK

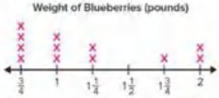
Student Practice Book, pp. 145–146

A group of friends picked blueberries. The weights, in pounds, of the amounts of blueberries each person picked are listed.

$\frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}, 2, \frac{3}{4}, 1, \frac{3}{4}, 1\frac{1}{4}, 1, \frac{3}{4}, 2, 1$

3. Make a line plot of the data.

Weight of Blueberries (pounds)



4. How many friends were in the group? __12__ friends
5. Which weight or weights of blueberries were picked most often? __$\frac{3}{4}$__ pound(s)
6. How many friends picked an amount of blueberries that weighed more than $1\frac{1}{2}$ pounds? __3__ friends
7. What is the heaviest weight of blueberries that one person picked? __2__ pounds

Math @ Home Activity Give your child 10 objects with lengths that are within 1 inch of each other. Have him or her measure the objects, record the lengths, and then create a line plot for the data. Ensure that your child includes key components of a line plot, such as a title, labeled tick marks, and an X for each data value. Have your child make observations about the data from the line plot.

Student Practice Book

Differentiation Resource Book, p. 146

Lesson **12-4 • Extend Thinking**

Represent Measurement Data on a Line Plot

Name ______________________

Nicole used some data to start making this line plot. The plot shows the hours a week spent reading by a group of students. However, she has not yet graphed all her data. Help her finish the line plot and then answer the questions.

1. There should be twice as many marks for $1\frac{1}{4}$ hours as what is currently marked.
2. There should be two more marks for 3 hours than what is marked for $2\frac{1}{4}$ hours.
3. There should be one less mark for $2\frac{3}{4}$ hours than what is now marked for $1\frac{1}{4}$ hours.
4. There is $\frac{1}{5}$ of the marks now shown for 3 hours marked for 1 hour.
5. How many students are represented on the line plot?
   22
6. How many students read less than 2 hours?
   8 students
7. How many students read 3 hours or more?
   7 students
8. Are there any outliers?
   Yes; the student that read for 4 hours.
9. How many students read for 2 hours?
   none or 0 students
10. What was the most common length of time spent reading?
    3 hours

Differentiation Resource Book

LESSON **12-5**

Solve Problems Involving Measurement Data on Line Plots

Learning Target

- I can solve problems using data in a line plot and perform operations on the data.

Standards ◆ Major ▲ Supporting ● Additional

Content

▲ **5.MD.B** Represent and interpret data.

▲ **5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Use operations on fractions for this grade to solve problems involving information presented in line plots.

Math Practices and Processes

MPP Reason abstractly and quantitatively.

MPP Attend to precision.

Vocabulary

Math Terms	Academic Term
data	emphasize
line plot	

Materials

The materials may be for any part of the lesson.

- blank number cubes
- index cards
- *Problem-Solving Tool* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students solve problems using data in a line plot and performing operations on the data.	• Students discuss solving problems using operations and line plot data using *amount* and the superlatives *greatest* and *least*. • To support maximizing linguistic and cognitive meta-awareness, ELs participate in MLR5: Co-Craft Questions and Problems.	• Students acknowledge different representations that can be used to complete a mathematical task, and reflect on the value of the similarities and differences.

Coherence

Previous	Now	Next
• Students displayed a data set of measurements in fractions of a unit and solved problems by using information presented in line plots (Grade 4). • Students represented and interpreted measurement data to eighths on a line plot (Unit 12).	• Students solve real-world problems involving data in fractional measurements on a line plot using operations.	• Students display numerical data in dot plots, histograms, and box plots and understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students extend their understanding of line plots and fraction operations to solve problems. *Conceptual understanding is not a specific element of rigor for this standard.*	• Students build procedural skills and proficiency with fraction operations and fluency in interpreting data on line plots to solve problems.	• Students apply their understanding of fraction operations and their interpretation of data on line plots to solve problems with real-world context.

GO ONLINE

Number Routine
Which Benchmark Is It Closest To? 5–7 min

Build Fluency Students build understanding of decimals as they identify the closest benchmark numbers to given decimals.

These prompts encourage students to talk about their reasoning:

- What are you being asked to do?
- What do you notice about the benchmarks? What patterns do you see?
- What do you notice about the given decimals? What did you think about the decimal?
- How did you determine the closest benchmark for the decimal?
- Which decimals were the most challenging to link to a benchmark? Why?
- How are decimals related to fractions?

Launch 5–7 min

Sense-Making Routine

Purpose Students begin to think about how they can redistribute groups to make equal amounts.

Notice & Wonder

- What do you notice?
- What do you wonder?

See the Appendix for a full description of the sense-making routines.

Teaching Tip You may want to have students draw groupings that match the objects in the image so that they can study the groups more closely.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of interpreting information provided visually and are based on possible comments and questions that students may make during the share out.

- How can you know how many tomatoes there are in all?
- How would you be able to regroup the tomatoes to make equal groups in each bowl? What steps would you take?

Math is... Mindset

- How can you show others that you value their ideas?

SEL **Social Awareness: Appreciate Diversity**

As students consider the Notice & Wonder routine, invite them to discuss different strategies they might use to redistribute to make equal groups. As students share their unique thought processes and ideas, emphasize the value of the differences as well as the similarities so students can understand and appreciate the importance of diversity.

Transition to Explore & Develop

Ask questions that get students thinking about how they can solve problems using data in a line plot and perform operations on the data.

ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about how we can use information in line plots to solve problems.

Explore & Develop 20 min

Learn

A tortilla maker put $4\frac{1}{2}$ cups of corn meal in ten bowls. The line plot shows the amount of corn meal in each of nine bowls.

How much corn meal is in the tenth bowl?

Corn Meal in Each Bowl (in cups)

Determine the amount of corn meal in each of the nine bowls.

2 bowls have $\frac{1}{8}$ cup each. $2 \times \frac{1}{8} = \frac{2}{8}$

2 bowls have $\frac{1}{4}$ cup each. $2 \times \frac{1}{4} = \frac{2}{4} = \frac{4}{8}$

3 bowls have $\frac{1}{2}$ cup each. $3 \times \frac{1}{2} = \frac{3}{2} = \frac{12}{8}$

1 bowl has $\frac{3}{4}$ cup. $\frac{3}{4} = \frac{6}{8}$

1 bowl has 1 cup. $1 = \frac{8}{8}$

$\frac{2}{8} + \frac{4}{8} + \frac{12}{8} + \frac{6}{8} + \frac{8}{8} = \frac{32}{8} = 4$

The nine bowls have 4 cups of corn meal.

Math is... Quantities
What could be another way to add the amounts of corn meal?

Subtract to determine the amount of corn meal in the tenth bowl.

$4\frac{1}{2} - 4 = \frac{1}{2}$

The tenth bowl has $\frac{1}{2}$ cup of corn meal.

You can solve problems by interpreting information given in line plots and then performing operations.

Work Together

Based on the line plot above, what is the difference between the greatest amount of flour in a bowl and the least amount of flour in a bowl? Explain your answer.

$\frac{7}{8}$ c; Sample answer: $1 - \frac{1}{8} = \frac{7}{8}$

184 Lesson 5 • Solve Problems Involving Measurement Data on Line Plots

1 Pose the Problem

ETP Pose Purposeful Questions

- What information does the line plot tell you?
- In your own words, explain what this problem is asking you to solve.

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Co-Craft Questions and Problems

Pair students and have them co-create a problem similar to the one on the Learn page. Have them work together to solve their problem and then trade their problem with another pair. After each pair solves the other pair's problems, have them form a group of four to check solutions and correct any mistakes that may have been made. Revisit the routine throughout the lesson for reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you solve problems using information given in line plots?

Key Takeaway

- Problems involving information presented in line plots can be solved by interpreting the data and then performing operations.

Work Together

Students solve a problem by interpreting information given in a line plot and performing an operation.

Common Error Students may subtract the least number of X's from the greatest number of Xs. Remind them of the meaning of the X's vs. the meaning of the labels on the number line in a line plot.

LOM Language of Math

Explain to students that *interpret* means to explain the meaning of something, whether it is data given in a line plot or words in a foreign language. An *interpreter* explains what someone means when they use a different language.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using information given in a line plot to solve problems.

Materials: *Problem-Solving Tool* Teaching Resource

Directions: Distribute copies of the *Problem-Solving Tool* Teaching Resource to each student or pairs. Have students solve the Pose the Problem.

ETP **Support Productive Struggle**

- What information do you need to find first?
- What strategies can you use to find the total amount of flour?
- How did you determine the total number of bowls?
- Is your answer reasonable? How do you know?

Math is... Quantities

- Why do you use division to find out how much flour should go in each bowl?

Students make sense of quantities and their relationships in problem situations.

Activity Debrief: Have groups share the plans they made before solving the problem as well as the steps they took to solve the problem. Encourage students to explain their thinking behind each step they took to solve the problem.

A PDF of the Teaching Resources are available in the Digital Teacher Center.

Guided Exploration

Students interpret information given in a line plot and use that information to solve a problem.

ETP **Facilitate Meaningful Mathematical Discourse**

Encourage students to make and share plans for solving the problem. Make sure they ask useful questions to improve each other's ideas. Ask:

- What are the steps you would perform to solve this problem?
- Can you understand other students' plans?
- How are their plans similar to yours? How are they different?

Have the students create an equation for the amount of flour. Ask:

- What should the operation be? Why?
- How should the numbers appear in the equation? Why?
- How should the unknown appear in the equation? Why?
- How does the equation represent the amount of flour in all the bowls?
- **Think About It:** What equation using addition only could you use to represent the amount of flour?

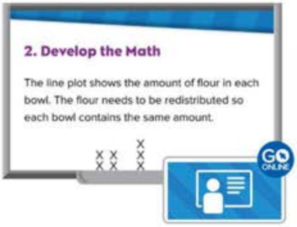
Have the students determine the number of bowls. Ask:

- How does the line plot help you determine the number of bowls of flour?

Math is... Quantities

- Why do you use division to find out how much flour should go in each bowl?

Students make sense of quantities and their relationships in problem situations.



EL **English Learner Scaffolds**

Entering/Emerging Support students' understanding of *so (that)*. Put thirty chips into two groups of 15. Say *I need to regroup these so there are three groups of ten.* Regroup. Continue using the chips to regroup, using *so* in your sentences. Finally, test student comprehension and give students twelve chips. Say, *Group these so there are four groups of three.*

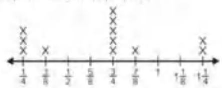
Developing/Expanding Support students' understanding of *so* (*that*). Put thirty chips into two groups of 15. Say, *I need to regroup these so there are three groups of ten.* Regroup. Continue using the chips to regroup, using *so* in your sentences. Finally, ask students to demonstrate the task using their own manipulatives and *so* in their sentences. Provide sentence frames for students who need more guidance.

Bridging/Reaching Ensure comprehension of *so* by asking students to group 15 chips so there are 3 chips per group. After they demonstrate, ask students to talk about other ways *so* is used. For example, to indicate a large amount like *so much*; or to indicate a result like *I have a lot of homework tonight, so I can't watch TV).* Allow students to use a dictionary if desired.

On My Own

Name ______

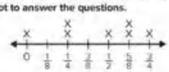
The line plot shows the weights of various mice. Use the line plot to answer the questions.



Mice Weight (in ounces)

1. What is the combined weight of the 4 lightest mice? $1\frac{1}{8}$ oz
2. What is the combined weight of the mice that weigh $\frac{3}{4}$ ounces? $3\frac{3}{4}$ oz
3. What is the combined weight of all the mice? $8\frac{1}{4}$ oz
4. What is the difference in weight between the heaviest mouse and the lightest mouse? 1 oz

The line plot shows the amount of rain that fell each day in a week. Use the line plot to answer the questions.

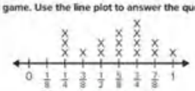


Daily Rainfall (in inches)

5. What was the total amount of rainfall in inches during the week? 3 in.
6. How many days did it rain during the week? 6 days
7. On the days it rained, what is the difference between the greatest and least amount of rainfall? $\frac{1}{2}$ in.
8. If the same amount of rain falls the following week, what is the total amount of rainfall over two weeks? 6 in.

Unit 12 • Measurement and Data 185

The line plot shows how much water each player drank during a basketball game. Use the line plot to answer the questions.



Water Drank (in gallons)

9. How many players drank water during the basketball game? 16 players
10. What is the difference between the greatest amount of water drank and the least amount of water drank? $\frac{3}{4}$ gal
11. **Error Analysis** Tony wants to find the total amount of water players drank during the game.
$\frac{1}{4}+\frac{3}{8}+\frac{1}{2}+\frac{5}{8}+\frac{3}{4}+\frac{7}{8}+1=4\frac{3}{8}$ gallons
Is Tony's work correct? Explain why or why not.
No, it is not correct. Sample answer: He did not multiply the number of gallons by the number of tick marks. The correct amount is $9\frac{3}{4}$ gallons.
12. **Extend Your Thinking** Why is being able to solve problems involving data on line plots helpful for analyzing data?
Sample answer: Solving problems involving data on line plots can help us find sums, differences, etc. which gives important information and can help explain information or give predictions about certain things.

Reflect

How can you use data displayed on a line plot to solve problems?
Answers may vary.

Math is... Mindset
How did you show others that you value their ideas?

186 Lesson 5 • Solve Problems Involving Measurement Data on Line Plots

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 1 Make sure students understand that they need to find the combined weight of the 4 lightest mice, not just the mice who weigh the least.

Item Analysis

Item	DOK	Rigor
1–10	2	Application
11–12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you use data represented on a line plot to solve problems?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you show others that you value their ideas?

Students reflect on how they practiced social awareness.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can solve problems using data in a line plot and perform operations on the data.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1a	2	Solve problems involving a line plot	5.MD.B.2
1b	2	Solve problems involving a line plot	5.MD.B.2
2	2	Solve problems involving a line plot	5.MD.B.2

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

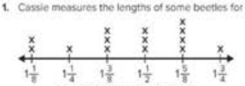
E Extend Thinking

Lesson 12-5

Exit Ticket

Name ______________________

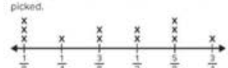
1. Cassie measures the lengths of some beetles for a project.



a. What is the difference in length between the longest and shortest beetles? $\frac{5}{8}$ inch

b. What is the total length of the beetles that are $1\frac{3}{8}$ inches long? $4\frac{1}{8}$ inches

2. The line plot shows the amounts of blueberries that 12 friends picked.



The friends decide to divide the blueberries so each friend has the same amount of blueberries. How many gallons of blueberries does each friend get?

$\frac{5}{12}$ gallon

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 233

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Line Plot Fun!

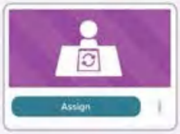
Work with students in pairs. Provide a number cube labeled with fraction measurements and 2–4 prepared line plot (index) cards with the same 6 fraction measurements. One student rolls the number cube. The other records an X above the value on a line plot card. After 10 rolls, students find the combined total of values. If students have difficulty, help them group the numbers with the same value and multiply to find the total, then add the products.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Line Plots with Operations (Halves)
- Line Plots with Operations (Quarters)
- Line Plots with Operations (Eighths)



INDEPENDENT WORK

Differentiation Resource Book, p. 147

Lesson 12-5 • Reinforce Understanding

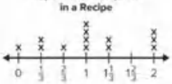
Solve Problems Involving Measurement Data on Line Plots

Name ____________________

Review

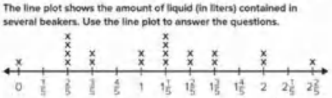
You can use information given in line plots to solve problems.

Cups of Flour Required in a Recipe



For example, you can find the total cups of flour needed to make all 13 recipes. $0 + \frac{1}{3} \times 2 + \frac{2}{3} + 1 \times 4 + 1\frac{1}{3} \times 2 + 2 \times 3$. This is $0 + \frac{2}{3} + \frac{2}{3} + 4 + \frac{8}{3} + 6 =$ 14 cups of flour.

The line plot shows the amount of liquid (in liters) contained in several beakers. Use the line plot to answer the questions.



1. What is the total amount of liquid contained in the beakers?
   22 liters
2. How much more liquid is in the 5 beakers containing the most liquid than is in the 5 beakers containing the least liquid?
   8 liters
3. If the liquid is redistributed equally, how much will each beaker hold?
   $1\frac{1}{10}$ liters

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Line Plot Task Cards Student practice creating and describing a line plot with fractional values.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 147–148

Lesson 12-5

Additional Practice

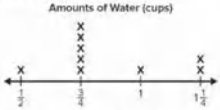
Name ____________________

Review

You can solve problems by interpreting information given in line plots and then performing operations.

Brett makes a line plot showing the amounts of water, in cups, that he drinks throughout the day. What is the total amount of water, in cups, that Brett drinks during the day?

Amounts of Water (cups)



To solve, add the amounts represented by each X.
Use multiplication to help.
$\frac{1}{2} + (5 \times \frac{3}{4}) + 1 + (2 \times 1\frac{1}{4}) = \frac{1}{2} + \frac{15}{4} + 1 + \frac{10}{4}$
Using a common denominator, $\frac{2}{4} + \frac{15}{4} + \frac{4}{4} + \frac{10}{4} = \frac{31}{4}$
As a mixed number, $\frac{31}{4} = 7\frac{3}{4}$.
Brett drank $7\frac{3}{4}$ cups of water throughout the day.

Use the line plot above for questions 1 and 2.

1. What is the difference between the greatest amount of water Brett drank and the least? $\frac{3}{4}$ cup
2. How many times did Brett get a drink of water during the day? 9 times

Student Practice Book

Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop fluency with finding volume.

Assign

Use It! Application Station

City of Trees Students create a line plot for plant growth data.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

STEM Adveture

Assign a digital simulation to apply skills and extend thinking.

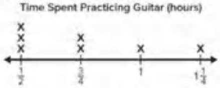
Assign

INDEPENDENT WORK

Student Practice Book, pp. 147–148

Sem made a line plot to show the time, in hours, that he spends practicing the guitar each day for a week.

Time Spent Practicing Guitar (hours)



3. For how many days did Sem record his time spent practicing? 7 days
4. What is the difference between the greatest amount of time Sem spent practicing and the least amount of time Sem spent practicing? $\frac{3}{4}$ hour
5. How much time did Sem spend practicing the guitar during the week? $5\frac{1}{4}$ hours
6. Next week, Sem plans to spend the same amount of time practicing, but plans to spend an equal amount of time each day. How much time should Sem spend practicing each day? $\frac{3}{4}$ hour(s)

Math @ Home Activity

Student Practice Book

Differentiation Resource Book, p. 148

Lesson **12-5 • Extend Thinking**

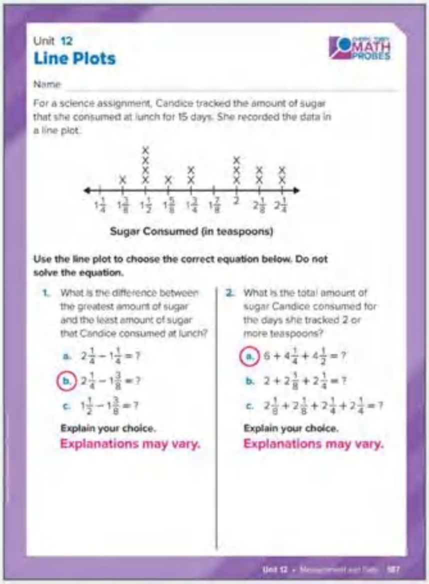
Solve Problems Involving Measurement Data on Line Plots

Name ____________________

Maurice used some data to start making this line plot. It shows the amount of liquid (in liters) contained in several beakers. However, he has not yet graphed all his data. Help him finish the line plot and then answer the questions.

1. There should be four times as many marks for $1\frac{2}{5}$ L as what is marked for $\frac{3}{5}$ L.
2. There should be one more mark for 1 L than what is now marked for $1\frac{2}{5}$ L.
3. There should be one less mark for $1\frac{3}{5}$ L than what is now marked for $1\frac{2}{5}$ L.
4. There should be twice as many marks for 2 L as there are for $\frac{2}{5}$ L.
5. How many total liters of water are there in the beakers? **26 liters**
6. If the liquid is redistributed equally, how much will each beaker hold? **$1\frac{3}{10}$ liters**
7. How many beakers hold more than 1 liter? How many liters are in these beakers? **12 beakers; $19\frac{3}{5}$ liters**
8. How many beakers hold less than 1 liter? How many liters are in these beakers? **3 beakers; $1\frac{2}{5}$ liters**
9. How many beakers are empty? **0**
10. Which problems would be affected by empty beakers? **#5, #6, #8, #9**

Differentiation Resource Book



Unit 12

Line Plots

Name

For a science assignment, Candice tracked the amount of sugar that she consumed at lunch for 15 days. She recorded the data in a line plot.

Use the line plot to choose the correct equation below. Do not solve the equation.

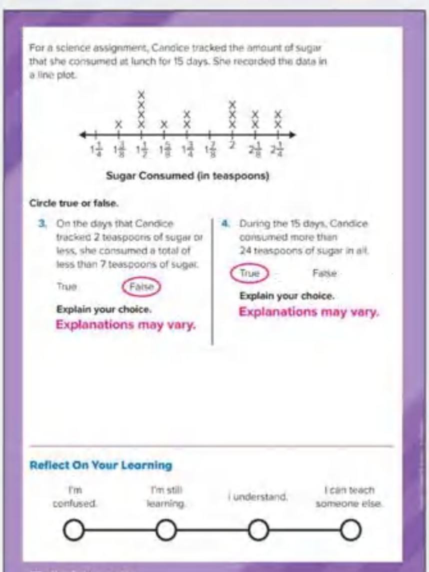
1. What is the difference between the greatest amount of sugar and the least amount of sugar that Candice consumed at lunch?
   a. $2\frac{1}{4} - 1\frac{1}{4} = ?$
   b. $2\frac{1}{4} - 1\frac{3}{8} = ?$
   c. $1\frac{1}{2} - 1\frac{3}{8} = ?$

   Explain your choice.
   Explanations may vary.

2. What is the total amount of sugar Candice consumed for the days she tracked 2 or more teaspoons?
   a. $6 + 4\frac{1}{4} + 4\frac{1}{2} = ?$
   b. $2 + 2\frac{1}{8} + 2\frac{1}{4} = ?$
   c. $2\frac{1}{8} + 2\frac{1}{8} + 2\frac{1}{4} + 2\frac{1}{4} = ?$

   Explain your choice.
   Explanations may vary.

Unit 12 • Measurement and Data 187



For a science assignment, Candice tracked the amount of sugar that she consumed at lunch for 15 days. She recorded the data in a line plot.

Circle true or false.

3. On the days that Candice tracked 2 teaspoons of sugar or less, she consumed a total of less than 7 teaspoons of sugar.
   True False

   Explain your choice.
   Explanations may vary.

4. During the 15 days, Candice consumed more than 24 teaspoons of sugar in all.
   True False

   Explain your choice.
   Explanations may vary.

Reflect On Your Learning

I'm confused. I'm still learning. I understand. I can teach someone else.

188 Math Probe • Line Plots

Analyze the Probe Formative Assessment

Targeted Concept Interpret a line plot and use operations with fractions to solve problems involving information presented in the line plot.

Targeted Misconceptions Some students have difficulty interpreting the meaning of the numbers along the scale of a line plot in connection with the meaning of each X that is marked above those numbers. They may think that the number of numbers listed along the scale reflects the total number of data points. Some students have difficultly estimating or determining sums and differences involving mixed numbers.

Authentic Student Work

Below are examples of students' explanations.

Sample A

1. What is the difference between the greatest amount of sugar and the least amount of sugar that Candice consumed at lunch?
   a. $2\frac{1}{4} - 1\frac{1}{4} = ?$
   b. $2\frac{1}{4} - 1\frac{3}{8} = ?$
   c. $1\frac{1}{2} - 1\frac{3}{8} = ?$

Explain your choice.

Well what the model is saying 2 1/4 are the highect amount and the 1 3/8 is the lowest.

Sample B

4. During the 15 days, Candice consumed more than 24 teaspoons of sugar in all.

   True False

Explain your choice.

$1\frac{3}{8}$ $1\frac{5}{8}$ 6 $3\frac{1}{2}$ 6 $4\frac{2}{8}$ $2\frac{2}{4}$

$1\frac{1}{2} \times 4$

$1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$

$1\frac{3}{4}$ $1\frac{3}{4}$

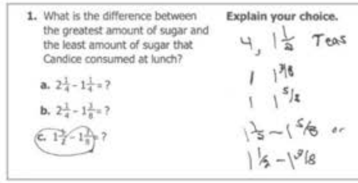
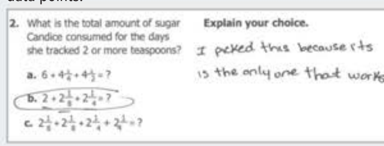
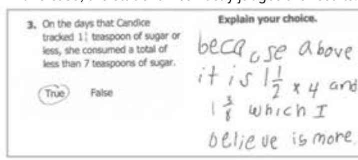
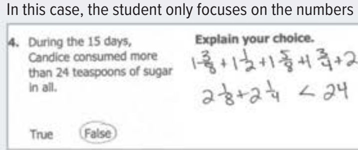
3 3

$5 + 12 + 6 = 23$

Fractions are more than one.

Collect and Assess Student Work

Collect and review student responses to determine possible misconceptions. See examples in If-Then chart.

IF incorrect...	THEN the student likely...	Sample Misconceptions
1. a or c	is unable to interpret a line plot: a) incorrectly uses $1\frac{1}{4}$ as the least amount recorded. c) is focused on the category with the most data points ($1\frac{1}{2}$) rather than on the greatest amount of sugar recorded ($2\frac{1}{4}$).	In this case, the student uses the numbers of data points shown by the Xs.  1. What is the difference between the greatest amount of sugar and the least amount of sugar that Candice consumed at lunch? a. $2\frac{1}{4} - 1\frac{1}{4} = ?$ b. $2\frac{1}{4} - 1\frac{1}{8} = ?$ c. $1\frac{1}{2} - 1\frac{1}{8} = ?$ Explain your choice. 4, $1\frac{1}{2}$ Teas 1 $1\frac{7}{8}$ 1 $1\frac{5}{8}$ $1\frac{1}{5} - 1\frac{5}{8}$ or $1\frac{1}{5} - 1\frac{7}{8}$
2. b or c	b) only considers the numbers along the scale, without considering the data points above them. c) only considers the two data points greater than 2 rather than including each data point for 2 and for those greater than 2.	In this case, the student focuses on the three numbers on the scale rather than the data points.  2. What is the total amount of sugar Candice consumed for the days she tracked 2 or more teaspoons? a. $6 + 4\frac{1}{4} + 4\frac{1}{2} = ?$ b. $2 + 2\frac{1}{8} + 2\frac{1}{4} = ?$ c. $2\frac{1}{8} + 2\frac{1}{8} + 2\frac{1}{4} + 2\frac{1}{4} = ?$ Explain your choice. I peked this because its is the only one that works.
3. True	is unable to interpret a line plot OR only considers the data point for the value that is less than $1\frac{1}{2}$ rather than also including the four data points for $1\frac{1}{2}$. OR incorrectly estimates the sum $1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{3}{8}$.	In this case, the student incorrectly judges the results.  3. On the days that Candice tracked $1\frac{1}{2}$ teaspoon of sugar or less, she consumed a total of less than 7 teaspoons of sugar. True False Explain your choice. because above it is $1\frac{1}{2}$ x 4 and $1\frac{3}{8}$ which I believe is more
4. False	is unable to estimate the sum of the data points OR only considers the numbers along the scale without considering the data points above them (the frequency of each category).	In this case, the student only focuses on the numbers in the scale.  4. During the 15 days, Candice consumed more than 24 teaspoons of sugar in all. True False Explain your choice. $1\frac{3}{8} + 1\frac{1}{2} + 1\frac{5}{8} + 1\frac{3}{4} + 2$ $2\frac{1}{8} + 2\frac{1}{4} < 24$

Many of the above difficulties result in a combination of correct and incorrect responses.
For correct responses, be sure to check for sound reasoning.

Take Action

Choose from the following resources or suggestions:

- Provide opportunities for students to work through the cycle of determining a question, collecting data about the question through measuring, organizing the collected data using a line plot, and interpreting the results.
- Provide opportunities for students to compare line plots with other types of graphs.
- Show two or more related line plots and true statements about each set of data. Have students match statements to the correct line plot.
- Provide several data sets and line plots. Ask students to determine which line plot represents which data set. Have students tell how they determined the matches.

Revisit the Probe After additional instruction. Have students review their initial answers to the probe. Use these questions for discussion:

- Are there any answers you would like to change? Explain why you might want to change them.
- Are there any questions that you still have about any of the exercises on this probe?

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Unit Review

Unit Review

Name ____________

Vocabulary Review

Choose the correct word(s) to complete the sentence.

capacity, convert, customary system, data, length, line plot, mass, metric system, weight

1. Information collected from a survey or experiment is called **data**. (Lesson 12-4)
2. The **metric system** is the measurement system based on powers of 10 with units such as meter, gram, and liter. (Lesson 12-2)
3. To **convert** a measurement to another measurement means to change the unit of measure used but not the quantity or amount. (Lesson 12-1)
4. The **capacity** is the amount a container can hold. (Lesson 12-1)
5. A measurement system that includes units such as foot, pound, and quart is the **customary system**. (Lesson 12-1)
6. A **line plot** is a type of graph that uses columns of Xs or dots above a number line to show data. (Lesson 12-4)
7. **Mass** measures the amount of matter in an object. (Lesson 12-2)
8. **Length** is a measure of distance. (Lesson 12-2)

Unit 12 • Measurement and Data 189

Review

9. What operation should you use to convert seconds to minutes? Explain your answer. (Lesson 12-1)
division; Sample answer: you are finding fewer of a greater unit

10. How many meters are equal to 3 kilometers? (Lesson 12-2)
3,000 m

11. Jolanna has $1\frac{1}{2}$ yards of decorative tape. She uses 1-inch pieces for her scrapbook. How many 1-inch pieces of decorative tape does she have? (Lesson 12-3)
A. 24 pieces
B. 36 pieces
C. 54 pieces
D. 90 pieces

12. It is recommended that a person sleep 8 hours every night. How many minutes does this person sleep in a year? (Lesson 12-1)
A. $48\frac{2}{3}$ minutes
B. 2,920 minutes
C. 175,200 minutes
D. 10,512,000 minutes

13. The art teacher has $3\frac{1}{4}$ gallons of paint for a mural on the wall. The students in fifth grade use $1\frac{1}{2}$ gallons. How many quarts of paint are left? (Lesson 12-3)
7 qt

14. How many meters equal 400 centimeters? (Lesson 12-2)
4 m

15. Catherine has a piece of fabric that is 3,200 centimeters long. She needs fabric pieces that are 1 meter long for her quilt. How can she determine the number of 1-meter long pieces she has for her quilt? (Lesson 12-4)
Sample answer: Divide by 100. 3,200 ÷ 100 = 32. Catherine has 32 1-meter pieces of fabric for her quilt.

16. Jamal picked 983 grams of blueberries. How many kilograms of blueberries did he pick? (Lesson 12-3)
0.983 kg

17. An Olympic-size pool is 50 meters long. How can you determine the length in centimeters? (Lesson 12-3)
Multiply 50 × 100 = 5,000. The length of the pool is 5,000 centimeters.

190 Unit 12 • Review

Students can complete the **Unit Review** to prepare for the **Unit Assessment**. Students may complete the Review in their Interactive eBook in the Digital Student Center.

Vocabulary Review

Item Analysis

Item	Lesson
1	12-4
2	12-2
3	12-4
4	12-1
5	12-1
6	12-1
7	12-4
8	12-1
9	12-2
10	12-1

Review

Item Analysis

Item	DOK	Lesson	Standard
11	3	12-1	5.MD.A.1
12	3	12-2	5.MD.A.1
13	2	12-3	5.MD.A.1
14	2	12-1	5.MD.A.1
15	2	12-3	5.MD.A.1
16	1	12-2	5.MD.A.1
17	3	12-3	5.MD.A.1
18	2	12-3	5.MD.A.1
19	2	12-3	5.MD.A.1

To review the lessons in this unit, have students watch the Math Replay video in their Digital Student Center.

Assign the Unit Review practice to students from the Digital Teacher Center.

GO ONLINE

MATH REPLAY

Item Analysis (continued)

Item	DOK	Lesson	Standard
20	1	12-4	5.MD.B.2
21	1	12-4	5.MD.B.2
22	1	12-4	5.MD.B.2
23	1	12-4	5.MD.B.2
24	1	12-4	5.MD.B.2
25	1	12-4	5.MD.B.2
26	2	12-4	5.MD.B.2
27	2	12-4	5.MD.B.2
28	2	12-5	5.MD.B.2
29	2	12-5	5.MD.B.2

Performance Task

Standard: 5.MD.A.1, 5.MD.B.2

Rubric (4 points)

Part A (DOK 2) – 2 points	
2	Student's work reflects a proficiency with converting units.
1	Student's work reflects developing proficiency with converting units.
0	Student's work reflects weak proficiency with converting units.

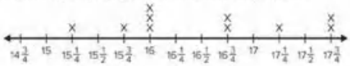
Part B (DOK 1) – 2 points	
2	Student's work reflects a proficiency with making line plots.
1	Student's work reflects developing proficiency with making line plots.
0	Student's work reflects weak proficiency with making line plots.

Reflect

The Reflect question provides an opportunity for students to express their understanding of the unit level focus question.

MATH REPLAY | GO ONLINE

The line plot shows the length of the best long jump for each athlete at a Track and Field meet. Use the line plot to answer the questions.



Best Long Jump (in feet)

18. How many athletes are represented on the line plot? (Lesson 12-4) **10 athletes**

19. How long is the longest jump? (Lesson 12-4) **$17\frac{3}{4}$ ft**

20. How long is the shortest jump? (Lesson 12-4) **$15\frac{1}{4}$ ft**

21. What measurement(s) occurred most often? (Lesson 12-4) **16 feet**

22. How many jumps are longer than 16 feet? (Lesson 12-4) **5 jumps**

23. How many jumps are 16 feet or shorter? (Lesson 12-4) **5 jumps**

24. What does no mark above a measurement mean? (Lesson 12-4) **Sample answer: The best jump for each athlete was not that length.**

25. What is the difference between the greatest jump length and the least jump length? (Lesson 12-5) **$2\frac{1}{2}$ ft**

Unit 12 • Measurement and Data 191

Performance Task

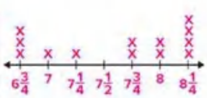
A town is redesigning a park. It will include a tree house.

Part A: The tree house, that the architect designed has a rectangular floor. He will use wooden tiles that are 20 centimeters wide and 40 centimeters long. How many tiles will he need for a floor that is 4 meters wide and 8 meters long?

Check students' work; 400 tiles

Part B: The architect plans to use wooden boards to build the walls. The boards will be different lengths. The construction manager needs to see what size boards he currently has to determine what he needs to purchase. Create a line plot to show his current inventory listed in the table.

Current Inventory	
Length (feet)	Total
$8\frac{1}{4}$	4
7	1
$7\frac{3}{4}$	2
$6\frac{3}{4}$	3
$7\frac{1}{4}$	1
8	2



Reflect

How can you use line plots to make decisions about a data set?

Answers may very.

192 Unit 12 • Performance Task

Fluency Practice

Unit 12

Fluency Practice

Name ____________________

Fluency Strategy

You can multiply 46 × 27 using an algorithm.

Step 1 Multiply 46 × 7.

$$\begin{array}{r} 46 \\ \times\ 27 \\ \hline 322 \end{array}$$

Step 2 Multiply 46 × 20.

$$\begin{array}{r} 46 \\ \times\ 27 \\ \hline 322 \\ +\ 920 \end{array}$$

Step 3 Add partial products.

$$\begin{array}{r} 46 \\ \times\ 27 \\ \hline 322 \\ +\ 920 \\ \hline 1{,}242 \end{array}$$

1. Use the algorithm to multiply.

$$\begin{array}{r} 86 \\ \times\ 92 \\ \hline 7{,}912 \end{array}$$

Fluency Flash

Use the area model to find the product.

2. 24 × 17 = 408

	10	7
20	200	140
4	40	28

3. 53 × 42 = 2,226

	40	2
50	2,000	100
3	120	6

Unit 12 • Measurement and Data 193

Fluency Check

What is the product or quotient?

4. 45 × 28 = 1,260
5. 478 × 4 = 1,912
6. 27 × 23 = 621
7. 67 × 98 = 6,566
8. 2,500 ÷ 5 = 500
9. 2,400 ÷ 4 = 600
10. 57 × 16 = 912
11. 275 × 7 = 1,925
12. 358 × 7 = 2,506
13. 64 × 9 = 576
14. 4,800 ÷ 6 = 800
15. 2,100 ÷ 7 = 300

Fluency Talk

Explain to a friend what partial products are.

Explanations may vary.

How can you multiply a 3-digit number by a single-digit factor?

Explanations may vary.

194 Unit 12 • Fluency Practice

Fluency practice helps students develop procedural fluency, that is, the "ability to apply procedures accurately, efficiently, and flexibly." Because there is no expectation of speed, students should not be timed when completing the practice activity.

Build Fluency **Objective** Students practice using an algorithm to multiply 2-digit numbers by 2-digit numbers.

Fluency Progression

Unit	Skill	Standard
1	Use Partial Sums to Add	4.NBT.B.4
2	Decompose by Place Value to Subtract	4.NBT.B.4
3	Use an Algorithm to Add	4.NBT.B.4
4	Use an Algorithm to Subtract	4.NBT.B.4
5	Choose a Strategy to Add	4.NBT.B.4
6	Choose a Strategy to Subtract	4.NBT.B.4
7	Multiply by Multiples of 10	5.NBT.B.5
8	Multiply by Multiples of 100	5.NBT.B.5
9	Divide Multiples of 10	5.NBT.B.6
10	Divide Multiples of 100	5.NBT.B.6
11	Use an Algorithm to Multiply (2- and 3-Digit Numbers by 1-Digit Numbers)	5.NBT.B.5
12	**Use an Algorithm to Multiply (2-Digit Numbers by 2-Digit Numbers)**	**5.NBT.B.5**
13	Choose a Strategy to Multiply	5.NBT.B.5
14	Choose a Strategy to Multiply	5.NBT.B.5

Fluency Expectations

Grade 4

- Add and subtract within 1,000,000.

Grade 5

- Multiply multi-digit whole numbers.

Grade 6

- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Performance Task

Track and Field

Students draw on their understanding of line plots and converting measurement. Use the rubric shown to evaluate students' work.

Standard: 5.MD.A.1, 5.MD.B.2

Rubric (8 points)

Part A (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with interpreting line plots and performing operations with the data. The student's answers are correct.
1 POINT	Student's work reflects developing proficiency with interpreting line plots and performing operations with the data. One of the student's answers is incorrect.
0 POINTS	Student's work reflects weak proficiency with interpreting line plots and performing operations with the data. The student's answers are incorrect.

Part B (DOK 3) – 2 points	
2 POINTS	Student's work reflects proficiency with solving multi-step problems. The student's answer and work are accurate, and their explanation is reasonable.
1 POINT	Student's work reflects developing proficiency with solving multi-step problems. Either the student's answer and work are not accurate, or their explanation is not reasonable.
0 POINTS	Student's work reflects weak proficiency with solving multi-step problems. The student's answer and work are not accurate, and their explanation is not reasonable.

Part C (DOK 3) – 2 points	
2 POINTS	Student's work reflects proficiency with using multiplication and division to convert among metric units. The student's explanation is reasonable.
1 POINT	Student's work reflects developing proficiency with using multiplication and division to convert among metric units. The student's explanation is partially reasonable.
0 POINTS	Student's work reflects weak proficiency with using multiplication and division to convert among metric units. The student's explanation is not reasonable.

Part D (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with converting customary units. The student's answer and work are accurate.
1 POINT	Student's work reflects developing proficiency with converting customary units. Either the student's answer or work is not accurate.
0 POINTS	Student's work reflects weak proficiency with converting customary units. The student's answer and work are not accurate.

Unit **12**

Performance Task

Name ______________________

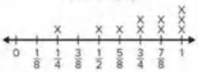
Track and Field

Metric City High School is hosting a track and field event for local athletes.

Part A

In preparation for the event, Josephine runs every morning before going to school. The following represents the fraction of a mile she completed each morning.

Distances (mi)



How does this line plot show which distance occurred most often?

1 mile has the most Xs

Find the total distance Josephine runs. Show your work.

Sample Answer: $7\frac{5}{8}$ miles; $\frac{1}{4} + \frac{1}{2} + \frac{5}{8} + (2 \times \frac{3}{4}) + (2 \times \frac{7}{8}) + (3 \times 1)$; $\frac{2}{8} + \frac{4}{8} + \frac{5}{8} + \frac{12}{8} + \frac{14}{8} + 3$; $\frac{37}{8} + 3 = 7\frac{5}{8}$

Part B

The local sports store is providing a special water bottle as a gift to each participating athlete. Each bottle holds 500 milliliters of water. If there are 23 athletes competing at the event, will a 10-liter jug of water be enough to fill each bottle before the event starts? Show your work and explain.

Sample answer: No, a 10-liter jug will not provide enough water.
23 bottles × 500 milliliters = 11,500 milliliters needed.
11,500 milliliters = 11.5 liters

Assessment Resource Book 235

Part C

The discus used in the throwing event weighs 2,000 grams. Marcus and Miriam converted 2,000 grams to kilograms using different methods. How would you respond to them?

Marcus' work:	Miriam's work:
2,000 g = ? kg	2,000 g = ? kg
$2{,}000 \times \frac{1}{1{,}000} =$	2,000 ÷ 1,000 = 2
$\frac{2{,}000}{1{,}000} = 2$	2,000 g = 2 kg
2,000 g = 2 kg	

Sample Answer: Both are correct. Marcus used the fact a gram is $\frac{1}{1,000}$ of a kilogram and multiplied the weight in grams by the factor $\frac{1}{1,000}$. Miriam used the fact that 1,000 grams is equal to 1 kilogram and divided the weight in grams by 1,000 to determine the weight in kilograms.

Part D

The longest distance event is the 6 mile race. If an athlete completes this event, how can you determine the race's length in feet? Show your work.

1 foot (ft) = 12 inches (in)
1 yard (yd) = 3 ft
1 mile (mi) = 1,760 yd

Sample answer: 6 × 1,760 = 10,560 yards
10,560 × 3 = 31,680 feet
6 miles = 31,680 feet

236 Assessment Resource Book

Unit Assessments

Two forms of the Unit Assessment, Form A and Form B, are available for either print or digital administration. The items on the two assessments are parallel items, assessing the same concept and standard. The table below provides the item analysis for both forms.

Both Unit Assessments are available in the Assessment Resource Book or as downloadable files from the Digital Teacher Center.

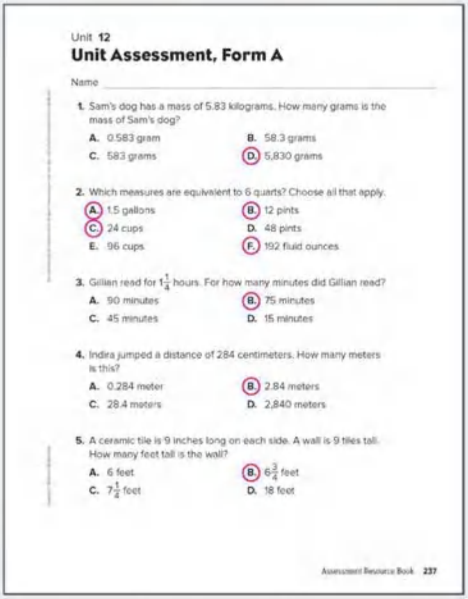
Data When students complete the Unit Assessment in the Digital Student Center, their responses are auto-scored.

Item Analysis

Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	2	12-2	Mass or Weight Conversion Word Problems	5.MD.A.1
2	1	12-1	Liquid Volume Conversions	5.MD.A.1
3	2	12-1	Length Conversions	5.MD.A.1
4	2	12-2	Length Conversions Word Problems	5.MD.A.1
5	2	12-3	Multi-Step Problems (Measurement)	5.MD.A.1
6	2	12-3	Multi-Step Problems (Measurement)	5.MD.A.1
7	2	12-3	Multi-Step Problems (Measurement)	5.MD.A.1
8a	2	12-4	Line Plots with Operations (Eighths)	5.MD.B.2
8b	2	12-4	Line Plots with Operations (Eighths)	5.MD.B.2
9a	2	12-5	Line Plots with Operations (Eighths)	5.MD.B.2
9b	2	12-5	Line Plots with Operations (Eighths)	5.MD.B.2
10	2	12-4	Line Plots with Operations (Quarters)	5.MD.B.2
11	2	12-2, 12-3	Multi-Step Problems (Measurement)	5.MD.A.1
12	2	12-2, 12-3	Multi-Step Problems (Measurement)	5.MD.A.1

GO ONLINE

Assign the digital Unit Assessment (Form A or B) to students or download and print PDFs from the Digital Teacher Center.



Unit 12

Unit Assessment, Form A

Name ____________

1. Sam's dog has a mass of 5.83 kilograms. How many grams is the mass of Sam's dog?
   A. 0.583 gram B. 58.3 grams
   C. 583 grams D. 5,830 grams

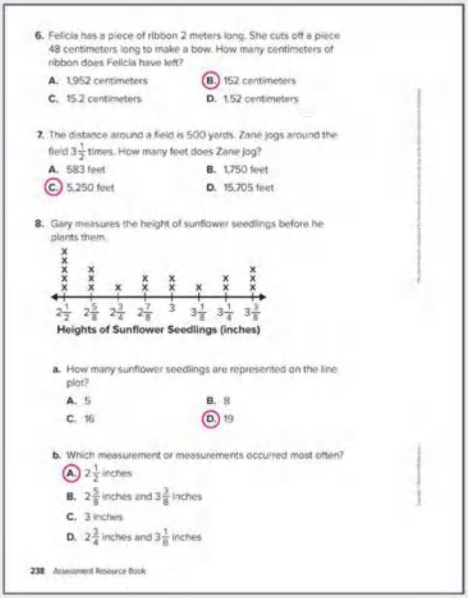
2. Which measures are equivalent to 6 quarts? Choose all that apply.
   A. 1.5 gallons B. 12 pints
   C. 24 cups D. 48 pints
   E. 96 cups F. 192 fluid ounces

3. Gillian read for $1\frac{1}{4}$ hours. For how many minutes did Gillian read?
   A. 90 minutes B. 75 minutes
   C. 45 minutes D. 15 minutes

4. Indira jumped a distance of 284 centimeters. How many meters is this?
   A. 0.284 meter B. 2.84 meters
   C. 28.4 meters D. 2,840 meters

5. A ceramic tile is 9 inches long on each side. A wall is 9 tiles tall. How many feet tall is the wall?
   A. 6 feet B. $6\frac{3}{4}$ feet
   C. $7\frac{1}{4}$ feet D. 18 feet

Assessment Resource Book 237



6. Felicia has a piece of ribbon 2 meters long. She cuts off a piece 48 centimeters long to make a bow. How many centimeters of ribbon does Felicia have left?
   A. 1,952 centimeters B. 152 centimeters
   C. 15.2 centimeters D. 1.52 centimeters

7. The distance around a field is 500 yards. Zane jogs around the field $3\frac{1}{2}$ times. How many feet does Zane jog?
   A. 583 feet B. 1,750 feet
   C. 5,250 feet D. 15,705 feet

8. Gary measures the height of sunflower seedlings before he plants them.

   a. How many sunflower seedlings are represented on the line plot?
      A. 5 B. 8
      C. 16 D. 19

   b. Which measurement or measurements occurred most often?
      A. $2\frac{1}{2}$ inches
      B. $2\frac{5}{8}$ inches and $3\frac{3}{8}$ inches
      C. 3 inches
      D. $2\frac{3}{4}$ inches and $3\frac{1}{8}$ inches

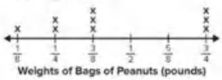
238 Assessment Resource Book

Unit **12**

Unit Assessment, Form A (continued)

Name ______________________________

9. Colette weighs the bags of peanuts the store receives from the warehouse.



a. What is the difference between the heaviest bag of peanuts and the lightest bag of peanuts?

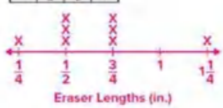
$\frac{5}{8}$ pound

b. How many pounds of peanuts are there in all?

4 pounds

10. Create a line plot to represent the data in the table.

Eraser Lengths (in.)			
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$1\frac{1}{4}$
$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$



Assessment Resource Book 239

11. Al, Ben, and Cal each measured their own height. Their measures are shown in the table.

Person	Height
Al	1,980 millimeters
Ben	185 centimeters
Cal	1.6 meters

From shortest to tallest, what is the order? Explain.

Cal, Ben, Al; Sample answer: Choose one unit, such as centimeters, and convert all measures to this unit. 1,980 millimeters = 198 centimeters and 1.6 meters = 160 centimeters. From shortest to tallest, the measures are 160 cm, 185 cm, and 198 cm, so the order is Cal, Ben, Al.

12. A cooler contains 5 gallons of fruit punch. After Brian fills his drink bottle, there are 76 cups left of fruit punch in the cooler. How many cups of fruit punch fit in Brian's drink bottle? Explain your answer.

4 cups; Sample answer: I multiplied the number of gallons by 16 to get the amount in cups. 5 gallons = 80 cups. I subtract the amount left from the 80 cups: 80 − 76 = 4.

240 Assessment Resource Book

Form B

Unit **12**

Unit Assessment, Form B

Name ______________________________

1. Cora adds 2.37 grams of baking soda to her chemistry experiment. How many milligrams of baking soda does Cora add?

A. 2,370 milligrams B. 237 milligrams
C. 23.7 milligrams D. 0.237 milligram

2. Which measures are equivalent to 10 quarts? Choose all that apply.

A. 160 cups B. 80 pints
C. 40 cups D. 20 pints
E. 2.5 gallons F. 320 fluid ounces

3. James worked on a puzzle for $1\frac{3}{4}$ hours. For how many minutes did James work on the puzzle?

A. 45 minutes B. 75 minutes
C. 105 minutes D. 135 minutes

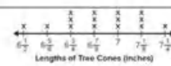
4. A frog jumped a distance of 312 centimeters. How many meters is this?

A. 3.12 meters B. 31.2 meters
C. 3,120 meters D. 31,200 meters

5. A ceramic tile is 9 inches long on each side. A wall is 6 tiles tall. How many feet tall is the wall?

A. $4\frac{1}{2}$ feet B. 12 feet
C. $15\frac{1}{2}$ feet D. 54 feet

Assessment Resource Book 241



a. How many tree cones are represented on the line plot?

A. 7 B. 10
C. 19 D. 20

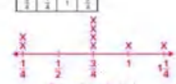
b. Which measurement or measurements occur most often?

A. $6\frac{1}{2}$ inches, $6\frac{5}{8}$ inches, and $7\frac{1}{4}$ inches
B. $6\frac{7}{8}$ inches
C. 7 inches and $7\frac{1}{8}$ inches
D. $7\frac{1}{4}$ inches

242 Assessment Resource Book

10. Create a line plot to represent the data in the table.

Eraser Lengths (in.)			
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$1\frac{1}{4}$
$\frac{3}{4}$	$\frac{1}{4}$	1	$\frac{3}{4}$



Assessment Resource Book 243

12. A paint bucket contains 5 gallons of paint. After Amelia paints her dresser, there are 64 cups left in the paint. How many cups of paint does Amelia use to paint her dresser? Explain your answer.

16 cups; Sample answer: I multiplied the number of gallons by 16 to get the amount in cups. 5 gallons = 80 cups. I subtract the amount left from the 80 cups: 80 − 64 = 16.

244 Assessment Resource Book

UNIT 13 PLANNER

Geometry

PACING: 10 days

LESSON		MATH OBJECTIVE	LANGUAGE OBJECTIVE	SOCIAL AND EMOTIONAL LEARNING OBJECTIVE
Unit Opener IGNITE! Tetrominoes Students explore polygons made from congruent connecting squares.				
13-1	Understand the Coordinate Plane	Students identify and describe features of a coordinate grid. Students use a coordinate plane to determine the ordered pair associated with a point.	Students discuss how they can describe features of the coordinate plane using *related*, *find,* and *ordered pair*.	Students collaborate with peers and contribute to group effort to achieve a collective mathematical goal.
13-2	Plot Ordered Pairs on the Coordinate Plane	Students plot ordered pairs on a coordinate plane.	Students explain how they can plot ordered pairs on a coordinate plane using the verbs *draw* and *label*.	Students set a focused mathematical goal and make a plan for achieving that goal.
13-3	Represent Problems on a Coordinate Plane	Students plot points that represent real-world situations. Students interpret coordinate values of points in the context of the situation.	Students talk about plotting points on the coordinate plane when given real-world data using the verbs *draw* and *label.*	Students identify and use mathematical tools to organize work.
13-4	Classify Triangles by Properties	Students classify triangles into categories and subcategories based on their properties. Students organize the categories and subcategories into a hierarchy.	Students talk about classifying triangles using *the same*, *different,* and *share*.	Students identify and discuss the emotions experienced during math learning.
13-5	Properties of Quadrilaterals	Students name quadrilaterals based on their properties.	Students explain how to identify quadrilaterals based on their properties with *know* and *makes*.	Students practice behavioral flexibility while working with peers to complete a challenging mathematical task.
Math Probe Ordered Pairs Students plot points on a coordinate plane.				
13-6	Classify Quadrilaterals by Properties	Students classify quadrilaterals into categories and subcategories based on their properties. Students organize the categories and subcategories into a hierarchy.	Students explain how to classify quadrilaterals into categories and subcategories based on their properties using *use* and *share*.	Students identify the information that is needed or most useful in order to complete a mathematical task.
Unit Review **Fluency Practice**				
Unit Assessment **Performance Task**				

FOCUS QUESTION:
How can I use the coordinate plane to identify and classify 2-dimensional figures?

LESSON	KEY VOCABULARY: Math Terms		KEY VOCABULARY: Academic Terms	MATERIALS TO GATHER	RIGOR FOCUS	STANDARD
13-1	coordinate plane ordered pair origin	*x*-axis *x*-coordinate *y*-axis *y*-coordinate	correspond emphasize	• *Understanding the Coordinate Plane* Teaching Resource	Conceptual Understanding, Procedural Skill & Fluency	5.G.A.1
13-2	coordinate plane ordered pair origin	*x*-axis *x*-coordinate *y*-axis *y*-coordinate	correspond quality	• blank number cubes • *Coordinate Plane* Teaching Resource	Conceptual Understanding, Procedural Skill & Fluency	5.G.A.1
13-3	origin *x*-axis *x*-coordinate	*y*-axis *y*-coordinate	accurate interpret	• *Coordinate Plane* Teaching Resource	Procedural Skill & Fluency	5.G.A.2
13-4	category equilateral triangle hierarchy	isosceles triangle property scalene triangle subcategory	evaluate suggest	• plastic straws • *Properties of Triangles* Teaching Resource	Conceptual Understanding	5.G.B.3, 5.G.B.4
13-5	attribute parallelogram property quadrilateral	rectangle rhombus square trapezoid	establish quality	• *Classifying Quadrilaterals* Teaching Resource	Conceptual Understanding	5.G.B.4
13-6	hierarchy parallelogram quadrilateral rectangle	rhombus square trapezoid Venn diagram	accurate analyze	• *Venn Diagram* Teaching Resource	Procedural Skill & Fluency	5.G.B.3, 5.G.B.4

Unit Overview

Focus

Coordinate Plane and Polygons

Using an *ordered pair of numbers* called *coordinates* identifies the location of a point on the coordinate plane. The point of intersection of the two axes on a coordinate plane has coordinates (0, 0) and is called the *origin*. By convention, the horizontal axis is called the *x*-axis; the vertical axis, the *y*-axis. The first coordinate in an ordered pair, the *x*-coordinate, tells the point's distance from the origin along the *x*-axis. The second coordinate, the *y*-coordinate, tells the point's distance from the origin along the *y*-axis. Students graph ordered pairs, interpret coordinate values of points in the context of a situation, and draw a line to connect points. Students make predictions about data points that are not specifically graphed on a line.

Then students build on their earlier work of classifying two-dimensional shapes based on properties. Students now look for structure as they classify two-dimensional figures in a hierarchy.

A hierarchy classifies figures into categories according to properties. Most often, a hierarchy includes a diagram showing relationships among categories and subcategories—with the most general category at the top. Each subcategory is more specific than the one above it—and has all the properties of the category above, with at least one additional property. Students build hierarchies of two-dimensional triangles and quadrilaterals. Triangles are sorted and classified by the lengths of their sides (equilateral, isosceles, scalene) and the sizes of their angles (acute, right, obtuse). A special case is noted as an isosceles triangle is one with at least two sides that are the same length, which makes an equilateral triangle a special case, or subcategory of isosceles.

Quadrilaterals are placed into hierarchies based on side length, congruency, and angle type. These hierarchies reinforce an understanding as to why a given polygon might have multiple names.

Coherence

What Students Have Learned

- Students classified two-dimensional figures based on whether or not it has parallel lines or angles of a particular size. (Grade 4)
- Students identified right triangles. (Grade 4)
- Students used graphical displays to show data. (Grade 4)

What Students Are Learning

- Students understand hierarchies and how the attributes of a category also apply to all of the subcategories.
- Students classify triangles and quadrilaterals based on properties.
- Students graph points on the coordinate plane.

What Students Will Learn

- Students draw polygons in a coordinate plane and find the lengths of size given the coordinates of vertices. (Grade 6)
- Students extend the coordinate plane to include rational numbers, including negative numbers. (Grade 6)
- Students graph points in all four quadrants of the coordinate plane and find distances between them. (Grade 6)

Rigor

Conceptual Understanding

Students develop understanding of
- the coordinate plane, its parts, and how to plot points on it;
- the attributes used to classify two-dimensional figures in a hierarchy.

Procedural Skill & Fluency

Students build proficiency with
- plotting points on a coordinate plane;
- representing problems by graphing on a coordinate plane;
- classifying 2-dimensional figures in a hierarchy based on properties.

Application

Students apply their knowledge of
- representing real-world problems by graphing in the first quadrant of the coordinate plane;
- interpreting coordinate values of points in the context of a situation.

Application is not a targeted element of rigor for the standards in this unit.

Effective Teaching Practices

Pose Purposeful Questions

Throughout instruction, purposeful questions draw students into the task and enables them to focus on what is important. Challenge students by posing questions that require more than one-word or short-phrase answers. Instead, ask them to explain their thinking or justify their solution method. Often students expect the questions from teachers to guide them to the exact answers. However, purposeful questioning does not guide them to a correct answer but rather steers them towards better mathematical reasoning. Students should show confidence in their own mathematical abilities and want to advance their own reasoning.

In order to pose purposeful questions throughout a lesson, consider the learning goal of the lesson and what will best allow students to express their thinking and reasoning in reply.

Throughout this unit, questioning should revolve around getting students to use their inductive and deductive reasoning to apply that hierarchy to classify figures.

For any question posed, consider the student responses it might elicit. If there is a way for a student to provide a minimal low-level response to the question or if there is a way for a student to misinterpret the question, refine it with more precision of language. By vetting the questions ahead of time, students will be challenged to focus on their task and the process it takes to arrive at an answer.

Posing purposeful questions will elicit not only what students know, but also what they understand about 2-dimensional figures. While they know much about them from previous grades, a purposeful line of questioning will expand their knowledge, providing them with a framework upon which to build.

Math Practices and Processes

Look for and Make Use of Structure

By seeking patterns and structure while learning, students will become more proficient in their abilities. Instead of memorizing several individual facts, students learn patterns and structure to minimize the number of facts that must be remembered.

Structure allows students to view their mathematical studies from a higher level. They can see the big picture rather than just the details. The structure gives them tools they can use to summarize what they have learned.

Making use of the structure empowers students to see an overview and understand how mathematical knowledge builds upon itself. Then, when learning a new topic or skill, they do not feel like they are starting from scratch. They recognize what they already know and how they are adding to that knowledge.

Without the structure, every mathematical task is a new one.

A hierarchy of figures, by definition, provides a structure upon which students will build their understanding of two-dimensional figures. As students progress through mathematics, they begin with recognizing two-dimensional shapes. Then they learn their attributes and compare them to one another. As they age, students learn a greater number of attributes and classifications for the same shapes that they have been working with throughout the elementary grades. In this module, students use this knowledge of structure to classify figures in hierarchies.

The structure of the hierarchy guides students to identify the number of ways a figure could be classified. For example, going up the hierarchy of quadrilaterals, a square is a rhombus, rectangle, parallelogram, and quadrilateral.

Social and Emotional Learning

What Skills Will We Develop?

- **Relationship Skills – Teamwork** (Lesson 13-1): When students work effectively as a team, they establish a stronger learning community.
- **Responsible Decision-Making – Reflect:** (Lesson 13-2): When students reflect, they can make connections between effort and achievement.
- **Self-Management – Organizational Skills** (Lesson 13-3): Organizing information and work can help students work through challenging mathematical tasks.
- **Self-Awareness – Accurate Self-Perception** (Lesson 13-4): Having accurate self-perception allows students to determine areas of strength as well as areas in which they need to focus and practice.
- **Social Awareness – Develop Perspective** (Lesson 13-5): Developing perspective can help students understand different ways of thinking.
- **Responsible Decision-Making – Evaluate** (Lesson 13-6): When students evaluate their own logic and reasoning, they can develop understanding that helps them make informed decisions.

Unit Overview

Language of Math

Classification Words

Students will use these key terms in this unit:

- **Coordinate plane*** (Lesson 13-1): The coordinate plane is formed by a horizontal number line and a vertical number line meeting at a right angle.
- **Equilateral, scalene, and isosceles triangle*** (Lesson 13-4): Students were introduced to these terms in the context of classifying triangles in Grade 4. Ask students to think about other attributes they can use to categorize triangles.
- **Hierarchy of figures*** (Lesson 13-4): While students have used characteristics to classify figures in previous grades, this term organizes their method of classification.
- **Ordered pair*** (Lesson 13-1): An ordered pair indicates the *x*-coordinate of the point, then the *y*-coordinate of the point, in that order.
- **Origin*** (Lesson 13-1): The point on the coordinate plane where the *x*-axis and *y*-axis meet, (0, 0), is called the origin.
- **Subcategory*** (Lesson 13-4): It describes a group that has common properties with a larger category.
- **Venn diagram*** (Lesson 13-6): This is a diagram that shows the relationships betwee**n items.**
- ***x*-axis*** (Lesson 13-1): This is the horizontal axis on the coordinate plane.
- ***y*-axis*** (Lesson 13-1): This is the vertical axis on the coordinate plane.

*This is a new term.

Math Language Development

A Focus on Speaking

When speaking about mathematics, students need to learn to be precise with their choice of words. In mathematics, specific terms relay images to the listener, and with a lack of precision, the wrong image can be depicted. Using precise language also simplifies the discussion because it eliminates the need to use extraneous or excessive words.

When communicating with teachers and peers about mathematical tasks, students should practice using good choices of vocabulary to be sure that their questions and/or their solutions are well-communicated. The precision of spoken language will eliminate much unnecessary confusion and focus students on the exact task at hand.

In this unit, students can use accurate and precise language to describe two-dimensional figures. They can use their new vocabulary to classify polygons with more specific attributes than just the number of sides. They can speak about polygons as:

- **Regular or irregular** Regular polygons have all sides congruent and all angles congruent.
- **Triangles** Triangles can be classified based on the number of sides or the types of angles. They can be further described by using both classifications, sides and angles.
- **Quadrilaterals** Two broad categories in the hierarchy of quadrilaterals include trapezoids and parallelograms. Students will use familiar descriptions such as rectangle, rhombus, or square, to further classify the parallelograms.

Any description that students give for a figure should be accurate enough that a picture of that figure could be drawn by the listener.

English Language Learner

In this unit, students are provided with a number of scaffolds to support their comprehension of the language used to present and explain strategies related to geometry. Because many of the words (*location*, *seconds, minutes*) and phrases (*how to get from* ______ *to* ______, *at least, properties, shared properties*) used in this unit could prove challenging to ELs, they are supported in understanding and using them so that the instruction is more accessible.

- Lesson 13-1 – *location*
- Lesson 13-2 – *how to get from* ______ *to* ______
- Lesson 13-3 – *seconds, minutes*
- Lesson 13-4 – *at least*
- Lesson 13-5 – *properties*
- Lesson 13-6 – *shared properties*

Unit Routines

Number Routines

Build Fluency The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the thinking habits of mind that are important for proficient doers of math.

Find the Missing Values

Purpose: Build identification of patterns and efficiency with solving equations while examining a list of related equations.

Overview: Students analyze a series of equations to look for patterns that they can use to determine the missing values in the equations. As students share their analyses and solutions, the teacher can reveal the missing values.

Greater Than or Less Than

Purpose: Build proficiency with number and place value sense; estimating and comparing skills.

Overview: Students use mental math to estimate or evaluate the value of given expressions and then compare the value of the expressions to a target benchmark number. Students share their solutions and thinking.

Would You Rather?

Purpose: Build flexibility with number sense and mental math operations; enhance decision-making.

Overview: Students choose between two options, both of which require mental computation. Students explain their choice and their rationale for their choice.

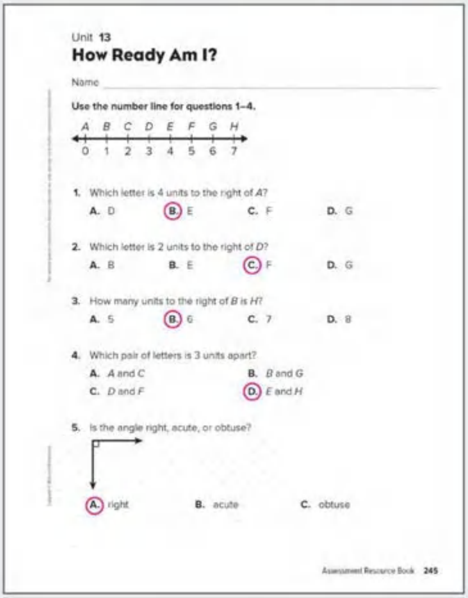
Sense-Making Routines

- **Notice & Wonder: What do you notice? What do you wonder?** (Lessons 13-1, 13-2, 13-6) In Lesson 13-1, students observe a coordinate grid in a real-world context. In Lesson 13-2, students are presented with a coordinate grid that displays a two-variable relationship in a real-world context. In Lesson 13-6, students view the hierarchy of quadrilaterals to help them understand the relationship of the types of quadrilaterals.
- **Notice & Wonder: What questions can you ask?** (Lesson 13-3) Students connect to real-world situations as they see the math in the story, quantities, and the relationships.
- **Notice & Wonder: What could the question be?** (Lessons 13-4, 13-5) Students explore the concept of defining attributes by looking at examples and non-examples of triangles in Lesson 13-4 and quadrilaterals in Lesson 13-5.

Math Language Routines

The Mathematical Language Routines used in this unit give teachers a structured, yet adaptable format for amplifying and developing students' social and academic language. For more information on the Math Language Routines, see the Appendix.

- Lesson 13-1 – Students participate in MLR8: Discussion Supports so that they can have a rich and inclusive discussion about coordinate planes.
- Lesson 13-2– Students participate in MLR1: Stronger and Clearer Each Time so that they can improve communication output.
- Lesson 13-3 – Students participate in MLR2: Collect and Display so that students' oral expressions can be captured into a stable, collective reference as they discuss representing problems on a coordinate plane.
- Lesson 13-4 – Students participate in MLR3: Critique, Correct, and Clarify so that they have an opportunity to analyze, reflect on, and develop a piece of mathematical writing about classifying triangles.
- Lesson 13-5 – Students participate in MLR1: Stronger and Clearer Each Time so that they have a structured and interactive opportunity to revise and refine their ideas while showing properties of quadrilaterals.
- Lesson 13-6 – Students participate in MLR5: Co-Craft Questions and Problems so that they can produce the language of mathematical questions related to classifying quadrilaterals by properties.


Unit 13

How Ready Am I?

Name

Use the number line for questions 1–4.

A B C D E F G H

0 1 2 3 4 5 6 7

1. Which letter is 4 units to the right of *A*?
A. D B. E C. F D. G

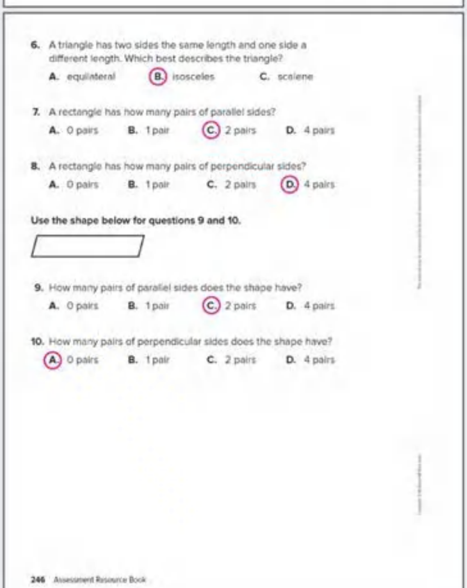
2. Which letter is 2 units to the right of *D*?
A. B B. E C. F D. G

3. How many units to the right of *B* is *H*?
A. 5 B. 6 C. 7 D. 8

4. Which pair of letters is 3 units apart?
A. *A* and *C* B. *B* and *G*
C. *D* and *F* D. *E* and *H*

5. Is the angle right, acute, or obtuse?
A. right B. acute C. obtuse

Assessment Resource Book 245


6. A triangle has two sides the same length and one side a different length. Which best describes the triangle?
A. equilateral B. isosceles C. scalene

7. A rectangle has how many pairs of parallel sides?
A. 0 pairs B. 1 pair C. 2 pairs D. 4 pairs

8. A rectangle has how many pairs of perpendicular sides?
A. 0 pairs B. 1 pair C. 2 pairs D. 4 pairs

Use the shape below for questions 9 and 10.

9. How many pairs of parallel sides does the shape have?
A. 0 pairs B. 1 pair C. 2 pairs D. 4 pairs

10. How many pairs of perpendicular sides does the shape have?
A. 0 pairs B. 1 pair C. 2 pairs D. 4 pairs

246 Assessment Resource Book

Administer the Readiness Diagnostic to determine your students' readiness for this unit.

Targeted Intervention

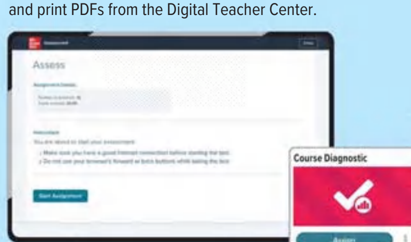
Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	1	Interpret a number line	Graph Whole Numbers	2.MD.B.6
2	1	Interpret a number line	Graph Whole Numbers	2.MD.B.6
3	1	Interpret a number line	Graph Whole Numbers	2.MD.B.6
4	1	Interpret a number line	Graph Whole Numbers	2.MD.B.6
5	1	Classify an angle	Angle Types	4.MD.C.5
6	2	Identify triangles by side lengths	Recognize Triangles by Sides	4.G.A.2
7	1	Identify parallel lines in a shape	Use Lines to Classify Shapes	4.G.A.2
8	1	Identify perpendicular lines in a shape	Use Lines to Classify Shapes	4.G.A.2
9	1	Identify parallel lines in a shape	Use Lines to Classify Shapes	4.G.A.2
10	1	Identify perpendicular lines in a shape	Use Lines to Classify Shapes	4.G.A.2

GO ONLINE

Assign the digital Readiness Diagnostic to students or download and print PDFs from the Digital Teacher Center.



Focus Question

Introduce the Focus Question: *How can I use the coordinate plane and identify and classify 2-dimensional figures?*

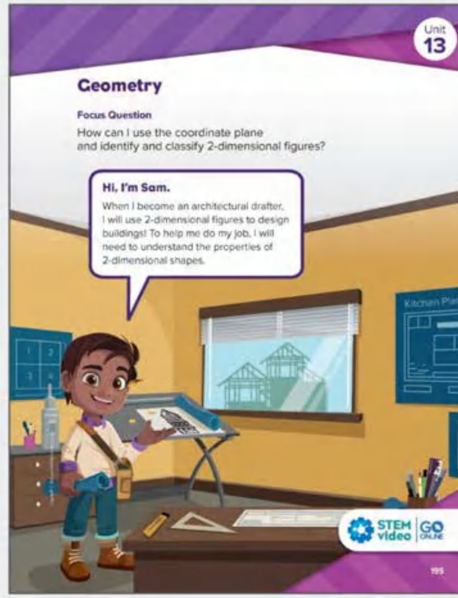
Ask students to think about what they know about 2-dimensional figures and graphing?

- What have you graphed before?
- What do you already know about 2-dimensional figures?

Remind students that at the end of the unit, they will reflect back on what they learned.

Family Letter

Each letter presents an overview of the math in the unit and home activities to support student learning.



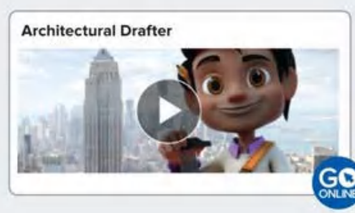
STEM in Action

Videos

Students can watch the two STEM videos.

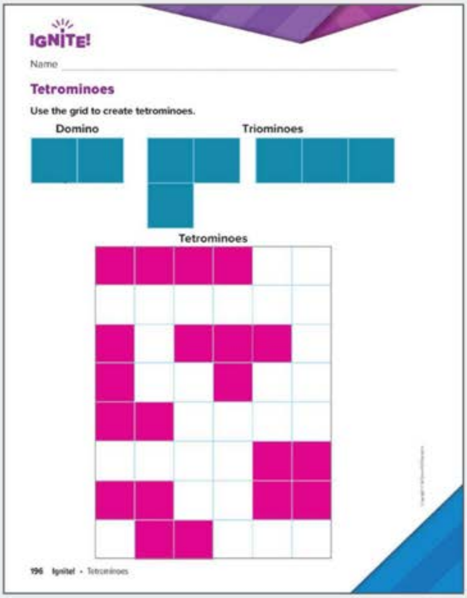
STEM Career: Architectural Drafter Students listen as Sam describes his aspirations to be an architect.

Sam Designs Windows Students watch to see how Sam uses two-dimensional figures to sketch a window.



STEM Project Card

Students can complete the STEM Project Card during their workstation time.



Ignite!

Tetrominoes

Students explore the polygons that are made from congruent connecting squares as an introduction to properties of 2-dimensional shapes.

Materials: square tiles or connecting cubes; color pencils

1. Describe *domino*. Explain that the word *triomino* comes from *tri-* + *domino*.
   - What do you suppose is a triomino?
2. Have students work in pairs to explore making more triominoes.
   - Try to draw or use tiles to make more triominoes. Be sure that each square shares at least one full side with another square.
   - Are you able to find triominoes that are different from those on the student page?
3. Explain that if a figure can be rotated (turned) or flipped to look like another figure, we consider two figures to be the same shape. So, it is possible to make only two triominoes.
4. Have students explore making *tetrominoes*.
   - A *tetromino* is made of four squares of the same size, where each square shares at least one full side with another square. Work to use tiles to make all possible tetrominoes. After you find a tetromino, draw it on the grid paper. You may want to shade each different tetromino in a different color.
5. Have students share their results to conclude that there are five possible tetrominoes. Discuss how rotating or flipping a figure demonstrates how two figures that may look different are actually the same shape.
6. Use questions such as these to promote a discussion on how students can classify the tetrominoes.
   - How are the shapes similar? How are they different?
   - Are there any familiar shapes?
   - Which shapes have four sides? Which have six sides? Which have eight sides?

Unit Resources At-A-Glance

Workstations

Reveal Math offers rich and varied resources that teachers can use to differentiate and enrich students' instructional experiences with the unit content. The table presents an overview of the resources available for the unit with recommendations for when to use.

Station	Activity	Description	Use After Lesson
Game Station	Game Station	Students build understanding of graphing in the coordinate plane and classification of triangles and quadrilaterals.	
		• **Coordinate Plane Race**	13-1
		• **Coordinate Plane Task Cards**	13-2
		• **Coordinate Plane Representation Race**	13-3
		• **Classifying Triangles Four in a Row**	13-4
		• **2-Dimensional Figures Sort**	13-5
		• **Hierarchy Sort**	13-6
Digital Station	Digital Game	**Submarine Plunge** Students practice dividing multi-digit numbers.	13-1
Application Station	**Have students complete at least one of the Use It! activities for this unit.**		
	STEM Project Card	**Drafting Tools for Accuracy** Students create their own drafting triangles and create a model of a covered bridge.	13-5
	Connection Card	**How Was That Created?** Students use Venn diagrams to research the properties of triangles and quadrilaterals in artwork.	13-6
	Real World Card	**Is This for Real?** Students research and use the TAARP method to determine website credibility.	13-1

Additional Resources

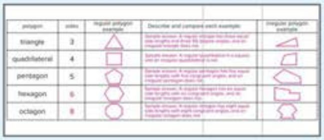
Use the resources below to provide additional support for this unit.

Vocabulary

Use the vocabulary cards to help students learn the vocabulary in this unit. Encourage students to write their own definitions of the key terms on the front side of the card.

Foldables

Use the unit foldables with Lessons 13-5 and 13-6.



Spiral Review

Students can complete the Spiral Review at any point during the unit as either a paper-and-pencil or digital activity.

Lesson	Standard
13-1	5,NF.A.1
13-2	5.NF.A.2
13-3	5.NF.B.3
13-4	5.NF.B.7
13-5	5.NF.B.6
13-6	5.NBT.B.5

LESSON **13-1**

Understand the Coordinate Plane

Learning Targets

- I can identify and describe features of a coordinate plane.
- I can use a coordinate plane to determine the ordered pair associated with a given point.

Standards ◆ Major ▲ Supporting ● Additional

Content

○ **5.G.A.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

Math Practices and Processes

MPP Construct viable arguments and critique the reasoning of others.

Vocabulary

Math Terms	Academic Terms
coordinate plane	correspond
ordered pair	emphasize
origin	
x-axis	
x-coordinate	
y-axis	
y-coordinate	

Material

The materials may be for any part of the lesson.

- *Understanding the Coordinate Plane* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students identify and describe features of the coordinate plane. • Students use the coordinate plane to determine the ordered pair associated with a point.	• Students discuss how to describe the coordinate plane using *related*, *find,* and *ordered pair*. • To support cultivating conversation, ELs participate in MLR8: Discussion Supports.	• Students collaborate with peers and contribute to group effort to achieve a collective mathematical goal.

Coherence

Previous	Now	Next
	• Students understand the coordinate plane and find the ordered pair for a given point.	• Students plot points on the coordinate plane (Unit 13). • Students understand rational and negative numbers as points on the number line and extend number line diagrams and coordinate axes to them (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of algebra to write ordered pairs that represent points on the coordinate plane.	• Students build proficiency with using the coordinate plane to determine ordered pairs that represent points on the coordinate plane.	• Students apply their understanding of ordered pairs to solve problems. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Greater Than or Less Than 5–7 min

Build Fluency Students build number sense as they estimate differences for each fraction subtraction expression, then compare each expression to a target fraction

These prompts encourage students to talk about their reasoning:

- What was your strategy for estimating the differences?
- How did you evaluate each expression?
- How did you know if your estimate was greater than or less than the target fraction?
- Why is it useful to compare values to $\frac{1}{2}$?
- How do you know that your estimate is reasonable?

Launch 5–7 min

Sense-Making Routine

Purpose Students study a real-world context that could be represented using the coordinate plane.

Notice & Wonder™

- What do you notice?
- What do you wonder?

Teaching Tip You may want to have students discuss if they have ever used an app or game that looked like the image, and if so, how they could move from one spot to another.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of the coordinate plane and are based on possible comments and questions that students may make during the share out.

- How could you describe the location of each coin using rows and columns?
- How are the locations of the coins using rows and columns similar? How are they different?

Math is... Mindset

- How can working as a team help you achieve your goal?

SEL Relationship Skills: Teamwork

Establish a positive classroom culture by providing students opportunities to work together to complete collective tasks. As students notice and wonder, encourage them to work together and build off the ideas of their peers. Invite students to participate in different ways so that each student can actively contribute to the team effort and establish a stronger learning community.

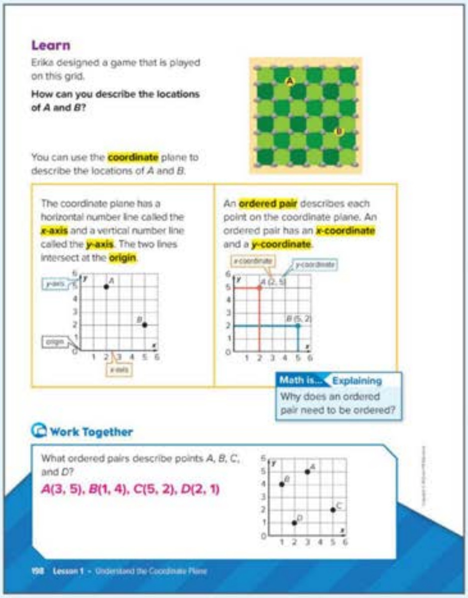
Transition to Explore & Develop

Guide students to think about what kind of representations they could use to describe the location of points *A* and *B*. As they make proposals, have them discuss the benefits and possible drawbacks of the representation.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about a way we can represent these coins.

Explore & Develop 20 min



Learn

Erika designed a game that is played on this grid.

How can you describe the locations of A and B?

You can use the **coordinate** plane to describe the locations of A and B.

The coordinate plane has a horizontal number line called the **x-axis** and a vertical number line called the **y-axis**. The two lines intersect at the **origin**.

An **ordered pair** describes each point on the coordinate plane. An ordered pair has an **x-coordinate** and a **y-coordinate**.

Math is... Explaining

Why does an ordered pair need to be ordered?

Work Together

What ordered pairs describe points A, B, C, and D?

A(3, 5), B(1, 4), C(5, 2), D(2, 1)

198 Lesson 1 • Understand the Coordinate Plane

1 Pose the Problem

MLR Discussion Supports

As students discuss the two questions, have them focus on the following: what they're trying to represent and what they already know. Restate statements they make as a question to seek clarification and provide vocabulary or grammar prompts for students who need more guidance.

ETP Pose Purposeful Questions

- What are you trying to represent?
- What information do you know about *A* and *B*?

2 Develop the Math

Choose the option that best meets your instructional goals.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How could you describe the coordinate plane to a friend?
- How can you determine the ordered pair that describes a point on the coordinate plane?

Key Takeaways

- The coordinate plane uses two perpendicular number lines. Their point of intersection is called the origin.
- The horizontal line is the *x*-axis, and the vertical line is the *y*-axis.
- The location of a point in the coordinate plane is indicated by two coordinates: the *x*-coordinate, which is its position relative to the *x*-axis, and the *y*-coordinate, which is its position relative to the *y*-axis. These two coordinates are called an ordered pair. The ordered pair for the origin is (0, 0).
- The *x*-coordinate of the point is always the first number in the ordered pair, and the *y*-coordinate is always the second number.

Work Together

Students work to name the ordered pairs of 4 points on the coordinate plane.

Common Error: Students may mix up the *x*-axis and the *y*-axis as they determine the ordered pairs for each point. Emphasize that the *x*-axis is the horizontal number line, and it will be helpful for them to find the *x*-coordinate first as that will be the first coordinate they name in the ordered pair. The *y*-axis is the vertical number line, and they should find the *y*-coordinate second.

LOM Language of Math

Students may be aware of the word *origin* from other contexts. An origin is the place or point where something begins. In effect, the coordinate plane begins at the origin. In the next lesson, students will see the origin is the point used to begin plotting any other point.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore describing locations on the coordinate plane.

Materials: *Understanding the Coordinate Plane* Teaching Resource

Directions: Provide copies of the *Understanding the Coordinate Plane* Teaching Resource. Explain to students that their task is to describe the location of each object.

ETP Support Productive Struggle

- What information do you think you need to accurately describe the location of an object?
- What do you think the labels along the edges represent?
- Can you use those labels to help you describe a location?

Activity Debrief: Have students share their descriptions of each object. Focus attention on different ways of describing, for example, some students may describe using the y-axis first, rather than the x-axis. Facilitate a discussion to ensure students understand what a coordinate plane is and how it can be used, as well as that ordered pairs can be used to describe locations on the coordinate plane.

Math is... Explaining

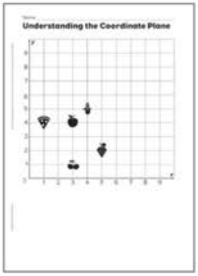
- Why does an ordered pair need to be ordered?

Students build a logical progression of statements to explore the truth of a statement.

Have students revisit the Pose the Problem question and discuss answers.

- How can you represent the locations of A and B?

A PDF of the Teaching Resource is available in the Digital Teacher Center.



Guided Exploration

Students understand the coordinate plane and find the ordered pair for a given point.

ETP Use and Connect Mathematical Representations

- How are the locations of the points in the coordinate plane related to their locations in the grid on the game?
- **Think About It:** Explain why (0, 0) is the ordered pair that represents the origin.

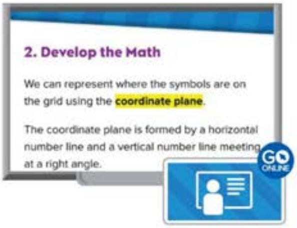
Have students determine the ordered pair that represents point B. Ask:

- How can you find the x-coordinate?
- How can you find the y-coordinate?
- How can you write the ordered pair?

Math is... Explaining

- Why does an ordered pair need to be ordered?

Students build a logical progression of statements to explore the truth of a statement.



EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *location*. Say *Let's find the location of classroom objects*. Point to the bookshelf. Say *The location of the [class dictionary] is the bookshelf*. Repeat with another object/location. Then repeat once more with another object/location, asking *Is the location of my [stapler] my desk or the closet?* Accept pointing.

Developing/Expanding Ensure understanding of *location*. Say *Let's find the location of classroom objects*. Point to the bookshelf. Say *The location of the [class dictionary] is the bookshelf*. Repeat with another object/location. Then repeat once more with another object/ location, asking *What is the location of my stapler?*

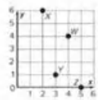
Bridging/Reaching Guide students to the top of Learn page. Tell them to read the word problem, focusing on *location*. Ask them to use it in a sentence that tells the class where something is. For example: *The location of my backpack is the closet*. Then ask students to brainstorm similar words they may already know (*position, place, spot, placement,* etc.). Allow students to use a dictionary or thesaurus if desired.

Practice & Reflect 10 min

On My Own

Name ____________________

Use the coordinate plane to answer exercises 1–7.



1. What ordered pair describes point *W*?
   (4, 4)

2. What ordered pair describes point *X*?
   (2, 6)

3. What ordered pair describes point *Y*?
   (3, 1)

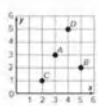
4. What ordered pair describes point *Z*?
   (5, 0)

5. What ordered pair describes the origin?
   (0, 0)

6. How did you find the *x*-coordinate for each ordered pair?
   Sample answer: I drew a line from the point to where it intersects with the *x*-axis and found the *x*-coordinate.

7. How did you find the *y*-coordinate for each ordered pair?
   Sample answer: I drew a line from the point to where it intersects with the *y*-axis and found the *y*-coordinate.

Charlie gave his friends these locations for a scavenger hunt. What are the ordered pairs that describe the locations on the coordinate plane?



8. Point *A* **(3, 3)**

9. Point *B* **(5, 2)**

10. Point *C* **(2, 1)**

11. **Error Analysis** Charlie tells his friends that point *D* is at (5, 4). His friends go to the wrong spot. Explain why.
    Sample answer: He put the coordinates in the wrong order. The ordered pair for point *D* should be (4, 5).

12. **Extend Your Thinking** A new point, *E*, is two units from point *A*. Give two possible ordered pairs for *E*.
    Sample answer: (3, 5) or (1, 3).

Reflect

How can you determine the ordered pair that describes a point on the coordinate plane?

Answers may vary.

Math is... Mindset

How did working as a team help you achieve your goal?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 4 Students may be confused by the fact that point *Z* is on the *x*-axis. Remind students that, as with the ordered pair for the origin (0, 0), 0 is an acceptable coordinate. However, make sure students understand that 0 is the *y*-coordinate, not the *x*-coordinate.

Item Analysis

Item	DOK	Rigor
1–5	1	Procedural Skill & Fluency
6–7	2	Conceptual Understanding
8–10	2	Application
11–12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you determine the ordered pair that represents a point on the coordinate plane?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did working as a team help you achieve your goal?

Students reflect on how they developed stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can identify and describe features of a coordinate plane.
- I can use a coordinate plane to determine the ordered pair associated with a given point.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Identify coordinates of points	5.G.A.1
2	1	Identify coordinates of points	5.G.A.1

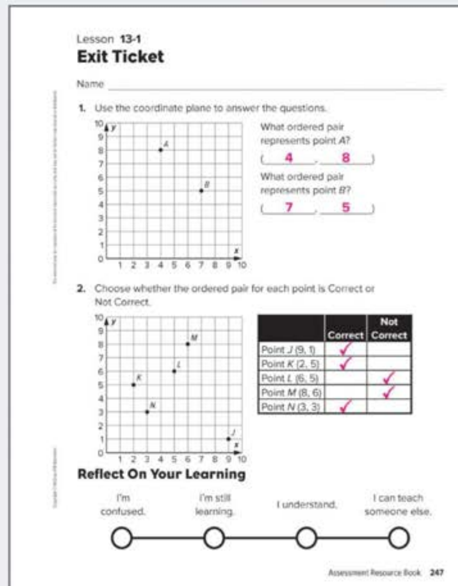
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
2 of 2	Additional Practice or any of the B or E activities
1 of 2	*Take Another Look* or any of the B activities
0 of 2	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 13-1

Exit Ticket

Name

1. Use the coordinate plane to answer the questions.

What ordered pair represents point A? (4, 8)

What ordered pair represents point B? (7, 5)

2. Choose whether the ordered pair for each point is Correct or Not Correct.

	Correct	Not Correct
Point J (9, 1)	✓	
Point K (2, 5)	✓	
Point L (6, 5)		✓
Point M (8, 6)		✓
Point N (3, 3)	✓	

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 247

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Find the Point

Work with students in small groups. Students each use a 10 by 10 section of the coordinate plane to mark a point. Students show the group, and the other students write the ordered pair for the point. Students compare results and use what they know about the coordinate plane to justify their answers. Make sure students recognize that the first number represents the horizontal position and the second number represents the vertical position.

GO ONLINE

Take Another Look Lesson

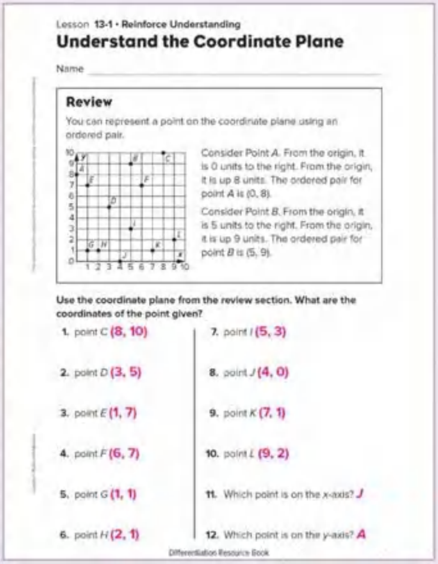
Assign the interactive lesson to reinforce targeted skills.

- Points on the Coordinate Plane

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 149


Lesson 13-1 • Reinforce Understanding

Understand the Coordinate Plane

Name

Review

You can represent a point on the coordinate plane using an ordered pair.

Consider Point *A*. From the origin, it is 0 units to the right. From the origin, it is up 8 units. The ordered pair for point *A* is (0, 8).

Consider Point *B*. From the origin, it is 5 units to the right. From the origin, it is up 9 units. The ordered pair for point *B* is (5, 9).

Use the coordinate plane from the review section. What are the coordinates of the point given?

1. point *C* (8, 10)
2. point *D* (3, 5)
3. point *E* (1, 7)
4. point *F* (6, 7)
5. point *G* (1, 1)
6. point *H* (2, 1)
7. point *I* (5, 3)
8. point *J* (4, 0)
9. point *K* (7, 1)
10. point *L* (9, 2)
11. Which point is on the *x*-axis? *J*
12. Which point is on the *y*-axis? *A*

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Coordinate Plane Race

Student practice identifying the ordered pairs represented by points on the coordinate plane.

GO ONLINE

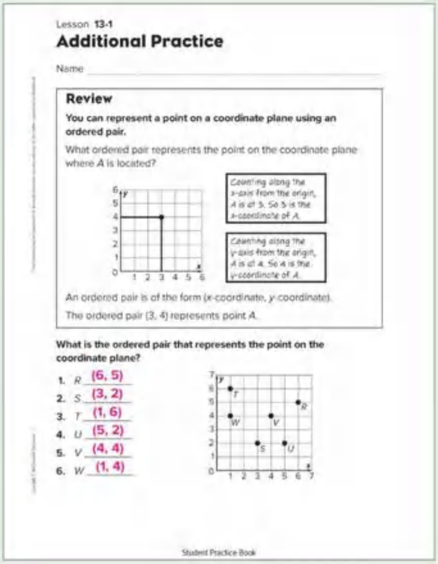
Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 149–150


Lesson 13-1

Additional Practice

Name

Review

You can represent a point on a coordinate plane using an ordered pair.

What ordered pair represents the point on the coordinate plane where *A* is located?

Counting along the *x*-axis from the origin, *A* is at 3. So 3 is the *x*-coordinate of *A*.

Counting along the *y*-axis from the origin, *A* is at 4. So 4 is the *y*-coordinate of *A*.

An ordered pair is of the form (*x*-coordinate, *y*-coordinate).

The ordered pair (3, 4) represents point *A*.

What is the ordered pair that represents the point on the coordinate plane?

1. *R* (6, 5)
2. *S* (3, 2)
3. *T* (1, 6)
4. *U* (5, 2)
5. *V* (4, 4)
6. *W* (1, 4)

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division with multi-digit numbers.

Assign

Use It! Application Station

How Was That Created? Students use Venn diagrams to research the properties of triangles and quadrilaterals in artwork. *The content of this card has concepts covered later in Lesson 13-6. You may want to assign this card to students ready to explore content covered later in this unit.*

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

GO ONLINE

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

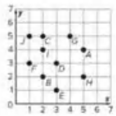
Assign

INDEPENDENT WORK

Student Practice Book, pp. 149–150

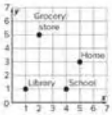
What is the ordered pair that represents the point on the coordinate plane?

7. A (5, 4)
8. B (2, 2)
9. C (2, 5)
10. D (3, 3)
11. E (3, 1)
12. F (1, 3)
13. G (4, 5)
14. H (5, 2)
15. I (2, 4)
16. J (1, 5)



Conrad uses a coordinate plane to represent locations around his town. What is the ordered pair that represents each location?

17. Home (5, 3)
18. School (4, 1)
19. Grocery Store (2, 5)
20. Library (1, 1)



Math @ Home Activity Create a coordinate plane that includes labels for the x- and y-axes. Give your child a marker and have them mark several points on the plane. Then have them give each point a different label. Work with your child to identify the ordered pair that represents each point on the coordinate plane. Ask your child to explain how they determined the ordered pairs.

Student Practice Book

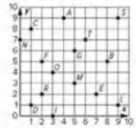
Differentiation Resource Book, p. 150

Lesson 13-1 • Extend Thinking

Understand the Coordinate Plane

Name ____________________

What point satisfies the condition given?



1. Which point has the same y-coordinate as point F? B
2. Which point has the same x-coordinate as point F? R
3. Which point has the same y-coordinate as point S? A
4. Which point is located on the x-axis? I
5. Which point is located on the y-axis. N
6. Which point has the same x-coordinate as point M? G
7. For which point is the x-coordinate 4? A
8. Which point can be found by counting 3 units up from the x-axis? M
9. Which point has the same x-coordinate as the point (7, 6)? E
10. What two words do the points spell? brain game

Differentiation Resource Book

LESSON **13-2**

Plot Ordered Pairs on the Coordinate Plane

Learning Target

- I can plot ordered pairs on a coordinate plane.

Standards ◇ Major △ Supporting ○ Additional

Content

○ **5.G.A.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate)

Math Practices and Processes

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	Academic Terms
coordinate plane	correspond
ordered pair	quality
origin	
x-axis	
x-coordinate	
y-axis	
y-coordinate	

Materials

The materials may be for any part of the lesson.

- blank number cubes
- *Coordinate Plane* Teaching Resource

Focus

Content Objective	Language Objectives	SEL Objective
• Students plot ordered pairs on a coordinate plane.	• Students explain how they can plot ordered pairs using the verbs *draw* and *label*. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students set a focused mathematical goal and make a plan for achieving that goal.

Coherence

Previous	Now	Next
• Students understood the coordinate plane and found the ordered pair for a given point (Unit 13).	• Students plot points on the coordinate plane given ordered pairs.	• Students plot data points from real-world situations on the coordinate plane and use it to interpret the data (Unit 13). • Students understand rational and negative numbers as points on the number line and extend number line diagrams and coordinate axes to them (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students understand that any point within the coordinate plane can be defined using a pair of numbers which indicate how far to travel from the origin.	• Students plot points on the coordinate plane by counting units from the *x* and *y*-axis.	• Students use points on the coordinate plane to represent real world situations. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Greater Than or Less Than 5–7 min

Build Fluency Students build number sense as they estimate differences for each fraction subtraction expression and compare each expression to a target fraction

These prompts encourage students to talk about their reasoning:

- What was your strategy for estimating the differences?
- How did you know if your estimate was greater than or less than the target fraction?
- How did you evaluate each expression?
- How do you know that your estimate is reasonable?

Purpose Students explore the relationships the coordinate plane can communicate.

Notice & Wonder

- What do you notice?
- What do you wonder?

Teaching Tip You may want to have students discuss the qualities of each animal they see in the plane with a partner before discussing what they notice and wonder about the plane as a class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of plotting points on the coordinate plane given ordered pairs and are based on possible comments and questions that students may make during the share out.

- How do you know what is represented on each axis?
- What does it mean if an animal is to the right on the "furry" axis and low on the "big" axis?
- What does it mean if an animal is to the left on the "furry" axis and high on the "big" axis?
- Where would you put a dog on this coordinate plane? Why?

Math is... Mindset

- How can being flexible in your thinking help you make good decisions?

SEL Responsible Decision-Making: Reflect

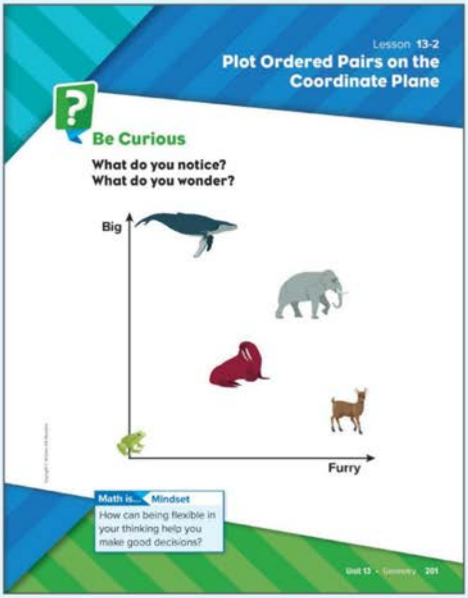
Before students begin the Notice & Wonder routine, invite them to share or write down a plan for how they will work toward answering the questions. Encourage students to reflect on how their planning and solving relate to plotting points on the coordinate plane.

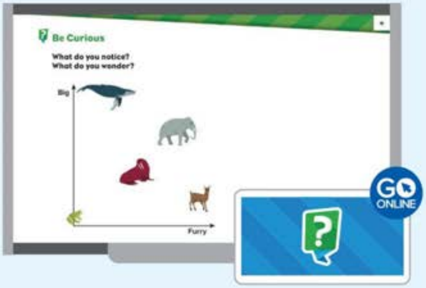
Transition to Explore & Develop

Ask students to think about how they would plot points on the coordinate plane given ordered pairs.

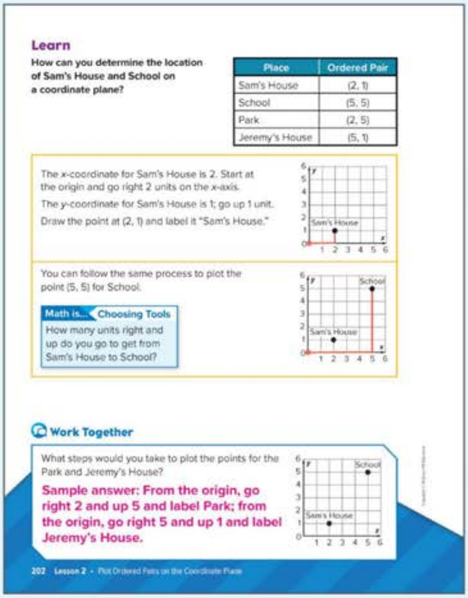
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can plot points on the coordinate plane when given an ordered pair.





Explore & Develop 20 min



Learn

How can you determine the location of Sam's House and School on a coordinate plane?

Place	Ordered Pair
Sam's House	(2, 1)
School	(5, 5)
Park	(2, 5)
Jeremy's House	(5, 1)

The x-coordinate for Sam's House is 2. Start at the origin and go right 2 units on the x-axis.

The y-coordinate for Sam's House is 1; go up 1 unit.

Draw the point at (2, 1) and label it "Sam's House."

You can follow the same process to plot the point (5, 5) for School.

Math is... Choosing Tools

How many units right and up do you go to get from Sam's House to School?

Work Together

What steps would you take to plot the points for the Park and Jeremy's House?

Sample answer: From the origin, go right 2 and up 5 and label Park; from the origin, go right 5 and up 1 and label Jeremy's House.

202 Lesson 2 • Plot Ordered Pairs on the Coordinate Plane

1 Pose the Problem

ETP Pose Purposeful Questions

- How are the locations of each place represented in the table? What does each number mean?
- What tools do you think you can use to help you solve the problem?
- Why would they each help you understand the problem?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about how to solve the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you plot a point on the coordinate plane when given an ordered pair?

Key Takeaway

- The coordinates of a point are used to plot the point on a coordinate plane.

Work Together

Students work together to plot the remaining two points in the table based on the ordered pairs.

Common Error: Students may mix up the two pairs or label each point incorrectly. Remind students that it is important to be accurate. Encourage students to check the location of each point after they have plotted it to make sure it matches the ordered pair in the table, paying particular attention to the x-coordinate (horizontal) vs. the y-coordinate (vertical).

LOM Language of Math

Students have likely heard the word *plot* in other contexts, such as when the discuss the plot of a movie or book. Explain that in mathematics, *plot* is a verb that means to mark the location of something on a graph.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore using ordered pairs to plot points on the coordinate plane.

Materials: *Coordinate Plane* Teaching Resource

Directions: Provide copies of the *Coordinate Plane* Teaching Resource. Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- What does the ordered pair tell you?
- Where should you start when plotting points on a coordinate plane?
- How can you determine how many units right to go on the *x*-axis to plot Sam's House?
- How can you determine how many units up to go on the *y*-axis to plot Sam's House?

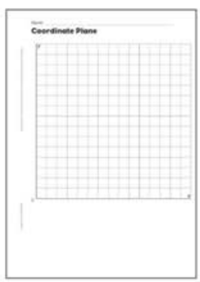
Math is... Choosing Tools

- How many units right and up do you go to get from Sam's House to school?

Students use the coordinate plane as a tool to help them solve a problem.

Activity Debrief: Have students share their solutions. Encourage students to defend their solutions using mathematically precise language.

A PDF of the Teaching Resource is available in the Digital Teacher Center.



Guided Exploration

Students plot points on the coordinate plane when given ordered pairs.

ETP Use and Connect Mathematical Representations

- Why should you start at the origin?
- How do you know what the *x*-coordinate for Sam's house is?
- How do you know what the *y*-coordinate for Sam's house is?
- **Think About It:** How would the location on the coordinate plane be different if the ordered pair for Sam's house was (1, 2)? Why?

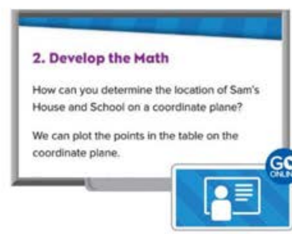
Have the students plot and label the point (5, 5). Ask:

- How many units to the right should you go on the *x*-axis? Why?
- How many units up should you go on the *y*-axis? Why?
- How should you draw the point?
- How should you label the point?

Math is... Choosing Tools

- How many units right and up do you go to get from Sam's house to school?

Students use the coordinate plane as a tool to help them solve a problem.

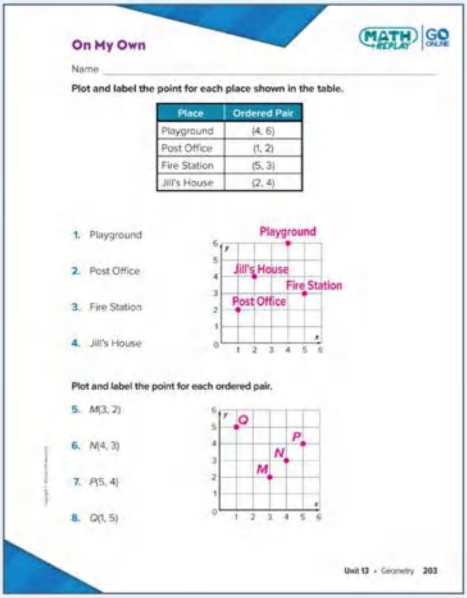


EL English Learner Scaffolds

Entering/Emerging Explain *how to get from* ____ *to* ____. Say *I'm going to show you how to get from the desk to the closet.* Show them by walking the route. Repeat again with new points A and B. Then repeat once more and ask *Did I show you how to get from the [door] to the [bookshelf] or how to get from the [bookshelf] to the [door]?*

Developing/Expanding Explain *how to get from* ____ *to* ____. Say *I'm going to show you how to get from the desk to the closet*. Show them by walking the route. Repeat again with new points A and B. Then ask students to show you how to get from a new point A and B that they choose, and to then complete the following sentence: *I showed you* ____ *the [door] to the [bookshelf].*

Bridging/Reaching Ask students to review the Math Is...In My World question on the Learn page. Ask students to explain how to get from different points A and B in the classroom. Allow students to interject, making corrections if necessary. For example, *No, you didn't show to get from the closet to the door - you showed how to get from the door to the closet.*



On My Own

Name

Plot and label the point for each place shown in the table.

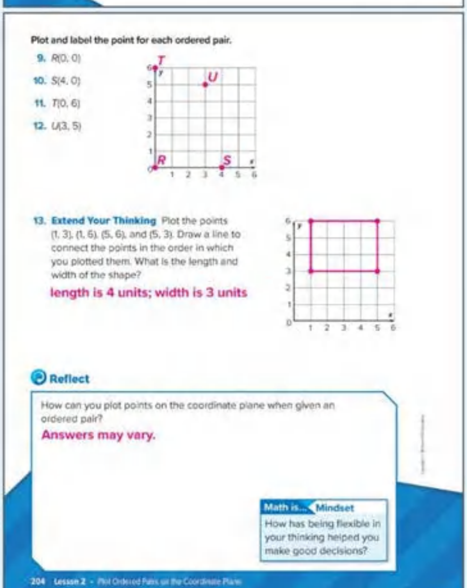
Place	Ordered Pair
Playground	(4, 6)
Post Office	(1, 2)
Fire Station	(5, 3)
Jill's House	(2, 4)

1. Playground
2. Post Office
3. Fire Station
4. Jill's House

Plot and label the point for each ordered pair.

5. M(3, 2)
6. N(4, 3)
7. P(5, 4)
8. Q(1, 5)

Unit 13 • Geometry 203



Plot and label the point for each ordered pair.

9. R(0, 0)
10. S(4, 0)
11. T(0, 6)
12. U(3, 5)

13. **Extend Your Thinking** Plot the points (1, 3), (1, 6), (5, 6), and (5, 3). Draw a line to connect the points in the order in which you plotted them. What is the length and width of the shape?

length is 4 units; width is 3 units

Reflect

How can you plot points on the coordinate plane when given an ordered pair?

Answers may vary.

Math is... Mindset

How has being flexible in your thinking helped you make good decisions?

204 Lesson 2 • Plot Ordered Pairs on the Coordinate Plane

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 9–11 Students may be confused by the use of 0 as a coordinate. Remind students that a coordinate of 0 means they are moving 0 units along that axis and will plot the point on the *x*-axis if the *y*-coordinate is 0, or on the *y*-axis if the *x*-coordinate is 0.

Item Analysis

Item	DOK	Rigor
1–4	2	Application
5—12	1	Procedural Skill & Fluency
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you plot points on the coordinate plane when given an ordered pair?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How has being flexible in your thinking helped you make good decisions?

Students reflect on how they practiced self-regulation.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can plot ordered pairs on a coordinate plane.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Plot points on a coordinate plane	5.G.A.1
2	2	Label points on a coordinate plane	5.G.A.1

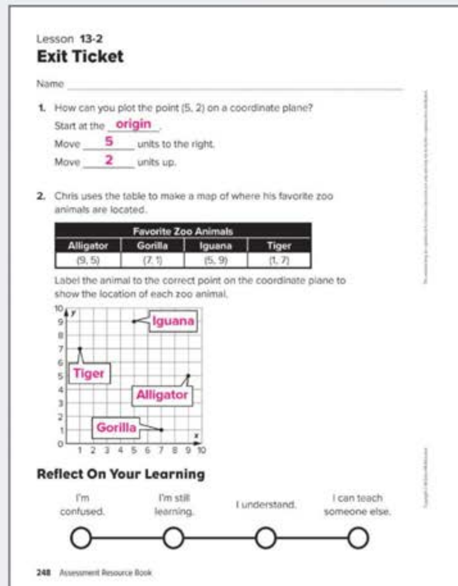
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
2 of 2	Additional Practice or any of the B or E activities
1 of 2	*Take Another Look* or any of the B activities
0 of 2	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking


Lesson 13-2
Exit Ticket

Name

1. How can you plot the point (5, 2) on a coordinate plane?
Start at the origin.
Move 5 units to the right.
Move 2 units up.

2. Chris uses the table to make a map of where his favorite zoo animals are located.

Favorite Zoo Animals			
Alligator	Gorilla	Iguana	Tiger
(9, 5)	(7, 1)	(5, 9)	(1, 7)

Label the animal to the correct point on the coordinate plane to show the location of each zoo animal.

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

248 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Plot 'n Roll

Work with students in small groups. Provide each student with a number cube and coordinate plane. Have a student roll twice to create an ordered pair. Then every student plots the point. Have students describe how they plotted the point. Make sure students understand that the first number represents the horizontal position and the second number represents the vertical position.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Points on the Coordinate Plane
- Identify Ordered Pairs

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 151

Lesson 13-2 • Reinforce Understanding

Plot Ordered Pairs on the Coordinate Plane

Name ______

Review

An ordered pair shows the x-coordinate of a point, followed by the y-coordinate of the point, in that order.

Plot the point (5, 2).

The x-coordinate is 5. This means we go 5 units to the right from the origin.

The y-coordinate is 2. This means we go up 2 units.

Mark the location with a point.

Plot and label the point for the ordered pair.

1. A (1, 1)
2. B (4, 6)
3. C (3, 2)
4. D (0, 5)
5. E (2, 3)

Plot and label the point for the ordered pair.

6. V (2, 0)
7. W (1, 5)
8. X (6, 3)
9. Y (4, 1)
10. Z (2, 4)

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Coordinate Plane Task Cards

Students practice plotting ordered pairs on the coordinate plane.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 151–152

Lesson 13-2

Additional Practice

Name ______

Review

You can plot a point on a coordinate plane if you are given an ordered pair.

How do you plot point A at (4, 3) on the coordinate plane?

The x-coordinate is 4 and the y-coordinate is 3. From the origin, move 4 units to the right, along the x-axis. Then move up 3 units, along the y-axis.

Label point A at (4, 3).

Plot the point for each ordered pair. Label with the given letter.

1. A (5, 3)
2. B (4, 1)
3. C (2, 5)
4. D (1, 2)
5. E (4, 5)
6. F (5, 2)
7. G (1, 4)
8. H (3, 4)
9. I (2, 1)
10. J (3, 3)

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division with multi-digit numbers.

Assign

Use It! Application Station

Is This for Real? Students research and use the TAARP method to determine website credibility.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

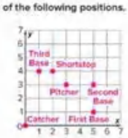
Assign

INDEPENDENT WORK

Student Practice Book, pp. 151–152

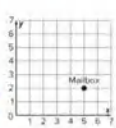
Plot and label the point for each of the following positions.

11. Catcher (0, 0)
12. Second Base (5, 3)
13. Pitcher (3, 3)
14. Shortstop (2, 4)
15. First Base (5, 1)
16. Third Base (1, 4)



17. Monica wants to plot the point (5, 2) on a coordinate grid to represent the position of her mailbox. Did she plot the point correctly? Explain.

Yes; Sample answer: To plot the point (5, 2), from the origin she moved to the right 5 units and then moved up 2 units.



Math @ Home Activity Using 10 index cards, write the name of a location on the front of each card and an ordered pair on the back of each card. Give your child the cards and a coordinate grid. Have them plot each location using a different color. Then randomly pick a card and have them explain how they plotted the point.

Student Practice Book

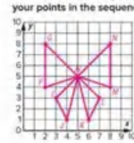
Differentiation Resource Book, p. 152

Lesson 13-2 • Extend Thinking

Plot Ordered Pairs on the Coordinate Plane

Name ____________________

Write the coordinate pair. Then plot and label the points. Connect your points in the sequence *HFGHIJHKLHMNH*.



1. Plot point F at (2, 4).
2. Point G has the same x-coordinate as point F, and point G has a y-coordinate of 8. G is at **(2, 8)**.
3. Point H is 5 units from the x-axis. The x-coordinate and the y-coordinate of point H are the same. H is at **(5, 5)**.
4. Point I is 3 units from the y-axis. The x-coordinate and the y-coordinate of point I are the same. I is at **(3, 3)**.
5. Plot point J at (4, 1).
6. Point K has the same y-coordinate as point J, and point K has an x-coordinate of 6. K is at **(6, 1)**.
7. Point L has the same y-coordinate as point I, and point L has an x-coordinate of 7. L is at **(7, 3)**.
8. Point M has the same y-coordinate as point F, and point M has an x-coordinate of 8. M is at **(8, 4)**.
9. Point N has the same x-coordinate as point M. The x-coordinate and the y-coordinate of point N are the same. N is at **(8, 8)**.

Differentiation Resource Book

LESSON **13-3**

Represent Problems on a Coordinate Plane

Learning Targets

- I can plot points that represent real-world situations.
- I can interpret coordinate values of points in the context of the situation.

Standards ◆ Major ▲ Supporting ● Additional

Content

○ **5.G.A** Graph points on the coordinate plane to solve real-world and mathematical problems.

○ **5.G.A.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Math Practices and Processes

MPP Model with mathematics.

Vocabulary

Math Terms	Academic Terms
ordered pair	accurate
x-axis	speculate
x-coordinate	
y-axis	
y-coordinate	

Materials

The materials may be for any part of the lesson.

- *Coordinate Plane* Teaching Resource
- transparent spinner

Focus

Content Objectives	Language Objectives	SEL Objective
• Students plot points that represent real-world situations. • Students interpret coordinate values of points in the context of the situation.	• Students talk about plotting points when given real-world data using *draw* and *label*. • To support sense-making, ELs participate in MLR2: Collect and Display.	• Students identify and use mathematical tools to organize work.

Coherence

Previous	Now	Next
• Students plotted points on the coordinate plane given ordered pairs (Unit 13).	• Students plot data points from real-world situations on the coordinate plane and use it to interpret the data.	• Students classify triangles into categories based on attributes, and understand the categories and subcategories using a hierarchy (Unit 13). • Students use the coordinate plane to draw polygons and find their side lengths, and apply these techniques in the context of solving real-world and mathematical problems (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students plot and understand points on the coordinate plane. *Conceptual Understanding is not a targeted element of rigor for this standard.*	• Students build proficiency in their use of the coordinate plane by plotting points to represent and solve problems.	• Students solve problems with real-world contexts. *Application is not a targeted element of rigor for this standard.*

Number Routine
Find the Missing Values

GO ONLINE

5–7 min

Build Fluency Students build number sense as they use solved equations to find unknown values in related equations. Reveal answers one level at a time after students discuss solutions.

Remind students that this is a mental activity and that they should look for patterns.

These prompts encourage students to talk about their reasoning:

- What do you know about adding fractions?
- How does each equation connect to the equation to the right?
- How does each equation connect to the equation below it?
- How can the pattern help you add fractions and mixed numbers using only mental math?

Launch 5–7 min

Sense-Making Routine

Purpose Students connect to real-world situations to think about representing the story, quantities, and relationships.

Notice & Wonder

- What question could you ask?

Teaching Tip You may want to have students discuss their questions with a partner and attempt to answer them before discussing students' question as a whole class.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of plotting data points from real-world situations on the coordinate plane and are based on possible comments and questions that students may make during the share out.

- What story could you tell about this image?
- What could you use to describe where an elevator is?
- How can you describe which direction the elevators might be going?

Math is... Mindset

- What strategies help you work more efficiently?

SEL Self-Management: Organizational Skills

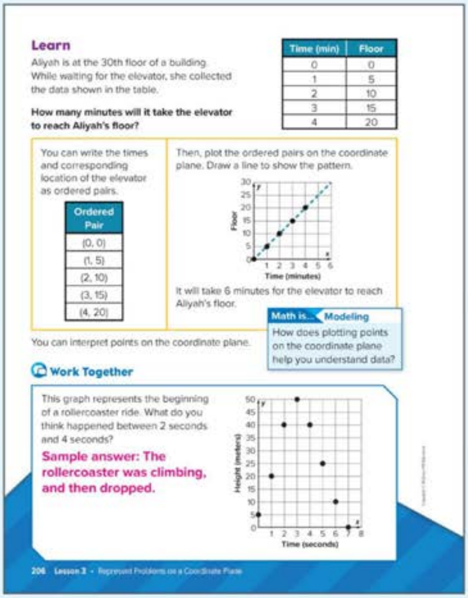
As students complete the Notice & Wonder routine, invite them to share how they organized their work or what they noticed about how you organized your work. For example, you may have written down the questions students asked in an organized list. Invite students to discuss the tools they may use to organize their work while representing problems on the coordinate plane. Encourage them to think about why this tool may be helpful for their work with representing problems on the coordinate plane.

Transition to Explore & Develop

Explain that the coordinate plane is a representation. Have students discuss ways in which they think it could be used. Ask questions that get students thinking about how the coordinate plane can represent real-world situations.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can plot data from real-world situations on the coordinate plane and use it to interpret the data.



Learn

Aliyah is at the 30th floor of a building. While waiting for the elevator, she collected the data shown in the table.

How many minutes will it take the elevator to reach Aliyah's floor?

Time (min)	Floor
0	0
1	5
2	10
3	15
4	20

You can write the times and corresponding location of the elevator as ordered pairs.

Ordered Pair
(0, 0)
(1, 5)
(2, 10)
(3, 15)
(4, 20)

Then, plot the ordered pairs on the coordinate plane. Draw a line to show the pattern.

It will take 6 minutes for the elevator to reach Aliyah's floor.

You can interpret points on the coordinate plane.

Math is... Modeling
How does plotting points on the coordinate plane help you understand data?

Work Together

This graph represents the beginning of a rollercoaster ride. What do you think happened between 2 seconds and 4 seconds?

Sample answer: The rollercoaster was climbing, and then dropped.

1 Pose the Problem

MLR Collect and Display

As students discuss the questions, record relevant words and phrases they may use such as *conclusions, data, corresponding floors,* and *ordered pairs*. Display the words for student reference. Use the student-generated expressions to help students make connections between student language and math vocabulary.

ETP Pose Purposeful Questions

- What information is presented in the table?
- How does the information in the table help you understand the location of the elevator at different times?

2 Develop the Math

Choose the option that best meets your instructional goals.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you plot points on the coordinate plane when given data in a table?
- How can you interpret data represented on the coordinate plane?

Key Takeaway

- Points on a coordinate plane can be used to represent and interpret real-world situations.

Work Together

Students work together to interpret data represented on the coordinate plane.

Common Misconception: Students may think that they should only be describing what happens at 3 seconds. Remind them that, in order to fully understand the data represented on the coordinate plane, it is important to look at what happens over a range of time.

LOM Language of Math

Ask students to discuss how they use the word *interpret* in other class subjects. Remind students that in math, we interpret information by explaining something that is not represented in words.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore making conclusions about real-world data after plotting the data points on the coordinate plane.

Materials: *Coordinate Plane* Teaching Resource

Directions: Provide copies of the *Coordinate Plane* Teaching Resource. Have students work together to solve the Pose the Problem.

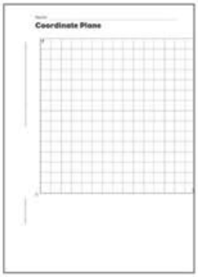
ETP Support Productive Struggle

- How can you use the data in the table to plot points on the coordinate plane?
- How should you label each axis?
- What does each point tell you about the location of the elevator?
- At which floor did the elevator start?
- Based on the data, which is the highest floor the elevator went to?
- What other conclusions can you make from this data?

Math is... Modeling

- How does plotting points on the coordinate plane help you understand data?

Students identify important quantities in a practical situation and map their relationships using tools, such as diagrams, two-way tables, graphs, flowcharts, and formulas.



Activity Debrief: Have groups share how they represented the data on the coordinate plane as well as any conclusions they were able to draw from the representation of the data.

A PDF of the Teaching Resource is available in the Teacher Digital Center.

Guided Exploration

Students plot points on the coordinate plane when given real-world data in a table and make conclusions about the data.

ETP Facilitate Meaningful Mathematical Discourse

- How can you determine which should be the x-coordinates and which should be the y-coordinates?
- Why does the order of the pairs matter?

Work with the students to plot the points. Have the students label the x-axis by ones and the y-axis by fives. Explain that they can do this to make the graph easier to draw and interpret. As students plot the points using the ordered pairs, ask:

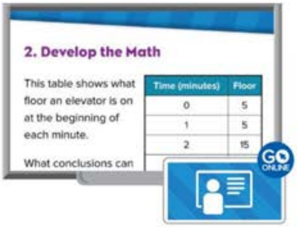
- How many units to the right should you go on the x-axis? Why?
- How many units up should you go on the y-axis? Why?
- How should you draw the point?
- How should you label the point?

- **Think About It:** Why do you think connecting the points will help you interpret the data?
- **Think About It:** When was the elevator going up? When was it going down? How do you know?

Math is... Modeling

- How does plotting points on the coordinate plane help you understand data?

Students identify important quantities in a practical situation and map their relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas.



EL English Learner Scaffolds

Entering/Emerging Explain *seconds/minutes.* Using a stopwatch, complete a task that takes over a minute; for example, putting away books. Show students the seconds and say *This took me [62] seconds to complete*. Pause, then say *There are 60 seconds in a minute. So it took me one minute and two seconds to complete.* Repeat with another task and ask *Did it take me [47] seconds or [47] minutes to complete?*

Developing/Expanding Explain *seconds/minutes.* Using a stopwatch, complete a task that takes over a minute. For example, putting away books. Show students the seconds and say *This took me [62] seconds to complete.* Pause, then say *There are 60 seconds in a minute. So it took me one minute and two seconds to complete.* Repeat with another task and ask *How many seconds did it take me to finish this task? How many minutes?*

Bridging/Reaching Ensure understanding of *seconds* and *minutes*. Ask students to explain how seconds relates to minutes (*60 seconds in one minute*). Then have them tell you how long it takes them to do some routine daily tasks, using seconds for tasks that take less than a minute, and minutes for tasks that take longer. Validate and provide correction as necessary.

Practice & Reflect 10 min

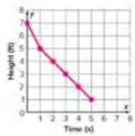
On My Own

Name ______

1. The table shows the time it took for a fifth-grade student to go down the slide at a park and their height from the ground while going down the slide. Write the time and corresponding heights as ordered pairs.
   (0, 7); (1, 5); (2, 4); (3, 3); (4, 2); (5, 1)

Time (seconds)	Height (feet)
0	7
1	5
2	4
3	3
4	2
5	1

2. Plot and connect the points on a coordinate plane.
   Check students' work.



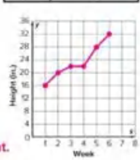
3. How tall is the slide?
   7 feet tall
4. How long does it take for the student to go down the slide?
   5 seconds
5. What happens between 0 seconds and 1 second?
   The student goes down 2 feet.
6. Where is the student after 5 seconds?
   1 foot off the ground

Unit 13 • Geometry 207

7. **STEM Connection** Poppy measures the height of a plant over several weeks and records it in the table. The plant is 14 inches tall before she begins recording. Write the weeks and corresponding heights as ordered pairs.
   (1, 16); (2, 20); (3, 22); (4, 22); (5, 28); (6, 32)

Week	Height (inches)
1	16
2	20
3	22
4	22
5	28
6	32

8. Plot and connect the points on the coordinate plane.
   Check students' work.



9. How much does the plant grow between Weeks 1 and 2?
   4 inches
10. What happens between Weeks 3 and 4?
   The plant remains the same height.
11. How much does the plant grow between before Poppy begins recording and Week 6?
   18 inches
12. **Extend Your Thinking** What are some real-world situations you could interpret from points represented on a coordinate plane?
   Sample answers: growth of students' height over time, distance a car travels on a road trip per day

Reflect

How are data presented on a coordinate plane helpful for understanding real-world situations?
Answers may vary.

Math is... Mindset
What strategies helped you work more efficiently?

208 Lesson 3 • Represent Problems on a Coordinate Plane

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 7 Because there is no "Week 0" in the table, students may think they do not need to plot a point having an *x*-coordinate of 0. Remind them to reread the problem and think about the information that is given about the height of the plant before Week 1.

Item Analysis

Item	DOK	Rigor
1–2	1	Procedural Skill & Fluency
3–6	2	Application
7–8	1	Procedural Skill & Fluency
9–11	2	Application
12	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How is data presented on a coordinate plane helpful for understanding real-world situations?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What strategies helped you work more efficiently?

Students reflect on how they practiced self-management.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can plot points that represent real-world situations.
- I can interpret coordinate values of points in the context of the situation.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Solve problems from a coordinate plane	5.G.A.2
2	2	Solve problems from a coordinate plane	5.G.A.2
3	2	Solve problems from a coordinate plane	5.G.A.2

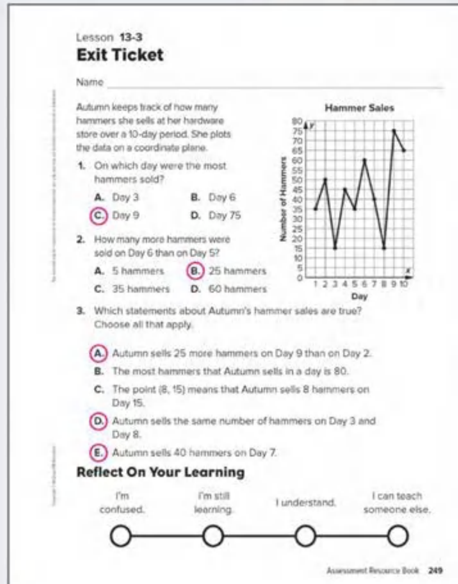
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking



Lesson 13-3
Exit Ticket

Name ____________

Autumn keeps track of how many hammers she sells at her hardware store over a 10-day period. She plots the data on a coordinate plane.

1. On which day were the most hammers sold?
   A. Day 3 B. Day 6
   C. Day 9 D. Day 75
2. How many more hammers were sold on Day 6 than on Day 5?
   A. 5 hammers B. 25 hammers
   C. 35 hammers D. 60 hammers
3. Which statements about Autumn's hammer sales are true? Choose all that apply.
   A. Autumn sells 25 more hammers on Day 9 than on Day 2.
   B. The most hammers that Autumn sells in a day is 80.
   C. The point (8, 15) means that Autumn sells 8 hammers on Day 15.
   D. Autumn sells the same number of hammers on Day 3 and Day 8.
   E. Autumn sells 40 hammers on Day 7.

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 249

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Spin 'n Plot

Students take turns spinning a spinner labeled 1 through 8 twice to create an ordered pair. The other students plot the point on the coordinate plane. The student who spun then describes how to locate the point from the origin as the others check their work. Make sure students realize that they can start with either the x or the y-coordinate, as long as they use the correct direction for each coordinate.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Real World and Mathematical Problems
- Graph Ordered Pairs
- Corresponding Terms as Ordered Pairs

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 153

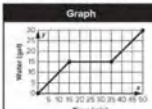
Lesson 13-3 • Reinforce Understanding

Represent Problems on a Coordinate Plane

Name ______

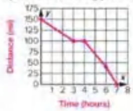
Review

Plotting points can help you understand real-world situations. The table shows the number of gallons of water in a tank over time.

Graph	Interpretation
	The tank was empty when filling began. After 15 min, filling stopped for 20 min. The tank had 30 gal of water after 50 min.

The table shows how many miles remain on a road trip to get to the final destination. Use the table to complete the following problems.

Time (hours)	Distance (mi)
0	150
3	100
4	100
6	40
7	0



1. Plot and connect the points on the coordinate plane.
2. How long did the road trip take to complete? **7 hours**
3. How many miles total was the road trip? **150 miles**
4. How far did the travelers drive before they stopped for a break? **50 miles**
5. How long did the travelers stop? **1 hour**

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Coordinate Plane Representation Race

Students practice interpreting information shown on the coordinate plane.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 153–154

Lesson 13-3

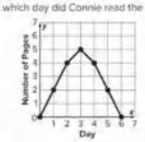
Additional Practice

Name ______

Review

You can interpret points on a coordinate grid to help you understand real-world problems.

The graph shows the number of pages Connie read over 7 days. On which day did Connie read the greatest number of pages?



The highest point along the y-axis, 5, shows the greatest number of pages read on one day. The x-coordinate of that point on the graph is 3.

Connie read the greatest number of pages on Day 3.

Use the graph above for questions 1–3.

1. How many pages did Connie read on Day 2? **4** pages
2. On which day(s) did Connie read 2 pages? Day(s) **1 and 5**
3. What does the point (6, 0) mean? **Sample answer: Connie did not read any pages on Day 6.**

Student Practice Book

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division with multi-digit numbers.

Use It! Application Station

Drafting Tools for Accuracy Students create their own drafting triangles and create a model of a covered bridge.

The content of this card has concepts covered later in Lesson 13-5. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

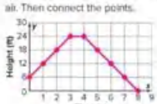
INDEPENDENT WORK

Student Practice Book, pp. 153–154

Will flies a drone in his yard. An app on his phone records the time the drone is in the air and its height. The table shows the results.

Drone Height	
Time (s)	Height (ft)
0	6
1	12
2	18
3	24
4	24
5	18
6	12
7	6
8	0

4. Plot the points on the coordinate grid to represent the height of the drone for each number of seconds that it is in the air. Then connect the points.



5. From what height does the drone take off? **6** feet

6. How high was the drone at 3 seconds? **24** feet

7. What does the point (7, 6) mean?
The drone was 6 feet above the ground after 7 seconds.

8. What was the drone doing between 3 seconds and 4 seconds after it took off?
Sample answer: It was hovering at a height of 24 feet.

Math @ Home Activity Create a table of values that could represent a situation your child is familiar with. Situations can include the number of minutes spent practicing an instrument or the number of minutes spent reading a book. Have your child plot the points and then connect the points with line segments. Point to different points on the graph, and ask your child to explain what the point means in relationship to the given context.

Student Practice Book

Differentiation Resource Book, p. 154

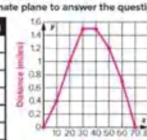
Lesson **13-3 • Extend Thinking**

Represent Problems on a Coordinate Plane

Name ____________________

A jogger runs from home to the store and then back. The table shows the distance from home the jogger is as time passes. Plot and connect the points on the coordinate plane to answer the questions.

Time (min)	Distance (miles)
0	0
10	0.4
20	1
30	1.5
40	1.5
50	1.2
60	0.7
70	0



1. What is the total distance of the jogger's run? **3 miles**
2. How long did it take the runner to run to the corner store? **30 minutes**
3. How long was the jogger at the corner store? **10 minutes**
4. How far did the jogger run in the first 20 minutes? **1 mile**
5. How far did the jogger run in the last 20 minutes? **1.2 miles**
6. How can you tell that the jogger ran fastest during the last 10 minutes? **It is where the line is steepest.**
7. When was the jogger running the slowest? **The jogger was running the slowest right after leaving the corner store**
8. How far away from home was the runner after 60 minutes? **0.7 miles**
9. How far away from the store was the runner after 60 minutes? **0.8 miles**

Differentiation Resource Book

LESSON **13-4**

Classify Triangles by Properties

Learning Targets

- I can classify triangles based on their properties into categories and subcategories.
- I can use properties of triangles to prove or disprove statements about triangles.

Standards ◇ Major △ Supporting ○ Additional

Content

○ **5.G.B** Classify two-dimensional figures into categories based on their properties.

○ **5.G.B.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

○ **5.G.B.4** Classify two-dimensional figures in a hierarchy based on properties.

Math Practices and Processes

MPP Model with mathematics.

Vocabulary

Math Terms	Academic Terms
category	evaluate
equilateral triangle	suggest
hierarchy	
isosceles triangle	
property	
scalene triangle	
subcategory	

Materials

The materials may be for any part of the lesson.

- plastic straws
- *Properties of Triangles* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students classify triangles into categories and subcategories based on their properties. • Students organize the categories and subcategories into a hierarchy.	• Students classify triangles using *the same*, *different*, and *share*. • To support cultivating conversation, ELs participate in MLR3: Critique, Correct, and Clarify.	• Students identify and discuss the emotions experienced during math learning.

Coherence

Previous	Now	Next
• Students classified two-dimensional figures based on their sides or angles, and recognized and identified right triangles (Grade 4). • Students plotted data points from real-world situations on the coordinate plane and used it to interpret the data (Unit 13).	• Students classify triangles into categories based on minimal defining attributes, and understand the categories and subcategories using a hierarchy.	• Students classify quadrilaterals into categories based on minimal defining attributes (Unit 13).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students extend their understanding of triangles through exploration of their properties.	• Students evaluate properties of triangles by creating a hierarchy. *Procedural Skill & Fluency is not a targeted element of rigor for this standard.*	• Students apply their understanding of triangles to sort triangles into groups. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Would You Rather?

5–7 min

Build Fluency Students develop number sense as they compare products of fractions and whole numbers to whole numbers.

These prompts encourage students to talk about their reasoning:

- What strategies did you use to choose the boxes?
- How did you decide whether to calculate or estimate a product?

Launch 5–7 min

Sense-Making Routine

Purpose Students explore the mathematical concept of defining attributes.

Notice & Wonder

- What could the question be?

Teaching Tip Before students begin discussing the image, you may want to have them discuss what they already know about triangles.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' noticing and wondering about the properties of shapes and are based on possible comments and questions that students may make during the share out.

- What do you notice about the shapes in "Examples"?
- What do you notice about the shapes in "Non-examples"?
- How can you describe what the shapes have in common?
- What kind of information would help you know more about the triangles? Explain.

Math is... Mindset

- How do your skills or interests help you with your work?

SEL Self-Awareness: Accurate Self-Perception

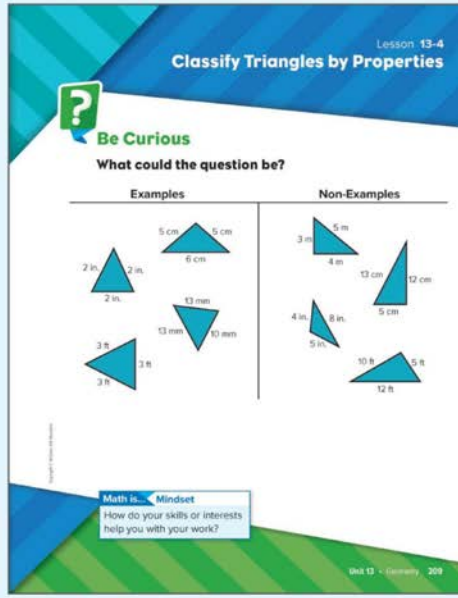
After students participate in the Notice & Wonder routine, invite them to share the self-perception they were experiencing. Encourage students to focus on the feelings they experienced when they were successful as well as when they were not. Their work throughout the lesson with classifying triangles by properties may be challenging, and they may feel unsuccessful. Remind students that these self-perceptions are neither right nor wrong, but how we deal with these perceptions can affect success with math work. Sharing and listening can help students build understanding of their own self-perceptions as well as the self-perceptions of others.

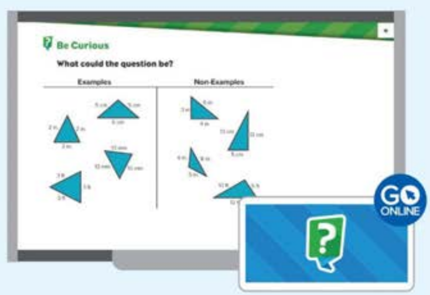
Transition to Explore & Develop

Ask questions that make students begin thinking about what makes a triangle look like it does.

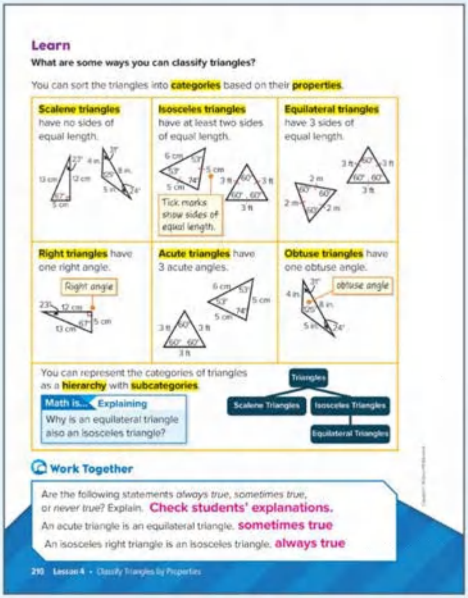
ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can classify triangles into categories based on their properties, and how we can organize those categories.





Explore & Develop 20 min



Learn

What are some ways you can classify triangles?

You can sort the triangles into **categories** based on their **properties**.

Scalene triangles have no sides of equal length.

Isosceles triangles have at least two sides of equal length.

Tick marks show sides of equal length.

Equilateral triangles have 3 sides of equal length.

Right triangles have one right angle.

Right angle

Acute triangles have 3 acute angles.

Obtuse triangles have one obtuse angle.

obtuse angle

You can represent the categories of triangles as a **hierarchy** with **subcategories**.

Triangles

Scalene Triangles

Isosceles Triangles

Equilateral Triangles

Math is... Explaining

Why is an equilateral triangle also an isosceles triangle?

Work Together

Are the following statements *always true*, *sometimes true*, or *never true*? Explain. **Check students' explanations.**

An acute triangle is an equilateral triangle. **sometimes true**

An isosceles right triangle is an isosceles triangle. **always true**

210 Lesson 4 • Classify Triangles by Properties

❶ Pose the Problem

ETP Pose Purposeful Questions

- How can you define the word *properties* in your own words?
- What are some ways you can classify shapes?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Critique, Correct, and Clarify

Make a false claim for students to critique. Write the following on the board: *A scalene triangle has three sides of equal length. Am I correct?* Ask students to correct the statement, explaining how they know it's incorrect. Revisit this routine throughout the lesson to provide reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you know if a triangle can be classified as scalene, isosceles, or equilateral?
- What is similar about categories and subcategories in a hierarchy? What is different?

Key Takeaways

- A property of a triangle is a defining attribute that remains the same for all triangles in that category.
- Properties of triangles include number of sides, number of angles, length of sides, and measures of angles.
- Triangles can be organized into a hierarchy based on the shared properties of groups of triangles.
- All triangles in a category share a set of properties.

Work Together

Students work together to determine if some sentences about triangles are always, sometimes, or never true.

Common Error: Students may think they need to know the degrees of the angles of a triangle in order to answer if an acute triangle is always, sometimes, or never an equilateral triangle. Make sure students understand that this Work Together is about classifying triangles by category and not evaluating specific triangles.

LOM Language of Math

A subcategory is "a part of" another category or "under" another category. The prefix *sub-* means "under" or "contained in." Have students think about other words with the prefix *sub-* to help them understand where in a hierarchy a subcategory can be found. A submarine operates under water, and a subcommittee is composed of some members of a larger committee.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore the properties of triangles and classify triangles based on their properties.

Materials: *Properties of Triangles* Teaching Resource

Directions: Provide copies of the *Properties of Triangles* Teaching Resource. Have students work together to solve the Pose the Problem. Before students begin sorting and classifying the triangles, facilitate a discussion to ensure students understand the meaning of the tick marks.

ETP Support Productive Struggle

- What do all of the triangles have in common?
- How are some triangles different from others?

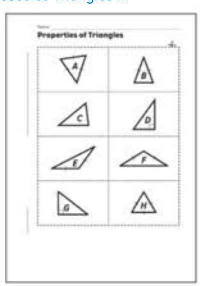
Activity Debrief: Have groups of students share how they sorted the triangles and discuss the properties of triangles that they used to sort them. Facilitate a discussion to ensure students understand the terms *category* and *properties*. Encourage students to label their categories as *scalene triangles*, *isosceles triangles*, and *equilateral triangles*. Have students review the *hierarchy* on the student page. Facilitate a discussion about *subcategories*.

Math is... Modeling

- Why is Equilateral Triangles under Isosceles Triangles in the hierarchy?

Students analyze the relationships represented in diagrams mathematically to draw conclusions.

A PDF of the Teaching Resource is available in the Teacher Digital Center.



Guided Exploration

Students classify triangles into categories based on minimal defining attributes, and understand the categories and subcategories using a hierarchy.

ETP Facilitate Meaningful Mathematical Discourse

- **Think About It:** How many properties do all triangles share? What are they?
- **Think About It:** Are all members of a subcategory members of the category above it? Why or why not?

Ask:

- What properties do all scalene triangles share with all triangles?
- What properties do all scalene triangles share that all triangles do not?
- **Think About It:** Why are the categories for scalene triangles and isosceles triangles not connected in the hierarchy?
- How is Isosceles Triangles a category and a subcategory?

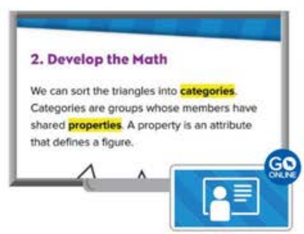
Ask:

- What properties do all equilateral triangles share with all triangles?
- What properties do all equilateral triangles share that all isosceles triangles do not?

Math is... Modeling

- Why is Equilateral Triangles under Isosceles Triangles in the hierarchy?

Students analyze the relationships represented in diagrams mathematically to draw conclusions.



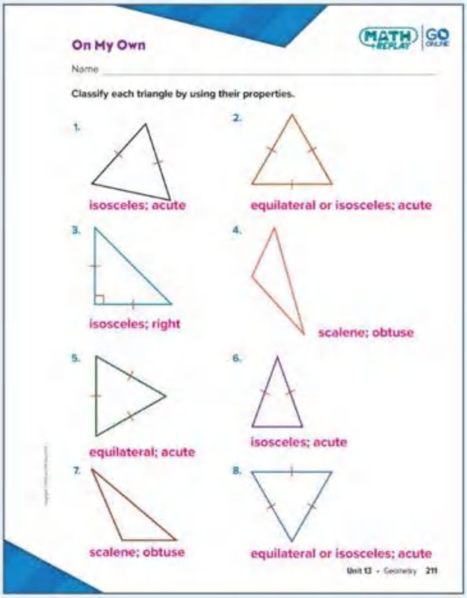
EL English Learner Scaffolds

Entering/Emerging Explain *at least*. Turn to the Learn page. Point to the relevant parts of the isosceles triangle while saying *Isosceles triangles must have at least two sides that are equal in length. They can't have just one.* Then draw three triangles, one with no equal sides, one with two, and one with three. For each, ask *Does this triangle have at least two equal sides?*

Developing/Expanding Explain *at least.* Turn to the Learn page. Point to the relevant parts of the isosceles triangle while saying *Isosceles triangles must have at least two sides that are equal in length. They can't have just one.* Then draw three triangles, one with no equal sides, one with two, and one with three. For each, say *Tell me about this triangle.* Provide the frame It is (not) an isosceles triangle because ____.

Bridging/Reaching Guide students to the Learn page and ask them to read the sentence on isosceles triangles, focusing on *at least*. Ask them to explain what it means (not less than, at the minimum) and compare it to the meanings of *the least/least* (smallest in amount or extent). Validate and correct as needed.

Practice & Reflect 10 min

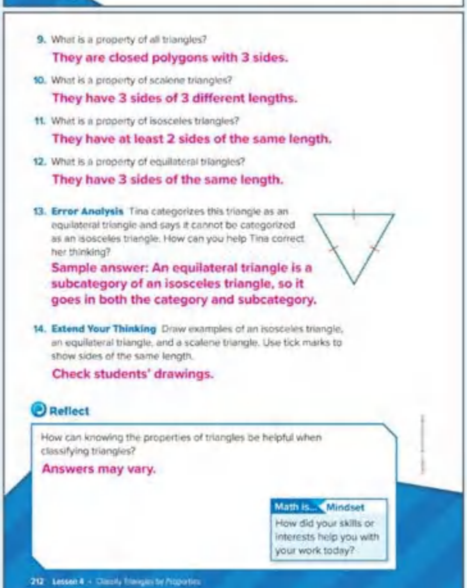

On My Own

Name

Classify each triangle by using their properties.

1. isosceles; acute
2. equilateral or isosceles; acute
3. isosceles; right
4. scalene; obtuse
5. equilateral; acute
6. isosceles; acute
7. scalene; obtuse
8. equilateral or isosceles; acute

Unit 13 • Geometry 211


9. What is a property of all triangles?
They are closed polygons with 3 sides.
10. What is a property of scalene triangles?
They have 3 sides of 3 different lengths.
11. What is a property of isosceles triangles?
They have at least 2 sides of the same length.
12. What is a property of equilateral triangles?
They have 3 sides of the same length.
13. **Error Analysis** Tina categorizes this triangle as an equilateral triangle and says it cannot be categorized as an isosceles triangle. How can you help Tina correct her thinking?
Sample answer: An equilateral triangle is a subcategory of an isosceles triangle, so it goes in both the category and subcategory.
14. **Extend Your Thinking** Draw examples of an isosceles triangle, an equilateral triangle, and a scalene triangle. Use tick marks to show sides of the same length.
Check students' drawings.

Reflect

How can knowing the properties of triangles be helpful when classifying triangles?

Answers may vary.

Math is... Mindset

How did your skills or interests help you with your work today?

212 Lesson 4 • Classify Triangles by Properties

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 2, 5, 8 Students may answer that these are isosceles triangles. While this is correct, encourage students to be as specific as possible as they classify the triangles by using the most precise name for each triangle.

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9–12	2	Conceptual Understanding
13–14	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can knowing the properties of triangles be helpful when classifying triangles?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did your skills or interests help you with your work today?

Students reflect on how they practiced self-awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can classify triangles based on their properties into categories and subcategories.
- I can use properties of triangles to prove or disprove statements about triangles.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Classify triangles based on side lengths	5.G.B.4
2	1	Classify triangles based on side lengths	5.G.B.4
3	2	Classify triangles based on side lengths	5.G.B.4

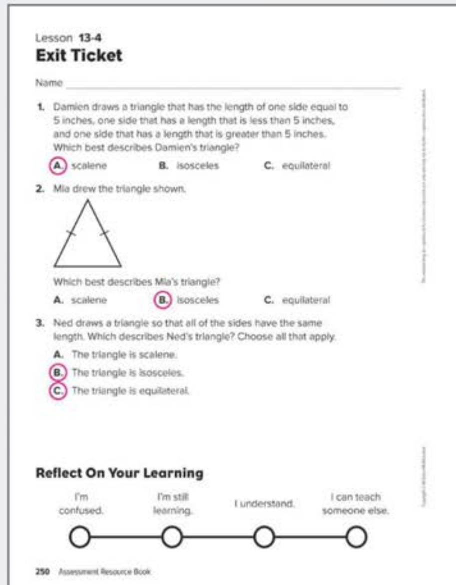
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 13-4
Exit Ticket

Name ____________________

1. Damien draws a triangle that has the length of one side equal to 5 inches, one side that has a length that is less than 5 inches, and one side that has a length that is greater than 5 inches. Which best describes Damien's triangle?
   A. scalene B. isosceles C. equilateral

2. Mia drew the triangle shown.

   Which best describes Mia's triangle?
   A. scalene B. isosceles C. equilateral

3. Ned draws a triangle so that all of the sides have the same length. Which describes Ned's triangle? Choose all that apply.
   A. The triangle is scalene.
   B. The triangle is isosceles.
   C. The triangle is equilateral.

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

250 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Pick-Up Triangles

Students take turns holding and releasing a bundle of plastic straws. The student looks for triangles formed when releasing the straws and names the type of triangles based on their properties. Other students must verify each triangle and its name. Each time, make sure the students agree that the triangle is scalene, isosceles, or equilateral before repeating the process.

GO ONLINE

Take Another Look Lessons

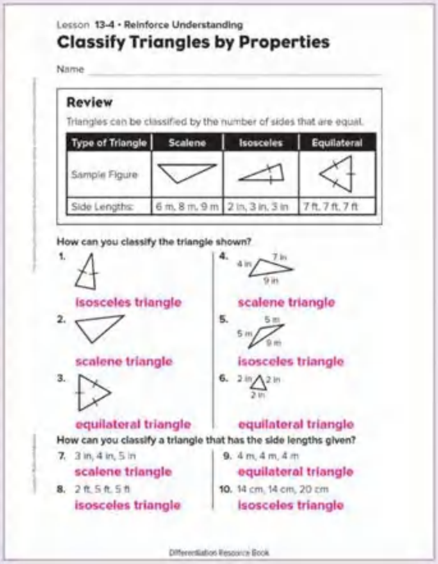
Assign the interactive lessons to reinforce targeted skills.

- Recognize Triangles by Angles
- Recognize Triangles by Sides

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 155



Lesson 13-4 • Reinforce Understanding

Classify Triangles by Properties

Name

Review

Triangles can be classified by the number of sides that are equal.

Type of Triangle	Scalene	Isosceles	Equilateral
Sample Figure			
Side Lengths	6 m, 8 m, 9 m	2 in, 3 in, 3 in	7 ft, 7 ft, 7 ft

How can you classify the triangle shown?

1. isosceles triangle
2. scalene triangle
3. equilateral triangle
4. 4 in, 7 in, 9 in — scalene triangle
5. 5 m, 5 m, 9 m — isosceles triangle
6. 2 in, 2 in, 2 in — equilateral triangle

How can you classify a triangle that has the side lengths given?

7. 3 in, 4 in, 5 in — scalene triangle
8. 2 ft, 5 ft, 5 ft — isosceles triangle
9. 4 m, 4 m, 4 m — equilateral triangle
10. 14 cm, 14 cm, 20 cm — isosceles triangle

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Classifying Triangles Four in a Row

Students practice classifying triangles.

GO ONLINE

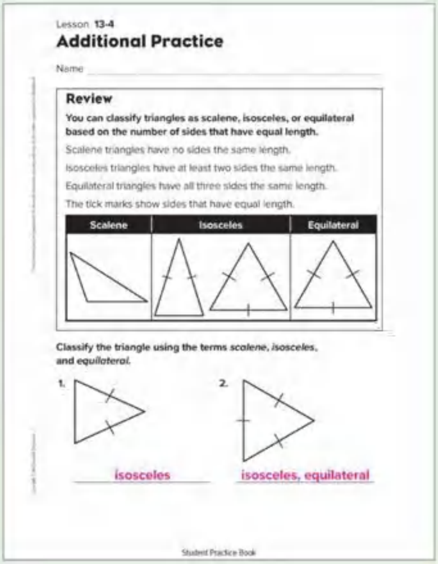
Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 155–156



Lesson 13-4

Additional Practice

Name

Review

You can classify triangles as scalene, isosceles, or equilateral based on the number of sides that have equal length.

Scalene triangles have no sides the same length.

Isosceles triangles have at least two sides the same length.

Equilateral triangles have all three sides the same length.

The tick marks show sides that have equal length.

Scalene	Isosceles	Equilateral

Classify the triangle using the terms *scalene*, *isosceles*, and *equilateral*.

1. isosceles
2. isosceles, equilateral

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division with multi-digit numbers.

Assign

Use It! Application Station

Is This for Real? Students research and use the TAARP method to determine website credibility.

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

GO ONLINE

Websketch Exploration

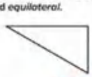
Assign a websketch exploration to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 155–156

Classify the triangle using the terms *scalene*, *isosceles*, and *equilateral*.

3. 

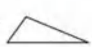
scalene

4. 

isosceles

5. 

isosceles, equilateral

6. 

scalene

7. Ezra draws a triangle that has the length of one side equal to 10 centimeters, one side that has a length that is less than 10 centimeters, and one side that has a length that is greater than 10 centimeters. What type of triangle does Ezra draw?

scalene

8. Ezra draws another triangle. This triangle has the length of one side that measures 3 inches, one side that measures 5 inches, and a third side that also measures 5 inches. What type of triangle does Ezra draw?

isosceles

Math @ Home Activity With your child, be on the lookout for different triangles that you may see in your everyday experiences. For example, you might notice that a Yield traffic sign is in the shape of an isosceles triangle. Look for other examples and classify the triangles according to the number of sides that are the same length.

Student Practice Book

Differentiation Resource Book, p. 156

Lesson 13-4 • Extend Thinking

Classify Triangles by Properties

Name ______

Which selection best describes the length of the 3rd side for the triangle given?

A. 6 m C. either 6 m or 7 m

B. 7 m D. neither 6 m or 7 m

1. scalene triangle
   2 m, 6 m, and B
2. isosceles triangle
   7 m, 7 m, and C
3. equilateral triangle
   6 m, 6 m and A
4. scalene triangle
   3 m, 5 m, and C m
5. Scalene triangle
   6 m, 7 m, and D
6. isosceles triangle
   6 m, 7 m, and C
7. isosceles triangle
   3 m, 7 m, B
8. equilateral triangle
   7 m, 7 m, and B

What is the least number of side lengths that must be changed in order to change the triangle classification?

9. To change an isosceles triangle into an equilateral triangle, change 1 side length(s).
10. To turn an isosceles triangle into a scalene triangle, change 1 side length(s).
11. To turn an equilateral triangle into an isosceles triangle, change 0 or 1 side length(s).
12. To turn a scalene triangle into an equilateral triangle, change 2 side length(s).

Differentiation Resource Book

LESSON 13-5

Properties of Quadrilaterals

Learning Target

- I can name quadrilaterals based on their properties.

Standards ◆ Major △ Supporting ○ Additional

Content

○ **5.G.B** Classify two-dimensional figures into categories based on their properties.

○ **5.G.B.4** Classify two-dimensional figures in a hierarchy based on properties.

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
attribute	establish
parallelogram	quality
property	
quadrilateral	
rectangle	
rhombus	
square	
trapezoid	

Materials

The materials may be for any part of the lesson.

- *Classifying Quadrilaterals* Teaching Resource

Focus

Content Objective

- Students name quadrilaterals based on their properties.

Language Objectives

- Students explain how to identify quadrilaterals based on their properties with *know* and *makes*.
- To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.

SEL Objective

- Students practice behavioral flexibility while working with peers to complete a challenging mathematical task.

Coherence

Previous

- Students classified two-dimensional figures based on their sides or angles, and recognized and identified right triangles (Grade 4).
- Students classified triangles into categories based on minimal defining attributes, and understand the categories and subcategories using a hierarchy (Unit 13).

Now

- Students classify quadrilaterals into categories based on minimal defining attributes.

Next

- Students understand the categories and subcategories of quadrilaterals using a hierarchy (Unit 13).

Rigor

Conceptual Understanding

- Students extend their understanding of quadrilaterals by working with quadrilaterals of various shapes and sizes.

Procedural Skill & Fluency

- Students begin to develop proficiency with identifying properties of quadrilaterals.

Procedural Skill & Fluency is not a targeted element of rigor for this standard.

Application

- Students apply their understanding of properties of quadrilaterals to identify them.

Application is not a targeted element of rigor for this standard.

GO ONLINE

Number Routine
Would You Rather?

5–7 min

Build Fluency Students develop number sense as they compare the values of expressions involving fractions.

These prompts encourage students to talk about their reasoning:

- What strategies did you use to compare the file sizes?
- How did you decide whether to calculate or estimate?

Purpose Students explore the mathematical concept of defining attributes of quadrilaterals.

Notice & Wonder

- What could the question be?

Teaching Tip Before students begin discussing the image, you may want to have them look for objects that have 4 sides around the classroom to start thinking about the defining attributes of quadrilaterals.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of the defining attributes of quadrilaterals and are based on possible comments and questions that students may make during the share out.

- Why do you think some shapes are labeled as "Examples"?
- Why do you think some shapes are labeled as "Non-Examples"?
- How can you describe what the shapes in "Examples" have in common?
- How can you describe what the shapes in "Non-Examples" have in common?
- What more do you want to know about the shapes?

Math is... Mindset

- How do you show that you understand your partner's point of view?

SEL Social Awareness: Develop Perspective

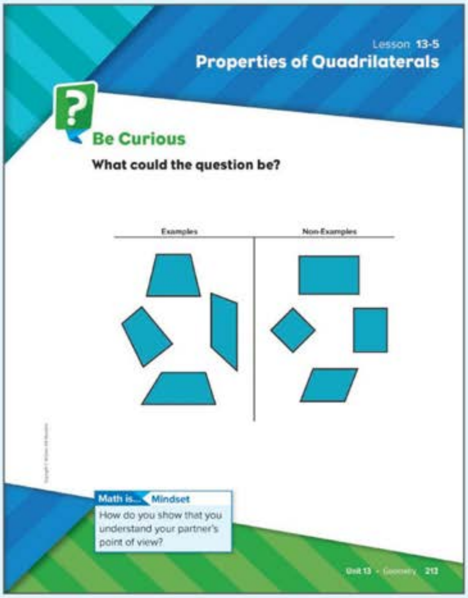
Developing perspective can help students understand different ways of thinking. After students engage in collaborative discourse around the Notice & Wonder routine, invite them to share how their ideas may have changed throughout the group discussion. Encourage students to think about how hearing the ideas of their peers may have helped them reach a new or different understanding of quadrilaterals. Use this discussion to encourage students to be open to the ideas and suggestions of their peers and remind them that their behavioral flexibility can allow them to learn from one another.

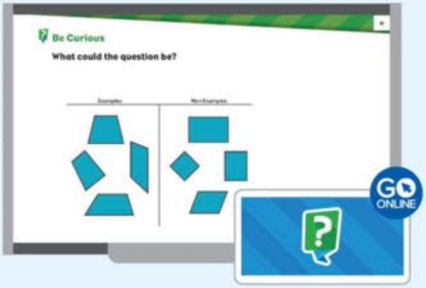
Transition to Explore & Develop

Encourage students to think about what all the shapes in the image have in common. Have them describe how they are alike and how they are different.

ETP Establish Mathematics Goals to Focus Learning

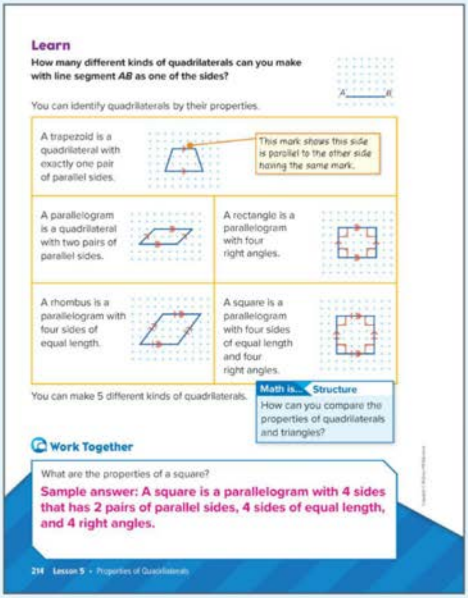
- Let's think about classifying quadrilaterals based on their properties.





Explore & Develop

20 min


Learn

How many different kinds of quadrilaterals can you make with line segment *AB* as one of the sides?

A B

You can identify quadrilaterals by their properties.

A trapezoid is a quadrilateral with exactly one pair of parallel sides.

This mark shows this side is parallel to the other side having the same mark.

A parallelogram is a quadrilateral with two pairs of parallel sides.

A rectangle is a parallelogram with four right angles.

A rhombus is a parallelogram with four sides of equal length.

A square is a parallelogram with four sides of equal length and four right angles.

You can make 5 different kinds of quadrilaterals.

Math is... Structure

How can you compare the properties of quadrilaterals and triangles?

Work Together

What are the properties of a square?

Sample answer: A square is a parallelogram with 4 sides that has 2 pairs of parallel sides, 4 sides of equal length, and 4 right angles.

214 Lesson 5 • Properties of Quadrilaterals

❶ Pose the Problem

ETP Pose Purposeful Questions

- How can you define a quadrilateral?
- How can the line segment be part of a quadrilateral?
- What kinds of quadrilaterals do you know now?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about how to answer the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- Are size or color properties of quadrilaterals? Why or why not?
- What properties do quadrilaterals have?

Key Takeaways

- A property of a quadrilateral is a defining attribute that remains the same for all quadrilaterals in that category.
- Properties of quadrilaterals include number of sides, number of angles, length of sides, measures of angles, and relationship of lines within the shape (parallel, perpendicular, intersecting).

Work Together

Students work together to describe the properties of a square.

Common Error: Students may initially leave out that a square always has 4 right angles. Ask students what quality distinguishes a square from a rhombus to elicit this part of the answer.

LOM Language of Math

Explain to students that quad means "four" and *lateral* means "side." So, the word *quadrilateral* means "a four-sided shape." Remind students that they can use word parts to determine the meaning of unknown words, in math or in other contexts.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore classifying quadrilaterals into categories based on minimal defining attributes.

Materials: *Classifying Quadrilaterals* Teaching Resource

Directions: Provide copies of the *Classifying Quadrilaterals* Teaching Resource. Before students begin classifying the quadrilaterals, facilitate a discussion to ensure students understand the meaning of the parallel marks.

ETP Support Productive Struggle

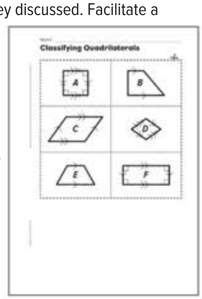
- What do all of the quadrilaterals have in common?
- How are some quadrilaterals the same as others?
- How are some quadrilaterals different from others?

Math is... Structure

- How can you compare the attributes of quadrilaterals and triangles?

Students look closely at the properties that define categories.

Activity Debrief: Have groups of students share the different properties of each quadrilateral that they discussed. Facilitate a discussion that outlines the minimal defining attributes of each quadrilateral, such as a rectangle is a parallelogram with four right angles.



Have students revisit the Pose the Problem question and discuss answers.

- How many different types of quadrilaterals can you make with line segment *AB* as one of the sides?

A PDF of the Teaching Resource is available in the Teacher Digital Center.

Guided Exploration

Students classify quadrilaterals into categories based on minimal defining attributes.

ETP Facilitate Meaningful Mathematical Discourse

- What makes a trapezoid different from a quadrilateral?
- What makes a parallelogram different from a quadrilateral?
- What makes a parallelogram different from a trapezoid?

Have the students use the figure to determine the properties of a rectangle. Ask:

- How many sides does this figure have? How do you know?
- How many pairs of parallel sides does this figure have? How do you know?
- How many right angles does this figure have? How do you know?

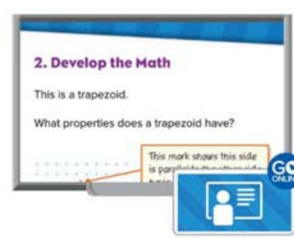
Have the students use the figure to determine the properties of a rhombus. Ask:

- How many sides does this figure have? How do you know?
- How many pairs of parallel sides does this figure have? How do you know?
- How many right angles does this figure have? How do you know?
- How many sides of the same length does this figure have? How do you know?

Math is... Structure

- How can you compare the attributes of quadrilaterals and triangles?

Students look closely at the properties that define categories.



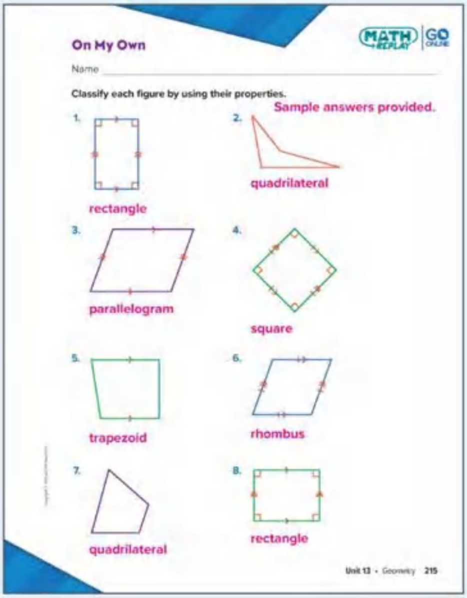
EL English Learner Scaffolds

Entering/Emerging Explain *properties.* Demonstrate reading a book. Say, *Some of the properties of this book are its color, its weight, and that it will burn.* Repeat with new examples. Then, ask students to use the examples to explain that a property is a characteristic of something that can be used to identify it.

Developing/Expanding Explain *properties.* Demonstrate reading a book. Say, *Some of the properties of this book are its color, its weight, and that it will burn.* Repeat with new examples. Then, ask students to use the examples to explain that a property is a characteristic of something that can be used to identify it. Ask students to complete the sentence: *Length of sides and measure of angles are are examples of* ______. (properties)

Bridging/Reaching Guide students to the Learn page and ask them to review the term *properties*. Ask them if they are familiar with one meaning of *property* as something that someone owns. Ask them to use the examples on this page to describe another meaning of *property*. (a characteristic that can be used to identify something). Once comprehension is validated, ask students to list properties of some of the shapes on the Learn page.

Practice & Reflect 10 min


On My Own

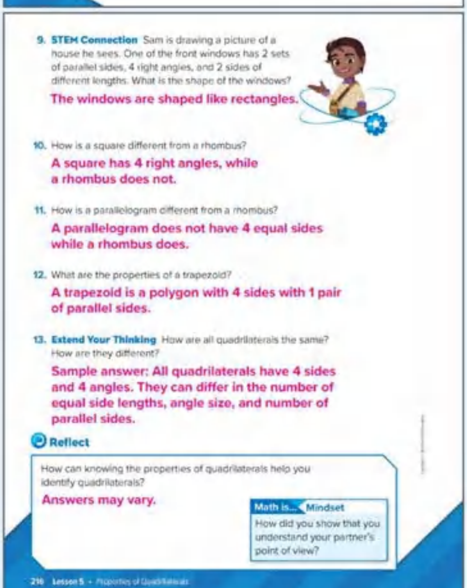
Name

Classify each figure by using their properties.

Sample answers provided.

1. rectangle
2. quadrilateral
3. parallelogram
4. square
5. trapezoid
6. rhombus
7. quadrilateral
8. rectangle

Unit 13 • Geometry 215


9. **STEM Connection** Sam is drawing a picture of a house he sees. One of the front windows has 2 sets of parallel sides, 4 right angles, and 2 sides of different lengths. What is the shape of the windows?
The windows are shaped like rectangles.

10. How is a square different from a rhombus?
A square has 4 right angles, while a rhombus does not.

11. How is a parallelogram different from a rhombus?
A parallelogram does not have 4 equal sides while a rhombus does.

12. What are the properties of a trapezoid?
A trapezoid is a polygon with 4 sides with 1 pair of parallel sides.

13. **Extend Your Thinking** How are all quadrilaterals the same? How are they different?
Sample answer: All quadrilaterals have 4 sides and 4 angles. They can differ in the number of equal side lengths, angle size, and number of parallel sides.

Reflect

How can knowing the properties of quadrilaterals help you identify quadrilaterals?
Answers may vary.

Math is... Mindset
How did you show that you understand your partner's point of view?

216 Lesson 5 • Properties of Quadrilaterals

Practice

ETP Build Procedural Fluency Conceptual from Understanding

Common Error: Exercises 2, 7 Students may be confused by a shape that has no parallel sides or sides of the same length. Ask students what they can call any polygon whose only property is "has 4 sides."

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9	2	Application
10–12	2	Conceptual Understanding
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can knowing the properties of quadrilaterals help you identify quadrilaterals?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you show that you understand your partner's point of view?

Students reflect on how they practiced social awareness.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can name quadrilaterals based on their properties.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Properties of quadrilaterals	5.G.B.4
2	1	Properties of quadrilaterals	5.G.B.4
3	2	Properties of quadrilaterals	5.G.B.4

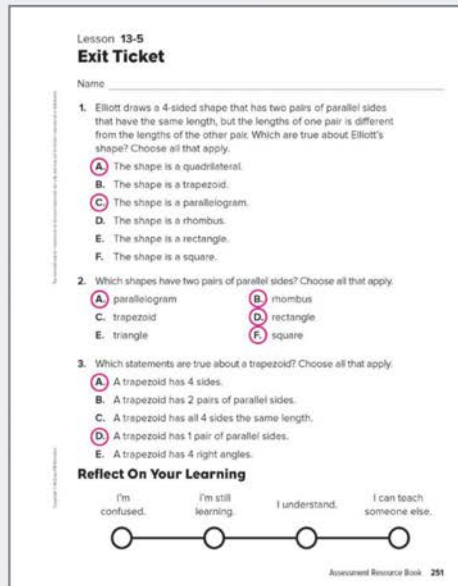
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding
B Build Proficiency
E Extend Thinking


Lesson 13-5
Exit Ticket

Name

1. Elliott draws a 4-sided shape that has two pairs of parallel sides that have the same length, but the lengths of one pair is different from the lengths of the other pair. Which are true about Elliott's shape? Choose all that apply.
A. The shape is a quadrilateral.
B. The shape is a trapezoid.
C. The shape is a parallelogram.
D. The shape is a rhombus.
E. The shape is a rectangle.
F. The shape is a square.

2. Which shapes have two pairs of parallel sides? Choose all that apply.
A. parallelogram
B. rhombus
C. trapezoid
D. rectangle
E. triangle
F. square

3. Which statements are true about a trapezoid? Choose all that apply.
A. A trapezoid has 4 sides.
B. A trapezoid has 2 pairs of parallel sides.
C. A trapezoid has all 4 sides the same length.
D. A trapezoid has 1 pair of parallel sides.
E. A trapezoid has 4 right angles.

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 251

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

What Shape Is It?

Work with students in groups. Have a student describe the properties of a specific type of quadrilateral. Then have the other students write down the name of the figure described. After each student has written an answer, work with the group to discuss the properties of the quadrilateral and come to a conclusion about which quadrilateral(s) might have been described.

GO ONLINE

Take Another Look Lessons

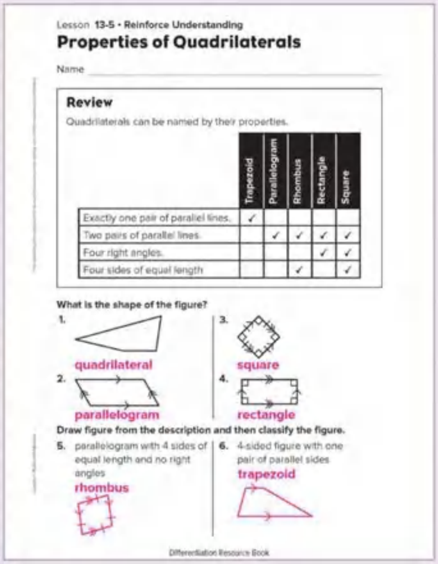
Assign the interactive lessons to reinforce targeted skills.

- Recognize Quadrilaterals
- Describe Quadrilaterals

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 157



Lesson 13-5 • Reinforce Understanding

Properties of Quadrilaterals

Name

Review

Quadrilaterals can be named by their properties.

	Trapezoid	Parallelogram	Rhombus	Rectangle	Square
Exactly one pair of parallel lines.	✓				
Two pairs of parallel lines.		✓	✓	✓	✓
Four right angles.				✓	✓
Four sides of equal length			✓		✓

What is the shape of the figure?

1. quadrilateral
2. parallelogram
3. square
4. rectangle

Draw figure from the description and then classify the figure.

5. parallelogram with 4 sides of equal length and no right angles
rhombus
6. 4-sided figure with one pair of parallel sides
trapezoid

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

2-Dimensional Figures Sort

Students practice classifying 2-dimensional figures.

GO ONLINE

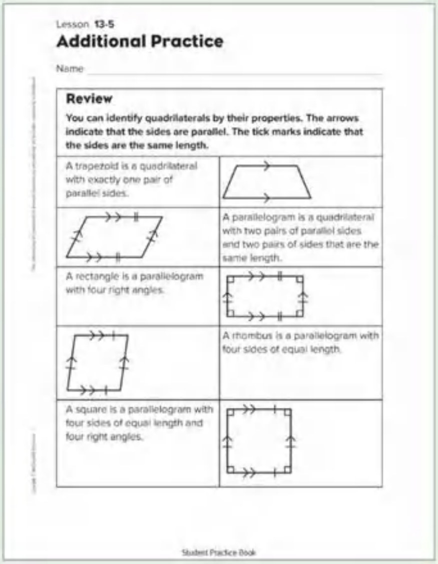
Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 157–158



Lesson 13-5

Additional Practice

Name

Review

You can identify quadrilaterals by their properties. The arrows indicate that the sides are parallel. The tick marks indicate that the sides are the same length.

A trapezoid is a quadrilateral with exactly one pair of parallel sides.

A parallelogram is a quadrilateral with two pairs of parallel sides and two pairs of sides that are the same length.

A rectangle is a parallelogram with four right angles.

A rhombus is a parallelogram with four sides of equal length.

A square is a parallelogram with four sides of equal length and four right angles.

Student Practice Book

Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division with multi-digit numbers.

Use It! Application Station

Drafting Tools for Accuracy Students create their own drafting triangles and create a model of a covered bridge.

GO ONLINE

Spiral Review

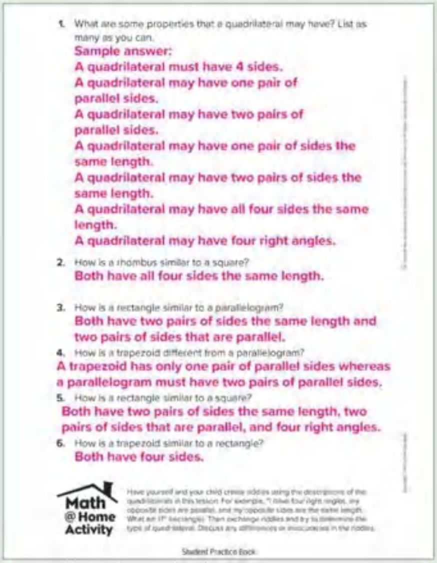
Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

INDEPENDENT WORK

Student Practice Book, pp. 157–158



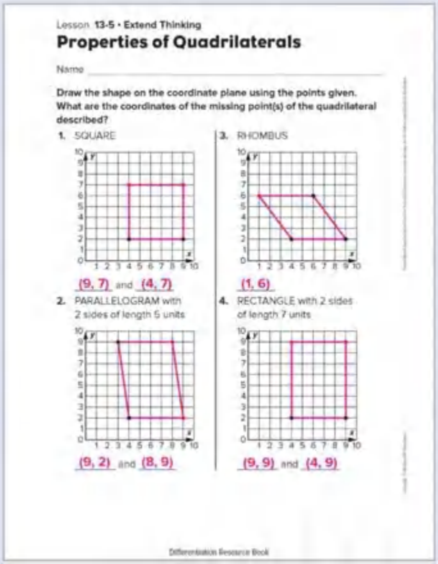
1. What are some properties that a quadrilateral may have? List as many as you can.
   Sample answer:
   A quadrilateral must have 4 sides.
   A quadrilateral may have one pair of parallel sides.
   A quadrilateral may have two pairs of parallel sides.
   A quadrilateral may have one pair of sides the same length.
   A quadrilateral may have two pairs of sides the same length.
   A quadrilateral may have all four sides the same length.
   A quadrilateral may have four right angles.
2. How is a rhombus similar to a square?
   Both have all four sides the same length.
3. How is a rectangle similar to a parallelogram?
   Both have two pairs of sides the same length and two pairs of sides that are parallel.
4. How is a trapezoid different from a parallelogram?
   A trapezoid has only one pair of parallel sides whereas a parallelogram must have two pairs of parallel sides.
5. How is a rectangle similar to a square?
   Both have two pairs of sides the same length, two pairs of sides that are parallel, and four right angles.
6. How is a trapezoid similar to a rectangle?
   Both have four sides.

Math @ Home Activity

Have yourself and your child create riddles using the descriptions of the quadrilaterals in this lesson. For example, "I have four right angles, my opposite sides are parallel, and my opposite sides are the same length. What am I?" (rectangle). Then exchange riddles and try to determine the type of quadrilateral. Discuss any differences or inaccuracies in the riddles.

Student Practice Book

Differentiation Resource Book, p. 158



Lesson 13-5 • Extend Thinking

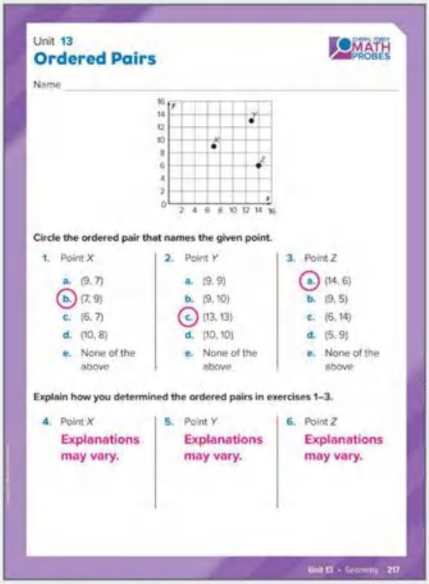
Properties of Quadrilaterals

Name ______

Draw the shape on the coordinate plane using the points given. What are the coordinates of the missing point(s) of the quadrilateral described?

1. SQUARE
   (9, 7) and (4, 7)
2. PARALLELOGRAM with 2 sides of length 5 units
   (9, 2) and (8, 9)
3. RHOMBUS
   (1, 6)
4. RECTANGLE with 2 sides of length 7 units
   (9, 9) and (4, 9)

Differentiation Resource Book


Unit 13
Ordered Pairs

Name

Circle the ordered pair that names the given point.

1. Point X
a. (9, 7)
b. (7, 9)
c. (6, 7)
d. (10, 8)
e. None of the above

2. Point Y
a. (9, 9)
b. (9, 10)
c. (13, 13)
d. (10, 10)
e. None of the above

3. Point Z
a. (14, 6)
b. (9, 5)
c. (6, 14)
d. (5, 9)
e. None of the above

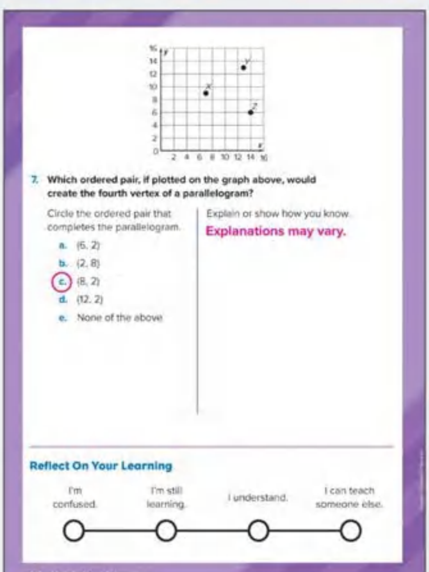
Explain how you determined the ordered pairs in exercises 1–3.

4. Point X
Explanations may vary.

5. Point Y
Explanations may vary.

6. Point Z
Explanations may vary.

Unit 13 • Geometry 217


7. Which ordered pair, if plotted on the graph above, would create the fourth vertex of a parallelogram?

Circle the ordered pair that completes the parallelogram.
a. (6, 2)
b. (2, 8)
c. (8, 2)
d. (12, 2)
e. None of the above

Explain or show how you know.
Explanations may vary.

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

218 Math Probe • Ordered Pairs

Analyze the Probe Formative Assessment

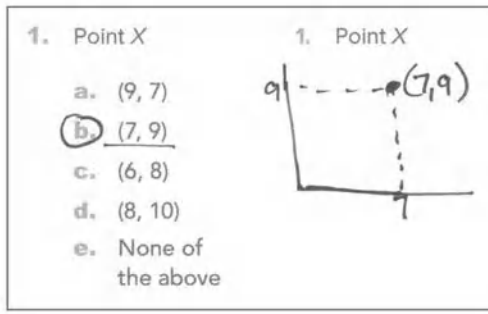
Targeted Concept Points in the coordinate plane are described as an ordered pair comprised of the *x*- and *y*-coordinates of the point's location. The location of a plane figure can also be described using the coordinates of its vertices.

Targeted Misconceptions Some students interchange the *x*- and *y*-coordinates or confuse the *x*- and *y*-axes. Students who have measurement difficulties may incorrectly count along the axes or make labeling errors. This is especially true when not all values along the axes are labeled.

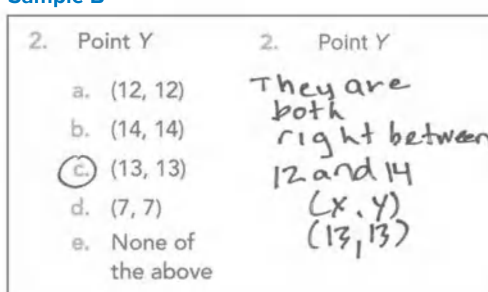
Authentic Student Work

Below are examples of correct students work and explanations.

Sample A

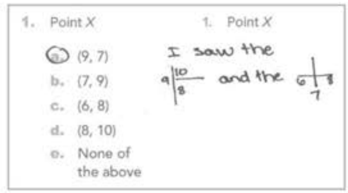
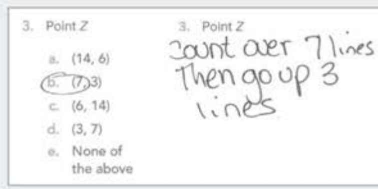
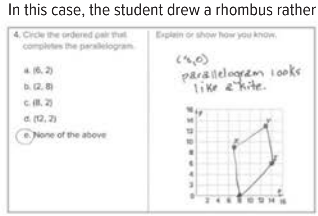


Sample B



Collect and Assess Student Work

Collect and review student responses to determine possible misconceptions. See examples in If-Then chart.

IF incorrect...	THEN the student likely...	Sample Misconceptions
1. a 3. c 4. b	has the x- and y-axes interchanged, or lists the coordinate as (y, x) rather than as (x, y).	In this case, the student interchanges the order. 
1. c, d 2. a, b, d 3. b, d 4. b, d	misinterprets the interval shown on the axes, counts incorrectly or "rounds" up or down to the closest whole number, or counts the spaces.	In this case, the student counts the lines, ignoring the scale. 
1. e 2. e 3. e 4. e	is not able to correctly name the point and has a different method for describing the points than any of those described above. In Exercise 4, the student may not know what a parallelogram looks like.	In this case, the student drew a rhombus rather than a parallelogram. 

Many of the above difficulties result in a combination of correct and incorrect responses.
For correct responses, be sure to check for sound reasoning.

Take Action

Choose from the following resources or suggestions:

- Revisit the graphing activities in Lessons 13-2 and 13-3.
- Have students locate points on horizontal and vertical number lines where some of the locations are not labeled. Connect the number lines to the coordinate plane. Watch for students who count hash marks rather than the intervals between hash marks.
- Create a coordinate plane on the classroom floor and have students walk on it to plot and name ordered pairs.
- Have students create mystery shapes on a coordinate grid and then provide their list of ordered pairs and instructions for a partner to use to construct the mystery shapes.
- Use color-coding to distinguish the two axes. Have students develop and share strategies for recalling which axis is which.

Revisit the Probe After additional instruction, have students review their initial answers to the probe. Use these questions for discussion:

- Are there any answers you would like to change? Explain.
- Are there any questions that you still have about any of the items on this probe?

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

LESSON **13-6**

Classify Quadrilaterals by Properties

Learning Targets

- I can classify quadrilaterals based on their properties into categories and subcategories.
- I can use properties of quadrilaterals to prove or disprove statements about quadrilaterals.

Standards ◇ Major △ Supporting ○ Additional

Content

○ **5.G.B** Classify two-dimensional figures into categories based on their properties.

○ **5.G.B.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

○ **5.G.B.4** Classify two-dimensional figures in a hierarchy based on properties.

Math Practices and Processes

MPP Model with mathematics.

Vocabulary

Math Terms	Academic Terms
hierarchy	accurate
parallelogram	analyze
quadrilateral	
rectangle	
rhombus	
square	
trapezoid	
Venn diagram	

Materials

The materials may be for any part of the lesson.

- *Venn Diagram* Teaching Resource

Focus

Content Objectives	Language Objectives	SEL Objective
• Students classify quadrilaterals into categories and subcategories based on their properties. • Students organize the categories and subcategories into a hierarchy.	• Students explain how to classify quadrilaterals based on their properties using *use* and *share*. • To support maximizing linguistic and cognitive meta-awareness, ELs participate in MLR5: Co-Craft Questions and Problems.	• Students identify the information that is needed or most useful in order to complete a mathematical task.

Coherence

Previous	Now	Next
• Students classified two-dimensional figures based on their sides or angles, and recognized and identified right triangles (Grade 4). • Students classified quadrilaterals into categories based on minimal defining attributes (Unit 13).	• Students understand the categories and subcategories of quadrilaterals using a hierarchy.	• Students generate two numerical patterns using two given rules, form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on the coordinate plane (Unit 14).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students extend their understanding of quadrilaterals by classifying quadrilaterals of various shapes and sizes. *Conceptual Understanding is not a targeted element of rigor for this standard.*	• Students begin to develop proficiency with classifying quadrilaterals based on their properties.	• Students apply their understanding of quadrilaterals to sort quadrilaterals into groups. *Application is not a targeted element of rigor for this standard.*

GO ONLINE

Number Routine
Would You Rather?

5–7 min

Build Fluency Students develop number sense as they compare the values of expressions involving fractions.

These prompts encourage students to talk about their reasoning:

- What strategies did you use to compare the amounts?
- How did you decide whether to calculate or estimate?
- What is another way to compare the amounts?

Launch 5–7 min

Sense-Making Routine

Purpose Students explore the relationships among the categories and subcategories of quadrilaterals.

Notice & Wonder

- What do you notice?
- What do you wonder?

Teaching Tip You may want to remind students of the definitions of *hierarchy*, *category*, and *subcategory* before they begin their discussion.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of the hierarchy of quadrilaterals and are based on possible comments and questions that students may make during the share out.

- How do you know what to call the type of relationship shown in the image?
- Why do you think the categories are organized in this way?
- What are the properties of the shapes listed in this image?

Math is... Mindset

- What helps you know that you have made good decisions?

SEL **Responsible Decision-Making: Evaluate**

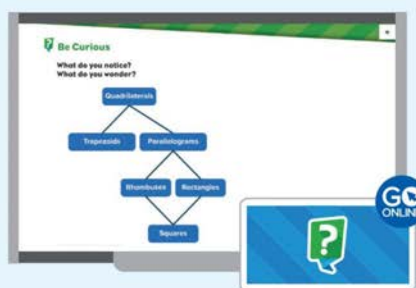
Help students develop strong learning habits by providing them opportunities to practice evaluating their decision-making skills. As students consider the Notice & Wonder routine, invite them to evaluate and share what information is most useful to identify the mathematical task at hand.

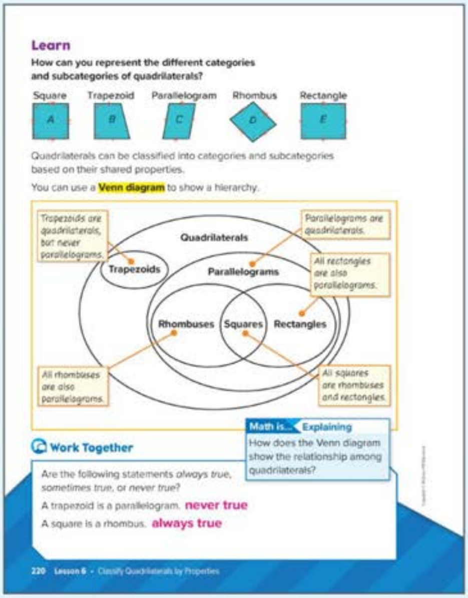
Transition to Explore & Develop

Ask questions that get students thinking about how a hierarchy can show the relationship among quadrilaterals.

ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about how we can organize the categories and subcategories of quadrilaterals using a hierarchy.





Learn

How can you represent the different categories and subcategories of quadrilaterals?

Square Trapezoid Parallelogram Rhombus Rectangle

A B C D E

Quadrilaterals can be classified into categories and subcategories based on their shared properties.

You can use a **Venn diagram** to show a hierarchy.

Math is... Explaining
How does the Venn diagram show the relationship among quadrilaterals?

Work Together

Are the following statements *always true, sometimes true,* or *never true?*

A trapezoid is a parallelogram. **never true**

A square is a rhombus. **always true**

220 Lesson 6 • Classify Quadrilaterals by Properties

❶ Pose the Problem

ETP Pose Purposeful Questions

- What properties of quadrilaterals do you know?
- How do those properties help you name quadrilaterals?
- How did you organize the categories and subcategories of triangles?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Co-Craft Questions and Problems

Pair students and have them co-create a problem similar to the one on the Learn page. Have them work together to solve their problem and then trade their problem with another pair. After each pair solves the other pair's problems, have them form a group of four to check solutions and correct any mistakes that may have been made. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- What are the subcategories of parallelograms? How do you know?
- What is different between the ways you can represent hierarchies? What is similar?

Key Takeaways

- Quadrilaterals can be organized into a hierarchy based on the shared properties of groups of quadrilaterals.
- All quadrilaterals in a category share a set of properties.

Work Together

Students work together to determine whether certain statements about quadrilaterals are always, sometimes, or never true.

Common Error: Students may think that a trapezoid can be classified as a parallelogram as it has one pair of parallel sides. Remind students that parallelograms always have two pairs of parallel sides.

LOM Language of Math

Remind students that two sides of a polygon are congruent if they are of the same length. *Congruent* means "in agreement." So, while in geometry *congruent* refers to the length of sides and shapes, students can use the word in other contexts to describe things that are the same size and shape.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore categories and subcategories of quadrilaterals to develop a hierarchy.

Materials: *Venn Diagram* Teaching Resource

Directions: Provide copies of the *Venn Diagram* Teaching Resource. Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- Which category of quadrilaterals has a property that none of the other quadrilaterals have? Where do you think that category is placed on the Venn diagram?
- Which categories of quadrilaterals share some properties? How does the Venn diagram show shared properties?
- Which category of quadrilaterals shares all of the properties of two different categories? Where do you think that category is placed on the Venn diagram?

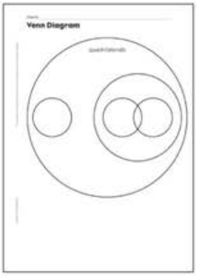
Math is... Modeling

- How are the representations for hierarchies of figures similar? How are they different?

Students map relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas.

Activity Debrief: Have students share their Venn diagrams and discuss how the Venn diagram represents the hierarchy of quadrilaterals.

A PDF of the Teaching Resources are available in the Teacher Digital Center.



Guided Exploration

Students explore the categories and subcategories of quadrilaterals using a hierarchy.

ETP Use and Connect Mathematical Representations

While discussing trapezoids, ask:

- What properties do all trapezoids share with all quadrilaterals?

While discussing parallelograms, ask:

- What properties do all parallelograms share that all quadrilaterals do not?

While discussing rectangles, ask:

- What properties do all rectangles share that all quadrilaterals do not?
- What properties do all rectangles share with all parallelograms?
- Is Rectangles also a subcategory of Quadrilaterals? Why or why not?

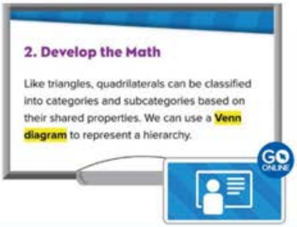
While discussing squares, ask:

- Why are squares also quadrilaterals, parallelograms, rhombuses, and rectangles?
- Why are squares not trapezoids?

Math is... Modeling

- How are the representations for hierarchies of figures similar? How are they different?

Students map relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas.



EL English Learner Scaffolds

Entering/Emerging Explain *shared properties,* Show a picture of a square and rectangle. Say *Let's talk about their shared properties.* Point to each shape and say *They both are rectangles.* Then point out the four sides on each shape and say *They both have four sides.* Point to the four sides. Repeat with new shapes listing different features and asking *Are these shared properties?*

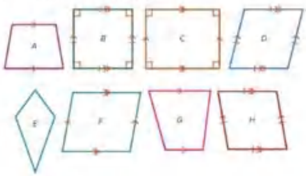
Developing/Expanding Explain *shared properties.* Show a picture of a square and rectangle. Say *Let's talk about their shared properties.* Point to each shape and say *They both are rectangles.* Then point out the four sides on each shape and say *They both have four sides.* Repeat with new shapes and ask students to tell you their shared properties. Provide sentence frames or guidance as necessary.

Bridging/Reaching Guide students to the Learn page and have students read the sentence about shared properties below the table. Ask students what they think that means (same features, etc.). Then show students different shapes and ask them to say what their shared properties are. Validate and correct as necessary.

On My Own

Name ______

Use the figures for Exercises 1–8. Identify the figures that could be classified into each subcategory.



1. quadrilaterals
Figures A, B, C, D, E, F, G, H
2. trapezoids
Figures A, G
3. parallelograms
Figures B, C, D, F, H
4. rectangles
Figures B, C
5. rhombuses
Figures B, D, H
6. squares
Figure B
7. How did you know how to classify each shape? Explain.
Sample answer: I classified the shapes based on number of parallel sides, number of sides of equal length, and number of right angles.
8. Did you classify any shapes into more than one category? If so, explain why. **Yes; Sample answer: All parallelograms are quadrilaterals so those shapes are in both categories; all rectangles are parallelograms so Shape C is in both categories; all rhombuses are parallelograms so those shapes are in both categories; and all squares are rhombuses so Shape B is in both categories.**

Unit 13 • Geometry 221

9. **STEM Connection** Hanna is helping cut some sheets of metal. She needs to cut them so that they have 4 sides with two pairs of parallel sides. Some need to have 4 right angles and some do not. How can she classify the sheets of metal?
The sheets with 4 right angles can be classified as rectangles and the sheets without right angles can be classified as parallelograms.

10. Which quadrilaterals always have 4 right angles?
squares and rectangles
11. Which quadrilaterals always have exactly 1 pair of parallel sides?
trapezoids
12. Which quadrilaterals always have 4 sides of equal length?
rhombuses and squares
13. **Extend Your Thinking** Why can a rectangle also be called a parallelogram?
A rectangle has all the properties of a parallelogram.

Reflect

How can knowing the hierarchy of quadrilaterals help you describe their properties?
Answers may vary.

Math is... Mindset
How did you know that you made good decisions?

222 Lesson 8 • Classify Quadrilaterals by Properties

Practice

ETP **Build Procedural Fluency from Conceptual Understanding**

Common Error: Exercises 1–6, 8 Students may be concerned about putting the same quadrilateral into multiple categories. Remind them that some of the shapes have properties that put them in multiple categories, and that it is acceptable to have the same shape in more than one category or subcategory.

Item Analysis

Item	DOK	Rigor
1–6	1	Procedural Skill & Fluency
7–8	2	Conceptual Understanding
9	2	Application
10—12	2	Conceptual Understanding
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can knowing the hierarchy of quadrilaterals help you describe their properties?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you know that you made good decisions?

Students reflect on how they practiced responsible decision-making.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can classify quadrilaterals based on their properties into categories and subcategories.
- I can use properties of quadrilaterals to prove or disprove statements about quadrilaterals.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Identify quadrilaterals in relation to other quadrilaterals	5.G.B.3
2	2	Identify quadrilaterals in relation to other quadrilaterals	5.G.B.3
3	2	Identify quadrilaterals in relation to other quadrilaterals	5.G.B.3

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

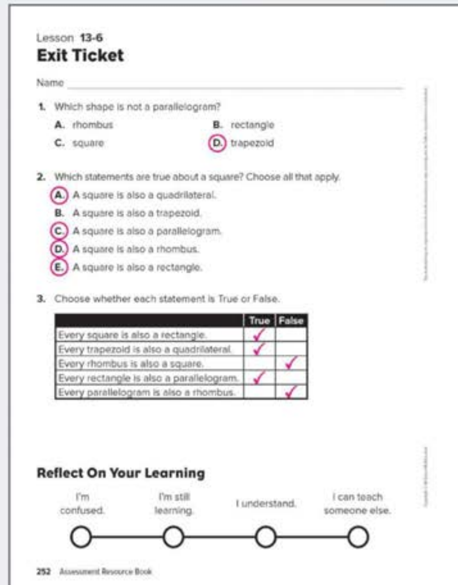
If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 13-6

Exit Ticket

Name ______

1. Which shape is not a parallelogram?

   A. rhombus
   B. rectangle
   C. square
   D. trapezoid

2. Which statements are true about a square? Choose all that apply.

   A. A square is also a quadrilateral.
   B. A square is also a trapezoid.
   C. A square is also a parallelogram.
   D. A square is also a rhombus.
   E. A square is also a rectangle.

3. Choose whether each statement is True or False.

	True	False
Every square is also a rectangle.	✓	
Every trapezoid is also a quadrilateral.	✓	
Every rhombus is also a square.		✓
Every rectangle is also a parallelogram.	✓	
Every parallelogram is also a rhombus.		✓

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

252 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

All or Some

Work with students in pairs. Provide the prompts "All ____ have ____" and "Some are ____." Partners complete the first sentence with a figure and a property of that figure and complete the second sentence with a second figure. For example: *All quadrilaterals have 4 sides. Some quadrilaterals are squares.* Make sure students recognize that different figures may have the same attribute.

GO ONLINE

Take Another Look Lesson

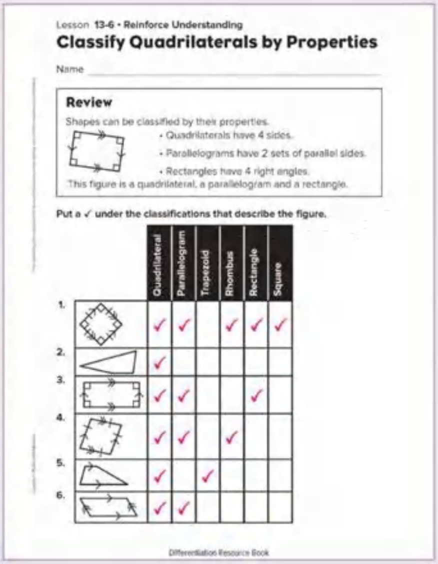
Assign the interactive lesson to reinforce targeted skills.

- Quadrilateral Hierarchy

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 159



Lesson 13-6 • Reinforce Understanding

Classify Quadrilaterals by Properties

Name

Review

Shapes can be classified by their properties.

- Quadrilaterals have 4 sides.
- Parallelograms have 2 sets of parallel sides.
- Rectangles have 4 right angles.

This figure is a quadrilateral, a parallelogram and a rectangle.

Put a ✓ under the classifications that describe the figure.

	Quadrilateral	Parallelogram	Trapezoid	Rhombus	Rectangle	Square
1.	✓	✓		✓	✓	✓
2.	✓					
3.	✓	✓			✓	
4.	✓	✓		✓		
5.	✓		✓			
6.	✓	✓				

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Hierarchy Sort

Students create a hierarchy of 2-dimensional figures.

GO ONLINE

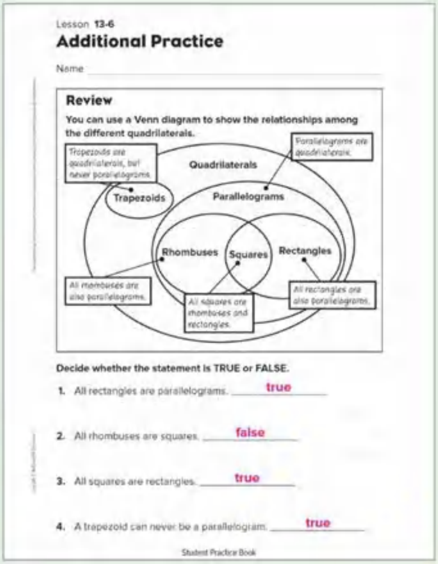
Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 159–160



Lesson 13-6

Additional Practice

Name

Review

You can use a Venn diagram to show the relationships among the different quadrilaterals.

Decide whether the statement is TRUE or FALSE.

1. All rectangles are parallelograms. true
2. All rhombuses are squares. false
3. All squares are rectangles. true
4. A trapezoid can never be a parallelogram. true

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital game to develop division with multi-digit numbers.

Assign

WORKSTATIONS

Use It! Application Station

How Was That Created? Students use Venn diagrams to research the properties of triangles and quadrilaterals in artwork.

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

GO ONLINE

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 159–160

5. A quadrilateral has two pairs of sides that are parallel. The quadrilateral also has four right angles. What shape could it be?
square, rectangle, or parallelogram

6. A quadrilateral has one pair of parallel sides. The quadrilateral also has one right angle. What shape could it be?
trapezoid

7. A quadrilateral has all four sides the same length. The quadrilateral does not have any right angles. What shape could it be?
rhombus or parallelogram

8. A quadrilateral has two pairs of sides that are the same length, but all four sides are not the same length. The quadrilateral does not have any right angles. What shape could it be?
parallelogram

9. Jesse draws a quadrilateral so that two sides measure 8 inches and the other two sides measure 5 inches. The shape has all right angles. What shape could it be?
rectangle or parallelogram

Math @ Home Activity Have yourself and your child create riddles using the descriptions of the quadrilaterals in this lesson. For example, "I have four right angles, my opposite sides are parallel, and my opposite sides are the same length. What am I?" (rectangle). Then exchange riddles and try to determine the type of quadrilateral. Discuss any differences or inaccuracies in the riddles.

Student Practice Book

Differentiation Resource Book, p. 160

Lesson 13-6 • Extend Thinking

Classify Quadrilaterals by Properties

Name ______________________

Is the statement *always* true, *sometimes* true, or *never* true?

1. A square is a rhombus. **always** true
2. A trapezoid is a rectangle. **never** true
3. A quadrilateral is a parallelogram. **sometimes** true
4. A rhombus is a parallelogram. **always** true
5. A parallelogram is a rhombus. **sometimes** true
6. A trapezoid is a parallelogram. **never** true
7. A rhombus is a trapezoid. **never** true
8. A trapezoid is a quadrilateral. **always** true
9. A rectangle is a square. **sometimes** true
10. A square is parallelogram. **always** true

What classification(s) make the statement true. Write all that apply. **Order may vary.**

11. A **square**, **rhombus**, and **rectangle** are always parallelograms.
12. A **trapezoid** is never a parallelogram.
13. A **quadrilateral**, **parallelogram**, and **rhombus** are sometimes a rectangle.
14. A **trapezoid**, **parallelogram**, **square**, **rhombus**, and **rectangle** are always quadrilaterals.
15. A **square** is always a rectangle.

Differentiation Resource Book

Unit Review

Name ____________________

Vocabulary Review

Choose the correct word(s) to complete each sentence.

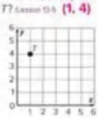
coordinate plane	parallelogram	subcategory	*x*-axis
ordered pair	square	trapezoid	*y*-axis
origin			

1. A **square** is a rectangle with four sides of equal length. (Lesson 13-5)
2. The **ordered pair** (2, 7) names the *x*-coordinate and *y*-coordinate of a point on the coordinate plane. (Lesson 13-1)
3. A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. (Lesson 13-5)
4. The **x-axis** is the horizontal number line on the coordinate plane. (Lesson 13-1)
5. The ordered pair (0, 0) represents the **origin** of the coordinate plane. (Lesson 13-1)
6. A **subcategory** is a subset of shapes of a category that share a certain property. (Lesson 13-4)
7. The **y-axis** is the vertical number line on the coordinate plane. (Lesson 13-1)
8. The **coordinate plane** is formed by a horizontal number line and a vertical number line intersecting and forming a right angle. (Lesson 13-1)
9. A **parallelogram** is a quadrilateral with two pairs of parallel sides. (Lesson 13-5)

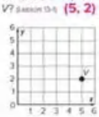
Unit 13 • Geometry 223

Review

10. What ordered pair represents point *T*? (Lesson 13-1) **(1, 4)**



11. What ordered pair represents point *V*? (Lesson 13-1) **(5, 2)**



12. What ordered pair represents the origin? (Lesson 13-1) **(0, 0)**

13. Which axis is used to find the *x*-coordinate? (Lesson 13-1)

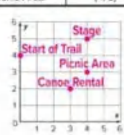
the *x*-axis

14. Which axis is used to find the *y*-coordinate? (Lesson 13-1)

the *y*-axis

15. Plot each location on the coordinate plane. (Lesson 13-2)

Feature	Ordered Pair
Start of Trail	(0, 4)
Canoe Rental	(3, 2)
Stage	(4, 5)
Picnic Area	(4, 3)



16. What are the steps in plotting the point (3, 10) on the coordinate plane? (Lesson 13-2)

Go right 3 units on the *x*-axis. Next, go up 10 units and plot the point.

224 Unit 13 • Review

Students can complete the **Unit Review** to prepare for the **Unit Assessment**. Students may complete the Review in their Interactive eBook in the Digital Students Center.

Vocabulary Review

Item Analysis

Item	Lesson
1	13-5
2	13-1
3	13-5
4	13-1
5	13-1
6	13-4
7	13-1
8	13-1
9	13-5

Review

Item Analysis

Item	DOK	Lesson	Standard
10	1	13-1	5.G.A.1
11	1	13-1	5.G.A.1
12	1	13-1	5.G.A.1
13	1	13-1	5.G.A.1
14	1	13-1	5.G.A.1
15	1	13-2	5.G.A.1
16	1	13-2	5.G.A.1

To review the lessons in this unit, have students watch the Math Replay video in their Digital Student Center.

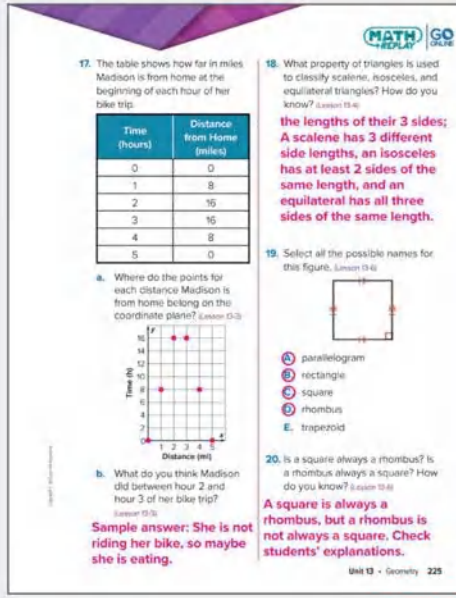
Assign the Unit Review practice to students from the Digital Teacher Center.

GO ONLINE

MATH REPLAY

Item Analysis (continued)

Item	DOK	Lesson	Standard
17a	2	13-3	5.G.A.2
17b	2	13-3	5.G.A.2
18	2	13-4	5.G.B.3, 5.G.B.4
19	1	13-6	5.G.B.3, 5.G.B.4
20	3	13-6	5.G.B.3, 5.G.B.4



MATH REPLAY | GO ONLINE

17. The table shows how far in miles Madison is from home at the beginning of each hour of her bike trip.

Time (hours)	Distance from Home (miles)
0	0
1	8
2	16
3	16
4	8
5	0

a. Where do the points for each distance Madison is from home belong on the coordinate plane? (Lesson 13-3)

b. What do you think Madison did between hour 2 and hour 3 of her bike trip? (Lesson 13-3)

Sample answer: She is not riding her bike, so maybe she is eating.

18. What property of triangles is used to classify scalene, isosceles, and equilateral triangles? How do you know? (Lesson 13-4)

the lengths of their 3 sides; A scalene has 3 different side lengths, an isosceles has at least 2 sides of the same length, and an equilateral has all three sides of the same length.

19. Select all the possible names for this figure. (Lesson 13-6)

A. parallelogram
B. rectangle
C. square
D. rhombus
E. trapezoid

20. Is a square always a rhombus? Is a rhombus always a square? How do you know? (Lesson 13-6)

A square is always a rhombus, but a rhombus is not always a square. Check students' explanations.

Unit 13 • Geometry 225

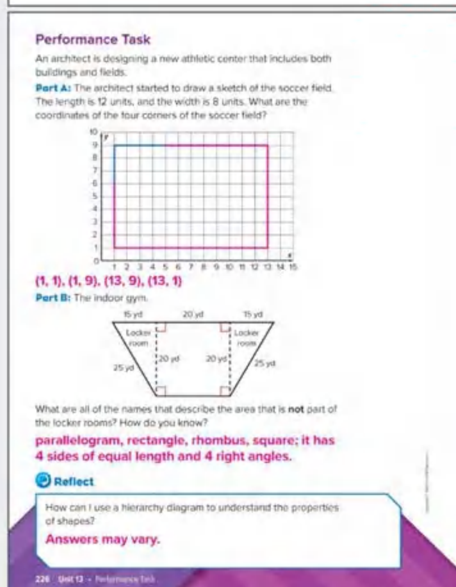
Performance Task

Standards: 5.G.A.1, 5.G.A.2, 5.G.B.3, 5.G.B.4

Rubric (4 points)

Part A (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with plotting points on a coordinate plane and classifying a quadrilateral.
1 POINT	Student's work reflects developing proficiency with plotting points on a coordinate plane and classifying a quadrilateral.
0 POINTS	Student's work shows weak proficiency with plotting points on a coordinate plane and classifying a quadrilateral.

Part B (DOK 3) – 2 points	
2 POINTS	Student's work reflects proficiency with classifying two-dimensional figures.
1 POINTS	Student's work reflects developing proficiency with classifying two-dimensional figures.
0 POINTS	Student's work shows weak proficiency with classifying two-dimensional figures.



Performance Task

An architect is designing a new athletic center that includes both buildings and fields.

Part A: The architect started to draw a sketch of the soccer field. The length is 12 units, and the width is 8 units. What are the coordinates of the four corners of the soccer field?

(1, 1), (1, 9), (13, 9), (13, 1)

Part B: The indoor gym.

What are all of the names that describe the area that is **not** part of the locker rooms? How do you know?

parallelogram, rectangle, rhombus, square; it has 4 sides of equal length and 4 right angles.

Reflect

How can I use a hierarchy diagram to understand the properties of shapes?

Answers may vary.

226 Unit 13 • Performance Task

Reflect

The Reflect question provides an opportunity for students to express their understanding of the unit level focus question.

Unit 13

Fluency Practice

Name ____________________

Fluency Strategy

You can choose any strategy to multiply. You can use an area model, partial products, or a strategy.

Area model

Decompose each factor by place value.

126 × 34

	100 +	20 +	6
30	3,000	600	180
+ 4	400	80	24

Add the partial products.

3,000 + 600 + 180 + 400 + 80 + 24 = 4,284

Fluency Flash

Complete the area model. Then solve.

1. 489 × 7 = 3,423

	400 +	80 +	9
7	2,800	560	63

2. 23 × 67 = 1,541

	60 +	7
20	1,200	140
+ 3	180	21

Unit 13 • Geometry 227

Fluency Check

What is the product?

3. 345 × 19 = 6,555
4. 439 × 76 = 33,364
5. 28 × 76 = 2,128
6. 364 × 5 = 1,820
7. 75 × 86 = 6,450
8. 257 × 32 = 8,224
9. 147 × 28 = 4,116
10. 59 × 48 = 2,832
11. 99 × 4 = 396
12. 284 × 63 = 17,892

Fluency Talk

When using an area model, how is multiplying a 2-digit number by a 1-digit number different than multiplying a 2-digit number by a 2-digit number?

Answers may vary.

When would you choose to use a strategy instead of an area model?

Answers may vary.

228 Unit 13 • Fluency Practice

Fluency practice helps students develop procedural fluency, that is, the "ability to apply procedures accurately, efficiently, and flexibly." Because there is no expectation of speed, students should not be timed when completing the practice activity.

Build Fluency **Objective** Students practice choosing a strategy to multiply.

Fluency Progression

Unit	Skill	Standard
1	Use Partial Sums to Add	4.NBT.B.4
2	Decompose by Place Value to Subtract	4.NBT.B.4
3	Use an Algorithm to Add	4.NBT.B.4
4	Use an Algorithm to Subtract	4.NBT.B.4
5	Choose a Strategy to Add	4.NBT.B.4
6	Choose a Strategy to Subtract	4.NBT.B.4
7	Multiply by Multiples of 10	5.NBT.B.5
8	Multiply by Multiples of 100	5.NBT.B.5
9	Divide Multiples of 10	5.NBT.B.6
10	Divide Multiples of 100	5.NBT.B.6
11	Use an Algorithm to Multiply (2- and 3-Digit Numbers by 1-Digit Numbers)	5.NBT.B.5
12	Use an Algorithm to Multiply (2-Digit Numbers by 2-Digit Numbers)	5.NBT.B.5
13	**Choose a Strategy to Multiply**	**5.NBT.B.5**
14	Choose a Strategy to Multiply	5.NBT.B.5

Fluency Expectations

Grade 4
- Add and subtract within 1,000,000.

Grade 5
- Multiply multi-digit whole numbers.

Grade 6
- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Performance Task

Shapes and the Coordinate Plane

Students draw on their understanding of the coordinate plane, classifying triangles, and classifying quadrilaterals. Use the rubric shown to evaluate students' work.

Standards: 5.G.A.1, 5.G.A.2, 5.G.B.3, 5.G.B.4

Rubric (10 points)

Parts A and B (DOK 3) – 4 points	
4 POINTS	Student's work reflects a proficiency with plotting ordered pairs and classifying triangles/quadrilaterals. The student was able to plot the point(s) and give an accurate location for the point(s).
2 POINTS	Student's work reflects a developing proficiency with plotting ordered pairs and classifying triangles/quadrilaterals. The student was either unable to plot the point(s) or give an accurate location for the point(s).
0 POINTS	Student's work reflects a weak proficiency with plotting ordered pairs and classifying triangles/quadrilaterals. The student was unable to plot the point(s) and give an accurate location for the point(s).

Part C (DOK 1) – 2 points	
2 POINTS	Student's work reflects a proficiency with plotting on the coordinate plane. The student accurately plotted the points and labeled the axes.
1 POINT	Student's work reflects a developing proficiency with plotting on the coordinate plane. The student labeled the axes but did not plot all the points accurately.
0 POINTS	Student's work reflects a weak proficiency with plotting on the coordinate plane. The student did not plot the points accurately or label the axes.

Part D (DOK 3) – 2 points	
2 POINTS	Student's work reflects a proficiency with analyzing the coordinate plane. The student's explanation is accurate.
1 POINT	Student's work reflects a developing proficiency with analyzing the coordinate plane. The student's explanation is partially accurate.
0 POINTS	Student's work reflects a weak proficiency with analyzing the coordinate plane. The student's explanation is not accurate.

Part E (DOK 3) – 2 points	
2 POINTS	Student's work reflects a proficiency with classifying quadrilaterals. The student's drawing and explanation are accurate.
1 POINT	Student's work reflects a developing proficiency with classifying quadrilaterals. Either the student's drawing or explanation is not accurate.
0 POINTS	Student's work reflects a weak proficiency with classifying quadrilaterals. The student's drawing and explanation are not accurate.

Unit 13

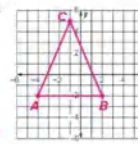
Performance Task

Name ______________________

Shapes and the Coordinate Plane

Part A

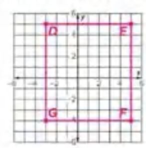
Naji is trying to draw an isosceles triangle. He is given point *A* at (−4, −2) and point *B* at (2, −2). Plot these two points, then help Naji make the isosceles triangle *ABC* by ploting point *C*. Give your coordinates for point *C*. It must be on the grid provided.



Student should give any point on the grid with −1 as the x-coordinate. Example given on the grid has point C at (−1, 5).

Part B

Miguel is trying to draw a rectangle. He is given point *D* at (−3, 5) and point *E* at (5, 5). Plot these two points, then help Miguel make the rectangle *DEFG* by plotting points *F* and *G*. Give your coordinates for points *F* and *G*. They must be on the grid provided.



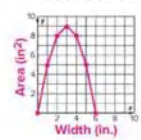
Student should give any two points on the grid with one having 5 as the x-coordinate and the other having −3 as the x-coordinate with their y-coordinates the same. Examples given on the grid have point F at (5, −4) and point G at (−3, −4).

Assessment Resource Book 253

Part C

The table shows the width and corresponding area of rectangles whose perimeter is 12 inches. Use the table to plot ordered pairs. Make sure to label the axes. Connect the points to help you interpret the data.

Width (in.)	Area (in^2)
0	0
1	5
2	8
3	9
4	8
5	5
6	0



Part D

For the rectangles in Part C, can one have an area of 10 in^2? Explain.

The rectangle cannot have an area of 10 in^2. The highest point on the graph has a y-coordinate value of 9. The maximum area the rectangle can have is 9 in^2.

Part E

Naji and Miguel are considering a particular shape. Naji identifies the shape as a rectangle. Miguel identifies the shape as a rhombus. They are both correct. How can this be? What properties does the shape have if both Naji and Miguel are correct? Include a diagram of the shape with your explanation.

In order for the shape to be classified as a rectangle, which is a parallelogram with all right angles, and also a rhombus, which is a parallelogram with all four sides the same length, it must be a square.

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Unit Assessments

Two forms of the Unit Assessment, Form A and Form B, are available for either print or digital administration. The items on the two assessments are parallel items, assessing the same concept and standard. The table below provides the item analysis for both forms.

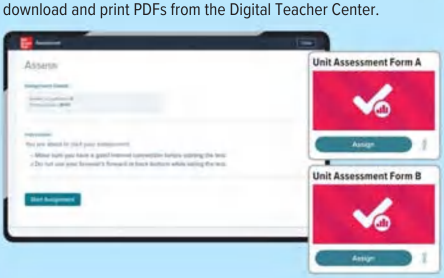
Both Unit Assessments are available in the Assessment Resource Book or as downloadable files from the Digital Teacher Center.

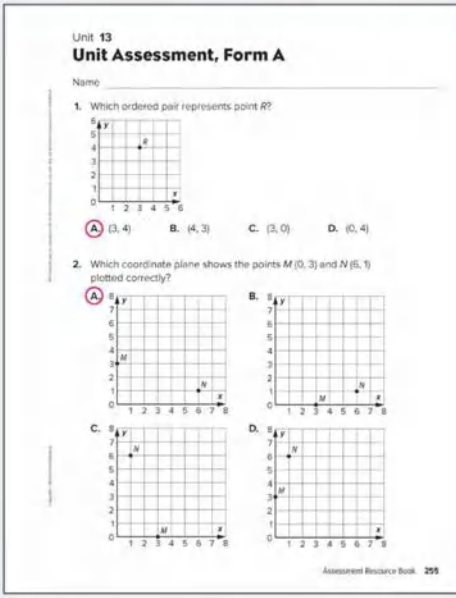
Data When students complete the Unit Assessment in the Digital Student Center, their responses are auto-scored.

Item Analysis

Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	1	13-1	Points on the Coordinate Plane	5.G.A.1
2	1	13-2	Identify Ordered Pairs	5.G.A.1
3	1	13-2	Identify Ordered Pairs	5.G.A.1
4	2	13-3	Real World and Mathematical Problems	5.G.A.2
5	2	13-3	Real World and Mathematical Problems	5.G.A.2
6	2	13-3	Real World and Mathematical Problems	5.G.A.2
7	2	13-4	Recognize Triangles by Sides	5.G.B.4
8	1	13-4	Recognize Triangles by Sides	5.G.B.3
9	2	13-5	Recognize Quadrilaterals	5.G.B.4
10	1	13-5	Describe Quadrilaterals	5.G.B.4
11	1	13-6	Quadrilateral Hierarchy	5.G.B.3
12	1	13-6	Quadrilateral Hierarchy	5.G.B.3
13	3	13-6	Quadrilateral Hierarchy	5.G.B.3

Assign the digital Unit Assessment (Form A or B) to students or download and print PDFs from the Digital Teacher Center.




Unit 13
Unit Assessment, Form A

Name ______

1. Which ordered pair represents point *R*?

(A.) (3, 4) B. (4, 3) C. (3, 0) D. (0, 4)

2. Which coordinate plane shows the points *M* (0, 3) and *N* (6, 1) plotted correctly?

(A.) B. C. D.

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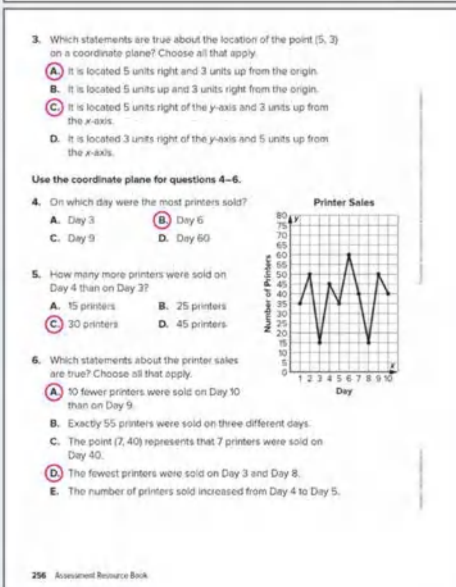
3. Which statements are true about the location of the point (5, 3) on a coordinate plane? Choose all that apply.

(A.) It is located 5 units right and 3 units up from the origin.
B. It is located 5 units up and 3 units right from the origin.
(C.) It is located 5 units right of the *y*-axis and 3 units up from the *x*-axis.
D. It is located 3 units right of the *y*-axis and 5 units up from the *x*-axis.

Use the coordinate plane for questions 4–6.

4. On which day were the most printers sold?

A. Day 3 (B.) Day 6
C. Day 9 D. Day 60

5. How many more printers were sold on Day 4 than on Day 3?

A. 15 printers B. 25 printers
(C.) 30 printers D. 45 printers

6. Which statements about the printer sales are true? Choose all that apply.

(A.) 10 fewer printers were sold on Day 10 than on Day 9.
B. Exactly 55 printers were sold on three different days.
C. The point (7, 40) represents that 7 printers were sold on Day 40.
(D.) The fewest printers were sold on Day 3 and Day 8.
E. The number of printers sold increased from Day 4 to Day 5.

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Unit 13
Unit Assessment, Form A (continued)

Name ______________________________

7. Daniel draws a triangle that has the length of one side equal to 18 centimeters, one side that has a length less than 18 centimeters, and one side that has a length greater than 18 centimeters. Which best describes Daniel's triangle?
   (A) scalene B. isosceles C. equilateral

8. Which statement is true about triangles?
   A. A scalene triangle may also be an isosceles triangle.
   B. An equilateral triangle may also be a scalene triangle.
   (C) An isosceles triangle may also be an equilateral triangle.

9. Faye draws a 4-sided shape that has one pair of parallel sides that are both 15 centimeters. The side lengths of the other pair of parallel sides are both 8 centimeters. Which statements are true about Faye's shape? Choose all that apply.
   (A) The shape is a quadrilateral.
   (B) The shape is a parallelogram.
   C. The shape is a trapezoid.
   D. The shape is a rectangle.
   E. The shape is a rhombus.
   F. The shape is a square.

10. Which statements are true about a rhombus? Choose all that apply.
   (A) A rhombus has 4 sides.
   (B) A rhombus has all sides the same length.
   C. A rhombus has 1 pair of parallel sides.
   (D) A rhombus has 2 pairs of parallel sides.

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11. Which statements are true about a rectangle? Choose all that apply.
   A. A rectangle is also a trapezoid.
   (B) A rectangle is also a quadrilateral.
   (C) A rectangle is also a parallelogram.
   D. A rectangle is also a rhombus.
   E. A rectangle is also a square.

12. Which statements are true about a square? Choose all that apply.
   (A) A square is also a quadrilateral.
   (B) A square is also a parallelogram.
   (C) A square is also a rhombus.
   (D) A square is also a rectangle.
   E. A square is also a trapezoid.

13. What are the relationships among quadrilaterals, trapezoids, and parallelograms? Explain.
   Sample answer: Trapezoids and parallelograms are both quadrilaterals because they both have four sides. Trapezoids are not parallelograms because they have only one pair of parallel sides, and parallelograms must have two pairs of parallel sides.

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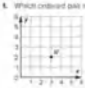
Form B

Unit 13
Unit Assessment, Form B

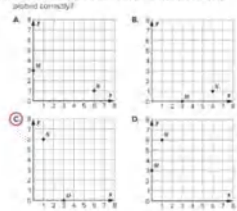
Name ______________________________

1. Which ordered pair represents point M?



   A. (0, 2) B. (2, 3) C. (3, 0) (D) (3, 2)

2. Which coordinate plane shows the points M (3, 0) and N (1, 6) plotted correctly?



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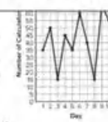

5. How many more calculators were sold on Day 6 than on Day 5?
   A. 15 calculators (B) 25 calculators
   C. 35 calculators D. 60 calculators

6. Which statements are true about the calculator sales? Choose all that apply.
   (A) 10 fewer calculators were sold on Day 10 than on Day 9.
   B. Exactly 25 calculators were sold on four different days.
   C. The point (2, 50) represents that 2 calculators were sold on Day 50.
   (D) The fewest calculators were sold on Day 3 and Day 8.
   (E) The number of calculators sold decreased from Day 6 to Day 7.

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9. Petra draws a 4-sided shape that has one pair of parallel sides that are both 5 inches. The side lengths of the other pair of parallel sides are both 3 inches. Which statements are true about Petra's shape? Choose all that apply.
   A. The shape is a square.
   B. The shape is a rectangle.
   C. The shape is a trapezoid.
   (D) The shape is a parallelogram.
   (E) The shape is a quadrilateral.
   F. The shape is a rhombus.

10. Which statements are true about a rhombus? Choose all that apply.
   (A) A rhombus has 4 sides.
   B. A rhombus has sides of two different lengths.
   (C) A rhombus has 2 pairs of parallel sides.
   (D) A rhombus has all sides the same length.

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are both quadrilaterals because they have four sides. Rectangles are also parallelograms because they have two pairs of parallel sides.

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UNIT 14 PLANNER

Algebraic Thinking

PACING: 10 days

LESSON		MATH OBJECTIVE	LANGUAGE OBJECTIVE	SOCIAL AND EMOTIONAL LEARNING OBJECTIVE
Unit Opener IGNITE! 5-4-3-2-1 Challenge Students explore how expressions can be interpreted in different ways.				
14-1	Write Numerical Expressions	Students write numerical expressions to represent calculations that are described using written statements.	Students explain how to write numerical expressions to represent a given word problem using *should*, *could*, and *use*.	Students exchange ideas for completing a mathematical task with a peer and reflect on the value of their similarities and differences.
14-2	Interpret Numerical Expressions	Students interpret numerical expressions without evaluating the expression.	Students discuss interpreting numerical expressions without evaluating the expression using *similar*, *different*, and *notice*.	Students recognize and respond appropriately to the emotions of others during collaborative math work.
14-3	Evaluate Numerical Expressions	Students use the order of operations to evaluate numerical expressions.	Students talk about using the order of operations to evaluate numerical expressions using the verb *help*.	Students demonstrate self-discipline through working through distractions to complete a mathematical task.
Math Probe Order of Operations Students identify which operation in an expression should be performed first.				
14-4	Numerical Patterns	Students generate two numerical patterns that follow two given rules. Students identify relationships between corresponding terms in the generated number patterns.	Students discuss the relationships between corresponding terms in the generated number patterns using the verbs *represent* and *determine*.	Students exercise creativity by solving a problem using more than one approach.
14-5	Relate Numerical Patterns	Students use a table to arrange corresponding terms of two numerical patterns. Students describe the relationship between corresponding terms in two numerical patterns.	Students discuss relationships between corresponding terms in two numerical patterns using the verbs *identify* and *use*.	Students self-motivate and sustain engagement to work independently to complete a challenging mathematical task.
14-6	Graphs of Numerical Patterns	Students plot ordered pairs consisting of the corresponding terms from two numerical patterns.	Students explain how to plot ordered pairs consisting of the corresponding terms from two numerical patterns using *can* and *should*.	Students discuss alternative strategies/methods for solving a mathematical problem and the value of flexible mathematical thinking.
Unit Review **Fluency Practice**				
Performance Task **Unit Assessment**				

FOCUS QUESTION:
How can I begin to think about algebra?

LESSON	KEY VOCABULARY: Math Terms	KEY VOCABULARY: Academic Terms	MATERIALS TO GATHER	RIGOR FOCUS	STANDARD
14-1	expression grouping symbol numerical expression parentheses	reflect suggest	• number cubes	Conceptual Understanding	5.OA.A.1, 5.OA.A.2
14-2	expression grouping symbol numerical expression parentheses	complex valid	• index cards	Conceptual Understanding	5.OA.A.1, 5.OA.A.2
14-3	evaluate order of operations	accurate contradiction	• cardstock	Conceptual Understanding, Procedural Skill & Fluency	5.OA.A.1
14-4	corresponding term numerical pattern rule (of a pattern)	emphasize transition	• two-color counters	Conceptual Understanding, Procedural Skill & Fluency	5.OA.B.3
14-5	corresponding term numerical pattern rule (of a pattern)	accurate inference	• number cubes	Conceptual Understanding, Procedural Skill & Fluency	5.OA.B.3
14-6	corresponding term numerical pattern	analyze speculate	• blank cubes • *Coordinate Plane* Teaching Resource • index cards	Conceptual Understanding, Procedural Skill & Fluency	5.OA.B.3

Focus

Order of Operations and Patterns on the Coordinate Plane

When evaluating expressions, the order of operations is needed so that everyone gets the same result. It is important for students to understand that we use grouping symbols when we want to do operations in an order that is different from what is required by the Order of Operations. Grouping symbols are also used to help clarify the order of operations when an expression contains many numbers and operations.

In this unit, students generate and extend numerical patterns and identify relationships between each set of corresponding terms.

A *numerical pattern* is an arrangement of numbers where the next number in the arrangement can be predicted based on a rule. A *rule* for a numerical pattern is a statement that describes how the numbers are related. A *sequence* is a list of numbers that follow a rule. A *term* is a number in a sequence. *Corresponding term*s are *terms* that appear in the same position in their respective patterns.

Consider the pattern shown.

Term	Pattern A	Pattern B
1	3	9
2	6	18
3	9	27
4	12	36
5	?	?

In the pattern, 9 and 27 are corresponding terms because eachis the third term in their pattern. The terms in Pattern B are 3 times as great as the corresponding terms in Pattern A. The rule for Pattern A is “Add 3” (Term 5 is 15); the rule for Pattern B is “Add 9” (Term 5 is 45). Using rules and tables to compare and extend two patterns sets the stage for graphing two patterns on a coordinate plane.

Coherence

What Students Have Learned

- **Expressions** Students evaluated simple numerical expressions. (Grade 4)
- **Expressions** Students used a given rule to extend a pattern. (Grade 4)
- **Patterns** Students explained features of patterns that are not obvious based on the rule. (Grade 4)

What Students Are Learning

- **Expressions** Students evaluate numerical expressions with grouping symbols.
- **Expressions** Students write numerical expressions without evaluating them.
- **Patterns** Students generate two numerical patterns from rules, graph them, and identify the relationship between them.

What Students Will Learn

- **Expressions** Students will write, read, and evaluate algebraic expressions. (Grade 6)
- **Expressions** Students will generate equivalent expressions by applying the order of operations. (Grade 6)
- **Patterns** Students will use variables to represent real-world quantities that change in relationship to one another. (Grade 6)

Rigor

Conceptual Understanding

Students develop understanding of
- Writing numerical expressions;
- Using grouping symbols in expressions;
- Generating two numerical patterns from given rules;
- Graphing two numerical patterns from given rules.

Procedural Skill & Fluency

Students build proficiency with
- Writing numerical expressions;
- Generating two numerical patterns from given rules;
- Graphing corresponding terms from numerical patterns.

Application

Students apply their knowledge of
- Expressions and patterns to solve real-world problems.

Application is not a targeted element of rigor for the standards in this unit.

Effective Teaching Practices

Use and Connect Mathematical Representations

In teaching mathematics, it is important to provide students with various representations and approaches to help students gain understanding of the concepts. As students approach concepts from different aspects, they begin to see connections between the written and verbal words and the physical, visual, and symbolic representations. Different types of representations speak differently to students of varying learning styles.

In the early grades, students learn about patterns by physically making them, such as standing, sitting, standing, sitting, etc. As students approach the concept of patterns from different modalities, they develop a broader understanding.

In this unit, students are learning to translate verbal and written expressions to numerical expressions and translate rules to number patterns. Students locate and describe points on the coordinate plane. Students are learning to interpret data on grids and tables.

When students learn to use and connect mathematical representations, they show a deeper understanding of the concepts and become better problem solvers.

- Use numerical expressions in problems to encourage students to discuss what the parts of the expression represent. Then reverse the activity to have students provide situations.
- Have students connect the data in a table with information on a coordinate grid. Have students form conclusions based on the table and compare the utility of displaying information using the different representations.

Math Practices and Processes

Attend to Precision

As students grow in their communication skills, they learn to use more clear and precise language in written and spoken terms. As students develop more understanding of concepts and skills, they develop a more precise vocabulary when referring to the mathematical concepts, diagrams, and figures.

Students use appropriate terminology when referring to expressions, patterns, sequences, graphs, and coordinate planes. They must be careful about identifying a rule for a sequence of numbers in a pattern or identifying the corresponding terms in a pattern.

Remind students that as they work, it is important to pay attention to details, use numbers and symbols precisely, and check that all numbers in a pattern follow the rule. Students need to attend to precision when plotting points and specifying locations of points. If students are not accurate, incorrect information may be communicated.

To help students build proficiency attending to precision, provide them with many opportunities to interact with others and different types of problems.

- Have students create situations in which other students must write numeric expressions to show how they interpret the situation.
- Have students identify a sequence of numbers in which others must identify the rule used.
- Have students plot ordered pairs on the coordinate grid. Make sure they understand that different levels of precision are needed for different purposes.
- Encourage students to communicate their ideas. Accurate communication promotes accurate conclusions.

Social and Emotional Learning

What Skills Will We Develop?

- **Relationship Skills – Communication** (Lesson 14-1): Students who can communicate effectively are more likely to build strong relationships and contribute to a positive classroom culture.
- **Social Awareness – Empathy** (Lesson 14-2): Students who can empathize with others are more able to build positive relationships.
- **Self-Management – Self-Discipline** (Lesson 14-3): Self-disciplined students can manage their impulses to focus on a mathematical task.
- **Self-Awareness – Recognize Strengths** (Lesson 14-4): When students recognize their own strengths, they can see themselves as resourceful and may be more willing to attempt to problem solve and help others.
- **Social Awareness – Respect Others** (Lesson 14-5): When students are respectful of one another, they strengthen their class community.
- **Responsible Decision-Making – Ethical Responsibility** (Lesson 14-6): Understanding rules and routines of the classroom environment can help students be responsible contributors to the learning community.

Unit Overview

Language of Math

Vocabulary

Students will use these key terms in this unit:

- **Corresponding terms*** (Lesson 14-4): Related terms in each of two numerical patterns are called corresponding terms. They have the same term number.
- **Evaluate*** (Lesson 14-3) To evaluate an expression means to find its value.
- **Grouping symbols*** (Lessons 14-1): Grouping symbols are used in numerical expressions to designate the order in which the operations should be completed. Parentheses, brackets, and braces are all types of grouping symbols.
- **Numerical expression*** (Lesson 14-1): A numerical expression contains numbers and operation or grouping symbols, but no unknown values.
- **Numerical patterns*** (Lesson 14-4): Students learn about numeric patterns in primary grades when skip counting, counting multiples, and identifying odd and even numbers.
- **Order of Operations*** (Lesson 14-3): A set of rules which gives the order in which operations are solved in an expression.
- **Parentheses*** (Lessons 14-1): A type of grouping symbol ( ) used in numerical expressions.
- **Rule*** (Lesson 14-4): Used to determine the terms in a pattern such as "add 5."

*This is a new term.

Math Language Development

A Focus on Reading

Reading in mathematics uses more than just words to convey a situation or a problem. Students must learn to adapt how they read to what they are reading and why they are reading it. Because of this, reading for math requires different strategies.

Reading in math is not always looking at words on a page going from left to right and comprehending the situation. It is often reading numbers and symbols, moving both horizontally and vertically in one problem. It is translating words into symbols, scanning information on a graph or a grid to interpret the meaning, or analyzing a diagram or graphic to understand and use to solve problems.

The meaning of a point on a grid or bar on a graph often depends on both its horizontal and vertical position. Keeping in mind the complexity of the math language, it is important to help students develop reading strategies to understand the math and make sense of the problems.

- Help students connect with prior knowledge by asking if they have read similar problems in another lesson.
- Have students discuss with classmates how to interpret a numerical expression based on a given situation.
- Have students take turns reading math rules or scenarios and have classmates write a numerical pattern and justify it for their classmates.
- Work with students to help them develop the vocabulary that relates to and describes operations and the parts of addition, subtraction, multiplication and division problems. Write and label expressions using these names. Then, post these labeled expressions for students to reference as they work.
- Have students write and then read descriptions of how to create ordered pairs from data and plot them as points on a coordinate plane. Have students share their graphs with classmates.

English Language Learner

In this unit, students are provided with a number of scaffolds to support their comprehension of the language used to present and explain strategies related to algebraic thinking. Because many of the words (*ordered*, *order*, *expression, corresponding*) and phrases (*to* ______, *use* ______, *You can* ______ *using* ______, *Use* ______ *to* ______, *write* ______ *as* ______) in this unit could prove challenging to ELs, they are supported in understanding and using them so that the instruction is more accessible.

Lesson 14-1 – *ordered, order*
Lesson 14-2 – *expression*
Lesson 14-3 – *to* ______, *use* ______
Lesson 14-4 – *You can* ______ *using* ______
Lesson 14-5 – *corresponding*
Lesson 14-6 – *write* ______ *as* ______

Unit Routines

Number Routines

Build Fluency The number routines found at the beginning of each lesson help students build number sense and operational fluency. They also help students develop the thinking habits of mind that are important for proficient doers of math.

Can You Make the Number?

Purpose: Build flexible thinking and efficiency with operations.

Overview: Students use all the given numbers to build expressions with a value matching the target number. Students can use a range of operations in their expressions. The teacher records students' expressions, then facilitates a discussion about students' expressions.

What's Another Way to Write It?

Purpose: Build flexibility with number sense and mental math operations.

Overview: Given a number, students generate expressions using operations that, when evaluated, have the same value as the number. The teacher records expressions as students share. Students then look for relationships amongst the expressions.

Would You Rather?

Purpose: Build flexibility with number sense and mental math operations and enhance decision-making.

Overview: Students choose between two options, both of which require mental computation. Students explain their choice and their rationale for their choice.

Sense-Making Routines

- **Notice & Wonder: How are they the same? How are they different?** (Lesson 14-2) Students are presented with 3 expressions and consider how the expressions are similar and different.
- **Notice & Wonder: What do you notice? What do you wonder?** (Lesson 14-5) Students observe two numerical patterns.
- **Notice & Wonder: What do you see?** (Lesson 14-3) Students discuss different ways to think about an image.
- **Notice & Wonder: What questions can you ask?** (Lesson 14-4) Students seek to understand the relationships in the image of a mathematical pattern.
- **Numberless Graph: What math do you see?** (Lesson 14-6) Students think about how the relationship between two patterns is shown in a graph.
- **Numberless Word Problem: What math do you see in this problem?** (Lesson 14-1) Students develop a better understanding of the problem itself by analyzing it without numbers.

Math Language Routines

The Mathematical Language Routines used in this unit give teachers a structured, yet adaptable format for amplifying and developing students' social and academic language. They can be used in ways that support real-time-, peer-, and self-assessment. For more information on the Math Language Routines, see the Appendix.

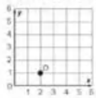
- Lesson 14-1: Students participate in MLR1: Stronger and Clearer Each Time.
- Lesson 14-2: Students participate in MLR5: Co-Craft Questions and Problems.
- Lesson 14-3: Students participate in MLR7: Compare and Connect.
- Lesson 14-4: Students participate in MLR2: Collect and Display.
- Lesson 14-5: Students participate in MLR4: Information Gap.
- Lesson 14-6: Students participate in MLR6: Three Reads.

Unit 14

How Ready Am I?

Name ______________________

1. Which ordered pair represents point *D*?



A. (1, 1) B. (1, 2)
C. (2, 2) D. (2, 1)

2. Pete buys 3 bags of apples. Each bag has 8 apples. He and some friends eat 10 of the apples. How many apples are left?

A. 1 apple B. 4 apples
C. 14 apples D. 34 apples

3. A classroom has 4 rows with 5 desks in each row. In the fifth row there are only 3 desks. How many desks are in the classroom?

A. 27 desks B. 23 desks
C. 20 desks D. 12 desks

4. Which rule describes the pattern?
1, 2, 4, 8, 16, 32, 64

A. Add 1 B. Add 4
C. Multiply by 2 D. Multiply by 4

5. Which are the next three terms in the sequence?
40, 36, 32, 28, 24, ____, ____, ____

A. 18, 12, 6 B. 20, 16, 12
C. 22, 20, 18 D. 28, 32, 36

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6. Which are the first five terms in the pattern?
Start with 5, add 3

A. 3, 6, 9, 12, 15
B. 3, 8, 13, 18, 23
C. 5, 8, 11, 14, 17
D. 5, 10, 15, 20, 25

7. Which pattern gives numbers that have alternate odd and even numbers?

A. Start with 4, add 3
B. Start with 3, add 4
C. Start with 5, multiply by 2
D. Start with 1, multiply by 3

8. Which pattern gives numbers that are all odd numbers?

A. Start with 2, multiply by 3
B. Start with 23, add 3
C. Start with 52, subtract 6
D. Start with 65, subtract 4

9. What is the 6th term in the pattern?
Start with 4, add 7

A. 11 B. 32 C. 39 D. 46

10. What is the 5th term in the pattern?
Start with 73, subtract 5

A. 48 B. 53 C. 58 D. 93

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Administer the Readiness Diagnostic to determine your students' readiness for this unit.

Targeted Intervention

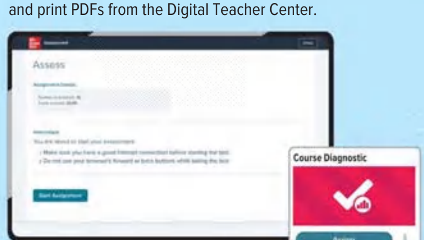
Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	1	Points on the coordinate plane	Points on the Coordinate Plane	5.G.A.1
2	2	Solve two-step problems	Multi-Step Word Problems	4.OA.A.3
3	2	Solve two-step problems	Multi-Step Word Problems	4.OA.A.3
4	2	Analyze numeric patterns	Extend Number Patterns	4.OA.C.5
5	2	Analyze numeric patterns	Extend Number Patterns	4.OA.C.5
6	2	Analyze numeric patterns	Extend Number Patterns	4.OA.C.5
7	2	Analyze numeric patterns	Extend Number Patterns	4.OA.C.5
8	2	Analyze numeric patterns	Extend Number Patterns	4.OA.C.5
9	2	Analyze numeric patterns	Extend Number Patterns	4.OA.C.5
10	2	Analyze numeric patterns	Extend Number Patterns	4.OA.C.5

GO ONLINE

Assign the digital Readiness Diagnostic to students or download and print PDFs from the Digital Teacher Center.



Focus Question

Introduce the Focus Question: *How can I begin to think about algebra?*

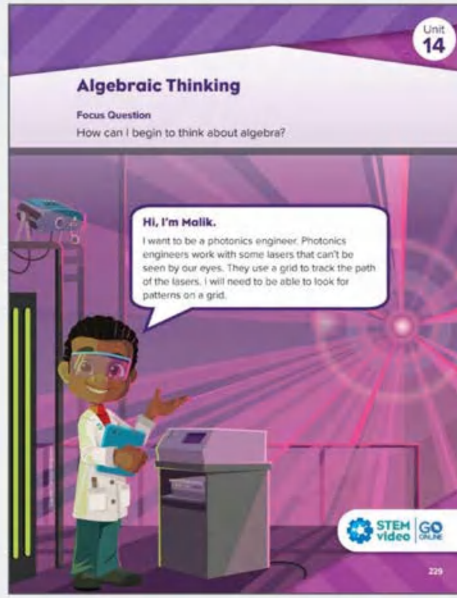
Ask students to think about what they know about algebra?

- What do you know about variables and expressions?
- What do you know about graphing on the coordinate plane?

Remind students that at the end of the unit, they will reflect back on what they learned.

Family Letter

Each letter presents an overview of the math in the unit and home activities to support student learning.



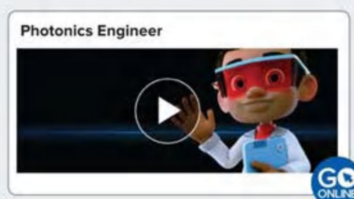
STEM in Action

Videos

Students can watch the two STEM videos.

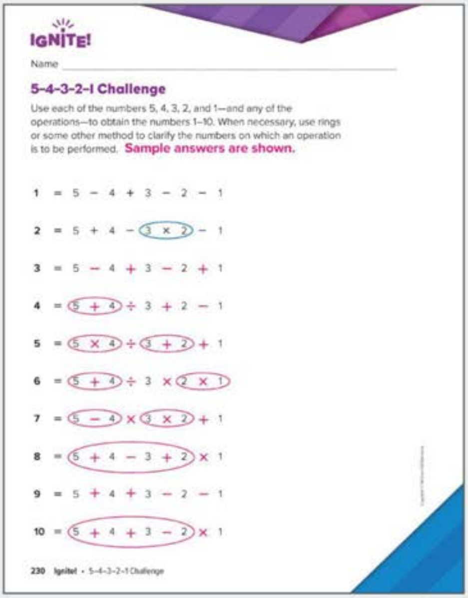
STEM Career: Photonics Engineer Students watch as Malik talks about his aspirations to be a photonics engineer.

Malik Uses a Graph Students watch as Malik and photonics engineers use graphs to help them do their work.



STEM Project Card

Students can complete the STEM project during their workstation time.



IGNITE!

Name

5-4-3-2-1 Challenge

Use each of the numbers 5, 4, 3, 2, and 1—and any of the operations—to obtain the numbers 1–10. When necessary, use rings or some other method to clarify the numbers on which an operation is to be performed. **Sample answers are shown.**

1 = 5 − 4 + 3 − 2 − 1

2 = 5 + 4 − 3 × 2 − 1

3 = 5 − 4 + 3 − 2 + 1

4 = 5 + 4 ÷ 3 + 2 − 1

5 = 5 × 4 ÷ 3 + 2 + 1

6 = 5 + 4 ÷ 3 × 2 × 1

7 = 5 − 4 × 3 × 2 + 1

8 = 5 + 4 − 3 + 2 × 1

9 = 5 + 4 + 3 − 2 − 1

10 = 5 + 4 + 3 − 2 × 1

230 Ignite! • 5-4-3-2-1 Challenge

Ignite!

5-4-3-2-1 Challenge

Students explore how expressions can be interpreted in different ways when the intended order in which the operations should be performed is not clear. Students work on a challenge that sets the stage for work with order of operations.

1. Ask students to consider the following expression and think about how it could lead to confusion.
   - Consider the expression $5 + 7 \times 3$. In what order would you have to perform the operations to get an answer of 26?
   - What other result could you get if you performed the operations in a different order? Explain.
   - Do you think it would be a good idea if expressions could have different answers depending on the order you choose to perform the operations?

2. Have students work in pairs to do the 5–4–3–2–1 Challenge on the student page.
   - The challenge is to use each of the numbers 5, 4, 3, 2, and 1, along with any operations, to get each number 1–10. Notice that the first two have been done for you.
   - Explain why $1 = 5 - 4 + 3 - 2 - 1$.
   - In the second problem, a ring is drawn to clarify the two numbers that are to be multiplied, rather than performing a string of operations beginning with $5 + 4$. Explain how the answer 2 is found.
   - See how many numbers from 3 through 10 you can produce. You may decide that in some cases you need to communicate which numbers go with which operation to avoid confusion. In those cases, use rings or some other method to group the operation with its intended numbers.

3. Have students share and compare their solutions to bring out multiple ways to produce some of the numbers.

You may wish to revisit this Ignite! later in the unit when parenthesis are introduced with order of operations.

Unit Resources At-A-Glance

Workstations

Reveal Math offers rich and varied resources that teachers can use to differentiate and enrich students' instructional experiences with the unit content. The table presents an overview of the resources available for the unit with recommendations for when to use.

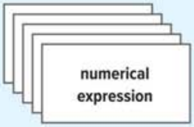
	Activity	Description	Use After Lesson
Game Station	Game Station	Students build understanding of numerical expressions and patterns.	
		• **Numerical Expressions Concentration**	14-1
		• **Numerical Expressions Task Cards**	14-2
		• **Order of Operations Showdown**	14-3
		• **Numerical Patterns Task Cards**	14-4
		• **Patterns on the Coordinate Plane Concentration**	14-5
		• **Patterns on the Coordinate Plane Concentration**	14-6
Digital Station	Digital Game	**Operation Station** Students practice applying the order of operations.	14-1
Application Station	**Have students complete at least one of the Use It! activities for this unit.**		
	STEM Project Card	**A Rule Created That?** Students use coordinate planes and coordinate pairs to create 3-D art.	14-6
	Connection Card	**Color by Number** Students use grid paper to create designs and numerical expressions.	14-3
	Real World Card	**Earning an Income** Students research 5 jobs and incomes, create a table, and plot the results on a coordinate plane.	14-6

Additional Resources

Use the resources below to provide additional support for this unit.

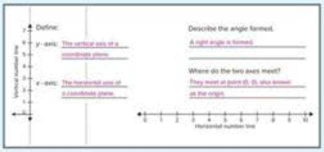
Vocabulary

Use the vocabulary cards to help students learn the vocabulary in this unit. Encourage students to write their own definitions of the key terms on the front side of the card.



Foldables

Use the unit foldables with Lesson 14-6.



Spiral Review

Students can complete the Spiral Review Practice at any point during the unit as either a paper-and-pencil or digital activity.

Lesson	Standard
14-1	5.NBT.A.3
14-2	5.NBT.A.4
14-3	5.NBT.B.6
14-4	5.NF.B.4
14-5	5.NF.B.5
14-6	5.NF.B.7

LESSON **14-1**

Write Numerical Expressions

Learning Target

- I can write numerical expressions to represent calculations that are described using written statements.

Standards ◆ Major ▲ Supporting ● Additional

Content

○ **5.OA.A** Write and interpret numerical expressions.

○ **5.OA.A.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

○ **5.OA.A.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Math Practices and Processes

MPP Attend to precision.

Vocabulary

Math Terms	Academic Terms
expression	reflect
grouping symbol	suggest
numerical expression	
parentheses	

Material

The materials may be for any part of the lesson.

- number cubes

Focus

Content Objective	Language Objectives	SEL Objective
• Students write numerical expressions to represent calculations that are described using written statements.	• Students explain how to write numerical expressions to represent a given word problem using *should*, *could*, and *use*. • To support optimizing output, ELs participate in MLR1: Stronger and Clearer Each Time.	• Students exchange ideas for completing a mathematical task with a peer and reflect on the value of their similarities and differences.

Coherence

Previous	Now	Next
	• Students write numerical expressions to represent calculations that are described using written statements.	• Students interpret numerical expressions without evaluating the expression (Unit 14). • Students write, read, evaluate, and generate and identify equivalent expressions in which letters stand for numbers (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of expressions as they begin to notice equations are two connected expressions.	• Students build procedural skill when interpreting numerical expressions. *Procedural Skill & Fluency is not a specific element of rigor for this standard.*	• Students apply understanding of numerical expressions when interpreting problems. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
Would You Rather?

5–7 min

Build Fluency Students develop number sense as they compare products of fractions and whole numbers.

These prompts encourage students to talk about their reasoning:

- What strategies did you use to solve the problem?
- How can you tell how many sandwiches each expression represents?

Launch 5–7 min

Sense-Making Routine

Purpose Students develop a better understanding of the underlying structure of a problem.

Numberless Word Problem

- What math do you see in this problem?

Teaching Tip You may want to have students write the math that they see on their own before beginning class discussion, that they can then revisit as they work their way through the problem in this lesson.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of writing numerical expressions and are based on possible comments and questions that students may make during the share out.

- What kind of information would you need to know in order to solve this problem? Explain.
- How can you determine which operation you could use to solve this problem? How did you reach that conclusion?

Math is... Mindset

- Why is it important to speak clearly and concisely?

SEL Relationship Skills: Communication

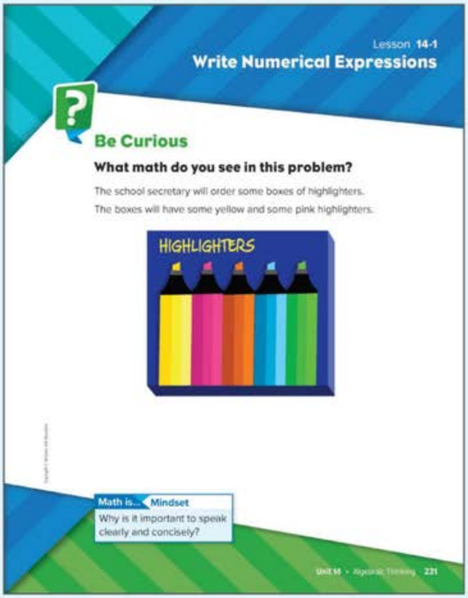
As students engage in collaborative discourse around the Numberless Word Problem routine, remind them that valuing the ideas of others is an important part of being an effective and respectful communicator. Students who can communicate effectively are more likely to build strong relationships and contribute to a positive classroom culture. Explain that one way to do this is by listening attentively when others are sharing their ideas about what math they see in the problem.

Transition to Explore & Develop

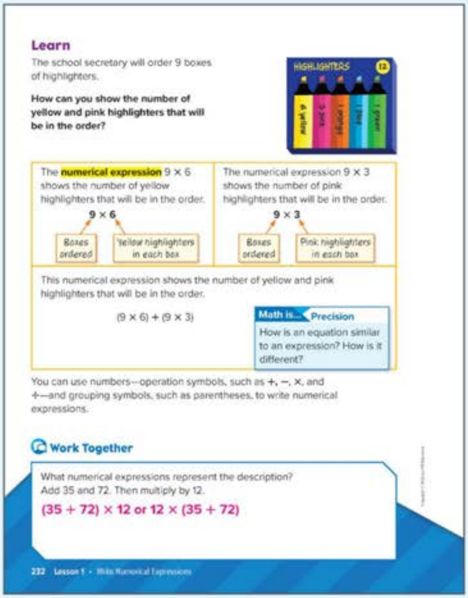
Help students make a connection between words and symbols. Ask questions that get students thinking about how they might be able to represent words with mathematical symbols.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can write numerical expressions that are described using words.



Explore & Develop 20 min



Learn

The school secretary will order 9 boxes of highlighters.

How can you show the number of yellow and pink highlighters that will be in the order?

The **numerical expression** 9 × 6 shows the number of yellow highlighters that will be in the order. **9 × 6** Boxes ordered; Yellow highlighters in each box	The numerical expression 9 × 3 shows the number of pink highlighters that will be in the order. **9 × 3** Boxes ordered; Pink highlighters in each box

This numerical expression shows the number of yellow and pink highlighters that will be in the order.

$(9 \times 6) + (9 \times 3)$

Math is... Precision
How is an equation similar to an expression? How is it different?

You can use numbers—operation symbols, such as +, −, ×, and ÷—and grouping symbols, such as parentheses, to write numerical expressions.

Work Together

What numerical expressions represent the description? Add 35 and 72. Then multiply by 12.

$(35 + 72) \times 12$ or $12 \times (35 + 72)$

232 Lesson 1

❶ Pose the Problem

ETP Pose Purposeful Questions

- How does the image help you understand the problem?
- What are some ways you could represent the problem? Explain.

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Stronger and Clearer Each Time

Pair students and have them work on a problem like the problem on the Learn page. Have them individually write about how to solve the problem. Then have them share their writing with their partner and fix mistakes. Revisit the routine throughout the lesson for reinforcement.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- What is a numerical expression?
- How are grouping symbols used in a numerical expression? Why might you need grouping symbols?
- How can you represent a problem given in words using a numerical expression?

Key Takeaways

- Numerical expressions consist of numbers, operations, and, if needed, grouping symbols.
- Numerical expressions can be used to record calculations that are described using verbal or written statements.

Work Together

Students work together to write a numerical expression when given a written description of a calculation.

Common Error: Students may be unsure of how to group the numbers in the numerical expression. Encourage students to look at the description of the calculation to help them understand how to use grouping symbols. Ask: *Which calculation would you perform first? Which would you perform second? How can you use grouping symbols to show the order of calculation?*

LOM Language of Math

Explain to students that, just as they can use an expression of words that conveys a specific meaning, an expression in math is like a sentence that tells a specific value. Expressions in language consist of words and punctuation, while expressions in math consist of numbers and symbols.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore interpreting numerical expressions as verbal descriptions.

Directions: Have students work in pairs. Each student writes a numerical expression that uses at least two operations. Then, pairs work together to create a verbal description that can be represented by each numerical expression. Remind students that they should not evaluate the numerical expressions, but rather only describe them.

ETP Support Productive Struggle

- What operation will you use in your numerical expression?
- What numbers will you use in your numerical expression?
- Are there any quantities that are grouped together? How is this shown in the numerical expression? How is this shown in the description?

Math is... Precision

- How is an equation similar to an expression? How is it different?

Students communicate precisely to others.

Activity Debrief: Have students share the numerical expressions and descriptions they wrote. Ask students how they knew when some quantities needed to be grouped together. Explain that grouping symbols, such as parentheses, can be used.

Have students revisit the Pose the Problem question and discuss answers.

- How can you show the number of yellow and pink highlighters that will be in the order?

Guided Exploration

Students write a numerical expression to represent a given word problem.

ETP Use and Connect Mathematical Representations

- **Think About It:** Why is multiplication used to show the number of yellow highlighters?
- How is an equation made of expressions?

Have the students create the numerical expression that shows the number of pink highlighters. Ask:

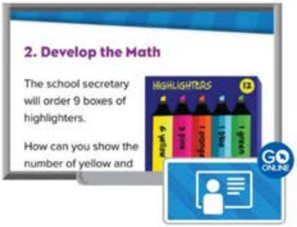
- What should the operation be? Why?
- How should the numbers appear in the numerical expression? Why?
- Should there be an equal sign in the numerical expression? Why or why not?

- **Think About It:** How do you know (9 × 6) + (9 × 6) is a numerical expression?

Math is... Precision

- How is an equation similar to an expression? How is it different?

Students communicate precisely to others.



EL English Learner Scaffolds

Entering/Emerging Explain *ordered/order.* Tell students *I'm hungry.* (Rub your stomach.) *I'm going to order a pizza and a drink.* Pretend to order on the phone. After completing the order, say *I ordered a pizza and a drink.* Then say *A pizza and soda were included in the order.* Repeat with a new order, and ask *Were [sandwiches] or [chicken] included in the order?*

Developing/Expanding Explain *ordered/order.* Tell students *I'm hungry.* (Rub your stomach.) *I'm going to order a pizza and a drink.* Pretend to order on the phone. After completing the order, say *I ordered a pizza and a drink.* Then say *A pizza and soda were included in the order.* Repeat with a new order and ask students to complete the following sentence: *[Sandwiches] were included in the* _____ (order).

Bridging/Reaching Guide students to the Learn page and ask them to review *ordered* on the page. Then ask students to explain the meaning of the verb *order* as it pertains to the Learn lesson (request something). Discuss with students the other meaning of *order* (arrange).

Practice & Reflect 10 min

On My Own

Name

What numerical expression represents the description?

1. Divide 40 by 5. Then, subtract 2.
   $(40 \div 5) - 2$
2. Multiply 4 and 8. Then, add 7.
   $(4 \times 8) + 7$
3. Add $2\frac{1}{2}$ and $4\frac{2}{3}$. Then, subtract $\frac{1}{8}$.
   $(2\frac{1}{2} + 4\frac{2}{3}) - \frac{1}{8}$
4. Add 4.8 and 5.6. Then, subtract the sum from 16.9.
   $16.9 - (4.8 + 5.6)$
5. Subtract $4\frac{1}{4}$ from $10\frac{2}{5}$. Then, divide by 3.
   $(10\frac{2}{5} - 4\frac{1}{4}) \div 3$
6. Subtract 8 from 32. Then, divide 48 by the difference.
   $48 \div (32 - 8)$
7. Add 6.7 and 8.25. Then, multiply by 11.2.
   $(6.7 + 8.25) \times 11.2$
8. Divide 24 by 6. Multiply 5 and 7. Then, add the quotient and the product.
   $(24 \div 6) + (5 \times 7)$
9. **Error Analysis** Christine is planting 48 marigolds. She will plant 12 of the flowers in pots and the rest in rows of 4 plants each. She wrote this numerical expression to represent the number of plants in each row.
   $48 - (12 \div 4)$
   How do you respond to Christine?
   The grouping symbols should be around 48 − 12 because that is the difference you need to find before dividing by 4.

10. The school cafeteria is making snack packs. Each pack will have the number of carrot sticks and celery sticks shown. What numerical expression represents how many carrot sticks and celery sticks are needed to make 25 snack packs?

   Sample answer: $(25 \times 4) + (25 \times 3)$
11. The principal is making 50 new student packets. Each packet contains 12 pencils and 5 pens. What numerical expression represents how many pencils and pens the principal needs to make the packets?
   Sample answer: $(12 + 5) \times 50$
12. Katie makes 49 cookies. She gives 4 to her sister and then divides the cookies up equally to give to her 9 friends. What numerical expression represents how many cookies each of her friends will get?
   $(49 - 4) \div 9$
13. **Extend Your Thinking** Write your own description and numerical expression.
   Sample answer: $(3 + 2) \times (5 + 10)$; the sum of three and two, times the sum of five and ten.

Reflect

How did you think like a mathematician while writing numerical expressions?

Answers may vary.

Math is... Mindset
How did speaking clearly and concisely help you share your ideas?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 4 Students may think that they should be subtracting 16.9 from the sum of 4.8 and 5.6. Encourage them to reread the description and ask, *In what order should you write the expression?*

Item Analysis

Item	DOK	Rigor
1–8	1	Procedural Skill & Fluency
9	3	Conceptual Understanding
10–12	2	Application
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How did you think like a mathematician while writing numerical expressions?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did speaking clearly and concisely help you share your ideas?

Students reflect on how they developed stronger relationship skills.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can write numerical expressions to represent calculations that are described using written statements.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Determine numerical expressions	5.OA.A.1, 5.OA.A.2
2	2	Determine numerical expressions	5.OA.A.1, 5.OA.A.2
3	2	Write numerical expressions	5.OA.A.1, 5.OA.A.2

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking

Lesson 14-1

Exit Ticket

Name

A coach buys 5 boxes of yogurt for team snacks. Each box contains 6 strawberry yogurts, 4 blueberry yogurts, and 8 vanilla yogurts.

1. Which is the numerical expression that represents the total number of strawberry and vanilla yogurts the coach buys?
   A. $(5 \times 6) + (5 \times 4)$ B. $(5 \times 6) + (5 \times 8)$
   C. $6 + 8 \times 5$ D. $5 \times 6 + 8$
2. Which is the numerical expression that represents the total number of strawberry and blueberry yogurts the coach buys?
   A. $6 + 4 \times 5$ B. $(5 \times 8) + (5 \times 4)$
   C. $5 \times 6 + 4$ D. $(5 \times 6) + (5 \times 4)$
3. Katie makes gift baskets. She has 134 scented soaps. She finds that 6 soaps are broken and can't be used. If she uses 8 scented soaps in each gift basket, write an expression that shows the number of gift baskets Katie will be able to make from the non-broken soaps.
   $(134 - 6) \div 8$

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 265

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Roll It, Write It!

Work with students in small groups. Write 2 numerical expressions to use as a scaffold. Have a volunteer roll 4 number cubes and complete the scaffold with those 4 digits. Then have students take turns saying the expression in words. Make sure students understand that there may be more than one correct way to read an expression.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Write Numerical Expressions

INDEPENDENT WORK

Differentiation Resource Book, p. 161

Lesson **14-1 • Reinforce Understanding**

Write Numerical Expressions

Name ______________________

Review

Grouping symbols, such as ( ), tell which expression to simplify first.

Consider the following description:
Subtract 3 from 9. Then divide 72 by the difference.

• 3 should be subtracted from 9 first, so 9 − 3 should be placed in ( ).	(9 − 3)
• 72 is then divided by the difference.	72 ÷ (9 − 3)

72 ÷ (9 − 3) is the numerical expression for the description:
Subtract 3 from 9. Then divide 72 by the difference.

Match the description with its numerical expression.

Description	Numerical Expression
Subtract 2 from 3. Then divide 12 by the difference.	12 − (2 ÷ 3)
Subtract 2 from 3. Then divide by 12.	(3 − 2) ÷ 12
Multiply 2 by 3. Divide 12 by 2. Then subtract the quotient from the product.	(2 + 3) × 12
Divide 2 by 3. Then multiply the quotient by 12.	(2 ÷ 3) × 12
Add 2 and 3. Then multiply by 12.	(2 × 3) − (12 ÷ 2)
Subtract 3 from 12. Then add the difference to 2.	12 ÷ (3 − 2)
Multiply 2 by 3. Divide 12 by 2. Then subtract the product from the quotient.	(12 ÷ 2) − (2 × 3)
Divide 2 by 3. Then subtract the quotient from 12.	(12 − 3) + 2

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Numerical Expressions Concentration

Students practice matching situations with numerical expressions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 161–162

Lesson **14-1**

Additional Practice

Name ______________________

Review

You can use numbers, operation symbols, such as +, −, ×, and ÷, and grouping symbols, such as ( ), to write numerical expressions.

Charissa cuts up some oranges into 30 slices. She gives 2 slices to her sister and then divides the remaining slices equally among 4 friends. Write a numerical expression to represent how many orange slices each friend will get.

First, subtract 2 from 30: 30 − 2

Then divide the result by 4: (30 − 2) ÷ 4

Each friend will receive the number of orange slices represented by the numerical expression (30 − 2) ÷ 4.

What numerical expression represents the description?

1. Multiply 6 and 7. Then add 5.
6 × 7 + 5 or (6 × 7) + 5

2. Subtract 2 from 8. Multiply the difference by 3.
(8 − 2) × 3

3. Add 4 and 7. Then divide 44 by the sum.
44 ÷ (4 + 7)

4. Divide 18 by 3. Multiply 4 and 5. Then add the quotient and the product.
(18 ÷ 3) + (4 × 5)

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital page to develop an understanding of the order of operations.

Use It! Application Station

A Rule Created That? Students use coordinate planes and coordinate pairs to create 3-D art.

The content of this card has concepts covered later in Lesson 14-6. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

INDEPENDENT WORK

Student Practice Book, pp. 161–162

5. Each bag of nuts and raisins contains 6 ounces of nuts and 4 ounces of raisins. Write a numerical expression to represent how many ounces of nuts and raisins are needed to make 20 bags of nuts and raisins.
   $(20 \times 6) + (20 \times 4)$

6. Kristin cuts several apples into 46 slices. She gives 6 to her brother and then divides the remaining apple slices equally among her 5 friends. Write a numerical expression to represent how many apple slices each of her friends will get.
   $(46 - 6) \div 5$

7. Greta plants her flowers in 5 rows of 8 plants, and then plants the remaining 3 flowers in another row. Write a numerical expression to represent how many flowers Greta planted.
   $(5 \times 8) + 3$

8. A set of pens contains pens that write with different colors of ink: 4 blue, 3 black, 2 red, and 1 purple. Write a numerical expression to represent how many pens a teacher will have if 12 sets of pens are ordered.
   $(12 \times 4) + (12 \times 3) + (12 \times 2) + (12 \times 1)$

Math @ Home Activity Identify situations in your everyday experiences where packages of different items may be purchased. Have your child identify the number of different items in each package. Then ask them to write a numerical expression to determine the number of specific items that will be in a certain number of packages.

Student Practice Book

Differentiation Resource Book, p. 162

Lesson 14-1 • Extend Thinking

Write Numerical Expressions

Name ______________________

Match the equivalent expressions. In column B, write how you could know the descriptions are equivalent expressions without fully solving them. Sample answers are shown

Column A	Column B
Subtract 3 from 7. Then multiply the difference by 8.	Multiply 3 and 3. Then subtract the quotient from 14. In both descriptions, 9 is subtracted from 14.
Add 2 and 6. Then divide 72 by the sum.	Multiply 4 and 5. Then add the product to 18. In both descriptions, 20 and 18 are added together.
Divide 72 by 8. Then subtract the quotient from 14.	Add 4 and 5. Subtract 7 from 9. Multiply the sum & difference. Both describe the product of 9 and 2.
Multiply 3 and 6. Then add the product to 20.	Subtract 10 from 14. Then multiply the difference by 8. In both, a difference of 4 is multiplied by 8.
Multiply 3 by 6. Divide 20 by 10. Add the product and the quotient.	Multiply 2 by 9. Divide 14 by 7. Add the product & quotient. Both describe the sum of 18 and 2.
Add 8 and 1. Subtract 3 from 5. Multiply the sum and the difference.	Multiply 2 and 4. Then divide 72 by the product. In both descriptions, 72 is divided by 8.

Differentiation Resource Book

LESSON **14-2**

Interpret Numerical Expressions

Learning Target

- I can interpret numerical expressions without evaluating them.

Standards

◆ Major ▲ Supporting ● Additional

Content

○ **5.OA.A** Write and interpret numerical expressions.

○ **5.OA.A.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

○ **5.OA.A.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
expression	complex
grouping symbol	valid
numerical expression	
parentheses	

Material

The materials may be for any part of the lesson.

- index cards

Focus

Content Objective	Language Objectives	SEL Objective
• Students interpret numerical expressions without evaluating the numerical expression.	• Students discuss interpreting numerical expressions without evaluating the numerical expression using *similar, different,* and *notice*. • To support maximizing linguistic and cognitive meta-awareness, ELs participate in MLR5: Co-Craft Questions and Problems.	• Students recognize and respond appropriately to the emotions of others during collaborative math work.

Coherence

Previous	Now	Next
• Students wrote numerical expressions to represent calculations that are described using written statements (Unit 14).	• Students interpret numerical expressions without evaluating the numerical expression.	• Students use the order of operations to evaluate numerical expressions (Unit 14). • Students write, read, evaluate, and generate and identify equivalent expressions in which letters stand for numbers (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students develop their understanding for how a numerical expression can represent the relationship between several values in a real-world context.	• Students gain proficiency as they practice interpreting numerical expressions. *Procedural Skill & Fluency is not a specific element of rigor for this standard.*	• All of the numerical expressions are interpreted within a real-world context. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
What's Another Way to Write It?

5–7 min

Build Fluency Students build number sense as they write three sums to represent the given fraction.

These prompts encourage students to talk about their reasoning:

- How did you determine numbers that have the given sum?
- How can you check your answers to be sure that they are correct?
- How are the different expressions with the same sum related?
- Whatis another way to think about the problem?

Purpose Students are presented with three numerical expressions and consider how the numerical expressions are similar and different.

Notice & Wonder™

- How are they the same?
- How are they different?

Teaching Tip Make sure students are not attempting to calculate each numerical expression. Rather, they should be exploring the numbers and symbols in each numerical expression and how they are similar to and different from one another.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' understanding of interpreting numerical expressions without evaluating them and are based on possible comments and questions that students may make during the share out.

- How do the numbers in the numerical expressions tell you what is similar and different about the numerical expressions?
- What do you notice about the operation symbols in each numerical expression?
- What do notice about the grouping symbols in each numerical expression?

Math is... Mindset

- How do you show you understand how others are feeling?

SEL Social Awareness: Empathy

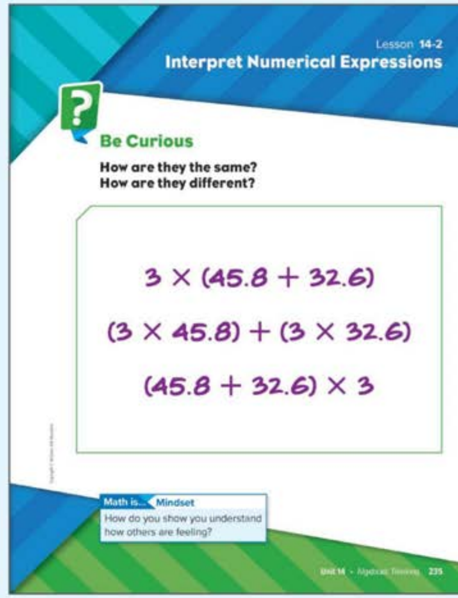
After the Notice & Wonder routine, invite students to share and discuss the emotions they have experienced as they worked to determine how the expressions were similar and how they were different. Collectively discuss how these emotions may make them feel or behave with empathy. Engaging in open discourse about their feelings can help students recognize, understand, and respond with empathy to the emotions of others.

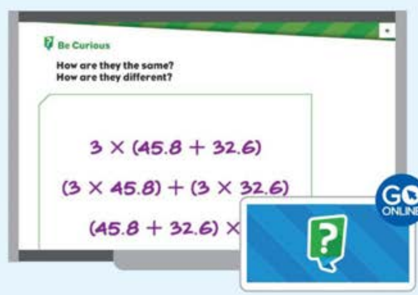
Transition to Explore & Develop

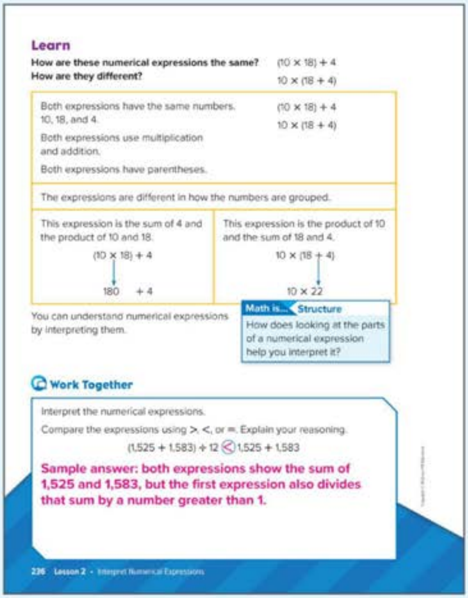
Guide students to think about how numerical expressions can be alike and how they can be different. Ask questions that get students thinking about what numerical expressions represent.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how we can interpret numerical expressions.







Learn

How are these numerical expressions the same? How are they different? (10 × 18) + 4 10 × (18 + 4)

Both expressions have the same numbers, 10, 18, and 4. (10 × 18) + 4 10 × (18 + 4)

Both expressions use multiplication and addition.

Both expressions have parentheses.

The expressions are different in how the numbers are grouped.

This expression is the sum of 4 and the product of 10 and 18.
(10 × 18) + 4
180 + 4

This expression is the product of 10 and the sum of 18 and 4.
10 × (18 + 4)
10 × 22

You can understand numerical expressions by interpreting them.

Math is... Structure
How does looking at the parts of a numerical expression help you interpret it?

Work Together

Interpret the numerical expressions.
Compare the expressions using >, <, or =. Explain your reasoning.
(1,525 + 1,583) ÷ 12 < 1,525 + 1,583

Sample answer: both expressions show the sum of 1,525 and 1,583, but the first expression also divides that sum by a number greater than 1.

236 Lesson 2 • Interpret Numerical Expressions

❶ Pose the Problem

ETP Pose Purposeful Questions

- What do you know about grouping symbols?
- Based on what you know about mathematics, can you make a conjecture?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Co-Craft Questions and Problems

Pair students and have them co-create a problem similar to the one on the Learn page. Have them work together to solve their problem and then trade their problem with another pair. After each pair solves the other pair's problems, have them form a group of four to check solutions and correct any mistakes.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you use the symbols in numerical expressions to interpret them?
- How can you compare numerical expressions by interpreting them?

Key Takeaways

- Numerical expressions show a relationship between and among quantities.
- Numerical expressions can be interpreted in terms of the relationship between and among quantities.

Work Together

Students may think the numerical expression on the left is greater because it contains multiple operations. Remind students to think about what happens during each operation and whether the result will be greater or lesser.

Common Misconception: Students may attempt to solve the problem by evaluating the numerical expressions. Remind them that they can solve the problem without evaluating by interpreting the numerical expressions.

LOM Language of Math

Students may have seen *interpret* in a non-math context. An interpreter translates one language to another. When you interpret expressions you translate from symbols to words.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how two numerical expressions are similar and different.

Directions: Have students work together to solve the Pose the Problem. Encourage students to use their understandings of parentheses, operations, and quantities to determine whether the numerical expressions represent the same value. Students should not evaluate the numerical expressions.

ETP Support Productive Struggle

- How can you describe what is happening in each numerical expression?
- How are your descriptions of the numerical expressions different?
- How can your descriptions of the numerical expressions help you compare them?

Math is... Structure

- How does looking at the parts of a numerical expression help you interpret it?

Students see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

Activity Debrief: Have students share the similarities and differences they discovered. Ask if any groups were able to make valid comparisons of the numerical expressions without evaluating.

Guided Exploration

Students explore how expressions are different by interpreting them.

ETP Facilitate Meaningful Mathematical Discourse

Have the students determine the similarities. Ask:

- What do you notice about the numbers in the numerical expressions?
- What do you notice about the operations in the numerical expressions?

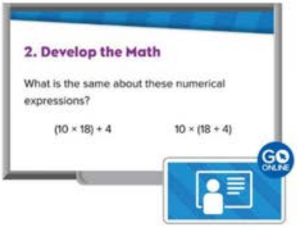
Have the students determine the differences. Ask:

- What do you notice about the grouping symbols in the numerical expressions?
- Why are $(10 \times 18) + 4$ and $10 \times (18 + 4)$ numerical expressions and not equations?
- **Think About It:** What do the grouping symbols tell you about the numerical expression $(10 \times 18) + 4$?
- **Think About It:** Why can you not interpret $(10 \times 18) + 4$ as *ten times eighteen plus four*?

Math is... Structure

- How does looking at the parts of a numerical expression help you interpret it?

Students see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.



EL English Learner Scaffolds

Entering/Emerging Explain the meaning of *expression*, as used in the lesson. Ask students what other meanings of *expression* they know. Have them mention facial expressions and make some of them. Tell students that there are other meanings of *expression*, and ask, *What types of feelings can you express?* Elicit from students that the meaning of *expression* in the lesson is quite different from the other meanings of *expression*.

Developing/Expanding Explain the meaning of *expression*, as used in the lesson. Ask students what other meanings of *expression* they know. Have them mention facial expressions and make some of them. Tell students that there are other meanings of *expression*, and ask, *What types of feelings can you express?* Elicit from students that the meaning of *expression* in the lesson is quite different from the other meanings of *expression*. Provide students with cards with numbers and operational signs on them and have them form *expressions*.

Bridging/Reaching Guide students to the Learn page and have them identify several *expressions*. Ask, *Why are these expressions*? Ask students what other meanings of *expression* they know. Have them mention facial expressions and expressing feelings. Ask, *What types of feelings can you express?* Elicit from students that the meaning of *expression* in the lesson is quite different from the other meanings of *expression*. Have students work with partners to write examples of numerical expressions.

Practice & Reflect 10 min

On My Own

Name

Write the description for each numerical expression.

1. $(9 \times 18) - 5$
Subtract 5 from the product of 9 and 18.

2. $9 \times (18 - 5)$
Subtract 5 from 18, then multiply the difference by 9.

3. $80 \div (20 \times 4)$
Divide 80 by the product of 20 and 4.

4. $(80 \div 20) \times 4$
Multiply the quotient of 80 divided by 20 by 4.

Compare the expressions using >, <, or =. Explain your reasoning.

5. $120 \div 12$ > $(120 \div 12) - 9$
Sample answer: The quotient of $120 \div 12$ is reduced by 9 in the second expression.

6. 50.5×7.2 > $(50.5 - 4.8) \times 7.2$
Sample answer: The 50.5 is reduced by 4.8 in the second expression.

7. $5\frac{3}{4} \times (2\frac{1}{8} + 3\frac{1}{2})$ = $(5\frac{3}{4} \times 2\frac{1}{8}) + (5\frac{3}{4} \times 3\frac{1}{2})$
Sample answer: Distributive Property; $5\frac{3}{4}$ is multiplied by both addends.

8. A store ordered 4,500 T-shirts and 4,500 sunglasses. Without doing any calculations, which costs more? Explain your reasoning.

sunglasses; Sample answer: Because 22 > 15, the product of 4,500 and 22 is greater than the product of 4,500 and 15.

Determine whether Expression A is 5 times as much as Expression B. Place a checkmark in the *Yes* or *No* column.

	Expression A	Expression B	Yes	No
9.	$5 \times (1\frac{1}{4} \times 4\frac{5}{8})$	$1\frac{1}{4} \times 4\frac{5}{8}$	✓	
10.	$(5 \times 4.39) + (5 \times 8.99)$	$4.39 + 8.99$	✓	
11.	$(65 \times 5) \times 2$	$(65 \times 2) \times 5$		✓
12.	$(3{,}492 - 2{,}482) \times 5$	$3{,}492 - 2{,}482$	✓	
13.	$(895 + 345) \div 5$	$895 + 345$		✓
14.	$6.71 \times (3.28 \times 5.16)$	6.71×3.28		✓

15. **Extend Your Thinking** Write a word problem that could be represented by each numerical expression.
$8 \times (4 + 2)$ $(8 \times 4) + 2$
Explain why the way the expressions are grouped impacts what happens in the word problem.
Answers may vary.

Reflect

How can you interpret numerical expressions without evaluating them?
Answers may vary.

Math is... Mindset
How did you show you understand how others are feeling?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 11 Students may be confused by the fact that the same operation is used twice in an expression. Encourage students to think about how the interpretation will change depending on the grouping.

Item Analysis

Item	DOK	Rigor
1–4	1	Procedural Skill & Fluency
5–7	2	Conceptual Understanding
8	2	Application
9–14	2	Procedural Skill & Fluency
15	3	Conceptual Understanding

Reflect

Students complete the Reflect question.
- How can you interpret numerical expressions without evaluating them?

Ask students to share their reflections with their classmates.

Math is... Mindset
- How did you show you understand how others are feeling?

Students reflect on how they practiced social awareness.

Learning Target

Ask students to reflect on the Learning Target of the lesson.
- I can interpret numerical expressions without evaluating them.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Assess 10 min

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Interpret numerical expressions	5.OA.A.1, 5.OA.A.2
2	1	Compare numerical expressions	5.OA.A.1, 5.OA.A.2
3	1	Compare numerical expressions	5.OA.A.1, 5.OA.A.2
4	1	Compare numerical expressions	5.OA.A.1, 5.OA.A.2
5	1	Compare numerical expressions	5.OA.A.1, 5.OA.A.2
6	1	Compare numerical expressions	5.OA.A.1, 5.OA.A.2
7	1	Interpret numerical expressions	5.OA.A.1, 5.OA.A.2

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

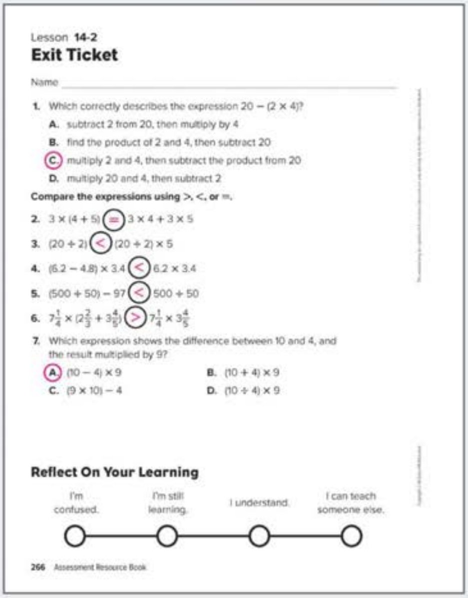
If students score	Then have students do
7 of 7	Additional Practice or any of the B or E activities
6 of 7	*Take Another Look* or any of the B activities
5 or fewer of 7	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 14-2
Exit Ticket

Name ______________________

1. Which correctly describes the expression $20 - (2 \times 4)$?
   A. subtract 2 from 20, then multiply by 4
   B. find the product of 2 and 4, then subtract 20
   C. multiply 2 and 4, then subtract the product from 20
   D. multiply 20 and 4, then subtract 2

Compare the expressions using >, <, or =.

2. $3 \times (4 + 5)$ = $3 \times 4 + 3 \times 5$
3. $(20 + 2)$ < $(20 + 2) \times 5$
4. $(6.2 - 4.8) \times 3.4$ < 6.2×3.4
5. $(500 + 50) - 97$ < $500 + 50$
6. $7\frac{1}{4} \times (2\frac{2}{3} + 3\frac{4}{5})$ > $7\frac{1}{4} \times 3\frac{4}{5}$
7. Which expression shows the difference between 10 and 4, and the result multiplied by 9?
   A. $(10 - 4) \times 9$ B. $(10 + 4) \times 9$
   C. $(9 \times 10) - 4$ D. $(10 \div 4) \times 9$

Reflect On Your Learning

I'm confused. I'm still learning. I understand. I can teach someone else.

266 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Telexpressions

Work with students in groups of 4. Have students sit in a line or in a circle. Give the first student a 3-number expression on an index card. He or she writes the expression in words. The second student writes an expression based on the first player's written description. The next student writes a description, and the next writes an expression. Discuss whether the final expression matches the original expression and why or why not.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Interpret the Magnitude of Expressions

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 163

Lesson 14-2 • Reinforce Understanding

Interpret Numerical Expressions

Name ____________

Review

Analyze the difference between the equations to determine whether Equation B is *greater than* or *less than* Equation A.

Equation A 10×8	Equation B $10 \times (8 - 5)$
• 10 is multiplied by 8	• 10 is multiplied by 3
• 10 is multiplied by a greater value.	• 10 is multiplied by a lesser value.
Equation A is greater than Equation B $10 \times 8 > 10 \times (8 - 5)$	

Which expression is greatest? Place a check mark next to the expression that is greatest. Then fill in the greater-than statement.

1. $120 \div 10$ ____ $(120 + 30) \div 10$ ✓
   $(120 + 30) \div 10$ > $120 \div 10$
2. $120 \div 10$ ✓ $120 \div (10 - 5)$ ____
   $120 \div 10$ > $120 \div (10 - 5)$
3. $50 + 3$ ____ $(50 + 3) \times 7$ ✓
   $(50 + 3) \times 7$ > $50 + 3$
4. $50 + 3$ ✓ $(50 - 10) + 3$ ____
   $50 + 3$ > $(50 - 10) + 3$
5. $20 - 4$ ✓ $(20 - 4) \div 8$ ____
   $20 - 4$ > $(20 - 4) \div 8$
6. $20 - 4$ ____ $(20 - 4) + 11$ ✓
   $(20 - 4) + 11$ > $20 - 4$

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Numerical Expressions Task Cards

Students practice writing situations that could be described by arithmetic expressions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 163–164

Lesson 14-2

Additional Practice

Name ____________

Review

You can understand the relationship between numbers by interpreting the numerical expressions.

What is the same about the numerical expressions $(20 \div 4) + 6$ and $20 \div (4 + 6)$? What is different?

The numerical expressions are the same in that both use the same numbers, 20, 4, and 6, the same operations, division and addition, and both use grouping symbols.

The grouping symbols, however, make the numerical expressions different because different numbers are grouped together.

The numerical expression $(20 \div 4) + 6$ means to divide 20 by 4, then add 6. The numerical expression $20 \div (4 + 6)$ means to divide 20 by the sum of 4 and 6.

Write the description for each numerical expression.

1. $(11 \times 9) + 5$
   Sample answer: The product of 11 and 9, then add 5.
2. $11 \times (9 + 5)$
   Sample answer: Multiply 11 by the sum of 9 and 5.
3. $20 - (12 \div 4)$
   Sample answer: The quotient of 12 and 4 subtracted from 20.
4. $(20 - 12) \div 4$
   Sample answer: The difference of 20 and 12, and the result divided by 4.

Student Practice Book

E Extend Thinking

Own It! Digital Station

Build Fluency **Games**

Assign the digital page to develop an understanding of the order of operations.

Use It! Application Station

Earning an Income Students research 5 jobs and incomes, create a table, and plot the results on a coordinate plane. *The content of this card has concepts covered later in Lesson 14-6. You may want to assign this card to students ready to explore content covered later in this unit.*

WORKSTATIONS

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

GO ONLINE

INDEPENDENT WORK

Student Practice Book, pp. 163–164

Compare the expressions using >, <, or =. Explain your reasoning.

5. $60 \div 10$ < $(60 \div 10) + 7$
Sample answer: The quotient $60 \div 10$ is increased by 7 in the second expression.

6. 40×6.5 > $(40 - 8) \times 6.5$
Sample answer: The factor 40 is reduced by 8 in the second expression.

7. $5 \times (4 + 3\frac{1}{2})$ = $(5 \times 4) + (5 \times 3\frac{1}{2})$
Sample answer: Distributive Property; 5 is multiplied by both addends.

8. $(20 \times 15) - 42$ < 20×15
Sample answer: The product 20×15 is reduced by 42 in the first expression.

Tell how the value of the first numerical expression compares to the value of the second numerical expression.

9. $512 + 259$ and $(512 + 259) \times 3$
Sample answer: The second expression is 3 times the first expression.

10. $(28 \times 43) + 12$ and 28×43
Sample answer: The first expression is 12 more than the second expression.

11. $(36 \div 4) - 3$ and $36 \div 4$
Sample answer: The first expression is 3 less than the second expression.

Math @ Home Activity

Student Practice Book

Differentiation Resource Book, p. 164

Lesson 14-2 • Extend Thinking

Interpret Numerical Expressions

Name ______________

Which expression is greatest? Rewrite the numerical expressions in order from least to greatest.

1.	$120 \div 10$	$(120 + 30) \div 10$	$120 \div (10 - 5)$	
	$120 \div (10 - 5)$ <	$120 \div 10$ <	$(120 + 30) \div 10$	
2.	$(50 - 10) + 3$	$(50 + 3) \times 7$	$50 + 3$	
	$(50 - 10) + 3$ <	$50 + 3$ <	$(50 + 3) \times 7$	
3.	$(20 - 4) \div 8$	$20 - 4$	$(20 - 4) + 11$	
	$(20 - 4) \div 8$ <	$20 - 4$ <	$(20 - 4) + 11$	
4.	15×4	$(15 \times 4) \div 10$	$(15 \times 4) \times 10$	
	$(15 \times 4) \div 10$ <	15×4 <	$(15 \times 4) \times 10$	
5.	$40 - 17$	$12 \times (40 - 17)$	$12 + (40 - 17)$	
	$40 - 17$ <	$12 + (40 - 17)$ <	$12 \times (40 - 17)$	
6.	$5 + 7$	$(5 + 7) \times 3$	$(5 + 7) - 7$	
	$(5 + 7) - 7$ <	$5 + 7$ <	$(5 + 7) \times 3$	

7. 18×12, $(18 \times (12 \div 3))$, and $(18 \times 12) \div 3$
$(18 \times (12 \div 3)) < (18 \times 12) \div 3 < 18 \times 12$

Differentiation Resource Book

LESSON **14-3**

Evaluate Numerical Expressions

Learning Target

- I can use the order of operations to evaluate numerical expressions.

Standards ◆ Major ▲ Supporting ● Additional

Content

○ **5.OA.A** Write and interpret numerical expressions.

○ **5.OA.A.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Math Practices and Processes

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	Academic Terms
evaluate	accurate
order of operations	contradiction

Material

The materials may be for any part of the lesson.

- cardstock

Focus

Content Objective	Language Objectives	SEL Objective
• Students use the order of operations to evaluate numerical expressions.	• Students talk about using the order of operations to evaluate numerical expressions using the verb *help*. • To support optimizing output, ELs participate in MLR7: Compare and Connect	• Students demonstrate self-discipline through working through distractions to complete a mathematical task.

Coherence

Previous	Now	Next
• Students interpreted numerical expressions without evaluating the numerical expression (Unit 14).	• Students use the order of operations to evaluate numerical expressions.	• Students generate two numerical patterns using rules and identify apparent relationships between corresponding terms in the patterns (Unit 14).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students gain understanding that not following an order will produce contradictory answers, but using the order of operations consistently produces a single value for a numerical expression.	• Students gain fluency and skill with the order of operations and handling grouping symbols as they follow the steps repeatedly with support throughout the lesson.	• Several of the numerical expressions are presented in a real-world context. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
What's Another Way to Write It? 5–7 min

Build Fluency Students build number sense as they write 3 differences to represent the number $\frac{1}{2}$.

These prompts encourage students to talk about their reasoning:

- How did you determine numbers that have a difference of $\frac{1}{2}$?
- How can you check your answers to be sure that they are correct?
- How are the subtraction expressions that have a difference of $\frac{1}{2}$ related?
- What is a different way to think about the problem?

Purpose Students discuss different ways to think about an image.

Notice & Wonder

- What do you see?

Teaching Tip You may want to have students draw pictures that represent how they see the picture before beginning a discussion as a whole class.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of using the order of operations to evaluate numerical expressions and are based on possible comments and questions that students may make during the share out.

- How can you describe the image?
- How can you describe the image in another way?
- How does the placement of the dice impact how you describe them?

Math is... Mindset

- What helps you get started on your work?

SEL **Self-Management: Self-Discipline**

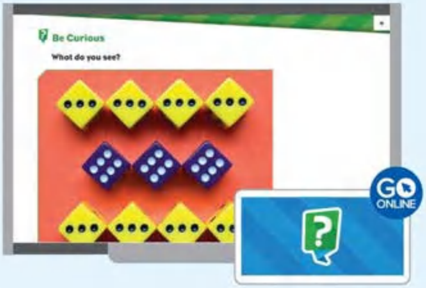
Help students develop strong learning habits by providing them opportunities to practice self-management. Before beginning the Notice & Wonder routine, discuss ways that students will manage distractions and stay focused on their work describing what they see.

Transition to Explore & Develop

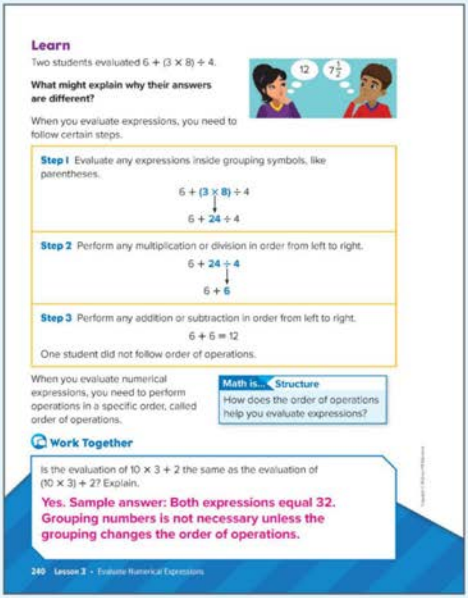
Ask questions that get students thinking about the specific order in which some activities have to take place. For example, they must put socks on before they put shoes on. Guide students to think about how we might need a specific order to do things in math.

ETP **Establish Mathematics Goals to Focus Learning**

- Let's think about the order of the steps we take to evaluate a numerical expression.



Explore & Develop 20 min



Learn

Two students evaluated $6 + (3 \times 8) \div 4$.

What might explain why their answers are different?

12

$7\frac{1}{2}$

When you evaluate expressions, you need to follow certain steps.

Step 1 Evaluate any expressions inside grouping symbols, like parentheses.

$$6 + (3 \times 8) \div 4$$
$$6 + 24 \div 4$$

Step 2 Perform any multiplication or division in order from left to right.

$$6 + 24 \div 4$$
$$6 + 6$$

Step 3 Perform any addition or subtraction in order from left to right.

$$6 + 6 = 12$$

One student did not follow order of operations.

When you evaluate numerical expressions, you need to perform operations in a specific order, called order of operations.

Math is... Structure
How does the order of operations help you evaluate expressions?

Work Together

Is the evaluation of $10 \times 3 + 2$ the same as the evaluation of $(10 \times 3) + 2$? Explain.

Yes. Sample answer: Both expressions equal 32. Grouping numbers is not necessary unless the grouping changes the order of operations.

240 Lesson 3 • Evaluate Numerical Expressions

❶ Pose the Problem

ETP Pose Purposeful Questions

- How can you interpret this numerical expression?
- Do you know any mathematical rules you can use to solve this problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Compare and Connect

Pair students and give them a problem similar to the Work Together problem on the Learn page. Ask each to work individually, evaluating a numerical expression, and then have them compare their strategies. Revisit this routine throughout the lesson to help students build proficiency.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How would you explain how the order of operations works to a friend?
- How can the order of operations help you evaluate a numerical expression?

Key Takeaway

- There is an order in which operations must be carried out when evaluating numerical expressions.

Work Together

Students work together to evaluate a numerical expression containing all four operations.

Common Error: Students may be confused as to where to start as there are no grouping symbols in this numerical expression. Remind them to think of the order in which they must perform the operations. Ask what they should do if there are no numerical expressions in grouping symbols.

LOM Language of Math

Students may wonder why they need to work from left to right when performing multiplication and division, and then addition and subtraction. Explain that evaluating a numerical expression is like reading a book: they must read from left to right in order to understand what is being said. But, math is like its own language, because they cannot just work from left to right, but must perform the operations in the correct order.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore the importance of the order of operations when evaluating numerical expressions.

Directions: Have students work together to evaluate the numerical expression presented in the Pose the Problem.

ETP Support Productive Struggle

- Can you write a verbal description to help you evaluate the numerical expression?
- How do the parentheses impact the description?
- Which operation did you perform first?
- How is your method the same as others? How is it different?
- Did you arrive as the same result as either student shown in the problem?

Activity Debrief: Have students share the steps they took to evaluate the numerical expression. Ask students to explain if they took different steps and arrived at different answers. Present to students the order of operations. Explain that they must follow this order when evaluating numerical expressions to calculate an accurate answer.

Math is... Choosing Tools

- How does the order of operations help you evaluate numerical expressions?

Students make sound decisions about when a tool might be helpful, recognizing both the insight to be gained and its limitations.

Guided Exploration

Students learn to evaluate numerical expressions using the order of operations.

ETP Facilitate Meaningful Mathematical Discourse

- **Think About It:** Why do you think operations have to be performed in a specific order?
- How can you use what you already know about grouping symbols to predict the order of operations?

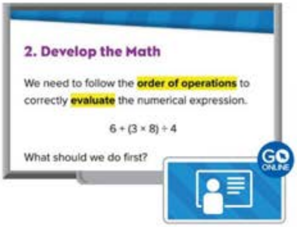
Have the students perform an error analysis. Ask:

- How do you think the one student got $7\frac{1}{2}$?
- How did that student follow the order of operations incorrectly?
- How does this problem help you understand why the order of operations is important?

Math is... Choosing Tools

- How does the order of operations help you evaluate numerical expressions?

Students make sound decisions about when a tool might be helpful, recognizing both the insight to be gained and its limitations.



EL English Learner Scaffolds

Entering/Emerging Explain *To* ___, *use* ___. Write an addition problem on the board. Say *To solve the problem, use a place value chart.* Solve the problem with a place value chart. Repeat with a new mathematical problem, using *To* ___, *use* ___ to explain how to solve it. Then repeat once more, asking students to answer the question *Did I use [counters] or [grouping] to solve the problem?*

Developing/Expanding Explain *To* ___, *use* ___. Write an addition problem on the board. Say *To solve the problem, use a place value chart.* Solve the problem with a place value chart. Repeat with a new mathematical problem, using *To* ___, *use* ___ to explain how to solve it. Then repeat once more, this time asking students to complete the sentence: ___ (To) *solve the problem* ___ (use) *counters.*

Bridging/Reaching Guide students to the Learn page and ask them to review the sentence *To evaluate a numerical expression, use the order of operations.* Ask students to come up with a new sentence explaining how to do something, using *To* ___, *use* ___. Allow students to interject, correcting as needed. For example, *No, I don't think that's right. To solve the problem, use a place value chart.*

Practice & Reflect 10 min

On My Own

Name ____________________

Which operation will you perform first to evaluate the expression? Explain your reasoning. Explanations may vary.

1. $25 - 5 \times (4 - 3)$ subtraction
2. $37 + 8 \div 2 - 5$ division
3. $\frac{3}{4} \times (2\frac{1}{2} + 6\frac{1}{4})$ addition
4. $100 \times 4 + 6 - 10$ multiplication

What is the solution? Show your work. Check students' work.

5. $3 + 7 \times 2 =$ 17
6. $(3 + 7) \times 2 =$ 20
7. $56 \div 8 - 3 + 2 \times 5 =$ 14
8. $56 \div (8 - 3 + 2) \times 5 =$ 40
9. $2\frac{3}{8} + 1\frac{1}{4} \times 6\frac{3}{4} - \frac{1}{2} =$ $10\frac{5}{16}$
10. $5.8 \times (6.75 + 3.25) \div 2 =$ 29

11. Which numerical expression is equal to 8?
    A. $24 \div 6 \times 4 + 7$
    B. $(24 \div 6) \times 4 + 7$
    C. $24 \div (6 \times 4) + 7$
    D. $24 \div 6 \times (4 + 7)$
12. Which numerical expression is equal to 1?
    A. $96 \div 12 \times 4 \div 2$
    B. $96 \div (12 \times 4) \div 2$
    C. $96 \div (12 \times 4 \div 2)$
    D. $96 \div 12 \times (4 + 2)$
13. **Error Analysis** Brenna evaluated this expression. How can you help Brenna correct her thinking?

    $36 \div 2 \times 9 \div 3 = \frac{2}{3}$

    Sample answer: Brenna multiplied first when she should have performed division and multiplication from left to right first; the answer is 54.
14. **Extend Your Thinking** Evaluate the expression. Then, explain how the use of grouping symbols could change the expression and how you evaluate it.

    $6 \div 2 + 9 \div 3$

    Sample answer: As written, the expression is equal to 6. Grouping symbols would change it only if there were parentheses around $2 + 9$; Then, the answer would be $\frac{2}{11}$.

Reflect

Why is following the order of operations important when evaluating numerical expressions?

Answers may vary.

Math is... Mindset

What helped you get started on your work?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 1 Students may think that, because they are supposed to solve multiplication and division before addition and subtraction, they should multiply or divide before performing any addition or subtraction inside the grouping symbols. Remind students that evaluating numerical expressions in the grouping symbols is the first step in the order of operations, regardless of the operation.

Item Analysis

Item	DOK	Rigor
1–4	1	Conceptual Understanding
5–12	2	Procedural Skill & Fluency
13–14	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- Why is following the order of operations important when evaluating numerical expressions?

Ask students to share their reflections with their classmates.

Math is... Mindset

- What helped you get started on your work?

Students reflect on how they practiced self-management.

Learning Target

Ask students to reflect on the Learning Target of the lesson.

- I can use the order of operations to evaluate numerical expressions.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Evaluate numerical expressions	5.OA.A.1
2	1	Evaluate numerical expressions	5.OA.A.1
3	1	Evaluate numerical expressions	5.OA.A.1
4	1	Evaluate numerical expressions	5.OA.A.1
5	1	Evaluate numerical expressions	5.OA.A.1

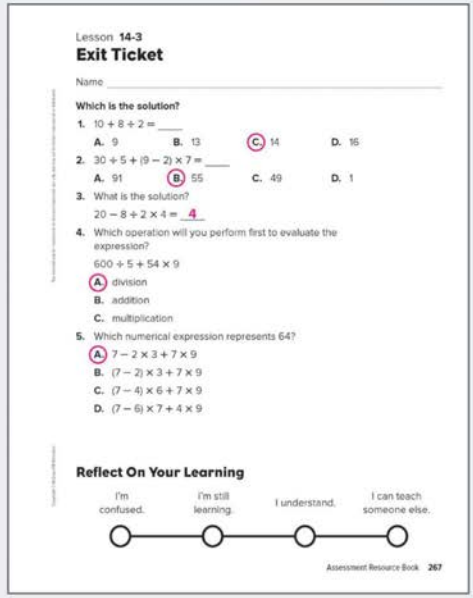
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
5 of 5	Additional Practice or any of the B or E activities
4 of 5	*Take Another Look* or any of the B activities
3 or fewer of 5	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 14-3
Exit Ticket

Name ____

Which is the solution?

1. $10 + 8 \div 2 =$ ____
   A. 9 B. 13 C. 14 D. 16
2. $30 \div 5 + (9 - 2) \times 7 =$ ____
   A. 91 B. 55 C. 49 D. 1
3. What is the solution?
   $20 - 8 \div 2 \times 4 =$ 4
4. Which operation will you perform first to evaluate the expression?
   $600 \div 5 + 54 \times 9$
   A. division
   B. addition
   C. multiplication
5. Which numerical expression represents 64?
   A. $7 - 2 \times 3 + 7 \times 9$
   B. $(7 - 2) \times 3 + 7 \times 9$
   C. $(7 - 4) \times 6 + 7 \times 9$
   D. $(7 - 6) \times 7 + 4 \times 9$

Reflect On Your Learning

I'm confused. | I'm still learning. | I understand. | I can teach someone else.

Assessment Resource Book 267

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Call the Doctor!

Write expressions with three operations and one set of grouping symbols on card stock cut into the shapes of gingerbread people. Give three students "Dr. Grouping," "Dr. Multiplydivide," and "Dr. Addsubtract" name tags. Each of these student takes turns evaluating the patient (expression) and completing the operations in the doctor's name. Make sure students understand the correct order of operations before they proceed.

GO ONLINE

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

- Parentheses
- Understand Order of Operations
- Apply Order of Operations

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 165

Lesson 14-3 • Reinforce Understanding

Evaluate Numerical Expressions

Name ______________________

Review

To evaluate a numerical expression, use the order of operations. Consider 11 − (16 + 8) ÷ 3.

Step 1	Evaluate any expressions inside grouping symbols.	$21 - (16 + 8) \div 3 \times 2$ $21 - 24 \div 3 \times 2$
Step 2	Perform any multiplication or division in order from left to right.	$21 - 24 \div 3 \times 2$ $21 - 8 \times 2$ $21 - 16$
Step 3	Perform any addition or subtraction in order from left to right.	5

So, by the order of operations, $21 - (16 + 8) \div 3 \times 2 = 5$.

Match the expression in Column A to its answer in Column B.

	Column A	Column B
1.	$30 \div 5 \times 2 + 5$	27
2.	$30 \div (5 \times 2) + 5$	18
3.	$30 \div 5 \times (2 + 5)$	8
4.	$30 - (2 + 4) \div 2$	14
5.	$30 - 2 + 4 + 2$	42
6.	$(30 - 2 + 4) \div 2$	16
7.	$2 \times 12 - 12 + 12 \div 2$	4
8.	$2 \times (12 - 12) + 8 \div 2$	17
9.	$2 \times 12 - (12 + 8) \div 2$	30

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Order of Operations Showdown

Students practice evaluating numerical expressions.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 165–166

Lesson 14-3

Additional Practice

Name ______________________

Review

You can evaluate a numerical expression using the order of operations.

Evaluate the numerical expression $4 + 6 \times (10 - 3)$.

Evaluate within grouping symbols first.

$4 + 6 \times (10 - 3) = 4 + 6 \times 7$

Perform any multiplication or division, in order from left to right.

$4 + 6 \times 7 = 4 + 42$

Perform any addition or subtraction, in order from left to right.

$4 + 42 = 46$

The numerical expression $4 + 6 \times (10 - 3)$ evaluates to be 46.

Which operation will you perform first to evaluate the expression? Explain your reasoning. Sample answers are given.

1. $32 + 7 \times (8 - 3)$
   subtraction; do any operation within grouping symbols first
2. $42 \div 10 + 5 - 2$
   division; multiplication and division is performed before addition or subtraction
3. $8 \div 2 \times 4 + 6$
   division; perform multiplication and division in order from left to right
4. $10 - 6 + 100 \times 4$
   multiplication; multiplication and division is performed before addition or subtraction

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital page to develop an understanding of the order of operations.

Assign

Use It! Application Station

Color by Number Students use grid paper to create designs and numerical expressions.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 165–166

Evaluate the numerical expression.

5. $10 - 5 + 2$ **7**
6. $6 + 12 \div 6$ **8**
7. $(3 + 4) \times 3$ **21**
8. $15 - (2 + 7) + 1$ **7**
9. $24 \div 2 \times 6 + 1$ **73**
10. $8 \div (2 \times 2) + 1$ **3**
11. $2 \times 9 - 8 + 1$ **11**
12. $14 - (6 + 7) + 4$ **5**
13. $42 \div 6 - 3 + 4 \times 5$ **24**
14. $4 + 36 \div (6 \div 3 + 4) \times 5$ **34**
15. $5 \times (12 - 2 \times 5) + 36 \div (10 - 6 + 2)$ **16**

Math @ Home Activity Write a 3- or 4-step numerical expression at the top of a sheet of paper. Give your child four different color pencils. Assign a color to each of the steps. Have your child evaluate the expression, using the correct color to show progression from one step to the next. Repeat the activity with a different expression.

Student Practice Book

Differentiation Resource Book, p. 166

Lesson **14-3 • Extend Thinking**

Evaluate Numerical Expressions

Name ______________________

Introduce ( ) in the numerical expression to make the equations true. If no ( ) are needed, say so. Show your work to evaluate the numerical expression.

1. $12 \div 6 \times 2 + 1$

$12 \div 6 \times 2 + 1 = 5$ no ( ) needed; $2 \times 2 + 1$ $= 4 + 1 = 5$	$12 \div (6 \times 2) + 1 = 2$ $12 \div 12 + 1$ $= 1 + 1 = 2$	$12 \div 6 \times (2 + 1) = 6$ $12 \div 6 \times 3$ $= 2 \times 3 = 6$

2. $2 + 20 \div 2 \times 5$

$(2 + 20) \div 2 \times 5 = 55$ $22 \div 2 \times 5$ $= 11 \times 5 = 55$	$2 + 20 \div 2 \times 5 = 52$ no ( ) needed; $2 + 10 \times 5$ $= 2 + 50 = 52$	$2 + 20 \div (2 \times 5) = 4$ $2 + 20 \div 10$ $= 2 + 2 = 4$

3. $16 - 4 + 9 - 3$

$16 - (4 + 9) - 3 = 0$ $16 - 13 - 3$ $= 3 - 3 = 0$	$16 - (4 + 9 - 3) = 6$ $16 - (13 - 3)$ $= 16 - 10 = 6$	$16 - 4 + 9 - 3 = 18$ no ( ) needed; $12 + 9 - 3$ $= 21 - 3 = 18$

4. $36 \div 2 \times 18 \div 3$

$36 \div (2 \times 18) \div 3 = \frac{1}{3}$ $36 \div 36 \div 3 =$ $1 \div 3 = \frac{1}{3}$	$36 \div (2 \times 18 \div 3) = 3$ $36 \div (36 \div 3)$ $= 36 \div 12 = 3$	$36 \div 2 \times 18 \div 3 = 108$ no ( ) needed; $18 \times 18 \div 3$ $= 324 \div 3 = 108$

Differentiation Resource Book

Unit 14

Order of Operations

MATH PROBES

Name

For each problem, determine which operation should be evaluated first. Do not perform the exact evaluation.

1. 4 + 3 × 9 − 1

Which expression should be evaluated first?

a. 4 + 3
b. 3 × 9
c. 9 − 1
d. Doesn't matter which expression is evaluated first

Explain or show your thinking.

Explanations may vary.

2. 24 ÷ 6 × 2 + 4

Which expression should be evaluated first?

a. 24 ÷ 6
b. 6 × 2
c. 2 + 4
d. Doesn't matter which is done first

Explain or show your thinking.

Explanations may vary.

Unit 14 • Algebraic Thinking 243

For each problem, determine which operation should be evaluated first. Do not perform the exact evaluation.

3. 8 + 3 × (4 − 1)

Which expression should be evaluated first?

a. 8 + 3
b. 3 × 4
c. 4 − 1
d. Doesn't matter which is evaluated first

Explain or show your thinking.

Explanations may vary.

4. 6 ÷ (3 + 3) × 4

Which expression should be evaluated first?

a. 6 ÷ 3
b. 3 + 3
c. 3 × 4
d. Doesn't matter which is evaluated first

Explain or show your thinking.

Explanations may vary.

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

244 Math Probe • Order of Operations

Analyze the Probe

Formative Assessment

Students determine which operation is evaluated first based on the order of operations. They circle one of 4 choices and justify their answers. Remind students not to actually perform the evaluation.

Targeted Concept Determine the operation to evaluate based on understanding the order of operations.

Targeted Misconceptions Some students do not correctly apply order of operations when evaluating a numerical expression. Some will evaluate from left to right regardless of the operation. Some will evaluate multiplication before division or addition before subtraction regardless of the order the operations appear in the expression. Students also sometimes think the answer will always be the same no matter the order in which the operations are evaluated.

Authentic Student Work

Below are examples of students' explanations.

Sample A

1. 4 + 3 × 9 − 1

Which expression should be evaluated first?

a. 4 + 3
b. 3 × 9
c. 9 − 1
d. Doesn't matter which expression is evaluated first

Explain or show your thinking.

Because you have to start with multiplacation. Then you add and subtract.

Sample B

4. 6 ÷ (3 + 3) x 4

Which expression should be evaluated first?

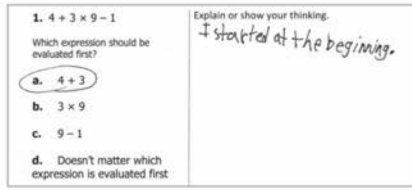
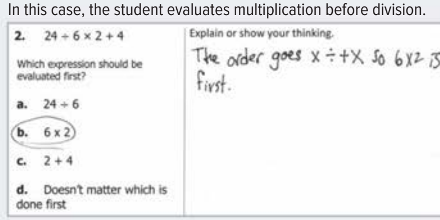
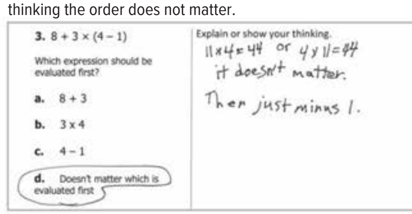
a. 6 ÷ 3
b. 3 + 3
c. 3 x 4
d. Doesn't matter which is evaluated first

Explain or show your thinking.

(3+3) is first Then 6÷6 Then 1x4.

Collect and Assess Student Work

Collect and review student responses to determine possible misconceptions. See examples in If-Then chart.

IF incorrect…	THEN the student likely…	Sample Misconceptions
1. a 3. a 4. a	evaluates expressions from left to right. Note that this incorrect application leads to a correct answer for Exercise 2 (choice a).	In this case, the student evaluated left to right ignoring the signs.  1. 4 + 3 × 9 − 1 Which expression should be evaluated first? a. 4 + 3 b. 3 × 9 c. 9 − 1 d. Doesn't matter which expression is evaluated first Explain or show your thinking. I started at the begining.
2. b 3. a 4. c	evaluates multiplication before division or addition before subtraction.	In this case, the student evaluates multiplication before division.  2. 24 ÷ 6 × 2 + 4 Which expression should be evaluated first? a. 24 ÷ 6 b. 6 x 2 c. 2 + 4 d. Doesn't matter which is done first Explain or show your thinking. The order goes x ÷ + X so 6x2 is first.
1. d 2. d 3. d 4. d	thinks changing the order of operations does not impact the result.	In this case, the student focuses on multiplication only while evaluating the expression thinking the order does not matter.  3. 8 + 3 × (4 − 1) Which expression should be evaluated first? a. 8 + 3 b. 3 x 4 c. 4 − 1 d. Doesn't matter which is evaluated first Explain or show your thinking. 11x4=44 or 4x11=44 it doesn't matter. Then just minus 1.

Many of the above difficulties result in a combination of correct and incorrect responses.
For correct responses, be sure to check for sound reasoning.

Take Action

Choose from the following resources or suggestions:

- Revisit order of operations activities in Lessons 14-1 through 14-4.
- To build understanding of grouping and operation symbols and the order of operations, ask students to write a story problem that matches a given expression.
- Rather than have students evaluate expressions, provide opportunities for students to sort expressions based on which operation is evaluated first. Include discussion and justification opportunities throughout the sort.
- Include sorts that require students to match story problem contexts and/or written statements with expressions.

Revisit the Probe After additional instruction, have students review their initial answers. Use these questions for discussion:

- Are there any answers you would like to change? Explain.
- Are there any questions that you still have about any of the items on this probe?

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

LESSON **14-4**

Numerical Patterns

Learning Targets

- I can generate two numerical patterns using two given rules.
- I can identify relationships between corresponding terms in the generated number patterns.

Standards ◆ Major ▲ Supporting ● Additional

Content

○ **5.OA.B** Analyze patterns and relationships.

○ **5.OA.B.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Math Practices and Processes

MPP Reason abstractly and quantitatively.

Vocabulary

Math Terms	Academic Terms
corresponding term	emphasize
numerical pattern	transition
rule (of a pattern)	

Material

The materials may be for any part of the lesson.

- two-color counters

Focus

Content Objectives	Language Objectives	SEL Objective
• Students generate two numerical patterns that follow two given rules. • Students identify relationships between corresponding terms in the generated number patterns.	• Students discuss relationships between corresponding terms in number patterns using the verbs *represent* and *determine.* • To support sense-making, ELs participate in MLR2: Collect and Display.	• Students exercise creativity by solving a problem using more than one approach.

Coherence

Previous	Now	Next
• Students generated a number or shape pattern that follows a given rule and identified apparent features of the pattern that were not explicit in the rule itself (Grade 4). • Students used the order of operations to evaluate numerical expressions (Unit 14).	• Students generate two numerical patterns using rules and identify apparent relationships between corresponding terms in the patterns.	• Students use a table to assist them in finding an apparent relationship between corresponding terms in two numerical patterns (Unit 14). • Students represent and analyze quantitative relationships between dependent and independent variables (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build on their understanding of algebra as they use expressions to identify relationships between corresponding terms.	• Students build proficiency with generating patterns using pattern rules to extend patterns and find corresponding terms.	• Students apply understanding of patterns to solve problems. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine

Can You Make the Number? 5–7 min

Build Fluency Students build number sense as they use 1, 3, 5, and 7 in any order and with any operations to make the target number 37.

Students must use all four numbers but can only use each number once in the combination. Encourage students to try to find more than one solution.

These prompts encourage students to talk about their reasoning:

- What number combination did you try first? How did you think about the numbers?
- What worked well for you? Where did you struggle?
- What was your strategy for getting to the target number?

Launch 5–7 min

Purpose Students explore relationships in a mathematical situation.

Notice & Wonder

- What question could you ask?

Teaching Tip You may want to provide students with counters of different colors so that they can model a relationship shown in the image on their own.

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of relationships between mathematical patterns and are based on possible comments and questions that students may make during the share out.

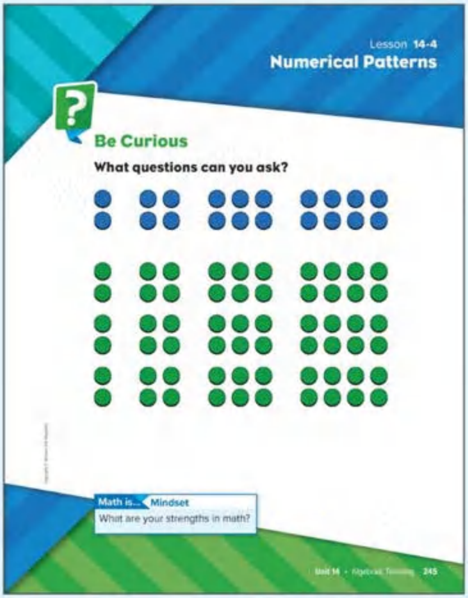
- How can you describe a relationship between the blue and green counters?
- What do you think would happen if a fifth group of blue and green counters were added? Explain your conclusion.

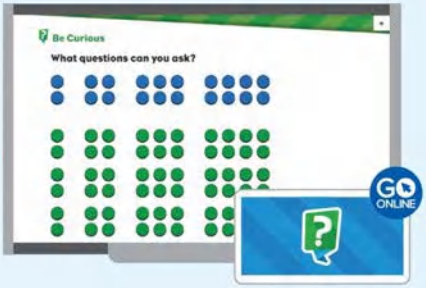
Math is... Mindset

- What are your strengths in math?

SEL Self-Awareness: Recognize Strengths

Before students begin the Notice & Wonder routine, invite them to think about their personal areas of strength in math. In addition to specific math skills, students may also acknowledge personal strengths that can help them with their math learning, such as listening, staying focused, or explaining. As students work with numerical patterns throughout the lesson, model giving positive feedback to help them acknowledge their personal strengths. Encourage students to recognize and acknowledge the strengths of their peers.





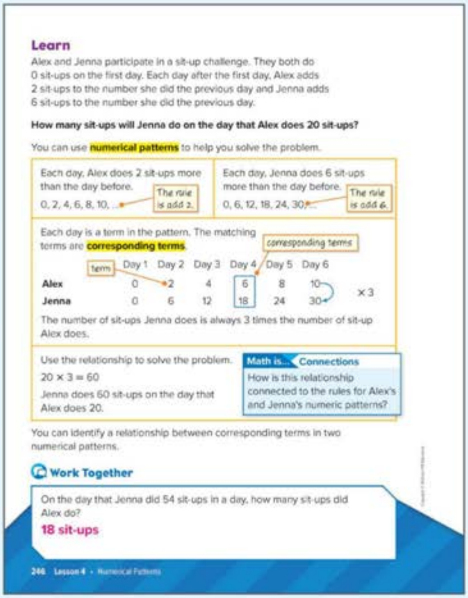
Transition to Explore & Develop

Ask questions that get students thinking about what a pattern is. As they discuss patterns, see if they naturally describe how patterns are formed. If not, they will learn more about this in the Develop section of the lesson.

ETP Establish Mathematics Goals to Focus Learning

- Let's think about how two patterns can be related.

Explore & Develop 20 min



Learn

Alex and Jenna participate in a sit-up challenge. They both do 0 sit-ups on the first day. Each day after the first day, Alex adds 2 sit-ups to the number she did the previous day and Jenna adds 6 sit-ups to the number she did the previous day.

How many sit-ups will Jenna do on the day that Alex does 20 sit-ups?

You can use **numerical patterns** to help you solve the problem.

Each day, Alex does 2 sit-ups more than the day before. The rule is add 2.
0, 2, 4, 6, 8, 10, ...

Each day, Jenna does 6 sit-ups more than the day before. The rule is add 6.
0, 6, 12, 18, 24, 30, ...

Each day is a term in the pattern. The matching terms are **corresponding terms**.

term	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
Alex	0	2	4	6	8	10
Jenna	0	6	12	18	24	30

corresponding terms

× 3

The number of sit-ups Jenna does is always 3 times the number of sit-ups Alex does.

Use the relationship to solve the problem.
$20 \times 3 = 60$
Jenna does 60 sit-ups on the day that Alex does 20.

Math is... Connections
How is this relationship connected to the rules for Alex's and Jenna's numeric patterns?

You can identify a relationship between corresponding terms in two numerical patterns.

Work Together

On the day that Jenna did 54 sit-ups in a day, how many sit-ups did Alex do?
18 sit-ups

246 Lesson 4 • Numerical Patterns

1 Pose the Problem

MLR Collect and Display

As students discuss the questions, record relevant words and phrases they may use such as *numerical patterns, rule, relationship between, generate,* and *corresponding terms.* Display the words for student reference. Use the student-generated expressions to help students make connections between student language and math vocabulary.

ETP Pose Purposeful Questions

- What tools can you use to represent the information you have?
- What information do you have about the number of sit-ups each girl adds each day? How can you use this information?

2 Develop the Math

Choose the option that best meets your instructional goals.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How do you use a pattern's rule to determine the terms in the pattern?
- How can you determine a relationship between corresponding terms in two numerical patterns?

Key Takeaways

- A numerical pattern is defined by a rule.
- Numerical patterns can be generated by the rule that defines them.
- There are often relationships between corresponding terms in numerical patterns.

Work Together

Students use the numerical patterns and rules they established during the Pose the Problem to answer how many sit-ups Alex did when given the number of sit-ups Jenna did.

Common Error: Students may think they need to multiply 54 by 3 as that was the operation they used in the Pose the Problem. Encourage students to pay attention as they read the Work Together problem to determine what they are trying to find. Ask: *How can you describe a relationship between the number of sit-ups Jenna does and the number of sit-ups Alex does?*

LOM Language of Math

Throughout the lesson, emphasize that *correspond means* "to be in agreement or conformity." Students will hear the word correspond in other contexts as well, for example, the number of desks in a classroom may *correspond* to the number of students in the class.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore numerical patterns to solve a problem.

Directions: Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- How can you find the number of sit-ups Alex did each day?
- How can you find the number of sit-ups Jenna did each day?
- Is there a relationship between Alex's sit-ups and Jenna's sit-ups? If so, describe it.
- Can you use that relationship to help you solve the problem?

Math is... Connections

- How is the relationship connected to the rules for Alex's and Jenna's numeric patterns?

Students make sense of quantities and their relationships in problem situations.

Activity Debrief: Have students share their solutions methods for solving the problem. Encourage students to identify similarities and differences among the solution methods.

Guided Exploration

Students generate two numerical patterns using their rules and identify an apparent relationship between corresponding terms in the patterns in order to solve a problem.

ETP Facilitate Meaningful Mathematical Discourse

- How is a numerical pattern similar to other patterns you know?

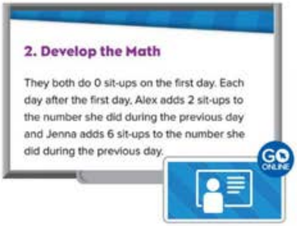
Have the students determine the terms in Jenna's numerical pattern. Ask:

- What is the rule for Jenna's numerical pattern? How do you know?
- If you know the number of sit-ups Jenna does on a day, what is the number of sit-ups she does on the next day?
- **Think About It:** Why are 12 in Alex's pattern and 36 in Jenna's pattern corresponding terms?

Math is... Connections

- How is the relationship you found connected to the rules for Alex's and Jenna's numeric patterns?

Students make sense of quantities and their relationships in problem situations.



EL English Learner Scaffolds

Entering/Emerging Explain *You can __ using __.* Write a problem on the board. Say *You can solve this problem using tens rods.* Demonstrate. Repeat with other problems and math tools, this time asking *How can I solve this problem?* Allow pointing to math tools.

Developing/Expanding Explain *You can __ using __.* Write a problem on the board. Say *You can solve this problem using tens rods.* Demonstrate. Repeat with other problems and math tools, this time asking *How can I solve this problem?* (using a [number line])

Bridging/Reaching Guide students to the Learn page and ask them to review the sentence *You can generate numerical patterns using rules.* Ask students to come up with their own sentence using this structure; for example, *You can solve addition problems using tens rods.* Validate and correct as needed.

On My Own

Name ________________

Use the information given for Exercises 1–8.

Quentin and Tyler are running laps on the school track. Each time they complete a lap, they do jumping jacks.

They both do 0 jumping jacks after the first lap.

Each lap, Quentin adds 1 jumping jack to the number of jumping jacks he did after the lap before.

Each lap, Tyler adds 4 jumping jacks to the number of jumping jacks he did after the lap before.

1. What is the rule for Quentin's numerical pattern?
add 1

2. What is the rule for Tyler's numerical pattern?
add 4

3. Write the first 5 terms of Quentin's numerical pattern.
0, 1, 2, 3, 4

4. Write the first 5 terms of Tyler's numerical pattern.
0, 4, 8, 12, 16

5. When Quentin does 4 jumping jacks after a lap, how many jumping jacks will Tyler do after that same lap?
16 jumping jacks

6. What is a relationship between corresponding terms in the two numerical patterns?
Multiply the number in Quentin's pattern by 4. The product is the number in Tyler's pattern.

7. How many jumping jacks will Tyler do after the lap when Quentin does 8 jumping jacks?
32 jumping jacks

8. How many jumping jacks will Quentin do after the lap when Tyler does 40 jumping jacks?
10 jumping jacks

Unit 14 • Algebraic Thinking 247

Use Numerical Patterns A and B for Exercises 9–12.

Numerical Pattern A: 0, 2, 4, 6, 8, 10, 12

Numerical Pattern B: 0, 6, 12, 18, 24, 30, 36

9. What is the rule for Pattern A?
add 2

10. What is the rule for Pattern B?
add 6

11. What is a relationship between the corresponding terms in the two numerical patterns?
Multiply the number in Pattern A by 3 and the product is the number in Pattern B.

12. When the number in Pattern A is 28, what will be the number in Pattern B? **84**

13. **Extend Your Thinking** Write two numerical patterns where a relationship between the corresponding terms is to multiply by 6. Start at 0 and write the first five terms for each numerical pattern and the rule for each numerical pattern.
Sample answer: 0, 2, 4, 6, 8 and 0, 12, 24, 36, 48; The rule for the first numerical pattern is add 2; The rule for the second numerical pattern is add 12.

Reflect

How can you explain the relationships between numerical patterns?

Answers may vary.

Math is... Mindset

How have you used your strengths today? What can you work to improve?

248 Lesson 4 • Numerical Patterns

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 8 Make sure students solve this problem not by using the relationship they wrote in item 6 (multiply by 4) but rather the inverse of that relationship (divide by 4), as they are starting with a term in Tyler's pattern and finding Quentin's corresponding term.

Item Analysis

Item	DOK	Rigor
1–8	2	Application
9–12	2	Procedural Skill & Fluency
13	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you explain the relationships between numerical patterns?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How have you used your strengths today? What can you work to improve?

Students reflect on how they practiced self-awareness.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can generate two numerical patterns using two given rules.
- I can identify relationships between corresponding terms in the generated number patterns.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Understand numerical patterns	5.OA.B.3
2	2	Understand numerical patterns	5.OA.B.3
3	2	Understand numerical patterns	5.OA.B.3
4	2	Apply numerical patterns	5.OA.B.3

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
4 of 4	Additional Practice or any of the B or E activities
3 of 4	*Take Another Look* or any of the B activities
2 or fewer of 4	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking

Lesson 14-4
Exit Ticket

Name ____

Darryl and Lena paint ceramic animals. On the first day, they paint 0 animals. Each day after the first day, Darryl paints 2 more animals than he did the previous day and Lena paints 4 more animals than she did the previous day.

1. What are the first five terms in Darryl's numerical pattern?
   0, 2, 4, 6, 8
2. What are the first five terms in Lena's numerical pattern?
   0, 4, 8, 12, 16
3. What is the relationship between corresponding terms in the two numerical patterns?
   A. Multiply the number in Lena's pattern by 2. The product is the number in Darryl's pattern.
   B. Multiply the number in Darryl's pattern by 2. The product is the number in Lena's pattern.
   C. Add 2 to Darryl's pattern. The sum is the number in Lena's pattern.
   D. Add 2 to Lena's pattern. The sum is the number in Darryl's pattern.
4. On the day when Lena paints 24 animals, how many animals does Darryl paint?
   A. 6 animals B. 12 animals
   C. 24 animals D. 48 animals

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

268 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

What's My Pattern?

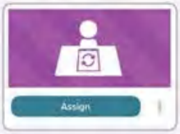
Work with students in groups. Have one student write a simple pattern rule using addition (such as + 4), but does not share the rule. Ask another student to say, "Number in is ____" followed by a number. The first student applies the rule and writes the "number out." Students continue until the group can identify the pattern rule. If students struggle finding the rule, ask "What can you add to the first number to get to the second number?"

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Create and Analyze the Pattern



INDEPENDENT WORK

Differentiation Resource Book, p. 167

Lesson 14-4 • Reinforce Understanding

Numerical Patterns

Name ____________________

Review

You can create numerical patterns to help you solve problems.

A:	0	4	8	12
	↓	↓	↓	↓
	÷2	÷2	÷2	÷2
	↓	↓	↓	↓
B:	0	2	4	6

A rule for a numerical pattern is an operation that takes a value in the pattern to the next value in the pattern.

"The rule for Pattern A is: Add 4."

The rule for Pattern B is: Add 2.

The relationship between 2 numerical patterns is the operation that takes a value in one pattern to the corresponding value in the other pattern.

If we are given a value in Pattern A, we can find the corresponding value in Pattern B by dividing the value by 2.

Use Numerical Patterns A and B for Exercises 1–5.

Numerical Pattern A: 0, 3, 6, 9, 12, 15, 18, 21

Numerical Pattern B: 0, 9, 18, 27, 36, 54, 63

1. What is the rule for Pattern A? Add 3
2. What is the rule for Pattern B? Add 9
3. What is the relationship between the corresponding terms in the two patterns?
   Multiply the number in Pattern A by 3.
4. When the number in Pattern A is 25, what will the number be in Pattern B?
   75
5. When the number in Pattern B is 90, what will the number be in Pattern B?
   30

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Numerical Patterns Task Cards

Students practice identifying pattern rules and finding missing terms.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 167–168

Lesson 14-4

Additional Practice

Name ____________________

Review

You can generate numerical patterns using rules and identify a relationship between corresponding terms in two numerical patterns.

Erika and Leo are picking apples. The first minute they each pick 0 apples. Then each minute after, Erika adds 4 apples to her basket and Leo adds 8 apples to his basket. When Erika has 16 apples in her basket, how many apples will Leo have in his basket?

Number of apples in Erika's basket each minute: 0, 4, 8, 12, **16**, 20

Number of apples in Leo's basket each minute: 0, 8, 16, 24, **32**, 40

When Erika has 16 apples in her basket, Leo will have 32 apples in his basket.

Refer to the numeric patterns for Erika and Leo above.

1. What is the rule for the number of apples in Erika's basket?
   Add 4
2. What is the rule for the number of apples in Leo's basket?
   Add 8
3. What is the relationship between corresponding terms in the two patterns? Sample answer: The number of apples in Leo's basket is 2 times the number of apples in Erika's basket.
4. When Leo has 48 apples in his basket, how many apples will Erika have in her basket? 24 apples

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital page to develop an understanding of the order of operations.

Assign

Use It! Application Station

A Rule Created That? Students use coordinate planes and coordinate pairs to create 3-D art.

The content of this card has concepts covered later in Lesson 14-6. You may want to assign this card to students ready to explore content covered later in this unit.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 167–168

5. Write the first six terms of the numerical pattern that starts at 0 and follows the rule Add 3. **0, 3, 6, 9, 12, 15**
6. Write the first six terms of the numerical pattern that starts at 0 and follows the rule Add 6. **0, 6, 12, 18, 24, 30**
7. Compare the numerical patterns. What is the relationship between corresponding terms in the two patterns? **Sample answer: The terms in the second pattern are 2 times the numbers in the first pattern.**

Rodney counts the value of his pennies. Diane counts the value of her nickels. They both start with 0 coins worth 0 cents.

Value of Rodney's pennies: 0, 1, 2, 3, 4, 5, 6

Value of Diane's nickels: 0, 5, 10, 15, 20, 25, 30

8. What is the rule for Rodney's pattern? **Add 1**
9. What is the rule for Diane's pattern? **Add 5**
10. What are the next three numbers in Rodney's pattern? **7, 8, 9**
11. What are the next three numbers in Diane's pattern? **35, 40, 45**
12. What is the relationship between corresponding terms in the two patterns? **Sample answer: The value of Diane's coins is 5 times the value of Rodney's coins.**
13. When Diane has 40 cents, what will be the value of Rodney's coins? **8** cents
14. When Rodney has 10 cents, what will be the value of Diane's coins? **50** cents

Math @ Home Activity Write two numerical patterns horizontally on a sheet of paper. Ask your child to identify the rule for each pattern, and have them show you how to write the next 3 terms for each pattern. Then have them explain how to find the relationship between corresponding terms in the two patterns, and predict the corresponding number in one pattern given a term in the other pattern.

Student Practice Book

Differentiation Resource Book, p. 168

Lesson 14-4 • Extend Thinking

Numerical Patterns

Name ____________________

In a certain game, players earn 3 tickets for every level completed after Level 1. Players do not receive any tickets for completing Level 1, however they must complete Level 1 before attempting Level 2.

1. Write the number of tickets awarded after completing each of the following levels.

   Level 1: **0** Level 4: **9**

   Level 2: **3** Level 5: **12**

   Level 3: **6** Level 6: **15**
2. Write the rule for the pattern. **Add 3**
3. How many tickets are awarded after completing Level 10? **27**
4. How can you find the number of tickets awarded after completing Level 100? **Sample answer: multiply the number of tickets by 3 then subtract 3; $100 \times 3 - 3 = 297$**

In a similar game, players earn 5 tickets for every level completed, including Level 1.

5. Write the number of tickets awarded after completing each of the following levels.

   Level 1: **5** Level 3: **15**

   Level 2: **10** Level 4: **20**
6. Write the rule for the pattern. **add 5**
7. How many tickets are awarded after completing Level 10? **50**
8. How can you find the number of tickets awarded after completing 100 levels? **Sample answer: multiply the number of tickets by 5; $100 \times 5 = 500$**

Differentiation Resource Book

LESSON **14-5**

Relate Numerical Patterns

Learning Targets

- I can arrange corresponding terms in two numerical patterns in a table.
- I can describe a relationship between corresponding terms in two numerical patterns.

Standards ◆ Major △ Supporting ● Additional

Content

○ **5.OA.B** Analyze patterns and relationships

○ **5.OA.B.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Math Practices and Processes

MPP Look for and make use of structure.

Vocabulary

Math Terms	Academic Terms
corresponding term	accurate
numerical pattern	inference
rule (of a pattern)	

Material

The materials may be for any part of the lesson.

- number cubes

Focus

Content Objectives	Language Objectives	SEL Objective
• Students use a table to arrange corresponding terms in two numerical patterns. • Students describe a relationship between corresponding terms in two numerical patterns.	• Students discuss relationships between corresponding terms in two numerical patterns using the verbs *identify* and use. • To support optimizing output, students participate in MLR4: Info Gap.	• Students self-motivate and sustain engagement to work independently to complete a challenging mathematical task.

Coherence

Previous	Now	Next
• Students generated a number or shape pattern that follows a given rule and identified apparent features of the pattern that were not explicit in the rule itself (Grade 4). • Students generated two numerical patterns using rules and identified apparent relationships between corresponding terms in the patterns (Unit 14).	• Students use a table to assist them in finding an apparent relationship between corresponding terms in two numerical patterns.	• Students form ordered pairs using corresponding terms from two numerical patterns, plot them on the coordinate plane, and use the graph to make conjecture (Unit 14). • Students represent and analyze quantitative relationships between dependent and independent variables (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students build understanding of algebra as they use expressions to describe relationships between corresponding terms.	• Students build proficiency with using pattern rules to extend patterns and find corresponding terms.	• Students apply understanding of patterns to solve problems. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine

Can You Make the Number? 5–7 min

Build Fluency Students build number sense as they use 0.1, 0.3, 0.5, and 0.7 in any order and using any operations to make the target number 25.

Students must use all four numbers but can only use each number once in the combination. Encourage students to look for more than one solution.

These prompts encourage students to talk about their reasoning:

- What was your strategy for getting to the target number?
- How could you make the target number if you didn't need to use all the numbers or if you could use them more than once?

Launch 5–7 min

Sense-Making Routine

Purpose Students focus on studying and comparing numerical patterns.

Notice & Wonder

- What do you notice?
- What do you wonder?

Teaching Tip You may want to relate the numerical patterns to a real-world scenario, as in the previous lesson, to help students think more concretely about the patterns

ETP Pose Purposeful Questions

The questions that follow may be asked in any order. They are meant to help advance students' exploration of how numerical patterns may be related and are based on possible comments and questions that students may make during the share out.

- What do you notice about each pattern?
- How can you describe a relationship between the patterns?
- What tools did you use to help you find a relationship between the two patterns?

Math is... Mindset

- How do you show others you respect their ideas?

SEL Social Awareness: Respect Others

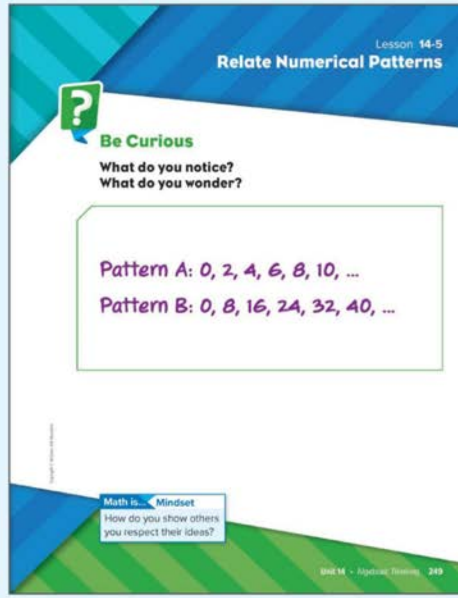
Begin the Notice & Wonder routine with a short time period for students to work with a partner. Invite students to think about strategies that can help them stay on task and work with others. In addition to developing a sense of respect for others, students will also be able to practice discipline, motivation, and focus.

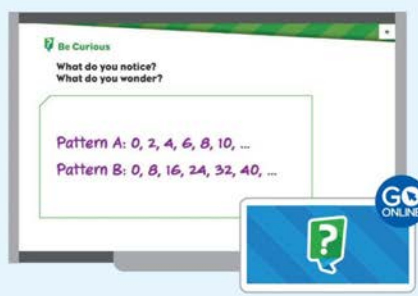
Transition to Explore & Develop

Ask students to tell you all they know about the numerical patterns. Guide students to find a relationship between the patterns, if it comes up organically. These questions will get students thinking about a relationship between the numerical patterns.

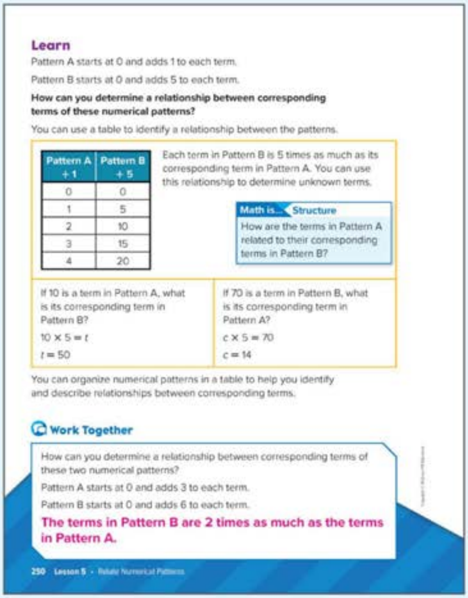
ETP Establish Mathematics Goals to Focus Learning

- Let's think about tools that can help us find a relationship between corresponding terms in two numerical patterns.





Explore & Develop 20 min



Learn

Pattern A starts at 0 and adds 1 to each term.

Pattern B starts at 0 and adds 5 to each term.

How can you determine a relationship between corresponding terms of these numerical patterns?

You can use a table to identify a relationship between the patterns.

Pattern A +1	Pattern B +5
0	0
1	5
2	10
3	15
4	20

Each term in Pattern B is 5 times as much as its corresponding term in Pattern A. You can use this relationship to determine unknown terms.

Math is... Structure
How are the terms in Pattern A related to their corresponding terms in Pattern B?

If 10 is a term in Pattern A, what is its corresponding term in Pattern B?

$10 \times 5 = t$

$t = 50$

If 70 is a term in Pattern B, what is its corresponding term in Pattern A?

$c \times 5 = 70$

$c = 14$

You can organize numerical patterns in a table to help you identify and describe relationships between corresponding terms.

Work Together

How can you determine a relationship between corresponding terms of these two numerical patterns?

Pattern A starts at 0 and adds 3 to each term.

Pattern B starts at 0 and adds 6 to each term.

The terms in Pattern B are 2 times as much as the terms in Pattern A.

250 Lesson 5 • Relate Numerical Patterns

1 Pose the Problem

ETP Pose Purposeful Questions

- How can knowing the rules for the numerical patterns help you determine a relationship between the corresponding terms?
- What tools do you know for representing numerical patterns?
- Why might you need to know a relationship between corresponding terms in numerical patterns?

2 Develop the Math

Choose the option that best meets your instructional goals.

MLR Info Gap

Pair students. Provide Partner A with a problem like the one on the Learn page. Provide Partner B with the information to carry out the problem. Instruct Partner B to ask A what information they need, and for A to respond, explaining why they need it. Have students continue until the problem is completed.

3 Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How is organizing numerical patterns in a table helpful?
- How can you determine unknown terms in a pattern using a relationship between corresponding terms?

Key Takeaway

- Organizing numerical patterns in a table can help identify relationships between corresponding terms.

Work Together

Students generate two numerical patterns when given the rules for each pattern and describe a relationship between corresponding terms in the patterns.

Common Error: If students do not write out the first few terms in each pattern, they may think a relationship between the terms is that adding 3 to the terms in Pattern *A* is how to determine the corresponding terms in Pattern *B*. Make sure students write out the terms in each pattern in a table to see a relationship between several corresponding terms.

LOM Language of Math

Ask students to define *rule,* and to explain how that word is used in the classroom. Review the math definition of *rule* and have students discuss similarities between the two definitions. Remind students that, just like they follow classroom rules, terms in a pattern follow the pattern's rule.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students generate numerical patterns when given rules and explore how they can describe a relationship between the patterns.

Directions: Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- What tools can you use to represent the patterns?
- How can using tools help you understand the patterns and look for relationships?
- How can you identify corresponding terms in the patterns? Explain why this is important.
- What is a relationship between the corresponding terms?

Math is... Structure

- How are the terms in Pattern *A* related to their corresponding terms in Pattern *B*?

Students can step back for an overview and shift perspective and view the apparent relationship in the "other direction," and make inferences about "inverses."

Activity Debrief: Have groups share their numerical patterns and the apparent relationships they discovered. Encourage students to show how they displayed the patterns and why they choose to use that method of organizing the corresponding terms. After students have shared their relationships, have them use their relationship to answer these questions.

- If 10 is a term in Pattern *A*, what is the corresponding term in Pattern *B*?
- If 70 is a term in Pattern *B*, what is the corresponding term in Pattern *A*?

Guided Exploration

Students generate numerical patterns when given rules, and use a table to assist them in describing the relationship between corresponding terms in the patterns. Students use that relationship to determine unknown terms when given a corresponding term.

ETP Use and Connect Mathematical Representations

Have the students determine the first 5 terms in the patterns. Ask:

- What is the first term in each pattern? How do you know?
- How do you determine the next 4 terms?

- **Think About It:** How does a table help you identify a relationship?

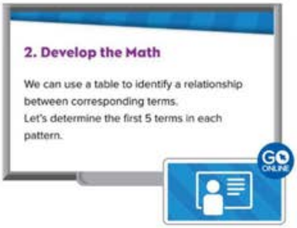
Have the students check their corresponding terms using another method (such as using the pattern rules to extend the Patterns). Make sure students communicate precisely, understand the approaches of others, and ask useful questions to improve each others' ideas. Ask:

- What is another way to find the term in Pattern *B* that corresponds to 10 in Pattern *A*?
- What is another way to find the term in Pattern *A* that corresponds to 70 in Pattern *B*?

Math is... Structure

- How are the terms in Pattern *A* related to their corresponding terms in Pattern *B*?

Students can step back for an overview and shift perspective and view the apparent relationship in the "other direction," and make inferences about "inverses."



EL English Learner Scaffolds

Entering/Emerging Ensure understanding of *corresponding* as it pertains to the lesson. Guide students to the Learn page and show them the Patterns table. Point to the 0 in Pattern *B's* column. Say *Zero is the corresponding term to zero* (pointing to each as you say the number). Repeat with 5 and 1. Then ask students *What is the corresponding term for 3: 10 or 15?*

Developing/Expanding Ensure understanding of *corresponding* as it pertains to the lesson. Guide students to the Learn page and show them the Patterns table. Point to the 0 in Pattern *B*'s column. Say *Zero is the corresponding term to zero* (pointing to each as you say the number). Repeat with 5 and 1. Then ask students *What is the corresponding term for 3?*

Bridging/Reaching Guide students to the Learn page and have them focus on the Patterns table. Ask them to name the corresponding term for each term in Pattern *A*. Then ask students to brainstorm other words for *corresponding (parallel, similar, etc.).* Allow them to use a dictionary or thesaurus if desired.

On My Own

Name ____________________

Describe a relationship between corresponding terms in Patterns A and B.

1. Pattern A starts at 0 and adds 4 to each term. Pattern B starts at 0 and adds 2 to each term.
   The terms in Pattern A are 2 times as much as the corresponding terms in Pattern B.
2. Pattern A starts at 0 and adds 3 to each term. Pattern B starts at 0 and adds 9 to each term.
   The terms in Pattern B are 3 times as much as the corresponding terms in Pattern A.
3. Pattern A starts at 0 and adds 20 to each term. Pattern B starts at 0 and adds 5 to each term.
   The terms in Pattern A are 4 times as much as the terms in Pattern B.

Use the table to answer Exercises 4–6.

Pattern A +2	Pattern B +8
0	**0**
2	**8**
4	16
6	24
8	**32**

4. Fill in the unknown terms in the table.
5. What is a relationship between the corresponding terms in Patterns A and B?
   Multiply the term in pattern A by 4 and the product is the term in Pattern B.
6. If a term in Pattern A is 20, what will be its corresponding term in Pattern B? **80**

7. Pattern A starts at 0 and adds 1 to each term. Pattern B starts at 0 and adds 6 to each term. If 5 is a term in Pattern A, what is its corresponding term in Pattern B? **30**
8. Pattern A starts at 0 and adds 4 to each term. Pattern B starts at 0 and adds 8 to each term. If 24 is a term in Pattern A, what is its corresponding term in Pattern B? **48**
9. Pattern A starts at 0 and adds 3 to each term. Pattern B starts at 0 and adds 12 to each term. If 72 is a term in Pattern B, what is its corresponding term in Pattern A? **18**
10. **STEM Connection** Saffron is baking bread. She wrote these numerical patterns to record the amount of water and flour needed.
    Water (in cups): 3, 4, 5, 6, ...
    Flour (in cups): 6, 8, 10, 12, ...
    How many cups of water is needed when using 48 cups of flour? **24 cups of water**

11. **Extend Your Thinking** A relationship between terms is that a term in Pattern A is $\frac{5}{4}$ times as much as its corresponding term in Pattern B. What could be the rules for each numerical pattern?
    Sample answer: The rule for Pattern A is add 5; the rule for Pattern B is add 4.

Reflect

How does knowing the rules of two numerical patterns help you determine an unknown corresponding term?
Answers may vary.

Math is... Mindset
How did you show others you respect their ideas?

Practice

Build Procedural Fluency from Conceptual Understanding

Common Error: Exercises 7-9 Students may struggle to answer these questions without first writing out several terms in each pattern. Encourage students to use a table to help them find the apparent relationships.

Item Analysis

Item	DOK	Rigor
1–3	1	Procedural Skill & Fluency
4–6	2	Conceptual Understanding
7–9	2	Procedural Skill & Fluency
10	2	Application
11	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How does knowing the rules of two numerical patterns help you determine an unknown corresponding term?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you show others you respect their ideas?

Students reflect on how they developed stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can arrange corresponding terms in two numerical patterns in a table.
- I can describe a relationship between corresponding terms in two numerical patterns.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

Exit Ticket ✓ Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	1	Relate numerical patterns	5.OA.B.3
2	1	Relate numerical patterns	5.OA.B.3
3	2	Relate numerical patterns	5.OA.B.3

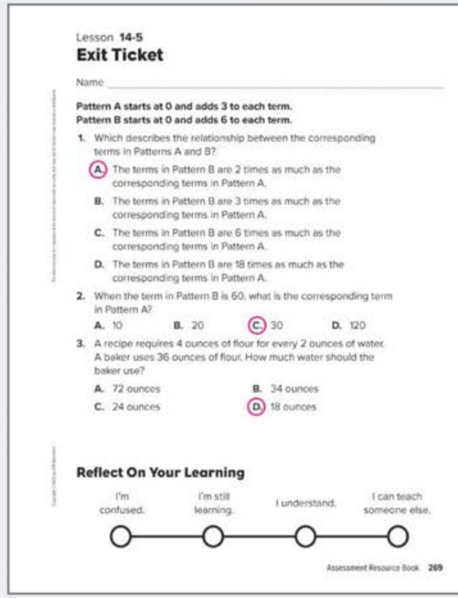
Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

- R Reinforce Understanding
- B Build Proficiency
- E Extend Thinking



Lesson 14-5

Exit Ticket

Name ____________________

Pattern A starts at 0 and adds 3 to each term.
Pattern B starts at 0 and adds 6 to each term.

1. Which describes the relationship between the corresponding terms in Patterns A and B?
   A. The terms in Pattern B are 2 times as much as the corresponding terms in Pattern A.
   B. The terms in Pattern B are 3 times as much as the corresponding terms in Pattern A.
   C. The terms in Pattern B are 6 times as much as the corresponding terms in Pattern A.
   D. The terms in Pattern B are 18 times as much as the corresponding terms in Pattern A.
2. When the term in Pattern B is 60, what is the corresponding term in Pattern A?
   A. 10 B. 20 C. 30 D. 120
3. A recipe requires 4 ounces of flour for every 2 ounces of water. A baker uses 36 ounces of flour. How much water should the baker use?
   A. 72 ounces B. 34 ounces
   C. 24 ounces D. 18 ounces

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

Assessment Resource Book 269

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

What's the Pattern?

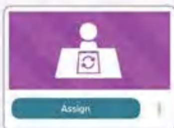
Each student rolls a number cube and generates a pattern with a rule starting at 0 and adding that number to create the first 5 terms in the pattern. Students should then compare their numerical patterns and discuss a relationship between corresponding terms in the patterns. If students describe a relationship using addition rather than multiplication, encourage them to find a relationship that works for every pair of terms in the same way.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Corresponding Terms as Ordered Pairs



INDEPENDENT WORK

Differentiation Resource Book, p. 169

Lesson 14-5 • Reinforce Understanding

Relate Numerical Patterns

Name ______________________

Review

A table can help you see the relationship between two patterns.

Pattern A: 0, 3, 6 **Pattern B:** 0, 9, 18

A: add 3	Operations	B: add 9
0	+ 0 or × 3 =	0
3	+ 6 or × 3 =	9
6	+ 12 or × 3 =	18

The relationship between A and B is the operation that is the same for all the corresponding terms.

Multiply the terms in Pattern A by 3.

Use the table to answer Exercises 1–3.

1. Fill in the missing terms in the table.
2. What is the relationship between the corresponding terms in Patterns A and B?
   Multiply the terms in Pattern A by 5.
3. When the term in Pattern A is 100, what will be the term in Pattern B?
   500

Pattern A +2	Pattern B +10
0	0
2	10
4	20
6	30
8	40

Use the table to answer Exercises 4–6.

4. Fill in the missing terms in the table.
5. What is the relationship between the terms in the table?
   Add 10 to the terms in Pattern A
6. When the term in Pattern A is 100, what will be the term in Pattern B?
   110

Pattern A +1	Pattern B +1
0	10
1	11
2	12
3	13
4	14

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Patterns on the Coordinate Plane Concentration

Students match situations, tables, and graphs.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

INDEPENDENT WORK

Student Practice Book, pp. 169–170

Lesson 14-5

Additional Practice

Name ______________________

Review

You can organize numerical patterns in a table to help you identify and describe relationships between corresponding terms and use this relationship to determine unknown terms.

Pattern A starts at 0 and adds 2 to each term.

Pattern B starts at 0 and adds 8 to each term.

What is the corresponding term in Pattern B when the term in Pattern A is 14?

Make a table to show the first 5 terms in each pattern.

Pattern A	0	2	4	6	8
Pattern B	0	8	16	24	32

Notice that the terms in Pattern B are 4 times the corresponding terms in Pattern A.

When 14 is the term in Pattern A, the corresponding term in Pattern B is $14 \times 4 = 56$.

Refer to the numeric patterns A and B above.

1. When the term in Pattern A is 22, what will be the corresponding term in Pattern B? 88
2. When the term in Pattern B is 48, what will be the corresponding term in Pattern A? 12
3. When the term in Pattern B is 200, what will be the corresponding term in Pattern A? 50
4. When the term in Pattern A is 100, what will be the corresponding term in Pattern B? 400

Student Practice Book

E Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital page to develop an understanding of the order of operations.

Assign

Use It! Application Station

Earning an Income Students research 5 jobs and incomes, create a table, and plot the results on a coordinate plane. *The content of this card has concepts covered later in Lesson 14-6. You may want to assign this card to students ready to explore content covered later in this unit.*

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 169–170

Use the patterns for problems 5–8.

Pattern A: Starts at 0 and adds 3 to each term.

Pattern B: Starts at 0 and adds 9 to each term.

5. Complete the table that shows the first six terms of the numerical patterns.

Pattern A	0	3	6	9	12	15
Pattern B	0	9	18	27	36	45

6. What is the relationship between the terms in Pattern B and the corresponding terms in Pattern A?
The terms in Pattern B are 3 times the corresponding terms in Pattern A.

7. When the term in Pattern A is 21, what will be the corresponding term in Pattern B? **63**

8. When the term in Pattern B is 90, what will be the corresponding term in Pattern A? **30**

9. A recipe requires 2 ounces of flour for every 6 ounces of water. A baker uses 12 ounces of flour. How many ounces of water should the baker use? **36** ounces

10. A restaurant uses 3 eggs in every omelet served. How many omelets were served if 24 eggs were used? **8** omelets

Math @ Home Activity Write two rules for numerical patterns on a sheet of paper. Ask your child to make a table showing the first 6 terms for each pattern. Have them explain the relationship between the corresponding terms in the two patterns. Then have them predict the corresponding number in one pattern given a term in the other pattern.

Student Practice Book

Differentiation Resource Book, p. 170

Lesson **14-5 • Extend Thinking**

Relate Numerical Patterns

Name ____________________

A baseball trading card company is running a special edition of card sets by packaging 2 classic player cards with every 4 current player cards.

1. Create a table that shows the relationship between the two type of cards by listing the number of current player cards and the corresponding number of number classic player cards.

# Classic Player Cards	# Current Player Cards
6	**12**
8	16
14	**28**

2. What is the rule for the number of current player cards?
add 4

3. What is the rule for the number of classic player cards?
add 2

4. What is the relationship between the terms in the table?
Divide the number of current player cards by 2.

5. How many classic playing cards will a special edition set that contains 64 of the current player cards?
32

6. Is it possible for a special edition set to contain exactly 10 of the current player cards? Explain how you know.
No; the number of current player cards must be a multiple of 4, and 10 is not a multiple of 4.

Differentiation Resource Book

LESSON 14-6

Graphs of Numerical Patterns

Learning Targets

- I can form ordered pairs consisting of corresponding terms from two numerical patterns.
- I can plot those ordered pairs on the coordinate plane.

Standards ◆ Major △ Supporting ● Additional

Content

○ **5.OA.B** Analyze patterns and relationships

○ **5.OA.B.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Math Practices and Processes

MPP Use appropriate tools strategically.

Vocabulary

Math Terms	Academic Terms
corresponding term	analyze
numerical pattern	speculate

Materials

The materials may be for any part of the lesson.

- blank cubes
- *Coordinate Plane* teaching resource
- index cards

Focus

Content Objective	Language Objectives	SEL Objective
• Students plot ordered pairs consisting of the corresponding terms from two numerical patterns.	• Students explain how to plot ordered pairs of corresponding terms from two numerical patterns using *can* and *should*. • To support sense-making, ELs participate in MLR3: Three Reads.	• Students discuss alternative strategies/methods for solving a mathematical problem and the value of flexible thinking.

Coherence

Previous	Now	Next
• Students generated a number or shape pattern that follows a given rule and identified apparent features of the pattern that were not explicit in the rule itself (Grade 4). • Students used a table to assist them in finding an apparent relationship between corresponding terms in two numerical patterns (Unit 14).	• Students form ordered pairs using corresponding terms from two numerical patterns, plot them on the coordinate plane, and use the graph to make conjectures.	• Students represent and analyze quantitative relationships between dependent and independent variables (Grade 6).

Rigor

Conceptual Understanding	Procedural Skill & Fluency	Application
• Students extend understanding by plotting ordered pairs and interpreting relationships between corresponding terms.	• Students develop proficiency with plotting points accurately and interpreting data shown on coordinate planes.	• Students solve problems with real-world contexts. *Application is not a specific element of rigor for this standard.*

GO ONLINE

Number Routine
Can You Make the Number? 5–7 min

Build Fluency Students build number sense as they use 0.1, 0.5, 0.7, and 0.9 in any order and with any operations in a numerical expression to make the target number 0.68.

Students must use all four numbers but can only use each number once. Encourage students to look for more than one solution.

These prompts encourage students to talk about their reasoning:

- What are some ways to think about 0.68?
- How did you use the order of operations? Why did you use grouping symbols? How is this helpful?

Launch 5–7 min

Sense-Making Routine

Purpose Students think about how a relationship between two patterns can be plotted on the coordinate plane.

Numberless Graph

- What math do you see?

Teaching Tip You may want to review with students all of the components of the coordinate plane before they begin class discussion.

ETP **Pose Purposeful Questions**

The questions that follow may be asked in any order. They are meant to help advance students' exploration of how a graph can show a relationship between two patterns and are based on possible comments and questions that students may make during the share out.

- Why do you think the points are joined by a line?
- What conjectures can you make about any other points from this example that would be plotted on that coordinate plane?

Math is... Mindset

- How do you act with your classmates to build safe classroom culture?

SEL **Responsible Decision-Making: Ethical Responsibility**

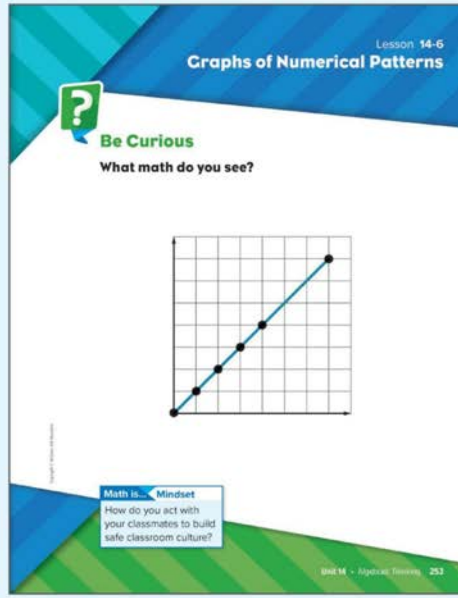
As students work through the Numberless Graph routine, have them think about the rules and routines of the classroom environment. Understanding these rules and routines of the classroom environment can help students be responsible contributors to the learning community. Remind them that thinking ethically can help them work responsibly through challenging problems.

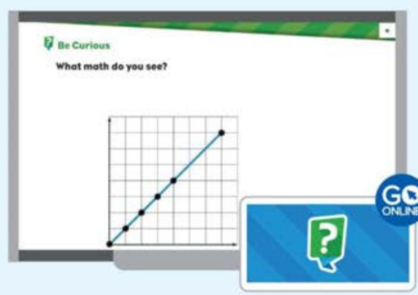
Transition to Explore & Develop

Question students to determine if they can relate the graph to two numerical patterns. Encourage them to tell you all they know about the relationships between numerical patterns.

ETP **Establish Mathematics Goals to Focus Learning**

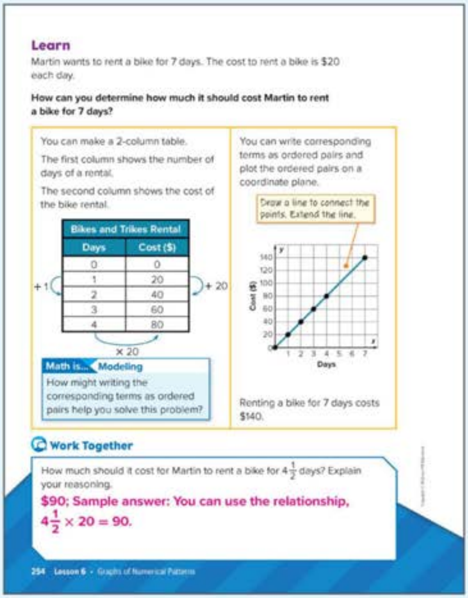
- Let's think about how we can use the coordinate plane to understand a relationship between two numerical patterns.





Explore & Develop

20 min


Learn

Martin wants to rent a bike for 7 days. The cost to rent a bike is $20 each day.

How can you determine how much it should cost Martin to rent a bike for 7 days?

You can make a 2-column table.

The first column shows the number of days of a rental.

The second column shows the cost of the bike rental.

Bikes and Trikes Rental

Days	Cost ($)
0	0
1	20
2	40
3	60
4	80

+ 1 + 20 × 20

Math is... Modeling
How might writing the corresponding terms as ordered pairs help you solve this problem?

You can write corresponding terms as ordered pairs and plot the ordered pairs on a coordinate plane.

Draw a line to connect the points. Extend the line.

Renting a bike for 7 days costs $140.

Work Together

How much should it cost for Martin to rent a bike for $4\frac{1}{2}$ days? Explain your reasoning.

$90; Sample answer: You can use the relationship, $4\frac{1}{2} \times 20 = 90$.

254 Lesson 6 • Graphs of Numerical Patterns

❶ Pose the Problem

ETP Pose Purposeful Questions

- What do you notice about the information given in the table?
- What information do you want to find?
- How can you use the information given in the table to think about the problem?

❷ Develop the Math

Choose the option that best meets your instructional goals.

MLR Three Reads

1st read: Instruct students to look at the Work Together problem on the Learn page. Ensure students understand the situation and key words: *rent, amounts*, and *charges*.

2nd read: Focus students' attention on the *How can*... question.

3rd read: Instruct students to brainstorm ways to solve the problem.

❸ Bring It Together

ETP Elicit and Use Evidence of Student Thinking

- How can you graph two numerical patterns on the coordinate plane?

Key Takeaway

- Ordered pairs can be formed using corresponding terms from two numerical patterns and graphed on the coordinate plane.

Work Together

Students work together to solve a problem involving an unknown term by using a corresponding term. Have students use a relationship and/or graph to explain their reasoning.

Common Error: Students may be confused as to how to solve the problem as $4\frac{1}{2}$ is not one of the terms of the pattern. Point out that finding a relationship allows you to make conjectures about values not in the patterns.

LOM Language of Math

Students may be aware of *coordinate* as a verb meaning organize or match, also. Point out that the noun (as in *x*-coordinate) and the verb are spelled the same but pronounced differently.

CHOOSE YOUR OPTION

Activity-Based Exploration

Students explore how they can represent numerical patterns on the coordinate plane.

Materials: *Coordinate Plane* Teaching Resource

Directions: Provide copies of the *Coordinate Plane* Teaching Resource. Have students work together to solve the Pose the Problem.

ETP Support Productive Struggle

- How can you describe the rules in the numerical patterns?
- How can you describe a relationship between corresponding terms in the numerical patterns?
- How can you write the corresponding terms as ordered pairs?
- How can you plot the points to represent the ordered pairs?

Math is... Choosing Tools

- How might writing the corresponding terms as ordered pairs help you solve this problem?

Students make sound decisions about when a tool like the coordinate plane might be helpful, recognizing both the insight to be gained from it and its limitations.

- What do you notice about the graph of these numerical patterns?

Activity Debrief: Have students share their solutions to the problem. Ensure students understand that another way to represent numerical patterns is to form and plot ordered pairs using the corresponding terms.

Guided Exploration

Students form ordered pairs using corresponding terms from two numerical patterns, plot them on the coordinate plane, and analyze the graph to draw conclusions.

ETP Facilitate Meaningful Mathematical Discourse

Have the students write the ordered pairs. Ask:

- What should the *x*- and *y*-coordinates be?

Have the students use the *Coordinate Plane* Teaching Resource to plot the ordered pairs. Ask:

- What label and numbers should go on the *x*-axis?
- What label and numbers should go on the *y*-axis?
- How can you plot each point?

- **Think About It:** Is there another way you can find the cost for 7 days? Explain how.

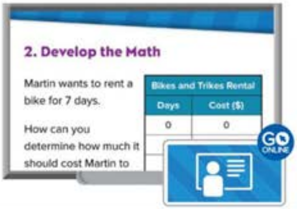
Have the students speculate about the linear functions they will learn about in Grade 6. Ask:

- What do you notice about the segments joining the points?
- Do you think other points having this relationship will be on this line?
- How could the line help you predict what other costs might be?

Math is... Choosing Tools

- How might writing the corresponding terms as ordered pairs help you solve this problem?

Students make sound decisions about when a tool like the coordinate plane might be helpful, recognizing both the insight to be gained from it and its limitations.



EL English Learner Scaffolds

Entering/Emerging Explain *write* _____ *as* _____. Guide students to the Learn page and have them look at the Bikes and Trikes Rental table. Point to the 1 and 20. Say *We can write corresponding terms as ordered pairs.* Write them as ordered pairs. Repeat with 2 and 40. Then point to 3 and 80 and ask *Can we write 3 and 80 as ordered pairs?* Point to 3 and 60 and ask *Can we write 3 and 60 as ordered pairs?*

Developing/Expanding Explain *write* _____ *as* _____. Guide students to the Learn page and have them look at the Bikes and Trikes Rental table. Point to the 1 and 20. Say *We can write corresponding terms as ordered pairs.* Write them as ordered pairs. Repeat with 2 and 40. Then ask students to complete the sentence: *We can* _____ (write) *3 and 60 as* _____ (ordered pairs).

Bridging/Reaching Guide students to the Learn page and ask students to focus on the sentence structure of *You can write the corresponding terms as ordered pairs.* Then ask students to think about what they've learned this year and to come up with a sentence with the same structure. For example: You can write fractions as decimals. Provide prompts for students who need more guidance.

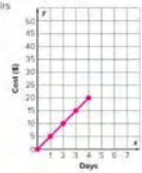
On My Own

MATH REPLAY | GO ONLINE

Name ______________________

1. The Scooters and Stuff Rental charges are shown in the table. Write the corresponding terms as ordered pairs and plot them on the coordinate plane.

Scooters and Stuff Rental		
Days	Cost ($)	Ordered Pair
0	0	(0, 0)
1	5	(1, 5)
2	10	(2, 10)
3	15	(3, 15)
4	20	(4, 20)



2. What is the rule for the pattern in the Days column of the table?
add 1

3. What is the rule for the pattern in the Cost ($) column of the table?
add 5

4. What is a relationship between the corresponding terms in the table?
Multiply days by 5.

5. How much should it cost to rent a scooter for 8 days?
$40

6. Write the ordered pair and plot the point on the coordinate plane for 8 days.
(8, 40); Check students' work.

7. How much should it cost to rent a scooter for $6\frac{1}{2}$ days?
$32.50

8. **STEM Connection** Malik learns that the light from a laser is stronger when the current is stronger. He is helping to make a laser where the rule for the current is add 10, and the rule for the light strength is add 2. Write the corresponding terms in a table, and then plot the points on the coordinate plane.
Check students' work.

9. **Extend Your Thinking** How does graphing numerical patterns help you understand the relationship between the patterns?
Answers may vary.

Reflect

How can you plot ordered pairs consisting of corresponding terms from two patterns?
Answers may vary.

Math is... Mindset
How did you and your classmates build a safe classroom culture?

Practice

ETP Build Procedural Fluency from Conceptual Understanding

Common Error: Exercise 1 Make sure students do not mix up the *x*-coordinate and *y*-coordinate for each ordered pair as they plot the points. Remind students that the *x*-axis is the horizontal axis, and the *y*-axis is the vertical axis.

Item Analysis

Item	DOK	Rigor
1–4, 6	1	Procedural Skill & Fluency
5, 7	2	Application
8	2	Application
9	3	Conceptual Understanding

Reflect

Students complete the Reflect question.

- How can you plot ordered pairs consisting of corresponding terms from two patterns?

Ask students to share their reflections with their classmates.

Math is... Mindset

- How did you and your classmates build a safe classroom culture?

Students reflect on how they developed stronger relationship skills.

Learning Targets

Ask students to reflect on the Learning Targets of the lesson.

- I can form ordered pairs consisting of corresponding terms from two numerical patterns.
- I can plot those ordered pairs on the coordinate plane.

To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.

GO ONLINE MATH REPLAY

Exit Ticket Formative Assessment

The *Exit Ticket* assesses students' understanding of lesson concepts.

Metacognitive Check *Reflect on Your Learning* allows students to think about their level of understanding of the lesson content on a scale of 1 to 4 with 4 being the highest confidence.

Exit Ticket Skill Tracker

Item	DOK	Skill	Standard
1	2	Develop ordered pairs to graph numerical patterns	5.OA.B.3
2	2	Graph numerical patterns	5.OA.B.3
3	2	Interpret graphs of numerical patterns	5.OA.B.3

Data Use students' scores on the *Exit Ticket* to assign the differentiated resources available. When students complete the *Exit Ticket* in the digital workspace, their responses are auto-scored.

Exit Ticket Recommendations

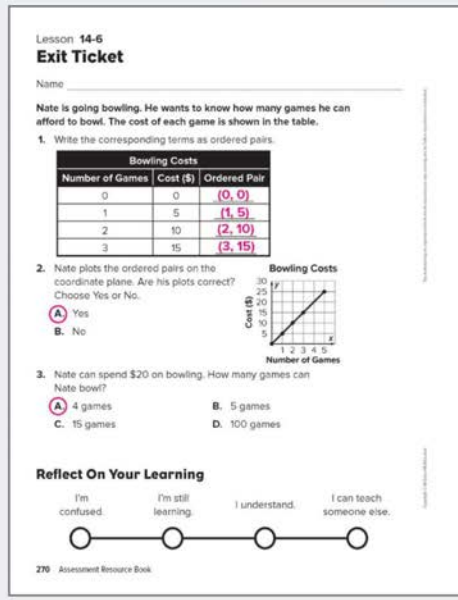
If students score	Then have students do
3 of 3	Additional Practice or any of the B or E activities
2 of 3	*Take Another Look* or any of the B activities
1 or fewer of 3	Small Group Intervention or any of the R activities

Key for Differentiation

R Reinforce Understanding

B Build Proficiency

E Extend Thinking



Lesson 14-6

Exit Ticket

Name

Nate is going bowling. He wants to know how many games he can afford to bowl. The cost of each game is shown in the table.

1. Write the corresponding terms as ordered pairs.

Bowling Costs		
Number of Games	Cost ($)	Ordered Pair
0	0	(0, 0)
1	5	(1, 5)
2	10	(2, 10)
3	15	(3, 15)

2. Nate plots the ordered pairs on the coordinate plane. Are his plots correct? Choose Yes or No.

A. Yes

B. No

Bowling Costs

Cost ($): 5, 10, 15, 20, 25, 30

Number of Games: 1, 2, 3, 4, 5

3. Nate can spend $20 on bowling. How many games can Nate bowl?

A. 4 games

B. 5 games

C. 15 games

D. 100 games

Reflect On Your Learning

I'm confused. — I'm still learning. — I understand. — I can teach someone else.

270 Assessment Resource Book

Differentiate

10 min Select resources based on your classroom setup and your students' needs.

R Reinforce Understanding

SMALL GROUP

Lining Up Points!

Provide a cube labeled with 6 different rules and a chart with *x*-coordinates listed on an index card. Each student rolls the cube and uses the rule to fill in the *y*-coordinates on the chart. Then each student plots the points on the coordinate plane in a unique color. Have students discuss relationships that they see between the sets of colored points. Help students recognize that all of the rules resulted in straight lines.

GO ONLINE

Take Another Look Lesson

Assign the interactive lesson to reinforce targeted skills.

- Graph Ordered Pairs

Assign

INDEPENDENT WORK

Differentiation Resource Book, p. 171

Lesson **14-6 • Reinforce Understanding**

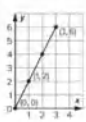
Graphs of Numerical Patterns

Name ____________________

Review

You can graph corresponding terms from two numerical patterns as ordered pairs on a coordinate plane.

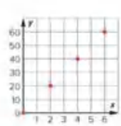
Pattern A	Pattern B	Ordered Pair
0	0	(0, 0)
1	2	(1, 2)
?	?	?
3	6	(3, 6)



The graph shows the missing values as the point (2, 4)

Use the table to answer Exercises 1–3.

Pattern A	Pattern B	Ordered Pair
0	0	(0, 0)
2	20	(2, 20)
4	40	(4, 40)
6	60	(6, 60)



1. Write the ordered pairs and plot the points on the graph.
2. Use the graph to predict the value of Pattern B when Pattern A is equal to 3. Write your answer as a coordinate pair.
   (3, 30)
3. What is the relationship between the corresponding terms?
   multiply Pattern A by 10

Differentiation Resource Book

B Build Proficiency

WORKSTATIONS

Practice It! Game Station

Patterns on the Coordinate Plane Concentration

Students match situations, tables, and graphs.

GO ONLINE

Interactive Additional Practice

Assign the digital version of the Student Practice Book.

Assign

INDEPENDENT WORK

Student Practice Book, pp. 171–172

Lesson **14-6**

Additional Practice

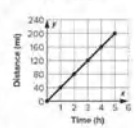
Name ____________________

Review

You can plot corresponding terms from numerical patterns as ordered pairs on a coordinate plane and use the data to solve problems.

The table shows the number of hours driven on a car trip and the number of miles traveled. The ordered pairs are graphed on the coordinate plane.

Number of hours	Number of miles	Ordered pair
1	40	(1, 40)
2	80	(2, 80)
3	120	(3, 120)
4	160	(4, 160)
5	200	(5, 200)



How many hours will it take to travel 240 miles?

Extend the line of the graph. When the line reaches 240 miles, along the *y*-axis, the *x*-coordinate of the point will be 6.

It will take 6 hours to travel 240 miles.

Refer to the data above for problems 1–4.

1. How many miles were traveled after 3 hours? 120 miles
2. How many hours did it take to travel 80 miles? 2 hours
3. What is the relationship between the number of hours spent traveling and the number of miles traveled?
   Multiply the number of hours traveled by 40.
4. After 10 hours, how many miles will be traveled? 400 miles

Student Practice Book

Extend Thinking

WORKSTATIONS

Own It! Digital Station

Build Fluency **Games**

Assign the digital page to develop an understanding of the order of operations.

Assign

Use It! Application Station

Color by Number Students use grid paper to create designs and numerical expressions.

GO ONLINE

Spiral Review

Assign the digital Spiral Review Practice to students or download and print PDFs of the Spiral Review Practice from the Digital Teacher Center.

Assign

Websketch Exploration

Assign a websketch exploration to apply skills and extend thinking.

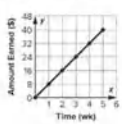
Assign

INDEPENDENT WORK

Student Practice Book, pp. 171–172

5. Fran earns $8.00 per hour at her job. Complete the table showing the number of hours Fran works and the amount of money she earns. Then graph the ordered pairs.

Number of Hours	Amount Earned ($)	Ordered pair
0	0	(0, 0)
1	8	(1, 8)
2	16	(2, 16)
3	24	(3, 24)
4	32	(4, 32)
5	40	(5, 40)



6. What is the rule for the pattern in the Number of Hours column in the table?
Add 1.
7. What is the rule for the pattern in the Amount Earned column in the table?
Add 8.
8. What is the relationship between the corresponding terms in the table? Multiply the number of hours by 8 to get the amount earned.
9. How much money does Fran earn for working 4 hours? $ 32
10. How many hours does Fran have to work in order to earn $40?
5 hours
11. How much money will Fran earn for working 8 hours? $ 64
12. How many hours does Fran have to work in order to earn $80?
10 hours

Math @ Home Activity

Student Practice Book

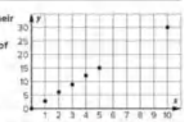
Differentiation Resource Book, p. 172

Lesson **14-6 • Extend Thinking**

Graphs of Numerical Patterns

Name ______________________________

The Santiago family is filling their car with gas at Gary's Gas Station. The costs per gallons of gas are shown on the graph.



Use the graph to answer the questions.

1. Fill in the table for the first 4 gallons.

Gallons	Cost ($)
0	0
1	3
2	6
3	9
4	12

2. How much will it cost to fill a tank with 5 gallons of gas?
$15
3. How much it will cost to fill a tank with 10 gallons of gas?
$30
4. What is the rule for the Cost column in the table?
add 3
5. What is the relationship between the number of gallons and the cost of gas?
Multiply the number of gallons by 3.
6. How much does gas cost per gallon at Gary's Gas Station?
$3.00 per gallon
7. How many Gallons of gas can be purchased with $50? Explain how you found your answer.
16 gallons; 10 gallons costs $30 and 6 gallons cost $18. $30 + $18 = $48 and 10g + 6g = 16g; so you can buy 16 gallons of gas with $50.

Differentiation Resource Book

Unit Review

Unit Review Name ______________

Vocabulary Review

Choose the correct word(s) to complete the sentence.

corresponding term	grouping symbols	order of operations
evaluate	numerical expression	parentheses
expression	numerical pattern	rule

1. A(n) __rule__ can be used to create a numerical pattern. (Lesson 14-5)
2. A(n) __numerical expression__ represents a number using only numbers and symbols, but not unknown values. (Lesson 14-1)
3. The rules that dictate the sequence in which the operations in an expression should be evaluated are called the __order of operations__. (Lesson 14-3)
4. When you __evaluate__ an expression, you calculate a value for the expression. (Lesson 14-3)
5. A(n) __expression__ can consist of numbers, operations, symbols, and unknown values. (Lesson 14-3)
6. A(n) __numerical pattern__ is a sequence of numbers generated by a rule. (Lesson 14-4)
7. __Grouping symbols__ are used to show where the group begins and where it ends. (Lesson 14-2)
8. __Corresponding terms__ are numbers in numerical patterns that appear in identical places. (Lesson 14-5)
9. The grouping symbol ( ), called __parentheses__, is used in an expression so it is evaluated first. (Lesson 14-2)

Unit 14 • Algebraic Thinking 257

Review

10. What numerical expression represents *three more than seven?* (Lesson 14-1) **7 + 3**
11. What operation is performed first? (Lesson 14-3)
    $8 + 16 \div 4 - 2$
    division
12. What are the rules for Pattern C and Pattern D? What is the relationship between the corresponding terms of Pattern C and Pattern D? (Lesson 14-5)

Pattern C	Pattern D
0	0
1	12
2	24
3	36
4	48

Pattern C: add 1. Pattern D: add 12. Multiply the terms in Pattern C by 12 to find the corresponding terms in Pattern D.

13. What is a verbal description for the numerical expression $100 + (5 \times 10)$? (Lesson 14-2)
    Sample answer: one hundred more than five multiplied by ten.
14. What expression represents *twelve less than eighteen?* (Lesson 14-1)
    18 − 12
15. What is a verbal description for the numerical expression $10 - (8 \div 4)$? (Lesson 14-2)
    Sample answer: ten minus the quotient of eight and four.
16. What operation is performed first? (Lesson 14-3)
    $12 \times (4 + 6) \div 6$
    addition
17. What is the value of the expression? (Lesson 14-3)
    $5 \times 25 - 18 \times 2$ **89**
18. Jared and Robert are playing different video games. Jared passes 2 levels each time he plays. Robert passes 3 levels each time he plays. When Jared passes 8 levels, how many levels will Robert have passed playing the same number of times? (Lesson 14-4) **12 levels**
19. Using words, compare these expressions. (Lesson 14-2)
    $(8 \times 4) + 6$ $\quad$ $8 \times (4 + 6)$
    Eight times the sum of four and six is greater than the sum of the product of eight and four plus six.

258 Unit 14 • Review

Students can complete the **Unit Review** to prepare for the **Unit Assessment.** Students may complete the Review in their Interactive eBook in the Digital Students Center.

Vocabulary Review

Item Analysis

Item	Lesson
1	14-5
2	14-1
3	14-3
4	14-3
5	14-3
6	14-4
7	14-2
8	14-5
9	14-2

Review

Item Analysis

Item	DOK	Lesson	Standard
10	1	14-1	5.OA.A.1, 5.OA.A.2
11	1	14-3	5.OA.A.1
12	2	14-5	5.OA.B.3
13	1	14-2	5.OA.A.1, 5.OA.A.2
14	1	14-1	5.OA.A.1, 5.OA.A.2
15	1	14-2	5.OA.A.1, 5.OA.A.2
16	1	14-3	5.OA.A.1
17	1	14-3	5.OA.A.1
18	2	14-4	5.OA.B.3
19	1	14-2	5.OA.A.1, 5.OA.A.2

To review the lessons in this unit, have students watch the Math Replay video in their Digital Student Center.

Assign the Unit Review practice to students from the Digital Teacher Center.

GO ONLINE

MATH REPLAY

Item Analysis (continued)

Item	DOK	Lesson	Standard
20a	2	14-5	5.OA.B.3
20b	2	14-6	5.OA.B.3
21	1	14-1	5.OA.A.1, 5.OA.A.2
22	1	14-2	5.OA.A.1, 5.OA.A.2
23	1	14-1	5.OA.A.1, 5.OA.A.2
24	1	14-3	5.OA.A.1
25	1	14-4	5.OA.B.3
26	1	14-3	5.OA.A.1

Performance Task

Standards: 5.OA.A.1, 5.OA.A.2, 5.OA.B.3

Rubric (2 points)

Part A (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with using expressions and patterns.
1 POINT	Student's work reflects developing proficiency with using expressions and patterns.
0 POINTS	Student's work shows weak proficiency with using expressions and patterns.

Part B (DOK 2) – 2 points	
2 POINTS	Student's work reflects proficiency with using expressions and patterns.
1 POINTS	Student's work reflects developing proficiency with using expressions and patterns.
0 POINTS	Student's work shows weak proficiency with using expressions and patterns.

Reflect

The Reflect question provides an opportunity for students to express their understanding of the unit level focus question.

MATH REPLAY | GO ONLINE

20. Clenna wants to know what she will earn mowing 8 lawns. The table shows the total amount she earns mowing different numbers of lawns.

Number of Lawns	Amount Earned ($)
0	0
1	30
2	60
3	90
4	120

a. How much will Clenna earn mowing 8 lawns? Use the coordinate plane to find the answer. (Lesson 14-5) **$240**

Lawn Mowing

b. How much will Clenna earn mowing 12 lawns? (Lesson 14-6) **$360**

21. What numerical expression represents *subtract eleven from twenty, then divide by three?* (Lesson 14-1) **$(20 - 11) \div 3$**

22. What is a verbal description for the numerical expression $(6 \div 2) + 3$? (Lesson 14-2)

Sample answer: divide six by two, then add three.

23. What numerical expression represents *add three and six, then multiply by twenty?* (Lesson 14-1)

$(3 + 6) \times 20$

24. What is the value of the expression? (Lesson 14-3)

$6 \times (8 - 3) + 14$ **44**

25. Pattern A starts at 0 and adds 4. Pattern B starts at 0 and adds 8. What is the term for Pattern B when Pattern A's term is 24? (Lesson 14-4) **48**

26. What is the value of the expression? (Lesson 14-3)

$250 - (12 \times 5) - 10 \times 2$ **170**

Performance Task

Malik has programmed a light show for a concert that will be played against a rectangular shaped screen above the stage.

Part A: Each time he flashes lights, Malik flashes red lights 2 more times and blue lights 3 more times. Start at 0 and write the next 4 terms of the sequences for the red and blue lights.

red lights: 0, 2, 4, 6, 8, ...
blue lights: 0, 3, 6, 9, 12, ...

Part B: The table shows the horizontal and vertical distances of a photograph in feet from the bottom left corner as it moves across the screen.

Horizontal Distance (ft)	Vertical Distance (ft)
0	0
1	4
2	8
3	12

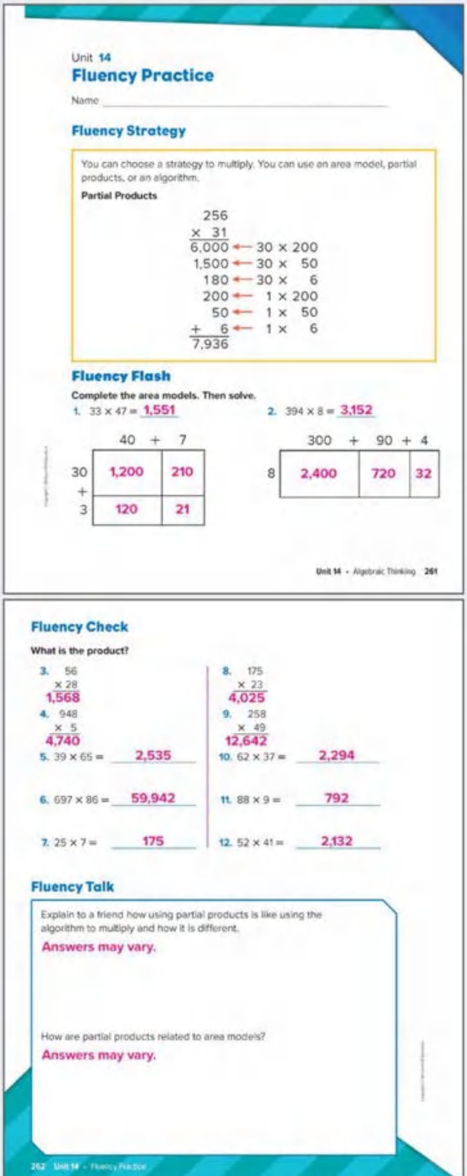
What is the rule for the Horizontal Distance and Vertical Distance? What is the relationship between the corresponding terms in the Horizontal and Vertical Distances?

Horizontal Distance: add 1, Vertical Distance: add 4. Multiply the horizontal distance by 4 to find the corresponding term in vertical distances.

Reflect

How can I use expressions to find a relationship between two sets of number patterns?

Answers may vary.



Unit 14

Fluency Practice

Name ____________________

Fluency Strategy

You can choose a strategy to multiply. You can use an area model, partial products, or an algorithm.

Partial Products

```
  256
×  31
6,000 ← 30 × 200
1,500 ← 30 ×  50
  180 ← 30 ×   6
  200 ←  1 × 200
   50 ←  1 ×  50
+   6 ←  1 ×   6
7,936
```

Fluency Flash

Complete the area models. Then solve.

1. 33 × 47 = 1,551

	40	7
30	1,200	210
3	120	21

2. 394 × 8 = 3,152

	300	90	4
8	2,400	720	32

Unit 14 • Algebraic Thinking 261

Fluency Check

What is the product?

3. 56 × 28 = 1,568
4. 948 × 5 = 4,740
5. 39 × 65 = 2,535
6. 697 × 86 = 59,942
7. 25 × 7 = 175
8. 175 × 23 = 4,025
9. 258 × 49 = 12,642
10. 62 × 37 = 2,294
11. 88 × 9 = 792
12. 52 × 41 = 2,132

Fluency Talk

Explain to a friend how using partial products is like using the algorithm to multiply and how it is different.

Answers may vary.

How are partial products related to area models?

Answers may vary.

262 Unit 14 • Fluency Practice

Fluency practice helps students develop procedural fluency, that is, the "ability to apply procedures accurately, efficiently, and flexibly." Because there is no expectation of speed, students should not be timed when completing the practice activity.

Build Fluency **Objective** Students practice choosing a strategy to multiply.

Fluency Progression

Unit	Skill	Standard
1	Use Partial Sums to Add	4.NBT.B.4
2	Decompose by Place Value to Subtract	4.NBT.B.4
3	Use an Algorithm to Add	4.NBT.B.4
4	Use an Algorithm to Subtract	4.NBT.B.4
5	Choose a Strategy to Add	4.NBT.B.4
6	Choose a Strategy to Subtract	4.NBT.B.4
7	Multiply by Multiples of 10	5.NBT.B.5
8	Multiply by Multiples of 100	5.NBT.B.5
9	Divide Multiples of 10	5.NBT.B.6
10	Divide Multiples of 100	5.NBT.B.6
11	Use an Algorithm to Multiply (2- and 3-Digit Numbers by 1-Digit Numbers)	5.NBT.B.5
12	Use an Algorithm to Multiply (2-Digit Numbers by 2-Digit Numbers)	5.NBT.B.5
13	Choose a Strategy to Multiply	5.NBT.B.5
14	**Choose a Strategy to Multiply**	**5.NBT.B.5**

Fluency Expectations

Grade 4
- Add and subtract within 1,000,000.

Grade 5
- Multiply multi-digit whole numbers.

Grade 6
- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Invoicing

Students draw on their understanding of numerical expressions and patterns. Use the rubric shown to evaluate students' work.

Standards: 5.OA.A.1, 5.OA.B.3

Rubric (8 points)

Part A (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with interpreting numerical expressions. The student's explanation is accurate.
1 POINT	Student's work reflects a developing proficiency with interpreting numerical expressions. The student's explanation is incomplete or contains a minor error.
0 POINTS	Student's work reflects a weak proficiency with interpreting numerical expressions. The student's explanation is inaccurate.

Part B (DOK 1) – 2 points	
2 POINTS	Student's work reflects a proficiency with evaluating numerical expressions. The student's answer and calculation are accurate.
1 POINT	Student's work reflects a developing proficiency with evaluating numerical expressions. The student's answer is inaccurate due to a minor error in calculation.
0 POINTS	Student's work reflects a weak proficiency with evaluating numerical expressions. The student's answer is incorrect due to several errors in calculation.

Part C (DOK 3) – 2 points	
2 POINTS	Student's work reflects a proficiency with describing rules in patterns and relationships between corresponding terms. The student's rules and relationship descriptions are accurate.
1 POINT	Student's work reflects a developing proficiency with describing rules in patterns and relationships between corresponding terms. The student's rules or relationship description is inaccurate.
0 POINTS	Student's work reflects a weak proficiency with describing rules in patterns and relationships between corresponding terms. The student's rules and relationship descriptions are inaccurate.

Part D (DOK 2) – 2 points	
2 POINTS	Student's work reflects a proficiency with graphing ordered pairs. The student accurately graphs the coordinates and has the correct cost to purchase 7 of Lure F.
1 POINT	Student's work reflects a developing proficiency with graphing ordered pairs. The student had an error in graphing the coordinates or the cost to purchase 7 of Lure F is incorrect.
0 POINTS	Student's work reflects a weak proficiency with graphing ordered pairs. The student inaccurately graphs the coordinates and the cost to purchase 7 of Lure F is incorrect.

Unit **14**

Performance Task

Name ______________________

Invoicing

Dwight is calculating an invoice for a customer who is buying some of his fishing lures. The table shows the type of lure and its normal price.

Lure Type	Normal Price
A	$5 each
B	$3 each
C	$6 each
D	$5 each
E	$4 each

Part A

Dwight is totaling the cost for the customer. He writes the following expression. From the expression, what was the customer's order?

$5(7 + 9) + 10 \times 6 + 8(3 + 4)$

The customer ordered 7 of lure A, 9 of lure D, 10 of lure C, 8 of lure B and 8 of lure E. Another possible answer is the customer ordered 7 of lure D, 9 of lure A, 10 of lure C, 8 of lure B and 8 of lure E.

Part B

Help Dwight calculate the total for the customer's invoice. Show your work.

$196; 5(7 + 9) + 10 × 6 + 8(3 + 4) = 5 × 16 + 10 × 6 + 8 × 7 = 80 + 60 + 56 = 140 + 56 = 196

Assessment Resource Book 271

Part C

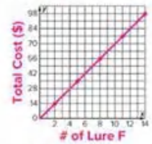
A week later, Dwight introduces Lure F for sale in his store. Describe the rules in each numerical pattern and the relationship between the corresponding terms in the patterns.

Pattern 1	Pattern 2
Number of Lure F	Total Cost ($)
2	$14
5	$35
8	$56
11	$77
14	$98

The rule for the pattern 1 is to add 3 to each term. The rule for pattern 2 is to add 21 to each term. The relationship between the corresponding terms in the patterns is to multiple the term of pattern 1 by 7 to get the corresponding term in pattern 2.

Part D

Graph the table from Part C on the grid provided. Be sure to label the axes. Using your graph, how much will it cost to purchase 7 of Lure F? **49**



272 Assessment Resource Book

Unit Assessments

Two forms of the Unit Assessment, Form A and Form B, are available for either print or digital administration. The items on the two assessments are parallel items, assessing the same concept and standard. The table below provides the item analysis for both forms.

Both Unit Assessments are available in the Assessment Resource Book or as downloadable files from the Digital Teacher Center.

Data When students complete the Unit Assessment in the Digital Student Center, their responses are auto-scored.

Item Analysis

Item	DOK	Lesson	Guided Support Intervention Lesson	Standard
1	1	14-1	Write Numerical Expressions	5.OA.A.1, 5.OA.A.2
2	2	14-1	Write Numerical Expressions	5.OA.A.1, 5.OA.A.2
3	1	14-2	Interpret the Magnitude of Expressions	5.OA.A.1, 5.OA.A.2
4	2	14-2	Interpret the Magnitude of Expressions	5.OA.A.1, 5.OA.A.2
5	1	14-3	Apply Order of Operations	5.OA.A.1
6	1	14-3	Apply Order of Operations	5.OA.A.1
7	2	14-2	Interpret the Magnitude of Expressions	5.OA.A.2
8	2	14-4	Create and Analyze the Pattern	5.OA.B.3
9	2	14-4	Create and Analyze the Pattern	5.OA.B.3
10	2	14-6	Graph Ordered Pairs	5.OA.B.3
11	2	14-6	Graph Ordered Pairs	5.OA.B.3
12	2	14-6	Graph Ordered Pairs	5.OA.B.3
13	1	14-5	Corresponding Terms as Ordered Pairs	5.OA.B.3
14	2	14-5	Corresponding Terms as Ordered Pairs	5.OA.B.3
15	3	14-3	Understand Order of Operations	5.OA.A.1

GO ONLINE

Assign the digital Unit Assessment (Form A or B) to students or download and print PDFs from the Digital Teacher Center.

Unit 14

Unit Assessment, Form A

Name ______________________

1. Which numerical expression represents the description?
   Add 3 and 5. Then subtract the sum from 20.
   A. $3 + 5 - 20$ B. $(3 + 5) - 20$
   C. $20 - 3 + 5$ D. $20 - (3 + 5)$

2. Cary makes snack bags using peanuts and raisins. He mixes 5 ounces of peanuts with 3 ounces of raisins. Which expression can be used to find how many ounces of peanuts and raisins Cary uses to make 6 snack bags?
   A. $6 \times 5 + 3$
   B. $6 \times 3 + 5$
   C. $6 + 5 \times 3$
   D. $(6 \times 5) + (6 \times 3)$

3. Which correctly describes the expression $15 - (3 \times 4)$?
   A. subtract 3 from 15, then multiply by 4
   B. find the product of 3 and 4, then subtract 15
   C. multiply 3 and 4, then subtract the product from 15
   D. multiply 15 and 4, then subtract 3

4. Write the description for the numerical expression.
   $7 \times (12 - 8)$
   Subtract 8 from 12, then multiply by 7.

5. Which is the solution?
   $6 + 8 \div 2 =$ ____
   A. 7 B. 10 C. 11 D. 12

Assessment Resource Book 273

6. What is the solution?
   $6 \div (2 + 1) \times 7 =$ **14**

7. Which situation can be represented by the numerical expression?
   $6 \times 3 + 2$
   A. Gertrude bought 6 hats that each cost 3 dollars and a pair of sunglasses that cost 2 dollars.
   B. Gertrude bought 6 hats and 2 pairs of sunglasses that each cost 3 dollars.
   C. Gertrude bought a hat that costs 6 dollars and 3 pairs of sunglasses that each cost 2 dollars.
   D. Gertrude bought 3 hats and 2 pairs of sunglasses that each cost 6 dollars.

Ken and Tamika are building a puzzle. On the first day, Ken and Tamika did not put together any of the puzzle pieces. Each day after the first day, Ken fits in 12 more pieces than he did the previous day and Tamika fits in 6 more pieces than she did on the previous day.

8. What are the first five terms in Tamika's numerical pattern?
   A. 0, 12, 24, 36, 48 B. 0, 12, 24, 48, 96
   C. 0, 6, 12, 18, 24 D. 0, 6, 12, 24, 48

9. On the day when Ken has fit in 60 puzzle pieces, how many puzzle pieces has Tamika fit in?
   A. 6 pieces B. 18 pieces
   C. 30 pieces D. 120 pieces

274 Assessment Resource Book

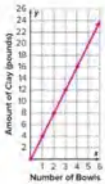
Unit 14
Unit Assessment, Form A (continued)

Name ______________________

Nicolaus is making bowls out of clay. He wants to know the number of bowls he can make with the amount of clay he has.

10. Write the corresponding terms as ordered pairs and plot them on the coordinate plane.

Clay Bowls		
Number of Bowls	Amount of Clay (pounds)	Ordered Pair
0	0	(0, 0)
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)



11. Which is the rule for showing the relationship between the number of bowls and the number of pounds of clay used?

A. Add 1 **B.** Add 4
C. Multiply by 4 **D.** Multiply by 2

12. Nicolaus makes 5 bowls. How many pounds of clay does he use?

A. 5 pounds **B.** 12 pounds
C. 16 pounds **D.** 20 pounds

13. Which describes the relationship between the corresponding terms in Patterns A and B?

Pattern A starts at 0 and adds 4 to each term. Pattern B starts at 0 and adds 8 to each term.

A. The terms in Pattern B are 4 more than the corresponding terms in Pattern A.

B. The terms in Pattern B are 8 more than the corresponding terms in Pattern A.

C. The terms in Pattern B are 2 times as much as the corresponding terms in Pattern A.

D. The terms in Pattern B are 4 times as much as the corresponding terms in Pattern A.

14. Kerri makes a bracelet that uses 4 red beads for every 2 blue beads. One bracelet Kerri makes uses 12 red beads. How many blue beads are on the bracelet?

A. 6 blue beads **B.** 10 blue beads
C. 18 blue beads **D.** 24 blue beads

15. What is the solution? Explain how to use order of operations to solve.

$72 \div (3 + 6) - 4 \times 2$

0; Sample answer: According to the Order of Operations we solve within the parentheses first: $72 \div (3 + 6) - 4 \times 2 = 72 \div 9 - 4 \times 2$. Then multiply and divide from left to right: $8 - 4 \times 2 = 8 - 8$. Then add and subtract from left to right: $8 - 8 = 0$.

Form B

Unit 14
Unit Assessment, Form B

Name ______________________

4. Write the description for the numerical expression.

Subtract 8 from 10, then multiply by 5.

Jim and June are starting to exercise by walking around the track. On the first day, Jim and June did not walk around the track. Each day after the first day, Jim walks 2 more laps around the track than he did the previous day and June walks 4 more laps around the track than she did the previous day.

15. What is the solution? Explain how to use order of operations to solve.

66; Sample answer: According to the Order of Operations we solve within the parentheses first: $10 \times (8 - 2) + 24 \div 4 = 10 \times 6 + 24 \div 4$. Then multiply and divide from left to right: $60 + 24 \div 4 = 60 + 6$. Then add and subtract from left to right: $60 + 6 = 66$.

Summative Assessment

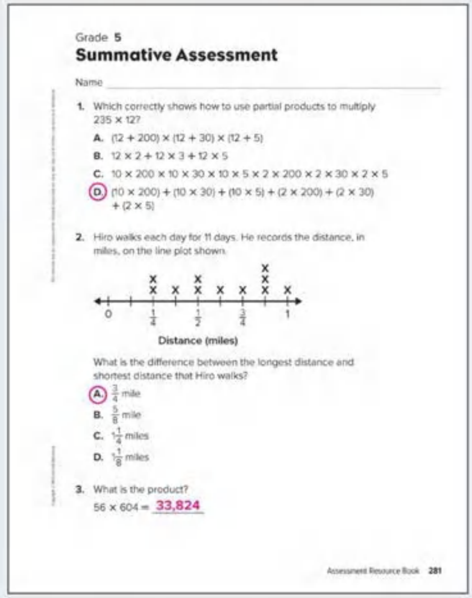
The Summative Assessment is available in both print and digital.

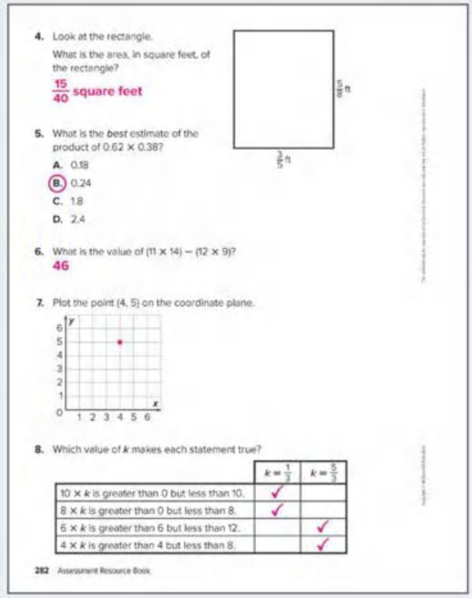
Data When students complete the Summative Assessment in the Digital Student Center, their responses are auto-scored.

GO ONLINE

Item Analysis

Item	DOK	Skill	Standard
1	2	Multiply multi-digit numbers	5.NBT.B.5
2	2	Use data from a line plot to solve word problems	5.MD.B.2
3	2	Multiply multi-digit numbers	5.NBT.B.5
4	2	Determine area with fraction side lengths	5.NF.B.4
5	2	Estimate products of decimals	5.NBT.B.7
6	2	Evaluate expressions with parentheses	5.OA.A.1
7	1	Plot ordered pairs on the coordinate plane	5.G.A.2
8	3	Interpret multiplication of fractions as scaling	5.NF.B.5
9	2	Determine volume using unit cubes	5.MD.C.4
10	2	Add fractions	5.NF.A.2
11	2	Estimate quotients	5.NBT.B.6
12	2	Relate fractions to division	5.NF.B.3
13	1	Read and write decimals	5.NBT.A.3.a
14	2	Represent multiplication of fractions and whole numbers	5.NF.B.4
15	3	Classify quadrilaterals	5.G.B.4
16	2	Divide decimals	5.NBT.B.7
17	2	Multiply multi-digit numbers	5.NBT.B.5
18	2	Interpret numerical expressions	5.OA.A.2
19	2	Estimate sums of decimals	5.NBT.B.7
20	2	Subtract decimals	5.NBT.B.7
21	2	Multiply decimals by powers of 10	5.NBT.A.1
22a	2	Use partial quotients to divide	5.NBT.B.6
22b	2	Divide multi-digit numbers	5.NBT.B.6
23	2	Multiply decimals	5.NBT.B.7
24	2	Estimate quotients of decimals	5.NBT.B.7
25	1	Solve volume word problems	5.MD.C.5
26	2	Solve measurement conversion word problems	5.MD.A.1
27	1	Add decimals	5.NBT.B.7
28	2	Round decimals	5.NBT.A.4
29	2	Multiply mixed numbers by whole numbers	5.NF.B.6
30	2	Multiply decimals	5.NBT.B.7
31	2	Subtract mixed numbers	5.NF.A.1
32	3	Form ordered pairs from patterns	5.OA.B.3
33	2	Divide whole numbers by fractions	5.NF.B.7
34	2	Add mixed numbers	5.NF.A.1
35	2	Relate multiplication and division	5.NBT.B.6





Grade **5**

Summative Assessment (continued)

Name ____________________

9. Jamal uses unit cubes to fill a box. The first layer of unit cubes is 4 unit cubes long and 6 unit cubes wide.

If the volume of the box is 72 cubic units, how many layers of unit cubes does Jamal put in the box? **3 layers**

10. What is the value of $\frac{3}{4} + \frac{5}{7}$? Fill in the missing numbers to solve.

$\frac{3}{4} + \frac{5}{7} = \frac{21}{28} + \frac{20}{28}$

$\frac{3}{4} + \frac{5}{7} = \frac{41}{28}$

$\frac{3}{4} + \frac{5}{7} = 1\frac{13}{28}$

11. Which quotient is the *best* estimate of 3,702 ÷ 41?

A. 70 **B.** 90 **C.** 100 **D.** 120

12. Which expression is equivalent to the fraction $\frac{4}{32}$?

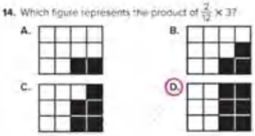
A. 4 × 32 **B.** 32 × 4 **C.** 32 ÷ 4 **D.** 4 ÷ 32

13. Write *five and thirty-seven thousandths* as a decimal.

5.037

14. Which figure represents the product of $\frac{2}{12} \times 3$?

A. B. C. D.



15. Which of these statements about a rectangle is *always* true?

A. A rectangle is a rhombus and a square.

B. A rectangle is a parallelogram and a square.

C. A rectangle is a quadrilateral and a parallelogram.

D. A rectangle is a quadrilateral and a rhombus.

16. What is the quotient?

0.36 ÷ 0.03 = **12**

17. Which method can be used multiply 982 × 11?

A. (900 × 11) + (80 × 11) + (2 × 11)

B. (900 × 80 × 2) × 11

C. (900 + 11) × (80 + 11) × (2 + 11)

D. (900 + 11) × (82 + 11)

18. Which statement *best* describes the expression 3 × (40 + 7)?

A. Three times forty plus seven

B. Forty plus seven divided by three

C. Three times the sum of forty and seven

D. Three times the difference of forty and seven

19. Nera biked 4.74 miles before work and 5.17 miles after work. Round each number to the nearest tenth to find a reasonable estimate for the total number of miles Nera biked.

4.7 + **5.2** = **9.9** miles

20. What is the difference of 7.08 − 5.52?

1.56

21. Fiorella has $0.50. Gwen has 10 times as much money. How much money does Gwen have?

A. $.05 **B.** $0.50 **C.** $5.00 **D.** $50.00

Grade **5**

Summative Assessment (continued)

Name ____________________

22. Beatrice uses partial quotients to solve 372 ÷ 5.

5) 372	
− **350**	**70**
22	
− **20**	**4**
2	**74**

a. Fill in the missing numbers to show how Beatrice can solve the problem.

b. What is the quotient?

372 ÷ 5 = **74** R **2**

23. What is the product of 42 × 0.8? Fill in the missing numbers to solve.

42 × 0.8 = 42 × 8 × **0.1**

42 × 0.8 = **33.6**

24. Which estimate is *most* reasonable for the quotient of 38.64 ÷ 12.8?

A. 5 **B.** 4

C. 3 **D.** 2

25. Hans wraps a gift box that has a base of 18 square inches and a height of 5 inches. What is the volume, in cubic inches, of the gift box Hans wraps?

90 cubic inches

26. Vickie makes 2 gallons of lemonade. She pours the lemonade equally into 4 pitchers. How much lemonade, in cups, is in each pitcher?

A. 2 cups **B.** 4 cups

C. 6 cups **D.** 8 cups

27. Tina uses partial sums to add 8.45 + 5.96. Fill in the missing numbers to complete her equations.

8 + 5 = 13

0.4 + **0.9** = **1.3**

0.05 + 0.06 = 0.11

13 + **1.3** + 0.11 = **14.41**

28. Mr. Tiller writes a number on the board. The number rounded to the nearest hundredth is 300. Which of these could be Mr. Tiller's number?

A. 299 **B.** 299.999

C. 300.01 **D.** 300.1

29. Melani does homework for $1\frac{5}{6}$ hours. Heidi does homework for twice as long as Melani. How long, in hours, does Heidi do homework?

$3\frac{4}{6}$ hours or equivalent

30. Use the fact 6 × 7 = 42 to fill in the correct products.

0.6 × 0.07 = **0.042**

6 × 0.7 = **4.2**

0.006 × 70 = **0.42**

31. Which equation shows a correct way to subtract $2\frac{4}{6} - 1\frac{1}{3}$?

A. $\frac{24}{6} - \frac{11}{3} = \frac{13}{3}$

B. $\frac{16}{6} - \frac{7}{6} = \frac{10}{6}$

C. $\frac{10}{3} - \frac{4}{3} = \frac{6}{3}$

D. $\frac{16}{6} - \frac{8}{6} = \frac{8}{6}$

32. Carlos generates Patterns W and Z using these rules:

- Pattern W: Start with 0 and add 7.
- Pattern Z: Start with 0 and subtract 4.

Which set of ordered pairs is generated from corresponding terms of Patterns W and Z?

A. (0, 0), (−7, 4), (−14, 8), (−21, 12)

B. (0, 0), (7, −4), (14, −8), (21, −12)

C. (0, −4), (7, −8), (14, −12), (21, −16)

D. (7, 0), (14, −4), (21, −8), (28, −12)

33. Beni has 4 feet of string to make keychains. If he uses $\frac{1}{2}$ foot of string for each keychain, how many keychains can Beni make?

8 keychains

34. Look at the equation.

$2\frac{3}{10} + 1\frac{1}{100} = 2 + 1 + \frac{\square}{100} + \frac{1}{100}$

What number goes in the box to make the equation true?

30

35. Which equation has the same unknown as 1,148 ÷ 28 = □?

A. 28 × □ = 1,148

B. 1,148 × □ = 28

C. □ ÷ 1,148 = 28

D. □ ÷ 1,148 = 28

Appendix

Appendix

Sense-Making Routines

Notice & Wonder™
Students are presented with an image or situation and are asked to share what they notice and wonder about the image or situation. Students are encouraged to notice and wonder about both math-related and non-math-related aspects of the image or situation.

This is primarily a whole-class, discussion-based brainstorming activity, one in which the noticings and wonderings of one student help spark additional noticings and wonderings among other students. To increase student participation in this whole-class brainstorming activity, allow students adequate time to write or draw any noticings and wonderings. This will help students control their impulses and expand their thinking. As you record students' thoughts, thank or acknowledge each student equally. Record all student suggestions. However, resist praising, restating, clarifying, or asking questions.

As students share, the teacher may want to record students' noticings and wonderings for all to see and to avoid redundancy of ideas. Allow for some non-math-related observations and questions, but eventually pose questions to get students thinking about a math focus. The support in the Teacher Edition offers prompts that can focus students' thinking on a math focus.

These activities are low floor-high ceiling activities and it is important that all students feel comfortable participating and valued for their participation.

Variations of the *Notice & Wonder* routine include:

- Tell me everything you can.
- What question could you ask?
- What could the question be?

How are they the same? How are they different?
In this variation of the routine, students are presented with two or more images or situations and consider how the images or situations are similar and different. The students share similarities and differences, some of which may be mathematical in nature, others non-mathematical.

As with the *Notice & Wonder* routine, this is also primarily a whole-class, discussion-based activity. As students can share both mathematical and non-mathematical similarities and differences, the teacher can record these for reference. If students do not bring up the intended math focus, prompts in the Teacher Edition can get students thinking about the focus.

Which Doesn't Belong?
Students are presented with a series of images, quantities, or numbers – usually four, and think about the question, "Which doesn't belong?" The activity has been designed to have multiple responses depending on which criteria or attribute the student is considering. Students are encouraged to think about as many different ways to identify the one that does not belong.

As students share their responses, they should be encouraged to share their reasoning for which one does not belong.

Guidance in the Teacher Edition can help the teacher direct students' thinking to the specific math focus for the lesson.

Is It Always True?
Students are presented with one or more images or situations and think about the relationship among the objects in the image or situation. Students then consider whether the relationship always holds or whether it/they are unique to the image or situation.

As with the other sense-making routines, this is primarily a whole-class, discussion-based brainstorming activity, one in which students share their thoughts around the generalizability of the relationship and their reasoning for their claims. As with other routines, allow students adequate time for them to reason through their own thinking. Students should be encouraged to write down or draw their thoughts and reasoning. This additional think time allows students to process the information presented.

Guidance in the Teacher Edition can help the teacher direct students' thinking to the specific math focus for the lesson that the sense-making routine is targeting.

Numberless Word Problems
Students are presented with a scenario or problem situation that suggests operations, but with no numbers. That is, instead of specifying quantities, the scenario or problem would indicate "some." For example, "Some dogs are in the dog park. Some dogs come into the dog park. Then some dogs leave the dog park." Students are expected to make sense of what is being described in the situation and explain which operations it represents. Without numbers, students are able to develop a better understanding of the underlying structure of the problem itself.

The prompts in the Teacher Edition offer options to extend and expand the discussion about the situation presented.

Number Routines

About How Much?

Purpose Build estimating skills.

Overview Students estimate the value of expressions (with operations) shown, explaining their strategies and thinking. The teacher records students' estimates, then reveals the value of the expression. Students analyze the estimates and discuss which are closest to the actual value of the expression.

Can You Make the Number?

Purpose Build flexible thinking and efficiency with operations.

Overview Students use all the given numbers to build expressions with a value of the given target number. Students can use a range of operations in their expressions. The teacher records students' expressions, then facilitates a discussion about students' expressions.

Decompose It

Purpose Build flexibility with numbers.

Overview Students come up with multiple (at least 3) ways to decompose given numbers and share their thinking for each decomposition. Teacher records decompositions then facilitates a discussion of patterns in the decompositions.

Find the Missing Values

Purpose Build their identification of patterns and their efficiency with solving equations as they examine a list of related equations.

Overview Students analyze a series of equations to look for patterns that they can use to determine the missing values in the equations. As students share their analyses and solutions, the teacher can reveal the missing values.

Find the Pattern, Make a Pattern

Purpose Build efficiency with recognizing and building patterns.

Overview Students determine the rule(s) for a given pattern, then use the rule(s) to continue the pattern or create a new pattern. The teacher records students' new patterns and facilitates a discussion to validate the pattern and its rules.

Greater Than, Less Than

Purpose Build proficiency with number and place value sense; estimating and comparing skills.

Overview Students use mental math to estimate or evaluate the value of given expressions and then compare the value of the expressions to a target benchmark number. Students share their solutions and thinking.

Math Pictures

Purpose Build number sense and mathematical awareness.

Overview Students respond to a prompt about an image.

What's Another Way to Write It?

Purpose Build flexibility with number sense and mental math operations.

Overview Given an expression, students come up with alternative expressions using the same or different operations that, when evaluated, have the same value. The teacher records expressions as students share. Students then look for relationships amongst the shared expressions.

Where Does It Go?

Purpose Build estimating skills using benchmarks.

Overview Students place a target number on number lines with different endpoints and justify their placement. In some instances, as a challenge, the target number may not actually belong on one of the number lines.

Which Benchmark Is It Closest To?

Purpose Enhance rounding and reasoning skills.

Overview Students determine to which benchmark the given number is closest and explain their reasoning.

Would You Rather?

Purpose Build flexibility with number sense and mental math operations; enhance decision-making.

Overview Students choose between two options, both of which require mental computation. Students explain their choice and their rationale for their choice.

Math Language Routines

MLR 1 Stronger and Clearer Each Time

Purpose To provide opportunities for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014).

Successive Pair Shares Students respond to a prompt either verbally or in writing. Then, students share their responses and their reasoning through successive pair shares, refining and revising their response after each pair share. Student-pairs are expected to provide details and to press their partners to provide details around their thinking. Students revise their responses after each pair share, incorporating as appropriate new ideas or language.

Convince Yourself, a Friend, a Skeptic Students think about an argument for three different audiences. They first justify the argument in a way that makes sense to them, then they explain what they know and think and how they know their argument is true to a classmate. For their third audience, students explain why they know their argument is true to a different classmate. Students' arguments may include words, pictures, numbers, and examples. Students should be prepared to offer counter-arguments.

MLR 2 Collect and Display

Purpose To help students build vocabulary and language when engaging in mathematical discourse.

Gather and Show Student Discourse (Dieckmann, 2017) While students are working in pairs or small groups, the teacher circulates and listens to student talk, capturing common or important words and phrases. The teacher organizes words and phrases captured in a visual display that the teacher and students can reference throughout the unit of study. The teacher can update and add to the visual display to show refinement in language students are using to discuss key concepts.

Number Talks (Humphreys & Parker, 2015) Number Talks have four parts: (1) Students are presented with a numeracy problem that they consider without written solution for a few minutes; (2) Students share their strategy for solving and their thinking about their strategy; (3) As students share their strategies and thinking, the teacher creates a visual display of students' strategies and thinking, noting common or important words or phrases shared; (4) The teacher facilitates a discussion analyzing the strategies and thinking, their benefits and drawbacks in different types of situations.

MLR 3 Critique, Correct, and Clarify

Purpose To have students analyze and reflect on a written or verbal mathematical argument.

Critique a Partial or Flawed Response The teacher presents a partial or flawed argument, explanation, or solution and has students come up ways to complete or fix the argument. Students can work individually or in pairs to propose improvements and/or additions to the argument. Students share their responses with a partner and based on feedback from their partner, they refine and revise their responses.

Always, Sometimes, Never The teacher presents a mathematical statement that students analyze to decide whether it is always, sometimes, or never true. If possible, the teacher presents the statement in a graphic organizer that can then be used to assess students' reasoning.

MLR 4 Information Gap

Purpose To create a need for students to communicate (Gibbons, 2002).

Info Gap Cards Each student-pair has two cards, Student A has Card A with the problem statement and Student B has Card B with data or information needed to solve the problem. Student A asks Student B for specific information that is needed to solve the problem. Before providing the information, Student B asks Student A to justify the need for information before providing the information requested (if available). Student A then explains how he or she will use the information to solve the problem.

Info Gap Games Student-pairs play a guessing or matching game. Student A shares the information and Student B uses his or her understanding of math concepts to guess what Student A is describing. For example, Student A may identify objects in the classroom that share a shape and Student B guesses the shape.

MLR 5 Co-Craft Questions and Problems

Purpose To allow students to use conversation skills and mathematical language to generate questions, problems, and situations.

Co-Craft Questions The teacher presents a scenario or part of a problem context and students come up with math-related questions that could be asked about the situation. Student-pairs share their questions, comparing and contrasting the questions each asked. The teacher can then facilitate a share-out of questions from the class, after which the actual question students will look to answer is revealed.

Co-Craft Problems Student-pairs work together to co-create problems. Student-pairs then solve their problems, and trade problems with another student-pair. Student-pairs solve the problems of others, checking their solution against that of the co-creators.

Co-Craft Situations The teacher presents a mathematical representation with no labels. Students work individually to write a story or scenario that matches the representation. Students then work with a partner to share their scenarios or situations, each explaining how their situations reflect the mathematical representation. Based on feedback from their partners, students revise their situations, adding details as appropriate.

MLR 6 Three Reads

Purpose To ensure that students make sense of problem situations and equip them with tools used to negotiate meaning (Kelemanik, Lucenta & Creighton, 2016).

Three Reads Students read a problem situation or scenario three times, each time with a different focus (1) Students read for comprehension, to understand the problem situation; (2) Students focus on the language used to present the mathematics with the goal of understanding what mathematics is most appropriate to use; (3) students think about which strategy or solution method would be appropriate.

Values/Units Chart (1) Students read through text and underline any words or phrases that represent a known or unknown value or amount. (2) They list these numbers, unknowns, and variables in the left column of their graphic organizer (Values). (3) After reading a second time, students write the meaning of the values in the right column of the graphic organizer (Units). (4) After the third read, students work in pairs to create mathematical expressions using only the right column.

MLR 7 Compare and Connect

Purpose To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.

Compare and Contrast Solution Strategies Students are given a problem to solve on their own. Students are then paired up and share their solution strategies with their partners, relating and connecting their partner's approach to their own approach. Some options include:

1. **Divide and conquer** Set students in pairs before they solve the problem. Each pair decides on two different ways to solve the problem; one partner does one way and the other partner does the other way.
2. **What is similar, what is different** After student-pairs share solution strategies, they identify what is similar and what is different about the approaches. Students can also discuss what worked with each approach.
3. **Mathematical focus** Student-pairs focus on specific mathematical relationships, operations, quantities and values.

Which One Doesn't Belong? Students are provided with sets of four numbers, equations, expressions, graphs, or geometric figures. Working in pairs, students decide together how to group the sets so that three of the items fit within a category they have created and one does not. Both partners should be prepared to explain to a different group how they agreed on a category and justify which item did not fit.

MLR 8 Discussion Supports

Purpose To facilitate rich discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009).

Whole Class Discussion Supports During whole class discussion, the teacher can use these strategies to support mathematical discourse:

- **Restating** The teacher restates students' ideas as questions to clarify meaning and model appropriate mathematical language
- **Press for Details** The teacher asks students to elaborate on an idea, expand an argument, or give an example.
- **Think Alouds** The teacher talks through their thinking about a mathematical concept.
- **Use multiple modalities** The teacher uses different modalities to show concepts.
- **Choral responses** The teacher has students practice common or important words or phrases through choral repetition.

Numbered Heads Together (1) The teacher has students count off by 4s (or the number of students he or she wants to have in a group. (2) The teacher then presents a question or problem and has students work in their groups according to their number to come up with an explanation or justification. (3) Each group reporter shares the group explanation and/or agree or disagree with the previous group reporter. Other members of the group are not allowed to talk or write, but the reporter can use the notes from the group discussion. The correct answer, if there is one, is revealed once all groups have presented.

Key Concepts and Learning Objectives

KEY CONCEPT Habits of Mind and Classroom Norms for Productive Math Learning

- Students make sense of problems and quantities and represent them different ways. (Unit 1)
- Students represent a real-world situation using mathematics. (Unit 1)
- Students construct an argument to explain their thinking with clear and appropriate terms. (Unit 1)
- Students use patterns to develop efficient strategies to solve problems. (Unit 1)
- Students tell their math biography and recognize the behaviors and attitudes that support a productive learning environment. (Unit 1)

KEY CONCEPT Operations with Fractions

- Students add, subtract, and multiply fractions, including mixed numbers, with unlike denominators. (Units 9, 10)
- Students find the area of a rectangle with fractional side lengths. (Unit 10)
- Students describe multiplication as scaling. (Unit 10)
- Students divide unit fractions by whole numbers and whole numbers by unit fractions. (Unit 11)

KEY CONCEPT Operations with Whole Numbers and Decimals

- Students describe the relationship between place value positions. (Unit 3)
- Students use an algorithm to multiply whole numbers. (Unit 5)
- Students divide multi-digit dividends by 2-digit divisors. (Unit 7)
- Students add, subtract, multiply, or divide decimals. (Units 4, 6, 8)
- Students solve word problems involving operations with whole numbers or decimals. (Units 4, 5, 6, 7, 8)

KEY CONCEPT Measurement and Data

- Students describe volume is an attribute of solid figures. (Unit 2)
- Students measure volume by counting unit cubes. (Unit 2)
- Students calculate the volume of rectangular prisms using formulas. (Unit 2)
- Students find the volume of composite solid figures. (Unit 2)
- Students convert measurement units within a given measurement system. (Unit 12)
- Students interpret data on a line plot. (Unit 12)

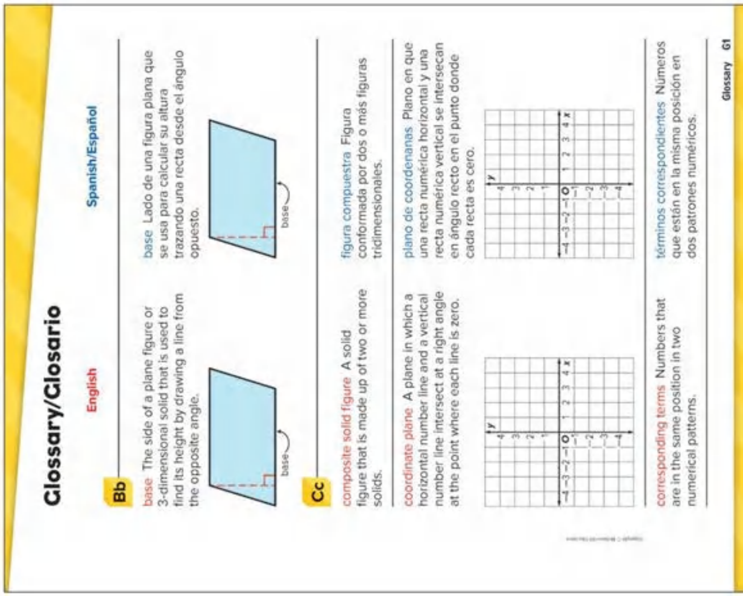
KEY CONCEPT Geometry

- Students identify and describe features of a coordinate plane. (Unit 13)
- Students graph points on the coordinate plane to solve problems. (Unit 13)
- Students classify 2-dimensional figures into categories based on their properties. (Unit 13)

KEY CONCEPT Algebraic Thinking

- Students write numerical expressions to represent calculations that are described using written statements. (Unit 14)
- Students interpret numerical expressions without evaluating them. (Unit 14)
- Students use the order of operations to evaluate numerical expressions. (Unit 14)
- Students generate two numerical patterns using two given rules. (Unit 14)
- Students identify apparent relationships between corresponding terms in the generated number patterns. (Unit 14)

Glossary/Glosario



Glossary/Glosario

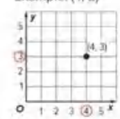
English	Spanish/Español
Bb	
base The side of a plane figure or 3-dimensional solid that is used to find its height by drawing a line from the opposite angle.	base Lado de una figura plana que se usa para calcular su altura trazando una recta desde el ángulo opuesto.
Cc	
composite solid figure A solid figure that is made up of two or more solids.	figura compuestra Figura conformada por dos o más figuras tridimensionales.
coordinate plane A plane in which a horizontal number line and a vertical number line intersect at a right angle at the point where each line is zero.	plano de coordenanas Plano en que una recta numérica horizontal y una recta numérica vertical se intersecan en ángulo recto en el punto donde cada recta es cero.
corresponding terms Numbers that are in the same position in two numerical patterns.	términos correspondientes Números que están en la misma posición en dos patrones numéricos.

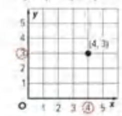
Glossary G1

Glossary/Glosario

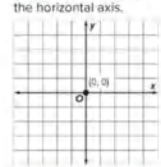
English	Spanish/Español
Ee	
evaluate To find the value of an algebraic expression by replacing variables with numbers.	**evaluar** Calcular el valor de una expresión algebraica reemplazando las variables con números.
exponent The number of times a base is multiplied by itself. In 3^2, the exponent is 2.	**exponente** Número de veces que la base se multiplica por sí misma. En 3^2, el exponente es 2.
expression A combination of numbers, variables, and operation symbols. 5 + 7	**expression** Combinación de números, variables y símbolos de operaciones. 5 + 7
Gg	
grouping symbol Parentheses ( ) or brackets [ ] that tell you where that group starts and ends. They help to determine the order when evaluating a numerical expression.	**símbolo de agrupación** Paréntesis ( ) o corchetes [ ] que indican el comienzo y el fin de un grupo. Sirven para ordenar y evaluar una expresión numérica.
Hh	
hierarchy of figures A classification of figures into categories and subcategories.	**jerarquía de figuras** Clasificación de figuras en categorías y subcategorías.
Nn	
numerical expression A combination of numbers and operations.	**expresión numérica** Combinación de números y operaciones.

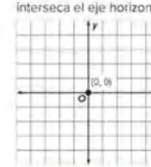
English	Spanish/Español
Oo	
order of operations Rules that tell what order to follow when evaluating expressions. The order of operations says: 1. Evaluate numerical expressions inside grouping symbols first. 2. Multiply and divide in order, from left to right. 3. Add and subtract in order, from left to right.	**orden de la operaciones** Reglas que indican qué orden seguir al evaluar una expresión: 1. Evalúa dentro de los paréntesis ( ). 2. Multiplica o divide de izquierda a derecha. 3. Suma o resta de izquierda a derecha.
ordered pair A pair of numbers that are the coordinates of a point in a coordinate plane or grid in this order (horizontal coordinate, vertical coordinate). Example: (4, 3)	**par ordenado** Par de números que son coordenadas de un punto en un plano de coordenadas o un cuadriculado, en este orden (coordenada horizontal, coordenada vertical). Ejemplo: (4, 3)





English	Spanish/Español
origin The point (0, 0) on a coordinate graph where the vertical axis meets the horizontal axis.	**origen** El punto (0,0) en una gráfica de coordenadas donde el eje vertical interseca el eje horizontal.





English	Spanish/Español

Pp

power of 10 A number obtained by raising 10 to a given exponent.

$10^2 = 10 \times 10 = 100$

potencia de 10 Un número que se obtiene elevando 10 a un exponente dado.

$10^2 = 10 \times 10 = 100$

Rr

range The difference between the greatest and the least numbers in a set of data.

rango La diferencia entre los números mayores y menores en un conjunto de datos.

Ss

subcategory Common properties within a larger category.

subcategoría Propiedades comunes dentro de una categoría más amplia.

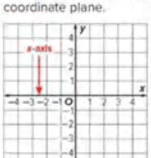
Vv

volume The number of cubic units needed to fill a 3-dimensional figure or solid figure.

volumen Número de unidades cúbicas necesarias para llenar una figura tridimensional o sólida.

Xx

x-axis The horizontal axis in a coordinate plane.



eje x Eje horizontal en una coordenada.

English	Spanish/Español

x-coordinate The first part of an ordered pair that indicates how far to the left or the right of the *y*-axis the corresponding point is.

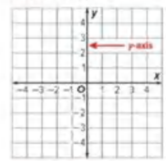
(1, 2): 1 unit to the right of the *y*-axis

coordenada x Primera parte de un par ordenado que indica la distancia a que está el punto correspondiente a la izquierda o a la derecha del eje *y*.

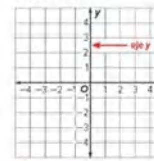
(1, 2): 1 unidad a la derecha del eje *y*

Yy

y-axis The vertical axis in a coordinate plane.



eje y El eje vertical en una coordenada.



y-coordinate The second part of an ordered pair that indicates how far above or below the *x*-axis the corresponding point is.

(1, 2): 2 units above the *x*-axis

coordenada y Segunda parte de un par ordenado que indica la distancia a que está el punto correspondiente por encima o por debajo del eje *x*.

(1, 2): 2 unidades por encima del eje *x*

Index

Key Terms are in *italics*.
1: = Volume 1 pages
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Reveal MATH

Math is... Mindset

What can you do to work well with others?

Math is...not just adding and subtracting.

Math is...

- working together
- finding patterns
- sharing ideas
- listening thoughtfully
- sticking with a challenge

Math is...
all around us.

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