Teacher Edition

Reveal MATH®

Course 1

McGraw Hill

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Send all inquiries to:
McGraw-Hill Education
STEM Learning Solutions Center
8787 Orion Place
Columbus, OH 43240

ISBN: 978-0-07-899720-4 *(Reveal Math, Course 1, Teacher Edition, Volume 1)*
MHID: 0-07-899720-8 *(Teacher Edition, Volume 1)*
ISBN: 978-0-07-667372-8 *(Interactive Student Edition, Volume 1)*
MHID: 0-07-667372-3 *(Interactive Student Edition, Volume 1)*

Printed in the United States of America.

4 5 6 7 8 9 WEB 25 24 23 22 21 20

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8 Area

9 Volume and Surface Area

10 Statistical Measures and Displays

Reveal Math® Guiding Principles

Academic research and the science of learning provide the foundation for this powerful K–12 math program designed to help reveal the mathematician in every student.

Reveal Math is built on a solid foundation of **RESEARCH** that shaped the **PEDAGOGY** of the program.

Reveal Math used findings from research on teaching and learning mathematics to develop its instructional model. Based on analyses of research findings, these areas form the foundational structure of the program:

- Rigor
- Productive Struggle
- Formative Assessment
- Rich Tasks
- Mathematical Discourse
- Collaborative Learning

Instructional Model

1 Launch

WARM UP

During the **Warm Up,** students complete exercises to activate prior knowledge and review prerequisite concepts and skills.

LAUNCH THE LESSON

In **Launch the Lesson**, students view a real-world scenario and image to pique their interest in the lesson content. They are introduced to questions that they will be able to answer at the end of the lesson.

EXPLORE

During the **Explore** activity, students work in partners or small groups to explore a rich, real-world or mathematical problem related to the lesson content.

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY

Reveal the full potential in every student!

2 Explore and Develop

LEARN

In the **Learn** section, students gain the foundational knowledge needed to actively work through upcoming Examples.

EXAMPLES & CHECK

Students work through **Examples** related to the key concepts and engage in mathematical discourse.

Students complete a **Check** after each Example as a quick formative assessment to help teachers adjust instruction as needed.

3 Reflect and Practice

EXIT TICKET

The **Exit Ticket** gives students an opportunity to convey their understanding of the lesson concepts.

PRACTICE

Students complete **Practice** exercises individually or collaboratively to solidify their understanding of lesson concepts or build proficiency with lesson skills.

Reveal Math Key Areas of Focus

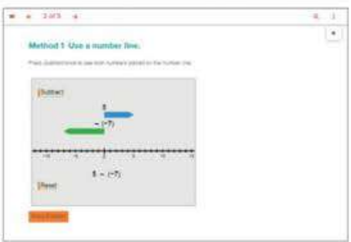
Reveal Math has a strong focus on rigor—especially the development of conceptual understanding—an emphasis on student mindset, and ongoing formative assessment feedback loops.

Rigor

Reveal Math has been thoughtfully designed to incorporate a balance of the three elements of rigor: conceptual understanding, procedural skills and fluency, and application.

Conceptual Understanding

Explore activities give all students an opportunity to work collaboratively and discuss their thinking as they build conceptual understanding of new topics. In the Explore activity to the left, students use algebra tiles to gain an understanding of operations with positive and negative integers.



Procedural Skills and Fluency

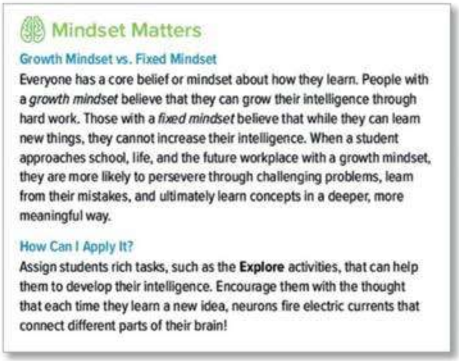
As students move through the lesson, they will use different strategies and tools to build procedural fluency. In the **Example** shown, students use **Web Sketchpad**® to develop proficiency with integer operations.

Application

Real-world examples and practice problems are opportunities for students to apply their learning to new situations. In the real-world example to the left, students apply their understanding of percents to solve a percent error problem.

Student Mindset

Mindset Matters tips located in each module provide specific examples of how *Reveal Math* content can be used to promote a growth mindset in all students. Another feature focused on promoting a growth mindset is **Ignite! Activities** developed by Dr. Raj Shah to spark student curiosity about why the math works. An **Ignite!** delivers problem sets that are flexible enough so that students with varying background knowledge can engage with the content and motivates them to ask questions, solve complex problems, and develop a can-do attitude toward math.



Mindset Matters

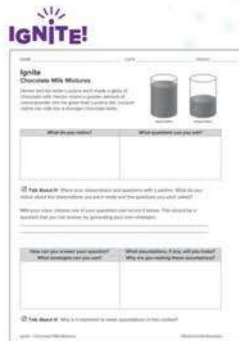
Growth Mindset vs. Fixed Mindset

Everyone has a core belief or mindset about how they learn. People with a *growth mindset* believe that they can grow their intelligence through hard work. Those with a *fixed mindset* believe that while they can learn new things, they cannot increase their intelligence. When a student approaches school, life, and the future workplace with a growth mindset, they are more likely to persevere through challenging problems, learn from their mistakes, and ultimately learn concepts in a deeper, more meaningful way.

How Can I Apply It?

Assign students rich tasks, such as the **Explore** activities, that can help them to develop their intelligence. Encourage them with the thought that each time they learn a new idea, neurons fire electric currents that connect different parts of their brain!

Teacher Edition Mindset Tip



Student Ignite! Activity

Formative Assessment

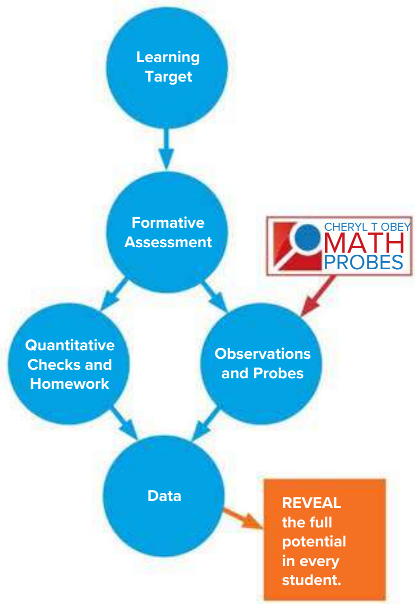
The key to reaching all learners is to adjust instruction based on each student's understanding. *Reveal Math* offers powerful formative assessment tools that help teachers to efficiently and effectively differentiate instruction for all students.

Math Probes

Each module includes a **Cheryl Tobey Formative Assessment Math Probe** that is focused on addressing student misconceptions about key math topics. Students can complete these probes at the beginning, middle, or end of a module. The teacher support includes a list of recommended differentiated resources that teachers assign based on students' responses.

Example Checks

Each example is followed by a formative assessment **Check** that students complete on their own that allows teachers to gauge students' understanding of the concept or skill presented. When students complete the Check, the teacher receives resource recommendations, which can be assigned to all students.



A Powerful Blended Learning Experience

The *Reveal Math* blended learning experience was designed to include purposeful print and digital components focused on sparking student curiosity and providing teachers with flexible implementation options.

Reveal Math has been thoughtfully developed to provide a rich learning experience no matter where a district, school, or classroom falls on the digital spectrum. All of the instructional content can be projected or can be accessed via desktop, laptop, or tablet.

Lesson

1 Launch

WARM UP

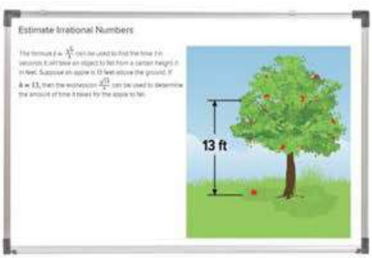
The **Warm Up** exercise can be projected on an interactive whiteboard.

LAUNCH THE LESSON

Launch the Lesson can be projected or assigned to students to access on their own devices.

EXPLORE

The **Explore** Activity can be projected while students record their observations in the Interactive Student Edition or can be assigned for students to complete on individual devices.



Launch the Lesson

Explore

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY
INTERACTIVE PRESENTATION
PRINT INTERACTIVE STUDENT EDITION

2 Explore and Develop

3 Reflect and Practice

LEARN

As students are introduced to the key lesson concepts, they can progress through the **Learn** by recording notes in their Interactive Student Edition or on their own devices.

EXAMPLES & CHECK

In their Interactive Student Edition or on an individual device, students work through one or more **Examples** related to key lesson concepts.

A **Check** follows each Example in either the Interactive Student Edition or on each student device.

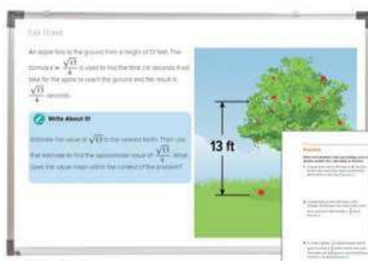
Aligned Digital Lesson Presentation to Interactive Student Edition

EXIT TICKET

The **Exit Ticket** is projected or accessed via student devices to provide students with lesson closure and an opportunity to revisit the lesson concepts.

PRACTICE

Assign students **Practice** problems from their Interactive Student Edition or create a digital assignment for them to work on their device in class or at home to solidify lesson concepts.



Exit Ticket

Practice

Supporting All Learners

The *Reveal Math* program was designed so that all students have access to:

- rich tasks that promote productive struggle,
- opportunities to develop proficiency with the habits of mind and thinking strategies of mathematicians, and
- prompts to promote mathematical discourse and build academic language.

Resources for Differentiating Instruction

When needed, resources are available to differentiate math instruction for students who may need to see a concept in a different way, practice prerequisite skills, or are ready to extend their learning.

AL

Approaching Level Resources

- Remediation Activities
- Extra Examples
- *Arrive Math* Take Another Look Mini Lessons

BL

Beyond Level Resources

- Beyond Level Differentiated Activities
- Extension Activities

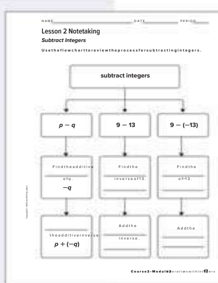
Resources for English Language Learners

Reveal Math also includes student and teacher resources to support students who are simultaneously learning grade-level math and building their English proficiency. Appropriate, research-based language scaffolds are also provided to support students as they engage in rigorous mathematical tasks and discussions.

ELL

English Language Learners

- Spanish Interactive Student Edition
- Spanish Personal Tutors
- Math Language-Building Activities
- Language Scaffolds
- *Think About It!* and *Talk About It!* Prompts
- Multilingual eGlossary
- Audio
- Graphic Organizers
- Web Sketchpad, Desmos, and eTools


NAME DATE PERIOD

Lesson 2 Notetaking

Subtract Integers

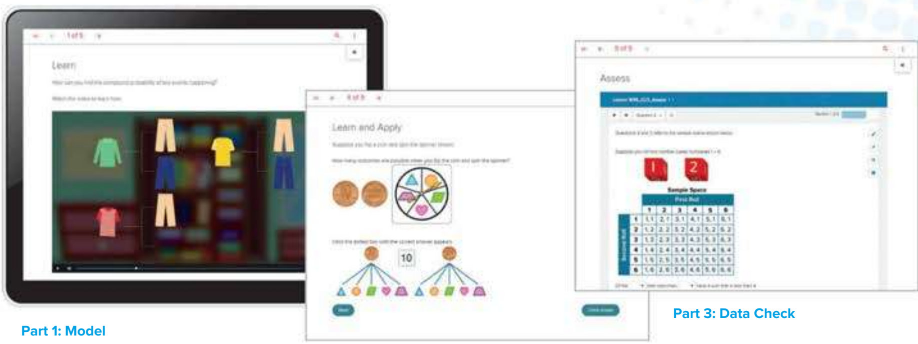
Use the flowchart to review the process for subtracting integers.

Course 2 · Module 3 Operations with Integers 17

Embedded Reteach Support
Arrive Math Booster Mini-Lessons

Reveal Math ensures a seamless connection for students who need extra topic support with embedded *Arrive Math Booster* mini-lessons. These mini-lessons, called *Take Another Look*, have been included in *Reveal Math* to provide students direct support related to the lesson objective.

- Teacher-assigned option based on Example Check results
- Digital, student-driven lesson
- Gradual release experience in three parts



Part 1: Model

Part 2: Interactive Practice

Part 3: Data Check

Arrive MATH BOOSTER

Complement *Reveal Math* with the K-8 *Arrive Math Booster* supplemental intervention to equip teachers with all the resources they need to supplement their instruction and meet the needs of all learners.

Digital mini-lessons

Utilize over 1,160 *Take Another Look* digital mini-lessons for every skill within the K-8 standards.

Hands-On Lesson

Complement the *Take Another Look* lessons with concrete modeling support using hands on, teacher-led activities.

Games

Engage students through exciting math games to become fluent in critical math skills.

Reveal Student Readiness with Individualized Learning Tools

Reveal Math incorporates innovative, technology-based tools that are designed to extend the teachers' reach in the classroom to help address a wide range of knowledge gaps, set and align academic goals, and meet student individualized learning needs.

LEARNSMART®

Topic Mastery

With embedded **LearnSmart**,® students have a built-in study partner for topic practice and review to prepare for multi-module, or mid-year tests.

LearnSmart's revolutionary adaptive technology measures students' awareness of their own learning, time on topic, answer accuracy, and suggests alternative resources to support student learning, confidence, and topic mastery.

ALEKS®

Individualized Learning Pathways

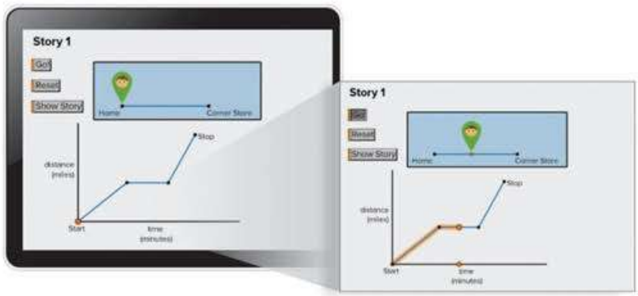
Learners of all levels benefit from the use of **ALEKS'** adaptive, online math technology designed to pinpoint what each student knows, does not know, and most importantly, what each student is ready to learn.

When paired with *Reveal Math,* **ALEKS** is a powerful tool designed to provide integrated instructionally actionable data enabling teachers to utilize *Reveal Math* resources for individual students, groups, or the entire classroom.

Activity Report

Powerful Tools for Modeling Mathematics

Reveal Math has been designed with purposeful, embedded digital tools to increase student engagement and provide unique modeling opportunities.

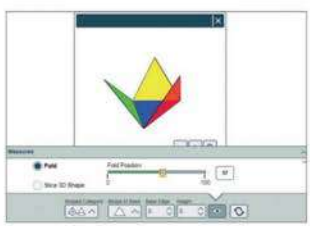


Web Sketchpad® Activities

The leading dynamic mathematics visualization software has now been integrated with **Web Sketchpad Activities** at point of use within *Reveal Math*. Student exploration (and practice) using **Web Sketchpad** encourages problem solving and visualization of abstract math concepts.

desmos

The powerful **Desmos** graphing calculator is available in *Reveal Math* for students to explore, model, and apply math to the real-world.



eTools

By using a wide-variety of digital **eTools** embedded within *Reveal Math,* students gain additional hands-on experience while they learn and teachers have the option to create problem-based learning opportunities.

Technology-Enhanced Items

Embedded within the digital lesson, technology-enhanced items—such as drag-and-drop, flashcard flips, or diagram completion—are strategically placed to give students the practice with common computer functions needed to master computer-based testing.

TYPE — SWIPE

DRAG & DROP — FLASHCARDS

eTOOLS — MULTI-SELECT

WATCH — EXPAND

Assessment Tools to Reveal Student Progress and Success

Reveal Math provides a comprehensive array of assessment tools to measure student understanding and progress. The digital assessment tools include next generation assessment items, such as multiple-response, selected-response, and technology-enhanced items.

Assessment

Reveal Math provides embedded, regular formative checkpoints to monitor student learning and provide feedback that can be used to modify instruction and help direct student learning using reports and recommendations based on resulting scores.

Summative assessments built in *Reveal Math* evaluate student learning at the module conclusion by comparing it against the state standards covered.

Formative Assessment Resources

- Cheryl Tobey Formative Assessment Math Probes
- Checks
- Exit Tickets
- Put It All Together

Summative Assessment Resources

- Module Tests
- Performance Tasks
- Benchmark Tests
- End-of-Course Tests
- LearnSmart

Or **Build Your Own** assessments focused on standards or objectives. Access to banks of questions, including those with tech-enhanced capabilities, enable a wide range of options to mirror high-stakes assessment formats.

Reporting

Clear, instructionally actionable data will be a click away with the *Reveal Math* Reporting Dashboard.

Activity Report Real-time class and student reporting of activities completed by the class. Includes average score, submission rate, and skills covered for the class and each student.

- **Item Analysis Report** Review a detailed analysis of response rates and patterns, answers, and question types in a class snapshot or by student.
- **Standards Report** Performance data by class or individual student is aggregated by standards, skills, or objectives linked to the related activities completed.

Activity Report

Professional Development Support for Continuous Learning

McGraw-Hill Education supports lifelong learning and demonstrates commitment to teachers with a built-in professional learning environment designed for support during planning or extended learning opportunities.

What You Will Find

- Best-practices resources
- Implementation support
- Teaching Strategies
- Classroom Videos
- Math Misconception Videos
- Content and Pedagogy Videos
- Content Progression Information

Why Professional Development is so Important

- Research-based understanding of student learning
- Improved student performance
- Evidence-based instructional best-practices
- Collaborative content strategy planning
- Extended knowledge of program how-to's

Reveal Math Expert Advisors

Cathy Seeley, Ed.D.

Austin, Texas

Mathematics educator, speaker, and writer, former Senior Fellow at the Charles A. Dana Center at The University of Texas at Austin, past President of NCTM, former Director of K-12 Mathematics for the State of Texas

Areas of expertise:
Mathematics Teaching, Equity, Assessment, STEM Learning, Informal Learning, Upside-Down Teaching, Productive Struggling, Mathematical Practices, Mathematical Habits of Mind, Family and Community Outreach, Mathematics Education Policy, Advocacy

"We want students to believe deeply that mathematics makes sense–in generating answers to problems, discussing their thinking and other students' thinking, and learning new material."

—Seeley, 2016, Making Sense of Math

Cheryl R. Tobey, M.Ed.

Gardiner, Maine

Senior Mathematics Associate at Education Development Center (EDC)

Areas of expertise:
Formative assessment and professional development for mathematics teachers; tools and strategies to uncovering misconceptions

"Misunderstandings and partial understandings develop as a normal part of learning mathematics. Our job as educators is to minimize the chances of students' harboring misconceptions by knowing the potential difficulties students are likely to encounter, using assessments to elicit misconceptions and implementing instruction designed to build new and accurate mathematical ideas."

— Tobey, et al 2007, 2009, 2010, 2013, 2104, Uncovering Student Thinking Series

Nevels Nevels, Ph.D.

Saint Louis, Missouri

PK-12 Mathematics Curriculum Coordinator for Hazelwood School District

Areas of expertise:
Mathematics Teacher Education; Student Agency & Identity; Socio-Cultural Perspective in Mathematics Learning

"A school building is one setting for learning mathematics. It is understood that all children should be expected to learn meaningful mathematics within its walls. Additionally, teachers should be expected to learn within the walls of this same building. More poignantly, I posit that if teachers are not learning mathematics in their school building, then it is not a school."

—Nevels, 2018

Raj Shah, Ph.D.

Columbus, Ohio

Founder of Math Plus Academy, a STEM enrichment program and founding member of The Global Math Project

Areas of expertise:
Sparking student curiosity, promoting productive struggle, and creating math experiences that kids love

"As teachers, it's imperative that we start every lesson by getting students to ask more questions because curiosity is the fuel that drives engagement, deeper learning and perseverance."

—Shah, 2017

Walter Secada, Ph.D.

Coral Gables, Florida

Professor of Teaching and Learning at the University of Miami

Areas of expertise:
Improving education for English language learners, equity in education, mathematics education, bilingual education, school restructuring, professional development of teachers, student engagement, Hispanic dropout and prevention, and reform

"The best lessons take place when teachers have thought about how their individual English language learners will respond not just to the mathematical content of that lesson, but also to its language demands and mathematical practices."

—Secada, 2018

Ryan Baker, Ph.D.

Philadelphia, Pennsylvania

Associate Professor and Director of Penn Center for Learning Analytics at the University of Pennsylvania

Areas of expertise:
Interactions between students and educational software; data mining and learning analytics to understand student learning

"The ultimate goal of the field of Artificial Intelligence in Education is not to promote artificial intelligence, but to promote education… systems that are designed intelligently, and that leverage teachers' intelligence. Modern online learning systems used at scale are leveraging human intelligence to improve their design, and they're bringing human beings into the decision-making loop and trying to inform them."

—Baker, 2016

Chris Dede, Ph.D.

Cambridge, Massachusetts

Timothy E. Wirth Professor in Learning Technologies at Harvard Graduate School of Education

Areas of expertise:
Provides leadership in educational innovation; educational improvements using technology

"People are very diverse in how they prefer to learn. Good instruction is like an ecosystem that has many niches for alternative types of learning: lectures, games, engaging video-based animations, readings, etc. Learners then can navigate to the niche that best fulfills their current needs."

—Dede, 2017

Dinah Zike, M.Ed.

Comfort, Texas

President of Dinah.com in San Antonio, Texas and Dinah Zike Academy

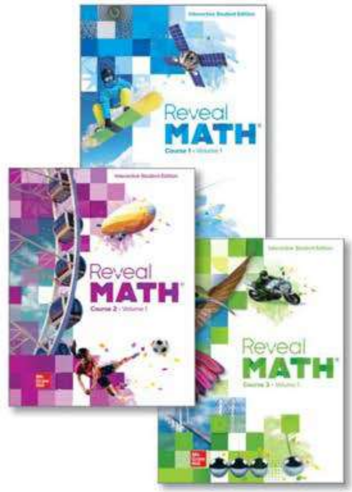
Areas of expertise:
Developing educational materials that include three-dimensional graphic organizers; interactive notebook activities for differentiation; and kinesthetic, cross-curricular manipulatives

"It is education's responsibility to meet the unique needs of students, and not the students' responsibility to meet education's need for uniformity."

—Zike, 2017, InRIGORating Math Notebooks

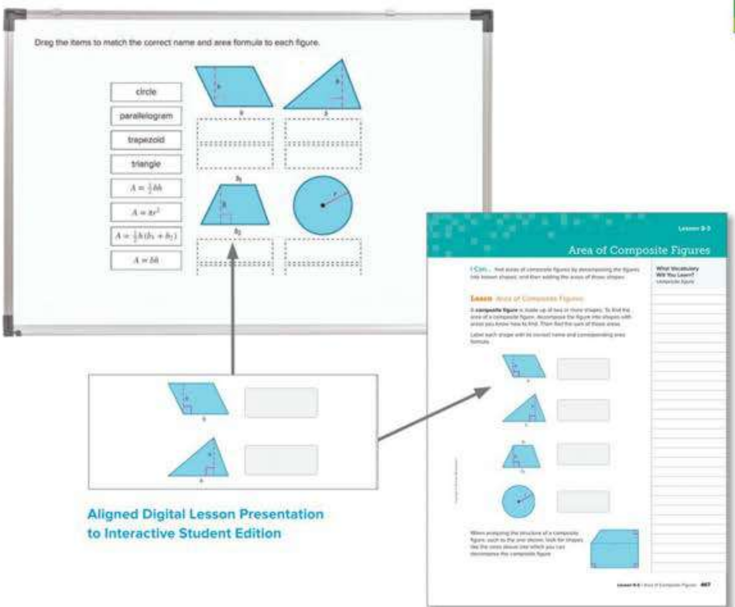
Reveal Everything Needed for Effective Instruction

Reveal Math provides both print and innovative, technology-based tools designed to address a wide range of classrooms. No matter whether you're in a 1:1 district, or have a classroom projector, *Reveal Math* provides you with the resources you need for a rich learning experience.



Blended Classrooms

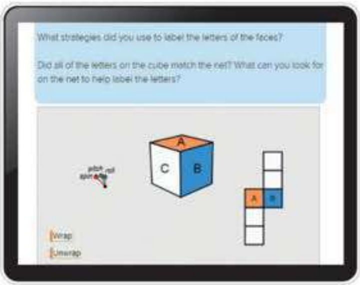
Focused on projection of the **Interactive Presentation**, students follow along taking notes and working through problems in their Interactive Student Edition during class time. Also included in the Interactive Student Edition is a glossary, **Foldables**® at point of use and in the back of the book, selected answers, and a reference sheet.



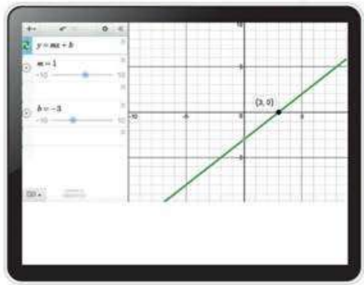
Aligned Digital Lesson Presentation to Interactive Student Edition

Digital Classrooms

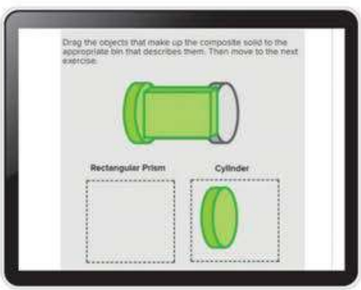
Projection is a focal point for key areas of the course with students interacting with the lesson using their own devices. Each student can access teacher-assigned sections of the lessons for **Explore** activities, **Learn** sections and **Examples**. Point of use videos, animations, as well as interactive content enable students to experience math in interesting and impactful ways.



Web Sketchpad

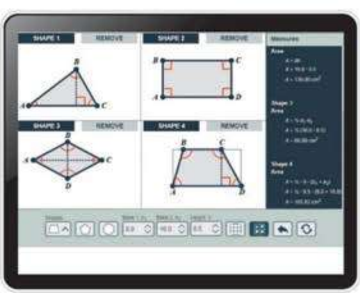


Desmos



Drag-and-Drop

Videos and Animations



eTools

In *Reveal Math*, R is for—

- Research
- Rigor
- Relevant Connections

Are you... READY to start?

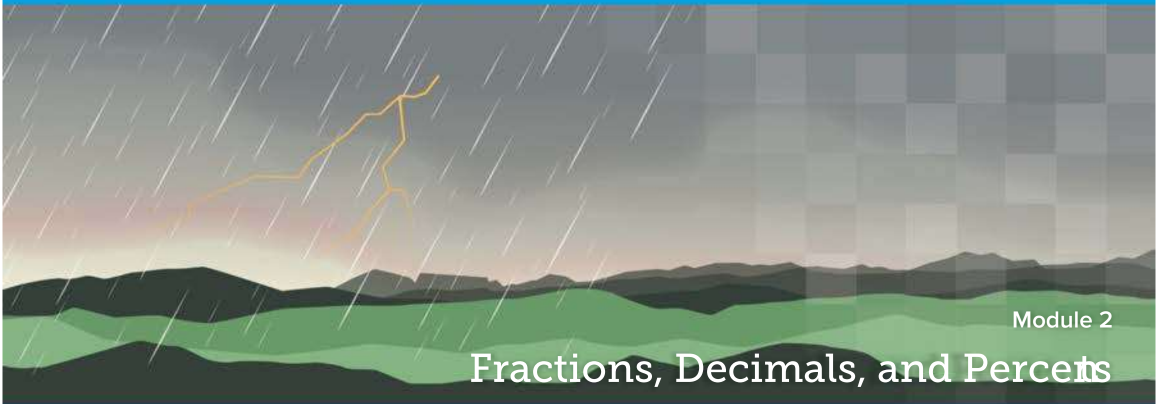
Module 1

Ratios and Rates

Essential Question

How can you describe how two quantities are related?

				Content Standards
		What Will You Learn?	1	
Lesson	1-1	Understand Ratios Explore Compare Two Quantities	3	6.RP.A.1
	1-2	Tables of Equivalent Ratios Explore Compare Equivalent Ratios	13	6.R.A.3, 6.RP.A.3.A
	1-3	Graphs of Equivalent Ratios	23	6.RP.A.3, 6.RP.A.3.A
	1-4	Compare Ratio Relationships	29	6.RP.A.3, 6.RP.A.3.A
	1-5	Solve Ratio Problems	37	6.RP.A.3
	1-6	Convert Customary Measurement Units	47	6.RP.A.3, 6.RP.A.3.D
	1-7	Understand Rates and Unit Rates Explore Compare Quantities with Different Units	57	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.A, 6.RP.A.3.B
	1-8	Solve Rate Problems	65	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.B
		Module 1 Review	73	



Module 2

Fractions, Decimals, and Percents

Essential Question

How can you use fractions, decimals, and percents to solve everyday problems?

				Content Standards
		What Will You Learn?	77	
Lesson	2-1	Understand Percents	79	Foundational for 6.RP.A.3, 6.RP.A.3.C
	2-2	Percents Greater Than 100% and Less Than 1%	85	Foundational for 6.RP.A.3, 6.RP.A.3.C
	2-3	Relate Fractions, Decimals, and Percents Explore Percents and Ratios	93	Foundational for 6.RP.A.3, 6.RP.A.3.C
	2-4	Find the Percent of a Number Explore Percent of a Number	103	6.RP.A.3, 6.RP.A.3.C
	2-5	Estimate the Percent of a Number	113	6.RP.A.3, 6.RP.A.3.C
	2-6	Find the Whole	121	6.RP.A.3, 6.RP.A.3.C
		Module 2 Review	129	

Module 3

Compute with Multi-Digit Numbers and Fractions

Essential Question

How are operations with fractions and decimals related to operations with whole numbers?

			Content Standards
		What Will You Learn? **133**	
Lesson	**3-1**	**Divide Multi-Digit Whole Numbers** **135**	6.NS.B.2
	3-2	**Compute With Multi-Digit Decimals** **143**	6.NS.B.3
	3-3	**Divide Whole Numbers by Fractions** .... **155** Explore Divide Whole Numbers by Fractions	6.NS.A.1
	3-4	**Divide Fractions by Fractions** **167**	6.NS.A.1
	3-5	**Divide with Whole and Mixed Numbers** .... **177** Explore Divide Fractions by Whole Numbers	6.NS.A.1
		Module 3 Review **187**	

Module 4

Integers, Rational Numbers, and the Coordinate Plane

Essential Question

How are integers and rational numbers related to the coordinate plane?

Lesson			Page	Content Standards
		What Will You Learn?	191	
Lesson	**4-1**	**Represent Integers**	193	6.NS.C.5, 6.NS.C.6, 6.NS.C.6.C
		Explore Represent Integers		
	4-2	**Opposites and Absolute Value**	199	6.NS.C.5, 6.NS.C.6, 6.NS.C.6.A, 6.NS.C.7, 6.NS.C.7.C
		Explore Opposites and Absolute Value		
	4-3	**Compare and Order Integers**	205	6.NS.C.7, 6.NS.C.7.A, 6.NS.C.7.B, 6.NS.C.7.C, 6.NS.C.7.D
	4-4	**Rational Numbers**	215	6.NS.C.6, 6.NS.C.6.C, 6.NS.C.7, 6.NS.C.7.A, 6.NS.C.7.C
	4-5	**The Coordinate Plane**	225	6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8
		Explore The Coordinate Plane		
	4-6	**Graph Reflections of Points**	237	6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8
		Explore Reflect a Point		
	4-7	**Absolute Value and Distance**	245	6.NS.C.8
		Explore Distance on the Coordinate Plane		
		Module 4 Review	255	

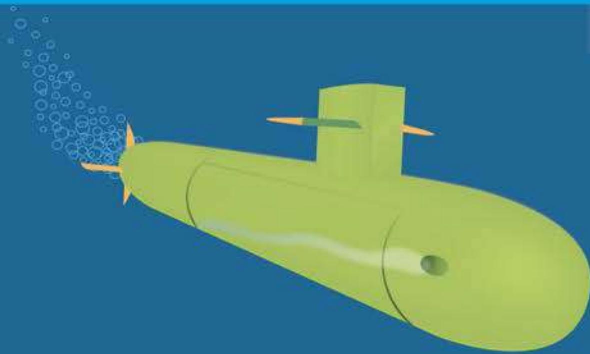
Module 5

Numerical and Algebraic Expressions

Essential Question

How can we communicate algebraic relationships with mathematical symbols?

				Content Standards
		What Will You Learn?	259	
Lesson	5-1	**Powers and Exponents**	261	6.EE.A.1
	5-2	**Numerical Expressions**	269	6.EE.A.1
	5-3	**Write Algebraic Expressions**	277	6.EE.A.2, 6.EE.A.2.A
		Explore Write Algebraic Expressions		6.EE.A.2.B, 6.EE.B.6
	5-4	**Evaluate Algebraic Expressions**	287	6.EE.A.2, 6.EE.A.2.C,
		Explore Algebraic Expressions		6.EE.B.6
	5-5	**Factors and Multiples**	295	6.NS.B.4
		Explore Greatest Common Factor		
		Explore Least Common Multiple		
	5-6	**Use the Distributive Property**	305	6.NS.B.4, 6.EE.A.3
		Explore Use Algebra Tiles to Model the Distributive Property		
	5-7	**Equivalent Algebraic Expressions**	315	6.EE.A.3, 6.EE.A.4
		Explore Properties and Equivalent Expressions		
		Module 5 Review	329	



Module 6

Equations and Inequalities

Essential Question

How are the solutions of equations and inequalities different?

Content Standards

What Will You Learn? **333**

Lesson 6-1 Use Substitution to Solve One-Step Equations .... **335** — 6.EE.B.5

6-2 One-Step Addition Equations .... **341** — 6.EE.B.6, 6.EE.B.7
- Explore Use Bar Diagrams to Write Addition Equations
- Explore One-Step Addition Equations

6-3 One-Step Subtraction Equations .... **351** — 6.EE.B.6, 6.EE.B.7
- Explore Use Bar Diagrams to Write Subtraction Equations

6-4 One-Step Multiplication Equations .... **359** — 6.EE.B.6, 6.EE.B.7
- Explore Use Bar Diagrams to Write Multiplication Equations

6-5 One-Step Division Equations .... **369** — 6.EE.B.6, 6.EE.B.7
- Explore Use Bar Diagrams to Write Division Equations

6-6 Inequalities .... **377** — 6.EE.B.5, 6.EE.B.8
- Explore Inequalities

Module 6 Review **391**

Module 7

Relationships Between Two Variables

Essential Question

What are the ways in which a relationship between two variables can be displayed?

		What Will You Learn?	**395**	**Content Standards**
Lesson	**7-1**	**Relationships Between Two Variables**	**397**	6.EE.C.9
		Explore Relationships Between Two Variables		
	7-2	**Write Equations to Represent Relationships Represented in Tables**	**405**	6.EE.C.9
		Explore Relationships with Rules that Require Two Steps		
	7-3	**Graphs of Relationships**	**415**	6.EE.C.9
	7-4	**Multiple Representations**	**423**	6.EE.C.9
		Module 7 Review	**429**	

Module 8

Area

Essential Question

How are the areas of triangles and rectangles used to find the areas of other polygons?

				Content Standards
		What Will You Learn?	433	
Lesson	8-1	**Area of Parallelograms**	435	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C
		Explore Area of Parallelograms		
	8-2	**Area of Triangles**	443	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C
		Explore Parallelograms and Area of Triangles		
		Explore Area of Triangles		
	8-3	**Area of Trapezoids**	451	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C
	8-4	**Area of Regular Polygons**	463	6.G.A.1
		Explore Area of Regular Polygons		
	8-5	**Polygons on the Coordinate Plane**	469	6.G.A.3
		Explore Explore the Coordinate Plane		
		Module 8 Review	479	

Module 9

Volume and Surface Area

Essential Question

How can you describe the size of a three-dimensional figure?

				Content Standards
		What Will You Learn?	483	
Lesson	9-1	Volume of Rectangular Prisms	485	6.G.A.2
	9-2	Surface Area of Rectangular Prisms	495	6.G.A.4
		Explore Cube Nets		
	9-3	Surface Area of Triangular Prisms	505	6.G.A.4
		Explore Non-Rectangular Prism Nets		
	9-4	Surface Area of Pyramids	517	6.G.A.4
		Module 9 Review	531	

Module 10

Statistical Measures and Displays

Essential Question

Why is data collected and analyzed and how can it be displayed?

			Content Standards
	What Will You Learn?	535	
Lesson 10-1	Statistical Questions Explore Collect Data	537	6.SP.A.1
10-2	Dot Plots and Histograms	543	6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A
10-3	Measures of Center Explore Mean	549	6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6. SP.B.5.C
10-4	Interquartile Range and Box Plots	561	6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C
10-5	Mean Absolute Deviation	569	6.SP.A.3, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B. 6. SP.B.5.C
10-6	Outliers Explore Mean, Median, and Outliers	575	6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C, 6.SP.B.5.D
10-7	Interpret Graphical Displays Explore Interpret Box Plots	583	6.SP.A.2, 6.SP.A.3, 6. SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C, 6.SP.B.5.D
	Module 10 Review	593	

Mathematical Overview for *Reveal Math*, Course 1

Reveal Math, Course 1, focuses on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Key Mathematical Understandings*, Grade 6

Ratios and Proportional Relationships (Domain 6.RP)

• Understand ratio concepts and use ratio reasoning to solve problems.

The Number System (Domain 6.NS)

• Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

• Compute fluently with multi-digit numbers and find common factors and multiples.

• Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations (Domain 6.EE)

• Apply and extend previous understandings of arithmetic to algebraic expressions.

• Reason about and solve one-variable equations and inequalities.

• Represent and analyze quantitative relationships between dependent and independent variables.

Geometry (Domain 6.G)

• Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability (Domain 6.SP)

• Develop understanding of statistical variability.

• Summarize and describe distributions.

From the Common Core State Standards for Mathematics

Standards for Mathematical Content, Grade 6

This correlation shows the alignment of *Reveal Math*, Course 1 to the Standards for Mathematical Content, Grade 6, from the Common Core State Standards for Mathematics. **Primary references are bold.** *Supporting references are italicized.*

Standards for Mathematical Content		Lesson(s)
6.RP Ratios and Proportional Relationships		
Understand ratio concepts and use ratio reasoning to solve problems. (Major Cluster)		
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**1-1**, *1-5*, *1-6*, *10-7*
6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] [1]Expectations for unit rates in this grade are limited to non-complex fractions.	**1-7**, **1-8**
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	**1-2**, **1-3**, **1-4**, **1-5**, **1-6**, **1-7**, **1-8**, **2-4**, **2-5**, **2-6**, *10-7*
	6.RP.A.3.A Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	**1-2**, **1-3**, **1-4**, **1-7**, *7-3*, *7-4*
	6.RP.A.3.B Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*	**1-7**, **1-8**
	6.RP.A.3.C Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	**2-4**, **2-5**, **2-6**
	6.RP.A.3.D Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	**1-6**

Standards for Mathematical Content		Lesson(s)
6.NS The Number System		
Apply and extend previous understandings of multiplication and division to divide fractions by fractions. (Major Cluster)		
6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**3-3**, **3-4**, **3-5**
Compute fluently with multi-digit numbers and find common factors and multiples. (Additional Cluster)		
6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.	**3-1**
6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**3-2**
6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*	**5-5**, **5-6**
Apply and extend previous understandings of numbers to the system of rational numbers. (Major Cluster)		
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**4-1**, **4-2**
6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	**4-1**, **4-2**, *4-3*, **4-4**, **4-5**, **4-6**, *4-7*, *6-6*, *7-3*, *7-4*
	6.NS.C.6.A Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., −(−3) = 3, and that 0 is its own opposite.	**4-2**, *4-6*
	6.NS.C.6.B Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	**4-5**, **4-6**
	6.NS.C.6.C Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	**4-1**, *4-3*, **4-4**, **4-5**, **4-6**, *6-6*, *7-3*, *7-4*

Standards for Mathematical Content		Lesson(s)
6.NS.C.7	Understand ordering and absolute value of rational numbers.	**4-2**, **4-3**, **4-4**, *4-7*
	6.NS.C.7.A Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right.*	**4-3**, **4-4**
	6.NS.C.7.B Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3°C > -7°C$ *to express the fact that* $-3°C$ *is warmer than* $-7°C$.	**4-3**, *4-4*
	6.NS.C.7.C Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* -30 *dollars, write* $\lvert -30 \rvert = 30$ *to describe the size of the debt in dollars.*	**4-2**, **4-3**, **4-4**, *4-7*
	6.NS.C.7.D Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* -30 *dollars represents a debt greater than 30 dollars.*	**4-3**
6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**4-5**, **4-6**, **4-7**
6.EE Expressions and Equations		
Apply and extend previous understandings of arithmetic to algebraic expressions. (Major Cluster)		
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	**5-1**, **5-2**
6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers.	**5-2**, **5-3**, **5-4**, *5-7*, *7-1*, **8-1**, **8-2**, **8-3**
	6.EE.A.2.A Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* $5 - y$.	**5-3**
	6.EE.A.2.B Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.*	**5-3**, *5-6*
	6.EE.A.2.C Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.	*5-2*, **5-4**, *7-1*, **8-1**, **8-2**, **8-3**
6.EE.A.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$.	**5-6**, **5-7**

Standards for Mathematical Content		Lesson(s)
6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*	**5-7**
Reason about and solve one-variable equations and inequalities. (Major Cluster)		
6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**6-1**, **6-6**
6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**5-3**, **5-4**, *6-1*, **6-2**, **6-3**, **6-4**, **6-5**, *6-6*, *7-2*, *7-3*, *7-4*, *9-1*, *10-3*
6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	**6-2**, **6-3**, **6-4**, **6-5**, *7-2*, *7-3*, *7-4*
6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**6-6**
Represent and analyze quantitative relationships between dependent and independent variables. (Major Cluster)		
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*	**7-1**, **7-2**, **7-3**, **7-4**

Standards for Mathematical Content		Lesson(s)
6.G Geometry		
Solve real-world and mathematical problems involving area, surface area, and volume. (Supporting Cluster)		
6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**8-1**, **8-2**, **8-3**, **8-4**, *8-5*
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**9-1**
6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**8-5**
6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**9-2**, **9-3**, **9-4**

Standards for Mathematical Content		Lesson(s)
6.SP Statistics and Probability		
Develop understanding of statistical variability. (Additional Cluster)		
6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**10-1**
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**10-4, 10-7**
6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	**10-3, 10-4, 10-5, 10-6, 10-7**
Summarize and describe distributions. (Additional Cluster)		
6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**10-2, 10-3, 10-4, 10-6, 10-7**
6.SP.B.5	Summarize numerical data sets in relation to their context, such as by:	*10-1*, **10-2, 10-3, 10-4, 10-5, 10-6, 10-7**
	6.SP.B.5.A Reporting the number of observations.	*10-1*, **10-2, 10-3, 10-5, 10-7**
	6.SP.B.5.B Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.	**10-3, 10-5, 10-7**
	6.SP.B.5.C Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	**10-3, 10-4, 10-5, 10-6, 10-7**
	6.SP.B.5.D Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**10-6, 10-7**

Standards for Mathematical Practice, Grade 6

This correlation shows the alignment of *Reveal Math*, Course 1 to the Standards for Mathematical Practice, from the Common Core State Standards.

	Standards for Mathematical Practice	Lesson(s)
MP1	**Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	A strong problem-solving strand is present throughout the program with an emphasis on having students explain to themselves and others the meanings of problems and plan their solution strategies. Look for the **Apply** problems and exercises labeled as **Persevere with Problems**. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-2, Apply • Lesson 3-1, Practice Exercise 15 • Lesson 3-3, Apply • Lesson 8-1, Apply • Lesson 9-1, Apply
MP2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Students are routinely asked to make sense of quantities and their relationships, and attend to the meaning of quantities as opposed to just computing with them. Students are often asked to decontextualize a real-world problem by representing it symbolically as an expression, equation, or inequality. Look for lessons addressing these algebraic topics and the exercises labeled as **Reason Abstractly**. Many ***Talk About It!*** question prompts ask students to reason about relationships between quantities. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-6, Example 1 • Lesson 5-3, Examples 2, 4, 5 • Lesson 6-2, Example 1 • Lesson 7-1, Example 2 • Lesson 7-3, Learn *Write an Equation from a Graph*

Standards for Mathematical Practice		Lesson(s)
MP3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	Students are required to justify their reasoning and to find the errors in another student's reasoning or work. Look for the **Apply** problems (Step 4) and the exercises labeled as **Make a Conjecture**, **Find the Error**, **Use a Counterexample**, **Make an Argument**, or **Justify Conclusions.** Many ***Talk About It!*** question prompts ask students to justify conclusions and/or critique another student's reasoning. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 2-3, Practice Exercises 16-17 • Lesson 8-2, Practice Exercises 11, 14 • Lesson 9-1, Practice Exercise 9 • Lesson 9-4, Example 2, *Talk About It!*
MP4	**Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Students apply the mathematics they know to solve real-world problems by using mathematical modeling. In the **Apply** problems, students determine their own strategy to solve application problems by choosing mathematical models to aid them. Look also for the exercises labeled as **Model with Mathematics.** In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 6-2, Example 1 • Lesson 6-4, Apply • Lesson 6-5, Apply • Lesson 7-2, Examples 1–2 • Lesson 7-2, Apply

Standards for Mathematical Practice		Lesson(s)
MP5	**Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	In addition to traditional tools such as estimation, mental math, or measurement tools, students are encouraged to use digital tools, such as Web Sketchpad, eTools, etc. to help solve problems. Students are routinely asked to compare and contrast methods, tools, and representations and note when one tool might be more advantageous to use than another. Look for selected *Talk About It!* prompts and exercises labeled as **Use Math Tools.** Many **Explore** activities ask students to select and use appropriate tools as they progress through the activities. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-4, Learn *Use Graphs to Compare Ratio Relationships* • Lesson 1-5, Learn *Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems* • Lesson 1-6, Learn *Convert Larger Units to Smaller Units* • Lesson 3-3, Examples 4-5 • Lesson 5-3, Explore activity *Write Algebraic Expressions*
MP6	**Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Students are routinely required to communicate precisely to partners, the teacher, or the entire class by using precise definitions and mathematical vocabulary. Look for the exercises labeled as **Be Precise.** Many ***Talk About It!*** question prompts ask students to clearly and precisely explain their reasoning. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 3-1, Learn *Divide Multi-Digit Numbers* • Lesson 4-4, Learn *Absolute Value of Rational Numbers, Talk About It!* • Lesson 6-2, Learn *Write Addition Equations, Talk About It!*

Standards for Mathematical Practice		Lesson(s)
MP7	**Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Students are routinely encouraged to look for patterns or structure present in problem situations. For example, students look for structure present in algebraic expressions and use the structure of three-dimensional figures to create nets Look for the exercises labeled as **Identify Structure.** Many ***Talk About It!*** question prompts ask students to study the structure of expressions and figures. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 4-6, Example 1, *Talk About It!* • Lesson 4-7, Learn *Find Vertical Distance, Talk About It!* • Lesson 5-3, Learn *Structure of Algebraic Equations, Talk About It!* • Lesson 5-3, Example 1 • Lesson 6-1, Learn *Equations, Talk About It!* • Lesson 9-2, Learn *Make a Net to Represent a Rectangular Prism, Talk About It!* • Lesson 9-3, Example 2
MP8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Students are encouraged to look for repeated calculations that lead them to sound mathematical conclusions. For example, students notice that division ends when a remainder is zero. Look for the exercises labeled as **Identify Repeated Reasoning.** Several ***Talk About It!*** question prompts ask students to look for repeated calculations. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 3-1, Example 2, *Talk About It!* • Lesson 4-2, Example 3 • Lesson 6-2, Explore activity *One-Step Addition Equations*

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can you describe how two quantities are related? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

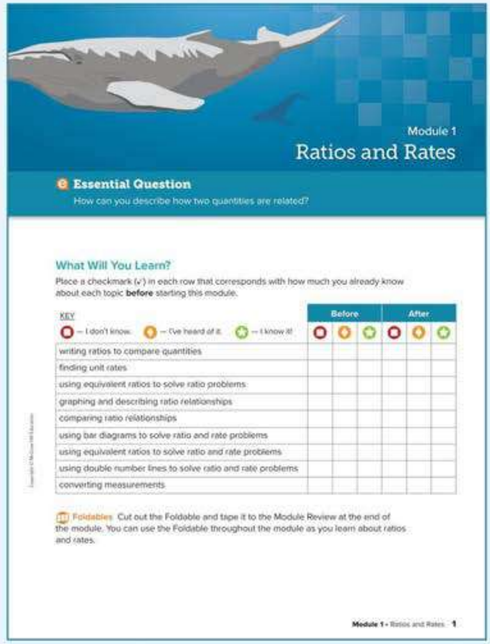
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of peanut butter production, baseball batting cages, and the eating habits of blue whales to introduce the idea of ratios and rates. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 1

Ratios and Rates

Essential Question

How can you describe how two quantities are related?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know	I've heard of it	I know it!	I don't know	I've heard of it	I know it!
writing ratios to compare quantities						
finding unit rates						
using equivalent ratios to solve ratio problems						
graphing and describing ratio relationships						
comparing ratio relationships						
using bar diagrams to solve ratio and rate problems						
using equivalent ratios to solve ratio and rate problems						
using double number lines to solve ratio and rate problems						
converting measurements						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about ratios and rates.

Module 1 • Ratios and Rates 1

Interactive Presentation

Module 1

Ratios and Rates

Module Goal

Use ratio and rate reasoning to solve real-world and mathematical problems.

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): **6.RP .A** Understand ratio concepts and use ratio reasoning to solve problems.

Standards for Mathematical Content:

6.RP.A.2 Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with $b \neq 0$, and use rate language in the context of a ratio relationship.

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to understand how a fraction can be used to express part of a whole, and need to be able to multiply and divide with whole numbers.

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students understood a fraction as part of a whole, and fraction equivalence. **3.NF.A.1, 4.NF.A.1**

Now
Students use ratio and rate reasoning to solve real-world and mathematical problems. **6.RP.A.1, 6.RP.A.2, 6.RP.A.3**

Next
Students will use ratio reasoning to find the percent of a number. **6.RP.A.3, 6.RP.A.3.C**

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of fractions and fraction equivalence to develop *understanding* of ratios and rates. They use this understanding to build *fluency* with finding equivalent ratios and rates, and finding unit rates. They also *apply* their understanding of ratios and rates to solve real-world problems.

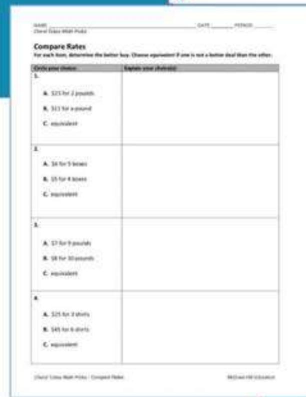
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
1-1	Understand Ratios	6.RP.A.1	2	1
1-2	Tables of Equivalent Ratios	6.RP.A.3, 6.RP.A.3.A	3	1.5
1-3	Graphs of Equivalent Ratios	6.RP.A.3, 6.RP.A.3.A	2	1
1-4	Compare Ratio Relationships	6.RP.A.3, 6.RP.A.3.A	1	0.5
1-5	Solve Ratio Problems	6.RP.A.3, *Also addresses 6.RP.A.1*	2	1
Put It All Together 1: Lessons 1-1 through 1-5			0.5	0.25
1-6	Convert Customary Measurement Units	6.RP .A.3, 6.RP.A.3.D, *Also addresses 6.RP.A.1*	2	1
1-7	Understand Rates and Unit Rates	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.A, 6.RP.A.3.B	2	1
1-8	Solve Rate Problems	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.B	2	1
Put It All Together 2: Lessons 1-6 through 1-8			0.5	0.25
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	20	10

Formative Assessment Math Probe
Ratios and Rates

Module Resource



Correct Answers: **1.** B; **2.** A; **3.** A; **4.** B

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which item is the better buy, and explain their choice.

Targeted Concept Ratios and rates can be compared by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams or equations.

Targeted Misconceptions

- Students compare the difference between the two quantities in each ratio.
- Students use an additive relationship to find comparative ratios.

Assign the probe after Lesson 8.

Collect and Assess Student Answers

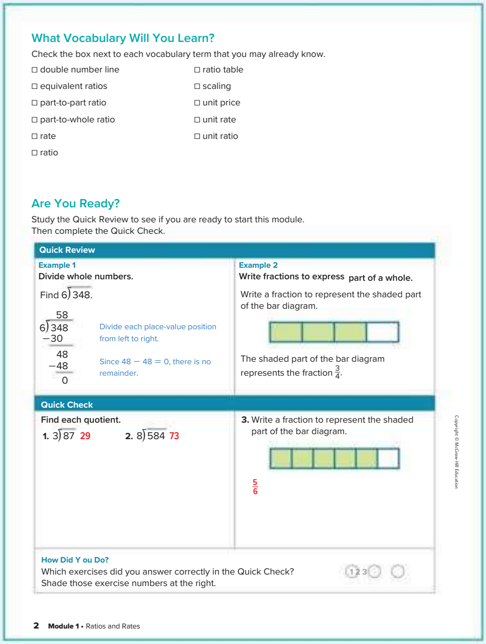
If the student selects...	Then the student likely...
2. C **3.** C	found the difference between the two quantities in the ratio. **Example:** For Exercises 2 and 3, the student chooses equivalent. because both ratios in each question have the same difference.
1. B **4.** A	added the number of items in one ratio to make it equivalent to the other, and added that same amount to the dollar amount. **Example:** For Exercise 1, the student chooses B, the correct answer, but reasons that B is equivalent to $12 for 2 pounds by adding 1 to each term. **Example:** For Exercise 4, the student determines that A is equivalent to $28 for 6 shirts by adding 3 to each term.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Ratios, Proportions, and Measurements
- Lesson 7, Examples 1–2
- Lesson 8, Examples 1–2

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.


What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ double number line	☐ ratio table
☐ equivalent ratios	☐ scaling
☐ part-to-part ratio	☐ unit price
☐ part-to-whole ratio	☐ unit rate
☐ rate	☐ unit ratio
☐ ratio	

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1
Divide whole numbers.

Find $6\overline{)348}$.

$$\begin{array}{r} 58 \\ 6\overline{)348} \\ -30 \\ \hline 48 \\ -48 \\ \hline 0 \end{array}$$

Divide each place-value position from left to right.

Since 48 − 48 = 0, there is no remainder.

Example 2
Write fractions to express part of a whole.

Write a fraction to represent the shaded part of the bar diagram.

The shaded part of the bar diagram represents the fraction $\frac{3}{4}$.

Quick Check

Find each quotient.

1. $3\overline{)87}$ 29 **2.** $8\overline{)584}$ 73

3. Write a fraction to represent the shaded part of the bar diagram.

$\frac{5}{6}$

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

1 2 3

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define Equivalent ratios are two ratios that express the same relationship between two quantities.

Example 12 : 6 is equivalent to 20 : 10

Ask What is an equivalent ratio to 9 : 81? Sample answers: 6 : 54, 5 : 45, 12 : 108

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- multiplying and dividing whole numbers
- understanding a fraction as part of a whole
- finding equivalent fractions

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Ratios, Proportions, and Measurements** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Growth Mindset vs. Fixed Mindset

Everyone has a core belief or mindset about how they learn. People with a *growth mindset* believe that they can grow their intelligence through hard work. Those with a *fixed mindset* believe that while they can learn new things, they cannot increase their intelligence. When a student approaches school, life, and the future workplace with a growth mindset, they are more likely to persevere through challenging problems, learn from their mistakes, and ultimately learn concepts in a deeper, more meaningful way.

How Can I Apply It?

Assign students rich tasks, such as the **Explore** activities, that can help them to develop their intelligence. Encourage them with the thought that each time they learn a new idea, neurons fire electric currents that connect different parts of their brain!

Learn Understand Ratios

Objective

Students will understand the concept of a ratio and how a ratio can be used to compare two quantities.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to attend to the value given for lemon juice and compare it to the total number of cups of lemonade.

As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the quantities and how changing them might affect the flavor of the lemonade.

Teaching Notes

SLIDE 2

Have students study the bar diagram to understand how the values in the table are used to make an additive comparison of cups of lemon juice to total cups of lemonade. Point out that this recipe makes 10 cups of lemonade. Suppose the quantities in the recipe are doubled in order to make 20 cups of lemonade. Ask students if there will still be 8 more cups of lemonade than cups of lemon juice. Students should use reasoning to determine that there would be 4 cups of lemon juice to make 20 cups of lemonade, which means there are 16, not 8, more cups of lemonade than lemon juice if the recipe is doubled. The additive comparison of *8 more cups of lemonade than cups of lemon juice* is only true for this first batch.

(continued on next page)

Lesson 1-1

Understand Ratios

I Can... show a ratio relationship between two quantities using different representations, and describe the relationship using correct mathematical language.

What Vocabulary Will Y ou Learn?
part-to-part ratio
part-to-whole ratio
ratio

Explore Compare Two Quantities

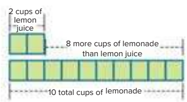
Online Activity Y ou will use Web Sketchpad to determine how many students and teachers should be on various buses to maintain the same relationship of one teacher for every eight students.

Learn Understand Ratios

The table shows the ingredients needed to make 10 cups of lemonade. How does the number of cups of lemon juice compare to the total number of cups of lemonade?

Ingredient	Number of Cups
Lemon Juice	2
Simple Syrup	1
Water	7

One way to make a comparison is to use a bar diagram. There are 8 more cups of lemonade than there are cups of lemon juice. This is an *additive comparison* because 2 + 8 = 10.



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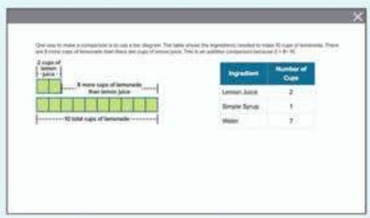
Interactive Presentation



Learn, Understand Ratios, Slide 2 of 4

Lesson 1-1

Understand Ratios

6.RP.A.1

LESSON GOAL

Students will understand the concept of a ratio.

1 LAUNCH

Launch the Lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Compare Two Quantities

Learn: Understand Ratios
Example 1: Understand Ratios
Learn: Part-to-Whole and Part-to-Part Ratios
Example 2: Part-to-Whole Ratios
Example 3: Part-to-Part Ratios
Apply: Fundraising

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: The Golden Ratio		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 1 of the *Language Development Handbook* to help your students build mathematical language related to understanding ratios and ratio language.

ELL You can use the tips and suggestions on page T1 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by solving problems by understanding the concept of a ratio.
Standards for Mathematical Content: **6.RP.A.1**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5**

Coherence

Vertical Alignment

Previous
Students understood a fraction as part of a whole, and fraction equivalence.
5.NF.B.3

Now
Students understand the concept of a ratio.
6.RP.A.1

Next
Students will use ratio tables and double number lines to find equivalent ratios.
6.RP.A.3, 6.RP.A.3.A

Rigor

The Three Pillars of Rigor

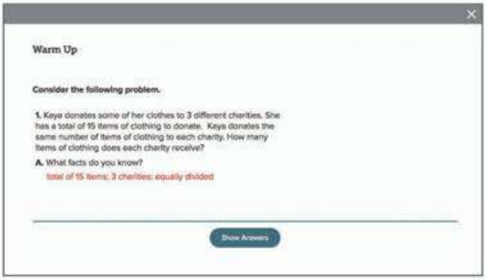
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of ratios and ratio language to describe the relationship between two quantities. They come to understand that ratios can be part-to-whole and part-to-part and write ratios in different forms that express different ratio relationships. They *apply* their understanding of ratios to solve real-world problems.

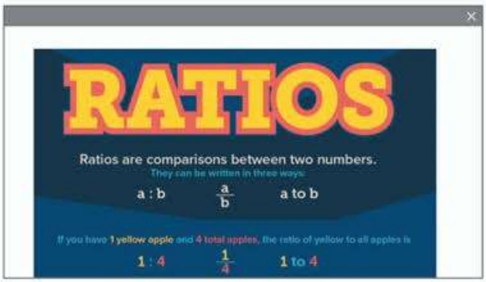
Mathematical Background

A ratio is a comparison between two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity. The phrases *for every* and *for each* are used to define and describe ratios. Ratios can be written in different ways and can be modeled using bar diagrams and other representations. A *part-to-whole ratio* compares one part of a group to the whole group. A *part-to-part ratio* compares one part of a group to another part of the same group.

Interactive Presentation



Warm Up



Launch the Lesson

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

• solving word problems (Exercises 1 and 2)

Answers

1A. total of 15 items; equally divided

1B. number of items each charity receives

1C. 5 items

1D. Multiply 5 by 3 to check that the total is 15.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about ratios and their real-world applications.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

• What are some everyday examples of where you might have heard the term *ratio* before? Sample answers: The ratio of wins to losses for a basketball team, the ratio of boys to girls in a class, the ratio of teachers to students at a school.

• What part of speech is the term *part-to-part*? If you knew that a *ratio* is a comparison of two quantities, what kind of ratio do you think is a *part-to-part ratio*? adjective; Sample answer: A *part-to-part ratio* is a special kind of ratio that might compare one part of a group to another part of the same group.

• What part of speech is the term *part-to-whole*? What kind of ratio do you think is a *part-to-whole ratio*? adjective; Sample answer: A *part-to-whole ratio* is a special kind of ratio that might compare one part of a group to the total.

Explore Compare Two Quantities

Objective

Students will use Web Sketchpad to explore how to maintain the same relationship between two quantities as one of the quantities changes.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to discuss why or why not the bus moves if there are 2 teachers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with various scenarios of the number of students and teachers on different buses and determine how many additional students or teachers need to be added in order to maintain the relationship of 1 teacher for every 8 students.

Inquiry Question

How can you use reasoning to maintain the same relationship between two quantities as one of the quantities changes? Sample answer: As one of the quantities changes, I can reason about how the relationship between the quantities must remain the same. For example, if the relationship between students and teachers is 1 teacher for every 8 students, then if there are 24 students, that means there are 3 groups of 8 students, and each group needs a teacher chaperone. So, 3 teachers are needed.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

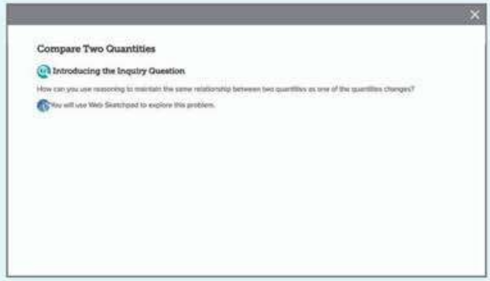
SLIDE 3

Mathematical Discourse

How many teachers did you place on the bus? Did the bus move? Why do you think the bus either moved or did not move? Sample answer: I placed two teachers on the bus and the bus moved, because the relationship 1 teacher for every 8 students is maintained. If there are 16 students, that is two groups of 8 students, and one teacher is needed to chaperone each group.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6

Explore, Slide 3 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to maintain the same relationship between two quantities as one of the quantities changes.

Interactive Presentation

Explore, Slide 5 of 6

TYPE

On Slide 6, students will respond to the Inquiry Question and can view a sample answer.

Explore Compare Two Quantities *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to use reasoning about the relationship between the number of teachers and students and what it means to maintain that relationship as the number of students or number of teachers changes. In order to maintain the relationship of 1 teacher for every 8 students, encourage them to think about groups of 8 students. If there are two groups of 8 students (16 students), then two teachers are needed. For every group of 8 students, one teacher is needed.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

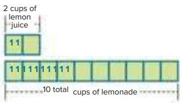
How many teachers did you place on the bus? Did the bus move? Why do you think the bus either moved or did not move? Sample answer: I placed three additional teachers on the bus and the bus moved, because the relationship 1 teacher for every 8 students is maintained. If there are 32 students, that is four groups of 8 students, and one teacher is needed to chaperone each group. This is a total of four teacher chaperones.

Your Notes

Another way to make a comparison is to use a ratio. A **ratio** is a comparison between two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity. The phrases *for every* and *for each* are used to define and describe ratios.

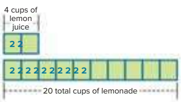
The relationships between the quantities of ingredients in recipes are examples of ratios. To make one batch of lemonade, 10 cups, you need 2 cups of lemon juice.

Ingredient	Number of Cups
Lemon Juice	2
Simple Syrup	1
Water	7



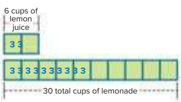
For every 2 cups of lemon juice, there are 10 total cups of lemonade. Each section represents 1 cup.

To make two batches of lemonade, 20 cups, how many cups of lemon juice will you need?



Double the quantities of lemon juice and lemonade to maintain the same ratio. Each section represents 2 cups. You need 4 cups of lemon juice.

To make three batches of lemonade, 30 cups, how many cups of lemon juice will you need?



Triple the quantities of lemon juice and lemonade to maintain the same ratio. Each section represents 3 cups. You need 6 cups of lemon juice.

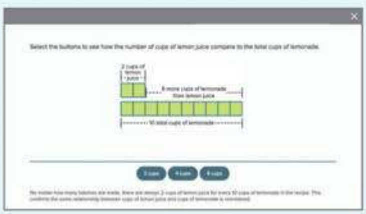
No matter how many batches are made, there are always 2 cups of lemon juice for every 10 cups of lemonade in the recipe. This confirms the same relationship between cups of lemon juice and cups of lemonade is maintained.

Talk About It!

If you did not maintain the same ratio of lemon juice to total cups of lemonade when making 2 or 3 batches, what might happen to your lemonade? Justify your response.

Sample answer: If there is too much lemon juice, the lemonade might be too sour. If there isn't enough, it might be too sweet.

Interactive Presentation



Learn, Understand Ratios, Slide 3 of 4

CLICK

On Slide 3, students select buttons to see how the cups of lemon juice compare to the total cups of lemonade.

Learn Understand Ratios *(continued)*

Teaching Notes

SLIDE 3

Students will learn that using a *ratio* is another way to compare quantities. You may wish to have students move through the slides that show how the bar diagrams can be used to compare the number of cups of lemon juice to the total number of cups of lemonade as the number of batches increases. Ask students the following questions.

- Why is the number of sections representing lemon juice and lemonade the same for each bar diagram? Sample answer: By keeping the number of sections the same, I can be sure that the ratio relationship between the two quantities is maintained.
- Why does the number labeled inside each section increase for each batch? What does this number represent? Sample answer: The number labeled inside each section indicates the number of cups each section represents, whether or lemon juice or lemonade. As the number of batches increases, this number increases.

Go Online to find a sample answer for the *Talk About It!* question on Slide 4.

DIFFERENTIATE

Language Development Activity ELL

Have students practice using the phrases *for every* and *for each* when describing ratio relationships. Students may be unsure when to use *for every* and when to use *for each*. You may wish to point out that the phrase *for every* is used when the second quantity is plural, and *for each* is used when the second quantity is singular. Some examples are shown.

- *For each cup* of simple syrup, there are 2 cups of lemon juice.
- *For every 2 cups* of lemon juice, there are 7 cups of water.

Knowing when to use *each* versus *every* can be confusing even among fluent English language speakers. Allow students space to make mistakes; the most important concept for them to grasp is the reasoning behind why these phrases are used when describing ratio relationships. Have students work with a partner to respond to the following questions, given the scenario presented in the Learn.

- Consider the phrase *for every 2 cups of lemon juice, there are 10 cups of lemonade*. Why do you think the phrase *for every* is necessary here? Sample answer: Without using *for every*, the relationship might not be maintained when making more batches of lemonade. By using *for every*, it is defining the connection between lemon juice and lemonade that persists for any number of batches.
- Write your own sentences comparing the quantities in the recipe that being with *for every* or *for each*. Sample answers given.
  For every 2 cups of lemon juice, there is 1 cup of simple syrup.
  For each cup of simple syrup, there are 7 cups of water.
  For every 7 cups of water, there are 10 total cups of lemonade.

Example 1 Understand Ratios

Objective

Students will use reasoning to determine if the same ratio is maintained.

Questions for Mathematical Discourse

SLIDE 2

AL What is a ratio? Sample answer: A ratio is a comparison between two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity.

OL Use ratio language to describe Pedro's original ratio of blue paint to yellow paint. For every 2 containers of blue paint, there are 3 containers of yellow paint.

BL A classmate wrote the ratio of blue paint to yellow paint as 3 : 2. Is this correct? Explain. no; Sample answer: The ratio is defined as comparing blue paint to yellow paint, so the ratio is 2 : 3.

BL In what context might it be correct to use the ratio 3 : 2 in this example? Why is it important to define the quantities used in a ratio? Sample answer: If you define the ratio as comparing yellow paint to blue paint, you can use the ratio 3 : 2. It is important to define the quantities you are using in a ratio, so that you know which number represents which quantity. The first quantity in the comparison is the first number in the ratio.

SLIDE 3

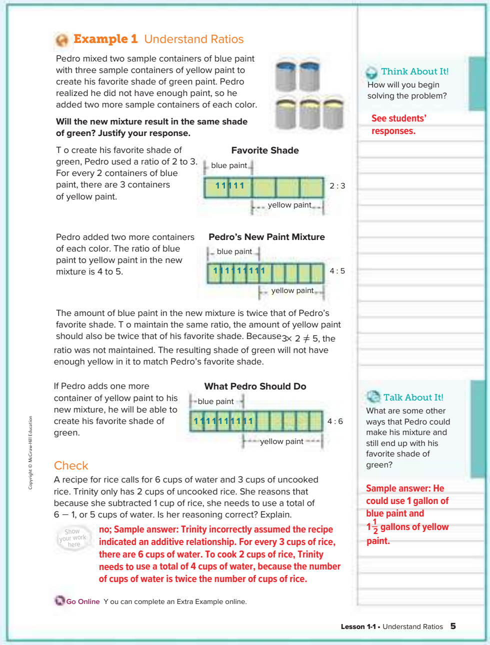
AL By what number can you multiply the original amount of blue paint to get the new amount of blue paint? Explain your reasoning. 2; Sample answer: The original amount of blue paint was 2 containers and the new amount is 4 containers, so the original is multiplied by 2.

OL What might be undesirable with the shade of the paint if Pedro uses the ratio of blue paint to yellow paint of 4 : 5? Sample answer: The paint could have more of a bluish tone than he likes.

BL If Pedro has 12 gallons of yellow paint, how many gallons of blue paint does he need to mix with the yellow paint in order to maintain the ratio? Explain your answer. 8 gallons; Sample answer: Since 12 is 3 × 4, multiply 2 by 4 to obtain the number of gallons of blue paint needed to maintain the ratio.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Understand Ratios

Pedro mixed two sample containers of blue paint with three sample containers of yellow paint to create his favorite shade of green paint. Pedro realized he did not have enough paint, so he added two more sample containers of each color.

Will the new mixture result in the same shade of green? Justify your response.

To create his favorite shade of green, Pedro used a ratio of 2 to 3. For every 2 containers of blue paint, there are 3 containers of yellow paint.

Pedro added two more containers of each color. The ratio of blue paint to yellow paint in the new mixture is 4 to 5.

The amount of blue paint in the new mixture is twice that of Pedro's favorite shade. To maintain the same ratio, the amount of yellow paint should also be twice that of his favorite shade. Because $3 \times 2 \neq 5$, the ratio was not maintained. The resulting shade of green will not have enough yellow in it to match Pedro's favorite shade.

If Pedro adds one more container of yellow paint to his new mixture, he will be able to create his favorite shade of green.

Think About It! How will you begin solving the problem?

See students' responses.

Talk About It! What are some other ways that Pedro could make his mixture and still end up with his favorite shade of green?

Sample answer: He could use 1 gallon of blue paint and $1\frac{1}{2}$ gallons of yellow paint.

Check

A recipe for rice calls for 6 cups of water and 3 cups of uncooked rice. Trinity only has 2 cups of uncooked rice. She reasons that because she subtracted 1 cup of rice, she needs to use a total of 6 − 1, or 5 cups of water. Is her reasoning correct? Explain.

Show your work here

no; Sample answer: Trinity incorrectly assumed the recipe indicated an additive relationship. For every 3 cups of rice, there are 6 cups of water. To cook 2 cups of rice, Trinity needs to use a total of 4 cups of water, because the number of cups of water is twice the number of cups of rice.

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 1, Understand Ratios, Slide 3 of 5

CLICK

On Slide 3, students move through the steps to compare the ratios.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Part-to-Whole and Part-to-Part Ratios

A **part-to-whole ratio** compares one part of a group to the whole group. The ratio 2 : 10 is a part-to-whole ratio because it compares the number of cups of lemon juice (the part) to the total number of cups of lemonade (the whole).

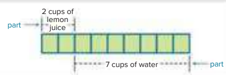
Ingredient	Number of Cups
Lemon Juice	2
Simple Syrup	1
Water	7

Model

Words	Ratio Notation
For every 2 cups of lemon juice, there are 10 total cups of lemonade.	part → 2 to 10 ← whole part → 2 : 10 ← whole part → $\frac{2}{10}$ ← whole

A **part-to-part ratio** compares one part of a group to another part of the same group. The ratio 2 : 7 is a part-to-part ratio because it compares the number of cups of lemon juice (one part) to the number of cups of water (another part) needed to make the lemonade.

Model



Words	Ratio Notation
For every 2 cups of lemon juice, there are 7 cups of water.	part → 2 to 7 ← part part → 2 : 7 ← part

Because a fraction represents a part of a whole, fraction notation is generally only used to represent part-to-whole ratios.

Talk About It!

No matter how many batches of lemonade are made, will there always be 2 cups of lemon juice for every 7 cups of water? Justify your response.

yes; Sample answer: When making batches of lemonade, the ratio relationship will always be maintained. So, if you make 2 batches, there will be 4 cups of lemon juice and 14 cups of water.

6 **Module 1** • Ratios and Rates

Interactive Presentation

Learn, Part-to-Whole and Part-to-Whole Ratios, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to learn about part-to-whole ratios.

FLASHCARDS

On Slide 2, students use Flashcards to learn about part-to-part ratios.

Learn Part-to-Whole and Part-to-Part Ratios

Objective

Students will understand the different kinds of ratios that can be used to compare quantities (part-to-whole and part-to-part).

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to attend to the meaning of each type of ratio. You may wish to have them describe other scenarios in which each type of ratio can be used.

Teaching Notes

SLIDE 1

Students will learn the definition of *part-to-whole ratio*. You may wish to have a volunteer use the Flashcards that illustrate how to model and write part-to-whole ratios of the cups of lemon juice to the cups of lemonade. Encourage students to understand the correspondences between the bar diagram, words, and ratio notation.

SLIDE 2

Students will learn the definition of *part-to-part ratio*. You may wish to have a student volunteer use the Flashcards that illustrate how to model and write part-to-part ratios of the cups of lemon juice to the cups of water. Have students compare and contrast part-to-whole and part-to-part ratios.

Talk About It!

SLIDE 2

Mathematical Discourse

Using the same recipe, write another ratio in which fraction notation would not be the best notation to use to represent the relationship. Explain. Sample answer: lemon juice to simple syrup; This ratio is a part-to-part relationship and fractions are used to represent part of a whole.

SLIDE 3

Mathematical Discourse

No matter how many batches of lemonade are made, will there always be 2 cups of lemon juice for every 7 cups of water? Justify your response. yes; Sample answer: When making batches of lemonade, the ratio relationship will always be maintained. So, if you make 2 batches, there will be 4 cups of lemon juice and 14 cups of water.

Example 2 Part-to-Whole Ratios

Objective

Students will write and use a part-to-whole ratio to find a new value for one quantity when the other quantity changes.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically In Step 2, encourage students to use the bar diagram to determine how many flowers each bar represents.

Questions for Mathematical Discourse

SLIDE 2

AL What is the ratio of tulips to total flowers? 4 to 12 or 4 : 12

AL How do we know that we need to use a part-to-whole ratio? Sample answer: The number of tulips is part of the total number of flowers.

OL How many flowers does each section represent? Each section represents 1 flower.

BL A classmate drew a bar diagram with 2 sections representing *tulips* and 6 sections representing *total flowers*. Is this a correct representation? Explain. How many flowers would each section represent? yes; Sample answer: If each section represents $\frac{1}{2}$ flower, the bar diagram correctly represents the ratio.

BL Can the ratio of tulips to total flowers be written using numbers that are less than 2 and 6? What is the ratio using these numbers? yes; Sample answer: 1 to 3 or 1 : 3.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

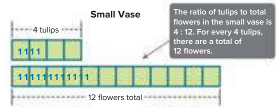
Example 2 Part-to-Whole Ratios

A florist is arranging flowers in vases to sell to her customers. She has two sizes of vases available: small and large. She wants the large vase to have the same ratio of flowers as the small vase.

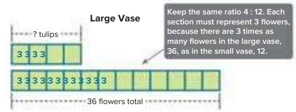
Small Vase	
Flower Quantity	
Carnations	6
Sunflowers	2
Tulips	4

If the large vase has a total of 36 flowers, how many are tulips?

Step 1 Use a bar diagram to represent the ratio of tulips to total flowers for the small vase.



Step 2 Use the same ratio to find the number of tulips in the large vase.



Each section in the diagram represents 3 flowers. There are four sections for tulips, so the large vase will contain 4 × 3, or 12 tulips.

Check

Refer to the table in Example 2. If the large vase has a total of 36 flowers, how many are carnations?

Show your work here

There are 18 carnations in a large vase.

Go Online Y ou can complete an Extra Example online.

Think About It! Why is the ratio of tulips to total flowers a part-to-whole ratio?

Sample answer: The ratio of tulips to total flowers is a part-to-whole ratio because it compares part of a group (tulips) to the whole group (total flowers).

Talk About It! Why does each section of the bar diagram have to represent the same amount, in this case, 3 flowers?

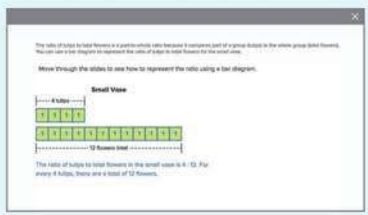
To maintain the same ratio, each part must remain constant.

Talk About It! Suppose the florist wanted to place the flowers in a medium vase, using the same ratio. What quantities of tulips and total flowers might be reasonable for a medium vase? Justify your response.

Sample answer: 8 tulips and 24 total flowers; See students' responses.

Lesson 1-1 • Understand Ratios 7

Interactive Presentation



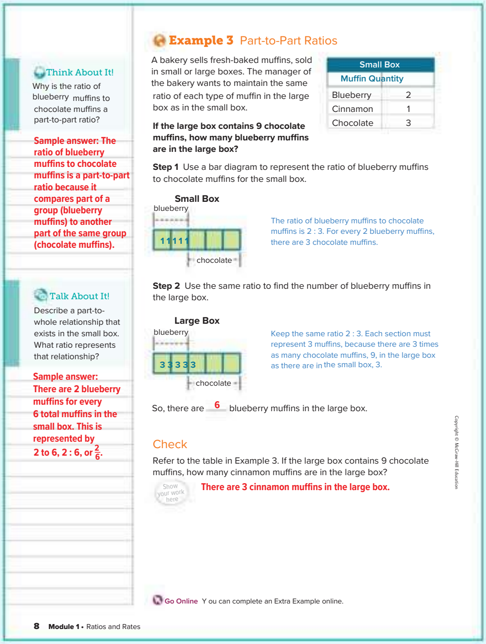
Example 2, Part-to-Whole Ratios, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to use ratio reasoning to determine the number of flowers in the large vase.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Part-to-Part Ratios

Think About It! Why is the ratio of blueberry muffins to chocolate muffins a part-to-part ratio?

Sample answer: The ratio of blueberry muffins to chocolate muffins is a part-to-part ratio because it compares part of a group (blueberry muffins) to another part of the same group (chocolate muffins).

Talk About It! Describe a part-to-whole relationship that exists in the small box. What ratio represents that relationship?

Sample answer: There are 2 blueberry muffins for every 6 total muffins in the small box. This is represented by 2 to 6, 2 : 6, or $\frac{2}{6}$.

A bakery sells fresh-baked muffins, sold in small or large boxes. The manager of the bakery wants to maintain the same ratio of each type of muffin in the large box as in the small box.

Small Box	
Muffin Quantity	
Blueberry	2
Cinnamon	1
Chocolate	3

If the large box contains 9 chocolate muffins, how many blueberry muffins are in the large box?

Step 1 Use a bar diagram to represent the ratio of blueberry muffins to chocolate muffins for the small box.

Small Box

blueberry 1 1 1 1 1 chocolate

The ratio of blueberry muffins to chocolate muffins is 2 : 3. For every 2 blueberry muffins, there are 3 chocolate muffins.

Step 2 Use the same ratio to find the number of blueberry muffins in the large box.

Large Box

blueberry 3 3 3 3 3 chocolate

Keep the same ratio 2 : 3. Each section must represent 3 muffins, because there are 3 times as many chocolate muffins, 9, in the large box as there are in the small box, 3.

So, there are 6 blueberry muffins in the large box.

Check

Refer to the table in Example 3. If the large box contains 9 chocolate muffins, how many cinnamon muffins are in the large box?

Show your work here

There are 3 cinnamon muffins in the large box.

Go Online Y ou can complete an Extra Example online.

8 Module 1 • Ratios and Rates

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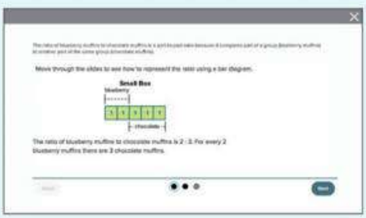
Interactive Presentation



Example 3, Part-to-Part Ratios, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to use ratio reasoning to determine the number of blueberry muffins in the large box.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Part-to-Part Ratios

Objective

Students will write and use a part-to-part ratio to find a new value for one quantity when the other quantity changes.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know a part-to-part ratio is needed? Sample answer: The number of blueberry muffins is part of the whole and the number of chocolate muffins is another part of the same whole.

OL Suppose a classmate said that for every 3 chocolate muffins, there are 2 blueberry muffins. Is this correct? Explain. yes; Sample answer: The ratio of chocolate muffins to blueberry muffins is 3 : 2.

BL Is it possible to create a box with 7 blueberry muffins and maintain the same ratio? no; Sample answer: The ratio of blueberry muffins to chocolate muffins is 2 : 3, to have 7 blueberry muffins you need to multiply by 3.5 ($2 \times 3.5 = 7$). Multiplying 3×3.5 is 10.5, or $10\frac{1}{2}$ muffins, and you need a whole number of muffins.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

Two rectangles are *similar* if the ratio of the width to the length is the same for each rectangle. Have students find the ratio of the width to the length for each of the following rectangles.

Rectangle	Width (units)	Length (units)	
A	4	3	4 : 3
B	4	6	4 : 6
C	12	9	12 : 9
D	6	9	6 : 9
E	3	4	3 : 4

1. Which rectangles are similar? Explain. A and C, B and D; Sample answer: The ratio of the width to the length for Rectangles A and C is 4 : 3 and the ratio of the width to the length for Rectangles B and D is 2 : 3.

2. The ratio of the length to the width for a rectangle is 3 : 5. What are possible dimensions of a rectangle that is similar to this one? Sample answer: width 6 in., length: 10 in.

Apply Fundraising

Objective

Students will come up with their own strategy to solve an application problem involving fundraising.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Do the amounts of the other ingredients matter in this problem?
- How many servings of granola do you expect to sell?
- How many cups of granola are included in each serving?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Fundraising

The students at Lake Meadow Middle School will sell bags of honey granola for a fundraising event. The table shows a recipe that makes 6 cups of granola. The students will place 3 cups of granola in each bag. If forty people are expected to buy one bag of granola each, how many cups of rolled oats do they need?

Honey Granola
4 cups rolled oats
1 cup chopped almonds
$\frac{2}{3}$ cup honey
1 cup coconut oil
$\frac{1}{2}$ teaspoon salt
1 tablespoon ground cinnamon
1 teaspoon vanilla extract

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

Show your work here

80 cups of rolled oats; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
How can you solve this problem another way?

See students' responses.

Interactive Presentation

Apply, Fundraising

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The ingredients needed to make two servings of a fruit smoothie are shown in the table. Suppose you have 12 cups of frozen strawberries. If you use the entire amount, how many cups of plain yogurt do you need to maintain the same ratio? How many servings will this make?

Ingredient	Cups
Plain Y ogurt	2
Fruit Juice	1
Frozen Strawberries	3

Show your work here

8 cups of plain yogurt; 8 servings

Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that shows your understanding of ratios. Include examples of each of the following in your graphic organizer.

- bar diagrams
- words
- ratio notation
- part-to-whole ratios
- part-to-part ratios

Record your observations here

See students' graphic organizers.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you describe how two quantities are related? In this lesson, students learned how to compare two quantities by using a ratio. Encourage them to discuss with a partner why different kinds of ratios (part-to-part and part-to-whole) might be used in different situations.

Exit Ticket

Refer to the Exit Ticket slide. Look around your classroom. Write two ratios, one part-to-whole and one part-to-part, that compare the quantities of two objects or people. Sample answer: The ratio 12 : 26 represents the part-to-whole ratio of boys to total students in my class. The ratio 12 : 14 represents the part-to-part ratio of boys to girls in my class.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 10–13
- Extension: The Golden Ratio
- ALEKS Ratios and Unit Rates

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8, 11, 12
- Extension: The Golden Ratio
- Personal Tutor
- Extra Examples 1–3
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	determine whether or not ratio relationships are maintained	1, 2
2	use part-to-whole ratios to solve problems	3, 4
2	use part-to-part ratios to solve problems	5, 6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems involving ratios	8, 9
3	higher-order and critical thinking skills	10–14

Common Misconceptions

When writing a ratio that uses the whole group as the second quantity, some students will fail to include the first quantity in the total. In Exercise 3, students are asked for the ratio of the number of cups of orange juice to the total number of cups of juice. The total number of cups of juice is $4 + 1 + 2 + 2 = 9$, but if students do not include the cups of orange juice (2) in their total, then they will think the ratio is $\frac{2}{7}$ instead of $\frac{2}{9}$. Encourage students to examine each ratio they write and be able to interpret it within the context of the problem.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. In Suri's coin purse, she has 6 dimes and 4 quarters. Martha has 5 dimes and 3 quarters. Suri thinks that the ratio of dimes to quarters in both purses is the same because they each have 2 more quarters than dimes. Is the same ratio of dimes to quarters maintained? Justify your response. (Example 1)

no; Sample answer: Suri's ratio is 6 : 4 and Martha's is 5 : 3.

2. In a trivia game, Levi answered 8 questions correctly out of 10 turns in the game. He then answered the next three questions correctly. He reasoned that because he added 3 to both the total questions and his correct responses, that the ratio of correct answers to total questions remained the same. Is he correct? Justify your response. (Example 1)

no; Sample answer: His original ratio was 8 : 10 and his new ratio is 11 : 13. The ratios are not the same.

3. Riley needs to make fruit punch for the family reunion. One batch of punch has the ingredients shown. If the punch bowl holds 27 cups, how many cups of orange juice will she need to keep the ratio in a full punch bowl the same? (Example 2)

Item	Cups
Cranberry Juice	4
Lemon Lime Soda	1
Orange Juice	2
Pineapple Juice	2

6 cups

4. A small fruit basket contains the fruits shown. A large basket has the same ratio of fruits as the small basket. If the large basket has 42 total pieces of fruit, how many are pears? (Example 2)

Type of Fruit	Amount
Apple	6
Orange	5
Pear	3

9 pieces

5. Mrs. Santiago is buying doughnuts for her office. Each box contains 6 glazed, 4 cream filled, and 2 chocolate flavored doughnuts. If there were 20 total cream filled doughnuts, how many chocolate doughnuts did she buy? (Example 3)

10 chocolate doughnuts

6. A small batch of trail mix contains 2 cups of raisins, 2 cups of peanuts, 1 cup of sunflower seeds, and 1 cup of chocolate coated candies. A large batch has the same ratio of ingredients as a small batch. If the large batch has 8 cups of peanuts, how many cups of sunflower seeds are in a large batch? (Example 3) **4 cups**

Test Practice

7. Open Response A football coach needs to divide 48 players into two groups. He wants the ratio of players in Group 1 to players in Group 2 to be 1 to 3. How many players will be in Group 2?

36 players

Apply *indicates multi-step problem

*8. T o make a homemade all-purpose household cleaner , you can mix the ingredients shown in the table. Samuel has 1 cup of rubbing alcohol and will use the entire amount. He plans to store the cleaning solution in containers that each hold a maximum of 6 cups. How many containers does he need? Write an argument to defend your solution.

All-Purpose Cleaner
1 cup vinegar
$\frac{1}{2}$ cup rubbing alcohol
1 gallon water (16 cups)

6 containers; Sample answer: The total solution will include 2 cups vinegar, 1 cup rubbing alcohol, and 32 cups water, which is a total of 35 cups. Five containers will hold 5 × 6, or 30 cups of solution. He will need a sixth container to hold the remaining 5 cups of solution.

*9. The table shows the ingredients needed to make one batch of homemade slime. Dodi has 2 cups of liquid starch and will use the entire amount. She plans to store the slime in containers that each hold a maximum of 6 fluid ounces. How many containers will she need? Write an argument to defend your solution. *(Hint: 2 cups = 16 fluid ounces)*

Ingredient	Amount (fl oz)
Glue	4
Liquid Starch	4
Water	4

8 containers; Sample answer: She has 2 cups, or 16 fluid ounces, of liquid starch. She will make 16 ÷ 4, or 4 batches of slime. Each batch makes 4 × 3, or 12 fluid ounces, so she will make a total of 48 fluid ounces of slime. If each container holds 6 fluid ounces, she needs 48 ÷ 6, or 8 containers.

Higher-Order Thinking Problems

10. MP **Find the Error** The ratio of quarts of white paint to red paint is 3 : 4. A student says that to maintain the same ratio, he will need 7 quarts of white paint if he has 8 quarts of red paint, because originally there was one more quart of red paint than white paint. Find the student's mistake and correct it.
Sample answer: The student misinterpreted the ratio of white paint to red paint as an additive comparison. He should think of the ratio 3 : 4, meaning that for every 3 quarts of white paint, there are 4 quarts of red paint. So, for 8 quarts of red paint, there should be 6 quarts of white paint.

11. MP **Justify Conclusions** Rowan found that 4 out of 28 students in her class bike to school. What is the ratio of students that bike to school to the number of students that do not bike to school? Write an argument to defend your solution.
4 : 24; Sample answer: If 4 students bike to school, then 28 — 4 or 24 students do not bike to school. The ratio is 4 : 24.

12. **Create** Write your own real-world problem involving part-to-whole or part-to-part ratios. Trade problems with a partner and solve each other's problem. Discuss with your partner how your knowledge of ratios helped you solve each problem.
See students' responses.

13. The ratio of the distance around a circle, the circumference, to the distance across a circle, the diameter , is represented by the Greek letter π. If the circumference of a circle is 6.28 inches and the diameter of the same circle is 2 inches, what is the approximate value of π to two decimal places?
$\frac{3.14}{1}$ **or 3. 14**

12 **Module 1** • Ratios and Rates

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students will construct an argument to explain why the student incorrectly determined the number of quarts of white paint.

In Exercise 11, students will find the ratio of the number of students that bike to school to the number of students that do not bike to school and will justify their response by presenting a reasoned defense.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 8–9 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 9 might be, "What is the total number of fluid ounces needed?"

Make sense of the problem.
Use with Exercise 10 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student added 4 quarts to both the red and white paint.

Learn Equivalent Ratios and Ratio Tables

Objective

Students will understand what it means for two ratios to be equivalent and how a ratio table can be used to display and find equivalent ratios.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others, 5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 3, encourage them to make a plausible argument for why it might be more useful to use a ratio table instead of a bar diagram.

Teaching Notes

SLIDE 1

Present students with the ratio relationship *for every two cups of Greek yogurt in a pizza dough recipe, there are three cups of self-rising flour*. Ask them how the ratio table displays this ratio relationship. You may wish to have them identify the three ratios that are displayed by the bar diagrams: 2 : 3, 4 : 6, and 6 : 9. Point out that these ratios are equivalent because they represent the same ratio relationship. Ask students how they can find the ratio 6 : 9 if they know the ratio 2 : 3. Students should note that they can multiply the flour quantity by 3 and the yogurt quantity by3. This concept is known as *scaling*. You may wish to ask students to use scaling to generate other ratios that represent this relationship.

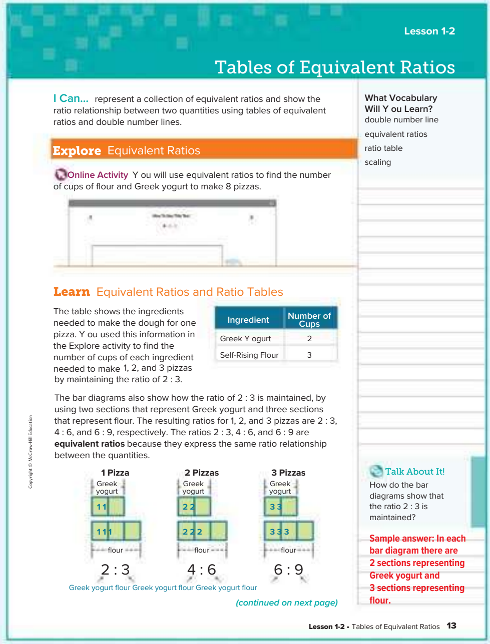
Talk About It!

SLIDE 1

Mathematical Discourse

How do the bar diagrams show that the ratio 2 : 3 is maintained? Sample answer: In each bar diagram there are 2 sections representing Greek yogurt and 3 sections representing flour.

(continued on next page)



Lesson 1-2

Tables of Equivalent Ratios

I Can... represent a collection of equivalent ratios and show the ratio relationship between two quantities using tables of equivalent ratios and double number lines.

What Vocabulary Will You Learn?
double number line
equivalent ratios
ratio table
scaling

Explore Equivalent Ratios

Online Activity You will use equivalent ratios to find the number of cups of flour and Greek yogurt to make 8 pizzas.

Learn Equivalent Ratios and Ratio Tables

The table shows the ingredients needed to make the dough for one pizza. You used this information in the Explore activity to find the number of cups of each ingredient needed to make 1, 2, and 3 pizzas by maintaining the ratio of 2 : 3.

Ingredient	Number of Cups
Greek Yogurt	2
Self-Rising Flour	3

The bar diagrams also show how the ratio of 2 : 3 is maintained, by using two sections that represent Greek yogurt and three sections that represent flour. The resulting ratios for 1, 2, and 3 pizzas are 2 : 3, 4 : 6, and 6 : 9, respectively. The ratios 2 : 3, 4 : 6, and 6 : 9 are **equivalent ratios** because they express the same ratio relationship between the quantities.

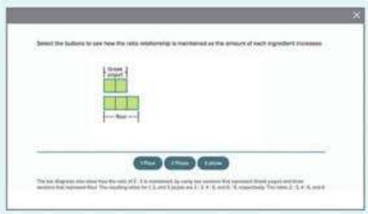
Talk About It!
How do the bar diagrams show that the ratio 2 : 3 is maintained?

Sample answer: In each bar diagram there are 2 sections representing Greek yogurt and 3 sections representing flour.

(continued on next page)

Lesson 1-2 • Tables of Equivalent Ratios 13

Interactive Presentation



Learn, Equivalent Ratios and Ratio Tables, Slide 1 of 3

CLICK

On Slide 1, students select the buttons to show the equivalent ratios.

Lesson 1-2

Tables of Equivalent Ratios

6.RP.A.3

LESSON GOAL

Students will use tables to find equivalent ratios.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Equivalent Ratios

Learn: Equivalent Ratios and Ratio Tables

Example 1: Scale Forward to Find Equivalent Ratios

Example 2: Scale Backward to Find Equivalent Ratios

Example 3: Scale in Both Directions

Example 4: Use a Double Number Line to Find Equivalent Ratios

Apply: Packaging

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

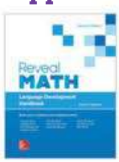
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 2 of the *Language Development Handbook* to help your students build mathematical language related to tables of equivalent ratios.



ELL You can use the tips and suggestions on page T2 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by solving problems by writing ratios to compare quantities.

Standards for Mathematical Content: **6.RP .A.3, 6.RP.A.3.A**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP7, MP8**

Coherence

Vertical Alignment

Previous

Students understood the concept of a ratio.
6.RP.A.1

Now

Students use tables to find equivalent ratios.
6.RP.A.3, 6.RP.A.3.A

Next

Students will use graphs to represent ratio relationships.
6.RP.A.3, 6.RP.A.3.A

Rigor

The Three Pillars of Rigor

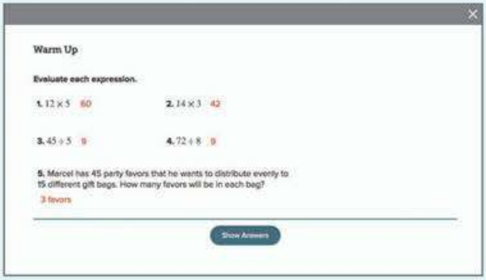
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their *understanding* of ratios as they explore ratio equivalence using ratio tables. They use the tables to build *fluency* with finding equivalent ratios by scaling forward or backward to find the desired ratio. They *apply* their understanding of equivalent ratios to solve real-world problems.

Mathematical Background

Equivalent ratios express the same ratio relationship between quantities. You can organize a collection of equivalent ratios into a table, called a *ratio table*. You can use tables and *scaling*, which is the process of multiplying or dividing each quantity in a ratio by the same number in order to generate equivalent ratios. Sometimes, it is beneficial to scale forward to find a desired equivalent ratio. Other times, it is beneficial to scale backward, and sometimes, you may need to scale in both directions.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- multiplying and dividing whole numbers (Exercises 1–5)

Answers

1. 60
2. 42
3. 9
4. 9
5. 3 favors

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about adjusting recipes in order to serve different numbers of people.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Based on what you know about number lines, for what do you think a *double number line* might be used? Sample answer: A double number line could be used to compare quantities in a ratio, similar to a bar diagram.
- What do you know about *equivalent fractions*? How could this help you infer what *equivalent ratios* are? Sample answer: Equivalent fractions look different but represent the same part of a whole. Equivalent ratios may look different but express the same relationship between two quantities.
- What is a *table* used for in mathematics? What can you infer about a *ratio table*? Sample answer: A table is used to organize and display information. A ratio table may organize information related to a specific ratio.
- What are some real-world examples where you might have heard the term *scaling* before? Sample answer: scaling the side of a mountain, weighing a specific weight

Explore Equivalent Ratios

Objective

Students will use tools to explore equivalent ratios.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the drag and drop activity to help model the number of cups of self-rising flour and yogurt needed to make 1, 2, and 3 pizzas. Students will generalize what they learned and use their observations to determine how to calculate quantities of ingredients for an additional number of pizzas. Students should be familiar with equivalence and the term *equivalent* from prior grades. In this activity, they will extend their understanding of equivalence to apply it to ratios.

Inquiry Question

How can you use a model to find equivalent ratios? Sample answer: I can use models such as counters or drawings to represent the different values in the ratio and then use multiple sets of these models to find equivalent amounts.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Record your results in a table. Describe to a classmate how you modeled the number of cups of each ingredient for different numbers of pizzas. Sample answer: I placed 2 of the metal cups and 3 of the glass cups in a group to represent one pizza. Then I added a second group of 2 metal cups and 3 glass cups to represent 2 pizzas. I added a third group to represent 3 pizzas.

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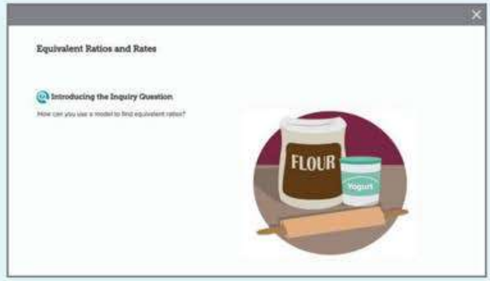
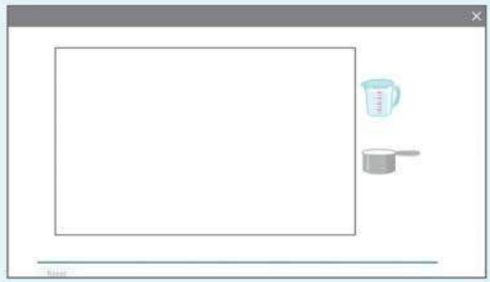
Interactive Presentation



Explore, Slide 1 of 5

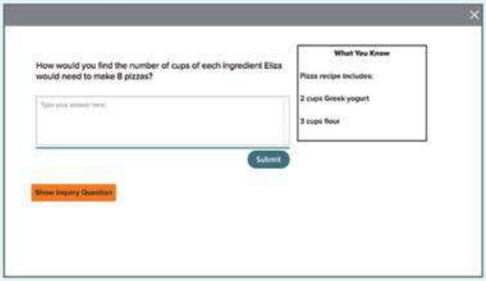


Explore, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag objects to model the ingredients needed to make 1, 2, and 3 pizzas.

Interactive Presentation



Explore, Slide 4 of 5

TYPE

On Slide 4, students explain how to find the amount of each ingredient needed to make 8 pizzas.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Equivalent Ratios *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use the drag and drop tool to model the simplified problem of making one pizza before considering the ingredients for two or more pizzas.

8 Look For and Express Regularity in Repeated Reasoning Encourage students to notice if any calculations are repeated as they complete the Explore.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

The ratios of cups of yogurt to cups of flour are *equivalent ratios*. Describe what it means for two ratios to be equivalent ratios.

Sample answer: Equivalent ratios use different numbers to describe the same ratio relationships between two quantities.

Your Notes

A table of equivalent ratios, or **ratio table**, is a collection of equivalent ratios that are organized in a table. Each column consists of a pair of quantities that have the same ratio as the pairs of quantities in the other columns.

In the ratio table shown, the ratios 2 : 3, 4 : 6, and 6 : 9 are all equivalent.

Greek Yogurt (c)	2	4	6
Flour (c)	3	6	9

Ratio tables show both an additive structure and a multiplicative structure.

+2 +2

Greek Yogurt (c)	2	4	6
Flour (c)	3	6	9

+3 +3

Add 2 to the cups of yogurt for each new column. Add 3 to the cups of flour for each new column.

×3 ×2

Greek Yogurt (c)	2	4	6
Flour (c)	3	6	9

×2 ×3

Multiply each of the original quantities by the same number to obtain the values in each of the other columns.

Talk About It!
Why might a ratio table be more advantageous to use than a bar diagram when finding the quantity of each ingredient needed to make 5 pizzas?

Sample answer: A ratio table can easily be extended to find equivalent ratios. Drawing bar diagrams can become cumbersome with larger values.

The process of multiplying each quantity in a ratio by the same number to obtain equivalent ratios is called **scaling**.

You can use scaling to extend the ratio table to find the number of cups of each ingredient needed to make additional pizzas. By doing so, you find more equivalent ratios.

×5 ×4

Greek Yogurt (c)	2	4	6	8	10
Flour (c)	3	6	9	12	15

×4 ×5

Continue the pattern by multiplying each of the original quantities by the same number to obtain the values in the other columns.

To make four pizzas, you need 8 cups of Greek yogurt and 12 cups of flour. To make five pizzas, you need 10 cups of Greek yogurt and 15 cups of flour.

The ratios 8 : 12 and 10 : 15 are equivalent to 2 : 3, 4 : 6, and 6 : 9.

Interactive Presentation

Learn, Equivalent Ratios and Ratio Tables, Slide 2 of 3

CLICK

On Slide 2, students click to reveal the additive and multiplicative structures of the ratio table.

Learn Equivalent Ratios and Ratio Tables *(continued)*

Teaching Notes

SLIDE 2

Present students with the ratio table showing the relationship between the number of cups of Greek yogurt and the number of cups of flour in the pizza dough recipe. You may wish to have a student volunteer reveal how ratio tables show both an additive structure and multiplicative structure. Encourage students to attend to the differences in structures.

SLIDE 3

After revealing the extended ratio table, you may wish to have students continue finding more equivalent ratios. Then ask students to find the cups of Greek yogurt and flour needed to make 13 pizzas and 18 pizzas.

Talk About It!

SLIDE 3

Mathematical Discourse

Why might a ratio table be more advantageous to use than a bar diagram when finding the quantity of each ingredient needed to make 5 pizzas?

Sample answer: A ratio table can easily be extended to find equivalent ratios. Drawing bar diagrams can become cumbersome with larger values.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of ratio tables, ask students to identify several other flour and yogurt ratio relationships that could appear in the table presented in the Learn, if it was extended.

Greek Yogurt	2	4	6
Self-rising flour	3	6	9

Sample answers: 8 cups of yogurt and 12 cups of flour, 10 cups of yogurt and 15 cups of flour, 12 cups of yogurt and 18 cups of flour

Example 1 Scale Forward to Find Equivalent Ratios

Objective

Students will scale forward to find equivalent ratios.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively, 5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the quantities given in the question and to understand how a ratio table with an additive structure can be used to help them solve the problem. Encourage them to see how either representation can be used as a tool to solve this problem.

Questions for Mathematical Discourse

SLIDE 2

AL How many drops of yellow correspond with 2 cups of icing? 6 drops

AL How many cups of icing correspond with the unknown value you are trying to find? 8 cups

OL Do you expect to need more than or less than 6 drops of yellow food coloring? Explain. more than 6 drops; Sample answer: 8 cups of icing is more than 2 cups, so the number of drops used for 8 cups of icing is more than the 6 drops used for 2 cups of icing.

BL Based on the equivalent ratio, how many cups of white icing should be mixed with 60 drops of yellow food coloring to make yellow icing? 20 cups

SLIDE 3

AL What number do we scale (multiply) 2 by to obtain 8? What does that tell us? 4; This is the number by which we need to multiply 6 drops of yellow.

OL Explain how you know that the number of drops of yellow will be greater than 18. Sample answer: $2 \times 3 = 6$ and $6 < 8$; The number by which I would multiply 6 drops of yellow must be greater than 3, which would yield a product greater than 18.

BL How many cups of icing are needed if 84 drops of yellow food coloring are used? Explain. 28; Sample answer: Multiply 6 by 14 to obtain 84 drops. Multiply 2 cups of icing by 14 to obtain 28 cups.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 1 Scale Forward to Find Equivalent Ratios

To make yellow icing, Amida mixes 6 drops of yellow food coloring with 2 cups of white icing.

How many drops of yellow food coloring should Amida mix with 8 cups of white icing to get the same shade of yellow?

Step 1 Create a ratio table with the given information.

For every 6 drops of yellow food coloring, there are 2 cups of icing. The unknown is the number of drops of yellow needed to mix with 8 cups of icing.

Drops of Yellow	6	?
Cups of Icing	2	8

Step 2 Scale forward to find how many drops of yellow Amida needs to mix with 8 cups of icing.

Drops of Yellow	6	24
Cups of Icing	2	8

×4 ×4

Because 2 × 4 = 8, multiply 6 by 4 to obtain 24.

The ratios 6 : 2 and 24 : 8 are equivalent ratios.

So, Amida should mix 24 drops of yellow food coloring with 8 cups of white icing to get the same shade of yellow.

Think About It! Should Amida add less than, more than, or the same number of drops, 6, of yellow food coloring to mix with the 8 cups of icing? Why?

more than; Sample answer: 8 cups > 2 cups, so she should add more than 6 drops.

Talk About It! How you can use a ratio table that shows an additive structure to solve this problem? Which structure, additive or multiplicative structure is more advantageous to use in this case? Explain.

Sample answer: Set up a table with the same row labels. The first column will be 6 drops of yellow for 2 cups of icing. Add 6 drops of yellow and 2 cups of icing until the number of drops of food coloring is found for 8 cups of icing. Using a multiplicative structure requires fewer operations than using an additive structure.

Check

In a batch of trail mix, there are 3 tablespoons of peanuts for every 2 tablespoons of sunflower seeds. How many tablespoons of sunflowers seeds are needed if you have 18 tablespoons of peanuts?

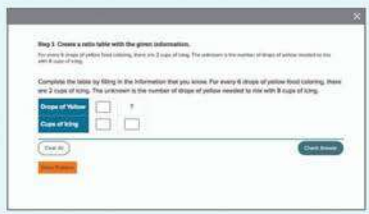
Show your work here

12 tablespoons

Go Online You can complete an Extra Example online.

Lesson 1-2 • Tables of Equivalent Ratios 15

Interactive Presentation



Example 1, Scale Forward to Find Equivalent Ratios, Slide 2 of 5

TYPE On Slide 2, students complete a table to show the relationship between the yellow food coloring and the cups of icing.

TYPE On Slide 3, students find the equivalent ratio.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Think About It!

How do you know that you cannot scale forward to solve this problem?

Sample answer: Because Aiko has less paint than in the original ratio.

Talk About It!

Suppose Aiko said that since she has half of the amount of red paint that Akeno has, she can mix that with half of the amount of yellow paint that Akeno has. Is she correct? Explain.

yes; Sample answer: Dividing by 2 is the same as finding half of an amount.

Example 2 Scale Backward to Find Equivalent Ratios

Akeno mixes three sample containers of yellow paint with four sample containers of red paint to create his favorite shade of orange paint. His little sister Aiko wants to create the same shade of orange paint, but she only has two sample containers of red paint.

What should Aiko do to create the same shade of orange paint?

Step 1 Create a ratio table with the given information.

For every 3 containers of yellow paint, there are 4 containers of red paint. The unknown is the amount of yellow paint needed to mix with 2 containers of red paint.

Yellow Paint (containers)	?	3
Red Paint (containers)	2	4

Step 2 Scale backward to find the equivalent ratio.

Yellow Paint (containers)	1.5	3
Red Paint (containers)	2	4

÷2 ÷2

Because 4 ÷ 2 = 2, divide 3 by 2 to obtain 1.5.

The ratios 1.5 to 2 and 3 to 4 are equivalent.

So, Aiko should mix 1.5 containers of yellow paint with 2 containers of red paint to create the same shade of orange paint.

Check

To make three loaves of banana bread, you need 9 bananas. How many bananas are needed to make one loaf of banana bread?

Show your work here

3 bananas

Go Online You can complete an Extra Example online.

16 Module 1 • Ratios and Rates

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Interactive Presentation

Example 2, Scale Backward to Find Equivalent Ratios, Slide 2 of 5

TYPE

On Slide 2, students complete the table using the given information.

CLICK

On Slide 3, students move through the slides to use a ratio table to scale backward to find equivalent ratios.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Scale Backward to Find Equivalent Ratios

Objective

Students will scale backward to find equivalent ratios.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities given in the problem to understand the relationship between the amount of yellow paint and the amount of red paint.

As students discuss the *Talk About It!* question on Slide 4, encourage them to explain why or why not Akeno is correct.

5 Use Appropriate Tools Strategically Students should understand how a ratio table can be a tool that can help them find equivalent ratios and solve real-world problems.

Questions for Mathematical Discourse

SLIDE 2

AL How many containers of yellow paint correspond with 4 containers of red paint? 3

OL How do you know that the ratio of 1.5 to 2 is equivalent to the ratio of 3 to 4? Sample answer: I can draw a bar diagram with 3 equal-size sections to represent yellow paint and 4 equal-size sections to represent red paint. If each section represents 0.5 containers, then there are 3 × 0.5, or 1.5 sections of yellow paint and 4 × 0.5, or 2 sections of red paint.

BL Compare and contrast scaling forward and scaling backward to find equivalent ratios. Sample answer: Scaling forward uses multiplication of whole numbers, while scaling backward uses division of whole numbers. Both are used to find equivalent ratios in ratio tables.

SLIDE 3

AL What is the operation that is performed in the ratio table, as you move through the slides? Why does that operation make sense? division; Sample answer: In order to find the number of containers of yellow paint needed for 2 containers of red paint, I can divide.

OL Can you subtract 2 from each quantity and still maintain the same ratio? Explain your reasoning. no; Sample answer: 3 − 2 = 1. The ratio 1 : 2 is not equivalent to the ratio 3 : 4.

BL Can you use multiplication to scale backward? Explain. yes; Sample answer: Instead of dividing by 2, I can multiply by $\frac{1}{2}$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Scale in Both Directions

Objective

Students will scale in both directions to find equivalent ratios.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of others As students discuss the *Talk About It!* question on Slide 4, they should be able to construct an argument to explain why scaling back was helpful.

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities given in the problem and to understand they can use a variety of scaling approaches (forward and backward) to solve the problem.

Questions for Mathematical Discourse

SLIDE 2

AL How many fluid ounces of fruit punch are needed to make one batch? How many scoops of ice cream are needed to make one batch? 9 fl oz; 6 scoops

OL Is the ratio of fruit punch to scoops of ice cream a part-to-whole ratio or a part-to-part ratio? part-to-part ratio

BL Suppose Natasha wants to make 2.5 batches of punch. How much of each ingredient would she need? 22.5 fl oz of punch, 15 scoops of ice cream, 7.5 L of soda

SLIDE 3

AL Is it possible to use a whole number to scale backward from 9 to 6? Explain. no; Sample answer: 9 is not evenly divisible by 6.

OL By what number could you divide both 9 and 6 to scale back? 3

OL Why would scaling by 1 not be helpful in this problem? Sample answer: Dividing by 1 does not scale back because the quotient is the original number.

BL Could you scale backward using a decimal? Explain. yes; Sample answer: 9 divided by 1.5 is 6, so I could also divide 6 by 1.5 to get 4.

BL How many scoops of ice cream are needed if Natasha has 15 fluid ounces of fruit punch? 10 scoops

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Scale in Both Directions

Natasha made raspberry punch for a party by mixing 9 fluid ounces of fruit punch, 3 liters of soda, and 6 scoops of raspberry ice cream. Halfway through the party, the punch bowl was empty.

If Natasha only has 6 fluid ounces of fruit punch left, how much ice cream does she need to make another batch of punch?

Step 1 Create a ratio table with the given information.

For every 9 fluid ounces of fruit punch, there are 6 scoops of raspberry ice cream. The unknown is the amount of ice cream needed to mix with 6 fluid ounces of fruit punch.

Fruit Punch (fl oz)	6	9
Ice Cream (scoops)	?	6

There is no whole number by which you can multiply 6 to obtain a product of 9.

Step 2 Scale backward to find an equivalent ratio.

÷3

Fruit Punch (fl oz)	3	6	9
Ice Cream (scoops)	2	?	6

÷3

To scale back, you can divide both 9 and 6 by 3. This results in the equivalent ratio 3 : 2.

Step 3 Use the equivalent ratio you found to scale forward to find the desired equivalent ratio.

×2

Fruit Punch (fl oz)	3	6	9
Ice Cream (scoops)	2	4	6

×2

To scale forward, you can multiply both 3 and 2 by 2. This results in the equivalent ratio 6 : 4.

So, Natasha should mix 4 scoops of raspberry ice cream with the remaining 6 fluid ounces of fruit punch.

Think About It! To mix with the remaining amount of fruit punch, will the number of scoops of ice cream that Natasha needs be less than, more than, or equal to 6? Explain.

less than; Sample answer: 6 fl oz < 9 fl oz, so she will need less than 6 scoops of ice cream.

Talk About It! Why was scaling back to find the equivalent ratio 3 : 2 helpful in solving the problem?

Sample answer: There is no whole number that can be used to scale from 9 to 6.

Check

Refer to Example 3. How many liters of soda should Natasha mix with the 6 fluid ounces of fruit punch?

Show your work here

2 liters

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 3, Scale in Both Directions, Slide 3 of 5

CLICK

On Slide 2, students complete the table using the given information.

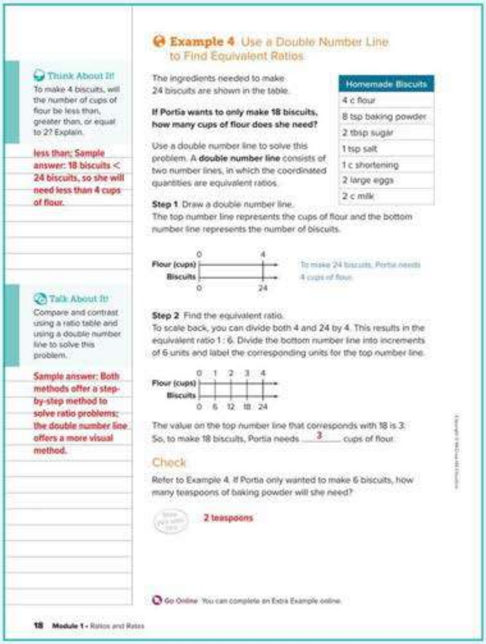
CLICK

On Slide 3, students move through the slides to see how to scale in both directions.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Think About It!
To make 4 biscuits, will the number of cups of flour be less than, greater than, or equal to 2? Explain.

less than; Sample answer: 18 biscuits < 24 biscuits, so she will need less than 4 cups of flour.

Talk About It!
Compare and contrast using a ratio table and using a double number line to solve this problem.

Sample answer: Both methods offer a step-by-step method to solve ratio problems; the double number line offers a more visual method.

Example 4 Use a Double Number Line to Find Equivalent Ratios

The ingredients needed to make 24 biscuits are shown in the table.

Homemade Biscuits
4 c flour
8 tsp baking powder
2 tbsp sugar
1 tsp salt
1 c shortening
2 large eggs
2 c milk

If Portia wants to only make 18 biscuits, how many cups of flour does she need?

Use a double number line to solve this problem. A **double number line** consists of two number lines, in which the coordinated quantities are equivalent ratios.

Step 1 Draw a double number line.
The top number line represents the cups of flour and the bottom number line represents the number of biscuits.

Step 2 Find the equivalent ratio.
To scale back, you can divide both 4 and 24 by 4. This results in the equivalent ratio 1 : 6. Divide the bottom number line into increments of 6 units and label the corresponding units for the top number line.

The value on the top number line that corresponds with 18 is 3. So, to make 18 biscuits, Portia needs 3 cups of flour.

Check

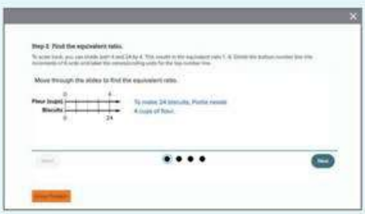
Refer to Example 4. If Portia only wanted to make 6 biscuits, how many teaspoons of baking powder will she need?

2 teaspoons

Go Online You can complete an Extra Example online.

18 Module 1 • Ratios and Rates

Interactive Presentation



Example 4, Use a Double Number Line to Find Equivalent Ratios, Slide 3 of 5

CLICK

On Slide 3, students move through the slides to determine the number of cups of flour.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Use a Double Number Line to Find Equivalent Ratios

Objective

Students will use a double number line to find equivalent ratios.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students should create a double number line so that the coordinated quantities on the number lines are equivalent ratios. They should be able to use the double number line to find the quantity that coordinates with 18 biscuits.

Questions for Mathematical Discourse

SLIDE 2

AL What does the top number line represent? the bottom number line? the cups of flour; the number of biscuits

OL What is the ratio of cups of flour to total biscuits? 4 : 24

OL What number on the top number line coordinates with 24 on the bottom number line? 4

BL What equivalent ratio will you use to find the number of cups of flour needed to make 18 biscuits? Explain. 1 : 6; Sample answer: I need to scale the ratio 4 : 24 backwards to the equivalent ratio 1 : 6 to find the cups of flour needed to make 18 biscuits.

SLIDE 3

AL Do you need to scale backwards or forwards to solve the problem? backwards

OL What increments should be used on the bottom number line? Explain. increments of 6; Sample answer: I need to include 18 on the bottom number line. Both 24 and 18 are divisible by 6, so I should label the number line in increments of 6.

OL What increments should be used on the top number line? Explain. increments of 1; Sample answer: I need the same number of tick marks on the top number line as are on the bottom number line, so they should be labeled by ones.

BL Suppose Portia only wanted to make 15 biscuits. How could you set up the double number line to find how many cups of flour she needs? Sample answer: The bottom number line should be labeled with increments of 3, and the top with halves. She would need $2\frac{1}{2}$ cups of flour.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Packaging

Objective

Students will come up with their own strategy to solve an application problem involving determining the cost of a bag of marbles.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How many total marbles are in the small bag?
- How can you write a ratio to help solve the problem?
- If you only know the number of green marbles, how can you use ratios to determine the number of blue, red, and orange marbles?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Packaging

A toy store sells assorted marbles, sold in small or large bags. The table shows the number of each color of marble in the small bag. The manager of the store wants to maintain the same ratio of each color of marble in the large bag as in the small bag. Each marble costs 20 cents. If the large bag contains 20 green marbles, how much does the large bag cost?

Color	Quantity
Blue	14
Red	12
Green	8
Orange	6

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$20; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How many red marbles are in the large bag? Provide a mathematical argument to support your answer.

30; Sample answer: There are 8 green marbles and 12 red marbles in a small bag. This means that for every 2 green marbles, there are 3 red marbles. Since the ratio of green to red is the same in the large bag, there will be 2 × 10 or 20 green marbles and 3 × 10 or 30 red marbles in the large bag.

Interactive Presentation

Apply, Packaging

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The table shows the number of slices of turkey and cheese in the regular Totally Turkey Sandwich at Dave's Deli.

Totally Turkey Sandwich (Regular)	
Ingredient	**Quantity**
Turkey Slices	3
Cheese Slices	2

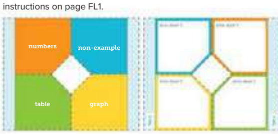
The ingredients are doubled in the large Totally Turkey Sandwich. On Wednesday, three times as many customers ordered the regular sandwich as the large sandwich. If 27 customers ordered the regular sandwich, how many total slices of turkey were used to make the sandwiches that day?

Show your work here

135 slices of turkey

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could describe how ratio tables are used to find equivalent ratios. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can you describe how two quantities are related?

In this lesson, students learned how to use tables and scaling to find equivalent ratios. Encourage them to discuss with a partner how equivalent ratios, such as $\frac{8 \text{ cats}}{12 \text{ dogs}}$ and $\frac{4 \text{ cats}}{6 \text{ dogs}}$, describe the same relationship between the two quantities.

Exit Ticket

Refer to the Exit Ticket slide. If a recipe serves 6 people and requires $1\frac{1}{2}$ cups of flour, how much flour do you need if you are serving 15 people? Write a mathematical argument that can be used to defend your solution. $3\frac{3}{4}$ cups of flour; Sample answer: If the recipe serves 6 people and requires $1\frac{1}{2}$ cups of flour, this means that I will need $\frac{1}{4}$ cup of flour for every person. To serve 15 people, I will need $3\frac{3}{4}$ cups of flour.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1–9 odd, 10–13
- ALEKS Ratios and Unit Rates

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–7, 9, 11, 13
- Personal Tutor
- Extra Examples 1–4
- ALEKS Equivalent Fractions

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- ArriveMATH Take Another Look
- ALEKS Equivalent Fractions

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	use scaling to find equivalent ratios	1–7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving equivalent ratios	9
3	higher-order and critical thinking skills	10–13

Common Misconception

When using scaling to determine whether or not two ratios are equivalent, students may have difficulty in determining whether they need to scale forward, backward, or both. Remind students that when using scaling, they should look at the relationship between the values. If you can multiply the original value by a whole number to obtain the new value, then you can scale forward. If you need to divide the original value by a whole number to obtain the new value, then you can scale backward. If you cannot multiply or divide the original value to obtain the final value, then you might need to multiply and then divide or vice versa in order to find the desired ratio.

Name Period Date

Practice

Go Online You can complete your homework online.

Use any strategy to solve each problem.

1. Jayden's snow cone machine makes 3 snow cones from 0.5 pound of ice. How many snow cones can be made with 5 pounds of ice? (Example 1)
   30 snow cones

2. Nyoko is having a pizza party. Two large pizzas serve 9 people. How many large pizzas should she order to serve 36 guests at the party? (Example 1)
   8 pizzas

3. The world record for the most number of speed skips in 60 seconds is 332 skips. If the record holder skipped at a constant ratio of seconds to skips, how many skips did she make in 15 seconds? (Example 2)
   83 skips

4. A recipe for homemade clay calls for 6 cups of water for every 12 cups of flour. How many cups of water are needed when 4 cups of flour are used? (Example 2)
   2 cups

5. Adrian decorated 16 cupcakes in 28 minutes. If he continues at this pace, how many minutes will it take him to decorate 56 cupcakes? (Example 3)
   98 minutes

6. A comic book store is having a sale. You can buy 20 comic books for $35. What is the cost of 8 comic books during the sale? (Example 3)
   $14

7. A certain store is selling packages of 10 pencils and 4 pens for back to school. The store manager wants to make a larger package in the same ratio. If the large package has 10 pens, how many pencils are in the large package? (Example 4)
   25 pencils

Test Practice

8. **Open Response** Ben made trail mix for his camping trip that contained 8 ounces of peanuts, 6 ounces of raisins, and 10 ounces of chocolate candies. He wants to make a larger batch for his next camping trip with 28 ounces of peanuts. How many ounces of raisins will he need?
   21 ounces of raisins

Apply *indicates multi-step problem

*9. The table shows the items in a family chicken taster meal at a restaurant. The restaurant wants to create a larger meal to accommodate larger groups of people. They also want to limit the number of chicken tenders to 15. If the ratio remains the same, how many biscuits are in the larger meal?

20 biscuits

Family Taster Meal
4 chicken sliders
6 chicken tenders
8 biscuits
1 pint of cole slaw

Higher-Order Thinking Problems

10. **Identify Structure** Generate a ratio table with at least two ratios equivalent to $\frac{\$10}{15 \text{ tickets}}$. Then describe how the table shows an additive structure and a multiplicative structure.

Sample answer:

Cost ($)	10	20	30
Number of Tickets	15	30	45

The table shows an additive structure by adding $10 to each entry in the first row and 15 tickets to each entry in the second row. It shows a multiplicative structure because you can multiply the values in the first column by 2 to find the number of tickets you can buy with $20 and then by 3 to find the number of tickets you can buy for $30.

11. **Justify Conclusions** There are 21 goats and 35 chickens on a farm. If 5 more goats and 5 more chickens are added, is the ratio of goats to chickens the same? Write an argument to defend your solution.

no; Sample answer: If 5 goats and 5 chickens are added, there would be 26 goats and 40 chickens on the farm, with a goat-to-chicken ratio of 13 : 20. The ratio of goats to chickens was originally 3 : 5 which is not equivalent to 13 : 20.

12. **Reason Inductively** A student said you can add the same number to both terms of a ratio to find an equivalent ratio. Is the student correct? Explain why or why not.

no; Sample answer: To find equivalent ratios, multiplication or division is used. Adding the same number changes the relationship between the two quantities.

13. **Create** Write and solve a real-world problem where you determine if two ratios are equivalent.

Sample answer: Seth's bouquet has 21 flowers with 15 roses. Keith's bouquet has 35 flowers with 25 roses. Are the ratios of roses to flowers the same? Y es, they both scale to 5 roses to 7 flowers.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 10, students can use the structure of a ratio table to generate equivalent ratios using additive and multiplicative reasoning.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students construct an argument to defend their response as to whether or not the ratio relationship was maintained.

2 Reason Abstractly and Quantitatively In Exercise 12, students reason with ratios to explain why or why not equivalent ratios can be found by adding the same value to both terms.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 9 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Interview a student.

Use with Exercise 11 Have pairs of students interview each other as they complete this problem. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 11 might be, "Without solving, do you think the ratios are the same?"

Learn Ratios as Ordered Pairs

Objective

Students will learn how to write ratios as ordered pairs and graph them on the coordinate plane.

Teaching Notes

SLIDE 1

You may wish to remind students how to extend a ratio table to write equivalent ratios. You may wish to also remind students about ordered pairs, a pair of numbers used to locate a point on the coordinate plane, before explaining that each pair of equivalent ratios can be expressed as an ordered pair. Encourage students to recognize the ordered pairs in the table. You may wish to have students write the ordered pairs as coordinates: (1, 3), (2, 6), (3, 9), and so on.

SLIDE 2

Students have graphed on the first quadrant of the coordinate plane in a previous grade. You may wish to remind students how to write the ratios in the table as ordered pairs and how to graph the ordered pairs.

Encourage students to study the structure of the graph representing the ratio relationship. Ask students what they notice. Some students may say the points seem to fall on an imaginary line that passes through the origin–note that this concept will be further developed in Grade 7. Encourage students to notice that each new point is 3 units up and 1 unit to the right of the previous point. Ask students how this relates to the ratio of olive oil to vinegar, 3 : 1. Students should notice that it is the same. This confirms the graph is the graph of a ratio relationship.

Talk About It!

SLIDE 3

Mathematical Discourse

Compare and contrast the ratio table and the graph. How do they both illustrate the same ratio relationship? How does the graph help you visualize the ratio relationship? Sample answer: The ratio table shows each equivalent ratio. The graph shows those ratios graphed on the coordinate plane. You can see that the graph is a ratio relationship because the points are in a straight line.

DIFFERENTIATE

Reteaching Activity AL

Students learned how to graph ordered pairs in the first quadrant of the coordinate plane in Grade 5. To review this, have them identify the x- and y-coordinates for the following ordered pairs and explain how to graph them on the coordinate plane.

(4, 7) x-coordinate: 4; y-coordinate: 7; Start at (0, 0). Move 4 units right and 7 units up.

(3, 1) x-coordinate: 3; y-coordinate: 1; Start at (0, 0). Move 3 units right and 1 unit up.

Lesson 1-3

Graphs of Equivalent Ratios

I Can... represent a collection of equivalent ratios as ordered pairs and graph the ratio relationship on the coordinate plane.

Learn Ratios as Ordered Pairs

You previously learned how to create a ratio table and extend it by finding equivalent ratios. You can also represent a ratio relationship by creating a table of ordered pairs and graphing the ordered pairs on the coordinate plane.

To make a simple salad dressing, you can use 3 cups of olive oil for every cup of vinegar. You can then add herbs, salt, and/or pepper for seasoning. This ratio relationship is shown in the table.

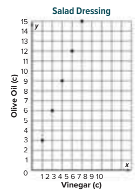
Vinegar (c), x	Olive Oil (c), y
1	3
2	6
3	9
4	12
5	15

Each pair of equivalent ratios can be expressed as an ordered pair. The x-coordinate represents the number of cups of vinegar. The y-coordinate represents the number of cups of olive oil.

Recall that to graph a point, start at the origin. Move right along the x-axis the number of units indicated by the x-coordinate. From that location, move up along the y-axis the number of units indicated by the y-coordinate. Place a dot at that location.

The graph illustrates the ratio relationship of the cups of olive oil to the cups of vinegar in the salad dressing.

What do you notice about the graphed points? You might notice that to travel from each point to the next point, you move up 3 units and to the right 1 unit. These are the same numbers in the ratio of 3 cups of olive oil for every 1 cup of vinegar.



Talk About It!
Compare and contrast the ratio table and the graph. How do they both illustrate the same ratio relationship? How does the graph help you visualize the ratio relationship?

Sample answer: The ratio table shows each equivalent ratio. The graph shows those ratios graphed on the coordinate plane. You can see that the graph is a ratio relationship because the points are in a straight line.

Interactive Presentation

Learn, Ratios as Ordered Pairs, Slide 1 of 3

eTOOLS

On Slide 2, students use the Coordinate Graphing eTool to graph the relationship on a coordinate plane.

Graphs of Equivalent Ratios

6.RP.A.3

LESSON GOAL

Students will use graphs to represent ratio relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Ratios as Ordered Pairs

Example 1: Graph Ratio Relationships

Example 2: Graph and Interpret Ratio Relationships

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 3 of the *Language Development Handbook* to help your students build mathematical language related to graphs of equivalent ratios.

ELL You can use the tips and suggestions on page T3 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by graphing tables of equivalent ratios.

Standards for Mathematical Content: **6.RP .A.3, 6.RP.A.3.A**

Standards for Mathematical Practice: **MP2, MP5, MP7**

Coherence

Vertical Alignment

Previous

Students used tables to find equivalent ratios.
6.RP.A.3, 6.RP.A.3.A

Now

Students use graphs to represent ratio relationships.
6.RP.A.3, 6.RP.A.3.A

Next

Students will use graphs and tables to compare ratio relationships.
6.RP.A.3, 6.RP.A.3.A

Rigor

The Three Pillars of Rigor

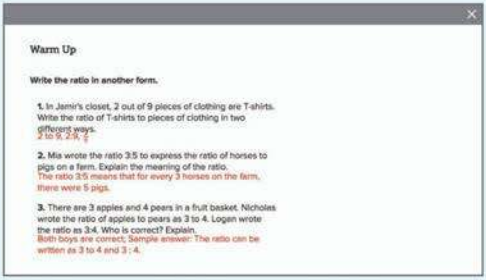
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to expand their *understanding* of ratio relationships by graphing ordered pairs that represent ratio relationships in the coordinate plane. They build *fluency* with graphing ordered pairs and *apply* their understanding of ratio relationships to solve real-world problems.

Mathematical Background

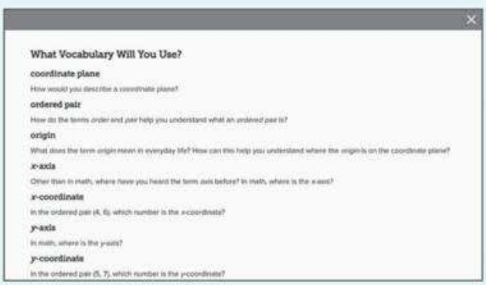
Equivalent ratios can be represented as ordered pairs and graphed on the coordinate plane. To graph a point, start at the origin. Move right along the x-axis the number of points indicated by the x-coordinate. From that location, move up along the y-axis the number of units indicated by the y-coordinate. Place a dot at that location. When a ratio relationship is graphed, the points fall along an imaginary line that passes through the origin. This concept will be further expanded on in Grade 7.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- understanding ratios (Exercises 1–3)

Answers

1. 2 to 9; 2 : 9
2. The ratio 3 : 5 means that for every 3 horses on the farm, there were 5 pigs.
3. Both boys are correct. The ratio can be written as 3 to 4 and 3 : 4.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about students participating in a read-a-thon.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- How would you describe a coordinate plane? A coordinate plane is a tool that can be used to visually display coordinates, which are points.
- How do the terms *order* and *pair* help you understand what an *ordered pair* is? Sample answer: An ordered pair represents the *pair* of *x*- and *y*-coordinates that are in a particular *order* (*x* first, followed by *y*). The ordered pair represents a point on the coordinate plane.
- What does the term *origin* mean in everyday life? How can this help you understand where the *origin* is on the coordinate plane? Sample answer: The origin of something means the start of something. On the coordinate plane, this is represented by the point (0, 0).

Your Notes

Think About It!
What is the ratio of charms to beads? Beads to charms?

1 : 6; 6 : 1

Example 1 Graph Ratio Relationships

Tamara is making charm bracelets for several friends. She uses 6 beads for every charm.

Generate the set of ordered pairs for the ratio relationship between the number of beads *y* and the number of charms *x* for a total of 1, 2, 3, and 4 charms. Then graph the relationship.

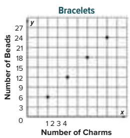
Part A Create a table of ordered pairs.

Let the *x*-coordinates represent the number of charms and the *y*-coordinates represent the number of beads.

Charms, *x*	Beads, *y*
1	6
2	12
3	18
4	24

Use scaling to complete the table to write the equivalent ratios for 2, 3, and 4 charms. The ordered pairs are (1, 6), (2, 12), (3, 18), and (4, 24).

Part B Graph the ordered pairs on the coordinate plane.



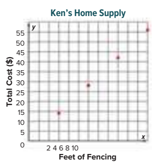
Talk About It!
What do you notice about the points on the graph?

Sample answer: They seem to fall on a line. Each new point is 6 units up from and 1 unit to the right of the previous point.

Check

Ken's Home Supply sells fencing that costs $14 for every 3 feet. Generate the set of ordered pairs for the ratio relationship between the cost *y* and the number of feet of fencing *x* for a total of 3, 6, 9, and 12 feet of fencing. Then graph the relationship.

(3, 14), (6, 28), (9, 42), (12, 56)



Go Online You can complete an Extra Example online.

24 Module 1 • Ratios and Rates

Example 1 Graph Ratio Relationships

Objective

Students will graph a ratio relationship on the coordinate plane.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities presented in the problem to generate the ordered pairs.

7 Look For and Make Use of Structure As students discuss the *Talk About It!* question on Slide 4, encourage them to study the structure of the graph as they respond to the question. Some students may notice the points fall on an imaginary line. Other students may notice that each new point is 6 units up and 1 unit to the right from the previous point.

Questions for Mathematical Discourse

SLIDE 2

AL What two quantities are being compared in the ratio? the number of charms to the number of beads for a bracelet

OL By what value can you multiply 6 to find the number of beads for 4 charms? 4

OL Suppose a classmate graphed the number of beads along the *x*-axis and the number of charms along the *y*-axis. Would this graph still represent the ratio of beads to charms? What would the graph look like? no; Sample answer: This graph would not represent the same ratio relationship between the two quantities. The graph would not be as steep, but the points would still fall on an imaginary line through the origin. Each new point would be 1 unit up and 6 units to the right from the previous point.

BL If Tamara made 5 bracelets, each with 5 charms, how many beads would she need? 150 beads

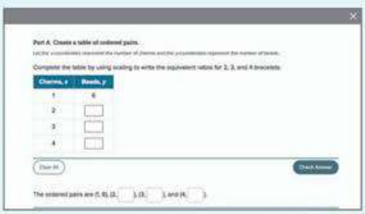
SLIDE 3

AL Explain how you would graph the ordered pair (1, 6) on the coordinate plane. Sample answer: Start at the origin. Move 1 unit to the right along the *x*-axis and 6 units up along the *y*-axis. Then graph the point.

OL What are the ordered pairs that need to be graphed on the coordinate plane? (1, 6), (2, 12), (3, 18), and (4, 24)

BL Can you use the graph to find the number of beads needed if Tamara has 6 charms? Explain. yes; Sample answer: I can extend the graph to include the value of 6 on the *x*-axis. Then I can draw a line through the points already on the graph to see what value corresponds with 6 on the *x*-axis.

Interactive Presentation



Example 1, Graph Ratio Relationships, Slide 2 of 5

CLICK

On Slide 2, students complete the ratio table.

eTOOLS

On Slide 3, students use the Coordinate Graphing eTool to graph the relationship on the coordinate plane.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Graph and Interpret Ratio Relationships

Objective

Students will graph tables of equivalent ratios and interpret the relationship between two quantities.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use the Coordinate Graphing eTool to graph the ordered pairs in order to describe the pattern they see.

Questions for Mathematical Discourse

SLIDE 2

AL How can you write the ordered pairs from the table? Sample answer: Let *x* represent the number of cups of flour and let *y* represent the number of cups of water.

OL For each batch of clay, are the ratios of water to flour equivalent? Explain. yes; Sample answer: They all have the same ratio, 2 to 4.

OL Suppose a classmate graphed the cups of water along the *x*-axis and the cups of flour along the *y*-axis. Would this graph still represent the ratio of water to flour? What would the graph look like? no; Sample answer: This graph would not represent the ratio of water to flour. The graph would not be as steep, but the points would still fall on an imaginary line through the origin. Each new point would be 4 units up and 2 units to the right from the previous point.

BL What are some other ordered pairs that could be plotted on the graph? Sample answers: (24, 12), (28, 14), (32, 16)

SLIDE 3

AL Explain how to graph the first ordered pair, (4, 2). Sample answer: Starting at the origin, move 4 units to the right, then 2 units up.

OL What are the ordered pairs that need to be graphed on the coordinate plane? (4, 2), (8, 4), (12, 6), (16, 8), and (20, 10).

BL Can you use the graph to determine how many batches of clay Sequoia could make if she has 10 cups of flour? Explain why or why not. no; Sample answer: I can use the graph to determine how many cups of water she would need for 10 cups of flour, but the graph does not directly show how many batches of clay she could make.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Graph and Interpret Ratio Relationships

To make one batch of homemade modeling clay that can be used in arts and crafts, Sequoia mixed the ingredients shown in the table.

Homemade Clay
4 cups flour
1 cups salt
2 cups water
food coloring

Graph the ratio relationship between the number of cups of water *y* and the number of cups of flour *x* for a total of 5 batches. Then describe the pattern in the relationship.

Part A Graph the ratio relationship.

Step 1 Generate a set of ordered pairs.

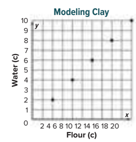
For every 4 cups of flour, there are 2 cups of water. Let the *x*-coordinates represent the number of cups of flour and the *y*-coordinates represent the number of cups of water.

Flour (c), *x*	Water (c), *y*	
4	2	1 batch
8	4	2 batches
12	6	3 batches
16	8	4 batches
20	10	5 batches

Use scaling to write the equivalent ratios for 1, 2, 3, 4, and 5 batches.

Step 2 Graph the relationship.

The *x*-coordinates increase from 4 to 20, so let each grid unit along the *x*-axis on the coordinate plane represent 2 units.



Part B Describe the pattern in the ratio relationship.

In the graph, the points appear to fall on a straight line. Each new point is 2 units up from and 4 units to the right of the previous point. This means that the number of cups of water increases by 2 cups as the number of cups of flour increases by 4 cups.

Think About It! How do you know that the relationship between flour and water is a ratio relationship?

Sample Answer: Each batch of clay will always have 4 cups of flour for every 2 cups of water.

Talk About It! Do you think that all ratio relationships will have graphs that appear to fall on a straight line? Why or why not?

yes; Sample answer: In a ratio relationship, you will also have the same relationship between the *x*- and *y*-values on the graph. This means that the points will always lie in a straight line.

Lesson 1-3 • Graphs of Equivalent Ratios 25

Interactive Presentation

Example 1, Graph Ratio Relationships, Slide 3 of 6

eTOOLS

On Slide 3, students use the Coordinate Graphing eTool to graph the relationship on the coordinate plane.

TYPE

On Slide 4, students identify a pattern in the graph.

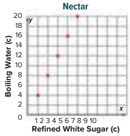
CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

T o make one batch of nectar to feed hummingbirds, Melanie added 4 cups of boiling water for every cup of refined white sugar.

Part A Graph the ratio relationship between cups of boiling water y and cups of refined white sugar x for a total of 1, 2, 3, 4, and 5 batches.

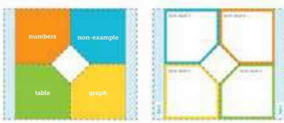


Part B Describe the pattern in the relationship.

Sample answer: In the graph, the points appear to fall on a straight line. Each new point is 4 units up from and 1 unit to the right of the previous point. This means that the cups of boiling water increase by 4 cups as the cups of sugar increase by 1 cup.

Go Online Y ou can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information in the tables and graphs sections about graphing ratios. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can you describe how two quantities are related?

In this lesson, students learned how to graph the relationship between two quantities, expressed as a ratio table, in the coordinate plane. Encourage them to discuss with a partner how a graph visually describes the relationship between two quantities. Have them compare and contrast using tables and graphs to represent ratio relationships.

Exit Ticket

Refer to the Exit Ticket slide. Who read the greatest number of pages per day? Predict how many pages each person will have read by the tenth day. Write a mathematical argument that can be used to defend your solution. Olivia; Sample answer: Olivia read 232 pages in 4 days or 58 pages per day. So, she will have read 58×10 or 580 pages after 10 days. Jackson read 260 pages in 5 days or 52 pages per day. So, he will have read 52×10 or 520 pages after 10 days.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1. 3, 5–8
- ALEKS® Ratios and Unit Rates

IF students score 66–89% on the Checks **THEN** assign: OL

- Practice, Exercises 1, 2, 6, 7
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Ordered Pairs

IF students score 65% or below on the Checks **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Ordered Pairs

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	graph tables of equivalent ratios and describe the relationship between two quantities	1, 2
2	extend concepts learned in class to apply them in new contexts	3, 4
3	higher-order and critical thinking skills	5–8

Common Misconception

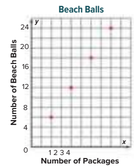
Some students may rush to complete a problem without carefully studying the scale on either or both axes of a graph. For example, in Exercise 1, students may incorrectly describe the pattern in the graph by saying that each point is 3 units up and 2 units to the right of the previous point. Be sure they study the structure of the graph carefully and attend to the precision of how the axes are labeled. In this exercise, each point is actually 6 units up from and 2 units to the right of the previous point, because the scale on the y-axis increases by 2.

Name ______ Period ______ Date ______

Practice

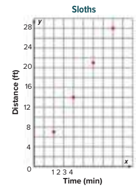
Go Online Y ou can complete your homework online.

1. Lulah is buying beach balls for her beach themed party. Each package contains 6 beach balls. Generate the set of ordered pairs for the ratio relationship between the number of beach balls y and the number of packages x for a total of 1, 2, 3, and 4 packages. Then graph the relationship on the coordinate plane and describe the pattern in the graph. (Examples 1 and 2)



(1, 6), (2, 12), (3, 18), (4, 24); Sample answer: The points appear to be in a straight line. Each point is 6 units up from and 1 unit to the right of the previous point. This means that the number of beach balls increases by 6 as the number of packages increases by 1.

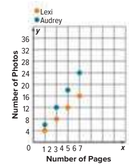
2. A sloth travels about 7 feet every minute. Generate the set of ordered pairs for the ratio relationship between the total distance traveled y and the number of minutes x for a total of 1, 2, 3, and 4 minutes. Then graph the relationship on the coordinate plane and describe the pattern in the graph. (Examples 1 and 2)



(1, 7), (2, 14), (3, 21), (4, 28); Sample answer: The points appear to be in a straight line. Each point is 7 units up from and 1 unit to the right of the previous point. This means that the distance travelled increases by 7 feet as the number of minutes increases by 1 minute.

3. Two friends are making scrapbooks. The number of photos Lexi and Audrey place on each page of their scrapbooks is shown in the graph. Describe the ratio relationship for each person.

Sample answer: The ratio of photos to pages for Lexi's scrapbook is 4 : 1. The ratio of photos to pages for Audrey's scrapbook is 6 : 1. Audrey uses more photos per page than Lexi.

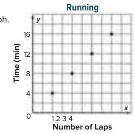


1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Test Practice

4. Multiselect Lacy is running laps around the track. The time in minutes and the number of laps ran are shown in the graph. Which of the following is true about the ratio relationship shown in the graph?

- ✓ Every 4 minutes, Lacy ran 1 lap.
- ☐ Lacy ran 8 laps in 2 minutes.
- ☐ It took Lacy 1 minute to run 4 laps.
- ✓ In 16 minutes, Lacy completed 4 laps.
- ✓ Based on the relationship, it would take Lacy 20 minutes to complete 5 laps.



Higher-Order Thinking Problems

5. Identify Structure There are 4 quarters for every one dollar and 10 dimes for every dollar. Without graphing, would the ratio of quarters to dollars or dimes to dollars appear to have a steeper line? Explain your reasoning.

dimes to dollars; Sample answer: The ratio of dimes to dollars is 10 : 1 and the ratio of quarters to dollars is 4 : 1. Since 10 is greater than 4, the ratio of dimes to dollars will have a steeper line.

6. What are the advantages of graphing when solving problems that involve ratios?

Sample answer: The graph allows you to see patterns, make predictions, and compare relationships more quickly than using another method.

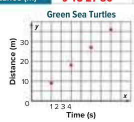
7. Reason Abstractly The table gives the number of beads needed to make bracelets of certain lengths. Suppose you graph the ordered pairs (bracelet length, number of beads) on the coordinate plane. Would the point (10.5, 42) make sense in this context? Explain.

Bracelet Length (in.)	7	8	9	10
Number of Beads	28	32	36	40

yes; Sample answer: A bracelet could have a length of 10.5 inches and 42 beads.

8. Multiple Relationships For every second, the average green sea turtle can swim 9 meters. Represent how far a green sea turtle can swim in 1, 2, 3 and 4 seconds in a table. Then graph the points on a coordinate plane.

Time (s)	1	2	3	4
Distance (m)	9	18	27	36



Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 5, students visualize the structures of the graphs of each ratio to determine which ratio's graph will appear to fall on a steeper line.

2 Reason Abstractly and Quantitatively In Exercise 6, students use reasoning to evaluate if an ordered pair would make sense in the given context.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Clearly explain your strategy.

Use with Exercise 8 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Learn Use Graphs to Compare Ratio Relationships

Objective

Students will understand how multiple ratio relationships can be compared by graphing them on the same coordinate plane.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to understand the benefit of graphing all three ratio relationships on the same coordinate plane, as opposed to three separate coordinate planes.

7 Look For and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to study the structure of the graph and reason about how reversing the order of the quantities in the ratio will alter the graph.

Teaching Notes

SLIDE 1

Students will understand how multiple ratio relationships can be compared by graphing them on the same coordinate plane. Have students begin by comparing the ratios of proteins to other ingredients for the three companies.

SLIDE 2

When graphing a ratio relationship, it is important to understand and maintain the order of the ratio. In this scenario, students are asked to compare the grams of protein to the cups of dog food. Specifically, they need to determine which company has the most protein in a cup of dog food. To graph this relationship, the number of grams of protein is represented by the y-coordinate, and the number of cups of dog food is represented by the x-coordinate. The company with the greatest ratio of grams of protein to cups of dog food will have the steepest line on the graph. You may wish to have students draw a dashed line through the points for each company to more clearly see which line is steepest.

Some students may instead graph the number of grams of protein along the x-axis and the number of cups of dog food along the y-axis. Be sure they understand that, while this is not incorrect, if they graph the quantities this way, the company with the *greatest* ratio of grams of protein to cups of dog food will actually have the *least* steepest line. Encourage them to understand how to read the graphs based on the way in which they graph the quantities.

Talk About It!

SLIDE 3

Mathematical Discourse

If the ratio compared the cups of dog food to the grams of protein, how would the graph change? Which line would be the steepest?

Sample answer: Grams of protein would be graphed on the x-axis and the cups of dog food would be graphed on the y-axis. Company C would have the steepest line.

Lesson 1-4

Compare Ratio Relationships

I Can... compare ratio relationships that are shown using different representations.

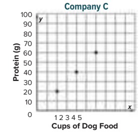
Learn Use Graphs to Compare Ratio Relationships

Ratios for ingredients in dog food vary among companies that manufacture it. Company A advertises 25 grams of protein for every cup of dog food. The relationship between protein and cups of dog food for two other companies is shown in the table and graph.

Company B

Dog Food (c), x	Protein (g), y
2	44
3	66
4	88

Company C



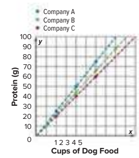
How can you compare the ratios of protein to cups of dog food for the three companies?

The ratios for each of the three companies is shown using a different representation. T o compare them, you can use the same representation for each, such as a graph.

The ratios for Company C are already graphed. For Company A, you can generate equivalent ratios to find the ordered pairs (1, 25), (2, 50), and (3, 75).

For Company B, the ordered pairs are (2, 44), (3, 66), and (4, 88).

Draw a dotted line through the points to determine which relationship has the steepest graph. The graph for Company A is the steepest, and the graph for Company B is steeper than the graph for Company C. This means that Company A has the greatest ratio of protein to cups of dog food.

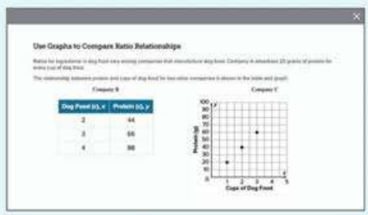


Talk About It!

If the ratio compared cups of dog food to protein, how would the graph change? Which line would be the steepest?

Sample answer: Protein would be graphed on the x-axis and cups of dog food would be graphed on the y-axis. Company C would have the steepest line.

Interactive Presentation



Learn, Use Graphs to Compare Ratio Relationships, Slide 1 of 3

Lesson 1-4

Compare Ratio Relationships

6.RP.A.3, 6.RP.A.3.A

LESSON GOAL

Students will use graphs and tables to compare ratio relationships.

1 LAUNCH

Launch the Lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Use Graphs to Compare Ratio Relationships
Example 1: Use Graphs to Compare Ratio Relationships
Learn: Use Tables to Compare Ratio Relationships
Example 2: Use Tables to Compare Ratio Relationships
Apply: Mixing Paint

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

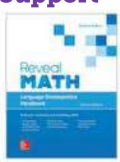
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 4 of the *Language Development Handbook* to help your students build mathematical language related to comparing ratio relationships.



ELL You can use the tips and suggestions on page T4 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by comparing ratios using graphs and ratio tables.
Standards for Mathematical Content: **6.RP .A.3, 6.RP.A.3.A**
Standards for Mathematical Practice: **MP1, MP3, MP4, MP5, MP7**

Coherence

Vertical Alignment

Previous
Students used graphs to represent ratio relationships.
6.RP.A.3, 6.RP.A.3.A

Now
Students use graphs and tables to compare ratio relationships.
6.RP.A.3, 6.RP.A.3.A

Next
Students solve real-world problems involving ratios.
6.RP.A.3

Rigor

The Three Pillars of Rigor

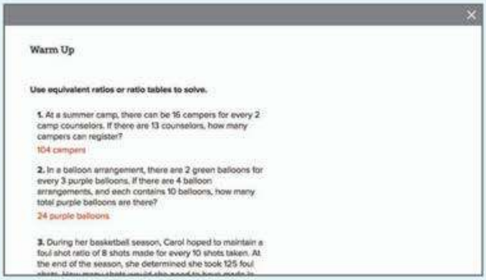
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of ratios. They begin to understand that ratio relationships can be represented in different forms, and that it can be easier to compare the ratio relationships when they are represented in the same form. They *apply* their understanding of ratios by comparing ratio relationships in real-world problems.

Mathematical Background

Ratio relationships can be described using words and represented in different forms, including tables and graphs. One way to compare two or more ratio relationships that are represented in different forms, is to represent them using the same form.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- using equivalent ratios or ratio tables to solve problems (Exercises 1–3)

Answers

1. 104 campers
2. 24 purple balloons
3. 100 foul shots made

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a student council comparing companies for a school fundraiser.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- When might it be helpful to compare *ratios*? Sample answer: It might be helpful to compare ratios when deciding on which company to use or which product to purchase.
- Since you can use a ratio table to organize a collection of equivalent ratios, how would using *ratio tables* help you compare several ratios? Sample answer: Using ratio tables could help me easily see many ratio relationships in a visually organized way.

Your Notes

Think About It!
Just by studying the table, which pizzeria, Slice of Pie or Paulo's Pizzeria, has more pepperonis on a 12-inch pizza?

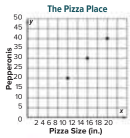
Paulo's Pizzeria

Example 1 Use Graphs to Compare Ratio Relationships

Paulo's Pizzeria advertises 24 pepperonis on every 12-inch pizza. The relationship of pepperonis to pizza size for two other pizzerias is shown in the table and graph.

Slice of Pie

Pizza Size (in.)	Pepperonis
10	15
12	18
14	21

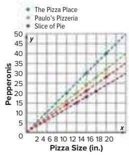


Which pizzeria advertises the greatest ratio of pepperonis to pizza size?

T o compare the three ratios, use the same representation for each, such as a graph. The ratios of pepperonis to pizza size for The Pizza Place are already graphed.

For Paulo's Pizzeria, use scaling to write the ordered pairs (8, 16), (10, 20), (12, 24), (14, 28), and (16, 32) to represent the ratio relationship.

For Slice of Pie, the ordered pairs are (10, 15), (12, 18), and (14, 21).



Draw dotted lines through the points. The graph for The Pizza Place is the steepest, and the graph for Paulo's Pizzeria is steeper than the graph for Slice of Pie.

This means that The Pizza Place has the greatest ratio of pepperonis to pizza size, in inches, followed by Paulo's Pizzeria, and then Slice of Pie.

Talk About It!
How many more pepperonis would be on an 18-inch pizza from The Pizza Place than on an 18-inch pizza from Paulo's Pizzeria? Justify your response.

9 pepperonis; Sample answer: By extending the relationships on the graph, you can see that The Pizza Place would have 45 pepperonis on an 18-inch pizza and Paulo's Pizzeria would have 36 pepperonis on an 18-inch pizza.

Check

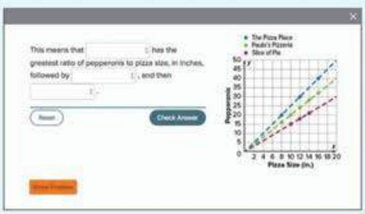
Refer to Example 1. A fourth pizzeria, Pizza Café, advertises 14 pepperonis for every 8-inch pizza. Graph the ratio relationship for Pizza Café on the graph above. Which pizzeria, Pizza Café or Slice of Pie, advertises the greater ratio of pepperonis to pizza size? Justify your response.

Show your work here

Pizza Café; Sample answer: The graph for Pizza Café is steeper than the graph for Slice of Pie.

Go Online Y ou can complete an Extra Example online.

Interactive Presentation



Example 1, Use Graphs to Compare Ratio Relationships, Slide 4 of 6

CLICK

On Slide 4, students identify the pizzeria with the greatest ratio of pepperonis to pizza size.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Use Graphs to Compare Ratio Relationships

Objective

Students will graph and compare multiple ratio relationships on the same coordinate plane.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to understand the benefit of graphing all three ratio relationships on the same coordinate plane, as opposed to three separate coordinate planes.

Questions for Mathematical Discourse

SLIDE 3

AL What models can you use to represnt the ratios? graph or table

OL After graphing the relationships for all three pizzerias on the same coordinate plane, what do you notice? Sample answer: The points for each pizzeria seem to fall in a straight line. The line for The Pizza Place appears to be the steepest.

BL Is there a pizza size with the same number of pepperonis for all three pizzerias? Explain. no; Sample answer: If two or more pizzerias have the same number of pepperonis for the same size pizza, the lines would intersect. On the graph, the only time the lines intersect is at 0, so there is no pizza size with the same number of pepperonis.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To extend students understanding of graphing and comparing ratio relationships, have them research the Internet, newspapers, or magazines, for real-world uses, such as comparing the ratio of gallons of gas to miles driven for various types of vehicles. You may wish to have students present their findings to the class.

1 CONCEPTUAL UNDERSTANDING   2 FLUENCY   3 APPLICATION

Learn Use Tables to Compare Ratio Relationships

Objective

Students will understand how ratio tables can be used to compare multiple ratio relationships.

Teaching Notes

SLIDE 1

Students are asked to determine which smoothie recipe has the greatest ratio of blueberries to strawberries. To compare the three relationships using ratio tables, students can generate equivalent ratios for each recipe until the quantity of strawberries is the same for all three recipes. It is important for students to understand that, because the number of blueberries is what they are most interested in, you want to hold the number of strawberries constant across the three recipes. When all three recipes have an equal number of strawberries, students can easily compare the number of blueberries. The recipe with the greatest number of blueberries, when the number of strawberries is constant, has the greatest ratio of blueberries to strawberries.

Go Online to find additional teaching notes, Teaching the Mathematical Practices, and a sample answer for the *Talk About It!* question.

Example 2 Use Tables to Compare Ratio Relationships

Objective

Students will use ratio tables to compare multiple ratio relationships.

Questions for Mathematical Discourse

SLIDE 2

AL What quantities are being compared? ounces of sunflower seeds to ounces of peanuts

OL By making which quantity the same will help you most in finding the greatest ratio of ounces of sunflower seeds to ounces of peanuts? Justify your response. ounces of peanuts; Sample answer: To determine which recipe has the greatest ratio of ounces of sunflower seeds to ounces of peanuts, I need to compare the quantity of sunflower seeds when the ounces of peanuts are the same in all three relationships.

BL Before writing equivalent ratios, can you determine the quantity for the ounces of peanuts you will need in order to compare sunflower seeds? Explain. Sample answer: The quantity for ounces of peanuts will need to be 12 in each relationship because 12 is evenly divisible by 3, 4, and 6.

(continued on next page)

Learn Use Tables to Compare Ratio Relationships

Another way to compare ratio relationships is to use tables. For example, a comparison of three smoothie recipes shows that Recipe A has a blueberry to strawberry ratio of 8 to 2, Recipe B has a ratio of 5 to 1, and Recipe C has a ratio of 10 to 3. Y ou can use tables of equivalent ratios to determine which recipe has the greatest ratio of blueberries to strawberries.

Recipe A

Blueberries	8 16 24
Strawberries	2 4 6

Recipe B

Blueberries	5 10 15 20 25 30
Strawberries	1 2 3 4 5 6

Recipe C

Blueberries	10 20 30
Strawberries	3 6 9

Use scaling to write equivalent ratios for each recipe. Y ou can compare the ratios when one of the quantities in each relationship is the same.

Recipe B has a ratio of 30 blueberries for every 6 strawberries, followed by Recipe A with a ratio of 24 to 6, and Recipe C with a ratio of 20 to 6. So, Recipe B has the greatest ratio of blueberries to strawberries.

Talk About It! If the ratio relationships were graphed with blueberries on the *y*-axis and strawberries on the *x*-axis, the line for which recipe would have the steepest line? Explain.

Sample answer: Recipe B would have the steepest line because it has the greatest ratio of blueberries to strawberries.

Example 2 Use Tables to Compare Ratio Relationships

Roman is considering different bird seeds to fill his bird feeder . Measured in ounces, Recipe A has a sunflower seed to peanut ratio of 2 to 3, Recipe B has a ratio of 3 to 4, and Recipe C has a ratio of 5 to 6.

Which recipe has the greatest ratio of ounces of sunflower seeds to ounces of peanuts?

Step 1 Create a ratio table for each recipe. Find equivalent ratios to compare the relationships.

Recipe A

Sunflower Seeds (oz)	2	4 6	8
Peanuts (oz)	3	6 9 12	

Recipe B

Sunflower Seeds (oz)	3	6 9 12	
Peanuts (oz)	4	8 12	16

Recipe C

Sunflower Seeds (oz)	5	10 15 20
Peanuts (oz)	6	12 18 24

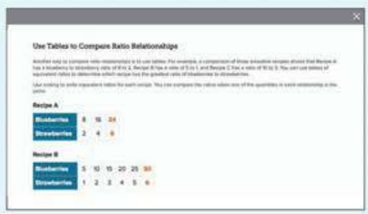
Think About It! Which quantity will you make equivalent in each ratio in order to compare the other quantity?

peanuts

(continued on next page)

Lesson 1-4 • Compare Ratio Relationships 31

Interactive Presentation



Learn, Use Tables to Compare Ratio Relationships, Slide 1 of 2

TYPE

On Slide 2 of Example 2, students generate equivalent ratios to compare the relationships.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Talk About It! Compare and contrast using graphs and using tables to compare ratio relationships.

See students' responses.

Step 2 Determine the recipe with the greatest ratio of sunflower seeds to peanuts.

Recipe A: 8 : 12 **Recipe B:** 9 : 12 **Recipe C:** 10 : 12

Because 10 is greater than 9 and 8, the recipe with the greatest ratio of sunflower seeds to peanuts is Recipe C.

Check

When working on homework, Bailey spends 15 minutes reading for every 20 minutes spent on math, Aisha spends 12 minutes reading for every 15 minutes of math, and Tyler spends 7 minutes reading for every 10 minutes of math. Which person has the greatest ratio of minutes spent on reading to minutes spent on math? Aisha

Bailey

Reading (min)	15	30	45			
Math (min)	20	40	60			

Aisha

Reading (min)	12	24	36	48		
Math (min)	15	30	45	60		

Tyler

Reading (min)	7	14	21	28	35	42
Math (min)	10	20	30	40	50	60

Go Online You can complete an Extra Example online.

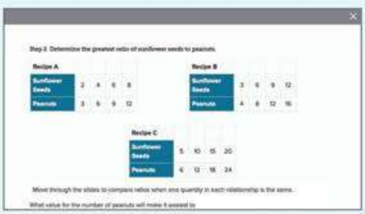
Pause and Reflect

Have you ever wondered when you might use the concepts you learn in math class? What are some everyday scenarios in which you might use what you learned today?

Record your observations here

See students' observations.

Interactive Presentation



Example 2, Use Tables to Compare Ratio Relationships, Slide 3 of 5

CLICK

On Slide 3, students move through the slides to identify the recipe with the greatest ratio of ounces of sunflower seeds to ounces of peanuts.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Use Tables to Compare Ratio Relationships *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL What is the greatest quantity of ounces of sunflower seeds when the number of ounces of peanuts are the same? 10 ounces of sunflower seeds

OL Which recipe has the greatest ratio of ounces of sunflower seeds to ounces of peanuts? Explain. Recipe C; Sample answer: When the number of ounces of peanuts is constant, 12, the recipe with the greatest number of ounces of sunflower seeds has the greatest ratio.

OL How would you alter your method if you were asked which recipe contains the greatest ratio of ounces of peanuts to ounces of sunflower seeds? Sample answer: Instead of generating equivalent ratios until the number of ounces of peanuts is constant, I can generate equivalent ratios until the number of ounces of sunflower seeds is constant. When the number of ounces of sunflower seeds is 30 for all three recipes, Recipe A has the greatest number of ounces of peanuts, 45. So, Recipe A has the greatest ratio of ounces of peanuts to sunflower seeds.

BL A classmate generated equivalent ratios until the number of ounces of peanuts was 24. Is this a valid method? Justify your response. yes; Sample answer: This is a valid method, because when the number of ounces of peanuts is 24, you can compare the number of ounces of sunflower seeds and arrive at the same solution. Recipe C has the greatest ratio. While this is a valid method, you can stop generating equivalent ratios when you find the first common value at 12.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Mixing Paint

Objective

Students will come up with their own strategy to solve an application problem involving mixing paint colors.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What strategies have you learned that can help solve the problem?
- What do you notice about the ratios for Marcus and Hiram? Can you tell whose paint mixture will have the most blue?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Mixing Paint

Three friends are each mixing containers of red and blue paint, according to the ratios shown, to create their favorite shades of purple paint. Each container is the same size. If each person uses 6 quarts of red paint, whose paint mixture will have the most blue?

	Marcus	Cassidy	Hiram
Red (qt)	2	3	2
Blue (qt)	3	4	2

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

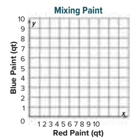
First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem. A coordinate grid is provided should you choose to use it.

Marcus; See students' work.



Talk About It!
Will the friend who has the most blue in his or her paint mixture always have the most blue, no matter how many quarts of red paint are used? Why or why not?

no; Sample answer: In the beginning, Cassidy uses more blue paint than Marcus, but when they use 6 quarts of red paint Marcus uses more than Cassidy.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

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Interactive Presentation

Apply, Mixing Paint

CHECK

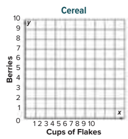
Students complete the Check exercise online to determine if they are ready to move on.

Check

Three cereal brands advertise the average number of berries for every cup of whole-grain cereal flakes as shown in the table. Each box is the same size. Which company advertises the greatest ratio of berries for every cup of flakes? **Brand A**

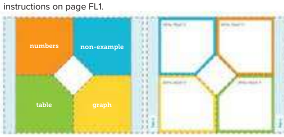
	Brand A	Brand B	Brand C
Cups of Flakes	1	2	3
Berries	5	6	12

A coordinate grid is provided should you choose to use it.



Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could write a short description of how to compare ratio relationships using tables and graphs. You may wish to have students share their Foldables with a partner to compare information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Which company should the student council choose if they want to earn the most money possible during the fundraiser? Write a mathematical argument that can be used to defend your solution. Company C; Sample answer: Using ratio tables, I can use scaling to find equivalent ratios for each company, then compare. For example, Company A will give \$4 for every \$24 sold, Company B will give \$4.80 for every \$24 sold, and Company C will give \$6 for every \$24 sold. So, Company C will be the best company to use because the council gets the most money back from them for selling the same dollar amount of gift wrap.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1–5 odd, 6–8
- ALEKS® Ratios and Unit Rates

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1, 2, 4, 8
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Ordered Pairs

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Ordered Pairs

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- AL Practice Form B
- OL Practice Form A
- BL Practice Form C

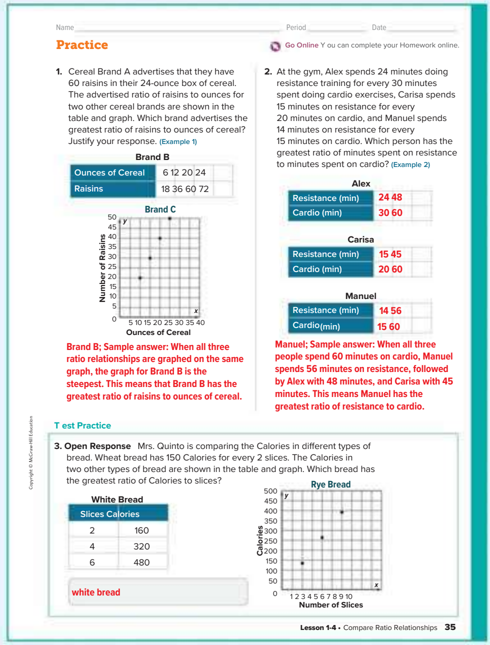
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	use a graph to compare multiple ratio relationships	1
2	use ratio tables to compare multiple ratio relationships	2
2	extend concepts learned in class to apply them in new contexts	3
3	solve application problems involving comparing ratio realtionships	4, 5
3	higher-order and critical thinking skills	6–8

Common Misconception

In Exercise 2, students might think that they can only use tables to compare the ratios. While this is the indicated method in the Exercise, students that are more visual learners may benefit from writing ordered pairs to represent each relationship and graphing the relationships on the same coordinate plane. If students are struggling with using tables, remind them that they can graph the relationships if needed.



Name Period Date

Practice

Go Online You can complete your Homework online.

1. Cereal Brand A advertises that they have 60 raisins in their 24-ounce box of cereal. The advertised ratio of raisins to ounces for two other cereal brands are shown in the table and graph. Which brand advertises the greatest ratio of raisins to ounces of cereal? Justify your response. (Example 1)

Brand B

Ounces of Cereal	6	12	20	24
Raisins	18	36	60	72

Brand B; Sample answer: When all three ratio relationships are graphed on the same graph, the graph for Brand B is the steepest. This means that Brand B has the greatest ratio of raisins to ounces of cereal.

2. At the gym, Alex spends 24 minutes doing resistance training for every 30 minutes spent doing cardio exercises, Carisa spends 15 minutes on resistance for every 20 minutes on cardio, and Manuel spends 14 minutes on resistance for every 15 minutes on cardio. Which person has the greatest ratio of minutes spent on resistance to minutes spent on cardio? (Example 2)

Alex

Resistance (min)	24	48
Cardio (min)	30	60

Carisa

Resistance (min)	15	45
Cardio (min)	20	60

Manuel

Resistance (min)	14	56
Cardio (min)	15	60

Manuel; Sample answer: When all three people spend 60 minutes on cardio, Manuel spends 56 minutes on resistance, followed by Alex with 48 minutes, and Carisa with 45 minutes. This means Manuel has the greatest ratio of resistance to cardio.

Test Practice

3. **Open Response** Mrs. Quinto is comparing the Calories in different types of bread. Wheat bread has 150 Calories for every 2 slices. The Calories in two other types of bread are shown in the table and graph. Which bread has the greatest ratio of Calories to slices?

White Bread

Slices	Calories
2	160
4	320
6	480

white bread

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Lesson 1-4 • Compare Ratio Relationships 35

Apply *indicates multi-step problem

*4. Mrs. Gonzalez wants to hire a catering company for her daughter's quinceañera. The ratios of the cost per person for a child and an adult for two different companies are shown in the table. Mrs. Gonzalez is planning on 25 adults and 12 children adding the party. How much less will it cost for her to hire Planning Pros than Party Time? **$19.50**

	Party Time	Planning Pros
Cost per Adult ($)	10.50	9.00
Cost per Child ($)	6.00	7.50

*5. Charlie, Beth, and Miguel all babysit kids in their neighborhood. The table shows the number of hours and the amount each of them earned last night. If each person babysits for 5 hours next weekend, which person will earn the most money? Use a coordinate plane if needed to solve. **Miguel**

	Charlie	Beth	Miguel
Number of Hours	3	4.5	4
Total Earned ($)	28.50	42.00	40.00

Higher-Order Thinking Problems

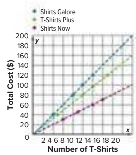
6. MP **Construct an Argument** Ratio relationships can be described with words or they can be displayed using bar diagrams, tables, and graphs. Which display is more advantageous to use when comparing ratio relationships? Explain your reasoning.

Sample answer: Graphs are more advantageous because I can visually see which relationship has a steeper line. The steeper the line, the greater the ratio.

7. Give an example of a ratio relationship that you have seen outside of school. How was the ratio relationship displayed, and why was the relationship displayed that way?

Sample answer: Three packages of hot dogs cost $9.50. The relationship was displayed in words because it's easier and faster for people to understand while shopping.

8. MP **Find the Error** Avery wants to order new practice T-shirts for her soccer team. The ratio of the total cost to the number of T-shirts purchased for three different stores is shown in the graph. Avery says that the shirts will cost less from Shirts Galore because the graph is steeper than the graphs of the other relationships. Find her mistake and correct it.



Sample answer: The graph of the relationship that is steepest represents the relationship that has the greatest ratio of total cost to number of T-shirts. To determine which company has the least cost, she should look for the graph that is the least steep.

36 Module 1 • Ratios and Rates

Teaching the Mathematical Processes

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 6, students construct an argument to defend their chosen display and why they think it is more advantageous than the other displays.

In Exercise 8, students diagnose and explain why Avery's solution is incorrect and then correct the solution.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.

Use with Exercises 4–5 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Be sure everyone understands.

Use with Exercises 6–7 Have students work in groups of 3-4 to solve the problem in Exercise 6. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 7.

Learn Use Bar Diagrams to Solve Ratio Problems

Objective

Students will understand that they can use a bar diagram to model and solve a real-world problem involving ratios.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students should understand how a bar diagram can be a tool that can help solve real-world problems involving ratios.

Go Online to have your students watch the video on Slide 1 if they would benefit from reviewing how to draw a bar diagram.

Teaching Notes

SLIDE 1

Present the problem and ask students to work with a partner to determine possible strategies they can use to solve it – without using a bar diagram. They may use any strategy, but must be able to explain why their strategy works. Then have them move through the slides that show how a bar diagram can be used to model and solve the problem. Have students compare this strategy to the one they chose.

Talk About It!

SLIDE 2

Mathematical Discourse

When thinking about the ratio of students who play sports to the total number of students, is it easier to think about 3 to 5, or 315 to 525? Explain. Sample answer: It is easier to think about the ratio 3 to 5 because I can visualize a lesser number of students better than a greater number.

DIFFERENTIATE

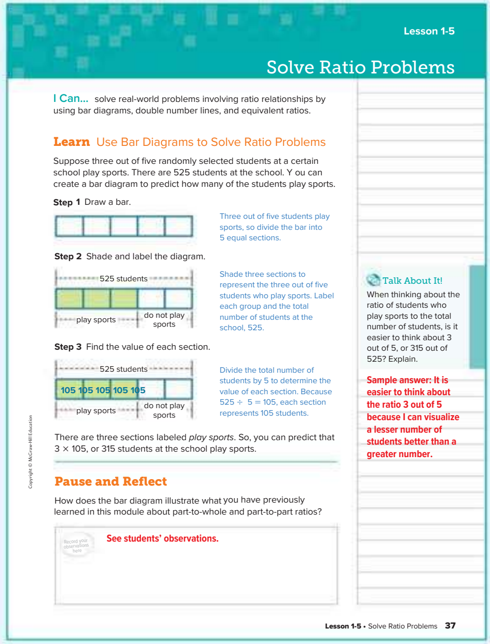
Language Development Activity ELL

For students that may be struggling to create bar diagrams to represent ratios, have them identify the part and the whole for each of the following ratios. Then have them identify the number of total sections and the number of sections that would be shaded in a corresponding bar diagram that represents the ratio.

6 out of 7 part: 6; whole: 7; total sections: 7; shaded sections: 6

2 : 9 part: 2; whole: 9; total sections: 9; shaded sections: 2

$\frac{5}{8}$ part: 5; whole: 8; total sections: 8; shaded sections: 5


Lesson 1-5

Solve Ratio Problems

I Can... solve real-world problems involving ratio relationships by using bar diagrams, double number lines, and equivalent ratios.

Learn Use Bar Diagrams to Solve Ratio Problems

Suppose three out of five randomly selected students at a certain school play sports. There are 525 students at the school. You can create a bar diagram to predict how many of the students play sports.

Step 1 Draw a bar.

Three out of five students play sports, so divide the bar into 5 equal sections.

Step 2 Shade and label the diagram.

Shade three sections to represent the three out of five students who play sports. Label each group and the total number of students at the school, 525.

Step 3 Find the value of each section.

Divide the total number of students by 5 to determine the value of each section. Because 525 ÷ 5 = 105, each section represents 105 students.

There are three sections labeled *play sports*. So, you can predict that 3 × 105, or 315 students at the school play sports.

Talk About It!

When thinking about the ratio of students who play sports to the total number of students, is it easier to think about 3 out of 5, or 315 out of 525? Explain.

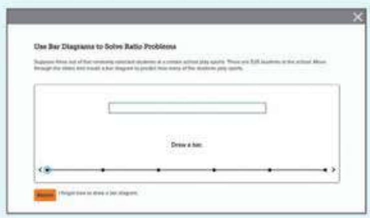
Sample answer: It is easier to think about the ratio 3 out of 5 because I can visualize a lesser number of students better than a greater number.

Pause and Reflect

How does the bar diagram illustrate what you have previously learned in this module about part-to-whole and part-to-part ratios?

See students' observations.

Interactive Presentation



Learn, Use Bar Diagrams to Solve Ratio Problems, Slide 1 of 2

WATCH

On Slide 1, students can watch a video that demonstrates how to draw a bar diagram.

CLICK

On Slide 1, students move through the slides to create a bar diagram that helps solve the given problem.

Lesson 1-5

Solve Ratio Problems

6.RP.A.3

LESSON GOAL

Students will solve real-world problems involving ratios.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Use Bar Diagrams to Solve Ratio Problems

Example 1: Use Bar Diagrams to Solve Ratio Problems

Example 2: Use Bar Diagrams to Solve Ratio Problems

Learn: Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems

Example 3: Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems

Apply: Inventory

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Determine if Figures are Similar		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 5 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving ratios.

ELL You can use the tips and suggestions on page T5 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by solving real-world problems involving ratios using equivalent ratios, double number lines, and bar diagrams.

Standards for Mathematical Content: **6.RP .A.3,** *Also addresses 6.RP.A.1*

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP7**

Coherence

Vertical Alignment

Previous

Students used graphs and tables to compare multiple ratio relationships.
6.RP.A.3, 6.RP.A.3.A

Now

Students solve real-world problems involving ratios.
6.RP.A.3

Next

Students will use ratio reasoning to convert between customary units of measurement.
6.RP.A.3, 6.RP.A.3.D

Rigor

The Three Pillars of Rigor

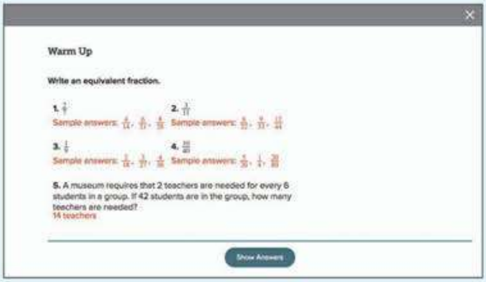
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students *apply* their *understanding* of ratio relationships to solve real-world problems. They build *fluency* with using different methods, such as bar diagrams, double number lines, and reasoning about equivalent ratios, as they solve problems.

Mathematical Background

Ratio problems can be solved by using a variety of methods, including the use of bar diagrams, double number lines, and reasoning about equivalent ratios. Bar diagrams and double number lines are both useful visual representations of the ratio relationship. Using bar diagrams and double number lines can help you understand the relationship between the two quantities. When the numbers are large or involve decimals or fractions, reasoning about equivalent ratios can be more advantageous.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing equivalent fractions (Exercises 1–4)
- solving real-world problems involving equivalent ratios (Exercise 5)

Answers

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about collecting surveys to gather and compare data.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

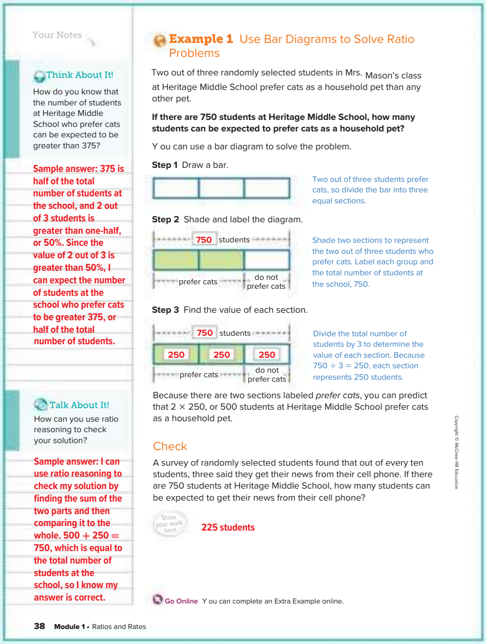
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does *equivalent* mean? What does this tell you about *equivalent fractions*? Sample answer: *Equivalent* means having the same value. For two fractions to be equivalent, they should be equal to the same value.
- You previously learned about ratios. Give a real-world example of a ratio. Sample answer: For every 1 dog at the animal shelter, there are 4 cats.


Your Notes

Think About It!
How do you know that the number of students at Heritage Middle School who prefer cats can be expected to be greater than 375?

Sample answer: 375 is half of the total number of students at the school, and 2 out of 3 students is greater than one-half, or 50%. Since the value of 2 out of 3 is greater than 50%, I can expect the number of students at the school who prefer cats to be greater 375, or half of the total number of students.

Talk About It!
How can you use ratio reasoning to check your solution?

Sample answer: I can use ratio reasoning to check my solution by finding the sum of the two parts and then comparing it to the whole. 500 + 250 = 750, which is equal to the total number of students at the school, so I know my answer is correct.

Example 1 Use Bar Diagrams to Solve Ratio Problems

Two out of three randomly selected students in Mrs. Mason's class at Heritage Middle School prefer cats as a household pet than any other pet.

If there are 750 students at Heritage Middle School, how many students can be expected to prefer cats as a household pet?

Y ou can use a bar diagram to solve the problem.

Step 1 Draw a bar.

Two out of three students prefer cats, so divide the bar into three equal sections.

Step 2 Shade and label the diagram.

750 students; prefer cats; do not prefer cats

Shade two sections to represent the two out of three students who prefer cats. Label each group and the total number of students at the school, 750.

Step 3 Find the value of each section.

750 students; 250 | 250 | 250; prefer cats; do not prefer cats

Divide the total number of students by 3 to determine the value of each section. Because 750 ÷ 3 = 250, each section represents 250 students.

Because there are two sections labeled *prefer cats*, you can predict that 2 × 250, or 500 students at Heritage Middle School prefer cats as a household pet.

Check

A survey of randomly selected students found that out of every ten students, three said they get their news from their cell phone. If there are 750 students at Heritage Middle School, how many students can be expected to get their news from their cell phone?

Show your work here

225 students

Go Online Y ou can complete an Extra Example online.

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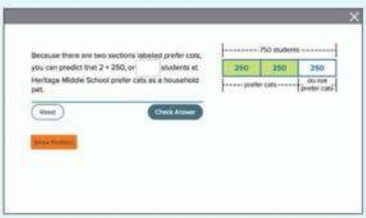
38 Module 1 • Ratios and Rates

Interactive Presentation



Example 1, Use Bar Diagrams to Solve Ratio Problems, Slide 3 of 5

CLICK

On Slide 2, students move through the slides to see how a bar diagram can be used to solve the problem.

TYPE

On Slide 3, students find the solution.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Use Bar Diagrams to Solve Ratio Problems

Objective

Students will use bar diagrams to solve real-world problems involving part-to-whole ratios.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to clearly explain how they can reason about the ratios 2 : 3 and 500 : 750 in order to determine if they are equivalent.

5 Use Appropriate Tools Strategically Encourage students to use the bar diagram as a tool to model and represent the problem. Students should understand how the bar diagram helps them visually make sense of the relationship between the quantities involved in the problem, in order to use reasoning to solve the problem.

Questions for Mathematical Discourse

SLIDE 2

AL Why is the bar divided into three sections? The ratio 2 to 3 is a part-to-whole ratio, and the whole is 3 students.

AL Why are two sections shaded and labeled *prefer cats*? The part that represents those who prefer cats is 2 students.

OL Why is the value of each section equal to 250? What does this represent? The total number of students is 750, and there are three equal parts; 750 ÷ 3 = 250. This represents the number of students in each of the three parts.

BL If 2 out of 5 students preferred cats, how would the bar diagram be altered? Sample answer: The bar diagram would be divided into 5 sections. Each section would represent 750÷ 5, or 150 students.

SLIDE 3

AL How many sections represent students that prefer cats? students that do not prefer cats? 2 sections; 1 section.

OL How can you check your answer? Sample answer: If 500 students prefer cats, and 250 do not prefer cats, the total is 750 students, which is correct.

BL How would the bar diagram and solution be altered if the total number of students were 900? Sample answer: Each section would be 300 students, and the number of students who prefer cats would be 600 students.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Use Bar Diagrams to Solve Ratio Problems

Objective

Students will use bar diagrams to solve real-world problems involving part-to-part ratios.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use the bar diagram as a tool to model and represent the problem. Students should understand how the bar diagram helps them visually make sense of the relationship between the quantities involved in the problem, in order to use reasoning to solve the problem.

7 Look For and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to use the structure of the bar diagram in their explanation. Because 4 sections represent the number of photos Maribel took, and 3 sections represent the number of photos Marcus took, there is 4 − 3, or 1 more section that represents the difference. Because each section represents 6 photos, Maribel took 6 more photos than Marcus.

Questions for Mathematical Discourse

SLIDE 2

AL Why is Marcus' label on the bar shorter than his sister's? For every 3 photos Marcus took, his sister took 4. His sister takes more photos.

AL Why is the bar divided into four equal sections? The ratio is 3 : 4. Divide the bar into 4 sections and shade 3 of them to represent the ratio.

OL Why does each section in the bar diagram represent 6 photos? Marcus took 18 photos. His bar diagram has 3 sections; 18 ÷ 3 = 6. Each section is equivalent, so they all represent 6 photos.

OL How do you know your answer is correct? Sample answer: The ratios 18 : 24 and 3 : 4 are equivalent because you can divide 18 and 24 both by 6 to obtain the quantities 3 and 4, respectively.

BL How would the bar diagrams be altered if Marcus took 24 photos? Sample answer: The label over Marcus' bar diagram would be 24, not 18. Each section would represent 24 ÷ 3, or 8 photos, instead of 6.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Use Bar Diagrams to Solve Ratio Problems

During their family vacation, Marcus took 18 photos on his cell phone. The ratio of the number of photos Marcus took to the number of photos his sister Maribel took is 3 to 4.

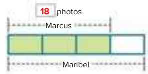
How many photos did Maribel take?

Y ou can use a bar diagram to solve the problem.

Step 1 Draw a bar.

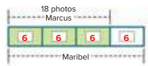
The ratio of the number of photos Marcus took to the number Maribel took is 3 : 4, so divide the bar into four equal sections.

Step 2 Shade and label the diagram.



Shade three sections to represent the ratio 3 : 4 and add labels for Marcus and Maribel. Because Marcus took 18 photos, label the three shaded sections as 18 photos.

Step 3 Find the value of each section.



Divide the total number of photos Marcus took by 3 to determine the value of each section. Because 18 ÷ 3 = 6, each section represents 6 photos.

There are four sections that represent the number of photos Maribel took. Multiply 6 by 4. So, Maribel took a total of 6 × 4, or 24 photos on their vacation.

Think About It! Is the number of photos Maribel took less than, greater than, or equal to 18? How do you know?

greater than; Sample answer: In the ratio 3 : 4, 3 represents photos Marcus took, while 4 represents photos Maribel took. Since 4 is greater than 3, Maribel took more photos than the 18 photos Marcus took.

Talk About It! How does the bar diagram indicate how many more photos Maribel took than Marcus?

Sample answer: The bar diagram shows that the whole bar is one section greater than the shaded portion. One section has a value of 6, so Maribel took 6 more photos than Marcus.

Check

A survey of randomly selected people found that the ratio of people who prefer oatmeal raisin cookies to those who prefer chocolate chip cookies is 3 to 5. If 27 people said that they prefer oatmeal raisin cookies, how many said they prefer chocolate chip? Draw a bar diagram to support your solution.

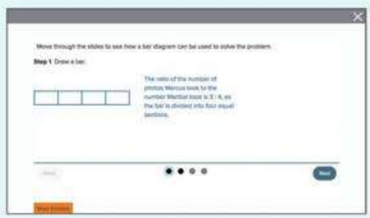
Show your work here

45 people; See students' work for bar diagram.

Go Online Y ou can complete an Extra Example online.

Lesson 1-5 • Solve Ratio Problems 39

Interactive Presentation



Example 2, Use Bar Diagrams to Solve Ratio Problems, Slide 2 of 4

CLICK

On Slide 2, students move through the slides to see how a bar diagram can be used to solve the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems

The manager of a small hotel determines that it takes 30 loads of laundry to clean the towels and sheets of the hotel's rooms each day. A large bottle of laundry detergent contains 150 ounces and the label indicates that the contents of the bottle can clean 75 loads. How many ounces of detergent are needed to clean the hotel's towels and sheets each day?

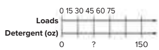
Y ou can represent this ratio relationship and solve the problem by using double number lines and equivalent ratios.

Method 1 Use a double number line.

Step 1 Draw a double number line.

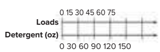
The top number line represents the number of loads of laundry. The bottom number line represents the number of ounces of detergent needed.

Mark the ratio of loads to detergent (75 : 150). Mark and label equal increments to show 30 loads.



Step 2 Find the equivalent ratio.

There are 5 equal sections. Because 150 ÷ 5 = 30, label equal increments of 30 on the bottom number line.



The value on the bottom number line that corresponds with 30 loads is 60 ounces of detergent.

So, 60 ounces of detergent are needed each day.

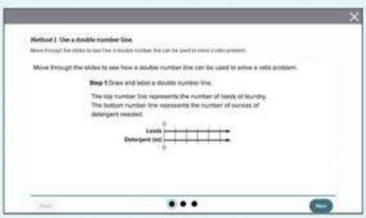
Talk About It!

Why might a bar diagram not be the best representation to help solve this problem?

Sample answer: A bar diagram might not be the best representation because the problem involves taking a portion of the original ratio, which might make it challenging to split up the bars in a bar diagram without getting confused.

(continued on next page)

Interactive Presentation



Learn, Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems, Slide 2 of 4

CLICK

On Slide 2, students move through the slides to learn how to use a double number line to solve ratio problems.

Learn Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems

Objective

Students will understand that they can use double number lines and equivalent ratios to solve a real-world problem involving ratios.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* questions, you may wish to have them try to draw a bar diagram to represent the problem. They may find it challenging as they determine how many sections to divide the bar into, and what each section represents. Some students may choose to use a double number line because it can be less complicated and can help them visually see the ratio relationships more immediately. Have students compare and contrast using a double number line and reasoning about equivalent ratios.

Teaching Notes

SLIDE 1

Present the real-world problem and ask students to work with a partner to determine possible strategies they can use to solve the problem. They must be able to explain why their strategy works. Then have them move through the slides that show how double number lines and equivalent ratios can be used to solve the problem. Have students compare this strategy to the one they chose.

Talk About It!

SLIDE 2

Mathematical Discourse

Why might a bar diagram not be the best representation to help solve this problem? Sample answer: A bar diagram might not be the best representation because the problem involves taking a portion of the original ratio, which might make it challenging to split up the bars in a bar diagram without getting confused.

(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of equivalent ratios, have them explain how to find an equivalent ratio for the following ratio. In this case, one ratio is not a whole-number multiple of the other.

$\frac{18}{63} = \frac{?}{105}$

Sample answer: 63 can be multiplied by $\frac{5}{3}$ to obtain 105. Multiply 18 by $\frac{5}{3}$ to find the unknown: $18 \cdot \frac{5}{3} = 30$.

Learn Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems *(continued)*

Teaching Notes

SLIDE 3

After moving through each of the methods shown, you may wish to have students discuss the similarities and differences, and determine scenarios in which one method might be more useful than the other.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 4

Mathematical Discourse

Compare and contrast using a double number line and equivalent ratios. Which method might be more advantageous to use if the numbers are large? Sample answer: A double number line helps visualize the equivalency, but using equivalent ratios might be more advantageous if the numbers are large because it is easier to perform operations on the numbers than it is to represent them on a double number line.

Example 3 Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems

Objective

Students will use equivalent ratios to solve real-world problems involving part-to-whole ratios.

Questions for Mathematical Discourse

SLIDE 2

AL What ratio represents the situation? 2 : 96

AL Is the ratio a part-to-part ratio or a part-to-whole ratio? part-to-whole

OL How do you know how many equal increments to mark? Sample answer: I need to start and end at 480 jars on the bottom number line. One of the increments needs to be at 96 jars. Because 480 is a multiple of 96, and $480 \div 96 = 5$, I need to mark 5 equal increments from 96 to 480. Counting the increment at 0, this is a total of 5 increments of 96 from 0 to 480.

BL How would the ratio change if two cases of peanut butter contained 230 jars? Sample answer: Instead of the second quantity being 96, it would be 230.

(continued on next page)

Method 2 Use equivalent ratios.

Write and solve an equation stating that two ratios are equivalent. Let *d* represent the unknown number of ounces of detergent needed to clean 30 loads of laundry.

$$\frac{30}{d} = \frac{75}{150}$$ (loads of laundry / ounces of detergent)

$$\frac{30}{d} = \frac{75}{150}$$ ÷ 2.5

Because $75 \div 2.5 = 30$, divide 150 by 2.5 to find the value of *d*.

$$\frac{30}{60} = \frac{75}{150}$$

$150 \div 2.5 = 60$; So, $d = 60$.

So, using either method, 60 ounces of detergent are needed to clean the hotel's towels and sheets each day.

Talk About It! Compare and contrast using a double number line and equivalent ratios. Which method might be more advantageous to use if the numbers are large?

Sample answer: A double number line helps visualize the equivalency, but using equivalent ratios might be more advantageous if the numbers are large because it is easier to perform operations on the numbers than it is to represent them on a double number line.

Example 3 Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems

The manager of a grocery store determines that an average of 480 jars of peanut butter are sold each week. Two cases of peanut butter contain 96 jars.

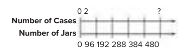
How many cases of peanut butter should the manager order each week?

Method 1 Use a double number line.

Step 1 Draw the double number line.

The top number line represents the number of cases of peanut butter. The bottom number line represents the number of jars of peanut butter.

Mark the ratio of cases to jars (2 : 96). Mark and label equal increments to show 480 jars.



Think About It! Can you solve this problem mentally without using any diagrams? Explain.

See students' responses.

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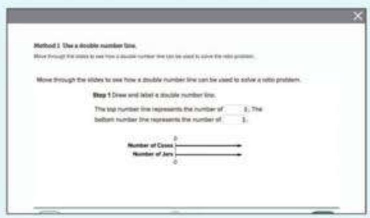
Interactive Presentation



Example 3, Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems, Slide 2 of 5

CLICK

On Slide 3 of the Learn, students move through the slides to use equivalent ratios to solve a problem involving ratios.

CLICK

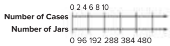
On Slide 2 of Example 3, students move through the slides to use a double number line to solve the problem.

CLICK

On Slide 2 of Example 3, students use a double number line to solve the problem.

Step 2 Find the equivalent ratio.

There are 5 equal sections. Label equal increments of 2 on the top number line.



The value on the top number line that corresponds with 480 jars is 10 cases. So, 10 cases should be ordered each week.

Method 2 Use equivalent ratios.

Write and solve an equation stating that two ratios are equivalent. Let c represent the unknown number of cases the manager should order each week.

number of cases → $\frac{2}{96} = \frac{c}{480}$ ← number of cases
number of jars → ← number of jars

$\frac{2}{96} \overset{\times 5}{=} \frac{c}{480}$ Because 96 × 5 = 480, multiply 2 by 5 to find the value of c.

$\frac{2}{96} = \frac{10}{480}$ 2 × 5 = 10; So, c = 10.

So, using either method, the manager should order **10** cases of peanut butter each week.

Talk About It!
How can you use scaling and a table of equivalent ratios to solve this problem?

Sample answer: I can set up a table beginning with the ratio 2 : 96 and ending with c : 480. Then I could use scaling to multiply 2 and 96 by the same number to obtain equivalent ratios until reaching 480.

Check

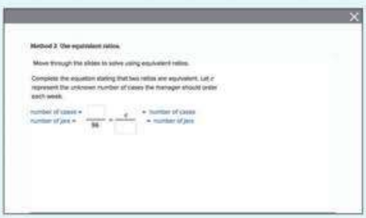
The manager of a bakery determines that an average of 112 loaves of cheese bread are sold each week. For every 2 loaves of cheese bread that are sold, about 3 loaves of whole wheat bread are sold. About how many loaves of whole wheat bread are sold each week?

Show your work here

168 loaves

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 3, Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems, Slide 3 of 5

CLICK

On Slide 3, students move through the steps to use equivalent ratios to solve the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why do you multiply both 2 and 96 by 5? I need to find the equivalent ratio, and 96 multiplied by 5 equals 480.

OL How can you determine by what number the quantity 2 needs to be multiplied to find the number of cases? Sample answer: Divide 480 by 96, which is 5. This means 96 × 5 is 480, so 2 must also be multiplied by 5.

OL What similarities do you notice between Methods 1 and 2? Sample answer: The ratio 2 : 96 is used in each method and this ratio is equivalent to a ratio containing an unknown value that is compared to 480.

BL How many cases should the manager buy if an average of 500 jars are sold each week? 11 cases

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Inventory

Objective

Students will come up with their own strategy to solve an application problem involving inventory at an office supply store.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoningof Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What information do you need in order to solve the problem? What information do you *not* need?
- How can you write a ratio to help solve the problem?
- How can you determine how many free reams of paper were given away?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Inventory

The manager of an office supply store decides to hold a *Buy 2, Get 1 Free* sale on all reams of paper. A *ream* of paper holds 500 sheets of paper. The sale is held for one week and a total of 154 reams of paper were sold (not including the ones given away for free). If each ream of paper cost the store $4.50, how much money did the store lose by giving away the free reams of paper?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

Show your work here

$346.50; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
Why do you think stores offer sales, such as *Buy 2, Get 1 Free?*

Sample answer: Stores might offer sales to attract more customers into their store, and to entice customers to purchase even more items. They may also have sales to get rid of inventory quickly, to make space for new items.

Lesson 1-5 • Solve Ratio Problems 43

Interactive Presentation

Apply, Inventory

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Math History Minute

Euphemia Haynes (1890–1980) was the first African-American woman to earn a Ph.D. in mathematics. She taught in the public school system of Washington, D.C. for 47 years and became the first woman to serve as chair of the city's School Board.

Check

The manager of a clothing store decides to hold a *Buy 1, Get 2 Free* sale on all pairs of socks. The sale is held for one week and a total of 182 pairs of socks were sold (not including the ones given away for free). If each pair of socks cost the store $2.50, how much money did the store lose by giving away the free socks?

Show your work here

$910

Go Online You can complete an Extra Example online.

Pause and Reflect

What are the advantages of using a bar diagram to solve ratio problems? When might it be more advantageous to use double number lines or equivalent ratios?

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you describe how two quantities are related?

In this lesson, students learned how to solve real-world ratio problems using bar diagrams and equivalent ratios. Encourage them to work with a partner to compare and contrast the two methods. Have them explain which method they prefer and why.

Exit Ticket

Refer to the Exit Ticket slide. If there are 480 students in your grade, how could you use this ratio to predict how many students in your grade prefer chocolate ice cream over vanilla? Explain how you solved the problem. Sample answer: Set up equivalent ratios and solve for the unknown. I can predict that about 320 students in my grade will prefer chocolate ice cream over vanilla.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 11–14
- Extension: Determine if Figures are Similar
- ALEKS Ratios and Unit Rates

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 9, 12, 13
- Extension: Determine if Figures are Similar
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- AL Practice Form B
- OL Practice Form A
- BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	use bar diagrams, double number lines, or equivalent ratios to solve real-world ratio problems	1–7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving ratio problems	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may incorrectly draw a bar diagram to solve a ratio problem. In Exercise 2, students may draw a bar diagram that identifies 720 students as the part rather than the whole. Some students may attempt to draw the bar diagram with 5 sections rather than 8. In either case, the students will most likely obtain an answer for the part that is greater than the whole. Encourage students to evaluate their answers within the context of the problem. A part that is greater than the whole does not make sense within the context of Exercise 2.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Use any strategy to solve each problem. (Examples 1–3)

1. A survey showed that 4 out of 5 students own a bicycle. Based on this result, how many of the 800 students in a school own a bicycle?
   640 students

2. A survey of Mr. Thorne's class shows that 5 out of 8 students will buy lunch today. Based on this result, how many of the 720 students in the school will buy today?
   450 students

3. The ratio of the number of baskets made by Tony to the number of baskets made by Colin is 2 to 3. Tony made 10 baskets. How many baskets did Colin make?
   15 baskets

4. In the school choir, there is 1 boy for every 4 girls. There are a total of 11 boys. How many girls are in the choir?
   44 girls

5. Liberty Middle School has 600 students. In Anna's class, 3 out of 8 students walk to school. How many students at the school can be expected to walk to school?
   225 students

6. Pine Hill Middle School has 300 students. In Zoey's class, 2 out of 5 students belong to a club. How many students at the school would you expect belong to a club?
   120 students

7. In a survey, the ratio of students who prefer popcorn to potato chips is 3 to 4. If the number of students surveyed who prefer popcorn is 360, how many preferred potato chips?
   480 students

Test Practice

8. **Open Response** In a neighborhood, the ratio of houses with swing sets to houses without swing sets is 3 to 5. If the number of houses with swing sets is 270, how many houses do not have swing sets?
   450 houses

Apply *indicates multi-step problem

*9. The manager of an art supply store decides to hold a *Buy 2, Get 1 Free* sale on tubes of watercolor paints. The sale is held for one week and a total of 280 tubes of paint were sold (not including the ones given away for free). If each tube of watercolor paint cost the store $7.25, how much money did the store lose by giving away the free tubes of paint?

$1,015

*10. The manager of a garden store decides to hold a *Buy 3, Get 1 Free* sale on vegetable plants. The sale is held for one week and a total of 636 vegetable plants were sold (not including the ones given away for free). If each plant cost the store $2.90, how much money did the store lose by giving away the free plants?

$614.80

Higher-Order Thinking Problems

11. **Construct an Argument** Determine if the following statement is *true* or *false*. Construct an argument to defend your response.

In equivalent ratios, if the numerator of the first ratio is greater than the denominator of the first ratio, then the numerator of the second ratio is less than the denominator of the second ratio.

false; Sample answer: For the ratios to be equivalent, they must be equivalent fractions. So, the numerator of the second fraction must also be greater than the denominator. Otherwise, the ratios are not equivalent.

12. Compare and contrast the use of bar diagrams and equivalent ratios to solve ratio problems.

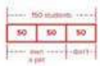
Sample answer: Both methods allow you to model the ratio and solve the problem. The bar diagram method provides a more visual representation of the problem, while the equivalent ratios method is more of a numerical representation of the problem and tends to be more efficient when working with larger numbers or fractions.

13. **Persevere with Problems** Suppose 20 out of 140 people said they play tennis and 1 out of every 9 of those players have a tennis coach. Using these same ratios, in a group of 504 people, predict how many you would expect to have a tennis coach. Explain how you made the prediction.

8 people; Sample answer: Using equivalent ratios, $\frac{20}{140} = \frac{?}{504}$. So, 72 people in a group of 504 would play tennis. Using equivalent ratios, $\frac{1}{9} = \frac{?}{72}$. So, 8 people out of those 72 would have a tennis coach.

14. Write and solve a real-world ratio problem that can be solved by using a bar diagram.

Sample answer: 2 out of 3 students in my class have a pet. Based on this result, how many of the 150 sixth-graders in my school own a pet? Draw a bar diagram to solve. 100 students



Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students determine whether a statement is true or false and construct an argument to defend their response.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 13, students determine a strategy they can use to make their prediction.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.
Use with Exercises 9–10 Have students work in pairs. Have students individually read Exercise 9 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 10.

Explore the truth of statements created by others.
Use with Exercises 11–14 Have students work in pairs. After completing the exercises, have students write two true statements about using bar diagrams and equivalent fractions to solve ratio problems and one false statement. An example of a true statement might be, “Bar diagrams can be used to solve part-to-whole ratio problems.” An example of a false statement might be “Equivalent ratios can only be used to solve part-to-part ratio problems.” Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. For each false statement, have them generate a counterexample. Have them discuss and resolve any differences.

Learn Unit Ratios and Measurement Conversions

Objective

Students will understand that they can use unit ratios to represent relationships between Customary units of measurement.

Teaching the Mathematical Practices

6 Attend to Precision As students refer to the Customary conversion chart to write unit ratios, make sure students are careful about the order of the quantities in their ratio. Remind them that in a unit ratio, the first quantity is compared to every 1 unit of the second quantity.

Teaching Notes

SLIDE 1

Students should be familiar with the Customary conversions represented in the table. Point out that they can use these conversions to write unit ratios, such as $\frac{3 \text{ feet}}{1 \text{ yard}}$. These unit ratios can help them convert measurements. You may wish to ask students to use the table to generate other unit ratios.

Talk About It!

SLIDE 2

Mathematical Discourse

What are some other unit ratios that you can describe from the conversions listed in the table?
Sample answers: 8 fl oz : 1 c, 2,000 lb : 1 T

DIFFERENTIATE

Language Development Activity ELL

Some of your students may be more familiar with the metric system than the Customary system, as the metric system is the standard throughout most parts of the world. You may want to spend more time reviewing the Customary measurement system for those students who are less familiar with it. Have students use the Internet or another source to research and describe in words how each of the following Customary measurement system relates to one of the standard metric system units. Sample responses are shown.

1 foot 1 foot is about 0.3 meter and 1 meter is about 3.3 feet, which means there are a little over 3 feet in 1 meter.

1 mile 1 mile is about 1.6 kilometers and 1 kilometer is about 0.6 mile, which means there is a little over half a mile in 1 kilometer.

1 inch 1 inch is about 2.5 centimeters and 1 centimeter is about 0.4 inch, which means there is a little under half an inch in 1 centimeter.

Lesson 1-6

Convert Customary Measurement Units

I Can... use ratio reasoning to convert between customary units of measurement.

What Vocabulary Will You Learn?
unit ratio

Learn Unit Ratios and Measurement Conversions

The table shows the Customary measurement conversions of length, weight, and capacity.

Customary Conversions

Type of Measure	Larger Unit	→	Smaller Unit
Length	1 foot (ft)	=	12 inches (in.)
	1 yard (yd)	=	3 feet
	1 mile (mi)	=	5,280 feet
Weight	1 pound (lb)	=	16 ounces (oz)
	1 ton (T)	=	2,000 pounds
Capacity	1 cup (c)	=	8 fluid ounces (fl oz)
	1 pint (pt)	=	2 cups
	1 quart (qt)	=	2 pints
	1 gallon (gal)	=	4 quarts

Each relationship listed in the table is a ratio relationship. Because there are 12 inches for every 1 foot, the relationship between number of inches and number of feet is a ratio relationship. The ratio of inches to feet is 12 : 1 or 12 to 1.

A **unit ratio** is a ratio in which the first quantity is compared to 1 unit of the second quantity. Each of the conversions can be written as unit ratios. Some examples of unit ratios are shown.

inches to feet	12 : 1
feet to yards	3 : 1
feet to miles	5,280 : 1

What unit ratio can you use to represent the relationship between ounces and pounds? 16 : 1

What unit ratio can you use to represent the relationship between pints and quarts? 2 : 1

What unit ratio can you use to represent the relationship between feet and miles? 5,280 : 1

Talk About It!
What are some other unit ratios that you can describe from the conversions listed in the table?
Sample answers: 8 fl oz : 1 c, 2,000 lb : 1 T

Lesson 1-6 • Convert Customary Measurement Units 47

Interactive Presentation

Customary Conversions

Type of Measure	Larger Unit	→	Smaller Unit
Length	1 foot (ft)	=	12 inches (in.)
	1 yard (yd)	=	3 feet
	1 mile (mi)	=	5,280 feet
Weight	1 pound (lb)	=	16 ounces (oz)
	1 ton (T)	=	2,000 pounds
Capacity	1 cup (c)	=	8 fluid ounces (fl oz)
	1 pint (pt)	=	2 cups
	1 quart (qt)	=	2 pints
	1 gallon (gal)	=	4 quarts

Learn, Unit Ratios and Measurement Conversions, Slide 1 of 2

TYPE

On Slide 2, students enter the unit ratios that represent each relationship.

Lesson 1-6

Convert Customary Measurement Units

6.RP.A.3

LESSON GOAL

Students will use ratio reasoning to convert between customary units of measurement.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Unit Ratios and Measurement Conversions
Learn: Convert Larger Units to Smaller Units
Example 1: Convert Larger Units to Smaller Units
Learn: Convert Smaller Units to Larger Units
Example 2: Convert Smaller Units to Larger Units
Apply: Soccer Practice

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 6 of the *Language Development Handbook* to help your students build mathematical language related to using ratio reasoning to convert measurements.

ELL You can use the tips and suggestions on page T6 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by solving real-world problems involving ratios and measurement units.
Standards for Mathematical Content: **6.RP .A.3, 6.RP.A.3.D,** *Also addresses 6.RP.A.1*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students solved real-world problems involving ratios.
6.RP.A.3, 6.RP.A.3.B

Now
Students use ratio reasoning to convert between customary units of measurement.
6.RP.A.3, 6.RP.A.3.D

Next
Students will use rates and unit rates to compare quantities.
6.RP.A.2, 6.RP.A.3, 6.RP.A.3.A, 6.RP.A.3.B

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their *understanding* of ratios and unit rates to the relationships among Customary measurement units of length, weight, and capacity. They build *fluency* with converting measurements within the Customary measurement system and *apply* their understanding of these measurement conversions to solve real-world problems.

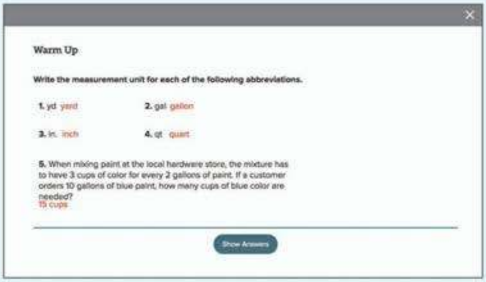
Mathematical Background

Unit ratios can be used to convert measurement units within the Customary measurement system.

- To convert a measurement to a smaller unit, multiply by the unit ratio.
- To convert a measurement to a larger unit, divide by the unit ratio.

Multiplying (or dividing) by a unit ratio is mathematically equivalent to using equivalent ratios to convert between units of measure.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- identifying abbreviations for Customary measurement units (Exercises 1–4)
- solving real-world problems involving equivalent ratios (Exercise 5)

Answers

1. yard
2. gallon
3. inch
4. quart
5. 15 cups

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about systems and units of measurement.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Given what you know about the terms *unit* and *ratio*, what can you infer about a *unit ratio*? Sample answer: A unit ratio will compare one quantity to one unit of another quantity.

msheldrake/123RF

Your Notes

Think About It!
Do you think the number of fluid ounces will be less than, greater than, or equal to 6? Why?

greater than; Sample answer: Fluid ounces is a smaller unit of measure, so there will be a greater quantity of that unit.

Talk About It!
Explain why the number of fluid ounces, 96, is greater than the number of pints, 6.

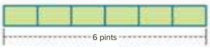
Sample answer: Fluid ounces is a smaller unit than pints. When converting a larger unit into a smaller unit, the number of smaller units will always be greater than the number of larger units.

Learn Convert Larger Units to Smaller Units

Y ou can use reasoning about ratios to convert a measurement from a larger unit to a smaller unit. The numerical value of the measurement is greater when a smaller unit is used. T o see why, consider the following problem. Suppose you want to know how many fluid ounces are in 6 pints.

Method 1 Use a bar diagram.

Step 1 Draw a bar to represent 6 pints.

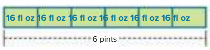


Divide the bar into six equal sections. Each section represents 1 pint.

Step 2 Find the number of cups.

Label each section as 2 cups, because there are 2 cups for every 1 pint.

Step 3 Find the number of fluid ounces.



For every 1 cup, there are 8 fluid ounces. This means that for every 2 cups, there are 16 fluid ounces.

Multiply 6 by 16 to find the number of fluid ounces that are in 6 pints. Because $6 \times 16 = 96$, there are 96 fluid ounces in 6 pints.

Method 2 Use unit ratios and equivalent ratios.

Step 1 Convert 6 pints to cups.

There are 2 cups in every 1 pint. The unit ratio of cups to pints is 2 : 1. Let *c* represent the unknown number of cups that are in 6 pints.

cups/pints $\frac{2}{1} = \frac{c}{6}$ cups/pints

$\frac{2}{1} \underset{\times 6}{\overset{\times 6}{=}} \frac{12}{6}$

Because $1 \times 6 = 6$, multiply 2 by 6 to find the value of *c*. There are 12 cups.

(continued on next page)

Interactive Presentation

Learn, Convert Larger Units to Smaller Units, Slide 2 of 3

CLICK

On Slide 2, students move through the slides to use a bar diagram to convert a larger unit to a smaller unit.

Learn Convert Larger Units to Smaller Units

Objective

Students will understand that they can use bar diagrams and unit ratios to convert larger units to smaller units.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students use reasoning about ratios to convert a measurement from a larger unit to a smaller unit, encourage them to make sense of the relationships between the unit measurements. Students should begin to understand and be able to explain the steps of the conversion more clearly as they progress through the Learn, so that they can then reason about the advantages and disadvantages of using bar diagrams, and unit ratios/equivalent ratios to solve problems.

5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 3, encourage them to reason about when it might be more beneficial to use equivalent ratios rather than a bar diagram. While a bar diagram helps to visualize the problem, if there are more than two conversions that are needed, the bar diagram might become complex and difficult to understand.

Teaching Notes

SLIDE 1

Students may need a reminder that the unit ratio *8 fluid ounces for every 1 cup* is used in the conversion, however, the bar diagram is divided into six equal sections, where each section represents 2 cups (1 pint). Because there are 8 fluid ounces in 1 cup, there are 16 fluid ounces in 2 cups. So, each section should be labeled as 2 cups.

SLIDE 2

Students should be familiar with setting up and using equivalent ratios to solve ratio problems. Ask students if they would arrive at the same solution to the problem if they had set up the equivalent ratios as $\frac{1}{2} = \frac{6}{c}$. Students should be able to reason that their solution would be the same, because the same ratio reasoning is applied.

Talk About It!

SLIDE 2

Mathematical Discourse

Explain why the number of fluid ounces, 96, is greater than the number of pints, 6. Sample answer: Fluid ounces is a smaller unit than pints. When converting a larger unit into a smaller unit, the number of smaller units will always be greater than the number of larger units.

(continued on next page)

Learn Convert Larger Units to Smaller Units *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

Compare the use of the bar diagram to using equivalent ratios. Which one is more advantageous to use to visualize the relationship? Sample answer: Both methods use the relationship between pints and cups and the relationship between cups and ounces. However, when you have to use more than 2 unit ratios, it might be better to use equivalent ratios.

Example 1 Convert Larger Units to Smaller Units

Objective

Students will use ratio reasoning to convert larger measurement units in the Customary system to smaller measurement units.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to check to make sure that the answer, 8 cups, makes sense in the context of the problem. They should use reasoning to make sense of each conversion from gallons to cups.

6 Attend to Precision Students should be careful about specifying the units of measure in each step of the problem.

Questions for Mathematical Discourse

SLIDE 2

AL What unit of measure are you given? What do you need to find? I know the gallons and I need to find the number of cups.

OL Why would we first convert gallons to quarts? Sample answer: I know the conversion from gallons to quarts, but not necessarily the conversion from gallons directly to cups.

OL How can you use reasoning to convert $\frac{1}{2}$ gallon to quarts? Sample answer: If one gallon is equal to 4 quarts, then half of a gallon is equal to 2 quarts.

BL Make an argument for how you could directly convert $\frac{1}{2}$ gallon to cups. Sample answer: One gallon equals $4 \times 2 \times 2$, or 16 cups. So, half of a gallon is equal to 8 cups.

(continued on next page)

Step 2 Convert 12 cups to fluid ounces.

There are 8 fluid ounces in every 1 cup. The unit ratio of fluid ounces to cups is 8 : 1. Let *f* represent the unknown number of fluid ounces.

fluid ounces → $\frac{8}{1} = \frac{f}{12}$ ← fluid ounces
cups → ← cups

$\frac{8}{1} = \frac{96}{12}$ (×12)

Because $1 \times 12 = 12$, multiply 8 by 12 to find the value of *f*. There are 96 fluid ounces.

Using either method, there are 96 fluid ounces in 6 pints.

Example 1 Convert Larger Units to Smaller Units

Marco needs to mix $\frac{1}{2}$ gallon of fertilizer with some soil before planting his tulip bulbs.

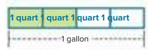
How many cups of fertilizer should Marco use?

Method 1 Use a bar diagram.

Step 1 Draw a bar to represent 1 gallon.

Divide the bar into two equal sections. Shade one section to represent $\frac{1}{2}$ gallon.

Step 2 Find the number of quarts.



There are 4 quarts in 1 gallon so there are 2 quarts in a $\frac{1}{2}$ gallon. Divide each half into two sections. Label each section as 1 quart.

Step 3 Find the number of pints.

For every 1 quart, there are 2 pints. Label each section as 2 pints.

(continued on next page)

Talk About It!
Compare the use of the bar diagram to using equivalent ratios. Which method is more advantageous to use to visualize the relationship?

Sample answer: Both methods use the relationship between pints and cups and the relationship between cups and ounces. However, when you have to use more than 2 unit ratios, it might be better to use equivalent ratios.

Talk About It!
Suppose Marco needed to find the number of cups that are in $\frac{1}{3}$ gallon. Why might a bar diagram not be the most advantageous method to use in this case?

Sample answer: There are 4 quarts in one gallon, so it might be difficult to divide a bar with 3 sections into 4 sections to show quarts.

Interactive Presentation

Example 1, Convert Larger Units to Smaller Units, Slide 2 of 5

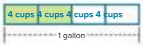
CLICK

On Slide 3 of the Learn, students move through the steps to use equivalent ratios to convert a larger unit to a smaller unit.

CLICK

On Slide 2 of Example 1, students move through the slides to use a bar diagram to convert a larger unit to a smaller unit.

Step 4 Find the number of cups. For every 1 pint, there are 2 cups. This means that for every 2 pints, there are 4 cups.



There are two shaded sections that each represent 4 cups. So there are 2×4 or 8 cups in $\frac{1}{2}$ gallon.

Method 2 Use unit ratios and equivalent ratios.

Step 1 Convert $\frac{1}{2}$ gallon to quarts. There are 4 quarts in every 1 gallon. The unit ratio of quarts to gallons is 4 : 1. Let q represent the unknown number of quarts.

quarts → $\frac{4}{1} = \frac{q}{\frac{1}{2}}$ ← quarts
gallons → ← gallons

$\frac{4}{1} = \frac{2}{\frac{1}{2}}$ (÷2)

Because $1 \div 2 = \frac{1}{2}$, divide 4 by 2 to find the value of q. There are 2 quarts.

Step 2 Convert 2 quarts to pints. There are 2 pints in every 1 quart. The unit ratio of pints to quarts is 2 : 1. Let p represent the unknown number of pints.

pints → $\frac{2}{1} = \frac{p}{2}$ ← pints
quarts → ← quarts

$\frac{2}{1} = \frac{4}{2}$ (×2)

Because $1 \times 2 = 2$, multiply 2 by 2 to find the value of p. There are 4 pints.

Step 3 Convert 4 pints to cups. There are 2 cups in every 1 pint. The unit ratio of cups to pints is 2 : 1. Let c represent the unknown number of cups.

cups → $\frac{2}{1} = \frac{c}{4}$ ← cups
pints → ← pints

$\frac{2}{1} = \frac{8}{4}$ (×4)

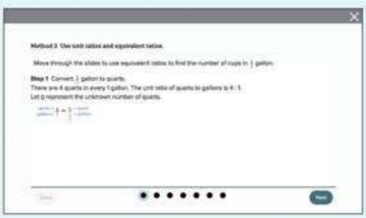
Because $1 \times 4 = 4$, multiply 2 by 4 to find the value of c. There are 8 cups.

So, Marco should use __8__ cups of fertilizer.

Talk About It!
Explain why it makes sense that the number of cups of fertilizer that are in $\frac{1}{2}$ gallon is greater than $\frac{1}{2}$.

Sample answer: Cups is a smaller unit than gallons. When converting a larger unit into a smaller unit, the number of smaller units will always be greater than the number of larger units.

Interactive Presentation



Example 1, Convert Smaller Units to Larger Units, Slide 3 of 5

CLICK

On Slide 3, students move through the slides to use equivalent ratios to convert a larger unit to a smaller unit.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Convert Larger Units to Smaller Units *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL What ratio is given? What unit ratio of the same measurements do you know? $\frac{1}{2}$ gallon to quarts; There are 4 quarts in 1 gallon.

OL How can you use equivalent ratios to find the unknown number of cups? I can set the ratio 4 quarts to 1 gallon equal to the ratio quarts q to $\frac{1}{2}$ gallon, then solve for q. I can continue to set unit ratios of smaller measurements equal to the ratios I find when solving for each new unknown measurement, until I reach the unit of cups.

BL If Marco needed to fertilize the tulip bulbs an additional time, how many total cups of fertilizer would he need? How many pints would that be? 16 cups; 8 pints

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Convert Smaller Units to Larger Units

Objective

Students will understand that they can use bar diagrams and unit ratios to convert smaller units to larger units.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students use reasoning about ratios to convert a measurement from a smaller unit to a larger unit, encourage them to make sense of the similarities and differences in the steps and methods between converting from smaller units to larger units and converting from larger units to smaller units. Students should be able to reason about the advantages and disadvantages of using bar diagrams, and unit ratios/equivalent ratios to solve problems.

5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 4, encourage them to use reasoning about the number of sections a bar diagram would need to have in order to convert 126 inches to yards.

Teaching Notes

SLIDE 1

Students should be familiar with the customary conversions represented in the table. Point out that they can use these conversions to write equivalent ratios, such as $\frac{1\text{ yd}}{3\text{ ft}}$ and $\frac{3\text{ ft}}{1\text{ yd}}$. As students will learn later in this Learn, they should choose the ratio that will allow them to divide out the common units.

SLIDE 2

Students may need help with determining which units to label as they create their bar diagram. Encourage them to think about the progression of unit measurements from smaller to larger. You may wish to ask them what is the next largest unit from inches, then the next largest unit from feet, and so on. You may wish to have them refer to the conversion chart at the beginning of this lesson as well.

(continued on next page)

Check

How many ounces are in $\frac{1}{4}$ pound? **4 oz**

Show your work here

Go Online Y ou can complete an Extra Example online.

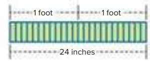
Learn Convert Smaller Units to Larger Units

Y ou can use reasoning about ratios to convert a measurement from a smaller unit to a larger unit. The numerical value of the measurement is less when a larger unit is used. T o see why, consider the following problem. Suppose you want to convert 24 inches to yards.

Method 1 Use a bar diagram.

Step 1 Find the number of feet.

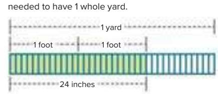
Draw a bar with 24 equal sections to represent 24 inches. For every 12 inches, there is 1 foot. Mark equal increments of 12 inches.



There are 2 whole feet in 24 inches.

Step 2 Find the number of yards.

For every 3 feet, there is 1 yard. There are only 2 feet. Another foot is needed to have 1 whole yard.



There are only two out of three sections shaded. So, there are 24 inches in $\frac{2}{3}$ yard.

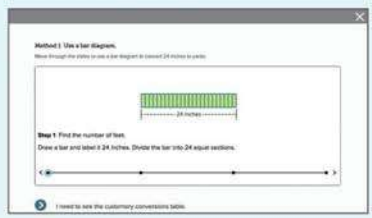
(continued on next page)

Talk About It!

Why might it not always be advantageous to use a bar diagram to convert measurement units? Would you choose to use a bar diagram to convert 126 inches to yards? Why or why not?

Sample answer: I would have to draw 126 sections to convert 126 inches to yards. I wouldn't use a bar diagram because it would take a long time to draw all of those sections. When there are larger numbers, it would be better to use equivalent ratios.

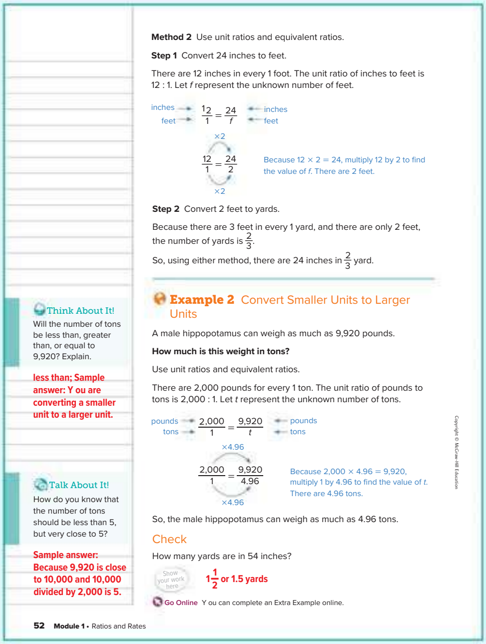
Interactive Presentation



Learn, Convert Smaller Units to Larger Units, Slide 2 of 4

CLICK

On Slide 2, students move through the slides to use a bar diagram to convert a smaller unit to a larger unit.



Method 2 Use unit ratios and equivalent ratios.

Step 1 Convert 24 inches to feet.

There are 12 inches in every 1 foot. The unit ratio of inches to feet is 12 : 1. Let *f* represent the unknown number of feet.

inches → $\frac{12}{1} = \frac{24}{f}$ ← inches
feet → ← feet

$\frac{12}{1} = \frac{24}{2}$ (×2)

Because 12 × 2 = 24, multiply 12 by 2 to find the value of *f*. There are 2 feet.

Step 2 Convert 2 feet to yards.

Because there are 3 feet in every 1 yard, and there are only 2 feet, the number of yards is $\frac{2}{3}$.

So, using either method, there are 24 inches in $\frac{2}{3}$ yard.

Example 2 Convert Smaller Units to Larger Units

A male hippopotamus can weigh as much as 9,920 pounds.

How much is this weight in tons?

Use unit ratios and equivalent ratios.

There are 2,000 pounds for every 1 ton. The unit ratio of pounds to tons is 2,000 : 1. Let *t* represent the unknown number of tons.

pounds → $\frac{2,000}{1} = \frac{9,920}{t}$ ← pounds
tons → ← tons

$\frac{2,000}{1} = \frac{9,920}{4.96}$ (×4.96)

Because 2,000 × 4.96 = 9,920, multiply 1 by 4.96 to find the value of *t*. There are 4.96 tons.

So, the male hippopotamus can weigh as much as 4.96 tons.

Check

How many yards are in 54 inches?

Show your work here

$1\frac{1}{2}$ or 1.5 yards

Go Online You can complete an Extra Example online.

Think About It! Will the number of tons be less than, greater than, or equal to 9,920? Explain.

less than; Sample answer: You are converting a smaller unit to a larger unit.

Talk About It! How do you know that the number of tons should be less than 5, but very close to 5?

Sample answer: Because 9,920 is close to 10,000 and 10,000 divided by 2,000 is 5.

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Interactive Presentation

Example 2, Convert Smaller Units to Larger Units, Slide 2 of 4

CLICK

On Slide 3 of the Learn, students move through the slides to use equivalent ratios to convert a smaller unit to a large unit.

TYPE

On Slide 2 of Example 2, students find the weight in tons.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Convert Smaller Units to Larger Units *(continued)*

Talk About It!

SLIDE 4

Mathematical Discourse

Why might it not always be advantageous to use a bar diagram to convert measurement units? Would you choose to use a bar diagram to convert 126 inches to yards? Why or why not? Sample answer: I would have to draw 126 sections to convert 126 inches to yards. I wouldn't use a bar diagram because it would take a long time to draw all of those sections. When there are larger numbers, it would be better to use equivalent ratios.

Example 2 Convert Smaller Units to Larger Units

Objective

Students will use ratio reasoning to convert smaller measurement units in the Customary system to larger measurement units.

Questions for Mathematical Discourse

SLIDE 2

AL What unit of measure are you given? What do you need to find? I know the pounds and I need to find the number of tons.

OL Why do we need to use the unit ratio of 2,000 pounds for every 1 ton? Sample answer: The unit of measurement given is pounds and I need to find the solution in tons.

OL Use reasoning to explain why the numerical value of the measurement is less than the given value. Sample answer: It takes more of a smaller unit to equal a larger unit, so the opposite is also true. It takes less of a larger unit such as tons, to equal a smaller unit, such as pounds.

BL Make an argument for why converting to a smaller unit is impractical in this real-world problem. Sample answer: A smaller unit such as ounces is impractical to convert to because a hippopotamus is such a large animal, that the number of ounces would be extremely large and not as easy to conceptualize.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Soccer Practice

Objective

Students will come up with their own strategy to solve an application problem involving the amount of water athletes drink during soccer practice.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What units of measurement are given? What unit ratios do you know for those units of measurement?
- Would using a bar diagram or unit ratios and equivalent ratios be more advantageous to use in this scenario?
- How will you find the total cost the coach will spend once you have converted the units to quarts?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Soccer Practice

The table shows the amount of drinking water each athlete drinks during one soccer practice. The coach buys bottles of water for $1.75 that each hold 1 liter of water. If 1 liter is equal to 1,000 milliliters, how much will the coach spend on water for one practice session?

Athlete	Amount (mL)
Deon	475
Sierra	350.5
Carmen	830
Mia	710.5
Ella	504

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$5.25; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! In the metric system, 1 liter = 1,000 milliliters and 1 kiloliter = 1,000 liters. How can you use ratio reasoning when converting measurements within the metric system?

Sample answer: Converting measurements in the metric system is similar to converting measurements in the Customary system. I can use the unit ratio $\frac{1000 \text{ mL}}{1 \text{ L}}$ to convert milliliters to liters.

Lesson 1-6 • Convert Customary Measurement Units 53

Interactive Presentation

Apply, Soccer Practice

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

On Tuesday, Joaquin drank 6 glasses of water each containing 10 fluid ounces. His goal was to drink 2 quarts. How many more fluid ounces does he need to drink in order to reach his goal?

Show your work here

4 fl oz

Go Online You can complete an Extra Example online.

Pause and Reflect

What are the advantages of using a bar diagram to convert Customary measurement units? When might it be more advantageous to use unit ratios and equivalent ratios?

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. How many teaspoons of oregano will you need to serve 16 people? If one teaspoon is one third of a tablespoon, how many tablespoons of oregano will you need to serve 16 people? Write a mathematical argument that can be used to defend your solution.

10 teaspoons; about $3\frac{1}{3}$ tablespoons; Sample answer: Multiply the number of teaspoons, 2.5, by 4, since 4 multiplied by 4 equals 16. This yields 10 teaspoons. Since each teaspoon is one third of a tablespoon, divide 10 by 3. This yields $3\frac{1}{3}$ tablespoons.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–11 odd, 13–16
- ALEKS U.S. Customary Units of Measurement

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–8, 11, 15, 16
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	use ratio reasoning to convert larger measurement units in the Customary system to smaller measurement units	1–4
2	use ratio reasoning to convert smaller measurement units in the Customary system to larger measurement units	5–8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving converting measurement units	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Students may incorrectly use a reciprocal relationship to convert a unit measurement. In Exercise 3, students may incorrectly think that there are 16 gallons in one cup rather than 16 cups in one gallon. Encourage students to understand the different sizes of unit measurements and use that information to determine the correct unit ratio.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Use any strategy to solve each problem. (Examples 1 and 2)

1. Mrs. Menary made $4\frac{1}{2}$ quarts of lemonade for a school party. How many fluid ounces of lemonade did she make?
   144 fluid ounces

2. A class walked 2.5 miles for a walk-a-thon. How many yards did the class walk?
   4,400 yards

3. The Martinez family has $\frac{3}{4}$ gallon of orange juice in the refrigerator. How many cups of orange juice are in the refrigerator?
   12 cups

4. A grand piano can weigh $\frac{1}{2}$ ton. How many ounces can a grand piano weigh?
   16,000 ounces

5. A female hippopotamus can weigh 48,000 ounces. How many tons can a female hippopotamus weigh?
   $1\frac{1}{2}$ tons

6. At soccer practice, Tracey's best kick traveled a distance of 1,200 inches. For how many yards did she kick the ball?
   $33\frac{1}{3}$ yards

7. An elephant can drink up to 6,400 fluid ounces of water a day. How many gallons of water can an elephant drink per day?
   50 gallons

8. A recipe for ice cream calls for 56 fluid ounces of milk. How many pints of milk are there in the recipe?
   $3\frac{1}{2}$ pints

Test Practice

9. One quart of strawberries weighs about 2 pounds. About how many quarts of strawberries would weigh $\frac{1}{4}$ ton?
   250 quarts

10. **Open Response** A mini fruit juice box contains 4 fluid ounces of juice. You need $2\frac{1}{2}$ quarts of fruit juice. How many mini fruit juice boxes will you need?
   20 mini fruit juice boxes

Apply *indicates multi-step problem

*11. At the grocery store, Mr. Arnett allowed each of his children to fill their own bag with trail mix for their hike. The table shows the amount of trail mix for each child. The trail mix costs $4.50 per pound. How much will Mr. Arnett pay for all the trail mix?

$15.75

Child	Amount of Trail Mix (oz)
Ava	15
Grayson	14
Mason	10
Tyler	17

*12. A hockey player needs to shoot a puck 55 meters from his current location to his opponent's goal to score a goal. After the shot, the puck is 120 centimeters from his opponent's goal. If there are 100 centimeters in 1 meter, how many meters did the puck travel?

53.8 meters

Higher-Order Thinking Problems

13. There are 60 minutes in one hour and 60 seconds in one minute. Using this information, explain how you could convert 20 miles per hour to feet per second.

Sample answer: First, convert 20 miles to feet. There are 5,280 × 20 or 105,600 feet in 20 miles. Then convert one hour to seconds. There are 60 × 60 or 3,600 seconds in one hour. So, $\frac{105{,}600 \text{ ft}}{3{,}600 \text{ s}} \approx \frac{29.3 \text{ ft}}{1 \text{ s}}$ or about 29.3 feet per second.

14. MP **Identify Structure** When converting from larger units such as quarts to smaller units such as cups, will the number of smaller units be greater than the number of larger units? Explain your reasoning.

yes; Sample answer: When converting from larger units to smaller units, there will be more of the smaller units to equal the larger units. For example, there are 4 cups in 1 quart.

15. The table shows the metric system conversions of length.

Larger Unit	→	Smaller Unit
1 kilometer (km)	=	1,000 meters (m)
1 meter	=	100 centimeters (cm)
1 centimeter	=	10 millimeters (mm)

How can you use ratio reasoning to find the number of centimeters in 2.2 kilometers?

Sample answer: I can use the equivalent ratios $\frac{1 \text{ km}}{1000 \text{ m}} = \frac{2.2 \text{ km}}{? \text{ m}}$ to find that 2.2 kilometers is equal to 2,200 meters. I can then use the equivalent ratios $\frac{1 \text{ m}}{100 \text{ cm}} = \frac{2200 \text{ m}}{? \text{ cm}}$ to convert meters to centimeters. So, 2.2 kilometers is equal to 100 × 2,200 or 220,000 centimeters.

16. MP **Find the Error** A student's work for converting 4 gallons to cups is shown. Find the mistake and correct it.

$$\frac{16 \text{ gallons}}{1 \text{ cup}} = \frac{4 \text{ gallons}}{d}$$

So, d is equal to $\frac{1}{4}$ cup.

Sample answer: The student's unit ratio is incorrect. There are 16 cups in one gallon, not 16 gallons in one cup. The correct answer is 4 gallons is equal to 64 cups.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 14, students use the structure and sizes of units to determine how the number of a certain unit will change when converted from a larger unit to a smaller unit.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students will explain why the conversion that another student completed is incorrect.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 11–12 Have students work in pairs. Have students individually read Exercise 11 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 12.

Clearly and precisely explain.

Use with Exercise 16 Have pairs of students prepare and practice their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of methods, such as using unit ratios or bar diagrams, in their responses.

Learn Understand a Rate and a Unit Rate

Objective

Students will understand how to compare quantities using rates and unit rates.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to use reasoning about the quantity 1.5 minutes and how that corresponds to minutes and seconds, or just seconds.

As students discuss the *Talk About It!* question on Slide 4, encourage them to reason about what each bar diagram represents and which one would be more beneficial to use if you wanted to find the unit rate in *minutes per lap* versus *laps per minute*.

Teaching Notes

SLIDE 2

Present the scenario and bar diagram to your students. Ask them to reason about the relationships represented in the bar diagram to find the number of minutes that Luciana can run *for each lap*. Students should be able to reason that Luciana can run each lap in 1.5 minutes, assuming she ran at a constant rate, because $4 \times 1.5 = 6$. Point out that this is a *unit rate*, because the first quantity (minutes) is compared to 1 unit of the second quantity (laps). Students may be familiar with rates and unit rates in their everyday lives, such as a car traveling at 65 miles per hour on the highway. Be sure students understand that a *rate* is a special kind of ratio in which the units are different. Many rates in the real-world involve time as one of the units.

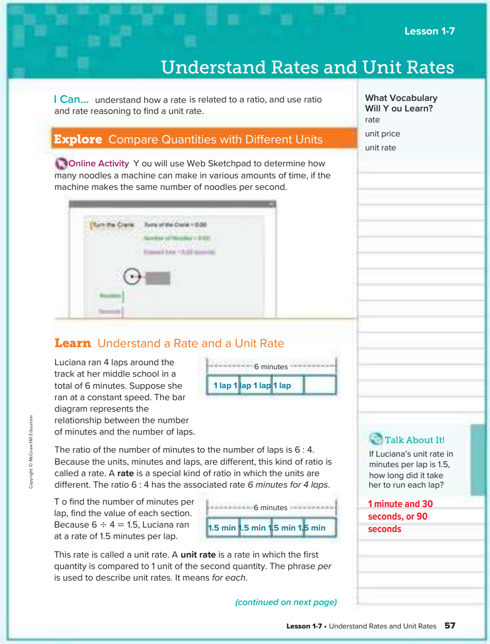
Talk About It!

SLIDE 2

Mathematical Discourse

If Luciana's unit rate in minutes per lap is 1.5, how long did it take her to run each lap? 1 minute and 30 seconds, or 90 seconds

(continued on next page)



Lesson 1-7

Understand Rates and Unit Rates

I Can... understand how a rate is related to a ratio, and use ratio and rate reasoning to find a unit rate.

What Vocabulary Will You Learn?
rate
unit price
unit rate

Explore Compare Quantities with Different Units

Online Activity You will use Web Sketchpad to determine how many noodles a machine can make in various amounts of time, if the machine makes the same number of noodles per second.

Learn Understand a Rate and a Unit Rate

Luciana ran 4 laps around the track at her middle school in a total of 6 minutes. Suppose she ran at a constant speed. The bar diagram represents the relationship between the number of minutes and the number of laps.

6 minutes: 1 lap | 1 lap | 1 lap | 1 lap

The ratio of the number of minutes to the number of laps is 6 : 4. Because the units, minutes and laps, are different, this kind of ratio is called a rate. A **rate** is a special kind of ratio in which the units are different. The ratio 6 : 4 has the associated rate *6 minutes for 4 laps*.

To find the number of minutes per lap, find the value of each section. Because $6 \div 4 = 1.5$, Luciana ran at a rate of 1.5 minutes per lap.

6 minutes: 1.5 min | 1.5 min | 1.5 min | 1.5 min

This rate is called a unit rate. A **unit rate** is a rate in which the first quantity is compared to 1 unit of the second quantity. The phrase *per* is used to describe unit rates. It means *for each*.

Talk About It! If Luciana's unit rate in minutes per lap is 1.5, how long did it take her to run each lap?

1 minute and 30 seconds, or 90 seconds

(continued on next page)

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Lesson 1-7 • Understand Rates and Unit Rates 57

Interactive Presentation

Learn, Understand a Rate and a Unit Rate, Slide 1 of 6

DIFFERENTIATE

Language Development Activity ELL

To further student's understanding of rates and unit rates, have them work with a partner to generate several different rates, some of which (but not all) are unit rates. Have them write each rate on a slip of paper. Then have them trade papers with another pair of students. Each pair should sort the rates as to whether or not they are unit rates, and explain their reasoning. Have the pairs check each other's work, and discuss and resolve any differences.

Lesson 1-7

6.RP.A.2, 6.RP.A.3

Understand Rates and Unit Rates

LESSON GOAL

Students will compare quantities by using unit rates.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Compare Quantities with Different Units

Learn: Understand a Rate and a Unit Rate
Example 1: Find a Unit Rate
Learn: Unit Price
Example 2: Find a Unit Price
Apply: Travel

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 7 of the *Language Development Handbook* to help your students build mathematical language related to understanding rates and unit rates.

ELL You can use the tips and suggestions on page T7 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by solving problems by finding unit rates to compare quantities.
Standards for Mathematical Content: **6.RP .A.2, 6.RP.A.3, 6.RP.A.3.A, 6.RP.A.3.B**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6**

Coherence

Vertical Alignment

Previous
Students used ratio reasoning to convert between Customary units of measurement.
6.RP.A.3, 6.RP.A.3.D

Now
Students use rates and unit rates to compare quantities.
6.RP.A.2, 6.RP.A.3, 6.RP.A.3.A, 6.RP.A.3.B

Next
Students will solve real-world problems involving rates.
6.RP.A.2, 6.RP.A.3, 6.RP.A.3.B

Rigor

The Three Pillars of Rigor

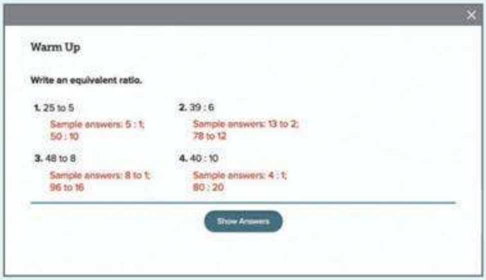
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of ratio relationships through rates and unit rates. They learn to use rate language to describe the relationships between quantities and start to build *fluency* with finding rates and unit rates. They *apply* their understanding of rates and unit rates to solve real-world problems.

Mathematical Background

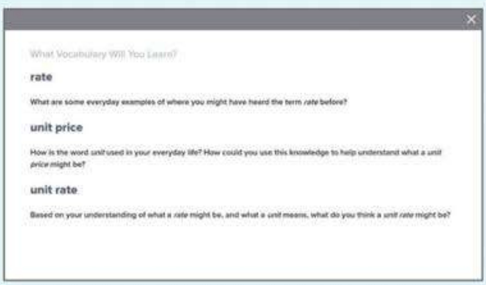
A *rate* is a ratio that compares two quantities with different types of units. A *unit rate* is a rate in which the first quantity is compared to 1 unit of the second quantity. The phrase *per* is used to describe unit rates. It means *for each*. Unit rates are used to solve problems involving best buys, unit prices, and finding other rates with the same unit rate.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing equivalent ratios (Exercises 1–4)

Answers

1. Sample answers: 5 to 1; 50 to 10
2. Sample answers: 13 to 2; 78 to 12
3. Sample answers: 6 to 1; 96 to 16
4. Sample answers: 4 to 1; 80 to 20

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about different-sized packages and prices of some food items in grocery stores.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What are some everyday examples of where you might have heard the term *rate* before? Sample answer: running or traveling at a fast or slow rate, or speed
- How is the word *unit* used in your everyday life? How could you use this knowledge to help understand what a *unit price* might be? Sample answer: We measure (length, weight, area, etc.) using units. A *unit price* may be the price for one item.
- Based on your understanding of what a rate might be, and what a unit means, what do you think a unit rate might be? Sample answer: A unit rate might be the rate per one quantity of something, such as the speed at which someone can run in one minute, or one hour.

Explore Compare Quantities with Different Units

Objective

Students will use Web Sketchpad to explore comparing quantities with different units.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the sketch that shows a crank that takes a certain amount of time and produces a certain number of noodles. They will write the relationship as a ratio before finding equivalent ratios. Throughout this activity, students will apply what they know to explore the idea of a unit rate.

Inquiry Question

How can you compare quantities with different units? Sample answer: I can use rates and unit rates to compare quantities with different units.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

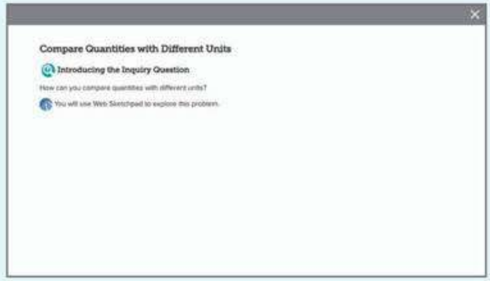
Mathematical Discourse

The units of the two quantities are different. This type of relationship is called a *rate*. A rate is a special type of ratio in which the two quantities being compared are different. Discuss some real-world examples of rates. 5 packages of cookies for $10

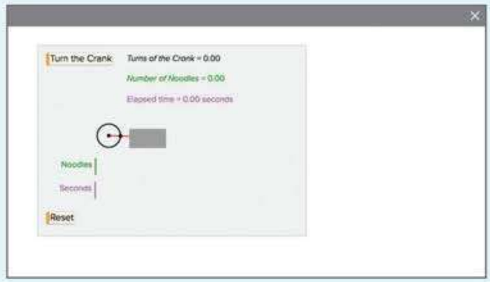
The machine made 7 noodles in 4 seconds. This is a constant rate. What do you think constant rate means? Sample answer: The machine is consistent. It produces the same number of noodles in a minute. This number does not change.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6

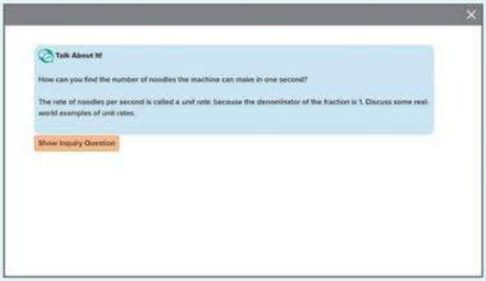


Explore, Slide 4 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to compare quantities with different rates.

Interactive Presentation



Explore, Slide 5 of 6

TYPE

a|

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Compare Quantities with Different Units *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to reason about the relationships between the elapsed time and the number of noodles produced by the machine.

6 Attend to Precision Students should be precise when talking about the different kinds of units, noodles, and seconds.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

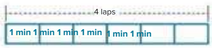
SLIDE 4

Mathematical Discourse

Without turning the crank, predict how many noodles the machine can make in 32 seconds. Explain. 56 noodles; Sample answer: If the machine makes 7 noodles in 4 seconds, it will make 7 × 8 or 56 noodles in 4 × 8 or 32 seconds.

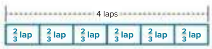
Your Notes

Luciana ran 4 laps in 6 minutes. Suppose you want to find how many laps she can run in 1 minute, at this same rate. The bar diagram represents the relationship between the number of laps, 4, and the number of minutes, 6.



The ratio of the number of laps to the number of minutes is 4 : 6, because Luciana ran 4 laps in 6 minutes. The ratio 4 : 6 has the associated rate *4 laps in 6 minutes*.

To find the number of laps per minute, find the value of each section. Because $4 \div 6 = \frac{4}{6}$, or $\frac{2}{3}$, Luciana ran at a rate of $\frac{2}{3}$ lap per minute.



The table summarizes ratios, rates, and unit rates.

Ratio		
Words	**Units**	**Examples**
a comparison between two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity	units can be alike or different	6 laps to 4 laps 6 : 4 4 laps in 6 minutes 4 : 6

Rate		
Words	**Units**	**Examples**
a special kind of ratio in which the units are different	units are different	6 minutes for 4 laps 4 laps in 6 minutes

Unit Rate		
Words	**Units**	**Examples**
a rate in which the first quantity is given for every 1 unit of the second quantity	units are different	1.5 minutes per lap $\frac{2}{3}$ lap per minute

Talk About It! How does this bar diagram compare to the one on the previous page? Do they represent the same relationship between the two quantities?

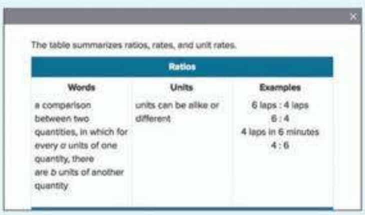
Sample answer: Both bar diagrams represent the same relationship between the number of laps and number of minutes. The choice to use either diagram depends on which unit rate you want to find (minutes per lap, or laps per minute).

Talk About It! Which unit rate, minutes per lap or laps per minute, would be helpful if you wanted to predict how many minutes it will take Luciana, at that rate, to run 5 laps? Why?

1.5 minutes per lap; Sample answer: I can multiply 1.5 by 5 to find the number of minutes, 7 .5.

58 Module 1 • Ratios and Rates

Interactive Presentation



Learn, Compare Quantities using Rates and Unit Rates, Slide 5 of 6

Learn Understand a Rate and a Unit Rate *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

How does this bar diagram compare to the one on the previous page? Do they represent the same relationship between the two quantities? Sample answer: Both bar diagrams represent the same relationship between number of laps and number of minutes. The choice to use either diagram depends on which unit rate you want to find (minutes per lap, or laps per minute).

Teaching Notes

SLIDE 4

Present the scenario and bar diagram to your students. Ask them to reason about the relationships represented in the bar diagram to find the number of laps that Luciana can run for each minute. Students should be able to reason that Luciana can run $\frac{2}{3}$ lap for each minute, assuming she ran at a constant rate, because $4 \div 6 = \frac{2}{3}$. If students struggle to reason, you may want to have them refer to their process earlier in this Learn, to note the use of division.

Talk About It!

SLIDE 6

Mathematical Discourse

Which unit rate, minutes per lap or laps per minute, would be helpful if you wanted to predict how many minutes it will take Luciana, at that rate, to run 5 laps? Why? 1.5 minutes per lap; Sample answer: I can multiply 1.5 by 5 to find the number of minutes, 7.5.

Go Online to find additional teaching notes.

DIFFERENTIATE

Language Development Activity ELL

Encourage students to spend time studying the table presented in the Learn that summarizes ratios, rates, and unit rates. Have students work with a partner to create a graphic organizer that includes examples of real-world ratios, rates, and unit rates. Have them draw a bar diagram that illustrates the relationship. Have them present their graphic organizers to another pair of students, and discuss and resolve any questions or discrepancies. You may wish to have students display their graphic organizers around the room.

Example 1 Find a Unit Rate

Objective

Students will find a unit rate in order to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should interpret the mathematical results within the context of the real-world problem and see whether the results make sense.

As students discuss the *Talk About It!* question on Slide 4, encourage them to understand the meaning of each of the quantities in the rate and use the rate to solve the problem.

Questions for Mathematical Discourse

SLIDE 2

AL Is the rate *1,590 wing flaps in 30 seconds* a unit rate? Explain. no; Sample answer: The unit rate should be written as the number of wing flaps *per* second (in 1 second).

OL Use reasoning to estimate the unit rate, in wing flaps per second, without performing any calculations. Sample answer: 1,590 is a little over fifty times 30, so the unit rate will be a little over 50.

BL Use reasoning to find the unit rate in flaps per second. 53 flaps per second

SLIDE 3

AL What must the denominator be of the rate that represents the unit rate? The denominator of any rate that represents a unit r ate should be the number 1.

OL Why do we divide both the numerator and denominator of the second rate by 30? The denominator in the rate needs to be 1, so that the equivalent rate is a unit rate; 30 divided by 30 is equal to 1.

BL How many wing flaps would the hummingbird have in 30 seconds, if its unit rate is 48 flaps per second? Explain how you solved the problem. 1,440 flaps; Sample answer: $\frac{48 \text{ flaps}}{1 \text{ second}} = \frac{s}{30 \text{ seconds}}$, $48 \times 30 = 1{,}440$ flaps

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 1 Find a Unit Rate

A scientist studying hummingbirds recorded that a hummingbird flapped its wings 1,590 times in 30 seconds during normal flight.

Assuming a constant rate, how many times did the hummingbird flap its wings per second?

Method 1 Use a ratio table.

Create a ratio table with the given information.

Scale backward to find the number of wing flaps per second.

Number of Wing Flaps	53	1,590
Number of Seconds	1	30

(÷30)

Method 2 Use equivalent rates.

Write and solve an equation stating that two rates are equivalent. Let s represent the unknown number of wing flaps per second.

wing flaps / seconds → $\frac{s}{1} = \frac{1{,}590}{30}$ ← wing flaps / seconds

(÷30) $\frac{s}{1} = \frac{1{,}590}{30}$ Because 30 ÷ 30 = 1, divide 1,590 by 30 to find the value of s.

(÷30) $\frac{53}{1} = \frac{1{,}590}{30}$ 1,590 ÷ 30 = 53; So, s = 53.

So, using either method, the hummingbird flapped its wings at a rate of 53 flaps per second.

Think About It! Why might a bar diagram not be the best method to use to find the unit rate?

Sample answer: The quantities given, 1,590 and 30, are large, so drawing a bar diagram would not be efficient.

Talk About It! At this rate, how many times would the hummingbird flap its wings in 2 minutes? Justify your response.

6,360 times; Sample answer: The number of flaps per minute is found by multiplying 53 flaps per second by 60 seconds. Then that unit rate is multiplied by 2, to find the number of laps in 2 minutes.

Check

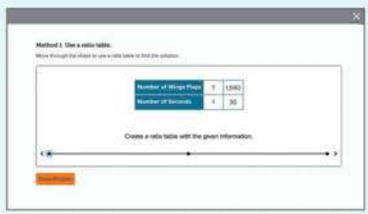
Refer to Example 1. The scientist also recorded that the hummingbird took 6,250 breaths over a period of 25 minutes. Assuming a constant rate, how many breaths per minute did the hummingbird take?

Show your work here

250 breaths per minute

Go Online You can complete an Extra Example online.

Interactive Presentation

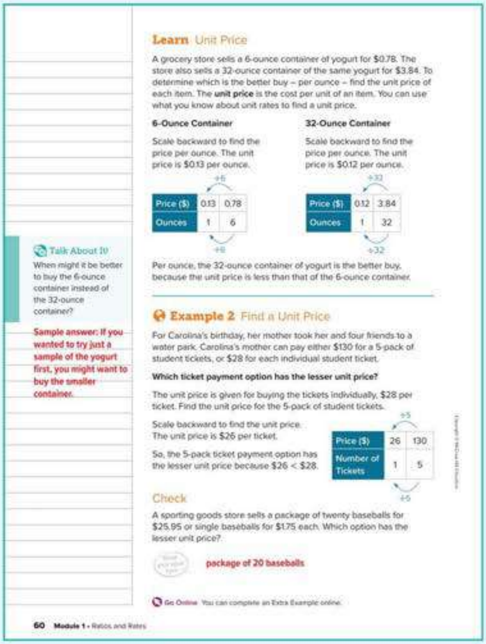


Example 1, Find a Unit Rate, Slide 2 of 5

CLICK On Slide 2, students move through the slides to use a ratio table to find the unit rate.

CLICK On Slide 3, students move through the steps to use equivalent rates to find the unit rate.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Learn Unit Price

A grocery store sells a 6-ounce container of yogurt for $0.78. The store also sells a 32-ounce container of the same yogurt for $3.84. To determine which is the better buy – per ounce – find the unit price of each item. The **unit price** is the cost per unit of an item. You can use what you know about unit rates to find a unit price.

6-Ounce Container

Scale backward to find the price per ounce. The unit price is $0.13 per ounce.

Price ($)	0.13	0.78
Ounces	1	6

÷6

32-Ounce Container

Scale backward to find the price per ounce. The unit price is $0.12 per ounce.

Price ($)	0.12	3.84
Ounces	1	32

÷32

Per ounce, the 32-ounce container of yogurt is the better buy, because the unit price is less than that of the 6-ounce container.

Talk About It! When might it be better to buy the 6-ounce container instead of the 32-ounce container?

Sample answer: If you wanted to try just a sample of the yogurt first, you might want to buy the smaller container.

Example 2 Find a Unit Price

For Carolina's birthday, her mother took her and four friends to a water park. Carolina's mother can pay either $130 for a 5-pack of student tickets, or $28 for each individual student ticket.

Which ticket payment option has the lesser unit price?

The unit price is given for buying the tickets individually, $28 per ticket. Find the unit price for the 5-pack of student tickets.

Scale backward to find the unit price. The unit price is $26 per ticket.

So, the 5-pack ticket payment option has the lesser unit price because $26 < $28.

Price ($)	26	130
Number of Tickets	1	5

÷5

Check

A sporting goods store sells a package of twenty baseballs for $25.95 or single baseballs for $1.75 each. Which option has the lesser unit price?

package of 20 baseballs

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 2, Find a Unit Price, Slide 2 of 3

TYPE

On Slide 2 of Example 2, students determine the unit price of dollars per ticket.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Unit Price

Objective

Students will learn how to find unit price in order to solve a real-world problem.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide1, encourage them to construct a plausible argument for why someone might choose to purchase the container with the greatest cost per ounce.

Teaching Notes

SLIDE 1

Present the real-world scenario to students and have them discuss how they can use what they know about *unit rates* to find the *unit price*. You may also wish to have a discussion about other situations students may have encountered that involve unit prices in their everyday lives.

Talk About It!

SLIDE 1

Mathematical Discourse

When might it be better to buy the 6-ounce container instead of the 32-ounce container? Sample answer: If you wanted to try just a sample of the yogurt first, you might want to buy the smaller container.

Example 2 Find a Unit Price

Objective

Students will find a unit price in order to solve a real-world problem.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you divide both the first quantity and second quantity by 5? I need to find the cost per ticket, and there are 5 tickets.

OL Why was it important to find the unit price? Sample answer: It is important to find the unit price so that the two purchase options can be compared to find the lesser price.

BL At the same unit price, what would the number of tickets need to be in order for the total cost to be $208? 8 tickets

Go Online

- Find additional teaching notes and the mathematical practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Travel

Objective

Students will come up with their own strategy to solve an application problem involving travel speeds.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

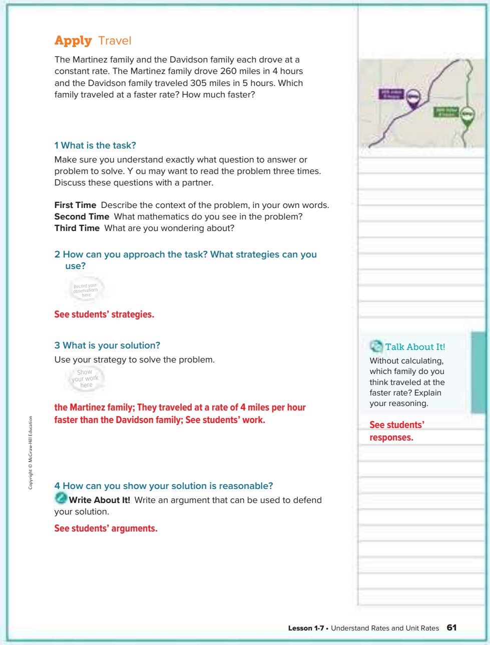
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What tools can you use to solve the problem?
- How might the unit rate help you?
- How can you compare the rates?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Travel

The Martinez family and the Davidson family each drove at a constant rate. The Martinez family drove 260 miles in 4 hours and the Davidson family traveled 305 miles in 5 hours. Which family traveled at a faster rate? How much faster?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

the Martinez family; They traveled at a rate of 4 miles per hour faster than the Davidson family; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Without calculating, which family do you think traveled at the faster rate? Explain your reasoning.

See students' responses.

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Lesson 1-7 • Understand Rates and Unit Rates 61

Interactive Presentation

Apply, Travel

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A runner is training for a half marathon. On Wednesday, she ran 6 miles in 50 minutes. On Thursday, she ran 4 miles in 32 minutes. Assume she ran at a constant rate each day. On which day did she run faster? By how much faster did she run?

Show your work here

Thursday; 0.05 mile per minute faster

Go Online You can complete an Extra Example online.

Pause and Reflect

How did what you learned in this lesson relate to a previous lesson or lessons in this module?

Record your observations here

See students' responses.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you describe how two quantities are related?

In this lesson, students learned how to compare quantities using rates and unit rates. Encourage them to work with a partner to compare and contrast ratios, rates, and unit rates.

Exit Ticket

Refer to the Exit Ticket slide. Which box is the better buy? Write a mathematical argument that can be used to defend your solution. The second box is the better buy; Sample answer: The unit price for the first box is $0.49 per bag. The unit price for the second box is $0.45 per bag, which is a lesser unit price.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 11–14
- ALEKS Ratios and Unit Rates

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 9, 12, 13
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find the unit rate in order to solve a real-world problem	1–4
2	find the unit price in order to solve a real-world problem	5–7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving rates and unit rates	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

When writing and solving an equation using equivalent rates, students might set up the given rate incorrectly. They might not keep the corresponding units in the same location across the equation. For example, in Exercise 4, students might incorrectly write the equation as $\frac{p}{1} = \frac{10}{4}$, where p = pounds. Students who consistently write the equation in this way might find greater success using a ratio table, where the units are more clearly labeled, thus making it easier to organize the data.

Name ________ Period ________ Date ________

Practice

Go Online Y ou can complete your homework online.

Use any strategy to solve each problem.

1. A hippopotamus can run 6 kilometers in 15 minutes. At this rate, how far can the hippopotamus run in 1 minute? (Example 1)

0.4 km per min

2. Imena earned $261 last week. If she worked 18 hours and earned the same amount each hour, how much was she paid per hour? (Example 1)

$14.50 per hour

3. A cat's heart beats approximately 45 times in 15 seconds. At this rate how many times does the cat's heart beat per second? (Example 1)

3 beats per second

4. Mr. Farley used 4 pounds of hamburger to make 10 hamburger patties of the same size. How many pounds of hamburger did he use per patty? (Example 1)

0.4 lb per hamburger patty

5. At the school festival, Heather can buy 25 game tickets for $10, or she can pay $0.50 per game ticket. Which option has the lesser price per ticket? (Example 2)

25 game tickets for $10

6. At a toy store, Colton canbuy a package of 6 mini footballs for $7 .50, or a package of 8 mini footballs for $9.60. Which option has the lesser price per mini football? (Example 2)

8 mini footballs for $9.60

7. The table shows the options Zoe's mother has for buying tickets to an adventure day camp for Zoe and 5 of her friends. Which option has the lesser cost per student ticket? (Example 2)

Adventure Camp Tickets	
Option	**Cost ($)**
6-pack of Student Tickets	126.00
Individual Student Ticket	21.50

6-pack of Student Tickets

T est Practice

8. Multiple Choice Which of the following offers the least price per ounce of shampoo?

A. $6 for 8 ounces of shampoo
B. $4 for 5 ounces of shampoo
C. $8 for 12 ounces of shampoo (selected)
D. $12 for 16 ounces of shampoo

Lesson 1-7 • Understand Rates and Unit Rates 63

Apply *indicates multi-step problem

*9. Nolan found two stores that sell filled party favor bags. The table shows his options. Which store has the lesser cost per filled bag? How much less?

Store	Number of Bags	Cost ($)
Party R Us	8	12
Celebrations	12	21

Party R Us; $0.25 less

*10. The Houck family and Roberts family took trains for their family vacations, traveling at constant rates. The Houck family's train traveled 552 miles in 6 hours and the Roberts family's train traveled 744 miles in 8 hours. Which family's train is traveling at a faster rate? How much faster?

Roberts family; 1 mph faster

Higher-Order Thinking Problems

11. Caleb paid $4.50 for 12 bagels. Describe a unit price for bagels that is greater than the unit price Caleb paid.

Sample answer: 1 bagel for $0.50

12. MP **Find the Error** A large box of spaghetti noodles contains 3 pounds and costs $2.40. A student said the unit cost is $1.20 per pound. Is the student correct? Explain.

no; Sample answer: The unit cost is equal to the cost divided by the number of pounds of spaghetti: $\frac{\$2.40}{3}$ or $0.80 per pound.

13. MP **Justify Conclusions** If you travel at a rate of 60 miles per hour, how many minutes will it take you to travel 1 mile? Write an argument that can be used to justify your conclusion.

1 min; Sample answer: There are 60 minutes in 1 hour, so 1 mile per minute is equivalent to 60 miles per hour.

14. MP **Reason Inductively** Suppose two boxes of cereal contain the same number of ounces but cost different amounts. Without computing, how can you determine which cereal will cost more per ounce of cereal? Explain.

Sample answer: If the number of ounces are the same, then the box that costs more will cost more per ounce.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others
In Exercise 12, students will critique the reasoning of another student who found an incorrect unit price.

In Exercise 13, students will find a unit rate using a different unit than the one given in the problem and will justify their answer by presenting a reasoned defense.

2 Reason Abstractly and Quantitatively In Exercise 14, students will reason about the method used to determine which box of cereal will cost more per ounce.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.
Use with Exercises 9–10 Have students work in groups of 3–4. After completing Exercise 9, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 10.

Make sense of the problem.
Use with Exercise 12 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that the spaghetti costs $1.20 per pound. Have each pair or group of students present their explanations to the class.

Learn Use Bar Diagrams to Solve Rate Problems

Objective

Students will understand that they can use bar diagrams to model and solve a real-world problem involving rates.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to understand that there are two rates presented in the Learn, so they need to draw two bar diagrams, one for each person. Once they correctly find the unit rate, they still need to find how many more miles Santiago can drive in 9 hours than Destiny.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use clear and precise mathematical language to explain how they can solve the problem another way.

Teaching Notes

SLIDE 1

Students may not readily see this problem as having multiple steps. Some students may correctly model the situation using a bar diagram, but neglect to finish the steps needed in order to answer the question. Encourage students to return to the problem scenario as they progress through the steps, to ensure that they use the unit rate found through use of the bar diagram, to find how many more miles Santiago can drive in 9 hours.

Talk About It!

SLIDE 2

Mathematical Discourse

Can you solve this rate problem another way? Explain. yes; Sample answer: After finding the unit rates, I can multiply each unit rate by 9 to determine the distance each person can drive in 9 hours. Then I can subtract the lesser distance from the greater distance to find how much farther Santiago can drive in 9 hours.

Lesson 1-8

Solve Rate Problems

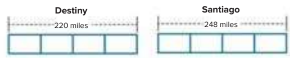
I Can... solve real-world problems involving rates and unit rates by using bar diagrams, double number lines, and equivalent rates.

Learn Use Bar Diagrams to Solve Rate Problems

Destiny drove 220 miles in 4 hours. Santiago drove 248 miles in 4 hours. At these rates, how many more miles can Santiago drive in 9 hours than Destiny? You can create bar diagrams to solve this rate problem.

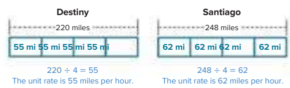
Step 1 Construct bar diagrams to represent the rates.

Draw two bars. Each bar represents the number of miles each person drove in 4 hours. Because each person drove 4 hours, divide each bar into 4 equal-size sections. Each section represents 1 hour.



Step 2 Find the unit rates.

Divide the total number of miles each person drove by the number of sections in the diagram to find the unit rate, the number of miles they drove per hour.



Destiny's unit rate is 55 miles per hour. Santiago's unit rate is 62 miles per hour.

Each hour, Santiago can drive 62 − 55, or 7 miles more than Destiny.

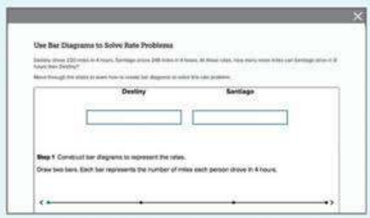
In 9 hours, Santiago can drive 9 × 7, or 63 miles more than Destiny.

Talk About It! Can you solve this rate problem another way? Explain.

yes; Sample answer: After finding the unit rates, I can multiply each unit rate by 9 to determine the distance each person can drive in 9 hours. Then I can subtract the lesser distance from the greater distance to find how much farther Santiago can drive in 9 hours.

Lesson 1-8 • Solve Rate Problems 65

Interactive Presentation



Learn, Use Bar Diagrams to Solve Rate Problems, Slide 1 of 2

CLICK

On Slide 1, students move through the slides to find the total number of miles each person drove.

Lesson 1-8

Solve Rate Problems

6.RP.A.3

LESSON GOAL

Students will solve real-world problems involving rates.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Use Bar Diagrams to Solve Rate Problems

Example 1: Use Bar Diagrams to Solve Rate Problems

Learn: Use Double Number Lines and Equivalent Rates to Solve Rate Problems

Example 2: Use Double Number Lines and Equivalent Rates to Solve Rate Problems

Apply: Bike-a-thon

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

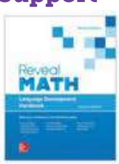
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Dimensional Analysis		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 8 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving rates.



ELL You can use the tips and suggestions on page T8 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by solving rate problems using ratios.

Standards for Mathematical Content: **6.RP .A.2, 6.RP.A.3, 6.RP.A.3.B**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous

Students solved real-world problems involving ratios.
6.RP.A.2, 6.RP.A.3, 6.RP.A.3.A, 6.RP.A.3.B

Now

Students solve real-world problems involving rates.
6.RP.A.2, 6.RP.A.3, 6.RP.A.3.B

Next

Students understand that a percent is a rate per 100.
6.RP.A.3.C

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students *apply* their *understanding* of rates and unit rates to solve real-world problems. They build *fluency* with different representations, such as bar diagrams, double number lines, and equivalent rates, as they solve problems.

Mathematical Background

Bar diagrams and double number lines are both useful visual representations to help solve problems involving *rates*. Using these visuals can help you understand the relationship between the two quantities. This becomes especially helpful when a comparison needs to be made between multiple rates represented in different problem scenarios. When the numbers are large or involve decimals or fractions, reasoning about equivalent ratios can be more advantageous.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding unit rates (Exercises 1–3)

Answers

1. 2 cups

2. 110 miles per hour

3. 1.5 books

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about video game arcades and the use of tokens instead of money.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- How would you use a *rate* involving distance in a sentence? Sample answer: We drove 450 miles in 7 hours.
- Where do you usually see *unit rates* being used in everyday life? Sample answer: Unit rates can be used to explain how fast a vehicle is moving. For example, a car may travel at a unit rate of 65 miles per hour.

Your Notes

Think About It!
Why do you need to know the sizes of the cans? Do you need to use that number when solving the problem?

Sample answer: In order to make a comparison, I need to know that the cans are the same size. I do not need to use the size of the cans in the calculations.

Talk About It!
How can you use estimation to help you solve this problem if you are in a store and do not have access to pencil, paper, or a calculator?

Sample answer: A case from the warehouse costs about \$10, so 6 cases costs about \$60. Twelve 3-packs from the grocery stores costs about 12 × \$6, or about \$72. The caterer will save about \$72 − \$60, or \$12.

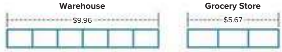
Example 1 Use Bar Diagrams to Solve Rate Problems

A warehouse sells 15-ounce cans of tomato sauce by the case. Each case contains 6 cans and sells for a price of \$9.96. At a local grocery store, three 15-ounce cans of the same brand of tomato sauce are on sale for \$5.67. A caterer needs to buy 36 cans.

How much will the caterer save by buying 36 cans from the warehouse instead of from the grocery store?

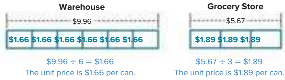
Step 1 Construct bar diagrams to represent each situation.

Draw two bars, one to represent the cost of tomato sauce cans at the warehouse, and one to represent the cost of tomato sauce cans at the grocery store. Each section represents one can.



Step 2 Find the unit prices.

Divide the total price for each by the number of cans to find the unit price, the price per can.



The caterer will save \$1.89 − \$1.66, or \$0.23 per can by buying from the warehouse instead of the grocery store. To buy 36 cans from the warehouse instead of the grocery store, the caterer will save 36 × \$0.23, or \$8.28.

Check

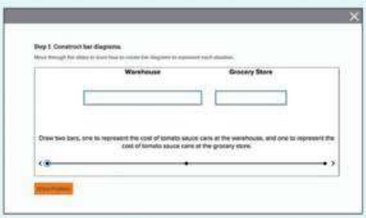
Miranda typed 325 words in 5 minutes, while Joseph typed 295 words in 5 minutes. At these rates, how many more words can Miranda type in 9 minutes than Joseph?

Show your work here

54 words

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Use Bar Diagrams to Solve Rate Problems, Slide 2 of 5

CLICK

On Slide 2, students move through the slides to see how a bar diagram is used to solve the problem.

TYPE

On Slide 3, students determine the solution.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Use Bar Diagrams to Solve Rate Problems

Objective

Students will use bar diagrams to model and solve a real-world problem involving rates.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to reason how using estimation would help solve the problem if they did not have any tools available to use.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we divide the bar for the warehouse into 6 sections, and the bar for the grocery store into 3 sections? Sample answer: Because the cans are sold in cases of 6 cans at the warehouse, and they are on sale in groups of 3 at the grocery store.

OL Describe another way you can solve this problem. Sample answer: I can multiply the cost of a case by 6 to find the total cost at the warehouse. Then, multiply the sale price of the cans at the grocery store by 12 to find the total cost at the store. Finally, subtract.

BL What is the savings per ounce? Round to the nearest cent. \$0.02

SLIDE 3

AL Why do you need to subtract \$1.66 from \$1.89? Sample answer: These are the unit prices, and I need to know the difference so I can determine the total amount of money the caterer saved by buying 36 cans from the warehouse.

OL How can you determine if your solution is accurate? Sample answer: I can multiply the cost per can at the warehouse by 36, and also the cost per can at the grocery store by 36. This will give me the two total amounts the caterer would spend at each location. I can then subtract the two amounts to determine how much the caterer saved. My solution should be the same as the difference between the two total amounts.

BL Suppose the warehouse increases the price of a case of 6 cans to \$10.98. Will the caterer still save money for 36 cans by buying them at the warehouse? Explain. yes; Sample answer: If the price increases to \$10.98 for 6 cans, the unit price would be \$1.83. So, \$1.89 − \$1.83 = \$0.06, which is the amount the caterer would save per can. Although the caterer will not save as much money as before, buying from the warehouse still saves the caterer money.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Use Double Number Lines and Equivalent Rates to Solve Rate Problems

Objective

Students will understand that they can use double number lines and equivalent rates to solve a real-world problem involving rates.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to understand that they can solve this problem by reasoning about the difference between the heartbeats. Because the problem asked for the difference in heartbeats for 6 minutes, they can begin the problem by finding the difference in heartbeats for 4 minutes. As they discuss the *Talk About It!* question on Slide 3, encourage them to consider alternative approaches to solving the problem.

5 Use Appropriate Tools Strategically Students will use two methods to solve this rate problem, a double number line and equivalent rates. Encourage them to understand the benefits of each method and the correspondences between them. The double number line allows students to visually see the rate relationship, while both methods essentially involve scaling by finding equivalent rates.

Teaching Notes

SLIDE 1

In this Learn, students understand that they can find the difference in heartbeats for 4 minutes to extend that to find the difference in heartbeats for 6 minutes. Some students may choose to find the unit rate for each animal first. Students will consider this method as they discuss the *Talk About It!* question on Slide 3.

Go Online to find additional teaching notes and the sample answer for the *Talk About It!* question.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of rate problems, have them explain whether using a double number line or equivalent ratios is a more advantageous method to solve the following problems.

1. Henry uses 5 gallons of gas in 2.5 hours. At this rate, how many gallons of gas will he use in 7.5 hours? double number line; Sample answer: 7.5 hours is 3 times 2.5 hours, so the total amount of gas can be found by multiplying 5 by 3.
2. Raelyn reads 7 pages every 12 minutes. At this rate, how many minutes will it take her to read 30 pages? equivalent ratios; Sample answer: 30 is not a multiple of 7, so a double number line is not the best method to use.

Learn Use Double Number Lines and Equivalent Rates to Solve Rate Problems

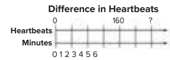
A veterinarian measured the number of heartbeats of her dog and cat for 4 minutes and recorded the results in the table. At these rates, how many more times does the cat's heart beat in 6 minutes than the dog?

Animal	Heartbeats
Dog	360
Cat	520

Method 1 Use a double number line.

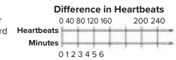
Step 1 Construct a double number line.

In four minutes, the cat's heart beats 520 – 360, or 160 more times than the dog's heart. Draw a double number line to represent this difference.



Step 2 Use scaling to find the unit rate.

Scale back to find the difference in heartbeats for 1 minute. Then scale forward to find the difference in heartbeats for 6 minutes.



The cat's heart beats 240 more times in 6 minutes than the dog's heart.

Method 2 Use equivalent rates.

Write and solve an equation. Let d represent the difference in heartbeats for 6 minutes. The difference in heartbeats for 4 minutes is 160 beats.

minutes → $\frac{6}{d} = \frac{4}{160}$ ← minutes
difference in heartbeats → ← difference in heartbeats

$$\frac{6}{d} = \frac{4}{160} \quad (\times 1.5)$$

Because $4 \times 1.5 = 6$, multiply 160 by 1.5.

$$\frac{6}{240} = \frac{4}{160}$$

$160 \times 1.5 = 240$; So, $d = 240$.

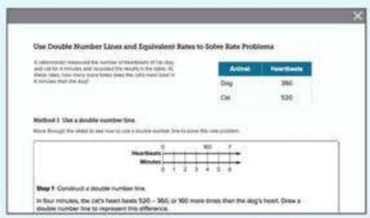
So, using either method, the cat's heart beats 240 more times in 6 minutes than the dog's heart.

Talk About It! A classmate stated that you can also find each animal's unit rate in heartbeats per minute first. Then multiply each unit rate by 6 minutes to determine the number of heartbeats in 6 minutes for each animal. Finally, subtract to find the difference. Is this method a valid method? Explain.

yes; Sample answer: You can find each unit rate first and then scale to find the number of heartbeats in 6 minutes before subtracting. Or, you can subtract first and then scale back and then forward to find the difference in heartbeats for 6 minutes. Either method is valid.

Lesson 1-8 • Solve Rate Problems 67

Interactive Presentation



Learn, Use Double Number Lines and Equivalent Rates to Solve Rate Problems, Slide 1 of 3

CLICK

On Slide 1, students move through the slides to see how a double number line can be used to solve the problem.

CLICK

On Slide 2, students move through the steps to use equivalent rates to solve the problem.

Think About It!
Will the price of a 15-pound bag be less than twice as much as the price for a 12-pound bag? Why or why not?

yes; Sample answer: 15 pounds is less than twice the weight of 12 pounds. Since the problem assumes the same rate, the 15-pound bag will cost less than twice the price of the 12-pound bag.

Talk About It!
The average weight of a Red Delicious apple is about 5 ounces. About how many apples would you expect to be in a 15-pound bag? Explain.

about 48 apples; Sample answer: 15 pounds is equivalent to 240 ounces; 240 ÷ 5 = 48

Example 2 Use Double Number Lines and Equivalent Rates to Solve Rate Problems

A bulk food store sells a 12-pound bag of Red Delicious apples for $18.

At this rate, what is the price of a 15-pound bag of apples?

Method 1 Use a double number line.

Step 1 Construct a double number line.

Draw a double number line to represent the price of a 12-pound bag. Mark equal increments on the bottom number line.

Price ($): 0, 18 ?
Pounds: 0 3 6 9 12 15

Step 2 Use scaling to find an equivalent rate.

Scale back to find the price for a 3-pound bag. Then scale forward to find the price for a 15-pound bag.

Price ($): 0 4.5 9 13.5 18 22.5
Pounds: 0 3 6 9 12 15

At this rate, the price of a 15-pound bag of apples is $22.50.

Method 2 Use equivalent rates.

Write and solve an equation. Let p represent the price of the 15-pound bag.

pounds → $\frac{15}{p} = \frac{12}{18}$ ← pounds
price ($) → ← price ($)

×1.25

$\frac{15}{p} = \frac{12}{18}$ Because 12 × 1.25 = 15, multiply 18 by 1.25 to find p.

×1.25

$\frac{15}{22.5} = \frac{12}{18}$ 18 × 1.25 = 22.5; So, p = 22.5.

So, using either method, the price of a 15-pound bag is $22.50.

Check

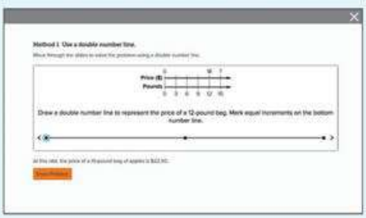
The manager of a small bakery determines that an average of 264 loaves of cinnamon raisin bread are sold every 12 weeks. At this rate, about how many loaves of cinnamon raisin bread are sold every 5 weeks?

Show your work here

about 110 loaves

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 2, Use Double Number Lines and Equivalent Rates to Solve Rate Problems, Slide 2 of 5

CLICK

On Slide 2, students move through the slides to see how a double number line is used to solve the problem.

CLICK

On Slide 3, students move through the steps to use equivalent rates to solve the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Use Double Number Lines and Equivalent Rates to Solve Rate Problems

Objective

Students will use double number lines and equivalent rates to solve a real-world problem involving rates.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to draw on their knowledge of converting units of Customary measurement to explain their reasoning.

5 Use Appropriate Tools Strategically Students will use two methods to solve this rate problem, a double number line and equivalent rates. Encourage them to understand the benefits of each method and the correspondences between them. The double number line allows students to visually see the rate relationship, while both methods essentially involve scaling by finding equivalent rates.

Questions for Mathematical Discourse

SLIDE 2

AL What units need to be represented in the double number line? price and pounds

OL Why do we mark and label the price of a 12-pound bag? Sample answer: We need to use this given rate to then be able to scale back and scale forward.

OL Why is the bottom number line divided into increments of 3 and not 1? Sample answer: Because 15 is 3 more than 12, and 12 is evenly divisible by 3, it is quicker to divide the bottom number line into increments of 3.

BL At this rate, what would be the price of an 18-pound bag of apples? $27.00

SLIDE 3

AL How can you represent the rate given in the problem to use in the equation? $\frac{12 \text{ pounds}}{18 \text{ dollars}}$

OL Why do we use $\frac{15}{p} = \frac{12}{18}$ instead of $\frac{p}{15} = \frac{12}{18}$? Sample answer: We must keep the same units in the same positions in the rates.

BL Which method is more advantageous to use in this problem? See students' responses.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Bike-a-thon

Objective

Students will come up with their own strategy to solve an application problem involving deciding on a bike-a-thon trail based on riding rate.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoningof Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What rate is given in the problem?
- Would using a bar diagram, double number lines, or equivalent rates be more advantageous to use in this scenario?
- Once you find the rate of minutes per mile, how will you use this to find the number of hours it will take her to ride each trail?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Bike-a-thon

Keshia can ride her bike 15 miles in 90 minutes. She wants to ride in a bike-a-thon that consists of two trail options, a 56-mile trail or a 36-mile trail. At her current rate, how many more hours will it take her to ride 56 miles than 36 miles? If she wants to ride for about 4 hours, which trail should she choose?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

2 hours longer; the 36-mile trail; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

If Keshia raises \$1.50 for each mile she rides, how much more money would she raise if she chose the 56-mile trail than the 36-mile trail? Explain.

\$30; Sample answer: 56 mi − 36 mi = 20 mi; 20 mi × \$1.50 = \$30

Interactive Presentation

Apply, Bike-a-thon

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Martin can run 6 miles in 60 minutes. He wants to run in either one of two upcoming races, a 4-mile race or a 12-mile race. At his current rate, how much longer will it take him to run the 12-mile race than the 4-mile race?

Show your work here

80 min, or 1 hour and 20 minutes

Go Online You can complete an Extra Example online.

Pause and Reflect

What are some problems or situations in which you may have encountered rates, such as a unit price or rate of travel, in your everyday life? How can you use your understanding of ratios and rates to solve everyday problems like these?

Record your observations here

See students' observations.

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70 Module 1 • Ratios and Rates

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you describe how two quantities are related?

In this lesson, students learned how to solve real-world problems involving rates using bar diagrams, double number lines, and equivalent rates. Encourage them to work with a partner to compare and contrast the three methods. Have them explain which method they prefer and why.

Exit Ticket

Refer to the Exit Ticket slide. What is the price of one package? How much would he spend if he bought 5 more packages? Write a mathematical argument that can be used to defend your solution. $4.98; $24.90; Sample answer: Three packages of tokens cost $14.95. To find the cost of one package, I used the equivalent ratio $\frac{\$14.95}{3 \text{ packages}} = \frac{?}{1 \text{ package}}$. Since $3 \div 3 = 1$, one package costs $14.95 ÷ 3 or about $4.98. To find the cost of five packages, I used the equivalent ratio $\frac{\$4.98}{1 \text{ package}} = \frac{?}{5 \text{ packages}}$. Since $1 \times 5 = 5$, five packages cost $4.98 × 5 or about $24.90.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–7 odd, 9–12
- Extension: Dimensional Analysis
- ALEKS Ratios and Unit Rates

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 7, 9, 11
- Extension: Dimensional Analysis
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	use bar diagrams, double number lines, and equivalent rates to solve real-world problems involving rates	1–5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving rate problems	7, 8
3	higher-order and critical thinking skills	9–12

Name_______ Period_______ Date_______

Practice

Go Online You can complete your homework online.

Use any strategy to solve each problem.

1. Mr. Anderson is ordering pizzas for a class pizza party. Pizza Place has a special where he can buy 3 large pizzas for $18.75. At Mario's Pizzeria, he can buy 4 large pizzas for $22. If he needs to buy 12 pizzas, how much will he save if he buys the pizzas from Mario's Pizzeria instead of Pizza Place? (Example 1)
   $9

2. Skylar and Rodrigo each recorded how far they traveled while skateboarding. Skylar traveled 65 feet in 5 seconds and Rodrigo traveled 108 feet in 8 seconds. How much farther did Rodrigo travel per second than Skylar? (Example 1)
   0.5 ft or 6 in.

3. Melissa is buying party favors to make gift bags. Supplies LTD sells a 5-pack of favors for $11.25 and Parties and More sells a 3-pack of favors for $8.25. At these rates, how much more will she spend if she buys 15 favors from Supplies LTD than Parties and More? (Example 1)
   $7.50

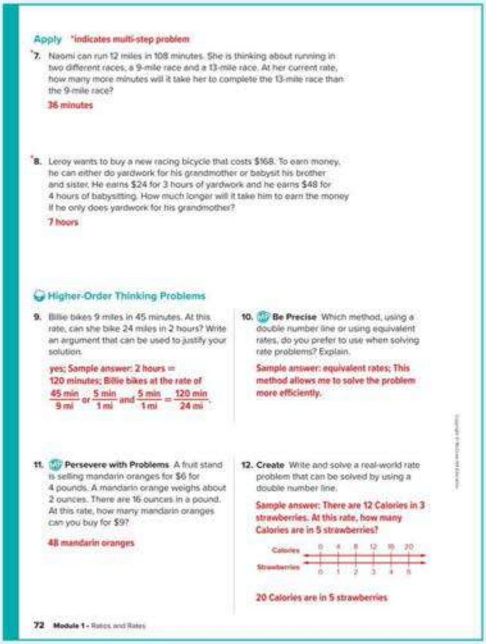
4. Tara can type 180 words in 4 minutes. At this rate, how many words would you expect her to type in 10 minutes? (Example 2)
   450 words

5. A bakery makes 260 donuts in 4 hours. At this rate, how many donuts can they make in 6 hours? (Example 2)
   390 donuts

Test Practice

6. **Open Response** While jumping rope, Juan jumped 24 times in 30 seconds. At this rate, how many times will he jump in 50 seconds?
   40 times

Lesson 1-8 • Solve Rate Problems **71**



Apply *indicates multi-step problem

*7. Naomi can run 12 miles in 108 minutes. She is thinking about running in two different races, a 9-mile race and a 13-mile race. At her current rate, how many more minutes will it take her to complete the 13-mile race than the 9-mile race?

36 minutes

*8. Leroy wants to buy a new racing bicycle that costs \$168. To earn money, he can either do yardwork for his grandmother or babysit his brother and sister. He earns \$24 for 3 hours of yardwork and he earns \$48 for 4 hours of babysitting. How much longer will it take him to earn the money if he only does yardwork for his grandmother?

7 hours

Higher-Order Thinking Problems

9. Billie bikes 9 miles in 45 minutes. At this rate, can she bike 24 miles in 2 hours? Write an argument that can be used to justify your solution.

yes; Sample answer: 2 hours = 120 minutes; Billie bikes at the rate of $\frac{45 \text{ min}}{9 \text{ mi}}$ or $\frac{5 \text{ min}}{1 \text{ mi}}$ and $\frac{5 \text{ min}}{1 \text{ mi}} = \frac{120 \text{ min}}{24 \text{ mi}}$.

10. **Be Precise** Which method, using a double number line or using equivalent rates, do you prefer to use when solving rate problems? Explain.

Sample answer: equivalent rates; This method allows me to solve the problem more efficiently.

11. **Persevere with Problems** A fruit stand is selling mandarin oranges for \$6 for 4 pounds. A mandarin orange weighs about 2 ounces. There are 16 ounces in a pound. At this rate, how many mandarin oranges can you buy for \$9?

48 mandarin oranges

12. **Create** Write and solve a real-world rate problem that can be solved by using a double number line.

Sample answer: There are 12 Calories in 3 strawberries. At this rate, how many Calories are in 5 strawberries?

20 Calories are in 5 strawberries

72 Module 1 • Ratios and Rates

Teaching the Mathematical Practices

6 Attend to Precision In Exercise 10, students must clearly and precisely explain the reasoning behind choosing the method that they prefer to use with solving real-world problems involving rates.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 11, students determine a strategy to solve a rate problem involving multiple steps.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 7 Have students work together to prepare a brief demonstration that illustrates why this problem might require multiple steps to solve. For example, before they can identify the difference between the two races, they have to find the unit rate in minutes per mile. Have each pair or group of students present their response to the class.

Create your own higher-order thinking problem.

Use with Exercises 9–12 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of equivalent ratios written as equations, tables, and graphs. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

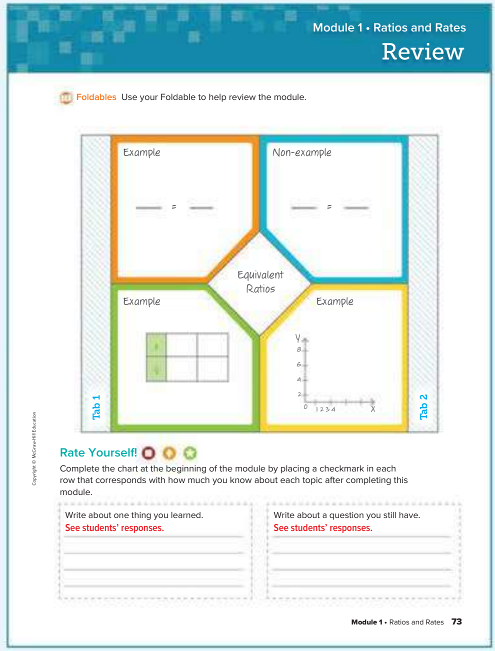
Assessment Resources

Put It All Together 1: Lessons 1-1 through 1-5
Put It All Together 2: Lessons 1-6 through 1-8
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Ratios and Proportional Relationships**.

- Ratios
- Rates
- Unit Rate
- Unit Cost
- Solve Problems: Ratio Tables
- Solve Problems: Unit Rates
- Solve Problems: Measurement Conversions


Module 1 • Ratios and Rates

Review

Foldables Use your Foldable to help review the module.

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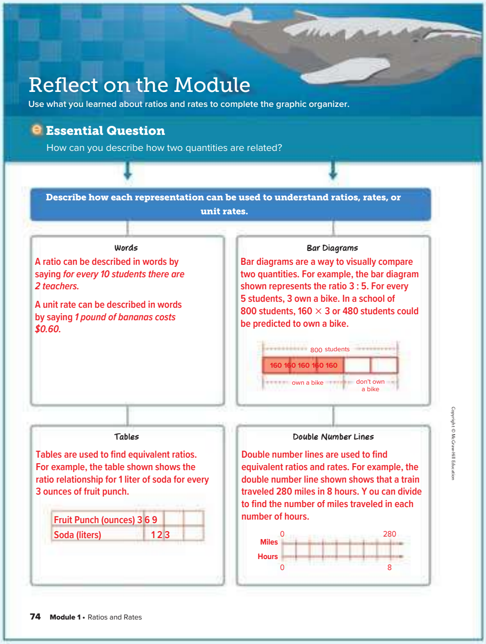
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 1 • Ratios and Rates 73


Reflect on the Module

Use what you learned about ratios and rates to complete the graphic organizer.

Essential Question

How can you describe how two quantities are related?

Describe how each representation can be used to understand ratios, rates, or unit rates.

Words

A ratio can be described in words by saying *for every 10 students there are 2 teachers.*

A unit rate can be described in words by saying *1 pound of bananas costs $0.60.*

Bar Diagrams

Bar diagrams are a way to visually compare two quantities. For example, the bar diagram shown represents the ratio 3 : 5. For every 5 students, 3 own a bike. In a school of 800 students, 160 × 3 or 480 students could be predicted to own a bike.

800 students
160 160 160 160 160
own a bike
don't own a bike

Tables

Tables are used to find equivalent ratios. For example, the table shown shows the ratio relationship for 1 liter of soda for every 3 ounces of fruit punch.

Fruit Punch (ounces)	3	6	9		
Soda (liters)	1	2	3		

Double Number Lines

Double number lines are used to find equivalent ratios and rates. For example, the double number line shown shows that a train traveled 280 miles in 8 hours. You can divide to find the number of miles traveled in each number of hours.

Miles 0 280
Hours 0 8

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can you describe how two quantities are related? See students' graphic organizers.

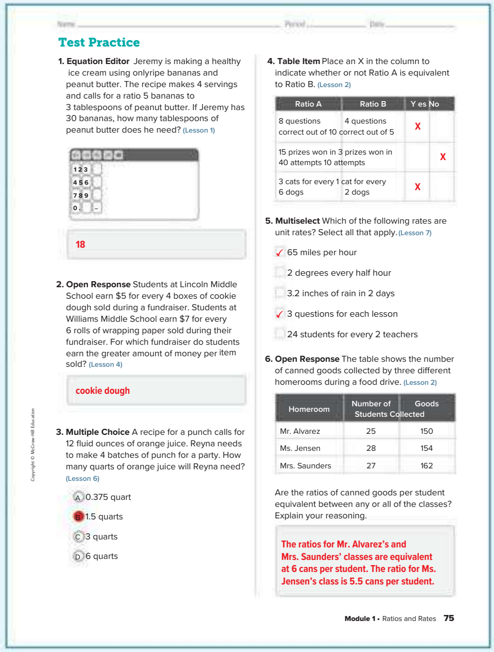
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–12 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	3, 10
Multiselect	Multiple answers may be correct. Students must select all correct answers.	5
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	1, 12
Table Item	Students complete a table by correctly classifying the information.	4
Grid	Students create a graph on an online coordinate plane.	8
Open Response	Students construct their own response in the area provided.	2, 6, 7, 9, 11

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.RP.A.1	1-1	1
6.RP.A.2	1-7, 1-8	5, 7, 9, 11
6.RP.A.3	1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8	2, 3, 4, 6, 8, 10, 11, 12
6.RP.A.3.A	1-2, 1-3, 1-4, 1-7, 1-8	2, 4, 5, 6, 7, 8, 9
6.RP.A.3.B	1-7, 1-8	5, 7, 9
6.RP.A.3.D	1-6	3, 12


Name ________ Period ________ Date ________

Test Practice

1. **Equation Editor** Jeremy is making a healthy ice cream using onlyripe bananas and peanut butter. The recipe makes 4 servings and calls for a ratio 5 bananas to 3 tablespoons of peanut butter. If Jeremy has 30 bananas, how many tablespoons of peanut butter does he need? (Lesson 1)

   18

2. **Open Response** Students at Lincoln Middle School earn $5 for every 4 boxes of cookie dough sold during a fundraiser. Students at Williams Middle School earn $7 for every 6 rolls of wrapping paper sold during their fundraiser. For which fundraiser do students earn the greater amount of money per item sold? (Lesson 4)

   cookie dough

3. **Multiple Choice** A recipe for a punch calls for 12 fluid ounces of orange juice. Reyna needs to make 4 batches of punch for a party. How many quarts of orange juice will Reyna need? (Lesson 6)

   A. 0.375 quart
   B. 1.5 quarts
   C. 3 quarts
   D. 6 quarts

4. **Table Item** Place an X in the column to indicate whether or not Ratio A is equivalent to Ratio B. (Lesson 2)

Ratio A	Ratio B	Yes	No
8 questions correct out of 10	4 questions correct out of 5	X	
15 prizes won in 40 attempts	3 prizes won in 10 attempts		X
3 cats for every 6 dogs	1 cat for every 2 dogs	X	

5. **Multiselect** Which of the following rates are unit rates? Select all that apply. (Lesson 7)

   ✓ 65 miles per hour
   ☐ 2 degrees every half hour
   ☐ 3.2 inches of rain in 2 days
   ✓ 3 questions for each lesson
   ☐ 24 students for every 2 teachers

6. **Open Response** The table shows the number of canned goods collected by three different homerooms during a food drive. (Lesson 2)

Homeroom	Number of Students	Goods Collected
Mr. Alvarez	25	150
Ms. Jensen	28	154
Mrs. Saunders	27	162

Are the ratios of canned goods per student equivalent between any or all of the classes? Explain your reasoning.

The ratios for Mr. Alvarez's and Mrs. Saunders' classes are equivalent at 6 cans per student. The ratio for Ms. Jensen's class is 5.5 cans per student.

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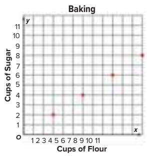
Module 1 • Ratios and Rates 75

7. Open Response Jessica jogged 4 laps around a track in 9 minutes, Luke jogged 8 laps in 27 minutes. Their rates can be expressed as the ratios $\frac{4 \text{ laps}}{9 \text{ minutes}}$ and $\frac{8 \text{ laps}}{27 \text{ minutes}}$. Are Jessica and Luke's rates equivalent? Explain. (Lesson 7)

no; Sample answer: Since the rates do not have the same unit rate, they are not equivalent.

8. Grid Kurt uses 3 cups of flour for every 2 cups of sugar in a recipe. Graph the ordered pairs to represent the cups of sugar needed if he uses 3, 6, 9, or 12 cups of flour. (Lesson 3)



9. Open Response Abigail surveyed 40 students about their favorite kind of movie. The results are shown in the table. If there are 200 students in the school, predict how many more students prefer action movies to scary movies. (Lesson 7)

Type of Movie	Number of Students
Action	14
Animated	3
Comedy	10
Drama	4
Scary	9

25 students

10. Multiple Choice Three out of 5 students at Maria's school participate in a school club or sport. There are 175 students at the school. Which of the following shows how equivalent fractions can be used to find the total number of students that participate in a school club or sport? (Lesson 5)

(A) $\frac{3}{5} = \frac{s}{175}$

(B) $\frac{3}{5} = \frac{175}{s}$

(C) $\frac{3}{175} = \frac{s}{5}$

11. Open Response A barge traveled 120 miles downstream in 8 hours. Then it traveled 100 miles upstream in 10 hours. (Lesson 8)

A. How did the rate of speed downstream compare to its rate of speed upstream?

rate of speed downstream = 15 mph; rate of speed upstream = 10 mph; The rate of speed downstream was faster than the rate of speed upstream.

B. What was the difference between the rates of speed?

5 miles per hour

12. Equation Editor Mr. Collins ordered 8,000 ounces of stone. How many tons of stone did he order? (Lesson 6)

$\frac{1}{4}$

![IGNITE!]

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question. They will complete a graphic organizer in the module review to help them answer the Essential Question.

How can you use fractions, decimals, and percents to solve everyday problems? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

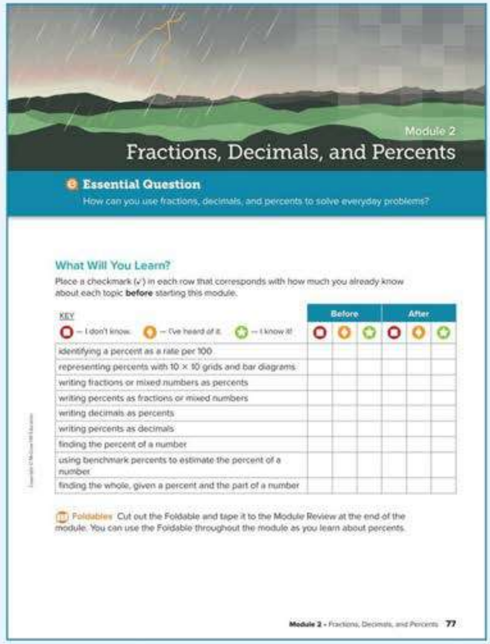
When to Use It Students will be directed to add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, at the end of the module, they can use it to help them study for the module assessment.

Launch the Module

For this module, the Launch the Module video uses the topics of weather, technology, and nutrition fact labels to introduce the idea of fractions, decimals, and percents. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 2

Fractions, Decimals, and Percents

Essential Question

How can you use fractions, decimals, and percents to solve everyday problems?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: I don't know. / I've heard of it. / I know it!

	Before			After		
	I don't know.	I've heard of it.	I know it!	I don't know.	I've heard of it.	I know it!
identifying a percent as a rate per 100						
representing percents with 10 × 10 grids and bar diagrams						
writing fractions or mixed numbers as percents						
writing percents as fractions or mixed numbers						
writing decimals as percents						
writing percents as decimals						
finding the percent of a number						
using benchmark percents to estimate the percent of a number						
finding the whole, given a percent and the part of a number						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about percents.

Module 2 • Fractions, Decimals, and Percents 77

Interactive Presentation

Module 2

Fractions, Decimals, and Percents

Module Goal

Learn about the relationship between fractions, decimals, and percents, and apply that relationship to finding the percent of a number.

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): 6.RP .A Understand ratio concepts and use ratio reasoning to solve problems.

Standards for Mathematical Content:

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.A.3.C Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- generate equivalent ratios
- express fractions as decimals

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students solved problems involving ratios and rates.
6.RP.A.1, 6.RP.A.2, 6.RP.A.3

Now
Students relate fractions, decimals, and percents, and find the percent of a number.
6.RP.A.3, 6.RP.A.3.C

Next
Students will use ratios to solve multi-step percent problems.
7.RP.A.3

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of fractions, decimals, ratios, and rates to build *fluency* with finding percents of a quantity. They *apply* their fluency with percents to solve real-world problems involving finding the whole, given the part and the percent.

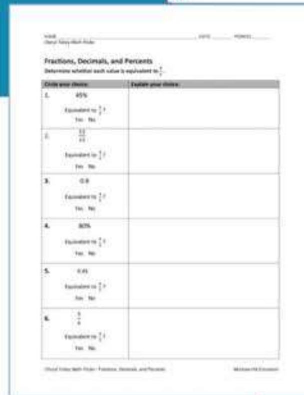
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
2-1	Understand Percents	Foundational for 6.RP.A.3, 6.RP.A.3.C	1	0.5
2-2	Percents Greater Than 100% and Less Than 1%	Foundational for 6.RP.A.3, 6.RP.A.3.C	1	0.5
2-3	Relate Fractions, Decimals, and Percents	Foundational for 6.RP.A.3, 6.RP.A.3.C	3	1.5
Put It All Together 1: Lessons 2-1 through 2-3			0.5	0.25
2-4	Find the Percent of a Number	6.RP.A.3, 6.RP.A.3.C	3	1.5
2-5	Estimate the Percent of a Number	6.RP.A.3, 6.RP.A.3.C	1	0.5
2-6	Find the Whole	6.RP.A.3, 6.RP.A.3.C	2	1
Put It All Together 2: Lessons 2-4 through 2-6			0.5	0.25
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	15	7.5

Formative Assessment Math Probe
Fractions, Decimals, and Percents

Module Resource


Fractions, Decimals, and Percents

Correct Answers: 1. No; **2.** Yes; **3.** Yes; **4.** Yes; **5.** No; **6.** No

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine whether each given value is equivalent to the given fraction, and explain their choice.

Targeted Concept Determining equivalent forms of fractions, decimals and percents involves conceptualizing the meaning of the different representations.

Targeted Misconceptions

- Students do not interpret the fraction bar as a division sign and instead substitute the fraction bar with the % sign and/or a decimal point.
- Students incorrectly interpret the value of the fraction by using the difference between the numerator and the denominator.

Assign the probe after Lesson 3.

Collect and Assess Student Answers

If the student selects...	**Then** the student likely...
1. Yes **5.** Yes	uses the fraction bar as a percent sign or as decimal point. (Students may also select no for 2 and 3 using this same incorrect reasoning.)
6. Yes	determines equivalence by subtracting the numerator from the denominator (both have a difference of 1).

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Fractions, Decimals, and Percents
- Lesson 1, Examples 1–4
- Lesson 2, Examples 1–4
- Lesson 3, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ benchmark percents

☐ percent

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Use part to whole ratios.

The ratio of strawberries to total ingredients in a recipe is 2 to 5. If you have 35 total ingredients, how many are strawberries?

strawberries → $\frac{2}{5} = \frac{s}{35}$ ← strawberries
total ingredients → ← total ingredients

$\frac{2}{5} \overset{\times 7}{=} \frac{s}{35}$ Because $5 \times 7 = 35$, multiply 2 by 7 to find the value of s.

$\frac{2}{5} = \frac{14}{35}$ $2 \times 7 = 14$; So, $s = 14$.

So, 14 strawberries are needed to maintain the ratio in the recipe.

Example 2
Use place value to write decimals in word form.

Write each decimal in word form.

0.3 The place value of the last digit, 3, is tenths.

word form: *three tenths*

2.15 The place value of the last digit, 5, is hundredths.

word form: *two and fifteen hundredths*

Quick Check

1. The ratio of cups of borax to total ingredients in a recipe for homemade laundry detergent is 2 : 6. If you need 24 total cups of laundry detergent, how many cups of borax do you need?
   8 cups

2. Write 0.212 in word form.
   two hundred twelve thousandths

3. Write 0.145 in word form.
   one hundred forty-five thousandths

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

① ② ③

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define A **percent** is a ratio that compares a number to 100.

Example There are 100 marbles in a bag, of which 34 are green, 17 are blue, 22 are red, and 27 are yellow. The ratio of blue marbles to the total number of marbles can be expressed as 17 to 100, or 17%.

Ask Write the ratio of yellow marbles to the total number of marbles as a percent. 27%

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- finding equivalent ratios
- solving word problems involving ratios and rates
- understanding rates
- making predictions using ratios

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Fractions, Decimals, and Percents** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

View Challenges as Opportunities

Part of cultivating a growth mindset in math involves viewing challenging problems or tasks as opportunities to learn and make new connections in your brain.

How Can I Apply It?

Encourage students to embrace challenges by trying problems that are thought provoking, such as the **Apply Problems** and **Higher-Order Thinking Problems** in the Practice section of each lesson. Remember to regularly remind students that each new challenge is an opportunity to grow! After each challenge, engage the class in a discussion about the positive outcomes or learning they experienced after they worked on a challenging problem.

Learn Use 10 × 10 Grids to Model Percents

Objective

Students will understand that 10 × 10 grids can be used to model percents.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the quantity that is shaded and use the fact that there are 100 squares to find the number of squares that are not shaded.

Teaching Notes

SLIDE 1

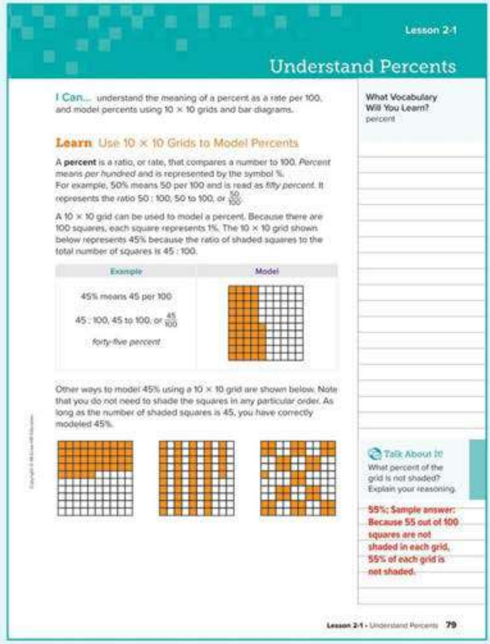
Students will learn the definition of a *percent*. A percent is a ratio that compares a number to 100. They will also learn the percent symbol and the meaning of the word percent. Encourage students to give another example of a percent, to read it, and to talk about what that number means out of 100. Students will also learn to use a 10 × 10 grid to model percents. Have students select the flashcards to view an example of a percent and its model.

Talk About It!

SLIDE 2

Mathematical Discourse

What percent of the grid is not shaded? Explain your reasoning. 55%; Sample answer: Because 55 out of 100 squares are not shaded in each grid, 55% of each grid is not shaded.



Lesson 2-1

Understand Percents

I Can... understand the meaning of a percent as a rate per 100, and model percents using 10 × 10 grids and bar diagrams.

What Vocabulary Will You Learn?
percent

Learn Use 10 × 10 Grids to Model Percents

A **percent** is a ratio, or rate, that compares a number to 100. *Percent* means *per hundred* and is represented by the symbol %. For example, 50% means 50 per 100 and is read as *fifty percent*. It represents the *ratio* 50 : 100, 50 to 100, or $\frac{50}{100}$.

A 10 × 10 grid can be used to model a percent. Because there are 100 squares, each square represents 1%. The 10 × 10 grid shown below represents 45% because the ratio of shaded squares to the total number of squares is 45 : 100.

Example	Model
45% means 45 per 100 45 : 100, 45 to 100, or $\frac{45}{100}$ *forty-five percent*	

Other ways to model 45% using a 10 × 10 grid are shown below. Note that you do not need to shade the squares in any particular order. As long as the number of shaded squares is 45, you have correctly modeled 45%.

Talk About It!
What percent of the grid is not shaded? Explain your reasoning.
55%; Sample answer: Because 55 out of 100 squares are not shaded in each grid, 55% of each grid is not shaded.

Lesson 2-1 • Understand Percents 79

Interactive Presentation

Learn, Use 10 × 10 Grids to Model Percents, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view an example of a percent and its model.

Lesson 2-1

Understand Percents

Foundational for 6.RP.A.3, 6.RP.A.3.C

LESSON GOAL

Students will use 10 × 10 grids and bar diagrams to model percents.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Use 10 × 10 Grids to Model Percents
Example 1: Identify the Percent
Example 2: Model the Percent
Learn: Use Bar Diagrams to Model Percents
Example 3: Identify the Percent
Example 4: Model the Percent

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

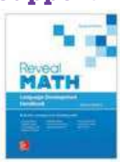
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Extension: Model Percents Using Fraction Models		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 9 of the *Language Development Handbook* to help your students build mathematical language related to understanding a percent as a rate per 100.



ELL You can use the tips and suggestions on page T9 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by using tools to model percents.
Standards for Mathematical Content: **Foundational for 6.RP .A.3, 6.RP.A.3.C**
Standards for Mathematical Practice: **MP2, MP3, MP5, MP7**

Coherence

Vertical Alignment

Previous
Students used decimal notation for fractions with denominators of 10 and 100.
4.NF.C.6

Now
Students use 10 × 10 grids and bar diagrams to model percents.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Next
Students will convert between percents, decimals, and fractions.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Rigor

The Three Pillars of Rigor

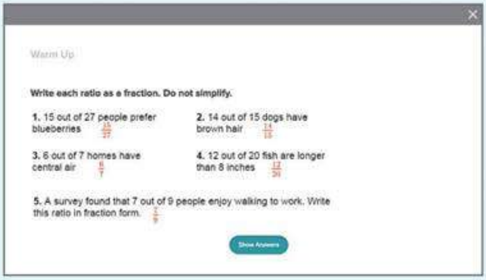
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their knowledge of fractions and ratios to develop *understanding* of percents. Using models such as bar diagrams and 10 × 10 grids, they come to understand that a percent is a ratio that compares a number to 100.

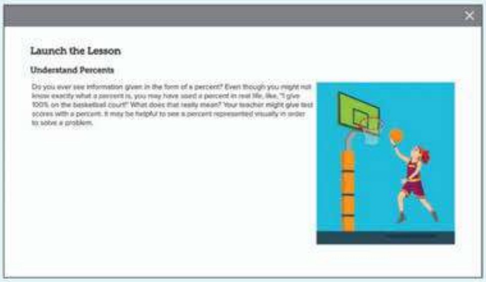
Mathematical Background

A *percent* is a ratio that compares a number to 100. It means *per hundred*. A percent can be modeled with a 10 × 10 grid by shading the number of squares corresponding to the percent. A bar diagram can also be used to model the percent by separating the bar into a number of sections equal to 100 divided by the greatest common factor of the percent and 100. The sections are shaded corresponding to the number of sections in the percent.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing ratios as fractions (Exercises 1–4)
- solving word problems involving ratios written as fractions (Exercise 5)

Answers

1. $\frac{15}{27}$
2. $\frac{14}{15}$
3. $\frac{6}{7}$
4. $\frac{12}{20}$
5. $\frac{7}{9}$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about where percents are seen in everyday life.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The term percent is believed to come from the Latin *per centum* which means "by the hundred". What are some other words that use the root word *cent*? What do those words mean? Sample answers: century means 100 years, cent means 1 penny (out of 100 pennies), centimeter is one hundredth of a meter, centennial means a 100th anniversary

Your Notes

Example 1 Identify the Percent

What percent is represented by the 10 × 10 grid?

Identify the number of shaded squares. How many squares are shaded? 50

Write the ratio that compares the number of shaded squares to the total number of squares.

The ratio is 50 : 100, 50 to 100, or $\frac{50}{100}$.

So, the percent represented by the 10 × 10 grid is 50 %.

Talk About It!
How can you quickly determine the number of shaded squares in the grid without counting every square?

Sample answer: Count the number of shaded rows, and then multiply by 10 because there are 10 squares in each row.

Check

What percent is represented by the 10 × 10 grid?

62%

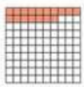
Go Online You can complete an Extra Example online.

Example 2 Model the Percent

In a recent survey, 17% of the people surveyed said that they have a magazine subscription.

Shade the 10 × 10 grid to model 17%.

17% means 17 per 100. There are 100 squares in a 10 × 10 grid. To model 17%, shade 17 squares on the grid.

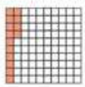


Talk About It!
When modeling a percent on a 10 × 10 grid, is there a specific order in which you must shade the squares? Explain.

no; Sample answer: I can shade any squares I choose. As long as the correct number of squares is shaded, the 10 × 10 grid will model the percent.

Check

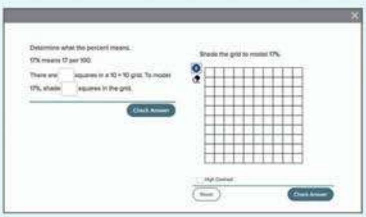
A middle school newspaper surveyed the student body and found that 14% of the students surveyed chose horses as their favorite animal. Shade the 10 × 10 grid to model 14%.



Go Online You can complete an Extra Example online.

80 Module 2 • Fractions, Decimals, and Percents

Interactive Presentation



Example 2, Model the Percent, Slide 2 of 4

TYPE

On Slide 2 of Example 1, students determine the percent modeled.

CLICK

On Slide 2 of Example 2, students shade a 10 × 10 grid to model 17%.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Identify the Percent

Objective

Students will identify the percent modeled by a 10 × 10 grid.

Questions for Mathematical Discourse

SLIDE 2

AL How many squares does the 10 × 10 grid have? What does each square represent within the context of the total grid? The 10 × 10 grid has 100 squares. Each square represents 1%.

OL Why did we use the fraction $\frac{50}{100}$? Sample answer: A percent can be easily written from a fraction with 100 in the denominator.

OL What fraction is equivalent to 50% of the grid being shaded? Explain how this makes sense. $\frac{50}{100}$ or $\frac{1}{2}$; Sample answer: $\frac{50}{100}$ is equivalent to $\frac{1}{2}$. This makes sense because 50 out of 100 squares are shaded, and this represents one half of the grid.

BL If 67 squares of a 10 × 10 grid are shaded, what is the percent that is modeled? 67%.

Example 2 Model the Percent

Objective

Students will model a percent using a 10 × 10 grid.

Questions for Mathematical Discourse

SLIDE 2

AL How can you write 17% as a fraction with a denominator of 100? $\frac{17}{100}$

AL How many squares will you shade? 17

OL How do you know how many squares to shade? Sample answer: 17% means 17 out of 100, so 17 squares should be shaded.

OL How do you know that you cannot shade 2 full columns? Sample answer: 2 full columns would be 2 × 10, or 20 squares. I only need to shade 17 squares.

BL How many squares of a 100 × 100 grid would be shaded to represent 17%? 1,700

BL How many more squares would you need to shade if the original given percent was 34%? 17 more squares

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Use Bar Diagrams to Model Percents

Objective

Students will understand how bar diagrams can be used to model percents.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others, 5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 4, encourage them to use mathematical reasoning to construct an argument for why it might not be advantageous to use a bar diagram to model 23%.

Teaching Notes

SLIDE 2

Students will view different bar diagrams to learn how to model a percent. You may wish to ask students to compare and contrast using 10×10 grids and bar diagrams to model percents. Have them discuss what considerations need to be made when drawing a bar diagram to model a given percent, such as whether or not the percent is a multiple of 5 or 10.

Talk About It!

SLIDE 2

Mathematical Discourse

Describe another way to divide a bar diagram to model 40%. Sample answer: Divide the bar diagram into 5 sections. Each section would represent 20%, so you would shade 2 of the 5 sections to model 40%.

SLIDE 4

Mathematical Discourse

Why might it not be advantageous to use a bar diagram to model a percent such as 23%? Sample answer: 23 is not a factor of 100, so you would have to divide the bar diagram into 100 sections to model 23%.

DIFFERENTIATE

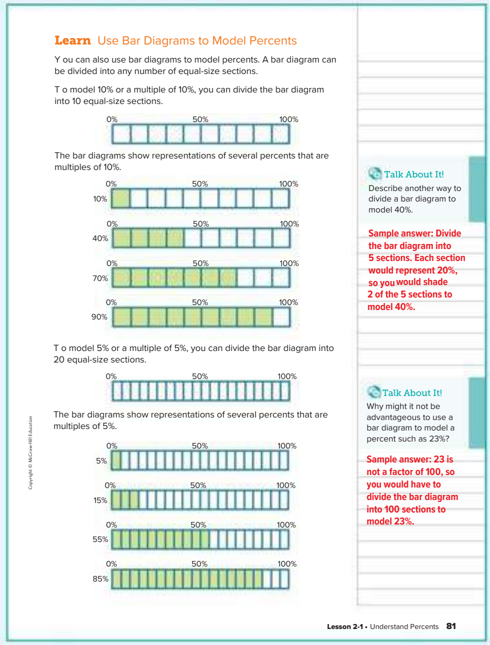
Enrichment Activity BL

To challenge students' understanding of modeling percents with bar diagrams, have them identify the least number of sections needed and the number of sections to shade in order to model each of the following percents.

28% 25 sections, 7 shaded

57% 100 sections, 57 shaded

20% 5 sections, 1 shaded



Learn Use Bar Diagrams to Model Percents

You can also use bar diagrams to model percents. A bar diagram can be divided into any number of equal-size sections.

To model 10% or a multiple of 10%, you can divide the bar diagram into 10 equal-size sections.

The bar diagrams show representations of several percents that are multiples of 10%.

To model 5% or a multiple of 5%, you can divide the bar diagram into 20 equal-size sections.

The bar diagrams show representations of several percents that are multiples of 5%.

Talk About It!
Describe another way to divide a bar diagram to model 40%.

Sample answer: Divide the bar diagram into 5 sections. Each section would represent 20%, so you would shade 2 of the 5 sections to model 40%.

Talk About It!
Why might it not be advantageous to use a bar diagram to model a percent such as 23%?

Sample answer: 23 is not a factor of 100, so you would have to divide the bar diagram into 100 sections to model 23%.

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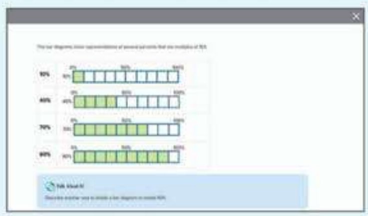
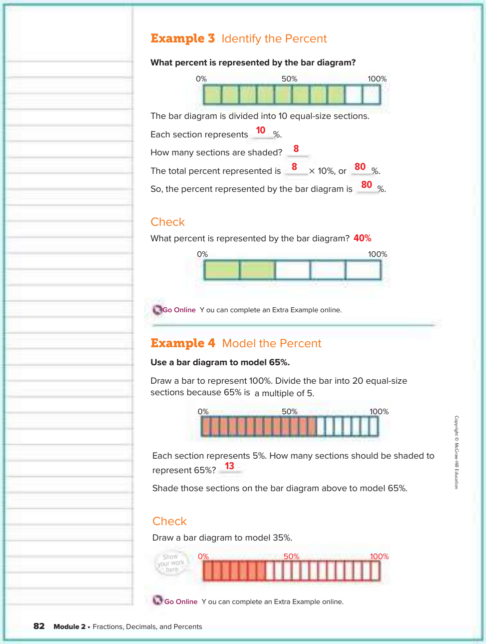
Interactive Presentation



Learn, Use Bar Diagrams to Model Percents, Slide 2 of 4



Example 3 Identify the Percent

What percent is represented by the bar diagram?

The bar diagram is divided into 10 equal-size sections.

Each section represents 10 %.

How many sections are shaded? 8

The total percent represented is 8 × 10%, or 80 %.

So, the percent represented by the bar diagram is 80 %.

Check

What percent is represented by the bar diagram? 40%

Go Online You can complete an Extra Example online.

Example 4 Model the Percent

Use a bar diagram to model 65%.

Draw a bar to represent 100%. Divide the bar into 20 equal-size sections because 65% is a multiple of 5.

Each section represents 5%. How many sections should be shaded to represent 65%? 13

Shade those sections on the bar diagram above to model 65%.

Check

Draw a bar diagram to model 35%.

Go Online You can complete an Extra Example online.

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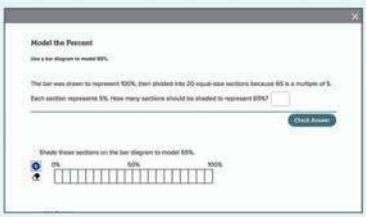
Interactive Presentation



Example 4, Model the Percent, Slide 1 of 2

CLICK

On Slide 2 of Example 3, students move through the steps to find the percent modeled by the bar diagram.

CLICK

On Slide 1 of Example 4, students shade a bar diagram to model the percent.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 3 Identify the Percent

Objective

Students will identify the percent modeled by a bar diagram.

Questions for Mathematical Discourse

SLIDE 2

AL How does the percent modeled by the eight shaded sections compare to 50%? Sample answer: The percent modeled by the eight shaded sections is greater than 50%.

OL Explain why each section of the bar diagram represents 10%. Sample answer: There are 10 total sections, and 100% divided by 10 equals 10%.

BL Can you use this bar diagram to represent 85%? Explain. yes; Sample answer: I can shade half of one more section. Since each section represents 10%, half of a section would represent 5%.

Example 4 Model the Percent

Objective

Students will model a percent by using a bar diagram.

Questions for Mathematical Discourse

SLIDE 1

AL Before you begin, do you think the shaded sections will cover less than half or more than half of the bar diagram? Explain. more than half; Sample answer: Half of the bar represents 50% and 65% > 50%.

OL Why is the bar divided into 20 sections of 5% each, instead of 10 sections of 10% each? Sample answer: Since 65% is divisible by 5, it makes sense to divide the bar into 20 sections of 5% each. If the bar was divided into 10 sections, each section would represent 10%, and 65 is not a multiple of 10.

BL How many sections of this bar diagram would need to be shaded if the percent given was 75%? Explain. 15 sections; Sample answer: Each section represents 5%, 75% divided by 5% is 15.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Exit Ticket

Refer to the Exit Ticket slide. Draw a 10 × 10 grid that models 85%. Then draw a bar diagram that models 85%. Explain the steps you used for each. See students' grids and diagrams; Sample answer: I drew a 10 × 10 grid and shaded 85 squares. Then I drew a bar diagram separated into 20 sections, since each section represents 5%, and 85 is a multiple of 5. To model 85%, I shaded 17 sections, since 5% × 17 = 85%.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

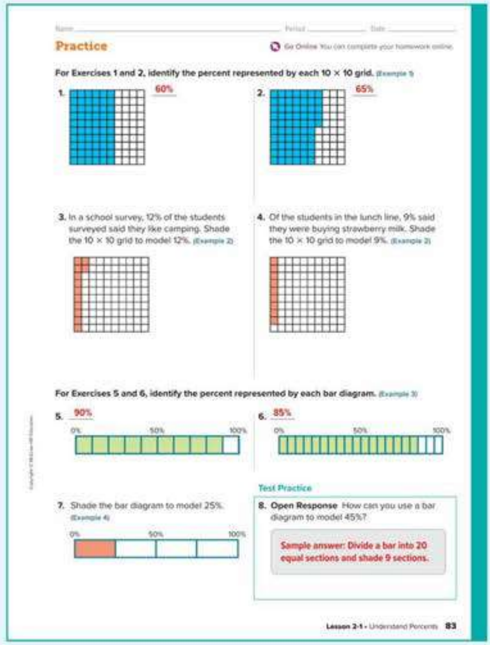
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify the percent modeled in a 10 × 10 grid	1, 2
1	model a percent using a 10 × 10 grid	3, 4
1	identify the percent modeled in a bar diagram	5, 6
2	model a percent using a bar diagram	7, 8
3	solve application problems involving modeling percents	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

When identifying the percent modeled by a bar diagram, students may incorrectly identify the percent because they do not count the total number of sections in the bar diagram. For example, in Exercise 6, students might say that the bar diagram represents 170% because there are 17 sections that are shaded. Remind students to first count the total number of sections to determine the value of each section. The bar diagram in Exercise 6 has 20 total sections, which means that each section represents 100% ÷ 20 or 5%, not 10%.



Name ______ Period ______ Date ______

Practice Go Online You can complete your homework online.

For Exercises 1 and 2, identify the percent represented by each 10 × 10 grid. (Example 1)

1. 60%
2. 65%

3. In a school survey, 12% of the students surveyed said they like camping. Shade the 10 × 10 grid to model 12%. (Example 2)

4. Of the students in the lunch line, 9% said they were buying strawberry milk. Shade the 10 × 10 grid to model 9%. (Example 2)

For Exercises 5 and 6, identify the percent represented by each bar diagram. (Example 3)

5. 90%
6. 85%

Test Practice

7. Shade the bar diagram to model 25%. (Example 4)

8. **Open Response** How can you use a bar diagram to model 45%?

Sample answer: Divide a bar into 20 equal sections and shade 9 sections.

Lesson 2-1 • Understand Percents **83**

Interactive Presentation

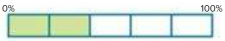
Exit Ticket

Foundational for 6.RP.A.3, 6.RP.A.3.C

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

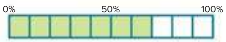
Apply *indicates multi-step problem

*9. The model shows the percent of students who voted for a tiger as the new school mascot. Did more than 50% of the students *not* vote for a tiger as the mascot? Write an argument that can be used to defend your solution.



yes; Sample answer: Each section of the model represents 20%. The 3 sections not shaded represent the percent of students who did not vote for the tiger. So, 20% × 3 = 60% and 60% is greater than 50%.

*10. The model shows the percent of baseball players on a team who plan to go to a baseball camp on Saturday. Can the coach say that more than 75% of his players are going to the camp? Write an argument that can be used to defend your solution.



no; Sample answer: Each section of the model represents 10%. So, 10% × 7 = 70% and 70% is not greater than 75%.

Higher-Order Thinking Problems

11. MP **Reason Abstractly** Suppose you divide a bar diagram into 25 equal-size sections and shade 5 sections. What percent is modeled in the diagram? Explain.

20%; Sample answer: Each section represents 4%. Since 5 sections are shaded, 5 × 4% = 20%.

12. MP **Find the Error** A student said that to write a percent as a fraction, write the number that comes before the percent symbol over a denominator of 100. Is the student correct? Justify your conclusion.

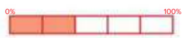
yes; Sample answer: A percent is a ratio that compares a number to 100.

13. MP **Make an Argument** Use an example to explain how you can model percents greater than 100%.

yes; Sample answer: T o model 110%, use two bar diagrams, each divided into 10 equal-size sections. Shade one bar diagram entirely to represent 100% and then shade 10% in the second bar diagram.

14. **Create** Write a real-world problem that involves a percent less than 50%. Then model the percent.

Sample answer: Of the students at the dance, 40% said they came with a friend.



MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 11, students use abstract reasoning to identify a percent modeled by a bar diagram.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students determine whether or not a statement made by another student is correct and justify their conclusion.

In Exercise 13, students construct an argument as to how percents greater than 100% can be modeled.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Interview a student.

Use with Exercises 9–10 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 9 might be, "What percent does each section of the model represent?"

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks **BL**
THEN assign:

- Practice, Exercises 7, 9, 11–14
- Extension: Model Percents Using Fraction Models
- ALEKS Understanding Percents

IF students score 66–89% on the Checks **OL**
THEN assign:

- Practice, Exercises 1–7, 9, 11, 14
- Extension: Model Percents Using Fraction Models
- Personal Tutor
- Extra Examples 1–4
- ALEKS Converting Between Fractions and Decimals

IF students score 65% or below on the Checks, **AL**
THEN assign:

- ALEKS Converting Between Fractions and Decimals

Learn Percents Greater Than 100%

Objective

Students will understand that 10 × 10 grids can be used to model percents greater than 100%.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 5, encourage them to discuss why the ratio that compares the rainfall in 2020 to the rainfall in 2019 is 100%. They should be able to use ratio reasoning to understand that because the amount of rainfall was the same for both years, the rainfall in 2020 is 100% (the same as) of the rainfall in 2019.

Teaching Notes

SLIDE 2

Have students recall that a percent is a ratio that compares a number to 100. You may wish to have students give another example of a percent and to talk about what that number means out of 100. When viewing the equivalent ratios, students should be able to justify why multiplying both the 3 and 4 in the ratio $\frac{3}{4}$ by 25 produces a ratio that represents 75%.

SLIDE 3

When viewing this representation of equivalent ratios, students should be able to reason that 125% is equivalent to $\frac{125}{100}$. You may wish to ask students to explain how they know this is an accurate representation of a percent greater than 100%. Encourage students to use the relationship between the part and whole in their explanation.

SLIDE 4

When modeling a percent that is greater than 100%, explain to students that they can use place value to determine how to use multiple 10 × 10 grids to model the percent. For example, when modeling 125%, students can examine the hundreds place to determine the number of whole 10 × 10 grids that need to be shaded. 125% = 100% + 25%. This means that one whole 10 × 10 grid should be shaded to represent 100%. There is 25% remaining. This means that 25 squares on a second 10 × 10 grid should be shaded, for a total of 125 shaded squares.

Talk About It!

SLIDE 5

Mathematical Discourse

Suppose the rainfall in 2020 is 5.0 inches. What percent compares the rainfall in 2020 to the rainfall in 2019? Explain why this makes sense. 100%; Sample answer: The rainfall in 2020 is equal to the rainfall in 2019, so the rainfall in 2020 is 100% of the rainfall in 2019.

Lesson 2-2

Percents Greater Than 100% and Less Than 1%

I Can... understand that percents can be greater than 100% or less than 1% and use 10 × 10 grids and bar diagrams to represent them.

Learn Percents Greater Than 100%

The table shows the total rainfall during April for a certain city for three different years.

Year	April Rainfall (in.)
2017	4.0
2018	3.0
2019	5.0

In 2018, it rained less than it did in 2017. To compare the rainfall in 2018 to that in 2017, use the ratio 3 : 4. Recall that a *percent* is a ratio that compares a number to 100. You can use equivalent ratios to show that the rainfall in 2018 was 75% of the rainfall in 2017.

$$\frac{3}{4} \overset{\times 25}{=} \frac{75}{100}$$

part, whole; percent

If the number being compared to 100 is less than 100, then the percent is less than 100%.

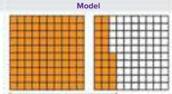
In 2019, it rained more than it did in 2017. To compare the rainfall in 2019 to that in 2017, use the ratio 5 : 4. You can use equivalent ratios to show that the rainfall in 2019 was 125% of the rainfall in 2017.

$$\frac{5}{4} \overset{\times 25}{=} \frac{125}{100}$$

part, whole; percent

If the number being compared to 100 is greater than 100, then the percent is greater than 100%.

Percents are greater than 100% when the number being compared to 100 is greater than 100. When the percent is greater than 100%, the part is greater than the whole.

Example	Model
125% means 125 per 100 125 : 100, 125 to 100, or $\frac{125}{100}$ *one hundred twenty-five percent*	 100% + 25% = 125%

Talk About It!

Suppose the rainfall in 2020 is 5.0 inches. What percent compares the rainfall in 2020 to the rainfall in 2019? Explain why this makes sense.

100%; Sample answer: The rainfall in 2020 is equal to the rainfall in 2019, so the rainfall in 2020 is 100% of the rainfall in 2019.

Lesson 2-2 • Percents Greater Than 100% and Less Than 1% 85

Interactive Presentation

Learn, Percents Greater Than 100%, Slide 4 of 5

FLASHCARDS

On Slide 4, students use Flashcards to view an example of a percent greater than 100% and its model.

Foundational for 6.RP.A.3, 6.RP.A.3.C

Percents Greater Than 100% and Less Than 1%

LESSON GOAL

Students will use 10 × 10 grids to model percents that are greater than 100% and less than 1%.

1 LAUNCH

Launch the Lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Percents Greater Than 100%

Example 1: Identify the Percent

Example 2: Model the Percent

Learn: Percents Less Than 1%

Example 3: Identify the Percent

Example 4: Model the Percent

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Arrive MATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 10 of the *Language Development Handbook* to help your students build mathematical language related to percents greater than 100% or less than 1%.

ELL You can use the tips and suggestions on page T10 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by using tools to model percents.

Standards for Mathematical Content: **Foundational for 6.RPA.3, 6.RP.A.3.C**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP5**

Coherence

Vertical Alignment

Previous

Students used tools to model percents.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Now

Students use 10 × 10 grids to model percents that are greater than 100% and less than 1%.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Next

Students will relate fractions, decimals, and percents.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Rigor

The Three Pillars of Rigor

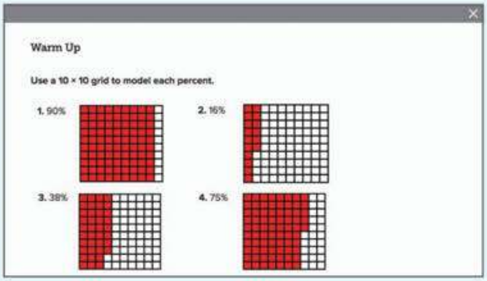
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of percents. They begin to understand that percents greater than 100% represent numbers greater than 1 and percents less than 1% represent numbers that are significantly less than the whole. They build *fluency* with modeling percents, and *apply* their understanding of percents to solve real-world problems.

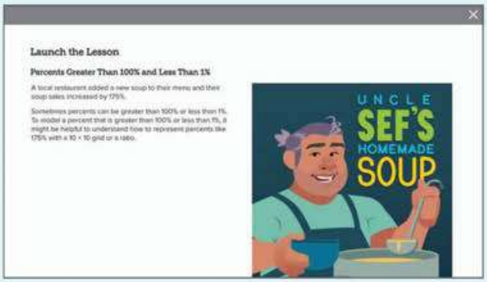
Mathematical Background

Students have used 10 × 10 grids and bar diagrams to model percents. A percent that is greater than 100% can be modeled with multiple 10 × 10 grids, while a percent that is less than 1% can be modeled in a close-up view of 1 square of a 10 × 10 grid.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- modeling percents (Exercises 1–4)

Answers

1–4. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a restaurant's increase in soup sales.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- A percent is a ratio, or rate, that compares a number of 100. Percent means *per hundred* and is represented by the symbol %. What are some examples of real-world percents? Sample answers: a sale in a store, tipping for service

Your Notes

Think About It!
How many total squares are in each grid?
100

Talk About It!
How can you quickly determine the number of shaded squares in the grid without counting every square?
Sample answer: I know that a completely shaded grid represents 100, and each column in the grid represents 10. So, I can count the number of whole grids for hundreds, the number of whole columns in the partially shaded grid for tens, and the number of squares in any partially shaded columns for ones, to obtain the total number of shaded squares without counting every square.

Example 1 Identify the Percent

What percent is represented by the 10 × 10 grids?

The percent compares the number of shaded squares to 100, because one whole grid contains 100 squares.

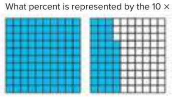
How many whole grids are shaded? 2
How many squares are shaded in the third grid? 63
How many squares are shaded altogether? 263

Write the ratio that compares the total number of shaded squares to one whole grid of 100 squares.

The ratio is 263 : 100, 263 to 100, or $\frac{263}{100}$.

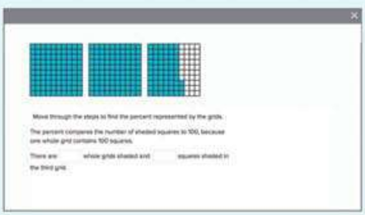
So, the percent represented by the 10 × 10 grids is 263 %.

Check
What percent is represented by the 10 × 10 grids? 137%



Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Identify the Percent, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to identify the percent modeled by the 10 × 10 grids.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Identify the Percent

Objective

Students will identify a percent, that is greater than 100%, modeled by 10 × 10 grids.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to reason about how shading a 10 × 10 grid can be used as a visual representation of a percent, given the definition of a percent as a rate or ratio per 100.

Questions for Mathematical Discourse

SLIDE 2

AL How many grids have all 100 squares shaded? 2 grids

AL How many squares are shaded on the last grid? 63 squares

OL Explain the method you will use to identify the percent that is modeled by the 10 × 10 grids. Sample answer: I will find the total number of squares shaded by knowing that if all the squares are shaded, the percent is 100%. Because two whole grids are shaded, part of the percent is 100% + 100% = 200%. Then I will count the number of squares in the last grid, 63. The percent modeled is 200% + 63%, or 263%.

BL Suppose you erase one-half of one shaded square in the last grid. What percent would the grids now represent? 262.5%

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

While presenting Example 2, students may struggle with the phrase *twice as much* and not connect its meaning with the phrase *two times as much*. You may wish to have students work with a partner to generate phrases that have the same meaning as *two times as much*, such as *twice as much*, or *double*. Point out that, depending on the context, instead of using *as much*, the phrase *as many* might be used. The phrase *as many* is used for countable objects, such as *the number of kittens is twice as many as the number of dogs* or *the number of ounces is twice as many at 3 weeks of age than at birth*. The phrase *as much* is used for objects that are not countable, such as *the kitten's weight is twice as much as it was at birth*. The number of *ounces* might be countable, but the term *weight* is not countable.

Example 2 Model the Percent

Objective

Students will use 10 × 10 grids to model percents greater than 100% and write a percent to represent a real-world context.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to reason about the differences in meaning of *increased by* versus *compared to*. As students discuss the *Talk About It!* question on Slide 3, encourage them to use that reasoning to help explain when it is correct to use 100% versus 200% when talking about the kitten's weight.

Questions for Mathematical Discourse

SLIDE 2

AL What number is in the hundreds place in the percent? tens? ones? 2; 0; 0

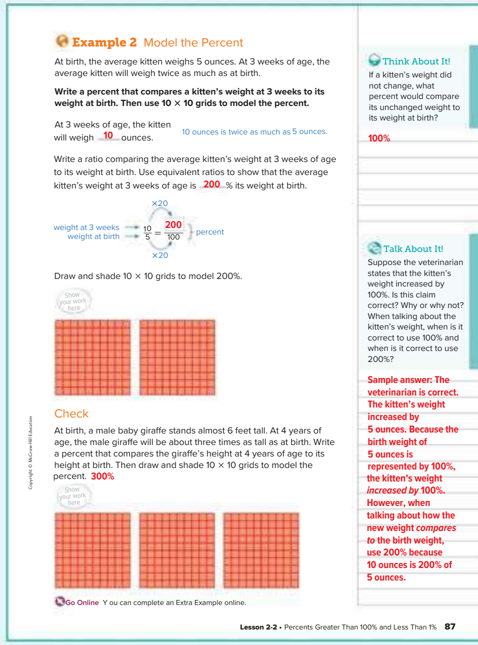
OL How can you set up the equivalent ratios to find the percent? Sample answer: Start with the ratio $\frac{10}{5}$. Set this equal to a ratio with 100 in the denominator. Because 5 × 20 = 100, multiply 10 × 20 to find the percent.

OL Suppose a classmate set up the ratio $\frac{5}{10}$ and found the percent to be 50%, not 200%. What did they do incorrectly? What does 50% represent in the context of the problem? Sample answer: They equated $\frac{5}{10}$ with $\frac{?}{100}$ and found the percent to be 50%. 50% actually represents the fact that the kitten's birth weight is 50% of its weight at 3 weeks of age.

BL What percent would represent the phrase *three times as much*? Describe a context in which some quantity is three times as much as another quantity, using the percent. 300%; Sample answer: My dog's weight is 300% of my cat's weight.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Model the Percent

At birth, the average kitten weighs 5 ounces. At 3 weeks of age, the average kitten will weigh twice as much as at birth.

Write a percent that compares a kitten's weight at 3 weeks to its weight at birth. Then use 10 × 10 grids to model the percent.

At 3 weeks of age, the kitten will weigh 10 ounces. 10 ounces is twice as much as 5 ounces.

Write a ratio comparing the average kitten's weight at 3 weeks of age to its weight at birth. Use equivalent ratios to show that the average kitten's weight at 3 weeks of age is 200 % its weight at birth.

weight at 3 weeks / weight at birth → $\frac{10}{5} = \frac{200}{100}$ percent (×20, ×20)

Draw and shade 10 × 10 grids to model 200%.

Show your work here

Check

At birth, a male baby giraffe stands almost 6 feet tall. At 4 years of age, the male giraffe will be about three times as tall as at birth. Write a percent that compares the giraffe's height at 4 years of age to its height at birth. Then draw and shade 10 × 10 grids to model the percent. 300%

Show your work here

Go Online You can complete an Extra Example online.

Think About It!
If a kitten's weight did not change, what percent would compare its unchanged weight to its weight at birth?

100%

Talk About It!
Suppose the veterinarian states that the kitten's weight increased by 100%. Is this claim correct? Why or why not? When talking about the kitten's weight, when is it correct to use 100% and when is it correct to use 200%?

Sample answer: The veterinarian is correct. The kitten's weight increased by 5 ounces. Because the birth weight of 5 ounces is represented by 100%, the kitten's weight *increased by* 100%. However, when talking about how the new weight *compares to* the birth weight, use 200% because 10 ounces is 200% of 5 ounces.

Lesson 2-2 • Percents Greater Than 100% and Less Than 1% 87

Interactive Presentation

Example 2, Model the Percent, Slide 2 of 4

CLICK

On Slide 2, students move through the slides to find and model the percent.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Percents Less Than 1%

Percents can also be less than 1%. Consider the following situation.

The distance from the center of Earth to the surface is also known as the *radius* of Earth. The radius of Earth is about 4,000 miles. The radius of the Sun is about 430,000 miles.

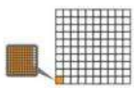
The ratio of Earth's radius to the Sun's radius is 4,000 : 430,000. Y ou can use equivalent ratios to show that the radius of Earth is about 0.93% of the Sun's radius. Because 430,000 divided by 4,300 is 100, divide 4,000 by 4,300. Round to the nearest hundredth.

$$\frac{4{,}000}{430{,}000} = \frac{0.93}{100}$$

(part → 4,000; whole → 430,000; ÷4,300 top and bottom; percent)

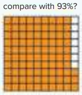
Percents are less than 1% when the number being compared to 100 is less than 1. When the percent is less than 1%, the part is significantly less than the whole. The radius of Earth is significantly less than the radius of the Sun.

On a 10 × 10 grid, 0.93% is represented by shading 93% of one grid square. One grid square represents 1% and 0.93% is less than 1%. Compared to 100%, 0.93% is significantly less.

Example	Model
0.93% means 0.93 per 100 0.93 : 100, 0.93 to 100, or $\frac{0.93}{100}$ *ninety-three hundredths of a percent*	

When thinking about how the size of Earth compares to the size of the Sun, it makes sense that Earth's radius is significantly less than the Sun's radius. Earth's radius is a little less than 1% of the Sun's radius.

Talk About It! A classmate used a 10 × 10 grid to model 0.93% as shown. What mistake did they make? How does 0.93% compare with 93%?



Sample answer: The classmate represented 93%, not 0.93%, on the grid. 0.93% is less than 1%, and 0.93% is significantly less than 93%, so less than 1 grid square should be shaded.

Interactive Presentation

Learn, Percents Less Than 1%, Slide 2 of 3

FLASHCARDS

On Slide 2, students use Flashcards to view an example of a percent less than 1% and its model.

Learn Percents Less Than 1%

Objective

Students will understand what a percent less than 1% means, and that 10 × 10 grids can be used to model percents less than 1%.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to use reasoning about percents as a ratio per 100 to explain why a grid that models a percent that is less than 1% will have less than 1 square shaded.

Teaching Notes

SLIDE 1

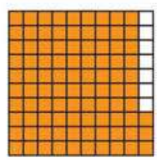
Students may not be able to determine whether the radius of Earth is the part or the whole. Because students are comparing the ratio of Earth's radius *relative* to the Sun's radius, Earth's radius is the part, and the Sun's radius is the whole, so the ratio is $\frac{4{,}000}{430{,}000}$.

Talk About It!

SLIDE 3

Mathematical Discourse

A classmate used a 10 × 10 grid to model 0.93% as shown. What mistake did they make? How does 0.93% compare with 93%?



Sample answer: The classmate represented 93%, not 0.93%, on the grid. 0.93% is less than 1%, and 0.93% is significantly less than 93%, so less than 1 grid square should be shaded.

Example 3 Identify the Percent

Objective

Students will identify a percent, that is less than 1%, modeled by 10 × 10 grids.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to reason about the amount of one grid square that is shaded relative to the entire grid. Because less than one grid square is shaded, the percent modeled is less than 1%.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use similar reasoning in their explanations. Have students take turns sharing their explanations until everyone in the class understands that the percent modeled is 0.25%, not 25%.

Questions for Mathematical Discourse

SLIDE 2

AL Of the entire grid, is more than one square, less than one square, or exactly one square shaded? less than one square

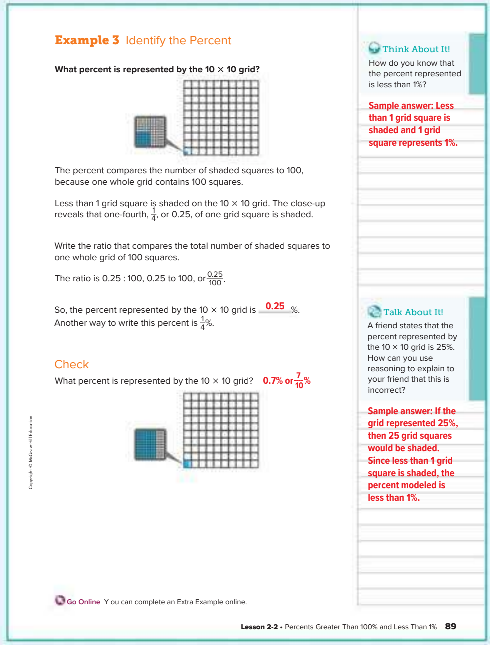
OL Because less than one square is shaded, what does this tell you about the percent? The percent is less than 1%.

OL If the close-up was not given of the one square, would you be able to state exactly what percent of the entire grid is shaded? Why or why not? no; Sample answer: I can estimate that it looks like about one-fourth of the one square is shaded, but I would not know exactly unless the close-up was given.

BL Describe how you could use a 10 × 10 grid to model 1.75%. Shade one square. Then shade three-fourths of a second square.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Example 3 Identify the Percent

What percent is represented by the 10 × 10 grid?

The percent compares the number of shaded squares to 100, because one whole grid contains 100 squares.

Less than 1 grid square is shaded on the 10 × 10 grid. The close-up reveals that one-fourth, $\frac{1}{4}$, or 0.25, of one grid square is shaded.

Write the ratio that compares the total number of shaded squares to one whole grid of 100 squares.

The ratio is 0.25 : 100, 0.25 to 100, or $\frac{0.25}{100}$.

So, the percent represented by the 10 × 10 grid is 0.25 %. Another way to write this percent is $\frac{1}{4}$%.

Check

What percent is represented by the 10 × 10 grid? 0.7% or $\frac{7}{10}$%

Think About It! How do you know that the percent represented is less than 1%?

Sample answer: Less than 1 grid square is shaded and 1 grid square represents 1%.

Talk About It! A friend states that the percent represented by the 10 × 10 grid is 25%. How can you use reasoning to explain to your friend that this is incorrect?

Sample answer: If the grid represented 25%, then 25 grid squares would be shaded. Since less than 1 grid square is shaded, the percent modeled is less than 1%.

Go Online You can complete an Extra Example online.

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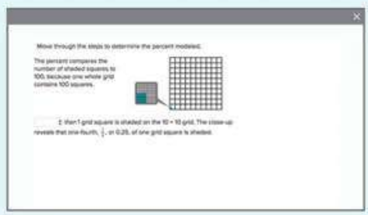
Lesson 2-2 • Percents Greater Than 100% and Less Than 1% 89

Interactive Presentation



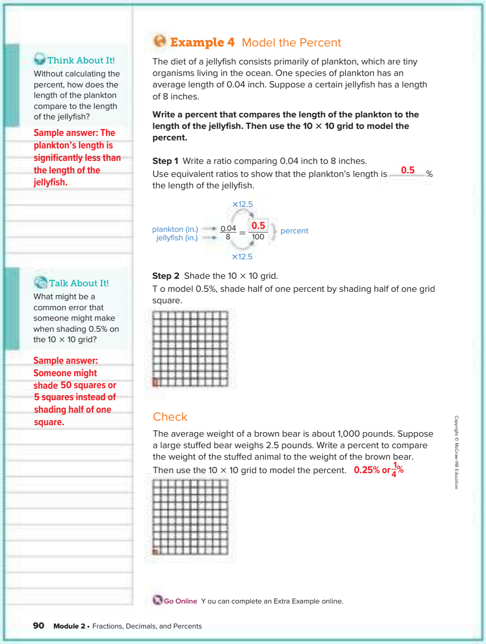
Example 3, Identify the Percent, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to identify the percent that is modeled by the 10 × 10 grid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! Without calculating the percent, how does the length of the plankton compare to the length of the jellyfish?

Sample answer: The plankton's length is significantly less than the length of the jellyfish.

Talk About It! What might be a common error that someone might make when shading 0.5% on the 10 × 10 grid?

Sample answer: Someone might shade 50 squares or 5 squares instead of shading half of one square.

Example 4 Model the Percent

The diet of a jellyfish consists primarily of plankton, which are tiny organisms living in the ocean. One species of plankton has an average length of 0.04 inch. Suppose a certain jellyfish has a length of 8 inches.

Write a percent that compares the length of the plankton to the length of the jellyfish. Then use the 10 × 10 grid to model the percent.

Step 1 Write a ratio comparing 0.04 inch to 8 inches.

Use equivalent ratios to show that the plankton's length is 0.5 % the length of the jellyfish.

$$\frac{0.04}{8} = \frac{0.5}{100} \quad \text{(×12.5)}$$

plankton (in.) / jellyfish (in.) → percent

Step 2 Shade the 10 × 10 grid.

T o model 0.5%, shade half of one percent by shading half of one grid square.

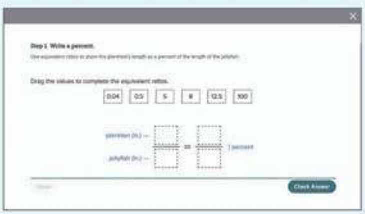
Check

The average weight of a brown bear is about 1,000 pounds. Suppose a large stuffed bear weighs 2.5 pounds. Write a percent to compare the weight of the stuffed animal to the weight of the brown bear. Then use the 10 × 10 grid to model the percent. 0.25% or $\frac{1}{4}$%

Go Online You can complete an Extra Example online.

90 Module 2 • Fractions, Decimals, and Percents

Interactive Presentation



Example 4, Model the Percent, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag to write the equivalent ratios.

CLICK

On Slide 3, students shade squares to model 0.5%.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Model the Percent

Objective

Students will use a 10 × 10 grid to model a percent less than 1%.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the percent, given the context of the problem, and not just perform the calculations. Because the plankton's length is significantly shorter than the length of the jellyfish, a percent of 50% or 5% would not make sense within this context. If the plankton's length was 50% of the jellyfish's length, the plankton's length would be 4 inches. If the plankton's length was 5% of the jellyfish's length, the plankton's length would be 0.4 inches (by using place-value reasoning and dividing 50% and 4 inches both by 10). Having students use this reasoning will help prepare them for upcoming lessons on finding the percent of a number.

As students discuss the *Talk About It!* question on Slide 4, encourage them to use similar reasoning to generate possible misconceptions. Ask students to explain why a student might have these misconceptions and how they can use reasoning to correct them.

Questions for Mathematical Discourse

SLIDE 2

AL What is the length of the plankton? the jellyfish? 0.04 in.; 8 in.

OL How can you set up the equivalent ratios to find the percent? Sample answer: Start with the ratio $\frac{0.04}{8}$. Set this equal to a ratio with 100 in the denominator. Because 8 × 12.5 = 100, multiply 0.04 by 12.5 to find the percent.

BL Write a percent that compares the length of the jellyfish to the length of the plankton. 20,000%

SLIDE 3

AL What number is in the tenths place in the percent? 5

OL What portion of one square should you shade? half

BL Would it be reasonable to use a bar diagram to find the percent instead of a 10 × 10 grid? Explain your reasoning. no; Sample answer: To model 0.5% using a bar diagram, I would need 200 sections, so that each section represents half of 1 percent. This would be very tedious and time consuming to create.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Exit Ticket

Refer to the Exit Ticket slide. Describe how you would model 175% with a 10 × 10 grid. Sample answer: Shade all 100 squares in one grid and 75 squares in a second grid.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

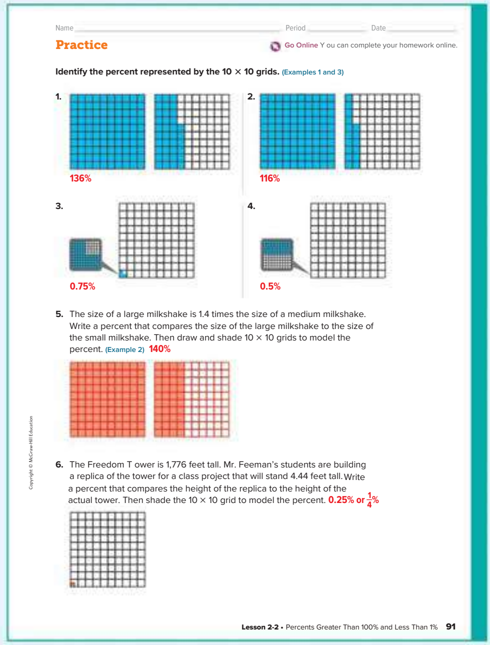
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify percents greater than 100% or less than 1% represented by 10 × 10 grids	1–4
2	model percents greater than 100% with 10 × 10 grids	5
2	model percents less than 1% with a 10 × 10 grid	6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems involving percents less than 1%	8
3	higher-order and critical thinking skills	9–12

Common Misconception

Students may think that because a percent is defined as a ratio that compares a number to 100, the number cannot be greater than 100 or less than 1. Because of this, they may only include the partially shaded grid when identifying the percent that is represented by the 10 × 10 grids. Remind students that percents can be any number and that they should include all of the shaded squares when writing the percent.



Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

Identify the percent represented by the 10 × 10 grids. (Examples 1 and 3)

1. 136%

2. 116%

3. 0.75%

4. 0.5%

5. The size of a large milkshake is 1.4 times the size of a medium milkshake. Write a percent that compares the size of the large milkshake to the size of the small milkshake. Then draw and shade 10 × 10 grids to model the percent. (Example 2) 140%

6. The Freedom T ower is 1,776 feet tall. Mr. Feeman's students are building a replica of the tower for a class project that will stand 4.44 feet tall. Write a percent that compares the height of the replica to the height of the actual tower. Then shade the 10 × 10 grid to model the percent. 0.25% or $\frac{1}{4}$%

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Interactive Presentation

Exit Ticket

T est Practice

7. **Equation Editor** A certain store's sales increased by 175% compared to the previous year. How many squares would be shaded on 10 × 10 grids to represent 175%?

175

Apply *indicates multi-step problem

*8. A bottle of cleaner states that it eliminates 0.999 of germs. For a magazine to recommend a cleaner to its readers, the percent of germs that it does not eliminate cannot exceed 1%. Would this cleaner be recommended by the magazine? Write an argument that can be used to defend your solution.

yes; Sample answer: The cleaner does not eliminate 0.001 or 0. 1% of germs. Since 0. 1%< 1%, the percent of germs that it does not eliminate is less than 1%.

Higher-Order Thinking Problems

9. **MP Persevere with Problems** The top running speed of a giraffe is 250% of the top speed of a squirrel. If a squirrel's top running speed is 12 miles per hour, find the speed of a giraffe.

30 mph

10. **MP Reason Inductively** A rational number is any number that can be written as a fraction with a numerator and denominator that are both whole numbers. Is a percent a rational number? Explain your reasoning.

yes; Sample answer: Every percent can be written as a fraction with a denominator of 100.

11. **MP Find the Error** A student said that to represent 0.2% with a 10 × 10 grid, you shade 2 squares in the grid. Find the student's error and correct it.

Sample answer: The student modeled 2%, not 0.2%. T o model 0.2%, only $\frac{1}{5}$ of one square should be shaded.

12. **Create** Write about a real-world situation involving a percent that is greater than 100% or a percent that is less than 1%. Then explain how you would use 10 × 10 grids to model the percent.

Sample answer: Tyrone's weekly salary is 110% of his previous salary; T o model the percent, use two 10× 10 grids and shade all 100 squares in the first grid and 10 squares in the second grid.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 9, students use mathematical reasoning to plan a strategy to determine the top running speed of a squirrel, given the running speed of a giraffe.

2 Reason Abstractly and Quantitatively In Exercise 10, students use reasoning to explain whether or not percents are rational numbers.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students explain why another student's solution is incorrect and then correct their solution.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Make sense of the problem.

Use with Exercise 11 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the Exercise thinks that 0.2% is equivalent to 2%. Have each pair or group of students present their explanations to the class.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 9–12
- ALEKS Percents, Decimals, and Fractions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8, 9, 11
- Personal Tutor
- Extra Examples 1–4
- ALEKS Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Understanding Percents

Learn Relate Percents to Fractions and Decimals

Objective

Students will understand that they can write a percent as a fraction and a decimal by first writing the percent as a rate per 100.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to use place-value reasoning to explain that while there may be many fractions that represent 35%, only one decimal represents 35%. Decimals are constructed using place-value.

Teaching Notes

SLIDE 1

Students will understand that they can write a percent as a fraction and as a decimal by first writing the percent as a rate per 100. Encourage students to use their knowledge of the definition of percent, equivalent ratios, and place-value reasoning as they progress through the slides of this Learn.

SLIDE 2

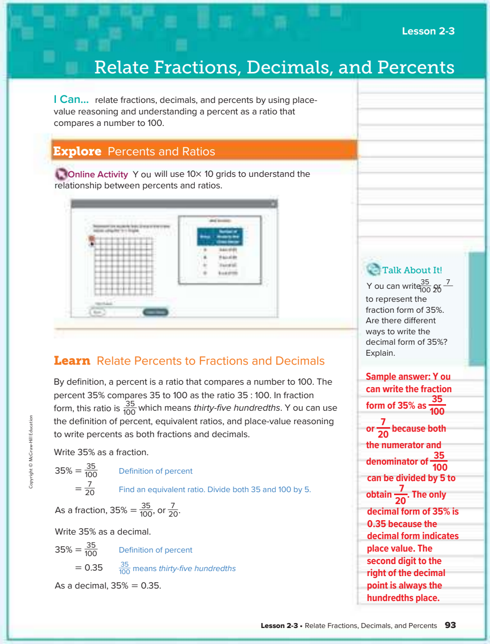
Before having students walk through the calculations, be sure they understand that a percent can be thought of as a ratio or rate per 100. Each time they need to write a percent as a fraction or decimal, encourage them to first write the percent as a ratio to 100. This will help them write the percent as a fraction with a denominator of 100. Because decimals are constructed using place-value, by writing the percent first as a ratio per 100, they can use place-value reasoning to express that value in decimal form.

Talk About It!

SLIDE 3

Mathematical Discourse

You can write $\frac{35}{100}$ or $\frac{7}{20}$ to represent the fraction form of 35%. Are there two different ways to write the decimal form of 35%? Explain. Sample answer: You can write the fraction form of 35% as $\frac{35}{100}$ or $\frac{7}{20}$ because both the numerator and denominator of $\frac{35}{100}$ can be divided by 5 to obtain $\frac{7}{20}$. The only decimal form of 35% is 0.35 because the decimal form indicates place value. The second digit to the right of the decimal point is always the hundredths place.



Lesson 2-3

Relate Fractions, Decimals, and Percents

I Can... relate fractions, decimals, and percents by using place-value reasoning and understanding a percent as a ratio that compares a number to 100.

Explore Percents and Ratios

Online Activity You will use 10× 10 grids to understand the relationship between percents and ratios.

Learn Relate Percents to Fractions and Decimals

By definition, a percent is a ratio that compares a number to 100. The percent 35% compares 35 to 100 as the ratio 35 : 100. In fraction form, this ratio is $\frac{35}{100}$ which means *thirty-five hundredths*. You can use the definition of percent, equivalent ratios, and place-value reasoning to write percents as both fractions and decimals.

Write 35% as a fraction.

$35\% = \frac{35}{100}$ Definition of percent

$= \frac{7}{20}$ Find an equivalent ratio. Divide both 35 and 100 by 5.

As a fraction, $35\% = \frac{35}{100}$, or $\frac{7}{20}$.

Write 35% as a decimal.

$35\% = \frac{35}{100}$ Definition of percent

$= 0.35$ $\frac{35}{100}$ means *thirty-five hundredths*

As a decimal, 35% = 0.35.

Talk About It! You can write $\frac{35}{100}$ or $\frac{7}{20}$ to represent the fraction form of 35%. Are there different ways to write the decimal form of 35%? Explain.

Sample answer: You can write the fraction form of 35% as $\frac{35}{100}$ or $\frac{7}{20}$ because both the numerator and denominator of $\frac{35}{100}$ can be divided by 5 to obtain $\frac{7}{20}$. The only decimal form of 35% is 0.35 because the decimal form indicates place value. The second digit to the right of the decimal point is always the hundredths place.

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Lesson 2-3 • Relate Fractions, Decimals, and Percents 93

Interactive Presentation

Learn, Relate Percents to Fractions and Decimals, Slide 2 of 3

CLICK

On Slide 2, students select the buttons to learn how to write a percent as a fraction and a decimal.

Foundational for 6.RP.A.3, 6.RP.A.3.C

Relate Fractions, Decimals, and Percents

LESSON GOAL

Students will relate fractions, decimals, and percents.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Percents and Ratios

Learn: Relate Percents to Fractions and Decimals
Example 1: Write Percents as Fractions and Decimals
Learn: Relate Fractions to Percents and Decimals
Example 2: Write Fractions as Percents and Decimals
Example 3: Write Mixed Numbers as Percents
Learn: Relate Decimals to Percents and Fractions
Example 4: Write Decimals as Percents and Fractions
Apply: School

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Find the Percent of a Population		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 11 of the *Language Development Handbook* to help your students build mathematical language related to fraction, decimal, and percent equivalencies.

ELL You can use the tips and suggestions on page T11 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by converting between fractions, decimals, and percents.

Standards for Mathematical Content: **Foundational for 6.RP.A.3, 6.RP.A.3.C**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP7**

Coherence

Vertical Alignment

Previous
Students used 10 × 10 grids and bar diagrams to model percents.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Now
Students relate fractions, decimals, and percents.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Next
Students will find the percent of a number.
6.RP.A.3, 6.RP.A.3.C

Rigor

The Three Pillars of Rigor

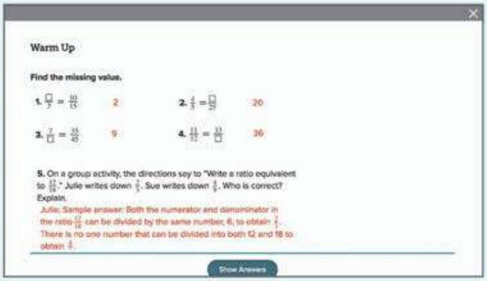
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of percents. They begin to understand the relationship between fractions, decimals, and percents and start to build *fluency* with conversions between them. They *apply* their understanding of fractions, decimals, and percents to solve real-world problems.

Mathematical Background

To write a fraction as a *percent*, find an equivalent fraction with a denominator of 100. The numerator is equal to the percent. Mixed numbers can be written as improper fractions and the same process applies. To write a percent as a fraction or mixed number, express the percent as a rate per 100 by writing the numerator as the percent without the percent sign and the denominator as 100. Then the fraction can be simplified or written as a mixed number, if necessary.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding equivalent ratios (Exercises 1–5)

Answers

1. 2

2. 20

3. 9

4. 36

5. See Warm Up slide online for correct answer.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a basketball team's ratios of free throw attempts.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- How would you describe 37% as a ratio? Sample answer: 37 out of 100

Explore Percents and Ratios

Objective

Students will explore writing ratios as percents.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the ratios of the number of students in different groups that chose soccer as their favorite sport. The students need to compare these ratios. Throughout this activity, students will use various strategies, including 10×10 grids and percents to compare the ratios. Students will use their observations to learn about how percents compare numbers differently than ratios.

Inquiry Question

Why is it helpful to write a ratio as a percent? Sample answer: When you have ratios with different denominators, you can compare them more easily as percents, because percents are given as parts per 100.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

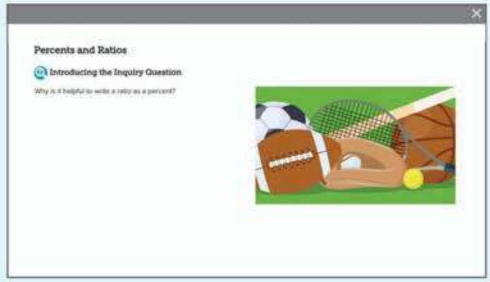
SLIDE 3

Mathematical Discourse

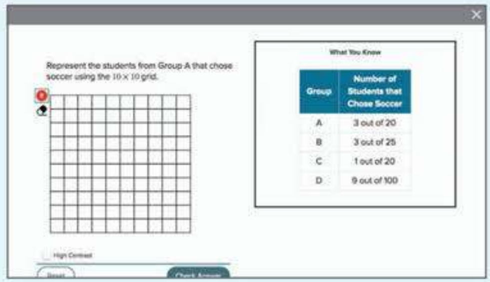
The grid has 100 squares, but the ratio is 3 out of 20. How can you determine the number of squares to shade? Sample answer: There are 5 groups of 20 in 100, so I can divide the grid into 5 sections of 20 squares and shade 3 out of 20 in each section.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8

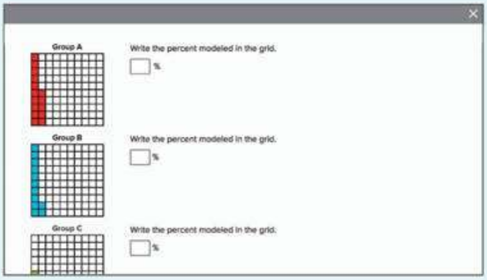


Explore, Slide 3 of 8

CLICK

On Slides 3 and 4, students shade a 10×10 grid to represent a ratio.

Interactive Presentation



Explore, Slide 7 of 8

CLICK

On Slides 5 and 6, students shade a 10 × 10 grid to represent a ratio.

CLICK

On Slide 7, students identify the group with the highest percent of students that chose soccer as their favorite sport.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Percents and Ratios *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use the grid to represent different ratios. Students should examine the similarities and differences between modeling ratios and modeling percents using 10 × 10 grids.

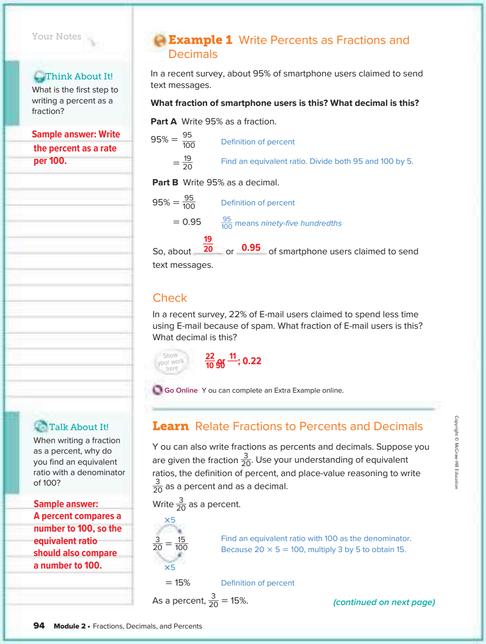
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Is it easier to compare the groups when looking at the models, the percents, or the ratios? Explain your reasoning. Sample answer: It is easier to compare the groups as percents because percents give you the parts per 100. The ratios are different sized groups, so it is more difficult to compare.



Your Notes

Think About It!
What is the first step to writing a percent as a fraction?

Sample answer: Write the percent as a rate per 100.

Example 1 Write Percents as Fractions and Decimals

In a recent survey, about 95% of smartphone users claimed to send text messages.

What fraction of smartphone users is this? What decimal is this?

Part A Write 95% as a fraction.

$95\% = \frac{95}{100}$ Definition of percent

$= \frac{19}{20}$ Find an equivalent ratio. Divide both 95 and 100 by 5.

Part B Write 95% as a decimal.

$95\% = \frac{95}{100}$ Definition of percent

$= 0.95$ $\frac{95}{100}$ means *ninety-five hundredths*

So, about $\frac{19}{20}$ or 0.95 of smartphone users claimed to send text messages.

Check

In a recent survey, 22% of E-mail users claimed to spend less time using E-mail because of spam. What fraction of E-mail users is this? What decimal is this?

Show your work here

$\frac{22}{100}$ or $\frac{11}{50}$; 0.22

Go Online You can complete an Extra Example online.

Learn Relate Fractions to Percents and Decimals

You can also write fractions as percents and decimals. Suppose you are given the fraction $\frac{3}{20}$. Use your understanding of equivalent ratios, the definition of percent, and place-value reasoning to write $\frac{3}{20}$ as a percent and as a decimal.

Talk About It!
When writing a fraction as a percent, why do you find an equivalent ratio with a denominator of 100?

Sample answer: A percent compares a number to 100, so the equivalent ratio should also compare a number to 100.

Write $\frac{3}{20}$ as a percent.

$\frac{3}{20} = \frac{15}{100}$ (×5) Find an equivalent ratio with 100 as the denominator. Because 20 × 5 = 100, multiply 3 by 5 to obtain 15.

$= 15\%$ Definition of percent

As a percent, $\frac{3}{20} = 15\%$.

(continued on next page)

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94 Module 2 • Fractions, Decimals, and Percents

Interactive Presentation

Example 1, Write Percents as Fractions and Decimals, Slide 2 of 3

TYPE

On Slide 2 of Example 1, students write a percent as a fraction.

CLICK

On Slide 2 of the Learn, students select the buttons to learn how to write a fraction as a percent and a decimal.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Example 1 Write Percents as Fractions and Decimals

Objective

Students will write percents as fractions and decimals by first writing the percent as a rate per 100.

Questions for Mathematical Discourse

SLIDE 2

AL Write 95% as a fraction with a denominator of 100. $\frac{95}{100}$

AL What is $\frac{95}{100}$ in word form? ninety-five hundredths

OL In this case, why is it beneficial to leave the fraction with a denominator of 100, before writing it as a decimal? Explain. Sample answer: Because the fraction already has a denominator of 100, it can easily be written as 0.95 using place-value reasoning.

BL Explain how you know your fraction is reasonable. Sample answer: 95% is close to 100%, and the numerator 19 is close to the denominator 20. So, my fraction is reasonable.

Go Online

- Find additional teaching notes and *Teaching the Mathematical Practices*.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Relate Fractions to Percents and Decimals

Objective

Students will understand that they can write a fraction as a percent and a decimal by first finding an equivalent ratio with 100 as the denominator.

Talk About It!

SLIDE 2

Mathematical Discourse

When writing a fraction as a percent, why do you find an equivalent ratio with a denominator of 100? Sample answer: A percent compares a number to 100, so the equivalent ratio should also compare a number to 100.

(continued on next page)

Learn Relate Fractions to Percents and Decimals *(continued)*

Teaching Notes

SLIDE 3

You may wish to pause the animation after the fraction $\frac{9}{15}$ is shown. Challenge students to come up with a strategy as to how they can write an equivalent fraction with 100 as the denominator. Have them share their strategies with the class. Then replay the animation and ask students to compare their strategy with the one shown. Ask students why it is beneficial to write $\frac{9}{15}$ as $\frac{3}{5}$, as opposed to other equivalent fractions, such as $\frac{18}{30}$, $\frac{27}{45}$, or $\frac{36}{60}$. Students should be able to reason that the denominators 30, 45, and 60 are not factors of 100, so those fractions do not help them get any closer to finding the percent.

Talk About It!

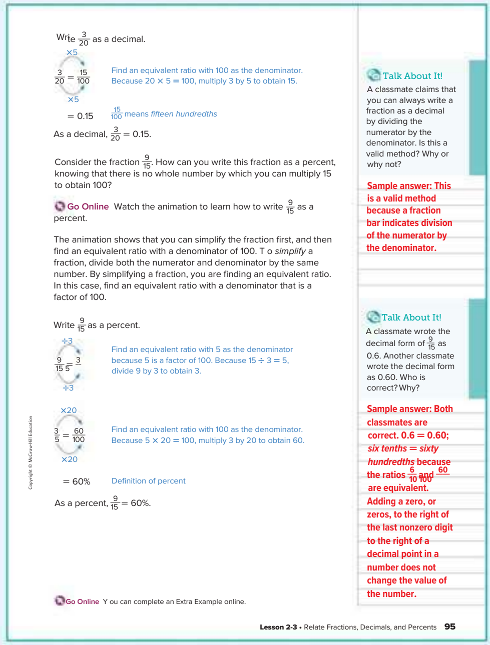
SLIDE 4

Mathematical Discourse

A classmate claims that you can always write a fraction as a decimal by dividing the numerator by the denominator. Is this a valid method? Why or why not? Sample answer: This is a valid method because a fraction bar indicates division of the numerator by the denominator.

A classmate wrote the decimal form of $\frac{9}{15}$ as 0.6. Another classmate wrote the decimal form as 0.60. Who is correct? Why? Sample answer: Both classmates are correct. 0.6 = 0.60; *six tenths* = *sixty hundredths* because the ratios $\frac{6}{10}$ and $\frac{60}{100}$ are equivalent. Adding a zero, or zeros, to the right of the last nonzero digit to the right of a decimal point in a number does not change the value of the number.

Go Online to find *Teaching the Mathematical Practices*.



Write $\frac{3}{20}$ as a decimal.

$\frac{3}{20} = \frac{15}{100}$ (×5) Find an equivalent ratio with 100 as the denominator. Because 20 × 5 = 100, multiply 3 by 5 to obtain 15.

$= 0.15$ $\frac{15}{100}$ means *fifteen hundredths*

As a decimal, $\frac{3}{20} = 0.15$.

Consider the fraction $\frac{9}{15}$. How can you write this fraction as a percent, knowing that there is no whole number by which you can multiply 15 to obtain 100?

Go Online Watch the animation to learn how to write $\frac{9}{15}$ as a percent.

The animation shows that you can simplify the fraction first, and then find an equivalent ratio with a denominator of 100. T o *simplify* a fraction, divide both the numerator and denominator by the same number. By simplifying a fraction, you are finding an equivalent ratio. In this case, find an equivalent ratio with a denominator that is a factor of 100.

Write $\frac{9}{15}$ as a percent.

$\frac{9}{15} = \frac{3}{5}$ (÷3) Find an equivalent ratio with 5 as the denominator because 5 is a factor of 100. Because 15 ÷ 3 = 5, divide 9 by 3 to obtain 3.

$\frac{3}{5} = \frac{60}{100}$ (×20) Find an equivalent ratio with 100 as the denominator. Because 5 × 20 = 100, multiply 3 by 20 to obtain 60.

$= 60\%$ Definition of percent

As a percent, $\frac{9}{15} = 60\%$.

Go Online Y ou can complete an Extra Example online.

Talk About It! A classmate claims that you can always write a fraction as a decimal by dividing the numerator by the denominator. Is this a valid method? Why or why not?

Sample answer: This is a valid method because a fraction bar indicates division of the numerator by the denominator.

Talk About It! A classmate wrote the decimal form of $\frac{9}{15}$ as 0.6. Another classmate wrote the decimal form as 0.60. Who is correct? Why?

Sample answer: Both classmates are correct. 0.6 = 0.60; *six tenths* = *sixty hundredths* because the ratios $\frac{6}{10}$ and $\frac{60}{100}$ are equivalent. Adding a zero, or zeros, to the right of the last nonzero digit to the right of a decimal point in a number does not change the value of the number.

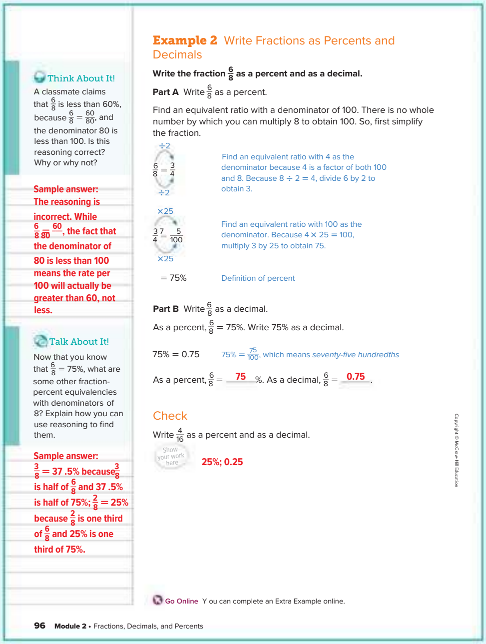
Lesson 2-3 • Relate Fractions, Decimals, and Percents 95

Interactive Presentation

Learn, Relate Fractions to Percents and Decimals, Slide 3 of 4

WATCH

On Slide 3, students watch an animation to learn how to write fractions as percents.



Example 2 Write Fractions as Percents and Decimals

Write the fraction $\frac{6}{8}$ as a percent and as a decimal.

Part A Write $\frac{6}{8}$ as a percent.

Find an equivalent ratio with a denominator of 100. There is no whole number by which you can multiply 8 to obtain 100. So, first simplify the fraction.

$\frac{6}{8} = \frac{3}{4}$ (÷2) — Find an equivalent ratio with 4 as the denominator because 4 is a factor of both 100 and 8. Because 8 ÷ 2 = 4, divide 6 by 2 to obtain 3.

$\frac{3}{4} = \frac{75}{100}$ (×25) — Find an equivalent ratio with 100 as the denominator. Because 4 × 25 = 100, multiply 3 by 25 to obtain 75.

= 75% — Definition of percent

Part B Write $\frac{6}{8}$ as a decimal.

As a percent, $\frac{6}{8}$ = 75%. Write 75% as a decimal.

75% = 0.75 — 75% = $\frac{75}{100}$, which means *seventy-five hundredths*

As a percent, $\frac{6}{8}$ = 75 %. As a decimal, $\frac{6}{8}$ = 0.75.

Check

Write $\frac{4}{16}$ as a percent and as a decimal. 25%; 0.25

Think About It! A classmate claims that $\frac{6}{8}$ is less than 60%, because $\frac{6}{8} = \frac{60}{80}$, and the denominator 80 is less than 100. Is this reasoning correct? Why or why not?

Sample answer: The reasoning is incorrect. While $\frac{6}{8} = \frac{60}{80}$, the fact that the denominator of 80 is less than 100 means the rate per 100 will actually be greater than 60, not less.

Talk About It! Now that you know that $\frac{6}{8}$ = 75%, what are some other fraction-percent equivalencies with denominators of 8? Explain how you can use reasoning to find them.

Sample answer: $\frac{3}{8}$ = 37.5% because $\frac{3}{8}$ is half of $\frac{6}{8}$ and 37.5% is half of 75%; $\frac{2}{8}$ = 25% because $\frac{2}{8}$ is one third of $\frac{6}{8}$ and 25% is one third of 75%.

Go Online You can complete an Extra Example online.

96 Module 2 • Fractions, Decimals, and Percents

Interactive Presentation

Example 2, Write Fractions as Percents and Decimals, Slide 2 of 5

CLICK On Slide 2, students move through the steps to write the fraction as a percent.

TYPE On Slide 3, students write the fraction as a decimal.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Write Fractions as Percents and Decimals

Objective

Students will write fractions as percents and decimals by first finding an equivalent ratio with 100 as the denominator.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantity $\frac{6}{8}$ and to understand how simplifying it first will allow them to write it as a fraction with a denominator of 100. Students should be able to reason that while 8 is not a factor of 100, 4 is a factor of 100.

Questions for Mathematical Discourse

SLIDE 2

AL Is the denominator of $\frac{6}{8}$ a factor of 100? no

AL Is $\frac{6}{8}$ in simplest form? Why or why not? no; Sample answer: It can be written as $\frac{3}{4}$.

OL Why do you simplify $\frac{6}{8}$ first? Sample answer: Simplifying $\frac{6}{8}$ to $\frac{3}{4}$ allows me to write an equivalent fraction with a denominator of 100 because 4 is a factor of 100.

BL Write three different fractions that are all equivalent to 75%. Sample answers: $\frac{9}{12}$, $\frac{15}{20}$, and $\frac{18}{24}$

SLIDE 3

AL Why did you start with 75%? Sample answer: The fraction was already written as 75%.

OL What mathematical operation do you use to convert 75% to 0.75? Explain. division; Sample answer: Divide 75 by 100.

BL What is another method you can use to write a fraction as a decimal? Sample answer: Divide the numerator by the denominator.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 3 Write Mixed Numbers as Percents

Objective

Students will write a mixed number as a percent.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantity $2\frac{9}{10}$ and to understand how first expressing it as an improper fraction will help in writing the quantity as a percent. Students should also be able to use reasoning to find the percent mentally.

Questions for Mathematical Discourse

SLIDE 2

AL What percent does the whole number 1 represent? 2? 100%; 200%

AL What is $\frac{9}{10}$ written as a percent? 90%

OL Why is it helpful to write $2\frac{9}{10}$ as an improper fraction? Sample answer: It aids in writing an equivalent fraction with a denominator of 100.

OL How can you use reasoning to write $2\frac{9}{10}$ as a percent, without writing the mixed number as an improper fraction first? Sample answer: The mixed number $2\frac{9}{10} = 2 + \frac{9}{10}$. The whole number 2 represents 200%. The fraction $\frac{9}{10}$ represents 90%. So, $2\frac{9}{10}$ represents 290%.

BL The *gyrfalcon* is a bird of prey, and the largest falcon in the falcon species. It has a top speed that is $1\frac{17}{20}$ times as fast as the top speed of a cheetah. Write $1\frac{17}{20}$ as a percent. 185%

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of writing fractions as percents, have them work with a partner to generate a list of fractions that satisfy each condition below. For each condition, they should generate at least 3 fractions. Have them explain how they determined the fractions in each list.

- fractions in which they can immediately write an equivalent fraction with a denominator of 100 (without simplifying the fraction first)
- fractions in which they should write the fraction in simplest form before writing an equivalent fraction with a denominator of 100

Example 3 Write Mixed Numbers as Percents

The cheetah is the fastest land mammal in the world. The peregrine falcon is the fastest bird in the world. The peregrine falcon's top speed is $2\frac{9}{10}$ times as fast as the top speed of a cheetah.

What percent represents this value?

Step 1 Write the mixed number as an improper fraction.

The fraction $2\frac{9}{10}$ is a mixed number that consists of a whole number part, 2, and a fractional part, $\frac{9}{10}$.

$2\frac{9}{10} = 2 + \frac{9}{10}$ — Write the mixed number as a sum.

$= \frac{10}{10} + \frac{10}{10} + \frac{9}{10}$ — $2 = 1 + 1$ and $1 = \frac{10}{10}$

$= \frac{29}{10}$ — Add.

Step 2 Find an equivalent ratio with 100 as a denominator.

$\frac{29}{10} = \frac{290}{100}$ (×10, ×10) — Find an equivalent ratio with 100 as the denominator. Because $10 \times 10 = 100$, multiply 29 by 10 to obtain 290.

$= 290\%$ — Definition of percent

So, the peregrine falcon's top speed is 290 % that of a cheetah's top speed.

Think About It! Is the top speed of the falcon greater than 200% that of the cheetah? How do you know?

yes; Sample answer: If the top speed of the falcon was 200% that of the cheetah, the falcon would be twice as fast, or two times as fast, as the cheetah. Since the falcon's top speed is $2\frac{9}{10}$ times as fast, the percent is greater than 200%.

Talk About It! How can you use mental math to express $2\frac{9}{10}$ as a percent?

Sample answer: $\frac{9}{10}$ is 90% and the whole number 2 means 200%. So, $2\frac{9}{10}$ as a percent is 90% + 200% or 290%.

Check

When blue whales feed, they can take in $1\frac{1}{25}$ times their body weight in food and water in one single gulp. What percent of their body weight is this?

Show your work here 104%

Go Online You can complete an Extra Example online.

Lesson 2-3 • Relate Fractions, Decimals, and Percents 97

Interactive Presentation

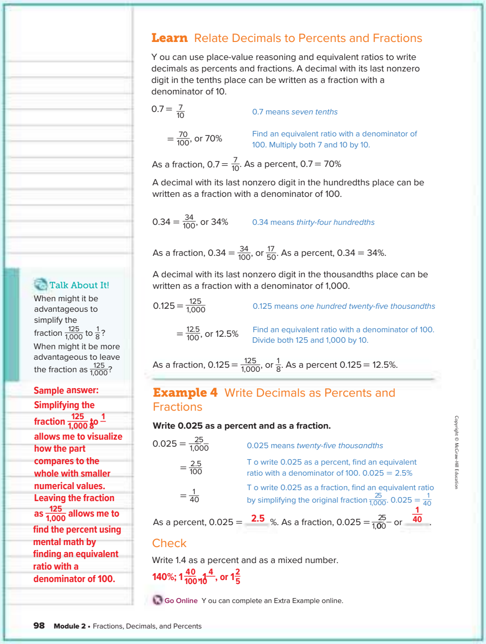
Example 3, Write Mixed Numbers as Percents, Slide 3 of 5

TYPE

On Slide 3, students write the mixed number as a percent.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Relate Decimals to Percents and Fractions

Y ou can use place-value reasoning and equivalent ratios to write decimals as percents and fractions. A decimal with its last nonzero digit in the tenths place can be written as a fraction with a denominator of 10.

$0.7 = \frac{7}{10}$ 0.7 means *seven tenths*

$= \frac{70}{100}$, or 70% Find an equivalent ratio with a denominator of 100. Multiply both 7 and 10 by 10.

As a fraction, $0.7 = \frac{7}{10}$. As a percent, 0.7 = 70%

A decimal with its last nonzero digit in the hundredths place can be written as a fraction with a denominator of 100.

$0.34 = \frac{34}{100}$, or 34% 0.34 means *thirty-four hundredths*

As a fraction, $0.34 = \frac{34}{100}$, or $\frac{17}{50}$. As a percent, 0.34 = 34%.

A decimal with its last nonzero digit in the thousandths place can be written as a fraction with a denominator of 1,000.

$0.125 = \frac{125}{1,000}$ 0.125 means *one hundred twenty-five thousandths*

$= \frac{12.5}{100}$, or 12.5% Find an equivalent ratio with a denominator of 100. Divide both 125 and 1,000 by 10.

As a fraction, $0.125 = \frac{125}{1,000}$, or $\frac{1}{8}$. As a percent 0.125 = 12.5%.

Talk About It!

When might it be advantageous to simplify the fraction $\frac{125}{1,000}$ to $\frac{1}{8}$? When might it be more advantageous to leave the fraction as $\frac{125}{1,000}$?

Sample answer: Simplifying the fraction $\frac{125}{1,000}$ to $\frac{1}{8}$ allows me to visualize how the part compares to the whole with smaller numerical values. Leaving the fraction as $\frac{125}{1,000}$ allows me to find the percent using mental math by finding an equivalent ratio with a denominator of 100.

Example 4 Write Decimals as Percents and Fractions

Write 0.025 as a percent and as a fraction.

$0.025 = \frac{25}{1,000}$ 0.025 means *twenty-five thousandths*

$= \frac{2.5}{100}$ T o write 0.025 as a percent, find an equivalent ratio with a denominator of 100. 0.025 = 2.5%

$= \frac{1}{40}$ T o write 0.025 as a fraction, find an equivalent ratio by simplifying the original fraction $\frac{25}{1,000}$. $0.025 = \frac{1}{40}$

As a percent, 0.025 = **2.5** %. As a fraction, $0.025 = \frac{25}{1,000}$ or $\frac{1}{40}$.

Check

Write 1.4 as a percent and as a mixed number.

140%; $1\frac{40}{100}$, $1\frac{4}{10}$, or $1\frac{2}{5}$

Go Online Y ou can complete an Extra Example online.

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Interactive Presentation

Example 4, Write Decimals as Percents and Fractions, Slide 1 of 2

CLICK

On Slide 1 of Example 4, students move through the steps to write the decimal as a percent and a fraction.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Relate Decimals to Percents and Fractions

Objective

Students will understand how to write a decimal as a percent and then as a fraction by first writing the decimal as a fraction with a denominator of 100.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 4

Mathematical Discourse

When might it be advantageous to simplify the fraction $\frac{125}{1,000}$ to $\frac{1}{8}$? When might it be more advantageous to leave the fraction as $\frac{125}{1,000}$? Sample answer: Simplifying the fraction $\frac{125}{1,000}$ to $\frac{1}{8}$ allows me to visualize how the part compares to the whole with smaller numerical values. Leaving the fraction as $\frac{125}{1,000}$ allows me to find the percent using mental math by finding an equivalent ratio with a denominator of 100.

Example 4 Write Decimals as Percents and Fractions

Objective

Students will write a decimal as a percent and then a fraction by first writing the decimal as a fraction with a denominator of 100.

Questions for Mathematical Discourse

SLIDE 1

AL What is the decimal 0.025 in word form? twenty-five thousandths

OL How do you know that *twenty-five thousandths* can be written as a fraction? Sample answer: *Thousandths* means that the last digit is in the thousandths place. *Twenty-five thousandths* means *twenty-five out of one thousand*, or $\frac{25}{1,000}$.

BL Write *thirty hundredths* as both a decimal and a percent. Can you write this number in word form differently? Explain. 0.30; 30%; yes; Sample answer: 0.30 also means *three tenths*.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply School

Objective

Students will come up with their own strategy to solve an application problem involving time spent studying.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

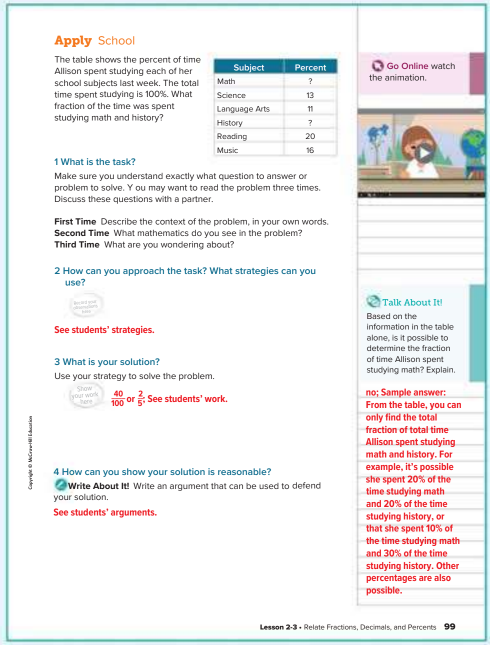
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What can we say about how the amount of time spent studying math compares to the amount of time spent studying history?
- How do you write a percent as a fraction?
- Can the fraction be simplified?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply School

The table shows the percent of time Allison spent studying each of her school subjects last week. The total time spent studying is 100%. What fraction of the time was spent studying math and history?

Subject	Percent
Math	?
Science	13
Language Arts	11
History	?
Reading	20
Music	16

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

$\frac{40}{100}$ **or** $\frac{2}{5}$**; See students' work.**

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online watch the animation.

Talk About It!

Based on the information in the table alone, is it possible to determine the fraction of time Allison spent studying math? Explain.

no; Sample answer: From the table, you can only find the total fraction of total time Allison spent studying math and history. For example, it's possible she spent 20% of the time studying math and 20% of the time studying history, or that she spent 10% of the time studying math and 30% of the time studying history. Other percentages are also possible.

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Lesson 2-3 • Relate Fractions, Decimals, and Percents 99

Interactive Presentation

Apply, School

WATCH

On Slide 1, students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Math History Minute

Graciano Ricalde Gamboa (1873–1942) was a Mexican mathematician who in 1910, achieved recognition for calculating the orbit of Halley's Comet. His precise calculations proved that the comet would not hit Earth, which was of great concern at the time. Halley's Comet follows a highly elliptical path and can be seen from Earth every 74–79 years.

Check

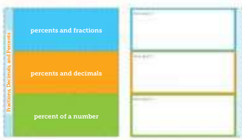
The table shows the percent of time Naima's soccer team spent on each skill during their last practice. The total time spent practicing is 100%. What fraction of the time was spent on crossing and passing? $\frac{9}{20}$

Skill	Percent
crossing	?
dribbling	21
heading	13
juggling	6
passing	?
shooting	15

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



100 **Module 2** • Fractions, Decimals, and Percents

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you use fractions, decimals, and percents to solve everyday problems? In this lesson, students learned how to write equivalent forms of fractions, decimals, and percents using the definition of percent as a rate per 100. Encourage them to brainstorm with a partner some real-world examples of when they might need to convert between fractions, decimals, and percents. For example, if they got 8 out of 10 problems correct on a quiz, that would be the same as a score of 80% on the quiz.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could describe how to convert between fractions, decimals, and percents. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. A basketball team made 12 out of 15 free throw attempts in their first game, 14 out of 25 in the second game, and 13 out of 20 in the third game. Has the team improved their free throw accuracy? Write a mathematical argument that can be used to defend your solution. Game #1: 80%; Game #2: 56%; Game #3: 65%; Sample answer: The team's accuracy decreased from the first game to the second game, and then increased from the second game to the third game.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 11, 13, 15, 16–19
- Extension: Find the Percent of a Population
- ALEKS Percents, Decimals, and Fractions

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–11, 15, 16, 17
- Personal Tutor
- Extra Examples 1–4
- ALEKS Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- ArriveMATH Take Another Look
- ALEKS Understanding Percents

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- AL Practice Form B
- OL Practice Form A
- BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	T opic	Exercises
1	write percents as fractions and decimals	1–3
1	write fractions as percents and decimals	4–6
1	write decimals as percents and fractions	7–9
2	convert between fractions, decimals, and percents	10–12
2	extend concepts learned in class to apply them in new contexts	13
3	solve application problems involving fractions, decimals, and percents	14, 15
3	higher-order and critical thinking skills	16–19

Common Misconception

Some students may have difficulty when writing mixed numbers as a percent because they do not include the whole number when writing the percent. In Exercise 5, remind students that they need to include the whole number when writing $1\frac{3}{4}$ as a percent. They can first write the fractional part as a percent and then add 100, since 1 is equal to 100%. Encourage students to think about the definition of a mixed number to understand that a mixed number will result in a percent that is greater than 100.

Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

Write each percent as a fraction in simplest form and as a decimal. (Example 1)

1. 45% $\frac{9}{20}$, 0.45
2. 72% $\frac{18}{25}$, 0.72
3. 80% $\frac{4}{5}$, 0.8

Write each fraction as a percent and as a decimal. (Examples 2 and 3)

4. $\frac{3}{20}$ 15%, 0. 15
5. $1\frac{3}{4}$ 175%, 1.75
6. $\frac{5}{8}$ 62.5%, 0.625

Write each decimal as a percent and as a fraction in simplest form. (Example 4)

7. 0.89 89%, $\frac{89}{100}$
8. 0.82 82%, $\frac{41}{50}$
9. 0.65 65%, $\frac{13}{20}$

10. About 0.41 of Hawaii's total area is water. Write 0.41 as a fraction and as a percent. $\frac{41}{100}$, 41%

11. Over the course of the basketball season, Zane's free throw average went up by 30%. Write 30% as a fraction and as a decimal. $\frac{3}{10}$, 0.3

12. There are 25 students in Muriel's class. Write a percent to represent the number of students that have brown eyes. Then write the percent as a fraction and as a decimal.

Eye Color	Number of Students
Blue	6
Brown	10
Green	7
Hazel	2

40%, $\frac{2}{5}$, 0.4

T est Practice

13. **Multiselect** Which of the following are equivalent to 85%? Select all that apply.

- [x] 0.85
- [x] $\frac{85}{100}$
- [ ] 0.8
- [x] $\frac{17}{20}$
- [ ] 85

Apply *indicates multi-step problem

*14. The table shows the results of a recent survey of sixth grade students at Potter Middle School about their favorite sports. What fraction of the students chose football or soccer?

$\frac{11}{20}$

Sport	Percent
Baseball	14
Football	20
Lacrosse	12
Soccer	35
Softball	8
Volleyball	11

*15. The table shows the percent of each type of pet owned by pet owners in a neighborhood. The total percent is equal to 100%. What fraction of the pets owned were cats and dogs?

$\frac{7}{10}$

Pet	Percent
Bird	4
Cat	?
Dog	?
Fish	14
Hamster	10
Snake	2

Higher-Order Thinking Problems

16. MP **Justify Conclusions** Determine if the following statement is *true* or *false*. Justify your conclusion.
Any decimal that ends with a digit in the hundredths place can be written as a fraction with a denominator that is divisible by both 2 and 5.

true; Sample answer: A decimal that ends in the hundredths place can be written with a denominator of 100. 100 is divisible by both 2 and 5. So, the denominator of every such decimal is divisible by 2 and 5.

17. MP **Reason Inductively** A sixth-grade class was surveyed about their favorite kind of drink. The results are shown in the table. Did chocolate milk and lemonade receive more than 50% of the votes? Explain.

Type of Drink	Percent (decimal)
Chocolate Milk	0.22
Iced T ea	0.05
Lemonade	0.24
Orange Juice	0.18
Sports Drink	0.31

no; Sample answer: $0.22 + 0.24 = 0.46$ and $0.46 = 46\%$. Since $46\% < 50\%$, chocolate milk and lemonade did not receive more than 50% of the votes.

18. MP **Persevere with Problems** Explain how you can write $25\frac{2}{5}\%$ as a decimal.

Sample answer: Since $\frac{2}{5}$ is equal to 0.4, I can write the percent as 25.4%. Then I can write the percent as the decimal 0.254.

19. MP **Identify Structure** When writing a fraction as a percent, how can you tell if the percent will be less than 100%, equal to 100%, or greater than 100%?

Sample answer: The percent will be less than 100% if the numerator is less than the denominator . The percent will be equal to 100% if the numerator and the denominator are equal. The percent will be greater than 100% if the numerator is greater than the denominator .

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students determine if a statement is true or false and justify their conclusion.

In Exercise 17, students use inductive reasoning to evaluate the results of a survey.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 18, students determine a strategy and explain how to write a mixed-number percent as a decimal.

7 Look for and Make Use of Structure In Exercise 19, students use the structure of a fraction to explain how to determine if an equivalent percent will be greater than or less than 100%.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.
Use with Exercises 14–15 Have students work in pairs. Have students individually read Exercise 14 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 15.

Be sure everyone understands.
Use with Exercises 16–17 Have students work in groups of 3–4 to solve the problem in Exercise 16. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 17.

Learn Find the Percent of a Number

Objective

Students will understand how to use bar diagrams, ratio tables, equivalent ratios, and double number lines to find the percent of a number.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, have them give advantages and disadvantages of dividing the bar into different numbers of sections.

Teaching Notes

SLIDES 1–2

You may wish to present the scenario to the class and ask them how they would find the number of people who prefer lemon, using any strategy they choose. They should be prepared to explain their strategy to the class. Have them compare their strategy to the one shown that uses a bar diagram. Ask them why the bar diagram is divided into ten equal sections, and what each section represents, both in terms of percent and in terms of the number of people. Students previously learned about part-to-whole ratios. Ask them what the part-to-whole ratio is of the number of people that preferred lemon sherbet, and how it is illustrated in the bar diagram.

Talk About It!

SLIDE 2

Mathematical Discourse

Why is the bar divided into 10 sections? Is there a different way you can divide the bar to solve the same problem? Explain. Sample answer: Since the percent is 20%, the bar is divided into 10 sections that each represent 10%. You could also divide the bar into 20 sections, and each section would then represent 5%.

(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling with creating a bar diagram to solve a percent problem, have them work with a partner to find the smallest number of sections that a bar diagram can be divided into to model each percent.

65% 20 sections

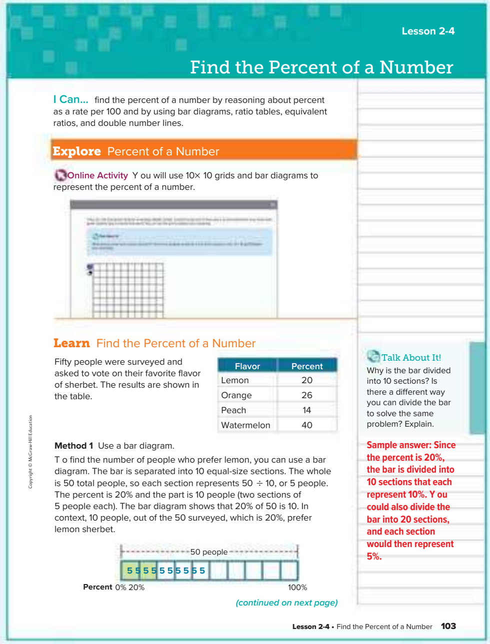
75% 4 sections

72% 25 sections

70% 10 sections

80% 5 sections

74% 50 sections



Lesson 2-4

Find the Percent of a Number

I Can... find the percent of a number by reasoning about percent as a rate per 100 and by using bar diagrams, ratio tables, equivalent ratios, and double number lines.

Explore Percent of a Number

Online Activity Y ou will use 10× 10 grids and bar diagrams to represent the percent of a number.

Learn Find the Percent of a Number

Fifty people were surveyed and asked to vote on their favorite flavor of sherbet. The results are shown in the table.

Flavor	Percent
Lemon	20
Orange	26
Peach	14
Watermelon	40

Method 1 Use a bar diagram.

T o find the number of people who prefer lemon, you can use a bar diagram. The bar is separated into 10 equal-size sections. The whole is 50 total people, so each section represents 50 ÷ 10, or 5 people. The percent is 20% and the part is 10 people (two sections of 5 people each). The bar diagram shows that 20% of 50 is 10. In context, 10 people, out of the 50 surveyed, which is 20%, prefer lemon sherbet.

(continued on next page)

Talk About It! Why is the bar divided into 10 sections? Is there a different way you can divide the bar to solve the same problem? Explain.

Sample answer: Since the percent is 20%, the bar is divided into 10 sections that each represent 10%. Y ou could also divide the bar into 20 sections, and each section would then represent 5%.

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Lesson 2-4 • Find the Percent of a Number 103

Interactive Presentation

Learn, Find the Percent of a Number, Slide 1 of 6

Lesson 2-4

Percent of a Number

6.RP.A.3, 6.RP.A.3.C

LESSON GOAL

Students will use bar diagrams, equivalent ratios, double number lines, and ratio tables to find the percent of a number.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Percent of a Number

Learn: Find the Percent of a Number
Example 1: Find the Percent of a Number
Example 2: Find the Percent of a Number
Example 3: Find the Percent of a Number
Example 4: Find the Percent of a Number
Apply: Book Fair

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Compare Multiple Discounts		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 12 of the *Language Development Handbook* to help your students build mathematical language related to finding the percent of a number.

ELL You can use the tips and suggestions on page T12 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by finding the percent of a number.
Standards for Mathematical Content: **6.RP .A.3, 6.RP.A.3.C**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students related fractions, decimals, and percents.
Foundational for 6.RP.A.3, 6.RP.A.3.C

Now
Students find the percent of a number. **6.RP.A.3, 6.RP.A.3.C**

Next
Students will estimate the percent of a number. **6.RP.A.3, 6.RP.A.3.C**

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their *understanding* of percents. They use the parts of a percent problem (part, whole, and percent), to build *fluency* with finding the percent of a number. They *apply* their fluency to solve real-world problems.

Mathematical Background

To find the percent of a number, first express the percent as a *rate per 100*. In other words, write the percent as a fraction with a denominator of 100. Simplify the fraction, if necessary, and then multiply the fraction by the number. For example, to find 32% of 514, think of 32% as the rate 32 per 100, or $\frac{32}{100}$. Multiply the rate by 514; $\frac{32}{100} \times 514 = 164.48$. You can also express the rate per 100 as a decimal and multiply it by the whole.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing fractions as percents (Exercises 1–4)
- understanding unit price (Exercise 5)

Answers

1. 80% **2.** 50%
3. 45% **4.** 75%
5. $2.51

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about percent discounts on items in stores.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What is an example of a *rate*? What might it mean to express a percent as a *rate per 100*? Sample answer: 120 miles in 2 hours; a percent compares a number to 100, so expressing a percent as a rate per 100 may mean to write it as a ratio using 100 as the whole.

Explore Percent of a Number

Objective

Students will use bar diagrams to explore percent of a number.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a scenario about 300 sixth-grade students. Throughout this activity, students will use a 10 × 10 grid to find the number of students that play a musical instrument. Students will then be given another scenario and they will use a double bar diagram to represent the number of students that play a musical instrument.

Inquiry Question

How can you use a model to find the percent of a number? Sample answer: I can use 10 × 10 grids or bar diagrams to find the percent of a number.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

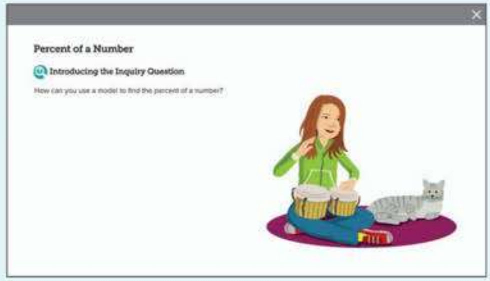
SLIDE 2

Mathematical Discourse

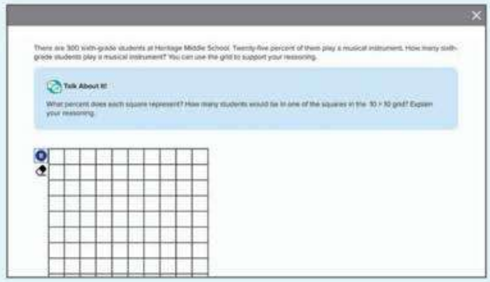
What percent does each square represent? How many students would be in one of the squares in the 10 × 10 grid? Explain your reasoning. Sample answer: Each square represents 1%. Each square also represents 3 students because 300 divided by 100 is 3.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6



Explore, Slide 2 of 6

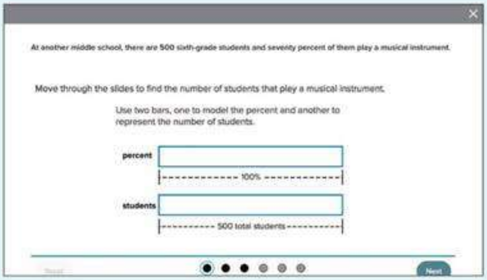
CLICK

On Slides 2 and 3, students use a 10 × 10 grid to represent percents.

TYPE

On Slide 3, students indicate the number of students represented by the shaded portion of the grid.

Interactive Presentation



Explore, Slide 4 of 6

CLICK

On Slide 4, students move through slides to find the number of students that play musical instruments.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Percent of a Number *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use a 10×10 grid and a double bar diagram to model finding the percent of a number.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

How could you use a bar diagram to represent the same situation?

Sample answer: I could divide the bar into 4 sections, each representing 25%. 300 divided by 4 is 75, so each section also represents 75 students.

Your Notes

Method 2 Use a ratio table.

You know that 100% of 50 is 50. You need to find 20% of 50. Scale back to find 20% of 50 by dividing both 100 and 50 by 5.

Percent	20	100
Number of People	10	50

÷5 ÷5

Method 3 Use equivalent ratios.

Let n represent the number of people who prefer lemon.

lemon → $\frac{n}{50} = \frac{20}{100}$ } percent
total surveyed →

÷2

$\frac{n}{50} = \frac{20}{100}$ Because 100 ÷ 2 = 50, divide 20 by 2.

÷2

$\frac{10}{50}\ \frac{20}{100}$ 20 ÷ 2 = 10; So, n = 10.

So, 10 people prefer lemon.

Method 4 Use a double number line.

Draw a double number line. The bottom number line represents the percent, so use increments of 10 to draw tick marks and label the percents. The top number line represents the number of people, so label the tick mark that corresponds with 100% on the bottom number line, with 50. Since there are 10 increments, the value of each tick mark on the top number line increases by 50 ÷ 10, or 5 units.

Number of People: 0 5 10 15 20 25 30 35 40 45 50
Percent: 0 10 20 30 40 50 60 70 80 90 100

The double number line shows that 20% corresponds to 10 people.

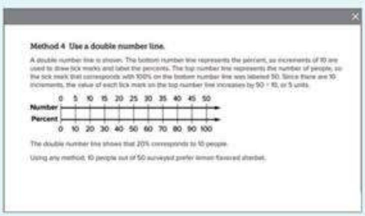
Using any method, 10 people out of 50 surveyed prefer lemon flavored sherbet.

Talk About It!

Which representation helps you to best visualize the problem? Can you think of a situation in which it might not be advantageous to use that representation?

See students' responses.

Interactive Presentation



Learn, Find the Percent of a Number, Slide 5 of 6

Learn Find the Percent of a Number *(continued)*

Teaching Notes

SLIDE 3

Although students have learned about ratio tables already, they still may need support in determining the appropriate labels and locations for the given data. Encourage students to look back at the scenario to identify the given data as the part, whole, or percent. Students may also find the bar diagram on the previous slide helpful as a comparison, as they work to understand the ratio table on this slide.

SLIDE 4

Some students may find using an equation to be their preferred method to solve the problem. You may wish to ask students to think of a scenario when an equation may not be the most advantageous to use. Students should be able to reason that if the whole does not go into 100 evenly, then using another method may be preferable.

SLIDE 5

At first, students may not want to learn four different methods for finding the percent of a number. Arrange students into groups of 3–4. Have each group study the four different methods, compare and contrast each method, and devise a reasoning behind when it would be advantageous to use each method. Then have each group share their findings with the class. Ask them how studying different methods helps them understand the mathematics involved in each, and to have a better understanding of what the percent of a number *means*, as opposed to memorizing a certain process or method without having that depth of understanding.

Talk About It!

SLIDE 6

Mathematical Discourse

Which representation helps you to visualize the problem? Can you think of a situation in which it might not be advantageous to use that representation? See students' responses.

Example 1 Find the Percent of a Number

Objective

Students will use a rate per 100 to find the percent of a number.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities and to understand which values represent the part, the percent, and the whole. While mental math may not always be a beneficial method when finding the percent of a number, it is very beneficial when it can be used, as in this Example. Understanding 15% of 300 as a rate of 15 for every 100 will help students solidify their understanding of what the percent of a number means, in general.

5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 5, encourage them to think about how to construct a bar diagram to solve this problem. They may compare the three methods and discuss which method is most efficient for solving this particular problem.

Questions for Mathematical Discourse

SLIDE 3

AL How can you interpret 15% as a comparison to 100? 15 out of 100

OL How can you determine if your answer is accurate? Sample answer: If 45 out of 300 students bring cheese as a snack, I can write it as a fraction and then simplify. Because, $\frac{45}{300} = \frac{15}{100}$, which is 15%, I know my answer is accurate.

BL If 14% of the students brought cheese as a snack, how many students brought cheese as a snack? 42 students

SLIDE 4

AL How can you represent the percent as a ratio? How can you represent the relationship between the part and the whole as a ratio? $\frac{15}{100}$; $\frac{n}{300}$

OL Would it be advantageous to simplify $\frac{15}{100}$ to $\frac{3}{20}$ before writing the equation? Explain. no; Sample answer: Because 100 is closer to the whole, 300, I will multiply by a smaller number, 3, to find the unknown part, than if I simplify the fraction first. If I use $\frac{3}{20}$, I will have to multiply by a larger number, 15.

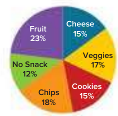
BL How many of the students do not bring cheese or veggies as a snack? 204 students

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 1 Find the Percent of a Number

The graph shows the types of snacks that students at Y ork Middle School bring with them to school. Suppose there are 300 students at the school.



How many of them bring cheese for a snack?

First, identify the part, the whole, and the percent. The part is unknown. The whole is 300. The percent is 15%.

Method 1 Use the rate per 100 and mental math.

The percent is 15%. This means, that for every 100 students, 15 of them bring cheese for a snack. This is the rate per 100.

$15 + 15 + 15$	There are three 100s in 300. For each 100, 15 students bring cheese as a snack.
$= 3 \times 15$	Write repeated addition as multiplication.
$= 45$	Multiply. 45 students bring cheese as a snack.

Method 2 Use equivalent ratios.

Write and solve an equation stating that the ratios are equivalent. Let *n* represent the number of students who bring cheese as a snack.

cheese / total students → $\frac{n}{300} = \frac{15}{100}$ } percent

$\frac{n}{300} = \frac{15}{100}$ (×3) — Because 100 × 3 = 300, multiply 15 by 3.

$\frac{45}{300} \frac{15}{100}$ — 15 × 3 = 45; So, $n = 45$.

So, using either method, 45 students bring cheese as a snack.

Check

Approximately 11% of the U.S. population is left-handed. If there are 700 students at a middle school, about how many of them are expected to be left-handed?

Show your work here: about 77 students

Go Online Y ou can complete an Extra Example online.

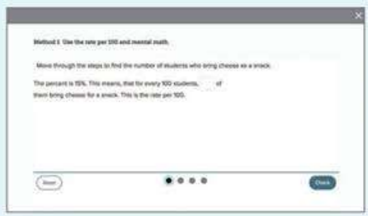
Think About It!
A classmate claims that because 15% is a little over 10% and 10% of 300 is 30, that 15% of 300 will be a little over 30. Do you think this reasoning is correct? Why or why not?

See students' responses.

Talk About It!
How can you use a bar diagram to find 15% of 300?

Sample answer: Draw a bar and divide it into 20 equal-size sections of 5% each. Then shade three sections to represent 15%. Each section also represents 15 because 300 ÷ 20 = 15. So, 15% of 300 is 15 + 15 + 15, or 45.

Interactive Presentation



Example 1, Find the Percent of a Number, Slide 3 of 6

TYPE
On Slide 2 of Example 1, students indicate the percent and the whole.

CLICK
On Slide 3 of Example 1, students move through the steps to use a rate per 100 to find the percent of a number.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
Is 30% of 240 less than, greater than or equal to 120? How do you know?

less than; Sample answer: 120 is half, or 50%, of 240. Since 30% < 50%, 30% of 240 will be less than 120.

Talk About It!
Now that you know 30% of 240, use mental math to find 60% of 240, 90% of 240, and 15% of 240.

Sample answer: Since 30% × 2 = 60%, 60% of 240 is 72 × 2 or 144. Since 30% × 3 = 90%, 90% of 240 is 72 × 3 or 216. Since 30% ÷ 2 = 15%, 15% of 240 is 72 ÷ 2 or 36.

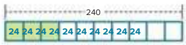
Example 2 Find the Percent of a Number

What is 30% of 240?

The part is unknown. The whole is 240. The percent is 30%.

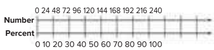
Method 1 Use a bar diagram.

Draw a bar diagram with 10 equal-size sections. The whole is 240, so each section represents 240 ÷ 10 or 24. Shade three sections to represent 30%. So, 30% of 240 is 24 + 24 + 24, or 72.



Method 2 Use a double number line.

Draw a double number line. The bottom number line represents the percent, so use increments of 10 to draw tick marks and label the percents. The top number line represents the number that corresponds with each percent, so label the tick mark that corresponds with 100% on the bottom number line with 240. Since there are 10 increments, the value of each tick mark on the top number line increases by 240 ÷ 10, or 24 units. So, 30% on the bottom number line corresponds with 72 on the top number line.



Method 3 Use equivalent ratios.

Write and solve an equation stating that the ratios are equivalent. Let *n* represent the unknown part.

part → whole → $\frac{n}{240} = \frac{30}{100}$ } percent

×2.4

$\frac{n}{240} = \frac{30}{100}$ Because 100 × 2.4 = 240, multiply 30 by 2.4.

×2.4

$\frac{72}{240} = \frac{30}{100}$ 30 × 2.4 = 72; So, $n = 72$.

So, using any method, 30% of 240 is **72**.

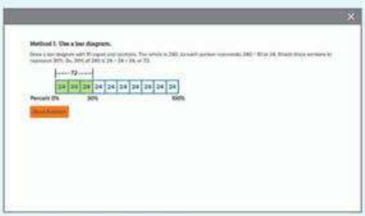
Check

What is 70% of 580? Use any strategy.

Show your work here **406**

Go Online Y ou can complete an Extra Example online.

Interactive Presentation



Example 2, Find the Percent of a Number, Slide 3 of 7

CLICK

On Slide 2, students identify the part, whole, and percent.

TYPE

On Slide 5, students indicate what number is 30% of 240.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find the Percent of a Number

Objective

Students will use bar diagrams, double number lines, and equivalent ratios to find the percent of a number.

Questions for Mathematical Discourse

SLIDE 2

AL What information is given? the percent and the whole

OL What value represents the whole? How do you know? 240; Sample answer: The question states *of 240*, which tells me this is the total amount, or the whole.

BL In the question *What is 12.5% of 36?*, what is the unknown? the part

SLIDE 3

AL Do you need to find the part, the whole, or the percent? the part

OL Can you use a different number of sections in the bar diagram? Explain. yes; Sample answer: 20 sections of 5% each

BL What is 70% of 240? How does the bar diagram help you? Sample answer: 7 sections of 24 equals 168.

SLIDE 4

AL What number on the top number line should correspond with 100 on the bottom number line? 240

OL How do you know what value to assign to each tick mark on the top number line? There are 10 intervals in the number line, so find 240 ÷ 10 to find the value for each tick mark.

BL How is using a bar diagram similar to using a double number line? Sample answer: In both, you have to divide the model into a certain number of sections, in this case, each section representing 10%, and then look at three sections to find 30% of the number.

SLIDE 5

AL What is the percent? What is the whole? 30%; 240

OL By what value do you need to multiply 100 to obtain 240? 2.4

BL Which method would you choose to use to find 35% of 240? Why? Sample answer: I prefer to use equivalent ratios, because it is less time consuming than drawing a model with 20 sections of 5% each.

Go Online

- Find additional teaching notes Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Find the Percent of a Number

Objective

Students will use ratio tables and equivalent ratios to find the percent of a number when the percent is greater than 100%.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of the part, in relation to the whole, 320, and not just perform the calculations. Students should be able to determine whether their answer is reasonable by reasoning that because 145% is greater than 100%, the part will be greater than 320.

Questions for Mathematical Discourse

SLIDE 3

AL When using a ratio table, how do you scale back to 1% from 100%? Sample answer: Divide both the percent and the part by 100.

OL Why is it beneficial to scale from 100% to 1% and then scale from 1% to 145%, as opposed to scaling directly from 100% to 145%? Sample answer: To scale directly from 100% to 145%, I need to multiply by 1.45. While this is valid, it might be more intuitive to divide by 100 first and then multiply by 145.

OL Explain why 145% of 320 is larger than 320. Sample answer: Because 145% is greater than 100%, 145% of 320 must be greater than 320.

BL How much greater is 245% of 320 than 145% of 320? Explain. 320; Sample answer: Each 100% is equal to 320, the whole. Therefore, since 245% is 100% more than 145%, 245% of 320 is 320 more than 145% of 320.

SLIDE 4

AL Explain how to estimate 145% of 320. Sample answer: 145% is close to 150% and 150% = 100% + 50%. 100% of 320 is 320. 50% of 320 is 160. So, 145% of 320 is about 320 + 160, or 480.

OL Without calculating, explain whether 145% of 320 is less than or greater than 320. Sample answer: Since 145% is greater than 100%, 145% of 320 will be greater than 320.

BL Now that you know 145% of 320 is 464, use mental math to find 72.5% of 320 and 290% of 320. Explain how you found these values. 232; 928; Sample answer: 72.5% is half of 145%, so half of 464 is 232. 290% is twice as great as 145%, so 2(464) = 928.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Find the Percent of a Number

What is 145% of 320?

The part is unknown. The whole is 320. The percent is 145%.

Method 1 Use a ratio table.

Y ou know that 100% of 320 is 320. Y ou need to find 145% of 320. Use a ratio table to scale back from 100% to 1%. Then scale forward from 1% to 145%.

Percent	1	100	145
Part	3.2	320	464

÷100, ×145

Because 100 ÷ 100 = 1, divide 320 by 100 to obtain 3.2. So, 1% of 320 = 3.2. Because 1 × 145 = 145, multiply 3.2 by 145 to obtain 464. So, 145% of 320 is 464.

Method 2 Use equivalent ratios.

Write and solve an equation stating the ratios are equivalent. Let n represent the unknown part.

part / whole: $\frac{n}{320} = \frac{145}{100}$ percent

×3.2

$\frac{n}{320} = \frac{145}{100}$ Because 100 × 3.2 = 320, multiply 145 by 3.2.

×3.2

$\frac{464}{320} = \frac{145}{100}$ 145 × 3.2 = 464; So, n = 464.

So, using either method, 145% of 320 is 464.

Check

What is 275% of 4? Use any strategy.

Show your work here 11

Go Online Y ou can complete an Extra Example online.

Think About It! Is 145% of 320 less than, greater than, or equal to 320? How do you know?

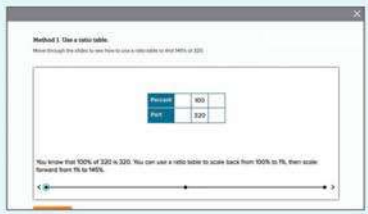
greater than; Sample answer: The percent is greater than 100% so the part will be greater than the whole.

Talk About It! Compare the part, 464, to the whole, 320. Does it make sense that 464 is greater than 320? Why or why not?

yes; Sample answer: The percent is greater than 100%, so the part should be greater than the whole.

Lesson 2-4 • Find the Percent of a Number 107

Interactive Presentation



Example 3, Find the Percent of a Number, Slide 3 of 6

DRAG & DROP

On Slide 2, students drag to identify the part, whole, and percent.

CLICK

On Slide 3, students move through the slides to use a ratio table to find 145% of 320.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
Why might it not be advantageous to use a bar diagram to find 0.25% of 58?

See students' responses.

Example 4 Find the Percent of a Number

What is 0.25% of 58?

The part is unknown. The whole is 58. The percent is 0.25%.

Method 1 Use a ratio table.

Y ou know that 100% of 58 is 58. Y ou need to find 0.25% of 58. Use a ratio table to scale back from 100% to 1%. Then scale back again from 1% to 0.25%.

Percent	0.25	1	100
Part	0.145	0.58	58

÷4 ÷100

Because 100 ÷ 100 = 1, divide 58 by 100 to obtain 0.58. So, 1% of 58 = 0.58. Because 1 ÷ 4 = 0.25, divide 0.58 by 4 to obtain 0.145. So, 0.25% of 58 is 0.145.

Method 2 Use equivalent ratios.
Write and solve an equation stating the ratios are equivalent. Let n represent the unknown part.

part / whole → $\frac{n}{58} = \frac{0.25}{100}$ } percent

×0.58

$\frac{n}{58} = \frac{0.25}{100}$ Because 100 × 0.58 = 58, multiply 0.25 by 0.58.

$\frac{0.145}{58} = \frac{0.25}{100}$ 0.25 × 0.58 = 0.145; So, $n = 0.145$.

So, using either method, 0.25% of 58 is 0. 145.

Talk About It!
Compare the part, 0.145, to the whole, 58. Does it make sense that 0.145 is significantly less than 58? Why or why not?

yes; Sample answer: The percent is less than 1%, so the part is significantly less than the whole.

Check

What is 0.55% of 220? Use any strategy.

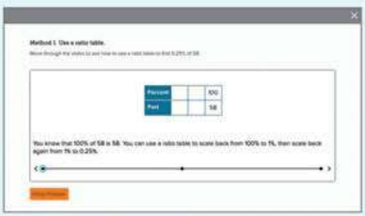
Show your work here

1.21

Go Online You can complete an Extra Example online.

108 Module 2 • Fractions, Decimals, and Percents

Interactive Presentation



Example 4, Find the Percent of a Number, Slide 3 of 6

CLICK

On Slide 2, students identify the part, whole, and percent.

CLICK

On Slide 3, students move through the slides to use a ratio table to find 0.25% of 58.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find the Percent of a Number

Objective

Students will use ratio tables and equivalent ratios to find the percent of a number when the percent is less than 1%.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to use reasoning to understand why it is beneficial to scale from 100% to 1% using the ratio table, and then scaling from 1% to 0.25%. Ask students if they can scale directly from 100% to 0.25% by dividing 100% by 400. While this is a valid method, it is not as intuitive as breaking up the scaling into two parts.

As students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of the percent, in relation to the whole, 58, and not just perform the calculations. 0.25% is less than 1%, so the part will be significantly less than 58. Students should use this reasoning to make sense of their solution.

Questions for Mathematical Discourse

SLIDE 3

AL What do you notice about the percent? Sample answer: It is less than 1%.

OL Without calculating, explain whether 0.25% of 58 is less than or greater than 58. Sample answer: Since 0.25% is less than 100%, 0.25% of 58 will be less than 58.

OL Explain how to estimate 0.25% of 58. Sample answer: 0.25% is one fourth of 1%. 1% of 58 is 0.58. So, 0.25% will be one fourth of 0.58. Since 0.58 is close to 0.6 and one fourth of 0.6 is 0.15, 0.25% of 58 is close to 0.15.

OL Why do we scale from 100% to 1% first? Sample answer: It is easier to scale back to 1% and then scale from 1% to 0.25% instead of scaling directly from 100% to 0.25%.

BL Now that you know that 0.25% of 58 is 0.145, use reasoning and mental math to find 0.75% of 58 and 0.25% of 29. Explain how you found these values. 0.435; 0.0725; Sample answer: Because 0.75% is 3 times as great as 0.25%, multiply 0.145 by 3 to obtain 0.435. Because 29 is half of 58, find half of 0.145 to obtain 0.0725.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Book Fair

Objective

Students will come up with their own strategy to solve an application problem involving attendance.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What information from the table is extra and not needed to solve the problem?
- Do you need to find the part, the whole, or the percent? Which method would you prefer to use to find the unknown?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Book Fair

Students were asked which night they planned on attending the book fair. The results of the survey are shown in the table. Twenty percent of the students who planned to attend on Wednesday attended on Thursday instead. Twenty-five percent of the students who planned to attend on Thursday attended on Wednesday instead. Which day, Wednesday or Thursday, had a greater actual attendance? By how many students?

Day	Number of Students
Monday	55
Tuesday	80
Wednesday	70
Thursday	112
Friday	65

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Thursday; 14 students; See students' work.

4 How can you show your solution is reasonable?

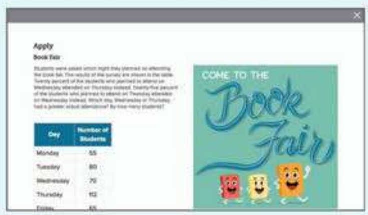
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
Would the solution be the same if 25% of the students who planned to attend Wednesday attended on Thursday, instead of 20%? Explain.

no; Sample answer: Since the percent is greater, the number of students switching to Thursday is greater.

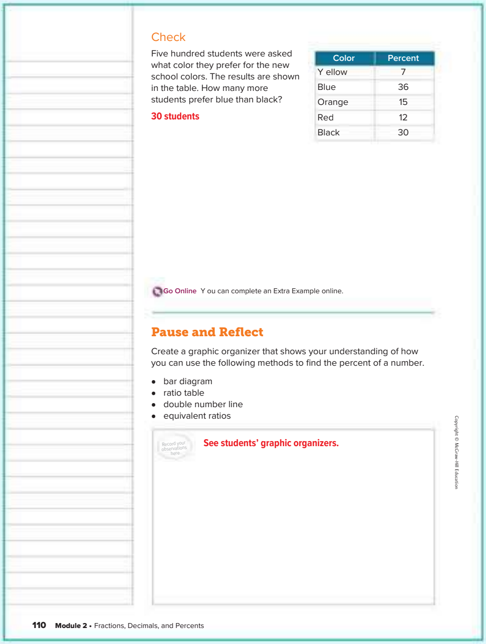
Interactive Presentation



Apply, Book Fair

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Five hundred students were asked what color they prefer for the new school colors. The results are shown in the table. How many more students prefer blue than black?

30 students

Color	Percent
Yellow	7
Blue	36
Orange	15
Red	12
Black	30

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that shows your understanding of how you can use the following methods to find the percent of a number.

- bar diagram
- ratio table
- double number line
- equivalent ratios

See students' graphic organizers.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you use fractions, decimals, and percents to solve everyday problems? In this lesson, students learned how to find the percent of a number using models. Encourage them to brainstorm with a partner some real-world examples of when they might need to find the percent of a number. For example, if a shirt that costs \$20 is discounted 10%, it would mean \$2 off the cost of the shirt.

Exit Ticket

Refer to the Exit Ticket slide. How much will you save if you buy the shirt on sale? Write a mathematical argument that can be used to defend your solution. \$17.46; Sample answer: You can use the equivalent ratios $\frac{30}{100} = \frac{7.485}{\$24.95}$ to find that the amount of discount is \$7.49. So, the sale price of the shirt is \$24.95 − \$7.49 or \$17.46.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1, 10, 12–15
- Extension: Financial Literacy: Compare Multiple Discounts
- ALEKS Percent of a Number

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–8, 10, 12–15
- Extension: Financial Literacy: Compare Multiple Discounts
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Understanding Percents

Ron Zmiri/123RF

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	T opic	Exercises
2	find the percent of a number	1, 2
1	use any method to find the percent of a number	3–8
2	extend concepts learned in class to apply them in new contexts	9
3	solve application problems involving percent of a number	10, 11
3	higher-order and critical thinking skills	12–15

Common Misconception

Some students may have trouble finding the percent of a number when the percent is less than 1. In Exercise 8, students may want to find 0.4 (40%) instead of 0.4%. Remind students that when using equivalent ratios they must maintain the decimal in the percent ratio $\left(\frac{n}{168} = \frac{0.4}{100}\right)$. Remind them that 1% of 168 is 1.68 and that 0.4% will be less than that.

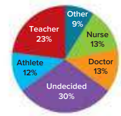
Name Period Date

Practice

Go Online Y ou can complete your homework online.

Use any strategy to solve each problem.

1. The graph shows the career interests of the students at Linda's school. Suppose there are 400 students at the school. How many of them want to be an athlete? (Example 1)



48 students

2. The graph shows the favorite activities of campers at a summer camp. Suppose there are 300 campers at the camp. How many campers favor fishing? (Example 1)

42 campers

Use any method to find the percent of each number. (Examples 2–4)

3. 15% of 240 = 36
4. 65% of 180 = 117
5. 250% of 82 = 205
6. 150% of 44 = 66
7. 0.15% of 350 = 0.525
8. 0.4% of 168 = 0.672

T est Practice

9. **Open Response** Kenzie is putting the family vacation videos onto a flash drive. The flash drive can hold 200 minutes of video. Kenzie has used 45% of the memory space already. How many minutes of the flash drive has she already used?

90 min

Lesson 2-4 • Find the Percent of a Number 111

Apply *indicates multi-step problem

*10. Students were asked which night they planned on going to the school festival. The results of the survey are shown in the table. If 18% of the students did not go on Friday, and 15% of the students did not go on Saturday, how many more students went on Friday than on Saturday?

Night	Number of Students
Friday	550
Saturday	480

43 students

*11. Students were surveyed about which school athletic event they were planning to attend this week. Of the students who said they were going to the football game, 25% did not attend. Of the students who stated they were going to the volleyball game, 20% did not attend. How many more students went to the football game than the volleyball game?

Event	Number of Students
Football Game	120
Gymnastics Meet	95
Volleyball Game	80

26 students

Higher-Order Thinking Problems

12. MP **Persevere with Problems** Olive is going to buy a scooter that costs $95. The sales tax rate is 8.5%. What is the total cost of the scooter including tax to the nearest cent?

$103.08

13. MP **Justify Conclusions** Is 18% of 30 the same as 30% of 18? Justify your conclusion.

yes; Sample answer: 18% of 30 is 5.4 and 30% of 18 is 5.4; 5.4 = 5.4.

14. MP **Identify Structure** How can you find 40% of 150 using mental math? Explain.

Sample answer: 40% can be represented as 10% + 10% + 10% + 10%. 10% of 150 is 15. 15 + 15 + 15 + 15 = 60; So, 40% of 150 is 60.

15. **Be Precise** Explain how the part of a whole can be greater than the whole itself. Use an example.

Sample answer: If the percent is greater than 100%, the part will be greater than the whole. For example, 125% of 12 is 15. The part, 15, is greater than the whole, 12.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 12, students apply their knowledge of percent of a number to find to find the total cost of an item including sales tax.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students determine whether or not the two expressions have the same value and justify their conclusion.

7 Look for and Make Use of Structure In Exercise 14, students use mental math to find 40% of 150. Encourage them to use structure in representing 40% as 10% + 10% + 10% + 10%.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 10–11 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 13 might be, "What does it mean that 18% of the students did not go on Friday?"

Clearly explain your strategy.
Use with Exercise 12 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would find the total cost with sales tax, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Learn Estimate the Percent of a Number

Objective

Students will learn how to use benchmark percents and rounding to estimate the percent of a number.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the percent given, 27%, in relation to the benchmark percent, 25%. They should be able to reason that because 25% < 27%, 25% of 40 is less than 27% of 40.

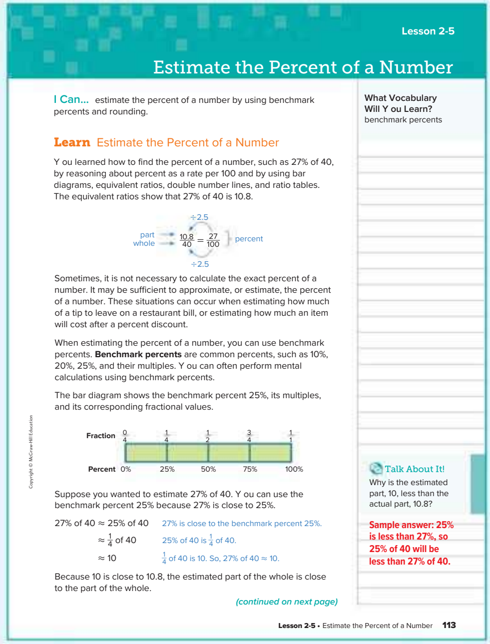
Talk About It!

SLIDE 3

Mathematical Discourse

Why is the estimated part, 10, less than the actual part, 10.8? Sample answer: 25% is less than 27%, so 25% of 40 will be less than 27% of 40.

(continued on next page)



Lesson 2-5

Estimate the Percent of a Number

I Can... estimate the percent of a number by using benchmark percents and rounding.

What Vocabulary Will Y ou Learn?
benchmark percents

Learn Estimate the Percent of a Number

Y ou learned how to find the percent of a number, such as 27% of 40, by reasoning about percent as a rate per 100 and by using bar diagrams, equivalent ratios, double number lines, and ratio tables. The equivalent ratios show that 27% of 40 is 10.8.

part, whole: $\frac{10.8}{40} = \frac{27}{100}$ percent ($\div 2.5$)

Sometimes, it is not necessary to calculate the exact percent of a number. It may be sufficient to approximate, or estimate, the percent of a number. These situations can occur when estimating how much of a tip to leave on a restaurant bill, or estimating how much an item will cost after a percent discount.

When estimating the percent of a number, you can use benchmark percents. **Benchmark percents** are common percents, such as 10%, 20%, 25%, and their multiples. Y ou can often perform mental calculations using benchmark percents.

The bar diagram shows the benchmark percent 25%, its multiples, and its corresponding fractional values.

Fraction	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{1}$
Percent	0%	25%	50%	75%	100%

Suppose you wanted to estimate 27% of 40. Y ou can use the benchmark percent 25% because 27% is close to 25%.

27% of 40 ≈ 25% of 40 — 27% is close to the benchmark percent 25%.
≈ $\frac{1}{4}$ of 40 — 25% of 40 is $\frac{1}{4}$ of 40.
≈ 10 — $\frac{1}{4}$ of 40 is 10. So, 27% of 40 ≈ 10.

Because 10 is close to 10.8, the estimated part of the whole is close to the part of the whole.

Talk About It!
Why is the estimated part, 10, less than the actual part, 10.8?

Sample answer: 25% is less than 27%, so 25% of 40 will be less than 27% of 40.

(continued on next page)

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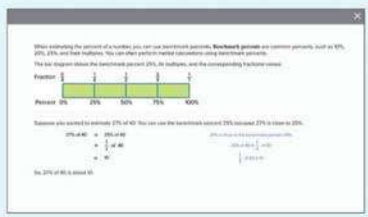
Lesson 2-5 • Estimate the Percent of a Number 113

Interactive Presentation



Learn, Estimate the Percent of a Number, Slide 2 of 6

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of why benchmark percents are useful, have them discuss with a partner why a percent such as 30% is a benchmark percent and 32% is not. Sample answer: The calculations with 30% are easier than the calculations for 32%. When written as a fraction, 30% is equal to $\frac{3}{10}$, so 30% of a number can be found by finding one tenth of the number and multiplying the result by 3. Finding one tenth of the number is the same as dividing by 10. When written as a fraction, 32% is equal to $\frac{32}{100}$ or $\frac{8}{25}$. Finding one twenty-fifth of a number is not as simple, and the result would have to be multiplied by 8 to find the part.

Lesson 2-5

Estimate the Percent of a Number

6.RP.A.3, 6.RP.A.3.C

LESSON GOAL

Students will estimate the percent of a number.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Estimate the Percent of a Number
Example 1: Estimate the Percent of a Number
Example 2: Estimate the Percent of a Number
Apply: Financial Literacy

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 13 of the *Language Development Handbook* to help your students build mathematical language related to estimating the percent of a number.

ELL You can use the tips and suggestions on page T13 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP.A** by estimating the percent of a number.
Standards for Mathematical Content: **6.RP.A.3, 6.RP.A.3.C**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5**

Coherence

Vertical Alignment

Previous
Students found the percent of a number.
6.RP.A.3, 6.RP.A.3.C

Now
Students estimate the percent of a number.
6.RP.A.3, 6.RP.A.3.C

Next
Students will find the whole given the percent and the part, using double number lines and equivalent ratios.
6.RP.A.3, 6.RP.A.3.C

Rigor

The Three Pillars of Rigor

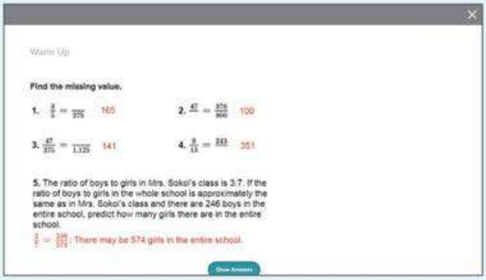
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students *apply* their *fluency* with percents to solve real-world problems involving estimating with percents.

Mathematical Background

To estimate a percent problem, we often use convenient *benchmark percents*. Benchmark percents are common percents to which we compare other percents: multiples of 10%, multiples of 5%, and commonly 25% and 75% because of their relationship to well-known fractions $\left(\frac{1}{4} \text{ and } \frac{3}{4}\right)$. To estimate a percent, round the whole to a convenient number, e.g. to the nearest 100, and round the percent to a convenient benchmark fraction, then multiply the rounded whole by the benchmark fraction. The use of bar diagrams and equivalent ratios is beneficial when estimating the percent of a number.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding equivalent fractions (Exercises 1–4)
- making predictions using ratios (Exercise 5)

Answers

1. 165
2. 100
3. 141
4. 351
5. $\frac{3}{7} = \frac{246}{574}$; There may be 574 girls in the entire school.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about survey results of theater moviegoers.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

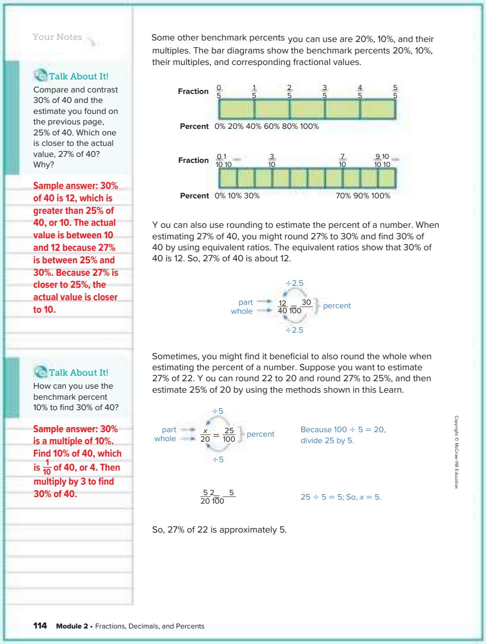
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Where have you seen the term *benchmark* before? Sample answer: A benchmark is a standard against which similar things can be measured. For example, you may have taken benchmark tests in school to see how well you have learned a certain set of skills.



Your Notes

Talk About It!
Compare and contrast 30% of 40 and the estimate you found on the previous page, 25% of 40. Which one is closer to the actual value, 27% of 40? Why?

Sample answer: 30% of 40 is 12, which is greater than 25% of 40, or 10. The actual value is between 10 and 12 because 27% is between 25% and 30%. Because 27% is closer to 25%, the actual value is closer to 10.

Talk About It!
How can you use the benchmark percent 10% to find 30% of 40?

Sample answer: 30% is a multiple of 10%. Find 10% of 40, which is $\frac{1}{10}$ of 40, or 4. Then multiply by 3 to find 30% of 40.

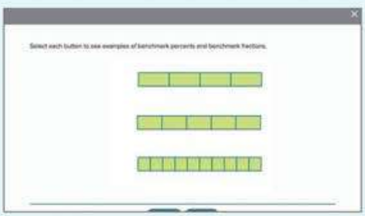
Some other benchmark percents you can use are 20%, 10%, and their multiples. The bar diagrams show the benchmark percents 20%, 10%, their multiples, and corresponding fractional values.

You can also use rounding to estimate the percent of a number. When estimating 27% of 40, you might round 27% to 30% and find 30% of 40 by using equivalent ratios. The equivalent ratios show that 30% of 40 is 12. So, 27% of 40 is about 12.

Sometimes, you might find it beneficial to also round the whole when estimating the percent of a number. Suppose you want to estimate 27% of 22. You can round 22 to 20 and round 27% to 25%, and then estimate 25% of 20 by using the methods shown in this Learn.

So, 27% of 22 is approximately 5.

Interactive Presentation



Learn, Estimate the Percent of a Number, Slide 4 of 6

Learn Estimate the Percent of a Number *(continued)*

Teaching Notes

SLIDE 4

Have students use the interactive tool to view examples of common benchmark percents and their related benchmark fractions. It will benefit students later on when solving real-world and mathematical problems to commit these common benchmark percents and fractions to memory.

Talk About It!

SLIDE 5

Mathematical Discourse

Compare and contrast 30% of 40 and the estimate you found on the previous page, 25% of 40. Which one is closer to the actual value, 27% of 40? Why? Sample answer: 30% of 40 is 12, which is greater than 25% of 40, or 10. The actual value is between 10 and 12 because 27% is between 25% and 30%. Because 27% is closer to 25%, the actual value is closer to 10.

Talk About It!

SLIDE 6

Mathematical Discourse

How can you use the benchmark percent 10% to find 30% of 40? Sample answer: 30% is a multiple of 10%. Find 10% of 40, which is $\frac{1}{10}$ of 40, or 4. Then multiply by 3 to find 30% of 40.

Example 1 Estimate the Percent of a Number

Objective

Students will use bar diagrams and equivalent ratios to solve a real-world problem that involves estimating the percent of a number.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should be able to use reasoning to estimate the percent and the whole in order to estimate the solution.

As students discuss the *Talk About It!* question on Slide 4, encourage them to think about how rounding both quantities affects the estimate, and why someone might choose to round differently in a real-world context.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we round $47.45 to $50.00? Sample answer: $47.45 is close to $50.00, and it is fairly easy to calculate with $50.00.

OL Why is 20% a good percent to use as an estimate for 18%? Sample answer: It is close to 18%, and it is a benchmark percent. It is not difficult to calculate 20% of a number.

OL Why is the bar diagram divided into 5 sections? Sample answer: The benchmark percent is 20%. Since 100 ÷ 20 = 5, the bar diagram has 5 sections.

BL Can the bar diagram be divided into a different number of sections? Explain your reasoning. yes; Sample answer: The diagram can be divided into 10 sections, each representing 10%.

SLIDE 3

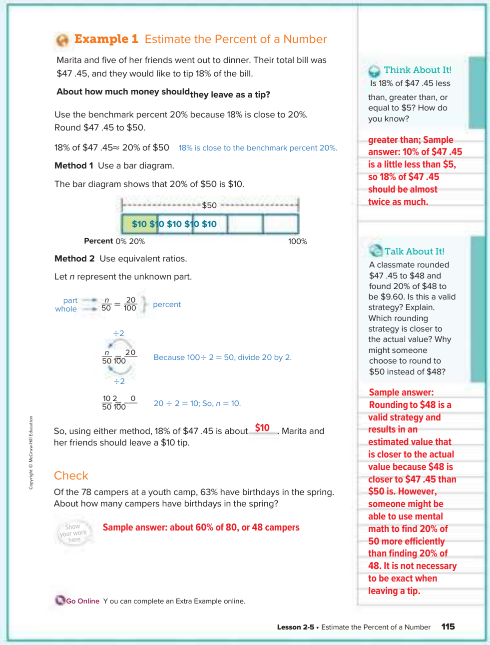
AL When writing the equivalent ratios, do you need to find the percent, part, or whole? part

OL Is the estimate less than or greater than the actual amount? Explain without calculating the actual amount. greater than; Sample answer: 20% > 18%, and $50.00 > $47.45, so the estimate is greater than the actual amount.

BL About how much would Marita tip if the total bill was $72.43? Explain. Sample answer: 20% is close to 18% and $70 is close to $72.43. 20% of $70 is $14, so Marita would tip about $14.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Estimate the Percent of a Number

Marita and five of her friends went out to dinner. Their total bill was $47 .45, and they would like to tip 18% of the bill.

About how much money should they leave as a tip?

Use the benchmark percent 20% because 18% is close to 20%. Round $47 .45 to $50.

18% of $47 .45 ≈ 20% of $50 — 18% is close to the benchmark percent 20%.

Method 1 Use a bar diagram.

The bar diagram shows that 20% of $50 is $10.

Method 2 Use equivalent ratios.

Let n represent the unknown part.

part / whole → $\frac{n}{50} = \frac{20}{100}$ } percent

$\frac{n}{50} = \frac{20}{100}$ (÷2) — Because 100 ÷ 2 = 50, divide 20 by 2.

$\frac{10}{50} = \frac{20}{100}$ — 20 ÷ 2 = 10; So, $n = 10$.

So, using either method, 18% of $47 .45 is about $10. Marita and her friends should leave a $10 tip.

Think About It! Is 18% of $47 .45 less than, greater than, or equal to $5? How do you know?

greater than; Sample answer: 10% of $47 .45 is a little less than $5, so 18% of $47 .45 should be almost twice as much.

Talk About It! A classmate rounded $47 .45 to $48 and found 20% of $48 to be $9.60. Is this a valid strategy? Explain. Which rounding strategy is closer to the actual value? Why might someone choose to round to $50 instead of $48?

Sample answer: Rounding to $48 is a valid strategy and results in an estimated value that is closer to the actual value because $48 is closer to $47 .45 than $50 is. However, someone might be able to use mental math to find 20% of 50 more efficiently than finding 20% of 48. It is not necessary to be exact when leaving a tip.

Check

Of the 78 campers at a youth camp, 63% have birthdays in the spring. About how many campers have birthdays in the spring?

Show your work here — Sample answer: about 60% of 80, or 48 campers

Go Online You can complete an Extra Example online.

Lesson 2-5 • Estimate the Percent of a Number 115

Interactive Presentation

Example 1, Estimate the Percent of a Number, Slide 2 of 5

TYPE

On Slide 3, students estimate 18% of $47.45.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
Do pet birds spend less than, greater than, or equal to 12 hours a day sleeping? Explain.

less than; Sample answer: 12 out of 24 hours is 50%. Since 40% < 50%, pet birds spend less than 12 hours a day sleeping.

Talk About It!
Why might it be more advantageous to use the benchmark percent 10% than 20%?

Sample answer: It is often easier to use mental math to find 10% of a number first.

Example 2 Estimate the Percent of a Number

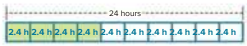
Most pet birds spend about 41% of the day sleeping.

About how many hours a day do they spend sleeping?

Y ou need to estimate 41% of 24, because there are 24 hours in a day. Because 41% is close to 40%, 41% of 24 ≈ 40% of 24.

Method 1 Use the benchmark percent 10%.

Draw a bar diagram with 10 equal-size sections. Each section represents 10%. The value of each section is 24 ÷ 10 or 2.4. So, 10% of 24 hours is 2.4 hours.



Multiply by 4 to find 40% of 24 hours.

40% of 24 = 4(10% of 24)	40% = 4(10%)
= 4(2.4)	10% of 24 = 2.4
= 9.6	Multiply.

Method 2 Use the benchmark percent 20%.

Draw a bar diagram with 5 equal-size sections. Each section represents 20%. The value of each section is 24 ÷ 5 or 4.8. So, 20% of 24 hours is 4.8 hours.

Multiply by 2 to find 40% of 24 hours.

40% of 24 = 2(20% of 24)	40% = 2(20%)
= 2(4.8)	20% of 24 = 4.8
= 9.6	Multiply.

So, using either method, 41% of 24 hours is about **9.6** hours. Pet birds spend about 9.6 hours a day sleeping.

Check

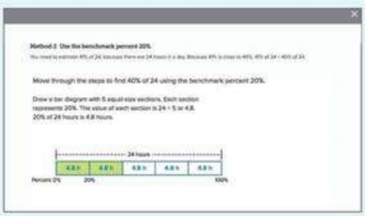
Estimate 76% of 122. Use any strategy.

Show your work here **Sample answer: 76% of 122 ≈ 75% of 120, or 90**

Go Online Y ou can complete an Extra Example online.

116 Module 2 • Fractions, Decimals, and Percents

Interactive Presentation



Example 2, Estimate the Percent of a Number, Slide 3 of 5

CLICK

On Slide 2, students move through the steps to estimate 41% of 24.

TYPE

On Slide 3, students estimate 41% of 24.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Estimate the Percent of a Number

Objective

Students will use a bar diagram to solve a real-world problem involving estimating the percent of a number.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Have students discuss the advantages of using the two different bar diagrams. Ask them to remark on when having more divisions of a bar diagram might not be advantageous.

Questions for Mathematical Discourse

SLIDE 2

AL The percent 41% is close to what benchmark percent? 40%

OL Why is the bar diagram divided into 10 sections? How many hours does each section of the bar diagram represent? Sample answer: Each section represents 10% and 10% · 10 = 100%; 2.4 hours

OL How do you know the answer is reasonable? Sample answer: 41% < 50% and 50% of 24 is 12. Because 9.6 < 12, the answer is reasonable.

BL How many hours per day do pet birds spend not sleeping? Explain. 14.4 hours; Sample answer: If a pet bird spends 9.6 hours per day sleeping, then it spends 24 — 9.6 or 14.4 hours per day not sleeping.

SLIDE 3

AL Why is the benchmark percent 20% 41% is close to 40%, and 40% is a multiple of the benchmark percent 20%.

AL How many 20s are in 40? 2

OL Using either method, will the estimate be greater than or less than the actual answer? Explain. less than; Sample answer: In both methods, 41% was rounded down to 40%. Since 40% < 41%, the estimate will be less than the actual solution.

BL Is there another benchmark percent that can be used instead of 10% or 20%? Is it an advantageous choice? Why or why not? yes; no; Sample answer: I can use 5%. It is not advantageous because I would have to divide the bar diagram into 20 sections.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Financial Literacy

Objective

Students will come up with their own strategy to solve an application problem involving sales tax.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Is the discount applied before or after the tax is applied?
- What benchmark percent can you use to estimate?
- How would you round the price of the service?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Financial Literacy

Sabrina takes her car to the car wash and chooses the Gold Star service that includes a wash, wax, and interior cleaning. This service normally costs $53.99, but is on special for $5.00 off. She must also pay a 6% sales tax, which is applied to the discounted price, and then added to find the total price. Estimate the total amount Sabrina paid at the car wash.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Sample answer: about $52.50; See students' work. (The sample answer is obtained by rounding $53.99 to $55, rounding 6% to 5%, and then finding 5% of $55 — $5, or $50.)

Talk About It!
Find the actual total amount. How close was the estimate? Why might it be helpful to estimate?

Sample answer: The actual price is $51.93. The estimate was close, with a difference of $0.57 . It is helpful to estimate, because it is faster than finding the exact amount.

4 How can you show that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Lesson 2-5 • Estimate the Percent of a Number 117

Interactive Presentation

Apply, Financial Literacy

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

There were 48,500 people at an amusement park on Monday. Forty-two percent of the people wanted to ride the new roller coaster. Twenty-three percent of those people decided not to ride the coaster because the line was too long. About how many people waited in line for the new roller coaster that day?

Show your work here

about 15,000 people

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe a situation in which you have estimated the percent of a number in your everyday life, or describe a situation in which you might do so in the future.

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you use fractions, decimals, and percents to solve everyday problems? In this lesson, students learned how to estimate the percent of a number using benchmark percents with bar diagrams and equivalent ratios. Encourage them to brainstorm with a partner some real-world examples of when they might need to estimate the percent of a number. Have them explain how they know when they need to find the actual percent of a number versus when they can use an estimate.

Exit Ticket

Refer to the Exit Ticket slide. Estimate the number of people who plan to see the movie. Write a mathematical argument that can be used to defend your solution. Sample answer: about 75 people; 27% is close to 25% and 298 is close to 300. $\frac{25}{100} \times 300 = 75$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 7–13 odd, 15–18
- ALEKS® Percent of a Number

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–11, 13, 15, 17
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–2
- ALEKS® Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ALEKS® Understanding Percents

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	estimate the percent of a number	1–6
2	estimate the percent of a number	7–11
2	extend concepts learned in class to apply them in new contexts	12
3	solve application problems involving fractions, decimals, and percents	13, 14
3	higher-order and critical thinking skills	15–18

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Sample answers given for 1–12.

For Exercises 1–11, estimate each percent. Show your estimation. (Examples 1 and 2)

1. 51% of 62 ≈ 30
   50% of 60 = 30
2. 26% of 78 ≈ 20
   25% of 80 = 20
3. 39% of 198 ≈ 80
   40% of 200 = 80
4. 78% of 148 ≈ 120
   80% of 150 = 120
5. 19% of 103 ≈ 20
   20% of 100 = 20
6. 98% of 59 ≈ 60
   100% of 60 = 60
7. Emilia and her three sisters went out to dinner. The total cost of their dinner was \$38.75. They want to leave a tip that is 23% of the total bill. About how much of a tip should they leave?
   about \$10; 25% of 40 = 10
8. Karl earned \$188 last month doing chores after school. If 68% of the money he earned was from doing yard work, about how much did Karl earn doing yard work?
   about \$140; 70% of 200 = 140
9. The concession stand at a football game served 288 customers. Of those customers, about 77% bought a hot dog. About how many customers bought a hot dog?
   about 225 customers; 75% of 300 = 225
10. In a recent season, the Chicago Cubs won 64% of the 161 regular season games they played. About how many games did they win?
    about 104 games; 65% of 160 = 104
11. The table shows how the 515 students at West Middle School get to school. About how many of the students walk to school?

Method	Percent of Students
Bus	53%
Car	21%
Walk	26%

about 125 students; 25% of 500 = 125

Test Practice

12. **Open Response** Carolyn's homeroom sold 207 magazine subscriptions. Of the magazine subscriptions sold, 28% were for fashion magazines. About how many fashion magazine subscriptions were sold?
    about 60 subscriptions

Apply *indicates multi-step problem

*13. Paul takes his dog to the groomer and selects the deluxe grooming package. He has a coupon for $10 off any grooming service. He must pay an 8% sales tax, which is applied to the discounted price, and then added to find the total price. Estimate the total amount Paul paid the dog groomer.

Grooming Package Cost ($)	
Regular	48.99
Deluxe	58.75

Sample answer: about $55

*14. A store purchases a television for $192 and adds $230 to set the sticker price. The store is having a sale where everything is 20% off the sticker price. Estimate the final price of the television.

Sample answer: about $344

Higher-Order Thinking Problems

15. There were 59,500 people who attended a football game. Twenty-four percent of the people received a voucher for a free water bottle. Six percent of those people never claimed their water bottle. About how many people claimed their water bottle?

about 14,250 people

16. **Reason Inductively** Zeb wants to buy a fishing pole regularly priced at $64. It is on sale for 60% off. Zeb estimates that he will save 60% of $60, or $36. Will the actual amount saved be more or less than $36? Explain.

more; Sample answer: Zeb rounded $64 down to $60, so the actual amount he will save will be a little more than $36.

17. Explain how you can estimate 39% of $197 .

Sample answer: First, round 39% to 40% and $197 to $200. Next, find 10% of $200, which is $20. Last, multiply $20 by 4 to find 40% of $200, or $80.

18. **Justify Conclusions** A store is having a 40% off sale. If you have $38, will you have enough money to buy an item that regularly sells for $65.99? Write an argument to justify your conclusion.

no; Sample answer: The sale price is about 60% of $65, or $39. Because $39 is more than $38, you do not have enough money.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 16, students analyze an estimate of a percent.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students determine whether or not they would have enough money to buy an item that is on sale and justify their conclusion.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.

Use with Exercises 13–14 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 13 might be, "The regular package costs $38.99 with the coupon." An example of a false statement might be, "The sales tax is added before the coupon is applied." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Solve the problem another way.

Use with Exercise 15 Have students work in groups of 3–4. After completing Exercise 15, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Learn Find the Whole

Objective

Students will understand how a bar diagram, a ratio table, a double number line, or equivalent ratios can be used to find the whole, given the part and the percent.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of how they can use the bar diagram to find the number of sixth graders who do not play a sport.

As students discuss the *Talk About It!* question on Slide 5, encourage them to discuss the disadvantages of using this method to solve this problem. Students should be able to reason that because there is no whole number by which they can multiply 60 to obtain 114, using another method, such as a ratio table, might be easier and more efficient.

Teaching Notes

SLIDE 1

You may wish to present the mathematical problem of finding the whole given that the part is 114 and the percent is 60%. Ask students to work with a partner to come up with possible strategies for finding the whole. Have students share their strategies with the class. Then have them complete the Learn and view the steps for using a bar diagram, a ratio table, or a double number line to find the whole, and compare that strategy with the one they used. You may wish to ask students how the methods from the Learn show other part, whole, and percent relationships other than the one asked for in the problem. For example, the double number line shows that 40% of 190 is 76.

Talk About It!

SLIDE 2

Mathematical Discourse

How can you use the bar diagram to find the number of sixth grade students who do *not* play a sport? Sample answer: There are 4 unshaded sections. These sections represent the students who do not play a sport. Each section represents 19 students and $19 \times 4 = 76$. So, 76 sixth grade students do not play a sport. You can also subtract 114 from the whole; $190 - 114 = 76$.

(continued on next page)

Lesson 2-6

Find the Whole

I Can... find the whole, given the part and the percent by using bar diagrams, ratio tables, double number lines, and equivalent ratios.

Learn Find the Whole

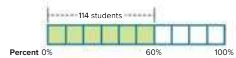
Sixty percent of the sixth grade students at Jackson Middle School play a sport. If 114 sixth grade students play a sport, how many sixth grade students are there in the school?

Y ou are given the part, 114 students, and the percent, 60%. Y ou need to find the whole. In other words, 60% of what number is 114?

Y ou can use bar diagrams, ratio tables, double number lines, and equivalent ratios to find the whole.

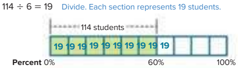
Method 1 Use a bar diagram.

Sixty is a multiple of 10 and 10 is a factor of 100. Draw a bar diagram with 10 equal-size sections of 10% each, because $10 \times 10 = 100$. Shade 6 sections to represent 60%. Label the shaded sections as 114 students, because 60% of the whole is 114.



Each section represents the same number of students. There are 6 shaded sections. Divide 114 by 6 to find the number of students represented by each section.

$114 \div 6 = 19$ Divide. Each section represents 19 students.



Because each section represents 19 students and there are 10 total sections, multiply 19 by 10 to find the total number of sixth grade students.

$19 \times 10 = 190$ Multiply. The whole is 190 students.

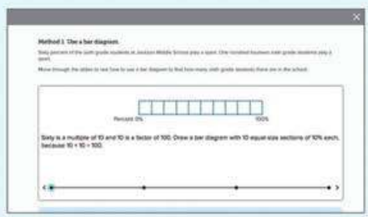
So, 60% of 190 is 114. There are 190 sixth grade students at the school.

Talk About It! How can you use the bar diagram to find the number of sixth grade students who do *not* play a sport?

Sample answer: There are 4 unshaded sections. These sections represent the students who do not play a sport. Each section represents 19 students and $19 \times 4 = 76$. So, 76 sixth grade students do not play a sport. Y ou can also subtract 114 from the whole; $190 - 114 = 76$.

(continued on next page)

Interactive Presentation



Learn, Find the Whole, Slide 2 of 5

CLICK

On Slide 2, students move through the slides to use a bar diagram to find the whole.

Lesson 2-6

Find the Whole

6.RP.A.3, 6.RP.A.3.C

LESSON GOAL

Students will find the whole given the percent and the part.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Find the Whole
Example 1: Find the Whole
Example 2: Find the Whole
Apply: Sales

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

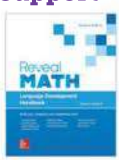
View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Find the Percent Given a Part and the Whole		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 14 of the *Language Development Handbook* to help your students build mathematical language related to finding the whole, given the part and the percent.

ELL You can use the tips and suggestions on page T14 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **6.RP .A** by finding the whole given the percent and the part.
Standards for Mathematical Content: **6.RP .A.3, 6.RP.A.3.C**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students estimated the percent of a number.
6.RP.A.3, 6.RP.A.3.C

Now
Students find the whole given the percent and the part.
6.RP.A.3, 6.RP.A.3.C

Next
Students will use proportional relationships to solve multi-step ratio and percent problems.
7.RP.A.3

Rigor

The Three Pillars of Rigor

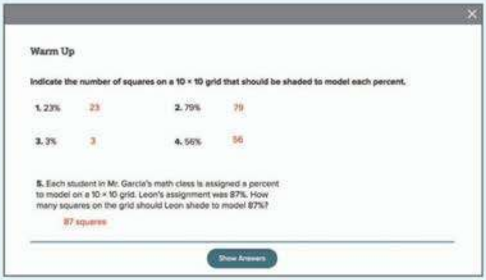
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students *apply* their *fluency* with percents to solve real-world problems that involve finding the whole. They build fluency with different representations, such as double number lines and equivalent ratios as they solve problems.

Mathematical Background

To find the whole, given the part and the percent, use bar diagrams, equivalent ratios, and double number lines. To use a double number line, the top number line is divided into equal parts with percents ranging from 0% to 100%, and the bottom contains the given part. The whole is the number associated with 100% on the number line. Alternatively, equivalent part-to-whole ratios can be written on one side and the fractional equivalent of the percent (as a rate per 100) on the other. Students can use what they know about equivalent ratios to find the missing whole.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- modeling percents with a 10 × 10 grid (Exercises 1–5)

Answers

1. 23
2. 79
3. 3
4. 56
5. 87 squares

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about finding the original price of a discounted item.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What other mathematical terms have the same root as the word *equivalent*? Use this to define *equivalent ratios*. Sample answer: equal, equation, equals sign; equivalent ratios are ratios that are equal in value but are expressed differently.

Your Notes

Method 2 Use a ratio table.

You know that 60% of some number is 114. Use a ratio table to scale back from 60% to 10%. Then scale forward from 10% to 100%.

Number of Students	19	114	190
Percent	10	60	100

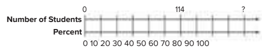
÷6, ×10

Because 60 ÷ 6 = 10, divide 114 by 6 to obtain 19. Because 10 × 10 = 100, multiply 19 by 10 to obtain 190. So, 60% of 190 is 114.

Method 3 Use a double number line.

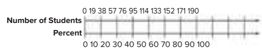
Step 1 Draw a double number line.

Draw a double number line. The bottom number line represents the percent, so use increments of 10 to draw tick marks and label the percents. The top number line represents the part of the whole that corresponds with each percent, so label the tick mark that corresponds with 60% on the bottom number line with 114.



Step 2 Find the whole.

Since there are 6 increments before 114, the value of each tick mark on the top number line increases by 114 ÷ 6, or 19 units.



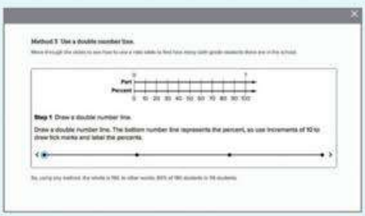
The double number line shows that 100%, or the whole, is 190.

So, using any method, the whole is 190. In other words, 60% of 190 students is 114 students.

Talk About It! A classmate let w represent the unknown whole and set up the equivalent ratios $\frac{114}{w} = \frac{60}{100}$. Is this method valid? Why might this method not be the most advantageous one to use in this case?

Sample answer: While this is a valid method, there is no whole number by which you can multiply 60 to obtain 114. You can use this method to multiply 60 by 1.9 to obtain 114, and then multiplying 100 by 1.9 to obtain 190. But it is not intuitive to know that 60 multiplied by 1.9 is 114.

Interactive Presentation



Learn, Find the Whole, Slide 4 of 5

CLICK

On Slide 3, students move through the slides to use a ratio table to find the whole.

CLICK

On Slide 4, students move through the slides to use a double number line to find the whole.

Learn Find the Whole *(continued)*

Teaching Notes

SLIDE 3

You may wish to ask students why it is necessary to scale back first, before scaling forward. Students should be able to reason that because there is no whole number by which you can multiply 60 to obtain 100, it is easier to scale back to 10, which can be multiplied by 10 to obtain 100.

Talk About It!

SLIDE 5

Mathematical Discourse

A classmate let w represent the unknown whole and set up the equivalent ratios $\frac{114}{w} = \frac{60}{100}$. Is this method valid? Why might this method not be the most advantageous one to use in this case? Sample answer: While this is a valid method, there is no whole number by which you can multiply 60 by to obtain 114. You can use this method by multiplying 60 by 1.9 to obtain 114, and then multiplying 100 by 1.9 to obtain 190. But it is not intuitive to know that 60 multiplied by 1.9 is 114.

DIFFERENTIATE

Reteaching Activity AL

For students that may be struggling with using double number lines to find the whole, explain how they can determine the number of sections in the double number line by analyzing the percent. Have students identify the number of sections needed in a double number line for each of the following percents. Have them draw number lines as needed to support their thinking.

50% 2   10% 10   15% 20   60% 5   85% 20

Example 1 Find the Whole

Objective

Students will use bar diagrams and equivalent ratios to solve a real-world problem involving finding the whole.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to reason about how a bar diagram helps them visualize the percent and the part, in order to find the whole.

7 Look For and Make Use of Structure Encourage students to analyze the structure of the bar diagram in order to recognize 75% as 3 parts out of a total of 4, where each part represents 25%.

Questions for Mathematical Discourse

SLIDE 3

AL Why is the bar diagram divided into four equal sections? The percent given is 75%, which is three-fourths, so divide the bar diagram into 4 sections.

OL Explain how you know that the whole is greater than 90. Sample answer: If 75% of his music library corresponds to 90 songs, then 100% of his music library must be greater than 90 songs.

OL How can you find the number of songs that each section represents? Sample answer: Since 90 represents three-fourths, divide 90 by 3 to find the value of each section.

BL A classmate claims that he could find the whole by dividing the bar diagram into 20 sections. Is the classmate correct? Explain. yes; Sample answer: Each section would represent 5%, or 6 songs, so the whole would be 6 · 20 or 120 songs.

SLIDE 4

AL When writing the equivalent ratios, do you need to find the percent, part, or whole? whole

OL By what number do you need to multiply 3 to obtain 90? 30

OL A classmate claims that Landon has 67.5 songs in his library. What was the likely mistake? Why is this answer not reasonable? Sample answer: The classmate likely found 75% of 90, thus treating 90 as the whole instead of the part. It is unreasonable to have 67.5 songs because you cannot have a fraction of a song.

BL Suppose Landon had 150 songs in his music library. What percent of his library would be country music? 60%

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 1 Find the Whole

Country music makes up 75% of Landon's music library.

If he has downloaded 90 country music songs, how many songs does Landon have in his music library?

The part is 90 country music songs. The percent is 75%. The whole, the number of songs he has in his library, is the unknown.

Method 1 Use a bar diagram.
Draw a bar diagram with 4 equal-size sections of 25% each. Shade 3 sections to represent 75%. Label the shaded sections as 90 songs.

How many songs are represented by each section? 30 songs

Label each section on the bar diagram.

How many songs are represented by the whole? 120 songs

Method 2 Use equivalent ratios.
Let w represent the whole.

part → whole → $\frac{90}{w} = \frac{75}{100}$ } percent

$\frac{90}{w} = \frac{3}{4}$ Simplify $\frac{75}{100}$ as $\frac{3}{4}$.

$\frac{90}{120} = \frac{3}{4}$ (×30) Because $3 \times 30 = 90$, multiply 4 by 30 to obtain 120. So, $w = 120$.

So, using either method, Landon has 120 songs in his music library.

Think About It! A classmate claims that because 75% is less than 100, Landon should have more than 90 music songs in his library. Do you think this reasoning is correct? Why or why not?

yes; Sample answer: If 75% of the whole is 90, then the whole must be greater than 90.

Talk About It! Explain why setting up the equation relating the equivalent ratios was advantageous to use in this example.

Sample answer: While there is no whole number by which you can multiply 75 to obtain 90, you can simplify the ratio $\frac{75}{100}$ to $\frac{3}{4}$. You can then multiply 3 by 30 to obtain 90.

Check

In the first year of ownership, a new car lost 20% of its value. If the car lost $4,200 of its value, how much did the car originally cost? Use any strategy.

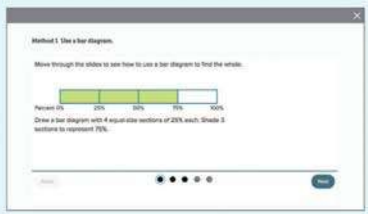
Show your work here

$21,000

Go Online You can complete an Extra Example online.

Lesson 2-6 • Find the Whole 123

Interactive Presentation



Example 1, Find the Whole, Slide 3 of 6

CLICK On Slide 3, students move through the steps to use a bar diagram to find the whole.

TYPE On Slide 4, students use equivalent ratios to find the whole.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
Is the whole less than, greater than, or equal to $15? How do you know?

greater than; Sample answer: The amount Marissa saved is 30% of the whole. Since 100% > 30%, the whole is greater than $15.

Example 2 Find the Whole

Marissa saved $15 because she bought a sweater that was on sale for 30% off.

What was the original price of the sweater?

The part is $15. The percent is 30%. The whole is the unknown.

Method 1 Use a bar diagram.
Draw a bar diagram with 10 equal-size sections of 10% each. Shade 3 sections to represent 30%. Label the shaded sections as $15.

How much money is represented by each section? $5

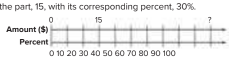
Label each section on the bar diagram.

How much money is represented by the whole? $50

Method 2 Use a double number line.

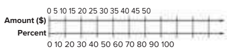
Step 1 Draw a double number line.

Label the part, 15, with its corresponding percent, 30%.



Step 2 Find the whole.

The value of each tick mark on the top number line increases by 15 ÷ 3, or 5 units. The number line shows that the whole, or 100%, is $50.



So, using either method, the original cost of the sweater was $50.

Talk About It!
Choose another strategy, such as a ratio table or an equation relating two equivalent ratios, to solve this problem. Compare and contrast the methods.

Sample answer: A bar diagram helps me to visually see the relationships among the part, percent, and whole. A ratio table or an equation relating equivalent ratios are efficient methods because you can use whole-number multiplication.

Check

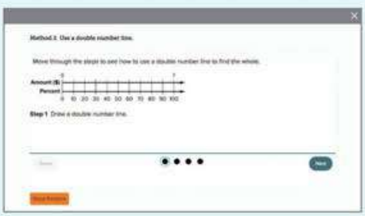
Kai calculates that he spends 15% of the school day in science class. If he spends 75 minutes in science class, how many minutes long is Kai's school day?

Show your work here

500 minutes

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 2, Find the Whole, Slide 4 of 6

CLICK
On Slide 3, students move through the steps to use a bar diagram to find the whole.

TYPE
On Slide 4, students use a double number line to find the whole.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find the Whole

Objective

Students will use bar diagrams and double number lines to solve a real-world problem that involves finding the whole.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities and to understand the correspondences between each method.

As students discuss the *Talk About It!* question on Slide 5, encourage them to discuss the advantages and disadvantages of using the different methods to solve this problem.

Questions for Mathematical Discourse

SLIDE 3

AL Was the sale 30% off the original price or 30% off $15? The sale was 30% off the original price.

AL Why is the bar diagram divided into 10 sections? The percent is 30%, and 30 is a multiple of 10.

OL How much does each section of the bar diagram represent? How do you know this? $5; Sample answer: 30% represents $15, and is represented by 3 sections. So, each section is $15 ÷ 3, or $5.

OL Where is the original price of the sweater represented on the bar diagram? It is the total of all ten sections of $5, or $50.

BL Suppose the percent discount was 35%. How would you change the bar diagram to find the whole? Sample answer: Divide the bar diagram into 20 sections, each representing about $2.14. Then multiply $2.14 by 20 to find the original price.

SLIDE 4

AL What number on the top number line corresponds with 30 on the bottom number line? 15

OL Why is the number line divided into 10 sections? Sample answer: The discount was 30%, so the number line should be divided into 10 equal sections of 10%.

BL A classmate says that the sweater originally cost 0.30 × $15, which is $4.50. Explain why this is not correct. Sample answer: The classmate is finding 30% of 15, and therefore treating 15 as the whole. However, 15 is the part. Realistically, the original price of the sweater cannot be $4.50 if she saved $15. That would mean she paid a negative amount.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Sales

Objective

Students will come up with their own strategy to solve an application problem involving selling bags of popcorn.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What percent of the bags of popcorn are represented by the cinnamon and cheese together?
- Which of the methods discussed in the lesson would be appropriate to use to solve the problem? Why?
- Why do you need to know the cost of each bag?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Sales

The table shows the percentage of each type of popcorn flavor at a specialty food store. A store clerk put all of the bags of cinnamon popcorn and cheese popcorn in a display in the front of the store. If the clerk put 60 bags in the front, how many bags of popcorn does the store have in all? If the store sells all of the bags of popcorn for $4.75 per bag, how much will the store earn in sales?

Flavor	Percent
Kettle Corn	60
Cinnamon	15
Caramel	10
Cheese	15

Go Online watch the animation.

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?
See students' strategies.

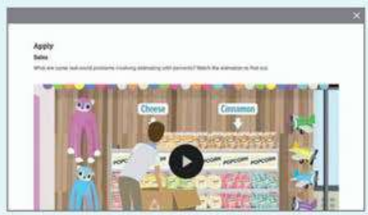
3 What is your solution?
Use your strategy to solve the problem.
The store has 200 total bags of popcorn, and the store will earn $950 in sales; See students' work.

4 How can you show that your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.
See students' arguments.

Talk About It!
How much more will the store earn in sales for selling all of the bags of kettle corn popcorn than caramel popcorn? Describe two different ways to solve this problem.
$475; See students' methods.

Lesson 2-6 • Find the Whole 125

Interactive Presentation



Apply, Sales

WATCH

On Slide 1, students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The table shows the percent of each type of puzzle in a toy store. During a sale, the store sold all of the 300-piece and 500-piece puzzles. If they sold 120 puzzles, how many puzzles did the store have before the sale? If they sell all of the puzzles for $8. 19 per puzzle, how much will the store make in sales?

Number of Pieces	Percent of Stock
300	50
500	30
750	15
1,000	5

Show your work here

150 puzzles; $1,228.50

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that shows your understanding of how you can use the following methods to find the whole, given the part and the percent.

- bar diagram
- ratio table
- double number line
- equivalent ratios

Record your observations here

See students' observations.

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Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. What was the cost of the drone before the sale? Write a mathematical argument that can be used to defend your solution. $200; Sample answer: The sale price of the drone is $50 which is 25% of the original price. I used equivalent ratios to find the sale price. $\frac{50}{?} = \frac{25}{100}$. Since $25 \times 2 = 50$, I multiplied 100 by 2 to obtain 200, the original price of the drone.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 11–14
- Extension: Find the Percent Given the Part and the Whole
- ALEKS® Percent Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 9, 11, 13
- Extension: Find the Percent Given the Part and the Whole
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–2
- ALEKS® Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Understanding Percents

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find the whole given the part and percent	1–7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving fractions, decimals, and percents	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconceptions

Students may attempt to find the whole given the percent and the part by incorrectly treating the part as the whole. In Exercise 1, students may find 80% of 20 and add the result to 20. Remind students that percents are rates per 100 (the whole) and that a double number line can be used to find the whole when it is unknown.

Name ________ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Use any strategy to solve each problem. (Examples 1 and 2)

1. Yolanda's club requires that 80% of the members be present for any vote. If at least 20 members must be present to have a vote, how many members does the club currently have?
   25 members

2. Action movies make up 25% of Sara's DVD collection. If she has 16 action DVDs, how many DVDs does Sara have in her collection?
   64 DVDs

3. Marcus saved $10 because he bought a baseball glove that was on sale for 40% off. What was the original price of the baseball glove?
   $25

4. Of the students in the marching band, 55% plan to go to the school dance. If there are 110 students in the marching band that are going to the dance, how many students are in the marching band?
   200 students

5. Melcher used 24% of the memory card on his digital camera while taking pictures at a family reunion. If Melcher took 96 pictures at the family reunion, how many pictures can the memory card hold?
   400 pictures

6. Mallorie has $12 in her wallet. If this is 20% of her monthly allowance, what is her monthly allowance?
   $60

7. The table shows the number of minutes Tim has for lunch and study hall. He calculates that these two periods account for 18% of the minutes he spends at school. How many minutes does he spend at school?

Period	Number of Minutes
Lunch	45
Study Hall	45

   500 minutes

Test Practice

8. **Open Response** The number of sixth grade students accounts for 35% of the total number of students enrolled in middle school. There are 245 sixth grade students. How many students are enrolled in the middle school?
   700 students

6.RP.A.3, 6.RP.A.3.C

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Apply *indicates multi-step problem

*9. Three different options for school lunch were offered on Friday. The table shows the percent of the total lunches sold for each option. If 270 students bought a cheese pizza or a pepperoni pizza, how many lunches were sold on Friday? If each lunch costs $3.50, how much money will the cafeteria earn from all of the lunches?

Option	Percent
Cheese Pizza	50
Pepperoni Pizza	40
Fried Chicken	10

300 lunches; $1,050

*10. The volleyball team is selling snack bags to raise money for new uniforms. The table shows the percent of the total bags sold for each type of snack bag. If they sold 210 bags of pretzels and cheese puffs, how many snack bags did they sell in all? If each snack bags costs $1.75, how much money did they raise?

Snack	Percent
Cheese Puffs	10
Corn Chips	15
Popcorn	25
Potato Chips	30
Pretzels	20

700 snack bags; $1,225

Higher-Order Thinking Problems

11. MP **Be Precise** Of the number of sixth grade students at a middle school, 120 prefer online magazines over print magazines. Of the number of seventh grade students, 140 prefer online magazines. A student said that this means a greater percent of seventh grade students prefer online magazines than sixth grade students. Is the student correct? Use precise mathematical language to explain your reasoning.

no; Sample answer: A percent compares the part to the whole. In this case, the only known value is the part. T o compare percents, the whole, the total number of sixth grade students and the total number of seventh grade students, must be known.

12. MP **Use Math Tools** In a photography club, about 48% of the members are girls. If there are 26 members who are girls, explain how you can use mental math to estimate the total number of people in the photography club?

Sample answer: Round 48% to 50% and round 26 to 25. Since 25 is about half of the members in the club, then 25+ 25 or 50 people is the approximate number of people in the club.

13. **Create** Write and solve a real-world problem where you use equivalent ratios to find the whole.

Sample answer: James's soccer team won 68% of the games they played. If they won 17 games, how many did they play? 25 games

14. If 10% of x is 100, how can you find the value of x?

Sample answer: Use equivalent fractions. $\frac{100}{x} = \frac{10}{100}$. Because 100 is 10 times 10, multiply 100 times 10. So, $x = 1{,}000$ because $100 \times 10 = 1{,}000$.

Teaching the Mathematical Practices

6 Attend to Precision In Exercise 11, students use precise mathematical language to explain why a comparison cannot be accurately made without using percents.

5 Use Appropriate Tools Strategically In Exercise 12, students use mental math to estimate the whole.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 9 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Clearly and precisely explain.

Use with Exercises 11 and 14 Have students work in pairs. Have students individually read Exercise 11 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 14.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of fractions, decimals, and percents. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

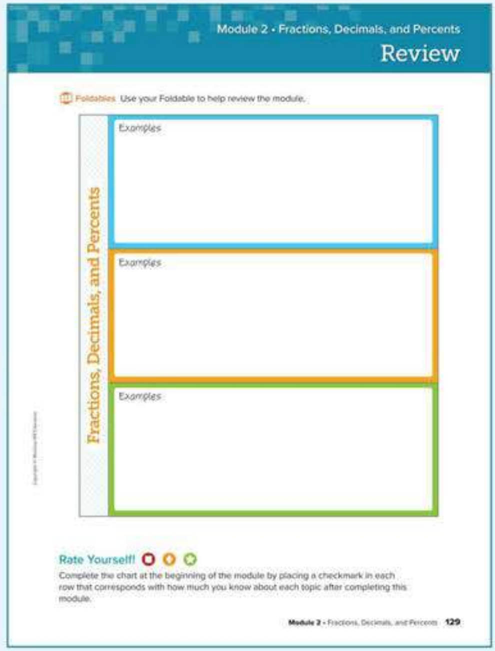
Assessment Resources

Put It All Together 1: Lessons 2-1 through 2-3
Put It All Together 2: Lessons 2-4 through 2-6
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Ratios and Proportional Relationships**.

- Ratios
- Rates
- Solve Problems: Unit Rates
- Solve Problems: Percent Rates


Module 2 • Fractions, Decimals, and Percents

Review

Foldables Use your Foldable to help review the module.

Fractions, Decimals, and Percents

Examples

Examples

Examples

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

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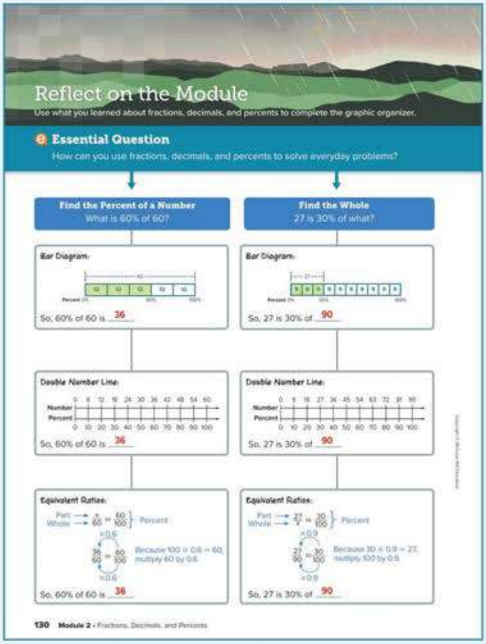
Reflect on the Module

Use what you learned about fractions, decimals, and percents to complete the graphic organizer.

Essential Question

How can you use fractions, decimals, and percents to solve everyday problems?

Find the Percent of a Number
What is 60% of 60?

Bar Diagram:

So, 60% of 60 is 36

Double Number Line:

Number 0 6 12 18 24 30 36 42 48 54 60

Percent 0 10 20 30 40 50 60 70 80 90 100

So, 60% of 60 is 36

Equivalent Ratios:

Part → $\frac{x}{60} = \frac{60}{100}$ } Percent
Whole →

$\frac{36}{60} = \frac{60}{100}$ (×0.6) Because 100 × 0.6 = 60, multiply 60 by 0.6.

So, 60% of 60 is 36

Find the Whole
27 is 30% of what?

Bar Diagram:

So, 27 is 30% of 90

Double Number Line:

Number 0 9 18 27 36 45 54 63 72 81 90

Percent 0 10 20 30 40 50 60 70 80 90 100

So, 27 is 30% of 90

Equivalent Ratios:

Part → $\frac{27}{x} = \frac{30}{100}$ } Percent
Whole →

$\frac{27}{90} = \frac{30}{100}$ (×0.9) Because 30 × 0.9 = 27, multiply 100 by 0.9.

So, 27 is 30% of 90

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can you use fractions, decimals, and percents to solve everyday problems? See students' graphic organizers.

Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–12 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 8
Multiselect	Multiple answers may be correct. Students must select all correct answers.	5
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	2
Open Response	Students construct their own response in the area provided.	3, 4, 6, 7, 9–12

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
Foundational for 6.RP.A.3	2-1, 2-2, 2-3	1–6
6.RP.A.3	2-4, 2-5, 2-6	7–12
Foundational for 6.RP.A.3.C	2-1, 2-2, 2-3	1–6
6.RP.A.3.C	2-4, 2-5, 2-6	7–12

Name ________ Period ________ Date ________

Test Practice

1. Multiple Choice What is 2.6% written as a decimal? (Lesson 2)

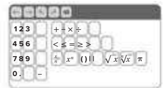
(A) 0.26

(B) 0.026

(C) 26

(D) 260

2. Equation Editor At a baking competition, 0.5 dishes were cooked by Room 102, $\frac{3}{10}$ were cooked by Room 104, and $\frac{1}{5}$ were cooked by Room 106. What fraction of the dishes were cooked by Rooms 102 and 104? (Lesson 1)

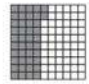


$\frac{4}{5}$

3. Open Response Vineisha earned 22 out of 20 points on her science quiz over the phases of the moon due to an extra credit question. What percent did she earn on the quiz? (Lesson 2)

110%

4. Open Response Refer to the grid shown below. (Lesson 2)



A. What percent of the grid is shaded?

42%

B. Write your answer from part A as a fraction and a decimal.

$\frac{42}{100}$ or $\frac{21}{50}$, 0.42

5. Multiselect Which number forms below are equivalent to 0.28? Select all that apply. (Lessons 1 and 4)

- [x] 28%
- [ ] $\frac{21}{80}$
- [x] $\frac{28}{100}$
- [x] $\frac{14}{50}$
- [ ] 28
- [x] $\frac{7}{25}$

6. Open Response At a food festival, $\frac{3}{8}$ of the dishes were from China. Another 12.5% of the dishes were from Japan. What percent of the dishes were from other countries? (Lesson 3)

50%

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7 . **Open Response**A basketball player made 40% of the shots she attempted. If she made 32 baskets, how many shots did she attempt? (Lesson 6)

80 shots

8. **Multiple Choice** A clothing store purchases a sweatshirt for $26 and adds $15 to set the sticker price. The store is having a sale where everything is on sale for 20% off. Choose the most reasonable estimate for the final price of a sweatshirt. (Lesson 5)

A $8.00

B $28.00

C $32.00

D $40.00

9. **Open Response** Three hundred students were surveyed about their favorite subject. The results are shown in the table below. How many more students prefer science than math? (Lesson 4)

Subject	Percent
Language Arts	15
Math	24
Science	33
Social Studies	21
Elective	7

27 students

10. **Open Response** The original price of a DVD is $11. The sale price is 30% off the original price. What is the sale price of the DVD? (Lesson 4)

$7 .70

11. **Open Response** The table shows the percent of total items sold for each type of ball sold at a sports equipment store in one week. (Lesson 6)

Type of Ball	Percent
Baseball	25
Basketball	35
Football	20
Soccer Ball	15
T ennis Ball	5

A. If they sold a total of 450 baseball and tennis balls, how many total items did the store sell in one week?

1,500 items

B. If each item is sold for $10.95, how much did the store have in sales?

$16,425

12. **Open Response** Twenty-one students in Michael's classroom are wearing jeans. There are 25 students in his class. Michael says that 80% of his class is wearing jeans. Is Michael correct? Explain your reasoning. (Lesson 4)

no; Sample answer: 21 is 84% of 25.

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IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How are operations with fractions and decimals related to operations with whole numbers? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

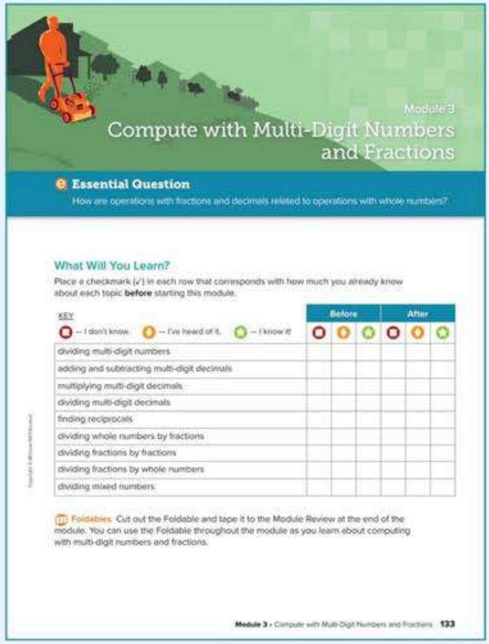
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of theater set design and business management to introduce the idea of computing with decimals and fractions. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 3

Compute with Multi-Digit Numbers and Fractions

Essential Question

How are operations with fractions and decimals related to operations with whole numbers?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
dividing multi-digit numbers						
adding and subtracting multi-digit decimals						
multiplying multi-digit decimals						
dividing multi-digit decimals						
finding reciprocals						
dividing whole numbers by fractions						
dividing fractions by fractions						
dividing fractions by whole numbers						
dividing mixed numbers						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about computing with multi-digit numbers and fractions.

Module 3 • Compute with Multi-Digit Numbers and Fractions 133

Interactive Presentation

Module 3

Compute with Multi-Digit Numbers and Fractions

Module Goal

Compute with multi-digit numbers and fractions.

Focus

Domain: The Number System

Major Cluster(s):

6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Standards for Mathematical Content:

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Also addresses 6.NS.B.2.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- fluently add, subtract, and multiply multi-digit whole numbers
- divide whole numbers with up to four-digit dividends and two-digit divisors
- perform operations with decimals to the hundredths place
- add, subtract, and multiply fractions
- divide whole numbers by unit fractions and vice versa using visual models

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students multiplied with fractions and mixed numbers and divided with unit fractions.
5.NF.B.4, 5.NF.B.6, 5.NF.B.7

Now
Students compute with multi-digit numbers and fractions
6.NS.A.1, 6.NS.B.2, 6.NS.B.3

Next
Students will extend previous understandings of numbers to the system of rational numbers.
6.NS.C.5, 6.NS.C.6, 6.NS.C.7, 6.NS.C.8

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of basic computation to develop *understanding* of computation with multi-digit numbers and fractions. They use this understanding to build *fluency* with the four basic operations involving whole numbers and decimals, and division of fractions and mixed numbers. They also *apply* their understanding of fractions to write and solve real-world story contexts.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

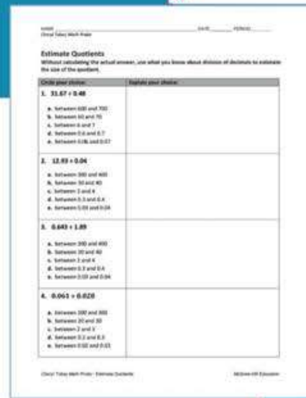
Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			**1**	**0.5**
3-1	Divide Multi-Digit Whole Numbers	6.NS.B.2	2	1
3-2	Compute with Multi-Digit Decimals	6.NS.B.3	2	1
Put It All Together 1: Lessons 3-1 and 3-2			**0.5**	**0.25**
3-3	Divide Whole Numbers by Fractions	6.NS.A.1	3	1.5
3-4	Divide Fractions by Fractions	6.NS.A.1	2	1
3-5	Divide with Whole and Mixed Numbers	6.NS.A.1	3	1.5
Put It All Together 2: Lessons 3-3, 3-4, and 3-5			**0.5**	**0.25**
Module Review			**1**	**0.5**
Module Assessment			**1**	**0.5**
		Total Days	16	8

Formative Assessment Math Probe
Estimate Quotients

CHERYL TOBEY MATH PROBES

Module Resource


Estimate Quotients

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students estimate the size of each quotient, without calculating. Exercises involve division with decimals.

Targeted Concept Reason about the value of numbers and understand the effects of division given those values.

Targeted Misconceptions

- Students may apply the whole number concept of "division makes quantities smaller".
- Students may have inaccuracies about the value of decimal numbers.

Assign the probe after Lesson 2.

Correct Answers: 1.b; **2.** a; **3.** d; **4.** c

Collect and Assess Student Work

If the student selects...	**Then** the student likely...
1. c, d, e **2.** c, d, e **3.** d, e **4.** e	overgeneralizes from operations with whole numbers, and reasons the rule "division makes smaller" applies to all numbers. **Example:** The student chooses a combination of these answers, because the quotient is greater than the dividend.
Various patterns of incorrect responses	applies incorrect reasoning about either the size of the decimal or the effect of the operation; and/or incorrectly applies an algorithm. **Example:** The student chooses c for Exercise 1 by incorrectly reasoning that dividing by one half is the same as dividing a quantity in half and option c is the closest answer to half of 31.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Decimals
- Lesson 2, Example 5

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ dividend
- ☐ multiplicative inverse
- ☐ divisor
- ☐ quotient
- ☐ Inverse Property of Multiplication
- ☐ reciprocal

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1	Example 2
Multiply whole numbers.	**Divide whole numbers.**
Find 13 × 15.	Find 323 ÷ 17 .
13 × 15	17)323 → 19
6 5 Multiply the ones.	−17 Divide the tens.
+130 Multiply the tens.	153, −153 Divide the ones.
195 Add.	0

Quick Check

1. Find 19 × 51. 969
2. Find 49 × 23. 1, 127
3. Find 539 ÷ 11. 49
4. Find 432 ÷ 16. 27

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

1 2 3 4

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine.

Define Two numbers whose product is 1 are called **multiplicative inverses**, or **reciprocals**.

Example The multiplicative inverse, or reciprocal of 9 is $\frac{1}{9}$. The multiplicative inverse of $\frac{5}{6}$ is $\frac{6}{5}$.

Ask What is the multiplicative inverse, or reciprocal of $\frac{7}{8}$? $\frac{8}{7}$

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- dividing whole numbers using the standard algorithm
- adding and subtracting multi-digit numbers
- multiplying fractions
- understanding inverse operations
- solving word problems involving the multiplication of fractions

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Decimals** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Collaborative Risk Taking

Some students may be averse to taking risks during math class, such as sharing an idea, strategy, or solution. They may worry about their grades or scores on tests, or some might feel less confident solving math problems, especially in front of their peers. Create a classroom environment where it is safe for students to take risks, including setting norms for how students will engage in classroom conversations. Encourage students to view mistakes as part of the path to success.

How Can I Apply It?

In the **Practice** section of each lesson, **Collaborative Practice** tips are provided for several exercises in the Teacher Edition. Assign those exercises and encourage students to take risks together as they solve problems, try new solution paths, and discuss their strategies.

When assigning the **Application Problems**, have students look for alternative approaches that can be used. Encourage them to view their solution process as one of refinement, as needed. They may try different paths, monitor their progress, and change course if necessary. This is part of the natural process of problem solving.

Learn Divide Multi-Digit Numbers

Objective

Students will understand the parts of a division problem.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to use the definitions of *quotient, dividend*, and *divisor* to accurately label the division problem as they complete the drag and drop activity on Slide 1.

Go Online to find additional teaching notes.

Example 1 Divide Multi-Digit Numbers

Objective

Students will fluently divide multi-digit whole numbers with whole number quotients.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to pay careful attention to each place-value position as they use the standard algorithm for division.

Questions for Mathematical Discourse

SLIDE 1

AL Identify the quotient, dividend, and divisor. The quotient is 2,145, the dividend is 25,740, and the divisor is 12.

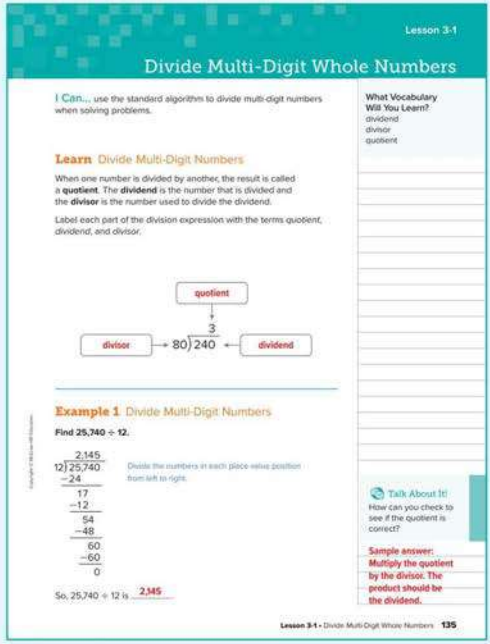
OL Why is the 2 in the quotient above the 5, instead of the 2, in 25,740? 12 cannot divide 2, but it can divide 25, so the 2 goes above the 5.

OL How can you check the quotient for reasonableness? Sample answer: Use estimation; $24{,}000 \div 12 = 2{,}000$, so the quotient should be close to 2,000.

BL If $25{,}740 \div 12 = 2{,}145$, how can you use mental math to find $2{,}145 \times 13$? Sample answer: I know that $2{,}145 \times 12 = 25{,}740$; Add another 2,145 to 25,740 to represent the 13th time that 2,145 is added. Since $25{,}740 + 2{,}145 = 27{,}885$, then $2{,}145 \times 13 = 27{,}885$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Lesson 3-1

Divide Multi-Digit Whole Numbers

I Can... use the standard algorithm to divide multi-digit numbers when solving problems.

What Vocabulary Will You Learn?
dividend
divisor
quotient

Learn Divide Multi-Digit Numbers

When one number is divided by another, the result is called a **quotient**. The **dividend** is the number that is divided and the **divisor** is the number used to divide the dividend.

Label each part of the division expression with the terms quotient, *dividend, and divisor.*

quotient → 3
divisor → 80)240 ← dividend

Example 1 Divide Multi-Digit Numbers

Find 25,740 ÷ 12.

```
    2,145
12)25,740
  -24
    17
   -12
     54
    -48
     60
    -60
      0
```

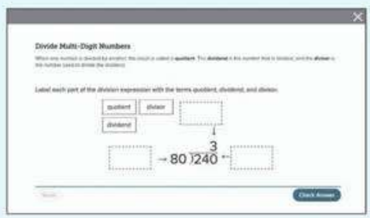
Divide the numbers in each place-value position from left to right.

So, 25,740 ÷ 12 is 2,145.

Talk About It!
How can you check to see if the quotient is correct?
Sample answer: Multiply the quotient by the divisor. The product should be the dividend.

Lesson 3-1 • Divide Multi-Digit Whole Numbers 135

Interactive Presentation



Learn, Divide Multi-Digit Numbers

DRAG & DROP On Slide 1 of the Learn, students drag terms to label the division problem.

TYPE On Slide 1 of Example 1, students determine the quotient.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Lesson 3-1

Divide Multi-Digit Whole Numbers

6.NS.B.2

LESSON GOAL

Students will find quotients of multi-digit whole numbers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Divide Multi-Digit Numbers
Example 1: Divide Multi-Digit Numbers
Learn: Divide Multi-Digit Numbers
Example 2: Divide Multi-Digit Numbers
Example 3: Divide Multi-Digit Numbers
Apply: Fundraising

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 15 of the *Language Development Handbook* to help your students build mathematical language related to division of multi-digit whole numbers.

ELL You can use the tips and suggestions on page T15 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System
Additional Cluster(s): In this lesson, students address additional cluster **6.NS.B** by finding quotients of multi-digit whole numbers.
Standards for Mathematical Content: **6.NS.B.2**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6, MP8**

Coherence

Vertical Alignment

Previous
Students divided four-digit dividends by two-digit divisors.
5.NBT.B.6

Now
Students find quotients of multi-digit whole numbers.
6.NS.B.2

Next
Students will perform operations on multi-digit decimals.
6.NS.B.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

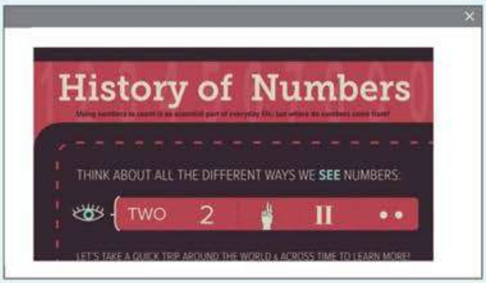
Conceptual Bridge In this lesson, students draw on their knowledge of division (gained in prior grades) to build *fluency* with dividing multi-digit whole numbers, with both whole number quotients and by annexing zeros in the decimal place. They *apply* their understanding of dividing multi-digit whole numbers to solve real-world problems.

Mathematical Background

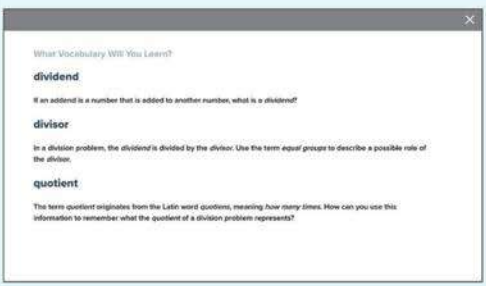
A division problem has three components: a *dividend*, a *divisor*, and a *quotient*. The dividend is the number being divided, the divisor is the number the dividend is being divided by, and the quotient is the result. Multi-digit whole numbers can be divided using the standard division algorithm. If the divisor does not divide the dividend evenly, the result can be written as a quotient and remainder, or zeros can be annexed and the standard division algorithm can be continued in order to write the quotient as a decimal.

Interactive Presentation

Warm Up



Launch the Lesson



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- dividing using the standard algorithm (Exercises 1–5)

Answers

1. 8
2. 9
3. 25
4. 42
5. $1.62

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the history of numbers.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- If an addend is a number that is added to another number, what is a *dividend*? a number that is divided by another number
- In a division problem, the *dividend* is divided by the *divisor*. Use the term *equal groups* to describe a possible role of the *divisor*. Sample answer: The divisor represents the number of equal groups into which the dividend is being divided.
- The term *quotient* originates from the Latin word *quotiens*, meaning *how many times*. How can you use this information to remember what the *quotient* of a division problem represents? Sample answer: the number of times the dividing number goes into the number being divided

Your Notes

Check

Find 52,428 ÷ 34. 1,542

Show your work here

Go Online You can complete an Extra Example online.

Learn Divide Multi-Digit Numbers

If two numbers do not divide evenly, you can write the quotient as a whole number with a remainder, or continue dividing by adding a decimal point to the right of the whole number and annexing zeros. Annex as many zeros as necessary to complete the division.

An example is shown. Compare long division using remainders and long division by annexing zeros.

With Remainders	Annexing Zeros
32 R20	32.8
25)820	25)820.0
−75	−75
70	70
−50	−50
20	200
	−200
	0

Talk About It! How do you know that 820 and 820.0 are equivalent?

Sample answer: Adding zeros to the end of a decimal does not change the value of the decimal.

Recall that a remainder can be written as a fraction with the remainder in the numerator and the dividend in the denominator. To check that 32 R 20 is equal to 32.8, first write the remainder as a fraction and then convert the fraction to a decimal.

$32 \text{ R } 20 = 32\frac{20}{25}$

$= 32\frac{20}{25}$ or 32.8

So, 820 ÷ 25 is 32 with a remainder of 20, or 32.8.

Interactive Presentation

Learn, Divide Multi-Digit Numbers, Slide 1 of 2

CLICK

On Slide 1, students compare the division algorithm using remainders with annexing zeros.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Divide Multi-Digit Numbers

Objective

Students will learn how to fluently divide multi-digit whole numbers by annexing zeros.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to reflect upon how annexing zeros to the end of a decimal affects, or doesn't affect, the value of the number.

Teaching Notes

SLIDE 1

Students learned about division with remainders in prior grades. You may wish to have them review the standard division algorithm using remainders. Point out that they can continue dividing by adding a decimal point to the right of the whole number and annexing zeros.

Have students select the buttons to examine the similarities and differences between using remainders and annexing zeros. Ask students what they notice. They should note that the whole number part of the quotient is the same, 32. When using remainders, the remainder is 20, which represents 20 out of 25 (the divisor). In other words, the quotient is $32\frac{20}{25}$. When continuing to divide by annexing zeros, the quotient is 32.8, which is the same as $32\frac{20}{25}$.

Talk About It!

SLIDE 2

Mathematical Discourse

How do you know that 820 and 820.0 are equivalent? Sample answer: Adding zeros to the end of a decimal does not change the value of the decimal.

DIFFERENTIATE

Reteaching Activity AL

Some students may have difficulty contextualizing the long division algorithm. Base-ten blocks can be an effective manipulative for helping students to understand how to divide multi-digit whole numbers. Have the students model the dividend in the expression 262 ÷ 8 using base-ten blocks. Then ask the following questions.

1. How did you model 262 with base-ten blocks? 2 hundreds, 6 tens, and 2 ones
2. Since you cannot divide the hundreds into 8 equal groups, regroup them into tens. How many tens are there altogether? 26
3. Divide the tens into eight groups. How many tens are in each group? How many are left over? 3 tens in each group with 2 left over
4. Regroup the remaining tens into ones. How many ones are there altogether? 22
5. Divide the ones into eight groups. How many ones are in each group? How many are left over? 2 ones in each group with 6 left over
6. What do the remaining 6 ones represent? the remainder

Example 2 Divide Multi-Digit Numbers

Objective

Students will fluently divide multi-digit whole numbers by annexing zeros.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to pay careful attention to each place-value position to ensure they annex zeros in the correct positions.

8 Look for and Express Regularity in Repeated Reasoning As students discuss the *Talk About It!* question on Slide 3, encourage them to look back at the division and make sense of the remainder of 0. If they were to keep dividing, they should notice that they would continue to repeat zeros, and this is why no further division needs to take place.

Questions for Mathematical Discourse

SLIDE 2

AL What does "annexing a zero" mean? Sample answer: Annexing a zero means to add a zero to the end of a number.

OL How could you check to make sure the quotient is correct? Sample answer: I can multiply 82.375 by 64 to make sure that their product is 5,272.

BL Would it be necessary to annex zeros if the divisor was 2? Explain. no; Sample answer: It would not be necessary to annex zeros, because 2 divides 5,272 evenly.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Divide Multi-Digit Numbers

Find 5,272 ÷ 64.

```
        82.375
64) 5,272.000     Divide from left to right. Annex zeros as needed.
   −5 12          Multiply 8 × 64, then subtract.
      152
     −128         Multiply 2 × 64, then subtract.
       240        There is a remainder. Annex a zero.
      −192        Multiply 3 × 64, then subtract.
        480       Annex a zero and continue dividing.
       −448       Multiply 7 × 64, then subtract.
         320      Annex a zero and continue dividing.
        −320      Multiply 5 × 64, then subtract.
           0      The remainder is 0.
```

So, 5,272 ÷ 64 is 82.375.

Think About It! How will you set up the division? See students' responses.

Talk About It! How do you know when you are done dividing? Sample answer: When the remainder is zero, no further division needs to take place.

Check

Find 16,047 ÷ 60. 267.45

Show your work here

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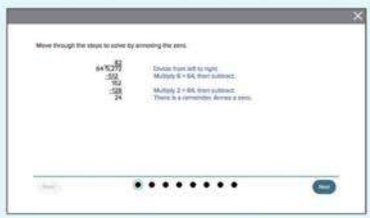
Go Online You can complete an Extra Example online.

Lesson 3-1 • Divide Multi-Digit Whole Numbers 137

Interactive Presentation



Example 2, Divide Multi-Digit Numbers, Slide 2 of 4

CLICK

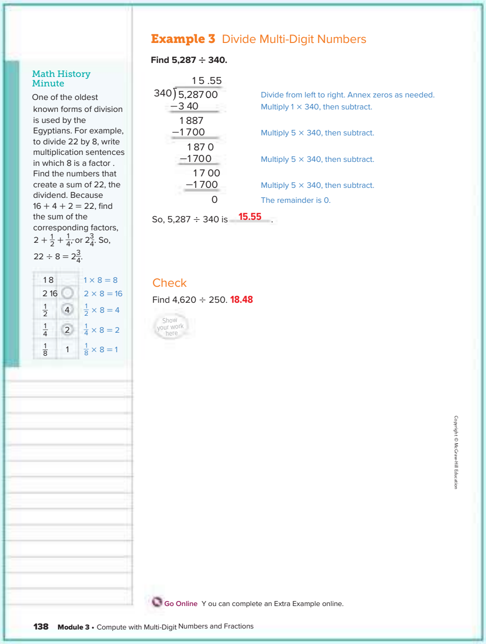
On Slide 2, students move through the steps to solve the division problem by annexing zeros.

TYPE

On Slide 2, students determine the quotient.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Math History Minute

One of the oldest known forms of division is used by the Egyptians. For example, to divide 22 by 8, write multiplication sentences in which 8 is a factor. Find the numbers that create a sum of 22, the dividend. Because $16 + 4 + 2 = 22$, find the sum of the corresponding factors, $2 + \frac{1}{2} + \frac{1}{4}$, or $2\frac{3}{4}$. So, $22 \div 8 = 2\frac{3}{4}$.

1	8	$1 \times 8 = 8$
2	16 (circled)	$2 \times 8 = 16$
$\frac{1}{2}$	4 (circled)	$\frac{1}{2} \times 8 = 4$
$\frac{1}{4}$	2 (circled)	$\frac{1}{4} \times 8 = 2$
$\frac{1}{8}$	1	$\frac{1}{8} \times 8 = 1$

Example 3 Divide Multi-Digit Numbers

Find 5,287 ÷ 340.

340)5,287.00 → 15.55	Divide from left to right. Annex zeros as needed.
−3 40	Multiply 1 × 340, then subtract.
1 887	
−1 700	Multiply 5 × 340, then subtract.
187 0	
−170 0	Multiply 5 × 340, then subtract.
17 00	
−17 00	Multiply 5 × 340, then subtract.
0	The remainder is 0.

So, 5,287 ÷ 340 is 15.55.

Check

Find 4,620 ÷ 250. 18.48

Show your work here

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Go Online You can complete an Extra Example online.

138 Module 3 • Compute with Multi-Digit Numbers and Fractions

Interactive Presentation

Example 3, Divide Multi-Digit Numbers, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to solve the division problem by annexing zeros.

TYPE

On Slide 1, students determine the quotient.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Divide Multi-Digit Numbers

Objective

Students will fluently divide multi-digit whole numbers by annexing zeros.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to pay careful attention to each place-value position to ensure they annex zeros in the correct positions.

Questions for Mathematical Discourse

SLIDE 1

AL Why do we place the 1 above the 8 of the dividend, 5,287? 340 cannot divide 5 or 52, but it can divide 528, so the 1 goes above the 8.

OL How do you know when you are done dividing? When the final remainder is zero, there is no other division to take place.

BL Find 5,287 ÷ 170. What do you notice about this divisor compared to 340? What do you notice about this quotient compared to the quotient of 5,287 ÷ 340? 31.1; Sample answer: 170 is half of 340, and the quotient 31.1 is twice the quotient 15.55.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Fundraising

Objective

Students will come up with their own strategy to solve an application problem involving making bags of cookies to sell for a fundraiser.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What operation is used when you need to separate items into different groups?
- How many of each type of cookie was donated for the sale?
- What do you need to do to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Fundraising

The table shows the number of cookies donated for the school bake sale. The cookies were placed into bags with one dozen cookies in each bag. How many bags of one dozen cookies were available to sell?

Bake Sale Cookies	
Type	Number
Chocolate Chip	125
Oatmeal	60
Peanut Butter	245
Sugar	116

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

45 bags; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Why is the final answer given as a whole number when the quotient is a decimal?

Sample answer: You won't make a partial bag of cookies, so you need to round the quotient down to the nearest whole number.

Interactive Presentation

Apply, Fundraising

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

There are 24 seats in each row of the middle school auditorium. The table shows the number of students from each grade who attended a concert. If the students fill each row in the auditorium, how many rows would be needed for all of the students? **35 rows**

Grade	Number of Students
Sixth	310
Seventh	256
Eighth	262

Show your work here

Go Online You can complete an Extra Example online.

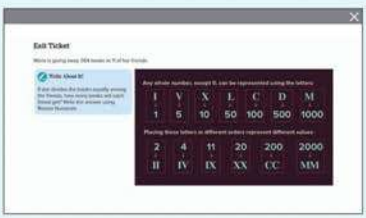
Pause and Reflect

When dividing whole numbers, the quotient can be written with a remainder, or you can annex zeros and continue dividing. How are these two methods similar? How are they different?

Record your observations here

See students' observations.

Interactive Presentation



Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Maria is giving away 264 books to 11 of her friends. If she divides the books equally among the friends, how many books will each friend receive? Write the answer using Roman Numerals. XXIV books

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 11, 13–17
- ALEKS Division

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 13, 14, 17
- Personal Tutor
- Extra Examples 1–3
- ALEKS Division

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Division

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	fluently divide multi-digit whole numbers with whole number quotients	1–3
1	fluently divide multi-digit whole numbers by annexing zeros	4–9
2	extend concepts learned in class to apply them in new contexts	10, 11
2	solve application problems involving the division of multi-digit numbers	12, 13
3	higher-order and critical thinking skills	14–17

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find each quotient. (Examples 1–3)

1. 52,080 ÷ 15 = 3,472
2. 38,480 ÷ 26 = 1,480
3. 648 ÷ 18 = 36
4. 3,409 ÷ 14 = 243.5
5. 8,890 ÷ 40 = 222.25
6. 3,120 ÷ 64 = 48.75
7. 6,750 ÷ 240 = 28.125
8. 4,415 ÷ 800 = 5.51875
9. 5,777 ÷ 160 = 36.10625

Test Practice

10. The table shows the distances between major cities. Mr. Santiago has a flight from Los Angeles to Toronto. If the plane travels at 520 miles per hour, how many hours long is the flight?

New York to Paris	3,636 miles
Los Angeles to Toronto	2,171 miles

4.175 hours

11. **Equation Editor** What is the value of the expression 3,082 ÷ 23?

134

Apply *indicates multi-step problem

*12. The table shows the number of each type of greeting card a gift shop had remaining at the end of the year. The store created bags with 15 random cards in each bag. How many complete bags of cards were they able to make?

37 bags

Card Type	Number of Cards
Anniversary	163
Birthday	258
Get Well	98
Thank You	47

*13. The table shows the number of each type of seed packet a garden center had remaining at the end of summer. Bags were created with 20 random seed packets in each bag. How many complete bags of seeds can be created?

24 bags

Seed Type	Number of Packets
Aster	40
Daisy	95
Pansy	160
Sunflower	125
Wildflower	70

Higher-Order Thinking Problems

14. Use the digits 9, 6, and 3 one time each in the following multi-digit division problem. Then rewrite the problem.

3 , 6 00 ÷ 9 0 = 40

3,600 ÷ 90 = 40

15. MP **Persevere with Problems** If the divisor is 60, what is the least four-digit dividend that would not have a remainder?

1,020

16. MP **Justify Conclusions** Determine if the following statement is *true* or *false*. Justify your conclusion.

The remainder in a division problem can equal the divisor.

false; Sample answer: The remainder cannot equal or be greater than the divisor. If the remainder is equal or greater, then the quotient should be increased by at least one.

17. How can you check that your quotient is correct when dividing multi-digit whole numbers?

Sample answer: Check your answer by multiplying the quotient by the divisor. Compare this answer to the dividend. They should be equal.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 15, students will have to try different division problems to determine the least four-digit dividend that would not have a remainder. They will need to progress methodically and persevere until they find the answer.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students will determine whether the remainder in a division problem can equal the divisor, and they will justify their conclusion.

Common Misconception

When finding the number of bags in Exercise 12, students may be tempted to divide the individual numbers by 15 and round. For example, they may divide 47 Thank-You cards by 15 to get 3 bags. They may also answer 4 if they think there is another bag necessary for the left over 2 Thank-You cards. After calculating the individual quotients, they may add them together to find the total number of bags. Encourage students to pay attention to the way the problem is worded, and ask them why they must add all of the cards together before dividing by 15. It will be helpful to demonstrate that the two methods produce two different answers.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 12–13 Have students work in pairs. Have students individually read Exercise 12 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 13.

Be sure everyone understands.

Use with Exercise 16 Have students work in groups of 3–4 to solve the problem in Exercise 16. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands why the statement is false. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class.

Learn Add and Subtract Multi-Digit Decimals

Objective

Students will learn how to fluently add and subtract multi-digit decimals when the number of decimal places is not the same.

MP Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to analyze the structure of an addition or subtraction expression involving decimals, in order to understand how they can annex zeros to ensure the same number of decimal places.

Teaching Notes

SLIDES 1–2

Present the addition problem on Slide 1, 45.16 + 21.384 and ask students what they notice. They should note that the decimals do not have the same number of decimal places. Ask students if there are any strategies they can use to write these numbers so that they have the same number of decimal places. Students should note that they can annex a zero to 45.16 without changing its value, thus writing the decimal as 45.160. They can then apply the same rules for adding decimals to the hundredths place to add 45.160 and 21.384 by lining up the decimal points and add the digits in the same place-value positions.

Talk About It!

SLIDE 3

Mathematical Discourse

How does annexing a zero help you correctly add or subtract the numbers? Sample answer: Annexing a zero allows you to align the place values of the numbers being added or subtracted.

DIFFERENTIATE

Language Development Activity ELL

If any of your students are struggling to add and subtract multi-digit decimals when the number of decimal places is not the same, encourage them to read aloud each decimal using the correct place-value positions. For example, to find 348.18+ 12.2, have students read aloud the expression as *three hundred forty-eight and eighteen hundredths plus twelve and two tenths*. The fact that the first decimal is to the hundredths place and the second decimal is to the tenths place should indicate they need to annex a zero to 12.2, so that it can be read twelve and twenty *hundredths* (12.20). Have them work with a partner to find the sum or difference of each of the following expressions using this strategy.

345.18 + 12.24 357.42	18.3 + 7.09 25.39
108.78 – 56.362 52.418	100.07 – 71.002 29.068

Lesson 3-2

Compute With Multi-Digit Decimals

I Can... solve problems by using the standard algorithms for addition, subtraction, multiplication, and division to compute with multi-digit decimals.

Learn Add and Subtract Multi-Digit Decimals

You have already added and subtracted decimals to the hundredths place. You can apply the same rules when adding and subtracting decimals to the thousandths place. First, align the decimal points, then annex zeros, if needed, so that both numbers have the same number of decimal places.

Find 45.16 + 21.384.

45.160
+ 21.384
Align the decimal points and annex a zero.

45.160
+ 21.384
66.544
Add numbers in the same place-value position.
Place the decimal point in the sum.

So, 45.16 + 21.384 is 66.544.

Find 32.94 – 15.386.

32.940
– 15.386
Align the decimal points and annex a zero.

32.940
– 15.386
17.554
Subtract as with whole numbers.
Place the decimal point in the difference.

So, 32.94 – 15.386 is 17.554.

Talk About It!
How does annexing a zero help you correctly add or subtract the numbers?

Sample answer: Annexing a zero allows you to align the place values of the numbers being added or subtracted.

Lesson 3-2 • Compute With Multi-Digit Decimals 143

Interactive Presentation

Learn, Add and Subtract Multi-Digit Decimals, Slide 1 of 3

CLICK

On Slide 1, students move through the steps to add and subtract the decimals.

Lesson 3-2

Compute with Multi-Digit Decimals

6.NS.B.3

LESSON GOAL

Students will perform operations on multi-digit decimals.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Add and Subtract Multi-Digit Decimals
Example 1: Add Multi-Digit Decimals
Example 2: Subtract Multi-Digit Decimals
Example 3: Subtract Multi-Digit Decimals
Learn: Multiply Decimals
Example 4: Multiply Multi-Digit Decimals
Learn: Divide Decimals
Example 5: Divide Multi-Digit Decimals
Apply: Shopping

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Compute with Multi-Digit Decimals and Whole Numbers		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 16 of the *Language Development Handbook* to help your students build mathematical language related to computations with decimals.

ELL You can use the tips and suggestions on page T16 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System

Additional Cluster(s): In this lesson, students address additional cluster **6.NS.B** by performing operations with multi-digit decimals.

Standards for Mathematical Content: **6.NS.B.3**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found quotients of multi-digit whole numbers.
6.NS.B.2

Now
Students perform operations on multi-digit decimals.
6.NS.B.3

Next
Students will divide whole numbers by fractions.
6.NS.A.1

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

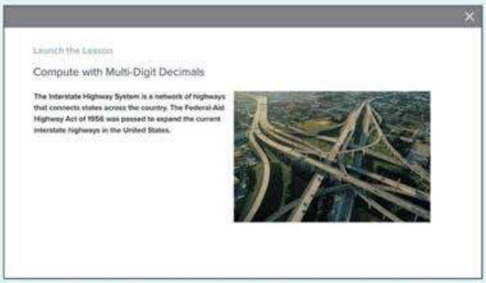
Conceptual Bridge In this lesson, students draw on their knowledge of whole number and decimal computation (gained in prior grades) to build *fluency* with adding, subtracting, multiplying, and dividing multi-digit decimals. They *apply* their understanding of computation with multi-digit decimals to solve real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation

Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- adding and subtracting multi-digit numbers (Exercises 1–4)
- subtracting multi-digit numbers (Exercise 5)

Answers

1. 3,785
2. 122,035
3. 71,346
4. 694
5. 29 ounces

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the cost of building the Interstate Highway System.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- An *annex* to a building is an addition or attachment to a main structure. How might you annex zeros to the number 20 without changing its value? Sample answer: Add a decimal and zeros after it, as in 20.00.

Your Notes

Example 1 Add Multi-Digit Decimals

Find 23.498 + 14.93. Check the solution.

Make an estimate. Round to the nearest whole number.

23.498 + 14.93 ≈ **23** + **15** or **38**

Find the sum.

  23.498
+ 14.930   Align the decimal points and annex a zero.
  38.428   Add. Place the decimal point in the sum.

So, 23.498 + 14.93 is **38.428**.

Check the solution.

Compare the solution to the estimate:

38.428 ≈ **38**   The solution is reasonable.

Talk About It!
Why is estimation useful when solving problems involving multi-digit decimals?

Sample answer: Estimation is used to ensure that the answer is reasonable.

Check

Find 356.725 + 142.4. **499. 125**

Show your work here

Go Online Y ou can complete an Extra Example online.

144 Module 3 • Compute with Multi-Digit Numbers and Fractions

Interactive Presentation

Example 1, Add Multi-Digit Decimals, Slide 1 of 3

CLICK

On Slide 1, students move through the steps to add the decimals.

TYPE

On Slide 1, students determine the sum.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Add Multi-Digit Decimals

Objective

Students will fluently add multi-digit decimals when the number of decimal places are not the same.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the vertical addition expression in order to know why and where the zero is annexed.

Questions for Mathematical Discourse

SLIDE 1

AL Why is it helpful to estimate first? Sample answer: If you estimate first, you can compare your actual answer to the estimate to make sure your answer is reasonable.

OL Why is annexing a zero helpful? Sample answer: Annexing a zero lets us add the decimals vertically with the same number of decimal places, keeping the digits that need to be added organized.

BL Suppose a classmate wrote the sum as 23.4980 + 14.93. What would you say to them about how they annexed the zero? Sample answer: By annexing a zero to 23.498, the numbers will still not have the same number of decimal places. It is more helpful to annex a zero to 14.93.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Subtract Multi-Digit Decimals

Objective

Students will fluently subtract multi-digit decimals when the number of decimal places are not the same.

MP Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the subtraction expression in order to determine where and how to annex zeros, so that the numbers have the same number of decimal places.

Questions for Mathematical Discourse

SLIDE 1

AL Explain how you can estimate the difference. Sample answer: 163.45 is close to 160, and 85.374 is close to 90. 160 − 90 = 70, so the difference should be close to 70.

OL How do you know where to annex a zero? Annex a zero to 163.45 because it has one less decimal place than 85.374.

OL How can you check your answer? Sample answer: Add 85.374 and 78.076 to make sure the sum is 163.45.

BL If 163.45 − 85.374 = 78.076, use reasoning to find 163.45 − 85.38. Explain. Sample answer: 85.38 is 0.006 more than 85.374, so I am subtracting 0.006 more from 163.45. The difference will be 0.006 less than 78.076, or 78.07.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Subtract Multi-Digit Decimals

Find 163.45 − 85.374. Check the solution.

Make an estimate. Round to the nearest ten.

163.45 − 85.374 ≈ **160** − **90** or **70**

Find the difference.

$$\begin{array}{r} 163.450 \\ -\ 85.374 \\ \hline 78.076 \end{array}$$

Align the decimal points and annex a zero.

Subtract. Place the decimal point in the difference.

So, 163.45 − 85.374 is **78.076**.

Check the solution.

Compare the solution to the estimate:

78.076 ≈ **70** The solution is reasonable.

Check

Find 356.18 − 142.257. **213.923**

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 2, Subtract Multi-Digit Decimals, Slide 1 of 2

CLICK

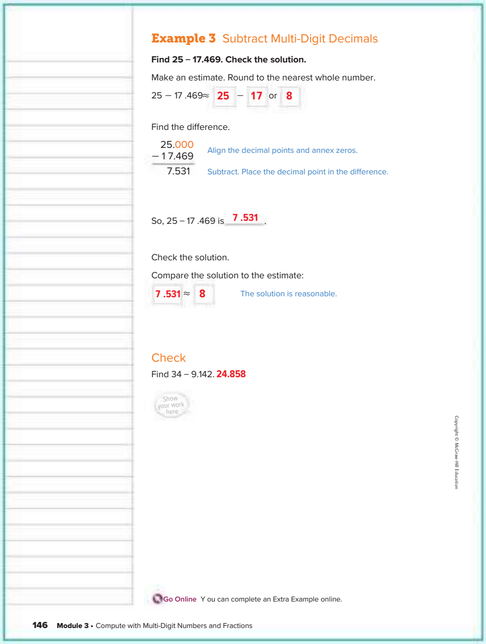
On Slide 1, students move through the steps to subtract the decimals.

TYPE

On Slide 1, students determine the difference.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Subtract Multi-Digit Decimals

Find 25 – 17.469. Check the solution.

Make an estimate. Round to the nearest whole number.

25 – 17.469 ≈ 25 – 17 or 8

Find the difference.

25.000 — Align the decimal points and annex zeros.
– 17.469
7.531 — Subtract. Place the decimal point in the difference.

So, 25 – 17.469 is 7.531.

Check the solution.

Compare the solution to the estimate:

7.531 ≈ 8 — The solution is reasonable.

Check

Find 34 – 9.142. 24.858

Show your work here

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Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3, Subtract Multi-Digit Decimals, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to subtract the decimals.

TYPE

On Slide 1, students determine the difference.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Subtract Multi-Digit Decimals

Objective

Students will fluently subtract multi-digit decimals when the number of decimal places are not the same.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the subtraction expression in order to determine where and how to annex zeros, so that the numbers have the same number of decimal places.

Questions for Mathematical Discourse

SLIDE 1

AL Explain how you can estimate the difference. Sample answer: 17.469 is close to 17. 25 – 17 = 8, so the difference should be close to 8.

OL How do you know where to annex zeros? Annex three zeros to 25 because it has three fewer decimal places than 17.469.

OL How can you check your answer? Sample answer: Add 7.531 and 17.469 to make sure the sum is 25.

BL If 25 – 17.469 = 7.531, use reasoning to find 25.02 – 17.469. Explain. Sample answer: 25.02 is 0.02 more than 25, so I am subtracting 17.469 from a greater number. The difference will be 0.02 greater than 7.531, or 7.551.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Multiply Decimals

Objective

Students will learn how to fluently multiply multi-digit decimals.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, encourage them to create a logical argument for why they need to align the decimal points when adding and subtracting, but not when multiplying.

Teaching Notes

SLIDE 1

Students will learn that the same strategy used to multiply whole numbers can be used when multiplying a decimal by a decimal. Students should note that to place the decimal point, they need to find the sum of the number of decimal places in each factor. The product has the same number of decimal places as the sum of the decimal places in each factor.

Talk About It!

SLIDE 2

Mathematical Discourse

When you add or subtract decimals, you need to align the decimal points. In multiplication, the decimal points are not aligned. Why don't you need to align the decimal points when multiplying? Sample answer: You add or subtract numbers in the same place-value position, so the decimal points are aligned. But you multiply each digit by every other digit, regardless of place-value position. So, the decimal points don't need to be aligned when multiplying.

Learn Multiply Decimals

When multiplying a decimal by a decimal, multiply as with whole numbers. To place the decimal point in the product, find the sum of the number of decimal places in each factor. The product has the same number of decimal places. If there are not enough decimal places in the product, annex zeros to the left of the first non-zero digit.

Find 0.014 × 3.7 .

```
  0.014   ← three decimal places
×   3.7   ← one decimal place
     98
+   420
 0.0518   ← Add. Then annex a zero to make four decimal places.
```

So, 0.014 × 3.7 is 0.0518.

Pause and Reflect

Are you ready to move on to the next Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

Record your observations here

See students' observations.

Talk About It!

When you add or subtract decimals, you need to align the decimal points. In multiplication, the decimal points are not aligned. Why don't you need to align the decimal points when multiplying?

Sample answer: You add or subtract numbers in the same place-value position, so the decimal points are aligned. But you multiply each digit by every other digit, regardless of place-value position. So, the decimal points don't need to be aligned when multiplying.

Interactive Presentation

Learn, Multiply Decimals, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to multiply the decimals.

Example 4 Multiply Multi-Digit Decimals

Find 0.067 × 1.42. Check your solution.

Make an estimate. Round to the nearest whole number.

0.067 × 1.42 ≈ 0 × 1 or 0

Find the product.

```
   0.067     Write the problem.
×   1.42     Multiply as with whole numbers.
    134
   268
+  67
 0.09514     Add. Then annex a zero to make five decimal places.
```

So, 0.067 × 1.42 is 0.09514.

Check the solution.

Compare the solution to the estimate:

0.09514 ≈ 0 The solution is reasonable.

Talk About It! Should the product of a number and 1.42 be larger or smaller than the original number? Explain your reasoning.

larger; Sample answer: Since 1.42 > 1.0, the product of any non-zero number and 1.42 will be larger than the original number.

Check

Find 14.7 × 11.361. **167 .0067**

Show your work here

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4, Multiply Multi-Digit Decimals, Slide 1 of 3

CLICK

On Slide 1, students move through the steps to multiply the decimals.

TYPE

On Slide 1, students determine the product

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Multiply Multi-Digit Decimals

Objective

Students will fluently multiply multi-digit decimals.

Teaching the Mathematical Practices

6 Attend to Precision Students should be able to calculate the product accurately and efficiently, and pay careful attention to placing the decimal point correctly in the product.

7 Look for and Make Use of Structure Encourage students to analyze the structure of each factor in order to determine the number of decimal places that need to be in the product.

Questions for Mathematical Discourse

SLIDE 1

AL Explain how you can estimate the product. Sample answer: 0.067 is close to 0, and any number multiplied by 0 is 0. So, the product should be a very small number close to zero.

OL Explain how to determine the number of decimal places there will be in the product. Sample answer: There are a total of 3 + 2, or 5 decimal places in the factors. So, the product will have 5 decimal places.

BL Find 0.067 × 1.42 × 6.1. Explain how you know the number of decimal places the product will have. 0.580354; Sample answer: There is a total of 3 + 2 + 1, or 6 decimal places in the factors. So, the product will have 6 decimal places.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Divide Decimals

Objective

Students will learn how to divide multi-digit decimals.

MP Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, they should use precise mathematical vocabulary, such as *power of ten* as they explain why the two expressions are equivalent.

7 Look for and Make Use of Structure Encourage students to analyze the structure of each expression, noting that 0.6 is a multiple of 0.006 and 12 is a multiple of 0.12.

Teaching Notes

SLIDE 1

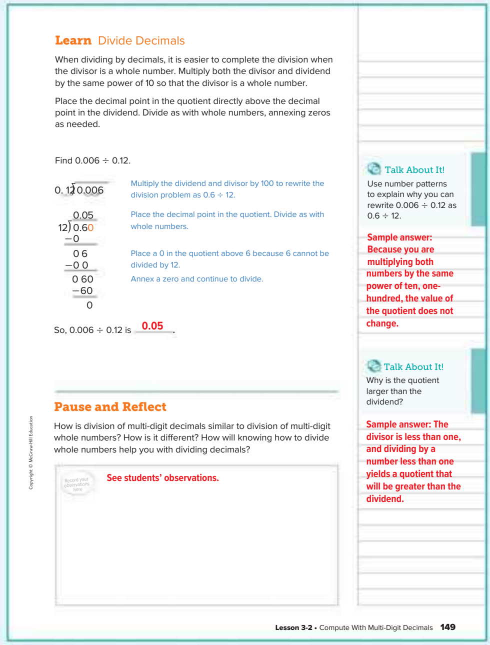
Present the division expression and ask students what they notice about the dividend and the divisor. Students should note that while neither of the numbers are whole numbers, they can rewrite the divisor 0.12 as 12 by multiplying 0.12 by a power of ten. Remind students that multiplying a number by a power of 10 has the same effect as moving the decimal point to the right, because the place-value system is based on powers of 10 (ones, tenths, hundredths, etc.). Be sure that students understand that if they multiply 0.12 by a power of ten, they must multiply 0.006 by the same power of ten.

Talk About It!

SLIDE 2

Mathematical Discourse

Use number patterns to explain why you can rewrite 0.006 ÷ 0.12 as 0.6 ÷ 12. Because you are multiplying both numbers by the same power of ten, one hundred, the value of the quotient does not change.



Learn Divide Decimals

When dividing by decimals, it is easier to complete the division when the divisor is a whole number. Multiply both the divisor and dividend by the same power of 10 so that the divisor is a whole number.

Place the decimal point in the quotient directly above the decimal point in the dividend. Divide as with whole numbers, annexing zeros as needed.

Find 0.006 ÷ 0.12.

0.12)0.006 — Multiply the dividend and divisor by 100 to rewrite the division problem as 0.6 ÷ 12.

12)0.60 with quotient 0.05 — Place the decimal point in the quotient. Divide as with whole numbers.

−0, 0 6, −0 0 — Place a 0 in the quotient above 6 because 6 cannot be divided by 12.

0 60, −60, 0 — Annex a zero and continue to divide.

So, 0.006 ÷ 0.12 is 0.05.

Talk About It! Use number patterns to explain why you can rewrite 0.006 ÷ 0.12 as 0.6 ÷ 12.

Sample answer: Because you are multiplying both numbers by the same power of ten, one-hundred, the value of the quotient does not change.

Talk About It! Why is the quotient larger than the dividend?

Sample answer: The divisor is less than one, and dividing by a number less than one yields a quotient that will be greater than the dividend.

Pause and Reflect

How is division of multi-digit decimals similar to division of multi-digit whole numbers? How is it different? How will knowing how to divide whole numbers help you with dividing decimals?

Record your observations here

See students' observations.

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Lesson 3-2 • Compute With Multi-Digit Decimals 149

Interactive Presentation

Learn, Divide Decimals, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to divide the decimals.

Example 5 Divide Multi-Digit Decimals

Find 60.927 ÷ 0.012.

Find the quotient.

$0.012\overline{)60.927}$ — Write the problem.

$0.012\overline{)60.927}$ — Multiply the dividend and divisor by 1,000 to eliminate the decimal point in the divisor.

```
    5077.25
12)60927.00
  −60
    09
   −0
    92
   −84
     87
    −84
      30
     −24
       60
      −60
        0
```

Place the decimal point in the quotient. Annex zeros and divide until there is a remainder of 0.

Place a zero in the quotient above 9 because 9 does not divide 12.

So, 60.927 ÷ 0.012 is 5,077.25.

Think About It! How will you set up the division? By what will you need to multiply both values to eliminate the decimal point in the divisor?

See students' responses.

Talk About It! Why is the quotient so much greater than the dividend?

Sample answer: When a number is divided by a lesser number, the quotient is always greater than the dividend.

Check

Find 2.943 ÷ 0.27. 10.9

Show your work here

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 5, Divide Multi-Digit Decimals, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to divide the decimals.

TYPE

On Slide 2, students determine the quotient.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Divide Multi-Digit Decimals

Objective

Students will fluently divide multi-digit decimals.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to compare the size of the quotient to the size of the dividend and divisor and ask themselves whether the extremely large quotient makes sense.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use the correct academic vocabulary (dividend, divisor, and quotient) in their explanations.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we multiply each number by 1,000? to eliminate the decimal point in the divisor

OL Compare the size of the quotient to the size of the dividend and divisor. What do you notice? Does this make sense? Sample answer: The divisor is an extremely small number. Dividing a quantity by an extremely small number means that the quotient will be very large. Since the quotient, 5,077.25, is very large, this makes sense.

BL Without calculating, explain how the quotient of 60.927 ÷ 0.00012 will compare to the quotient in this example. Sample answer: The quotient will be 100 times greater, or 507,725, because the new divisor is 100 times smaller.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Shopping

Objective

Students will come up with their own strategy to solve an application problem involving shopping at a farmer's market.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

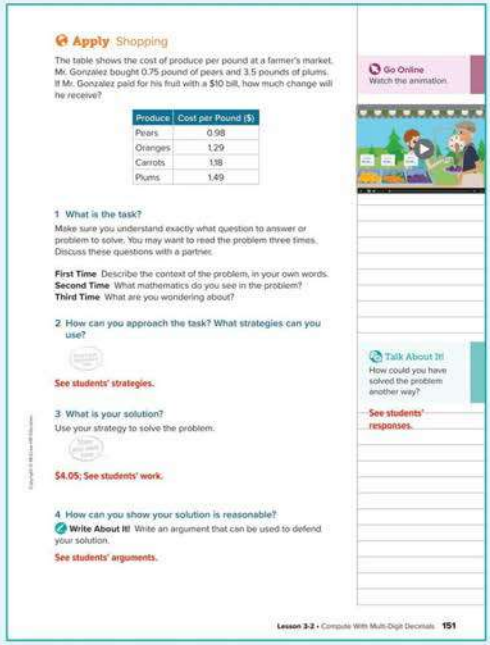
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What types of things can you buy at a farmer's market?
- How would you find the amount he spent on pears and plums?
- What information in the table isn't needed to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Shopping

The table shows the cost of produce per pound at a farmer's market. Mr. Gonzalez bought 0.75 pound of pears and 3.5 pounds of plums. If Mr. Gonzalez paid for his fruit with a $10 bill, how much change will he receive?

Produce	Cost per Pound ($)
Pears	0.98
Oranges	1.29
Carrots	1.18
Plums	1.49

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

$4.05; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It! How could you have solved the problem another way?

See students' responses.

Lesson 3-2 • Compute With Multi-Digit Decimals 151

Interactive Presentation

Apply, Shopping

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

There are two types of granola being sold at a local grocery store. Jerome wants to buy 1.5 pounds of cranberry granola for \$5.99 per pound and 0.9 pound of dark chocolate granola for \$7 .99 per pound. If Jerome pays for his granola with a \$20 bill, how much change will he receive? **\$3.82**

Show your work here

Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Where did you encounter difficulty in this lesson, and how did you deal with it? Write down any questions you still have.

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How are operations with fractions and decimals related to operations with whole numbers?

In this lesson, students practiced fluency in adding, subtracting, multiplying, and dividing decimals. Encourage them to work with a partner to compare and contrast these operations with operations with whole numbers. For example, have them compare and contrast how they would simplify each of the expressions 0.086×3.15 and 86×315.

Exit Ticket

Refer to the Exit Ticket slide. How many miles of road were built in the recent year as part of the interstate highway system? Write a mathematical argument that can be used to defend your solution.

$14{,}300 \times 3.28 = 46{,}904$; Sample answer: The number of miles of road built in the recent year is 3.28 times greater than the 14,300 miles of road built in 1962.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 13, 15–18
- Extension: Compute with Multi-Digit Decimals and Whole Numbers
- ALEKS Addition and Subtraction, Multiplication, Division

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 13, 15–18
- Extension: Compute with Multi-Digit Decimals and Whole Numbers
- Personal Tutor
- Extra Examples 1–5
- ALEKS Place Value and Ordering

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Place Value and Ordering

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Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	T opic	Exercises
1	fluently add multi-digit decimals	1, 2
1	fluently subtract multi-digit decimals	3, 4
1	fluently multiply multi-digit decimals	5, 6
1	fluently divide multi-digit decimals	7, 8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving multi-digit decimals	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

When adding or subtracting multi-digit decimals, students often line up the last decimal place of each number rather than lining up the decimal points. In Exercise 2, ask each student to read the place values as they add the two numbers together: "8 tenths plus 4 tenths" rather than simply "8 plus 4". This will help emphasize that the reason behind aligning the decimal points is to ensure that the numbers in the same place-value position are added together.

Name Period Date

Practice

Go Online Y ou can complete your homework online.

Find each sum. (Example 1)

1. 34.672 + 15.31 = **49.982**

2. 152.875 + 35.4 = **188.275**

Find each difference. (Examples 2 and 3)

3. 139.65 – 59.623 = **80.027**

4. 352.37 – 231.975 = **120.395**

Find each product. (Example 4)

5. 0.025 × 1.24 = **0.031**

6. 17 .15× 1.062 = **18.2133**

Find each quotient. (Example 5)

7. 32.674 ÷ 0.016 = **2,042. 125**

8. 3.825 ÷ 0.25 = **15.3**

9. The table shows the number of miles Roberto hiked each weekend. How many more miles did he hike on weekend two than on weekend one? **8.52 mi**

Weekend	Miles Hiked
One	21.48
Two	30

T est Practice

10. Equation Editor What is the value of the expression 2,965.7 + 5.8?

2971.5

Apply *indicates multi-step problem

*11. The table shows the cost per pound of food items you can buy in bulk at a grocery store. Mrs. Linden bought 1.25 pounds of dried fruit and 0.5 pound of cereal. If Mrs. Linden paid for her items with a $5 bill, how much change will she receive?

Item	Cost per Pound ($)
Beans	2.86
Cereal	2.38
Dried Fruit	1.84
Rice	0.52

$1.51

*12. Chloe is making hair bows to sell at a craft show. The table shows the cost per yard of different types of ribbon. Chloe bought 5.5 yards of satin ribbon and 3.8 yards of tulle. If Chloe paid with a $20 bill, how much change will she receive?

Ribbon	Cost per Y ard ($)
Chiffon	5.88
Satin	1.50
Lace	3.29
Tulle	2.25

$3.20

Higher-Order Thinking Problems

13. MP **Construct an Argument** Explain how you can mentally determine if the product of 5.5 and 0.95 is less than, greater than, or equal to 5.5?

Sample answer: Since the decimal 0.95 is less than 1, the product of 5.5 × 0.95 must be less than 5.5 × 1 or 5.5.

14. MP **Persevere with Problems** Brand A dish detergent costs $2.48 for a 21.6-ounce bottle. Brand B costs $1.55 for a 12.6-ounce bottle. Which brand costs less per ounce?

Brand A

15. Explain how you know that the sum of 26.541 and 14.2 will be greater than 40.

Sample answer: If you add the whole numbers, the sum is 40. The sum of the decimals will be added to 40 which will make the sum greater than 40.

16. MP **Find the Error** A student is multiplying 1.02 × 2.55. Find the student's mistake and correct it.

$$\begin{array}{r} 1.02 \\ \times\ 2.55 \\ \hline 510 \\ 5100 \\ +\ 20400 \\ \hline 260.10 \end{array}$$

Sample answer: The student placed the decimal point, as in addition. The student needs to count the total number of decimal places to the right of the decimal, which is 4. The correct answer is 2.6010 or 2.601.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students construct an argument for why a product must be less than 5.5 by comparing one of the factors to 1.

In Exercise 16, students explain why another student's solution is incorrect and then correct the solution.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 14, students make a plan for solving a problem involving unit costs for two different products and the division of multi-digit decimals.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.

Use with Exercise 11 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Make sense of the problem.

Use with Exercise 16 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that there are only two decimal places in the product. Have each pair or group of students present their explanations to the class.

Learn Reciprocals

Objective
Students will understand that multiplicative inverses, or reciprocals, are two numbers with a product of 1.

Teaching the Mathematical Practices
7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to analyze the structure of the fractions in order to compare the numerator and denominator of each fraction.

Go Online to find additional teaching notes.

Talk About It!
SLIDE 2

Mathematical Discourse
The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses or reciprocals. What are the similarities and differences between the two numbers?
Sample answer: Both fractions share the same values in their numerators and denominators, 2 and 3. They are different because their numerators and denominators are reversed.

Example 1 Find Reciprocals

Objective
Students will find the reciprocal of a unit fraction.

Teaching the Mathematical Practices
6 Attend to Precision Encourage students to know and use the mathematical terms *reciprocal*, *multiplicative inverse*, and *Inverse Property of Multiplication* in order to find the reciprocal of the unit fraction.

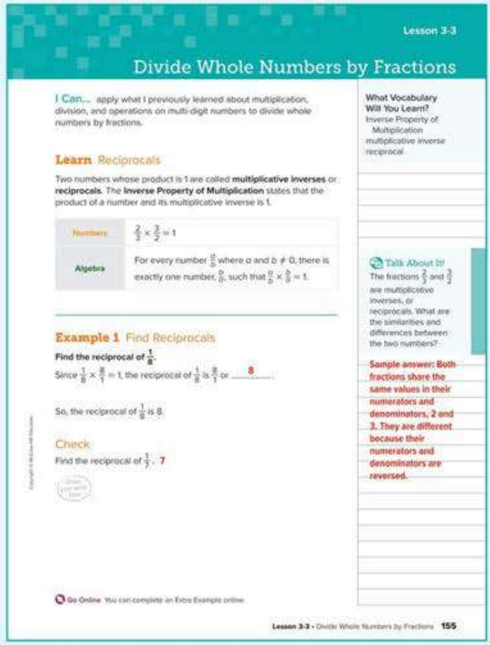
Questions for Mathematical Discourse
SLIDE 1

AL In your own words, what is a reciprocal? Sample answer: The reciprocal of a number is the number by which you need to multiply the original number to obtain a product of 1.

OL Use your knowledge of multiplying fractions to explain why the product of $\frac{1}{8}$ and $\frac{8}{1}$ is equal to 1. Sample answer: To multiply fractions, multiply the numerators and multiply the denominators. $1 \times 8 = 8$, and $8 \times 1 = 8$. The product is $\frac{8}{8}$, which simplifies to 1.

BL What is the reciprocal of $\frac{1}{80}$? 80? 80; $\frac{1}{80}$

Go Online
- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 3-3

Divide Whole Numbers by Fractions

I Can... apply what I previously learned about multiplication, division, and operations on multi-digit numbers to divide whole numbers by fractions.

What Vocabulary Will You Learn?
Inverse Property of Multiplication
multiplicative inverse
reciprocal

Learn Reciprocals

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. The **Inverse Property of Multiplication** states that the product of a number and its multiplicative inverse is 1.

Numbers	$\frac{2}{3} \times \frac{3}{2} = 1$
Algebra	For every number $\frac{a}{b}$ where a and $b \neq 0$, there is exactly one number, $\frac{b}{a}$, such that $\frac{a}{b} \times \frac{b}{a} = 1$.

Talk About It! The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses, or reciprocals. What are the similarities and differences between the two numbers?

Sample answer: Both fractions share the same values in their numerators and denominators, 2 and 3. They are different because their numerators and denominators are reversed.

Example 1 Find Reciprocals

Find the reciprocal of $\frac{1}{8}$.

Since $\frac{1}{8} \times \frac{8}{1} = 1$, the reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$ or 8.

So, the reciprocal of $\frac{1}{8}$ is 8.

Check

Find the reciprocal of $\frac{1}{7}$. 7

Go Online You can complete an Extra Example online.

Lesson 3-3 • Divide Whole Numbers by Fractions 155

Interactive Presentation

Learn, Find Reciprocals, Slide 1 of 2

FLASHCARDS On Slide 1 of the Learn, students use Flashcards to learn about reciprocals.

TYPE On Slide 1 of Example 1, students determine the reciprocal.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Lesson 3-3

Divide Whole Numbers by Fractions

6.NS.A.1

LESSON GOAL

Students will divide whole numbers by fractions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Reciprocals
Example 1: Find Reciprocals
Example 2: Find Reciprocals of Fractions
Example 3: Find Reciprocals of Whole Numbers

Explore: Divide Whole Numbers by Fractions

Learn: Divide Whole Numbers by Fractions
Example 4: Divide Whole Numbers by Fractions
Example 5: Divide Whole Numbers by Fractions
Apply: Cooking

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 17 of the *Language Development Handbook* to help your students build mathematical language related to division of whole numbers by fractions.

ELL You can use the tips and suggestions on page T17 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.A** by dividing whole numbers by fractions.
Standards for Mathematical Content: **6.NS.A.1**
Standards for Mathematical Practice: **MP1, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students performed operations on multi-digit decimals.
6.NS.B.3

Now
Students divide whole numbers by fractions.
6.NS.A.1

Next
Students will divide fractions by fractions.
6.NS.A.1

Rigor

The Three Pillars of Rigor

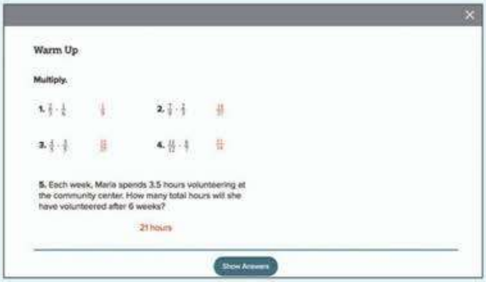
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of multiplicative inverses to build *fluency* with dividing whole numbers by fractions, using visual models and the standard algorithm. They *apply* their understanding of dividing whole numbers by fractions to solve real-world problems.

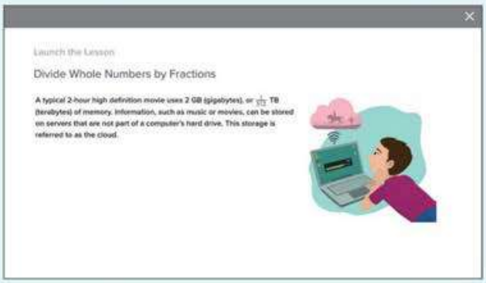
Mathematical Background

Two numbers whose product is 1 are called *multiplicative inverses* or *reciprocals*. The reciprocal of a fraction $\frac{a}{b}$, where $a \neq 0$ and $b \neq 0$, is $\frac{b}{a}$ because $\frac{a}{b} \times \frac{b}{a} = 1$. The reciprocal of a whole number a is $\frac{1}{a}$ because $a \times \frac{1}{a} = 1$. To divide a whole number by a fraction, multiply the whole number by the reciprocal of the fraction.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying fractions (Exercises 1–4)
- multiplying decimals and whole numbers (Exercise 5)

Answers

1. $\frac{1}{9}$
2. $\frac{14}{27}$
3. $\frac{12}{25}$
4. $\frac{11}{14}$
5. 21 hours

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about terabytes and gigabytes.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- How would you describe the word *inverse* using your own words? What is the inverse operation to multiplication? opposite direction or position; division
- Based on the meanings of the terms *inverse* and *multiplicative,* describe what you think a *multiplicative inverse* might be. Sample answer: Inverse means opposite and multiplicative refers to multiplication. A multiplicative inverse might be something that is related to the opposite of multiplication.
- What does the term *reciprocate* mean in everyday life? Sample answer: to return a favor, or respond to an action or gesture by making a corresponding one

Your Notes

Example 2 Find Reciprocals of Fractions

What number multiplied by $\frac{3}{4}$ has a product of 1?

$\frac{3}{4} \times \frac{4}{3} = 1$

So, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

Check

What number multiplied by $\frac{4}{7}$ has a product of 1? $\frac{7}{4}$

Show your work here

Example 3 Find Reciprocals of Whole Numbers

Find the reciprocal of 5.

The whole number 5 can be written as the fraction $\frac{5}{1}$.

Since $\frac{5}{1} \times \frac{1}{5} = 1$, the reciprocal is $\frac{1}{5}$.

So, the reciprocal of 5 is $\frac{1}{5}$.

Check

Find the reciprocal of 4. $\frac{1}{4}$

Show your work here

Talk About It!

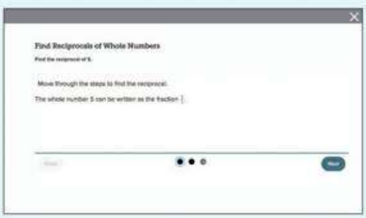
Can you write any whole number as a fraction? Explain.

yes; Sample answer: Any whole number can be written as a fraction with the whole number in the numerator and 1 in the denominator because any number divided by 1 is the number itself.

Go Online You can complete an Extra Example online.

156 Module 3 • Compute with Multi-Digit Numbers and Fractions

Interactive Presentation



Example 3, Find Reciprocals of Whole Numbers, Slide 1 of 3

TYPE

On Slide 1 of Example 2, students enter the missing value that gives a product of 1.

CLICK

On Slide 1 of Example 3, students move through the steps to find the reciprocal.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 2 Find Reciprocals of Fractions

Objective

Students will find the reciprocal of a fraction that is not a unit fraction.

Questions for Mathematical Discourse

SLIDE 1

AL Is this fraction a unit fraction? Explain. no; Sample answer: A unit fraction has 1 as the numerator. The numerator of this fraction is 3.

OL Explain why you know that the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$. Sample answer: The product of $\frac{3}{4}$ and $\frac{4}{3}$ is equal to 1, because $3 \times 4 = 12$, $4 \times 3 = 12$, and $\frac{12}{12}$ simplifies to 1.

BL What is the reciprocal of $\frac{31}{42}$? $\frac{42}{31}$

Example 3 Find Reciprocals of Whole Numbers

Objective

Students will find the reciprocal of a whole number.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the whole number, noting that any whole number can be written as a fraction with 1 in the denominator.

Questions for Mathematical Discourse

SLIDE 1

AL How can any whole number be written as a fraction? Any whole number can be written as a fraction by placing the whole number in the numerator and 1 in the denominator.

OL Your friend wrote 5 as a fraction as $\frac{1}{5}$. Explain the error. Sample answer: Instead of placing 5 in the numerator and 1 in the denominator, the friend reversed their positions. 5 and $\frac{1}{5}$ are not equivalent. The number 5 written as a fraction is $\frac{5}{1}$.

BL What is the reciprocal of the reciprocal of 6? Explain. 6; Sample answer: The reciprocal of 6 is $\frac{1}{6}$, and the reciprocal of $\frac{1}{6}$ is 6.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Divide Whole Numbers by Fractions

Objective

Students will understand that visual models and equations can be used to divide whole numbers by fractions.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language in their explanations.

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to analyze the structure of the visual model of the division equation $3 \div \frac{3}{4} = 4$ to make connections between the visual model and the equation.

Teaching Notes

SLIDE 1

As students move through each slide that illustrates how a model can be used to divide 3 by $\frac{3}{4}$, have them pause and reflect at each step. Ask them to explain how the model represents each step in the process. Some sample questions to help facilitate discussion are shown.

Why are there three bars drawn in the first step? The dividend is 3.

Why is each bar divided into fourths in the second step? The denominator of the divisor is 4.

Why is it important to identify how many groups of three-fourths there are? The divisor is *three fourths*. The number of groups of *three fourths* that are in the whole number *three* represents the quotient.

How does the bar diagram illustrate the quotient? The quotient is 4, the number of groups of *three fourths* there are in the whole number *three*.

Talk About It!

SLIDE 1

Mathematical Discourse

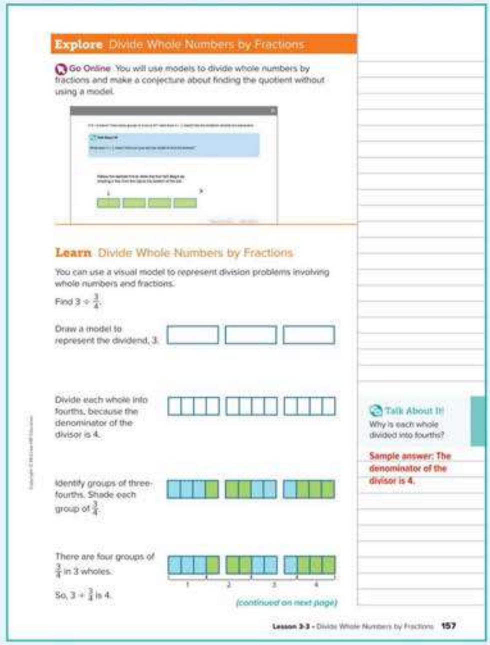
Why is each whole divided into fourths? Sample answer: The denominator of the divisor is 4.

(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of using visual models to divide whole numbers by fractions, have them work with a partner to generate at least 3 expressions that involve dividing a whole number by a fraction. Then have them create their own visual models that illustrate the division and help them find the quotient. Have them trade their visual models with another pair of students. Each pair should determine the division expression and quotient that is represented by the model. Have pairs of students discuss and resolve any differences.



Explore Divide Whole Numbers by Fractions

Go Online You will use models to divide whole numbers by fractions and make a conjecture about finding the quotient without using a model.

Learn Divide Whole Numbers by Fractions

You can use a visual model to represent division problems involving whole numbers and fractions.

Find $3 \div \frac{3}{4}$.

Draw a model to represent the dividend, 3.

Divide each whole into fourths, because the denominator of the divisor is 4.

Identify groups of three-fourths. Shade each group of $\frac{3}{4}$.

There are four groups of $\frac{3}{4}$ in 3 wholes.

So, $3 \div \frac{3}{4}$ is 4.

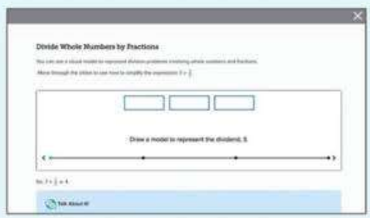
(continued on next page)

Talk About It! Why is each whole divided into fourths?

Sample answer: The denominator of the divisor is 4.

Lesson 3-3 • Divide Whole Numbers by Fractions 157

Interactive Presentation



Learn, Divide Whole Numbers by Fractions, Slide 1 of 3

CLICK

On Slide 1 of the Learn, students move through the slides to view how a model can represent the division.

Explore Divide Whole Numbers by Fractions

Objective

Students will explore how to use models to divide whole numbers by fractions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Building upon their understanding of division with whole numbers, and division with whole numbers and unit fractions from prior grades, students will explore division of whole numbers by unit fractions using bar diagrams. They will make a conjecture as to how to divide whole numbers by unit fractions without using a bar diagram.

Inquiry Question

How is dividing whole numbers by fractions similar to dividing whole numbers by whole numbers? Sample answer: With both, I need to find out how many groups of the divisor there are in the dividend.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

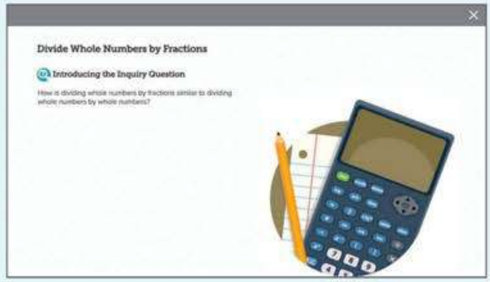
Talk About It!

SLIDE 3

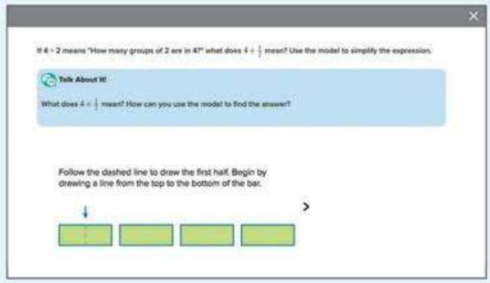
Mathematical Discourse

What does $4 \div \frac{1}{2}$ mean? How can you use the model to find the answer? Sample answer: It means how many groups of $\frac{1}{2}$ are in 4. I can divide each of the four bars in half, then count how many halves make up the four bars.

Interactive Presentation



Explore, Slide 1 of 7

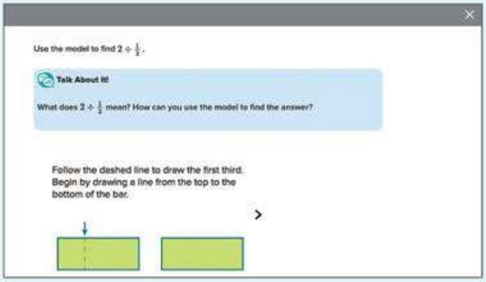


Explore, Slide 3 of 7

CLICK

On Slide 3, students separate all of the bars into halves.

Interactive Presentation



Explore, Slide 4 of 7

CLICK

On Slide 4, students separate the bar into thirds.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Divide Whole Numbers by Fractions *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students should be able to explain the benefit of using the interactive tool to separate the bar diagrams into halves or thirds; it can help them visualize the result and simplify the expression.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

What does $2 \div \frac{1}{3}$ mean? How can you use the model to find the answer?

Sample answer: It means how many groups of $\frac{1}{3}$ are in 2. I can divide each of the bars into thirds, then count how many thirds make up the two bars.

You can also use an equation to solve division problems involving whole numbers and fractions. Recall that multiplication and division are inverse operations, so you can divide a whole number by a fraction by multiplying the whole number by the reciprocal of the fraction.

$3 \div \frac{3}{4} = \square$ Write the equation.

$3 \div \frac{3}{4} = \frac{3}{1} \div \frac{3}{4}$ Write the whole number as a fraction.

$= \frac{3}{1} \times \frac{4}{3}$ Multiply by the reciprocal of $\frac{3}{4}$, $\frac{4}{3}$.

$= \frac{\cancel{3}^{1}}{1} \times \frac{4}{\cancel{3}_{1}}$ Divide by common factors.

$= \frac{1 \times 4}{1 \times 1}$ Simplify.

$= \frac{4}{1}$ or 4 Multiply.

So, $3 \div \frac{3}{4}$ is 4.

Talk About It!

Describe how the visual model supports the equation.

Sample answer: The bar diagram shows that when 3 is divided into groups of $\frac{3}{4}$ there are 4 groups of $\frac{3}{4}$. So, $3 \div \frac{3}{4} = 4$.

Pause and Reflect

Did you struggle with any of the concepts in this Learn? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

Record your observations here

See students' observations.

Interactive Presentation

Learn, Divide Whole Numbers by Fractions, Slide 2 of 3

CLICK

On Slide 2, students use an equation and their understanding of reciprocals to perform the division.

Learn Divide Whole Numbers by Fractions *(continued)*

Teaching Notes

SLIDE 2

Point out that visual models are not the only methods that can be used to divide a whole number by a fraction. By setting up an equation where the quotient is the unknown, students can use their understanding of reciprocals to find the quotient. Have students move through the steps for dividing the whole number by the fraction, being sure that they can clearly explain each step.

Talk About It!

SLIDE 3

Mathematical Discourse

Describe how the visual model supports the equation. Sample answer: The bar diagram shows that when 3 is divided into groups of $\frac{3}{4}$ there are 4 groups of $\frac{3}{4}$. So, $3 \div \frac{3}{4} = 4$.

Example 4 Divide Whole Numbers by Fractions

Objective

Students will divide whole numbers by fractions when the quotients are whole numbers.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically In Method 1, encourage students to draw a visual model to help them represent and simplify the division expression. In Method 2, students use an equation to find the quotient. As they discuss the *Talk About It* question on Slide 4, encourage them to analyze each method, note correspondences, and make an argument for which one might be more advantageous to use given different division problems.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we draw a model to represent 2? The whole number dividend is 2.

OL Why do we divide each whole into thirds? We need to see how many groups of $\frac{2}{3}$ there are in 2. Since the denominator of $\frac{2}{3}$ is 3, we divide each whole into thirds.

OL How many groups of two thirds are there in the diagram? Explain. 3; Sample answer: Each square represents one third, so group the squares by 2. Each group represents two thirds. There are three groups of two thirds.

BL Explain how to use a visual model to find $5 \div \frac{2}{3}$. Sample answer: Draw a model to represent 5 and divide each whole into thirds. There are 7 groups of $\frac{2}{3}$ and 1 section of $\frac{1}{3}$ is one half of two thirds. So, $5 \div \frac{2}{3} = 7\frac{1}{2}$

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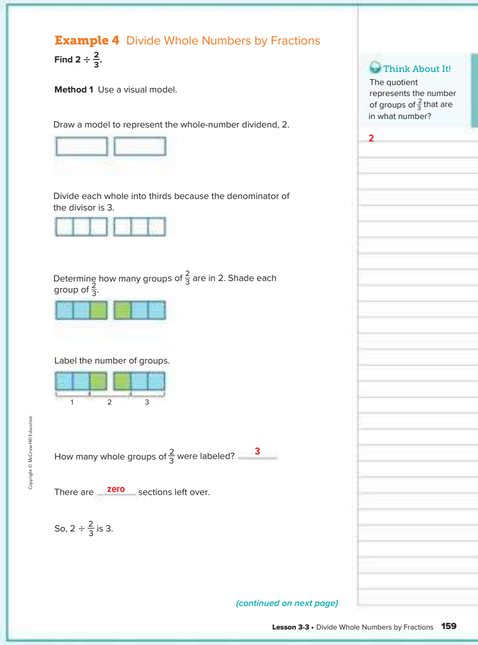

Example 4 Divide Whole Numbers by Fractions

Find $2 \div \frac{2}{3}$.

Method 1 Use a visual model.

Draw a model to represent the whole-number dividend, 2.

Divide each whole into thirds because the denominator of the divisor is 3.

Determine how many groups of $\frac{2}{3}$ are in 2. Shade each group of $\frac{2}{3}$.

Label the number of groups.

1 2 3

How many whole groups of $\frac{2}{3}$ were labeled? 3

There are zero sections left over.

So, $2 \div \frac{2}{3}$ is 3.

Think About It! The quotient represents the number of groups of $\frac{2}{3}$ that are in what number? 2

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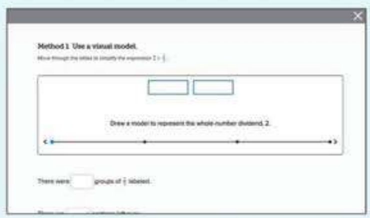
Lesson 3-3 • Divide Whole Numbers by Fractions 159

Interactive Presentation



Example 4, Divide Whole Numbers by Fractions, Slide 2 of 5

CLICK

On Slide 2, students view how a model can be used to divide (Method 1).

Method 2 Use an equation.

$2 \div \frac{2}{3} = \square$ Write the equation.

$2 \div \frac{2}{3} = \frac{2}{1} \div \frac{2}{3}$ Write the whole number as a fraction.

$= \frac{2}{1} \times \frac{3}{2}$ Multiply by the reciprocal of $\frac{2}{3}$, $\frac{3}{2}$.

$= \frac{\cancel{2}^{1}}{1} \times \frac{3}{\cancel{2}_{1}}$ Divide by common factors.

$= \frac{1 \times 3}{1 \times 1}$ Simplify.

$= 3$ Multiply.

So, $2 \div \frac{2}{3}$ is 3.

Talk About It!
Compare and contrast the two methods used to find $2 \div \frac{2}{3}$.

Sample answer: Using the visual model gives a visual explanation for the quotient. Both methods are valid approaches to finding the quotient.

Check
Find $4 \div \frac{2}{5}$. **10**

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 4, Divide Whole Numbers by Fractions, Slide 3 of 5

CLICK

On Slide 3, students view how an equation can be used to divide (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Divide Whole Numbers by Fractions *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why do we write the whole number as a fraction? so that we can multiply it by $\frac{3}{2}$.

OL Why do we multiply by the reciprocal? Dividing by a fraction is equivalent to multiplying by its reciprocal.

BL Use this method to find $5 \div \frac{2}{3}$.
$5 \div \frac{2}{3} = \frac{5}{1} \div \frac{2}{3} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2}$, or $7\frac{1}{2}$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example

DIFFERENTIATE

Reteaching Activity AL

Some students may have difficulty remembering the steps for using a reciprocal to divide a whole number by a fraction. Have students create a flowchart that lists all of the steps. The flowchart would include (1) write the whole number as a fraction, (2) rewrite the division problem as a multiplication problem by finding the reciprocal of the second fraction, (3) multiply the fractions by multiplying the numerators and multiplying the denominators, and (4) simplify the answer, if necessary. Have them use the flowchart to find the quotient of the following.

$5 \div \frac{2}{3}$ $7\frac{1}{2}$ $7 \div \frac{4}{5}$ $8\frac{3}{4}$ $2 \div \frac{3}{10}$ $6\frac{2}{3}$

Example 5 Divide Whole Numbers by Fractions

Objective

Students will divide whole numbers by fractions when the quotients are not whole numbers.

MP Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Method 2, encourage students to use an equation to find the quotient. Then have them compare the two methods.

As students discuss the *Talk About It!* question on Slide 5, encourage them to understand the benefits of each method and identify the correspondences between them.

5 Use Appropriate Tools Strategically In Method 1, encourage students to draw a visual model to help them represent and simplify the division expression.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we draw a model to represent 4 wholes? The whole number dividend is 4.

OL Why do we divide each whole into fourths? We need to see how many groups of $\frac{3}{4}$ there are in 4. Since the denominator of $\frac{3}{4}$ is 4, we divide each whole into fourths.

OL How many groups of three fourths are there in the diagram? Explain. $5\frac{1}{3}$; Sample answer: There are 5 whole groups of three fourths, and one section left over. The one section left over is one fourth of one whole, but one third of three fourths. So, there are $5\frac{1}{3}$ groups of three fourths in 4.

BL Suppose a classmate said the quotient was $5\frac{1}{4}$. Describe the error they made. Sample answer: They interpreted the one section left over as $\frac{1}{4}$. That one section is $\frac{1}{4}$ of one whole, but $\frac{1}{3}$ of three fourths.

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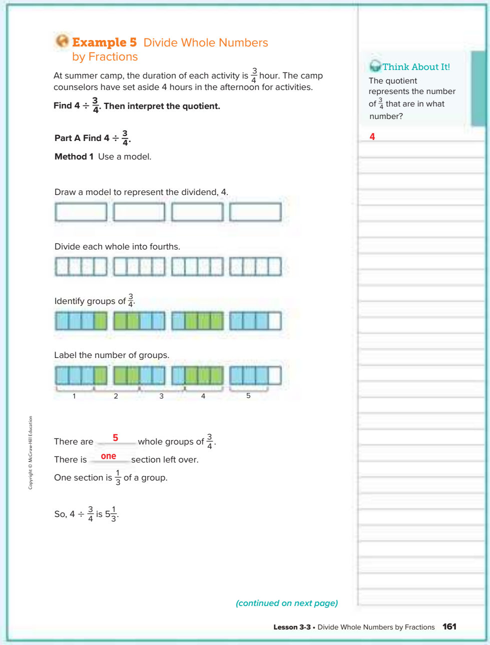

Example 5 Divide Whole Numbers by Fractions

At summer camp, the duration of each activity is $\frac{3}{4}$ hour. The camp counselors have set aside 4 hours in the afternoon for activities.

Find $4 \div \frac{3}{4}$. Then interpret the quotient.

Part A Find $4 \div \frac{3}{4}$.

Method 1 Use a model.

Draw a model to represent the dividend, 4.

Divide each whole into fourths.

Identify groups of $\frac{3}{4}$.

Label the number of groups.

There are 5 whole groups of $\frac{3}{4}$.

There is one section left over.

One section is $\frac{1}{3}$ of a group.

So, $4 \div \frac{3}{4}$ is $5\frac{1}{3}$.

Think About It! The quotient represents the number of $\frac{3}{4}$ that are in what number? 4

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Lesson 3-3 • Divide Whole Numbers by Fractions 161

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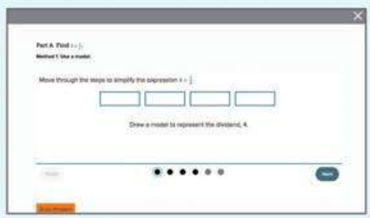
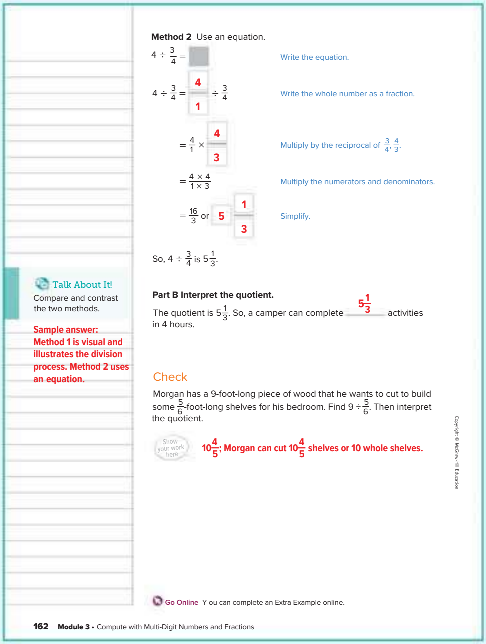
Interactive Presentation



Example 5, Divide Whole Numbers by Fractions, Slide 2 of 6

CLICK

On Slide 2, students view how a model can be used to divide (Method 1).



Method 2 Use an equation.

$4 \div \frac{3}{4} = \square$ Write the equation.

$4 \div \frac{3}{4} = \frac{4}{1} \div \frac{3}{4}$ Write the whole number as a fraction.

$= \frac{4}{1} \times \frac{4}{3}$ Multiply by the reciprocal of $\frac{3}{4}$, $\frac{4}{3}$.

$= \frac{4 \times 4}{1 \times 3}$ Multiply the numerators and denominators.

$= \frac{16}{3}$ or $5\frac{1}{3}$ Simplify.

So, $4 \div \frac{3}{4}$ is $5\frac{1}{3}$.

Talk About It!
Compare and contrast the two methods.

Sample answer: Method 1 is visual and illustrates the division process. Method 2 uses an equation.

Part B Interpret the quotient.

The quotient is $5\frac{1}{3}$. So, a camper can complete $5\frac{1}{3}$ activities in 4 hours.

Check

Morgan has a 9-foot-long piece of wood that he wants to cut to build some $\frac{5}{6}$-foot-long shelves for his bedroom. Find $9 \div \frac{5}{6}$. Then interpret the quotient.

Show your work here

$10\frac{4}{5}$; Morgan can cut $10\frac{4}{5}$ shelves or 10 whole shelves.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 5, Divide Whole Numbers by Fractions, Slide 3 of 6

CLICK

On Slide 3, students move through the slides to divide the whole number by the fraction (Method 2).

TYPE

On Slide 3, students determine the quotient.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Divide Whole Numbers by Fractions (*continued*)

Questions for Mathematical Discourse

SLIDE 3

AL Explain why the quotient will be greater than the dividend. Sample answer: Dividing a number by a number that is between 0 and 1 will result in a quotient that is greater than the original number.

OL How can you check to make sure that the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$? Sample answer: Find $\frac{4}{3} \times \frac{3}{4}$ to make sure that the product is equal to 1.

BL A classmate claims that $4 \div \frac{3}{4} = \frac{3}{16}$. Describe the error they made. The classmate found the reciprocal of the dividend, 4, and multiplied it by $\frac{3}{4}$.

SLIDE 4

AL What do the dividend 4 and the divisor $\frac{3}{4}$ represent within the context of the problem? The dividend 4 represents the total number of hours for activities, and the divisor $\frac{3}{4}$ represents the number of hours for each activity.

OL Why can you only complete part of the sixth activity? $5\frac{1}{3}$ means $5 + \frac{1}{3}$, so 5 whole activities can be completed plus $\frac{1}{3}$ of the sixth activity.

BL How many activities can be scheduled if each activity is $\frac{1}{2}$ hour? 8; $4 \div \frac{1}{2} = 8$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example

Apply Cooking

Objective

Students will come up with their own strategy to solve an application problem involving following a recipe.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

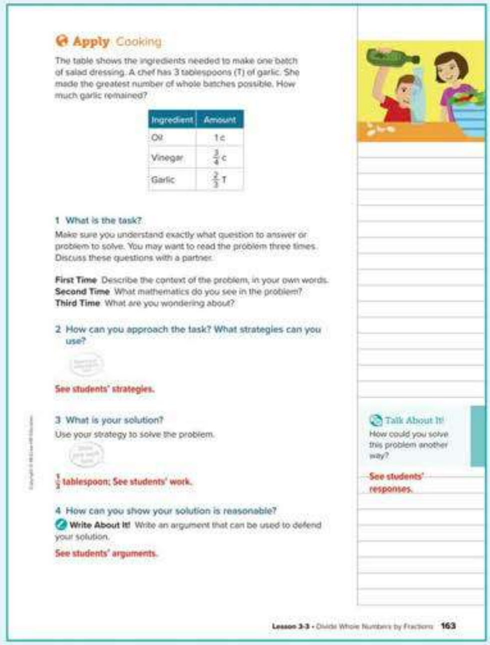
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What measurements are given in the table?
- What operation do you need to use to determine the number of batches the chef can make?
- What information in the table isn't needed to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Cooking

The table shows the ingredients needed to make one batch of salad dressing. A chef has 3 tablespoons (T) of garlic. She made the greatest number of whole batches possible. How much garlic remained?

Ingredient	Amount
Oil	1 c
Vinegar	$\frac{3}{4}$ c
Garlic	$\frac{2}{3}$ T

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$\frac{1}{3}$ tablespoon; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How could you solve this problem another way?

See students' responses.

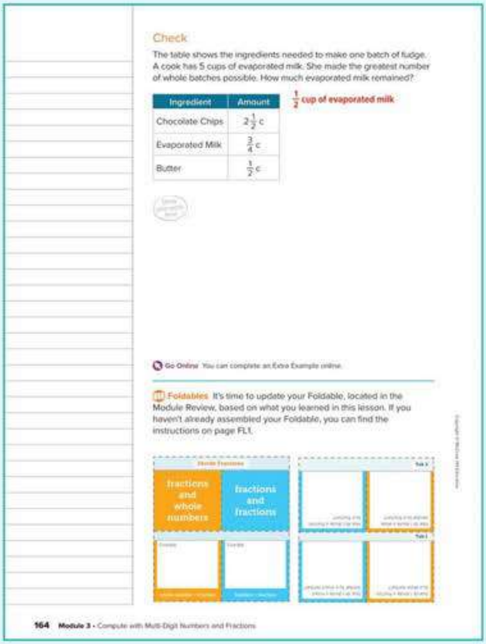
Lesson 3-3 • Divide Whole Numbers by Fractions 163

Interactive Presentation

Apply, Cooking

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows the ingredients needed to make one batch of fudge. A cook has 5 cups of evaporated milk. She made the greatest number of whole batches possible. How much evaporated milk remained?

Ingredient	Amount
Chocolate Chips	$2\frac{1}{2}$ c
Evaporated Milk	$\frac{3}{4}$ c
Butter	$\frac{1}{2}$ c

$\frac{1}{2}$ cup of evaporated milk

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

fractions and whole numbers

fractions and fractions

164 Module 3 • Compute with Multi-Digit Numbers and Fractions

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could write descriptions of the different methods used to divide a whole number by a fraction. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with fractions and decimals related to operations with whole numbers? In this lesson, students learned how to divide whole numbers by fractions using models and equations. Encourage them to work with a partner to compare and contrast dividing whole numbers by fractions with dividing whole numbers. For example, have them compare and contrast how they would simplify each of the expressions $6 \div \frac{3}{4}$ and $6 \div \frac{3}{1}$, or $6 \div 3$.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you knew you had 3 terabytes of cloud storage space. Find $3 \div \frac{1}{512}$ and interpret the quotient. 1,536; Sample answer: The quotient represents the number of typical 2-hour high definition movies that you could store in 3 terabytes of cloud storage space.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or above on the Checks,
THEN assign:

- Practice, Exercises 10, 12, 14–17
- ALEKS® Division with Fractions

OL

IF students score 66–89% on the Checks,
THEN assign:

- Practice, Exercises 1–9, 12, 14, 15
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Multiplication with Fractions

AL

IF students score 65% or below on the Checks,
THEN assign:

- ArriveMATH Take Another Look
- ALEKS® Multiplication with Fractions

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

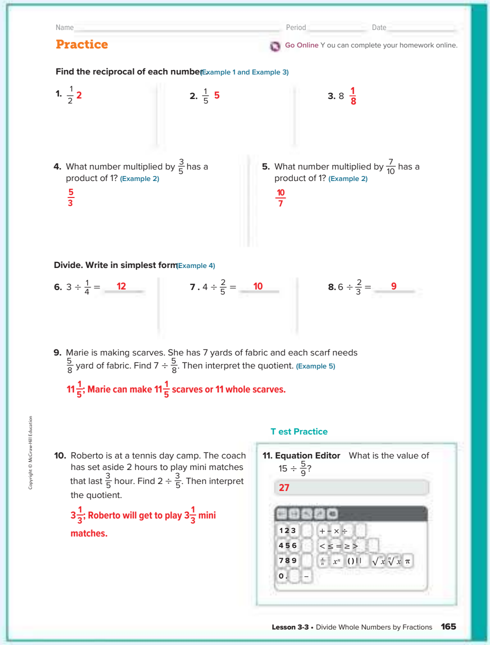
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find reciprocals of unit fractions	1, 2
1	find reciprocals of whole numbers	3
1	find reciprocals of fractions	4, 5
1	divide whole numbers by fractions	6–8
2	divide whole numbers by fractions when the quotients are not whole numbers	9
2	extend concepts learned in class to apply them in new contexts	10, 11
3	solve application problems involving the division of whole numbers by fractions	12, 13
3	higher-order and critical thinking skills	14–17

Common Misconception

When writing whole numbers as fractions, students might mistakenly write the whole number a as the fraction $\frac{a}{a}$ because it represents "one whole" instead of as the fraction $\frac{a}{1}$. Remind students that whole numbers are written with the whole number in the numerator and the number 1 in the denominator. If students continue to struggle with this concept, have them write a whole number as each type of fraction and then perform the division to show that the two fractions are not equivalent.


Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

Find the reciprocal of each number (Example 1 and Example 3)

1. $\frac{1}{2}$ **2**
2. $\frac{1}{5}$ **5**
3. 8 $\frac{1}{8}$

4. What number multiplied by $\frac{3}{5}$ has a product of 1? (Example 2) $\frac{5}{3}$
5. What number multiplied by $\frac{7}{10}$ has a product of 1? (Example 2) $\frac{10}{7}$

Divide. Write in simplest form (Example 4)

6. $3 \div \frac{1}{4} =$ **12**
7. $4 \div \frac{2}{5} =$ **10**
8. $6 \div \frac{2}{3} =$ **9**

9. Marie is making scarves. She has 7 yards of fabric and each scarf needs $\frac{5}{8}$ yard of fabric. Find $7 \div \frac{5}{8}$. Then interpret the quotient. (Example 5)
$11\frac{1}{5}$; Marie can make $11\frac{1}{5}$ scarves or 11 whole scarves.

T est Practice

10. Roberto is at a tennis day camp. The coach has set aside 2 hours to play mini matches that last $\frac{3}{5}$ hour. Find $2 \div \frac{3}{5}$. Then interpret the quotient.
$3\frac{1}{3}$; Roberto will get to play $3\frac{1}{3}$ mini matches.

11. **Equation Editor** What is the value of $15 \div \frac{5}{9}$?
27

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Lesson 3-3 • Divide Whole Numbers by Fractions 165

Apply *indicates multi-step problem

*12. The table shows the amount of each ingredient Jacob is using to make one pizza. If he has 11 cups of mozzarella cheese and makes the greatest number of whole pizzas possible, how much mozzarella cheese remains?

Ingredient	Amount
Mozzarella Cheese	$\frac{3}{4}$ c
Sauce	$\frac{1}{2}$ c

$\frac{1}{2}$ cup

*13. The table shows the ingredients for one batch of barbeque sauce. Anne has 9 cups of ketchup and makes the greatest number of whole batches of barbeque sauce possible. How much ketchup remains?

Ingredient	Amount
Brown Sugar	$\frac{1}{4}$ c
Cider Vinegar	$\frac{1}{2}$ c
Ground Cumin	1 tsp
Ketchup	$\frac{2}{3}$ c
Pepper	1 tsp

$\frac{1}{3}$ cup

Higher-Order Thinking Problems

14. **Find the Error** A student is solving $9 \div \frac{3}{4}$. Find the student's mistake and correct it.

$9 \div \frac{3}{4} = \frac{9}{1} \times \frac{3}{4}$

$= \frac{27}{4}$ or $6\frac{3}{4}$

Sample answer: The student did not multiply by the reciprocal of $\frac{3}{4}$ which is $\frac{4}{3}$. $\frac{9}{1} \times \frac{4}{3} = \frac{36}{3}$ or 12.

15. Zach has 20 sub sandwiches for a party. Each sub sandwich is going to be cut into thirds. Zach needs 55 sandwich pieces. Will he have enough sandwich pieces? Justify your answer.

yes; Sample answer: $20 \div \frac{1}{3} = \frac{20}{1} \times \frac{3}{1}$ or 60, which is greater than 55. So, Zach will have enough sandwich pieces.

16. **Persevere with Problems** In a $\frac{3}{4}$-mile relay race, each runner on one team runs $\frac{3}{16}$ mile. How many runners are on one team?

4 runners

17. Identify the whole number whose reciprocal has a decimal equivalent between 0.2 and 0.3. Explain.

4; Sample answer: The reciprocal of 4 is $\frac{1}{4}$, which is equal to 0.25 and $0.2 < 0.25 < 0.3$.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 16, students develop a plan for using the division of fractions by a whole number to solve a problem in which the whole number is missing rather than the quotient.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students explain why another student's solution is incorrect and then correct the solution.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercises 12–13 Have students work in groups of 3–4. After completing Exercise 12, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 13.

Clearly and precisely explain.

Use with Exercise 15 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as using fraction models or multiplying by the reciprocal in their responses.

Learn Divide Fractions by Fractions

Objective

Students will understand that they can use various strategies to divide fractions by fractions.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the first *Talk About It!* question on Slide 3, encourage them to make sense of the visual model on the previous slides, and how it can be used to divide the fractions.

6 Attend to Precision As students discuss the second *Talk About It!* question on Slide 3, remind them to draw a model with precision and use clear mathematical language when explaining the meaning of the reciprocal.

Teaching Notes

SLIDE 1

Students previously learned how to divide a whole number by a fraction, using both visual models and equations. Present the division expression $\frac{1}{2} \div \frac{1}{3}$, and ask students how this expression is different than dividing a whole number by a fraction. Students should note that both numbers are fractions. Have students move through the slides to see how a visual model can be used to help divide the two fractions. Ask them to explain each step in the process. Some sample questions to help facilitate discussion are shown.

In the first step, how does the model represent the dividend? The dividend is $\frac{1}{2}$ and the model has 1 out of 2 bars shaded to represent $\frac{1}{2}$.

In the second step, why is the model divided into thirds? The denominator of the divisor is 3.

Why is it important to identify how many groups of one-thirds there are in one half? The divisor is *one third*. The number of groups of *one thirds* that are in *one half* represents the quotient.

How does the bar diagram illustrate the quotient? There is 1 group of *one third* plus $\frac{1}{2}$ of another *one third* in the shaded section that represents *one half*. This means the quotient is $1 + \frac{1}{2}$, or $1\frac{1}{2}$.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how to divide a fraction by a fraction.

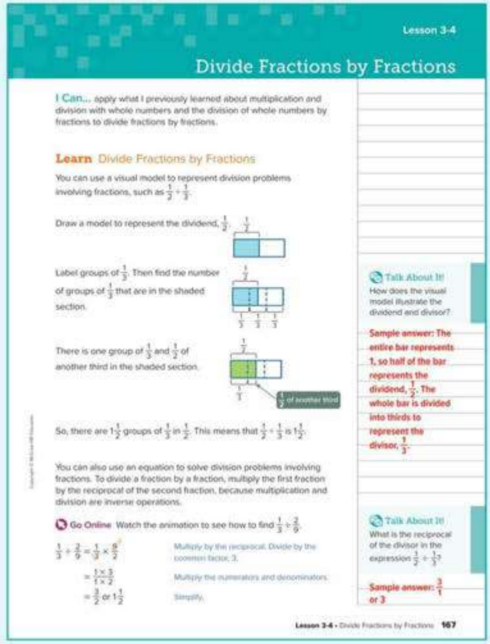
Talk About It!

SLIDE 3

Mathematical Discourse

How does the visual model illustrate the dividend and divisor? Sample answer: The entire bar represents 1, so half of the bar represents the dividend, $\frac{1}{2}$. The whole bar is divided into thirds to represent the divisor, $\frac{1}{3}$.

What is the reciprocal of the divisor in the expression $\frac{1}{2} \div \frac{1}{3}$? $\frac{3}{1}$ or 3



Lesson 3-4

Divide Fractions by Fractions

I Can... apply what I previously learned about multiplication and division with whole numbers and the division of whole numbers by fractions to divide fractions by fractions.

Learn Divide Fractions by Fractions

You can use a visual model to represent division problems involving fractions, such as $\frac{1}{2} \div \frac{1}{3}$.

Draw a model to represent the dividend, $\frac{1}{2}$.

Label groups of $\frac{1}{3}$. Then find the number of groups of $\frac{1}{3}$ that are in the shaded section.

There is one group of $\frac{1}{3}$ and $\frac{1}{2}$ of another third in the shaded section.

So, there are $1\frac{1}{2}$ groups of $\frac{1}{3}$ in $\frac{1}{2}$. This means that $\frac{1}{2} \div \frac{1}{3}$ is $1\frac{1}{2}$.

You can also use an equation to solve division problems involving fractions. To divide a fraction by a fraction, multiply the first fraction by the reciprocal of the second fraction, because multiplication and division are inverse operations.

Go Online Watch the animation to see how to find $\frac{1}{3} \div \frac{2}{9}$.

$\frac{1}{3} \div \frac{2}{9} = \frac{1}{3} \times \frac{9}{2}$ Multiply by the reciprocal. Divide by the common factor, 3.

$= \frac{1 \times 3}{1 \times 2}$ Multiply the numerators and denominators.

$= \frac{3}{2}$ or $1\frac{1}{2}$ Simplify.

Talk About It! How does the visual model illustrate the dividend and divisor?

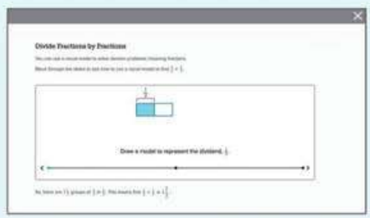
Sample answer: The entire bar represents 1, so half of the bar represents the dividend, $\frac{1}{2}$. The whole bar is divided into thirds to represent the divisor, $\frac{1}{3}$.

Talk About It! What is the reciprocal of the divisor in the expression $\frac{1}{2} \div \frac{1}{3}$?

Sample answer: $\frac{3}{1}$ or 3

Lesson 3-4 • Divide Fractions by Fractions 167

Interactive Presentation



Learn, Divide Fractions by Fractions, Slide 1 of 3

CLICK

On Slide 1, students move through the steps to use a model to find the quotient of two fractions.

WATCH

On Slide 2, students watch an animation that explains how to use reciprocals to find the quotient of two fractions.

Lesson 3-4

Divide Fractions by Fractions

6.NS.A.1

LESSON GOAL

Students will divide fractions by fractions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Divide Fractions by Fractions
Example 1: Divide Fractions by Fractions
Example 2: Find and Interpret Quotients
Learn: Write Story Contexts
Example 3: Write Story Contexts
Apply: Food

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

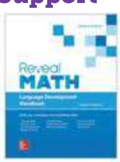
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Use Mental Math to Solve Problems with Division of Fractions		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 18 of the *Language Development Handbook* to help your students build mathematical language related to division of fractions.



ELL You can use the tips and suggestions on page T18 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.A** by dividing fractions by fractions.
Standards for Mathematical Content: **6.NS.A.1**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students divided whole numbers by fractions.
6.NS.A.1

Now
Students divide fractions by fractions.
6.NS.A.1

Next
Students will divide with whole and mixed numbers.
6.NS.A.1

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

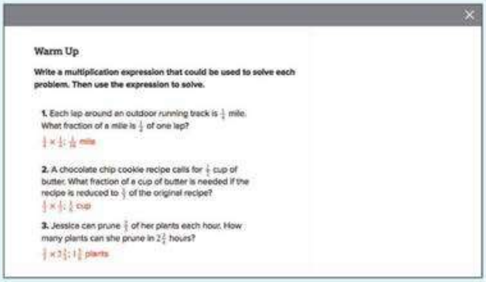
Conceptual Bridge In this lesson, students continue to develop *understanding* of division by fractions. They use visual models and the standard algorithm to build *fluency* with dividing fractions by fractions. They *apply* their understanding to write and solve real-world story contexts.

Mathematical Background

To divide $\frac{a}{b}$ by $\frac{c}{d}$, multiply $\frac{a}{b}$ by the reciprocal of $\frac{c}{d}$.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, given $b, c, d \neq 0$

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- multiply fractions (Exercises 1–3)

Answers

1. $\frac{1}{4} \times \frac{1}{4}$; $\frac{1}{16}$ mile
2. $\frac{1}{2} \times \frac{1}{3}$; $\frac{1}{6}$ cup
3. $\frac{2}{3} \times 2\frac{3}{4}$; $1\frac{5}{6}$ plants

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the use of fractions in carpentry.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

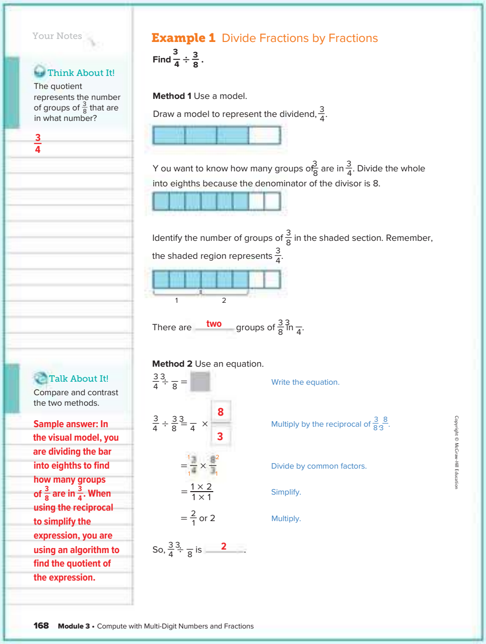
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What is the *quotient* of 6 divided by 3? 2
- How can you describe a *reciprocal*? Sample answer: Two numbers are reciprocals if they have a product of 1.



Your Notes

Think About It!
The quotient represents the number of groups of $\frac{3}{8}$ that are in what number?

$\frac{3}{4}$

Talk About It!
Compare and contrast the two methods.

Sample answer: In the visual model, you are dividing the bar into eighths to find how many groups of $\frac{3}{8}$ are in $\frac{3}{4}$. When using the reciprocal to simplify the expression, you are using an algorithm to find the quotient of the expression.

Example 1 Divide Fractions by Fractions

Find $\frac{3}{4} \div \frac{3}{8}$.

Method 1 Use a model.

Draw a model to represent the dividend, $\frac{3}{4}$.

You want to know how many groups of $\frac{3}{8}$ are in $\frac{3}{4}$. Divide the whole into eighths because the denominator of the divisor is 8.

Identify the number of groups of $\frac{3}{8}$ in the shaded section. Remember, the shaded region represents $\frac{3}{4}$.

1 2

There are two groups of $\frac{3}{8}$ in $\frac{3}{4}$.

Method 2 Use an equation.

$\frac{3}{4} \div \frac{3}{8} =$ ☐ Write the equation.

$\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$ Multiply by the reciprocal of $\frac{3}{8}$, $\frac{8}{3}$.

$= \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}}$ Divide by common factors.

$= \frac{1 \times 2}{1 \times 1}$ Simplify.

$= \frac{2}{1}$ or 2 Multiply.

So, $\frac{3}{4} \div \frac{3}{8}$ is 2.

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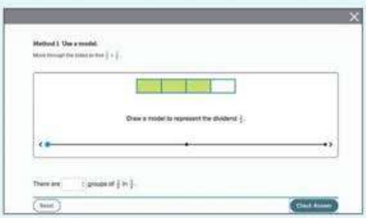
168 **Module 3** • Compute with Multi-Digit Numbers and Fractions

Interactive Presentation



Example 1, Divide Fractions by Fractions, Slide 2 of 5

CLICK

On Slide 2, students use a model to divide the fractions (Method 1).

TYPE

On Slide 3, students use an equation to divide the fractions (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Divide Fractions by Fractions

Objective

Students will divide fractions by fractions.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
As students discuss the *Talk About It* question on Slide 4, encourage them to analyze each method, note correspondences, and make an argument for which one might be more advantageous to use given different division problems.

5 Use Appropriate Tools Strategically In Method 1, encourage students to draw a visual model to help them find the quotient. In Method 2, encourage students to use an expression to find the quotient, and compare the two methods. They should see that both methods are valid approaches to finding the quotient.

Questions for Mathematical Discourse

SLIDE 2

AL Why was a model drawn to represent $\frac{3}{4}$? The model is drawn to represent $\frac{3}{4}$ because $\frac{3}{4}$ is the dividend.

OL Why do we divide the whole into eighths? The divisor is $\frac{3}{8}$, and the denominator of the divisor is 8.

OL How many groups of $\frac{3}{8}$ are in the shaded section? Why do we not count the two sections left over? Explain. 2 groups; Sample answer: The shaded section represents the dividend, so we do not want to count the two sections left over that are not part of the dividend.

BL What would the dividend need to be for the quotient to be $2\frac{2}{3}$? Explain. 1; Sample answer: If the dividend was one whole, then we could count the two sections left over as part of the quotient. The quotient would represent 2 whole groups of $\frac{3}{8}$ plus two-thirds of another group.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find and Interpret Quotients

Objective

Students will divide fractions by fractions and interpret the quotients.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should be able to interpret the quotient within the context of the problem, noting that, since Asahi wants to deliver whole batches of cookies, he cannot make $3\frac{1}{3}$ batches of cookies.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to clearly explain what the quotient represents within the context of the problem, and what the problem is actually asking for.

Questions for Mathematical Discourse

SLIDE 2

AL Explain, in your own words, why the expression $\frac{5}{6} \div \frac{1}{4}$ represents the problem. Sample answer: Since there is $\frac{5}{6}$ pound of sugar, I need to divide that by the amount of sugar required in each batch, $\frac{1}{4}$, to find the number of batches Asahi can make.

OL After finding the quotient, have you completed everything you needed to do for this example? Explain. Sample answer: No, I need to interpret the quotient in terms of the context of the problem.

BL How many more batches could Asahi make if each batch required $\frac{1}{8}$ pound of sugar? Explain. 3 more batches; Sample answer: $\frac{1}{8}$ is half of $\frac{1}{4}$, so Asahi can make twice as many batches (6 batches total) if he only needs half as much sugar for each batch.

SLIDE 3

AL What is the quotient? $3\frac{1}{3}$

OL Can Asahi actually make $3\frac{1}{3}$ batches of cookies? Explain. yes; Sample answer: He can make $3\frac{1}{3}$ batches of cookies, but since he wants to deliver whole batches of cookies, he will not make any partial batches.

BL If the quotient was $3\frac{2}{3}$ would we round up to say that Asahi can make 4 batches of cookies? Explain. no; Sample answer: As long as the quotient is less than 4, Asahi cannot actually make 4 batches of cookies.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

Find $\frac{7}{9} \div \frac{2}{3}$. $\frac{7}{6}$ or $1\frac{1}{6}$

Show your work here

Go Online You can complete an Extra Example online.

Example 2 Find and Interpret Quotients

Asahi is making cookies. There is $\frac{5}{6}$ pound of sugar left in the canister. Each batch of cookies requires $\frac{1}{4}$ pound of sugar. He wants to deliver one batch to each of his neighbors. How many neighbors will receive cookies?

Write and solve an equation that models the situation. Then interpret the quotient.

Part A Write and solve an equation.

The expression $\frac{5}{6} \div \frac{1}{4}$ represents the number of batches he can make, since Asahi has $\frac{5}{6}$ pound of sugar left, and each batch of cookies requires $\frac{1}{4}$ pound of sugar.

$\frac{5}{6} \div \frac{1}{4} = \square$ Write the equation.

$\frac{5}{6} \div \frac{1}{4} = \frac{5}{6} \times \frac{4}{1}$ Multiply by the reciprocal of $\frac{1}{4}$, $\frac{4}{1}$.

$= \frac{5}{\cancel{6}_3} \times \frac{\cancel{4}^2}{1}$ Divide by common factors.

$= \frac{5 \times 2}{3 \times 1}$ Simplify.

$= \frac{10}{3}$ Multiply.

So, $\frac{5}{6} \div \frac{1}{4}$ is $3\frac{1}{3}$

Part B Interpret the quotient.

Because Asahi wants to deliver whole batches of cookies, he is only able to make __3__ batches of cookies.

Think About It! What is the divisor? What is the dividend?

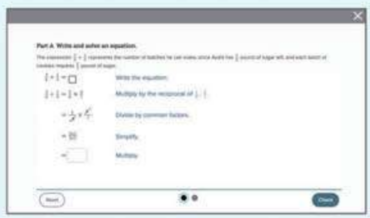
$\frac{1}{4}$; $\frac{5}{6}$

Talk About It! Why do the quotient and the solution of the word problem differ?

Sample answer: The quotient shows the expression $\frac{5}{6} \div \frac{1}{4}$ simplified, but the solution of the word problem is how many whole batches of cookies Asahi will deliver.

Lesson 3-4 • Divide Fractions by Fractions 169

Interactive Presentation



Example 2, Find and Interpret Quotients, Slide 2 of 5

TYPE

On Slide 2, students determine the quotient.

TYPE

On Slide 3, students interpret the quotient within the context of the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Check

Jasmine is mixing paint colors. She has $\frac{3}{4}$ gallon of blue paint. She needs $\frac{1}{6}$ gallon for each new color that she is mixing. Write and solve an equation that models the situation. Then interpret the quotient.

Part A

Write and solve an equation.

$\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$

Part B

Interpret the quotient. **Jasmine can make 4 new colors.**

Show your work here

Go Online You can complete an Extra Example online.

Learn Write Story Contexts

You can write a story context, or word problem, to represent any division problem. You can then solve the problem using a model or equation.

For the expression $\frac{4}{5} \div \frac{1}{10}$, you can write a story context by describing each piece of the division problem.

Write the dividend and divisor into the correct location in the story context.

Navid is hanging pictures in his room and has $\frac{4}{5}$ foot of tape to use. He uses $\frac{1}{10}$ foot of tape to hang each photo. How many photos can he hang on the wall?

Interactive Presentation

Learn, Write Story Contexts

CLICK

Students view how a story context can represent a division expression.

Learn Write Story Contexts

Objective

Students will understand how a story context can be written to represent an expression involving the division of fractions.

Teaching Notes

SLIDE 1

Present the division expression and the story context. Prior to selecting the *Dividend* and *Divisor* buttons, have students make a conjecture as to how the story context represents the division expression. Encourage students to make the connection between the dividend, divisor, and unknown quotient and how they are represented in the real-world scenario. You may wish to have students generate other story contexts that can represent this division expression.

DIFFERENTIATE

Language Development Activity ELL

Some students may struggle with writing story contexts for a division problem, because they have difficulty understanding what division means. One way to think of division is to find the number of objects in each equal-size group, when the total is known, and the number of groups is known. Another way to think of division is to find the number of groups, when the total is known and the number of objects in each group is known. For the following division problem, have students work with a partner to write a story context that represents the problem and explain what each quantity represents.

$3\frac{1}{4} \div \frac{1}{2}$ Sample answer: A recipe calls for $\frac{1}{2}$ cup of flour. Kenneth has $3\frac{1}{4}$ cups of flour. How many batches of the recipe can he make? The number of batches, the quotient, represents the number of groups. The number of cups of flour in each batch represents the number of objects in each group.

Example 3 Write Story Contexts

Objective

Students will write a story context for a problem involving division of fractions.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to contextualize the division expression by applying it to many different real-world contexts.

As students discuss the *Talk About It!* question on Slide 4, encourage them to use the context of the same word problem that they chose earlier in the example to explain what the division expression would mean within that context.

Questions for Mathematical Discourse

SLIDE 2

AL In the expression $\frac{2}{3} \div \frac{1}{6}$, identify the dividend and divisor. The dividend is $\frac{2}{3}$ and the divisor is $\frac{1}{6}$.

OL If $\frac{2}{3}$ is the dividend and $\frac{1}{6}$ is the divisor, what might this mean when writing a context? Sample answer: This means that $\frac{2}{3}$ will represent the beginning amount that we start within a story context, and $\frac{1}{6}$ will represent how many groups of that dividend we will have.

BL Write a different story context that is not included in this example. Then solve the problem. Sample answer: Mimi has $\frac{2}{3}$ yard of fabric. Each craft she makes uses $\frac{1}{6}$ yard of fabric. How many crafts can she make? She can make 4 crafts.

SLIDE 3

AL Why is 6 the reciprocal of $\frac{1}{6}$? $\frac{1}{6} \times \frac{6}{1} = 1$

OL Why is it helpful to keep $\frac{6}{1}$ written as a fraction, and not write it as a whole number? Sample answer: I can keep myself organized and not incorrectly multiply.

BL How might a classmate obtain the incorrect quotient 9? Sample answer: The classmate may have multiplied the fractions, and then found the reciprocal, instead of multiplying by the reciprocal of $\frac{1}{6}$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Write Story Contexts

Write a story context for $\frac{2}{3} \div \frac{1}{6}$. Then find the quotient.

Part A Write a story context.

T o write a story context for the division expression, consider the following situation.

Mimi is very active. She loves to cook, has a couple of hobbies, and has tasks around the house. Choose one the activities shown. Then write a story context using your choice.

cooking dinner | doing laundry painting
making pasta | feeding birds | swimming

Sample answer: Mimi has $\frac{2}{3}$ pound of sunflower seeds. Each day, she feeds the cardinals in her yard $\frac{1}{6}$ pound of seeds. How many days will she be able to feed the birds?

Think About It! How would you begin writing the problem?

See students' responses.

Part B Solve.

$\frac{2}{3} \div \frac{1}{6} = \square$ Write the equation.

$\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1}$ Multiply by the reciprocal of $\frac{1}{6}$, $\frac{6}{1}$.

$= \frac{2}{\cancel{3}_1} \times \frac{\cancel{6}^2}{1}$ Divide by common factors.

$= \frac{2 \times 2}{1 \times 1}$ Simplify.

$= 4$ Multiply.

So, $\frac{2}{3} \div \frac{1}{6}$ is 4.

Talk About It! If $\frac{2}{3} \div \frac{1}{6} = 4$, what does this mean in the context of the same word problem that you chose?

See students' responses.

Lesson 3-4 • Divide Fractions by Fractions 171

Interactive Presentation

Example 3, Write Story Contexts, Slide 2 of 5

CLICK On Slide 2, students choose a story context from several possibilities.

TYPE On Slide 3, students divide the fractions to solve the problem.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

Write a story context for $\frac{5}{6} \div \frac{1}{12}$. Then find the quotient.

Show your work here

Sample answer: Marin has $\frac{5}{6}$ pound of potpourri. She wants to divide the potpourri into bags that each contain $\frac{1}{12}$ pound. How many bags will she be able to make?; 10 bags

Go Online You can complete an Extra Example online.

Pause and Reflect

When dividing with fractions, explain why you can multiply the dividend by the reciprocal of the divisor to find the quotient. Can this method be used to divide two whole numbers? Explain your reasoning.

Record your observations here

See students' observations.

DIFFERENTIATE

Language Development Activity ELL

To further students' understanding of writing story contexts that represent division of fractions, have them work with a partner to refer to the Check Exercise that accompanies Example 3, and create arguments for why the story contexts in answer choices A and B do not represent the division expression $\frac{5}{6} \div \frac{1}{12}$. Have them present their arguments to another pair of students, or to the entire class. Some students may be uncomfortable speaking in front of others. Encourage them to make appropriate eye contact, and articulate their thoughts clearly and loudly enough for others to hear.

Apply Food

Objective

Students will come up with their own strategy to solve an application problem involving making snack bags of different kinds of nuts.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Which types of nuts are in the snack bags?
- Do you need the information about the almonds and the cashews?
- What operation is used to find the number of whole servings of walnuts that are in each bag?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Food

Alfonso is making snack bags with different types of nuts as shown in the table. Each snack bag contains $\frac{1}{8}$ pound of one type of nut. How many more whole servings of walnuts can he make than peanuts?

Type of Nut	Weight (lb)
Almonds	$\frac{1}{2}$
Cashews	$\frac{1}{4}$
Peanuts	$\frac{2}{5}$
Walnuts	$\frac{3}{4}$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

3 more servings; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Why is the solution 3 more servings of walnuts instead of $2\frac{4}{5}$ more servings?

Sample answer: The question asks for the number of whole servings. Since there are 3 whole servings of peanuts, you would subtract 3, not $3\frac{1}{5}$ from 6 servings of walnuts.

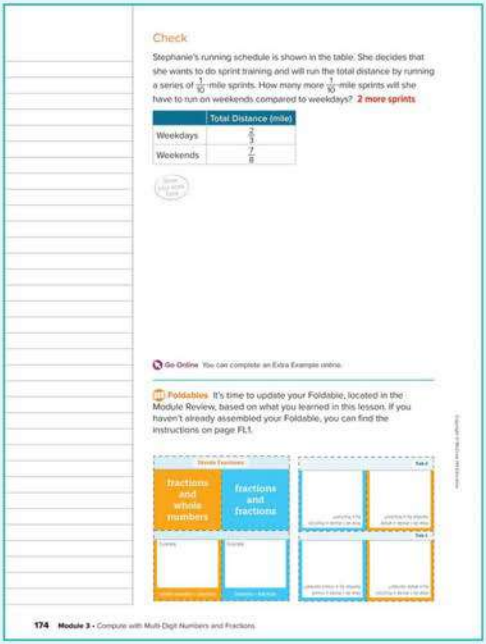
Lesson 3-4 • Divide Fractions by Fractions 173

Interactive Presentation

Apply, Food

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Stephanie's running schedule is shown in the table. She decides that she wants to do sprint training and will run the total distance by running a series of $\frac{1}{10}$-mile sprints. How many more $\frac{1}{10}$-mile sprints will she have to run on weekends compared to weekdays? 2 more sprints

	Total Distance (mile)
Weekdays	$\frac{2}{3}$
Weekends	$\frac{7}{8}$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

174 Module 3 • Compute with Multi-Digit Numbers and Fractions

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could write descriptions of the different methods used to divide a fraction by another fraction. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with fractions and decimals related to operations with whole numbers? In this lesson, students learned how to divide fractions by fractions using models and equations. Encourage them to work with a partner to compare and contrast dividing fractions with dividing whole numbers. For example, have them compare and contrast how they can use models to simplify each of the expressions $\frac{7}{8} \div \frac{1}{2}$, $\frac{1}{2} \div \frac{7}{8}$, and $8 \div 2$.

Exit Ticket

Refer to the Exit Ticket slide. Find $\frac{3}{4} \div \frac{1}{6}$ to determine how many pieces of wood can be cut from the plank. Interpret the quotient within the context of the problem. $\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$; Sample answer: The number of pieces of wood must be a whole number. So, the carpenter can cut four pieces. There will not be enough left over to cut a fifth piece.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or above on the Checks,
THEN assign:

- Practice, Exercises 5–9 odd, 10–13
- Extension: Solve Problems with Division of Fractions
- ALEKS Division with Fractions

OL

IF students score 66–89% on the Checks,
THEN assign:

- Practice, Exercises 1–6, 9, 11, 13
- Extension: Solve Problems with Division of Fractions
- Personal Tutor
- Extra Examples 1–3
- ALEKS Multiplication with Fractions

AL

IF students score 65% or below on the Checks,
THEN assign:

- ArriveMATH Take Another Look
- ALEKS Multiplication with Fractions

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	T opic	Exercises
1	divide fractions by fractions	1–3
2	divide fractions by fractions and interpret the quotients	4, 5
2	write a story context for a problem involving division of fractions	6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems involving the division of fractions by fractions	8, 9
3	higher-order and critical thinking skills	10–13

Common Misconception

Students may find the reciprocal of the first number rather than the second number when dividing fractions. In Exercise 2, have students write out what the number sentence means: "How many ninths are in one third?" Students should be able to draw a bar diagram to answer the question, and this will help them see that finding the reciprocal of the first number leads to an incorrect answer.

Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

Divide. Write in simplest form (Example 1)

1. $\frac{5}{6} \div \frac{5}{12} =$ **2**

2. $\frac{1}{3} \div \frac{1}{9} =$ **3**

3. $\frac{3}{7} \div \frac{1}{14} =$ **6**

4. Romeo had $\frac{3}{4}$ pound of fudge left. He divided the remaining fudge into $\frac{5}{16}$-pound bags. Write and solve an equation that models the situation. Then interpret the quotient. (Example 2)

$\frac{3}{4} \div \frac{5}{16} = 2\frac{2}{5}$; Romeo can make 2 whole bags.

5. Chelsea has $\frac{7}{8}$ pound of butter to make icing. Each batch of icing needs $\frac{1}{4}$ pound of butter. Write and solve an equation that models the situation. Then interpret the quotient. (Example 2)

$\frac{7}{8} \div \frac{1}{4} = 3\frac{1}{2}$; Chelsea can make 3 whole batches of icing.

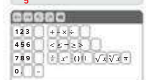
6. Write a story context for $\frac{5}{6} \div \frac{1}{6}$. Then find the quotient. (Example 3)

Sample answer: A nature trail is $\frac{5}{6}$ mile long. There are information markers every $\frac{1}{6}$ mile. How many information markers are there?; 5 markers

T est Practice

7. **Equation Editor** What is the value of the expression $\frac{2}{5} \div \frac{1}{6}$?

$2\frac{2}{5}$



Lesson 3-4 • Divide Fractions by Fractions 175

Apply *indicates multi-step problem

*8. A teacher is making bags of different colors of modeling clay. The table shows the amount of each color she has available. Each color will be divided into $\frac{3}{16}$-pound bags. How many more bags of purple can she make than yellow?

1 more bag

Color	Weight (lb)
Green	$\frac{1}{2}$
Purple	$\frac{15}{16}$
Red	$\frac{2}{3}$
Y ellow	$\frac{3}{4}$

*9. Mateo is making bookmarks with different colored ribbon. The amount of each color he has is shown in the table. Each bookmark will be $\frac{1}{6}$-yard long. How many more orange bookmarks can he make than aqua bookmarks?

1 more bookmark

Color	Length (yd)
Aqua	$\frac{3}{4}$
Orange	$\frac{9}{10}$
Y ellow	$\frac{15}{16}$

Higher-Order Thinking Problems

10. MP **Make a Conjecture** Can the quotient of two positive fractions be less than 1? Explain.

yes; Sample answer: When the dividend is less than the divisor, the quotient is less than 1. For example, $\frac{1}{2}$ is less than $\frac{3}{5}$. So, $\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$ and $\frac{5}{6} < 1$.

11. The length of a race is $\frac{9}{10}$ mile. Andrew wants to place a flag every $\frac{1}{3}$ mile. He has 3 flags. Does he have enough flags? Explain.

yes; Sample answer: $\frac{9}{10} \div \frac{1}{3} = \frac{9}{10} \times \frac{3}{1} = \frac{27}{10} = 2\frac{7}{10}$. He only needs 2 flags. So, he has enough.

12. MP **Persevere with Problems** Lannie has $5\frac{1}{2}$ cups of chocolate chips. She needs $1\frac{3}{4}$ cups to make one batch of chocolate chip cookies. How many batches of chocolate chip cookies can she make?

3 batches

13. Write a division problem involving the division of two positive fractions whose quotient is equal to 1. Show that your problem is correct.

Sample answer: $\frac{7}{8} \div \frac{7}{8} = \frac{7}{8} \times \frac{8}{7} = \frac{56}{56} = 1$

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students conjecture about whether or not the quotient of two positive fractions can be less than 1, and students are asked to construct an argument to defend their conjecture.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 12, students complete a problem in which they have to understand what the problem is asking, write mixed numbers as fractions, divide the fractions, and interpret their answer in the proper context.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercises 8–9 Have students work in groups of 3–4 to solve the problem in Exercise 8. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 9.

Create your own higher-order thinking problem.

Use with Exercises 10–13 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Learn Divide Fractions by Whole Numbers

Objective

Students will understand that they can use various strategies to divide a fraction by a whole number.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 3, encourage them to describe the similarities and difference between using a bar diagram and the reciprocal of a whole number to divide a fraction by a whole number, even though both methods are valid approaches and yield the correct solution. Ask them which method might be more advantageous for different division problems.

Teaching Notes

SLIDE 1

Students previously learned how to divide a whole number by a fraction, and how to divide a fraction by a fraction, using both visual models and equations. Present the division expression $\frac{3}{5} \div 2$, and ask students how this expression is different from the other ones they have seen. Students should note that the divisor is a whole number, not a fraction. Have students move through the slides to see how a visual model can be used to divide the whole number by the fraction. Ask them to explain each step in the process. Some sample questions to help facilitate discussion are shown.

In the first step, how does the model represent the dividend? The dividend is $\frac{3}{5}$ and the model has 3 out of 5 bars shaded to represent $\frac{3}{5}$.

In the third step, why is each fifth separated into two sections? The divisor is 2.

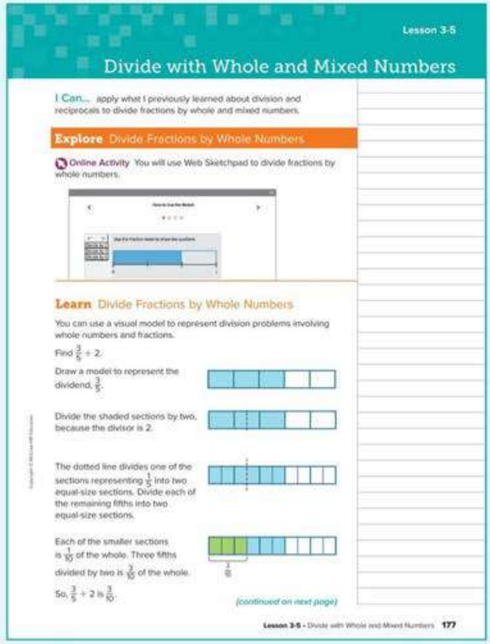
In the fourth step, how does the model illustrate dividing $\frac{3}{5}$ by 2? There are 10 total sections, and 6 of them were shaded to represent $\frac{3}{5}$. Dividing those 6 sections into 2 groups results in 3 sections in each group. Of the whole, this represents $\frac{3}{10}$, or 3 out of the 10 total sections.

(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of using visual models to divide fractions by whole numbers, have them work with a partner to generate at least 3 expressions that involve dividing a fraction by a whole number. Then have them create their own visual models that illustrate the division and help them find the quotient. Have them trade their visual models with another pair of students. Each pair should determine the division expression and quotient that is represented by the model. Have pairs of students discuss and resolve any differences.


Lesson 3-5

Divide with Whole and Mixed Numbers

I Can... apply what I previously learned about division and reciprocals to divide fractions by whole and mixed numbers.

Explore Divide Fractions by Whole Numbers

Online Activity You will use Web Sketchpad to divide fractions by whole numbers.

Learn Divide Fractions by Whole Numbers

You can use a visual model to represent division problems involving whole numbers and fractions.

Find $\frac{3}{5} \div 2$.

Draw a model to represent the dividend, $\frac{3}{5}$.

Divide the shaded sections by two, because the divisor is 2.

The dotted line divides one of the sections representing $\frac{1}{5}$ into two equal-size sections. Divide each of the remaining fifths into two equal-size sections.

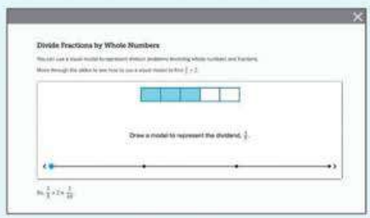
Each of the smaller sections is $\frac{1}{10}$ of the whole. Three fifths divided by two is $\frac{3}{10}$ of the whole.

So, $\frac{3}{5} \div 2$ is $\frac{3}{10}$.

(continued on next page)

Lesson 3-5 • Divide with Whole and Mixed Numbers 177

Interactive Presentation



Learn, Divide Fractions by Whole Numbers, Slide 1 of 3

CLICK

On Slide 1, students view how to use a visual model to solve the division problem.

Lesson 3-5

Divide with Whole and Mixed Numbers

6.NS.A.1

LESSON GOAL

Students will divide with whole and mixed numbers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Divide Fractions by Whole Numbers

Learn: Divide Fractions by Whole Numbers
Example 1: Divide Fractions by Whole Numbers
Learn: Divide Mixed Numbers
Example 2: Divide Mixed Numbers
Example 3: Divide Mixed Numbers
Apply: Decorating

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

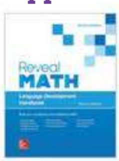
View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Compute with Fractions, Decimals, and Whole Numbers		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 19 of the *Language Development Handbook* to help your students build mathematical language related to division of fractions by whole numbers.

ELL You can use the tips and suggestions on page T19 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.A** by dividing with whole and mixed numbers.
Standards for Mathematical Content: **6.NS.A.1**
Standards for Mathematical Practice: **MP1, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students divided fractions by fractions.
6.NS.A.1

Now
Students divide with whole and mixed numbers.
6.NS.A.1

Next
Students will extend previous understandings of numbers to the system of rational numbers.
6.NS.C.7, 6.NS.C.8

Rigor

The Three Pillars of Rigor

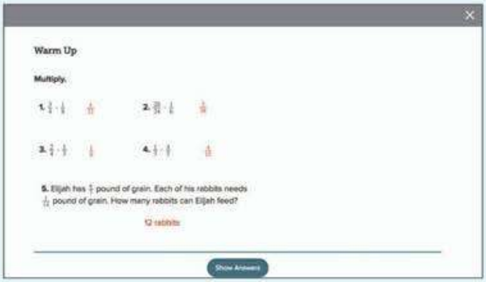
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their *understanding* of division to include whole numbers, fractions, and mixed numbers. They use visual models and standard algorithms to build *fluency* with division of whole and mixed numbers. They *apply* their understanding of division of fractions to write and solve real-world story contexts.

Mathematical Background

To divide a fraction by a whole number, multiply the fraction by the reciprocal of the whole number. To perform division with mixed numbers, first write them as fractions. A mixed number can be written as a fraction by writing the whole number portion as a fraction and then finding the sum of the two fractions.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying fractions (Exercises 1–4)
- solving word problems involving dividing fractions (Exercise 5)

Answers

1. $\frac{3}{32}$
2. $\frac{5}{36}$
3. $\frac{1}{6}$
4. $\frac{4}{15}$
5. 12 rabbits

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about fractions of cords of firewood.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Give an example of a dividend. Sample answer: An example of a dividend would be 12 in the scenario "12 divided by 2 is 6".
- Describe a *divisor* in your own words. Sample answer: A divisor is the number that divides the dividend. An example of a divisor is 2 in the scenario "12 divided by 2 is 6".

Explore Divide Fractions by Whole Numbers

Objective

Students will use Web Sketchpad to explore how to divide fractions by whole numbers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with multiple division problems. Throughout this activity, students will use a sketch to model the division. Students will use their observations to compare division of fractions by whole numbers to division of whole numbers by fractions.

Inquiry Question

How is dividing fractions by whole numbers similar to dividing whole numbers by fractions? Sample answer: With both, I need to write the whole number as a fraction and then multiply the dividend by the reciprocal of the divisor.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

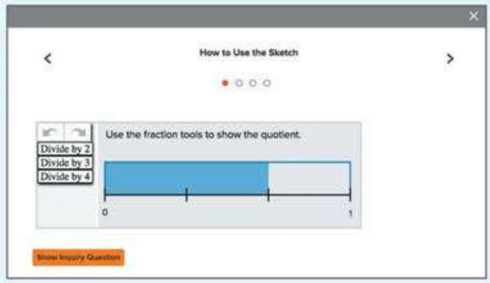
Mathematical Discourse

What is the quotient? What steps did you take in order to evaluate the expression? $\frac{1}{3}$; Sample answer: I can use the "Divide by 2" tool to divide the $\frac{2}{3}$ section into two equal parts.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 5

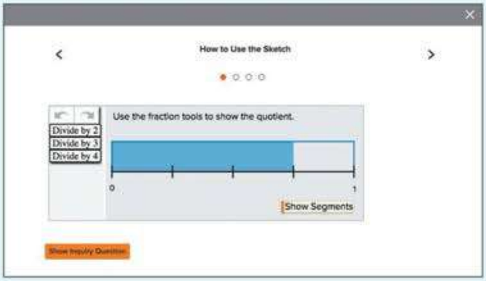


Explore, Slide 2 of 5

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to divide fractions by whole numbers.

Interactive Presentation



Explore, Slide 4 of 5

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Divide Fractions by Whole Numbers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and examine bar diagrams in order to divide fractions by whole numbers. Encourage them to understand the advantages of using this tool and ask them if there are situations when it might not be advantageous.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What is the quotient? What are some similarities and differences you notice between the parts of the expression, the dividend, divisor, and quotient? $\frac{3}{8}$; Sample answer: The numerator of the quotient is the same as the numerator of the dividend. The denominator of the quotient is twice as large as the denominator of the dividend. The two is the divisor.

Your Notes

You can also use an equation to solve division problems involving whole numbers and fractions. To divide a fraction by a whole number, multiply the fraction by the reciprocal of the whole number.

Find $\frac{3}{5} \div 2$.

$\frac{3}{5} \div 2 = \square$ Write the equation.

$= \frac{3}{5} \div \frac{2}{1}$ Write the whole number as a fraction.

$= \frac{3}{5} \times \frac{1}{2}$ Multiply by the reciprocal of $\frac{2}{1}$, $\frac{1}{2}$.

$= \frac{3 \times 1}{5 \times 2}$ Multiply the numerators and denominators.

$= \frac{3}{10}$ Simplify.

So, $\frac{3}{5} \div 2$ is $\frac{3}{10}$.

Talk About It!

Compare and contrast the two methods.

Sample answer: The bar diagram is visual and illustrates the division process. Using an equation to find the quotient uses the reciprocal of the whole number.

Pause and Reflect

Did you struggle with any of the concepts in this Learn? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

Record your observations here

See students' observations.

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Interactive Presentation

Learn, Divide Fractions by Whole Numbers, Slide 2 of 3

CLICK

On Slide 2, students view how to use an equation to solve the division problem.

TYPE

On Slide 2, students enter the missing value to solve the division equation.

Learn Divide Fractions by Whole Numbers *(continued)*

Teaching Notes

SLIDE 2

Point out that visual models are not the only methods that can be used to divide a fraction by a whole number. By setting up an equation where the quotient is the unknown, students can use their understanding of reciprocals to find the quotient. Have students move through the steps for dividing the fraction by the whole number, being sure that they can clearly explain each step.

Talk About It!

SLIDE 3

Mathematical Discourse

Compare and contrast the two methods. Sample answer: The bar diagram is visual and illustrates the division process. Using an equation to find the quotient uses the reciprocal of the whole number.

Example 1 Divide Fractions by Whole Numbers

Objective

Students will divide fractions by whole numbers.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically In Method 1, encourage students to draw a visual model to help them find the quotient. In Method 2, encourage students to use an equation to find the quotient, and compare the two methods. They should see that both methods are valid approaches to finding the quotient. Ask them which method might be more advantageous for different division problems.

Questions for Mathematical Discourse

SLIDE 2

AL What operation will be used to find how many pounds of cashews are in each package? How do you know? Division will be used because we know the number of packages and need to find how many pounds of cashews are in each package.

OL Which number is the dividend? Which number is the divisor? Explain. Sample answer: The dividend is $\frac{3}{4}$ because we are trying to find the amount of cashews in each bag. The divisor is 12 because this is how many bags Faye has.

BL What expression would model the problem if Faye was dividing $\frac{5}{9}$ pound of cashews into 18 packages? $\frac{5}{9} \div 18$

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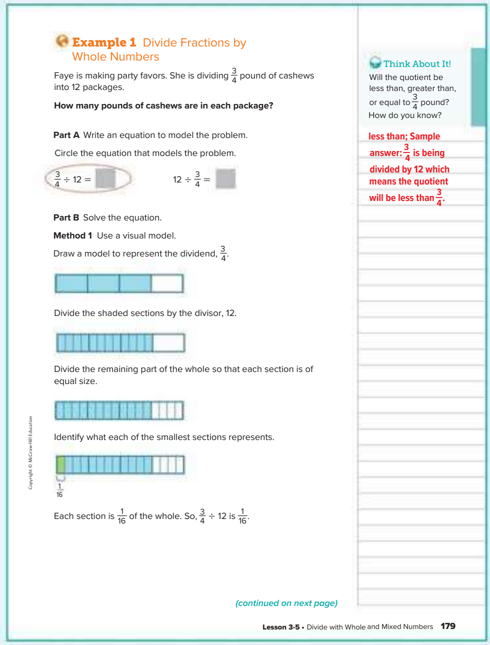

Example 1 Divide Fractions by Whole Numbers

Faye is making party favors. She is dividing $\frac{3}{4}$ pound of cashews into 12 packages.

How many pounds of cashews are in each package?

Part A Write an equation to model the problem.

Circle the equation that models the problem.

$\frac{3}{4} \div 12 = \square$ (circled) $\qquad 12 \div \frac{3}{4} = \square$

Part B Solve the equation.

Method 1 Use a visual model.

Draw a model to represent the dividend, $\frac{3}{4}$.

Divide the shaded sections by the divisor, 12.

Divide the remaining part of the whole so that each section is of equal size.

Identify what each of the smallest sections represents.

$\frac{1}{16}$

Each section is $\frac{1}{16}$ of the whole. So, $\frac{3}{4} \div 12$ is $\frac{1}{16}$.

Think About It!
Will the quotient be less than, greater than, or equal to $\frac{3}{4}$ pound? How do you know?

less than; Sample answer: $\frac{3}{4}$ is being divided by 12 which means the quotient will be less than $\frac{3}{4}$.

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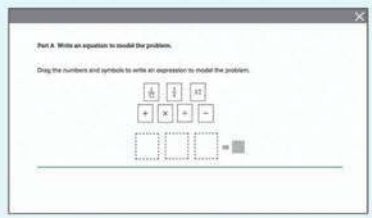
Lesson 3-5 • Divide with Whole and Mixed Numbers 179

Interactive Presentation



Example 1, Divide Fractions by Whole Numbers, Slide 2 of 6

DRAG & DROP

On Slide 2, students drag the numbers and symbols to write an expression to model the problem.

CLICK

On Slide 3, students use a visual model to divide (Method 1).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Method 2 Use an equation.

$\frac{3}{4} \div 12 = \square$ — Write the equation.

$\frac{3}{4} \div 12 = \frac{3}{4} \div \frac{12}{1}$ — Write the whole number as a fraction.

$= \frac{3}{4} \times \frac{1}{12}$ — Multiply by the reciprocal of $\frac{12}{1}$, $\frac{1}{12}$.

$= \frac{\overset{1}{\cancel{3}}}{4} \times \frac{1}{\underset{4}{\cancel{12}}}$ — Divide by common factors.

$= \frac{1 \times 1}{4 \times 4}$ — Simplify.

$= \frac{1}{16}$ — Multiply.

There are $\frac{1}{16}$ pound(s) of cashews in each package.

Talk About It!
Compare and contrast the two methods.

Sample answer: Using a model shows a visual representation of how to find the solution, while the equation is a numerical representation.

Check

Ernesto is making designs for classroom bulletin boards. He is cutting $\frac{3}{4}$-yard of fabric into 6 pieces of the same length. Write and solve an equation to find the length of each piece of fabric.

Show your work here

$\frac{3}{4} \div 6 = \frac{1}{8}; \frac{1}{8}$ yd

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 1, Divide Fractions by Whole Numbers, Slide 4 of 6

TYPE

On Slide 4, students use an equation to divide (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Divide Fractions by Whole Numbers *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Into how many sections will the bar initially be divided? Explain. Sample answer: The bar will be divided into 4 sections to represent the denominator of $\frac{3}{4}$.

OL Why are the first three sections divided into 12 sections? The first three sections represent $\frac{3}{4}$ pound of cashews. The three sections were divided into 12 smaller sections to represent the 12 packages.

BL Suppose Faye had $\frac{7}{8}$ pound of cashews. What would the first piece of the bar look like? The first piece of the bar will be 8 sections, with 7 of these sections being shaded.

SLIDE 4

AL What does each number in the expression represent in context of the problem? $\frac{3}{4}$ is the dividend. It represents the amount of cashews Faye has. 12 is the divisor. It represents the amount of bags into which she is dividing the cashews.

OL How can you check that $\frac{1}{12}$ is the reciprocal of 12? Sample answer: To check to make sure $\frac{1}{12}$ and 12 are reciprocals, I need to make sure their product is 1.

BL Suppose Faye has $\frac{1}{2}$ pound of walnuts and wants to add them equally into the individual bags of cashews. How much will each individual bag weigh when it contains both cashews and walnuts? Explain. $\frac{5}{48}$ lb; Sample answer: Each bag will have $\frac{1}{2} \div 12 = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24}$ pound of walnuts added, so each bag will have $\frac{1}{16} + \frac{1}{24}$ or $\frac{5}{48}$ pound of nuts.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Divide Mixed Numbers

Objective

Students will learn how to divide with mixed numbers.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the mixed number $2\frac{1}{4}$ is written as the fraction $\frac{9}{4}$. Ask students to explain how they know that $2\frac{1}{4} = \frac{9}{4}$. Students should note that the mixed number $2\frac{1}{4}$ is equivalent to $2 + \frac{1}{4}$. They should be able to explain that the whole number portion of the mixed number, 2, can be written as $\frac{8}{4}$. Since $\frac{8}{4} + \frac{1}{4} = \frac{9}{4}$, the mixed number $2\frac{1}{4}$ can be written as $\frac{9}{4}$. Encourage students to notice the similarities and the differences between dividing with mixed numbers and dividing just with fractions.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to divide with mixed numbers.

Example 2 Divide Mixed Numbers

Objective

Students will divide a mixed number by a whole number.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to calculate efficiently and accurately in order to find the quotient of a mixed number and a whole number. They should pay attention to their final answer, being sure to write it in simplest form.

Questions for Mathematical Discourse

SLIDE 1

AL Why do we write the mixed number and the whole number as fractions? so that we can divide as with fractions

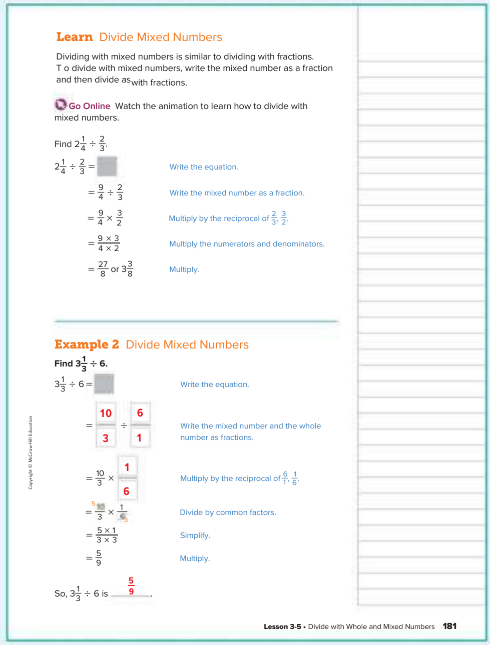
OL Why do we multiply by the reciprocal? To divide by a fraction, multiply by its reciprocal.

OL Explain why it makes sense that the quotient is less than the dividend. Sample answer: Any number divided by a number greater than 1, will result in a number that is less than itself.

BL Find $2\frac{1}{2} \div 1\frac{1}{4}$. $2\frac{1}{2} \div 1\frac{1}{4} = \frac{5}{2} \div \frac{5}{4} = \frac{5}{2} \times \frac{4}{5} = 2$

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Divide Mixed Numbers

Dividing with mixed numbers is similar to dividing with fractions. To divide with mixed numbers, write the mixed number as a fraction and then divide as with fractions.

Go Online Watch the animation to learn how to divide with mixed numbers.

Find $2\frac{1}{4} \div \frac{2}{3}$.

$2\frac{1}{4} \div \frac{2}{3} = \square$	Write the equation.
$= \frac{9}{4} \div \frac{2}{3}$	Write the mixed number as a fraction.
$= \frac{9}{4} \times \frac{3}{2}$	Multiply by the reciprocal of $\frac{2}{3}$, $\frac{3}{2}$.
$= \frac{9 \times 3}{4 \times 2}$	Multiply the numerators and denominators.
$= \frac{27}{8}$ or $3\frac{3}{8}$	Multiply.

Example 2 Divide Mixed Numbers

Find $3\frac{1}{3} \div 6$.

$3\frac{1}{3} \div 6 = \square$	Write the equation.
$= \frac{10}{3} \div \frac{6}{1}$	Write the mixed number and the whole number as fractions.
$= \frac{10}{3} \times \frac{1}{6}$	Multiply by the reciprocal of $\frac{6}{1}$, $\frac{1}{6}$.
$= \frac{\overset{5}{\cancel{10}}}{3} \times \frac{1}{\underset{3}{\cancel{6}}}$	Divide by common factors.
$= \frac{5 \times 1}{3 \times 3}$	Simplify.
$= \frac{5}{9}$	Multiply.

So, $3\frac{1}{3} \div 6$ is $\frac{5}{9}$.

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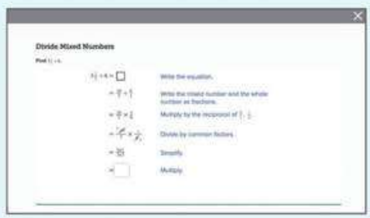
Lesson 3-5 • Divide with Whole and Mixed Numbers 181

Interactive Presentation



Example 2, Divide Mixed Numbers, Slide 1 of 2

WATCH In the Learn, students watch an animation that explains how to divide with mixed numbers.

TYPE On Slide 1 of Example 2, students determine the quotient.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

Find $2\frac{1}{2} \div 3$. Write in simplest form. $\frac{5}{6}$

Show your work here

Example 3 Divide Mixed Numbers

Find $4\frac{2}{3} \div 1\frac{3}{4}$.

$4\frac{2}{3} \div 1\frac{3}{4} = \square$ — Write the equation.

$= \frac{14}{3} \div \frac{7}{4}$ — Write the mixed numbers as fractions.

$= \frac{14}{3} \times \frac{4}{7}$ — Multiply by the reciprocal of $\frac{7}{4}$.

$= \frac{\overset{2}{\cancel{14}}}{3} \times \frac{4}{\underset{1}{\cancel{7}}}$ — Divide by common factors.

$= \frac{2 \times 4}{3 \times 1}$ — Simplify.

$= \frac{8}{3}$ or $2\frac{2}{3}$ — Multiply.

So, $4\frac{2}{3} \div 1\frac{3}{4}$ is $2\frac{2}{3}$.

Check

Find $2\frac{3}{8} \div 1\frac{1}{4}$. Write in simplest form. $1\frac{9}{10}$

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 3, Divide Mixed Numbers, Slide 1 of 2

TYPE

On Slide 1, students determine the quotient.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Divide Mixed Numbers

Objective

Students will divide a mixed number by a mixed number.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to calculate efficiently and accurately in order to find the quotient of two mixed numbers. They should pay attention to their final answer, being sure to write it in simplest form.

Questions for Mathematical Discourse

SLIDE 1

AL Why do we write both mixed numbers as fractions? so that we can divide as with fractions

OL Instead of simplifying before multiplying, can you multiply first, and then simplify? Explain. yes; Sample answer: Either method will result in equivalent quotients.

BL Find $12\frac{1}{2} \div 10\frac{3}{4}$. Write as a mixed number.

$12\frac{1}{2} \div 10\frac{3}{4} = \frac{25}{2} \div \frac{43}{4} = \frac{25}{2} \times \frac{4}{43} = \frac{50}{43} = 1\frac{7}{43}$

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Decorating

Objective

Students will come up with their own strategy to solve an application problem involving area of mirrors.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

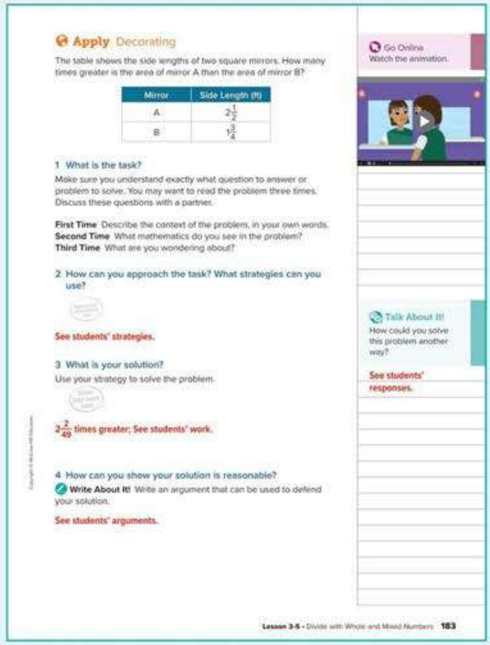
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you find the area of a square?
- Are the side lengths of a square equal or different?
- Which mirror will have the greater area?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Decorating

The table shows the side lengths of two square mirrors. How many times greater is the area of mirror A than the area of mirror B?

Mirror	Side Length (ft)
A	$2\frac{1}{2}$
B	$1\frac{3}{4}$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$2\frac{2}{49}$ times greater; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

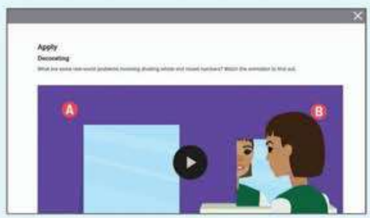
Go Online Watch the animation.

Talk About It! How could you solve this problem another way?

See students' responses.

Lesson 3-5 • Divide with Whole and Mixed Numbers 183

Interactive Presentation



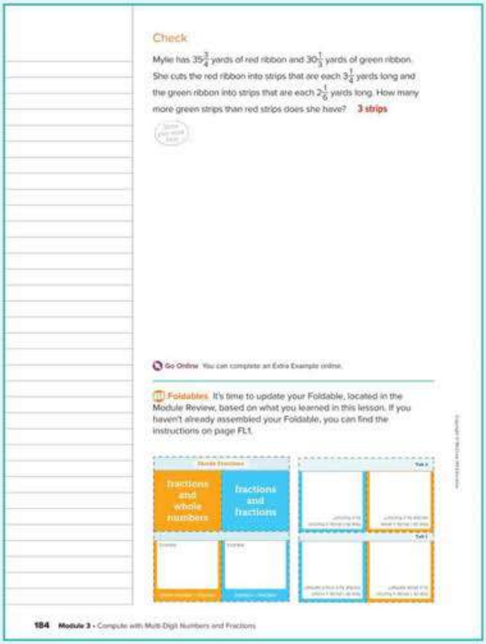
Apply, Decorating

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Mylie has $35\frac{3}{4}$ yards of red ribbon and $30\frac{1}{3}$ yards of green ribbon. She cuts the red ribbon into strips that are each $3\frac{1}{4}$ yards long and the green ribbon into strips that are each $2\frac{1}{6}$ yards long. How many more green strips than red strips does she have? 3 strips

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

fractions and whole numbers

fractions and fractions

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could write descriptions of the different methods used to divide with whole and mixed numbers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with fractions and decimals related to operations with whole numbers? In this lesson, students learned how to divide fractions by whole numbers using models and equations, and how to divide with mixed numbers. Encourage them to work with a partner to compare and contrast dividing fractions with dividing whole numbers. For example, have them compare and contrast how they can use models to simplify each of the expressions $\frac{3}{4} \div \frac{2}{3}$, $\frac{3}{4} \div 2$, $3 \div \frac{2}{3}$, and $3 \div 2$.

Exit Ticket

Refer to the Exit Ticket slide. Find the fraction of a cord that they can burn each day, assuming they burn an equal amount each day. Write a mathematical argument that can be used to defend your solution.

The campers can burn $\frac{1}{30}$ of a cord each day; Sample answer: $\frac{1}{3} \div 10 = \frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or above on the Checks
THEN assign:

- Practice, Exercises 1, 9, 11, 13–16
- Extension: Compute with Fractions, Decimals, and Whole Numbers
- ALEKS Division with Fractions

OL

IF students score 66–89% on the Checks
THEN assign:

- Practice, Exercises 1–8, 11, 14, 16
- Extension: Compute with Fractions, Decimals, and Whole Numbers
- Personal Tutor
- Extra Examples 1–3
- ALEKS Multiplication with Fractions

AL

IF students score 65% or below on the Checks
THEN assign:

- ArriveMATH Take Another Look
- ALEKS Multiplication with Fractions

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

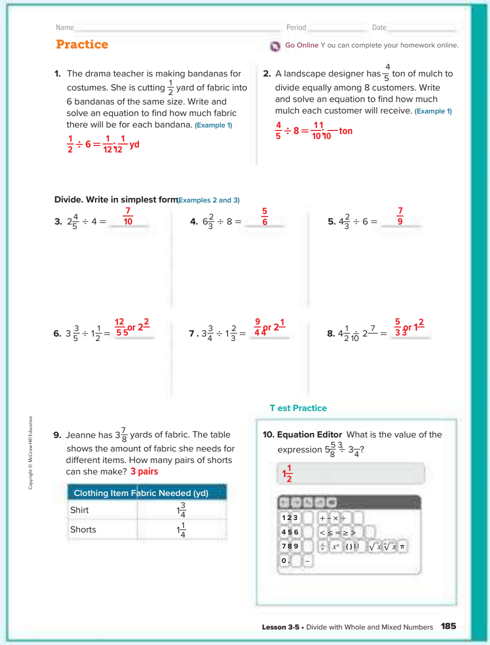
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	divide fractions by whole numbers	1, 2
1	divide a mixed number by a whole number	3–5
1	divide a mixed number by a mixed number	6–8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving division with whole and mixed numbers	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

When dividing whole numbers, fractions, and mixed numbers, students should first write the whole numbers and mixed numbers as fractions. After writing as fractions, some students might forget to multiply by the reciprocal of the divisor. Remind students that when dividing fractions, you need to perform the inverse operation of division, which is multiplying by the reciprocal, to find the quotient.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The drama teacher is making bandanas for costumes. She is cutting $\frac{1}{2}$ yard of fabric into 6 bandanas of the same size. Write and solve an equation to find how much fabric there will be for each bandana. (Example 1)
   $\frac{1}{2} \div 6 = \frac{1}{12}$; $\frac{1}{12}$ yd

2. A landscape designer has $\frac{4}{5}$ ton of mulch to divide equally among 8 customers. Write and solve an equation to find how much mulch each customer will receive. (Example 1)
   $\frac{4}{5} \div 8 = \frac{1}{10}$; $\frac{1}{10}$ ton

Divide. Write in simplest form. (Examples 2 and 3)

3. $2\frac{4}{5} \div 4 =$ $\frac{7}{10}$
4. $6\frac{2}{3} \div 8 =$ $\frac{5}{6}$
5. $4\frac{2}{3} \div 6 =$ $\frac{7}{9}$
6. $3\frac{3}{5} \div 1\frac{1}{2} =$ $\frac{12}{5}$ or $2\frac{2}{5}$
7. $3\frac{3}{4} \div 1\frac{2}{3} =$ $\frac{9}{4}$ or $2\frac{1}{4}$
8. $4\frac{1}{2} \div 2\frac{7}{10} =$ $\frac{5}{3}$ or $1\frac{2}{3}$

9. Jeanne has $3\frac{7}{8}$ yards of fabric. The table shows the amount of fabric she needs for different items. How many pairs of shorts can she make? **3 pairs**

Clothing Item	Fabric Needed (yd)
Shirt	$1\frac{3}{4}$
Shorts	$1\frac{1}{4}$

Test Practice

10. **Equation Editor** What is the value of the expression $5\frac{5}{8} \div 3\frac{3}{4}$?
    $1\frac{1}{2}$

Apply *indicates multi-step problem

*11. Kara and Nathan are each painting a poster for the school dance. Their posters have the dimensions shown in the table. How many times greater is the area of Kara's poster than Nathan's?

Student	Poster Length (ft)	Poster Width (ft)
Nathan	$1\frac{1}{2}$	$1\frac{1}{2}$
Kara	$3\frac{3}{4}$	$3\frac{3}{4}$

$6\frac{1}{4}$ times greater

*12. Mrs. Brown is putting different colored sand into cups for her 4 daughters to make sand art bottles. The total amount of each color she has is shown in the table. If each color is divided equally among the daughters, how much more pink sand will be available for each girl than purple sand?

Sand Color	Weight (lb)
Blue	$\frac{15}{16}$
Pink	$\frac{3}{4}$
Purple	$\frac{1}{2}$
Turquoise	$\frac{7}{8}$

$\frac{1}{16}$ pound

Higher-Order Thinking Problems

13. Create Write and solve a real-world problem that involves the division of two mixed numbers.

Sample answer: A bag contains $22\frac{1}{2}$ cups of flour. A recipe for pancakes uses $1\frac{1}{4}$ cups of flour. How many batches of pancakes can be made with one bag of flour? 18 batches

14. Find $2\frac{1}{10} \div 1\frac{1}{5}$. How can you determine if your quotient is reasonable? Explain.

Sample answer: Use estimation. Round each mixed number: $2 \div 1 = 2$. The actual quotient is $1\frac{3}{4}$ which is close to 2. So, the answer is reasonable.

15. MP Persevere with Problems Without dividing, explain whether the quotient of $\frac{9}{10} \div 3$ is greater than or less than the quotient of $\frac{9}{10} \div 2$.

less than; Sample answer: $\frac{9}{10} \div 3$ is divided into more parts than $\frac{9}{10} \div 2$. Since it is divided into more parts, each part represents a lesser amount. So, $\frac{9}{10} \div 2 > \frac{9}{10} \div 3$.

16. MP Reason Inductively Without computing, which expression is greater, $20 \times \frac{1}{2}$ or $20 \div \frac{1}{2}$? Explain your reasoning.

$20 \div \frac{1}{2}$; Sample answer: Multiplying 20 by a number less than 1 results in a number that is less than 20. Dividing 20 by a number less than 1 results in a number that is greater than 20.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 15, students make sense of a problem involving two quotients and carry out a plan to determine which is greater without performing the division.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students construct an argument to defend whether the product or the quotient is greater without performing the computation.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 11 Have students work together to prepare a brief demonstration that illustrates why this application problem might require multiple steps to solve. For example, before they determine how many times greater one poster is than the other, they have to find the area of each poster. Have each pair or group of students present their response to the class.

Listen and ask clarifying questions.
Use with Exercises 14–15 Have students work in pairs. Have students individually read Exercise 14 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 15.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of multiplication and division of fractions with fractions, mixed numbers, and whole numbers. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

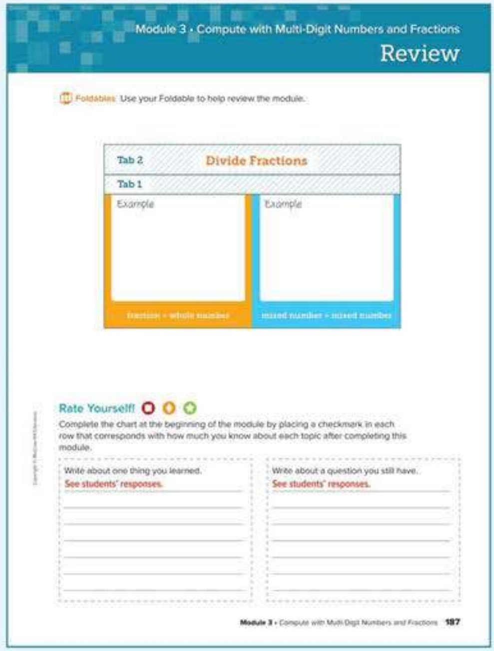
Assessment Resources

Put It All Together 1: Lessons 3-1 and 3-2
Put It All Together 2: Lessons 3-3, 3-4, and 3-5
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **The Number System.**

- Multiplication and Division of Fractions
- Division of Whole Numbers
- Decimal Operations


Module 3 • Compute with Multi-Digit Numbers and Fractions

Review

Foldables Use your Foldable to help review the module.

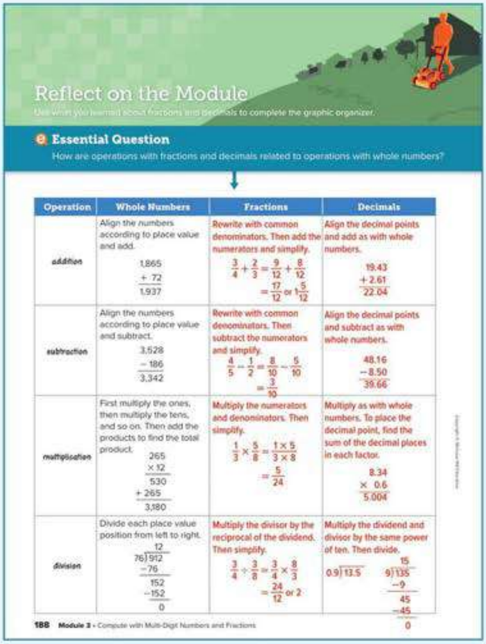
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 3 • Compute with Multi-Digit Numbers and Fractions 187



Reflect on the Module

Use what you learned about fractions and decimals to complete the graphic organizer.

Essential Question

How are operations with fractions and decimals related to operations with whole numbers?

Operation	Whole Numbers	Fractions	Decimals
addition	Align the numbers according to place value and add. 1,865 + 72 = 1,937	Rewrite with common denominators. Then add the numerators and simplify. $\frac{3}{4}+\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}$ or $1\frac{5}{12}$	Align the decimal points and add as with whole numbers. 19.43 + 2.61 = 22.04
subtraction	Align the numbers according to place value and subtract. 3,528 − 186 = 3,342	Rewrite with common denominators. Then subtract the numerators and simplify. $\frac{4}{5}-\frac{1}{2}=\frac{8}{10}-\frac{5}{10}=\frac{3}{10}$	Align the decimal points and subtract as with whole numbers. 48.16 − 8.50 = 39.66
multiplication	First multiply the ones, then multiply the tens, and so on. Then add the products to find the total product. 265 × 12: 530 + 265 = 3,180	Multiply the numerators and denominators. Then simplify. $\frac{1}{3}\times\frac{5}{8}=\frac{1\times 5}{3\times 8}=\frac{5}{24}$	Multiply as with whole numbers. To place the decimal point, find the sum of the decimal places in each factor. 8.34 × 0.6 = 5.004
division	Divide each place value position from left to right. 912 ÷ 76 = 12: −76, 152, −152, 0	Multiply the divisor by the reciprocal of the dividend. Then simplify. $\frac{3}{4}\div\frac{3}{8}=\frac{3}{4}\times\frac{8}{3}=\frac{24}{12}$ or 2	Multiply the dividend and divisor by the same power of ten. Then divide. 13.5 ÷ 0.9; 135 ÷ 9 = 15: −9, 45, −45, 0

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How are operations with fractions and decimals related to operations with whole numbers? See students' graphic organizers.

Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–13 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	5, 8, 12
Multiselect	Multiple answers may be correct. Students must select all correct answers.	10
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	2, 3, 7
Open Response	Students construct their own response in the area provided.	1, 4, 6, 9, 11, 13

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.NS.A.1	3-3, 3-4, 3-5	5–13
6.NS.B.2	3-1	1, 2
6.NS.B.3	3-2	3, 4

Name ______ Period ______ Date ______

Test Practice

1. **Open Response** In Jamal's county, there are 60 farms that cover about 8,370 acres of land. If the farms are all approximately the same size, how many acres is each farm? Explain how can you solve the problem. (Lesson 1)

139.5 acres; Divide 8,370 by 60 to find that each farm is 139.5 acres.

2. **Equation Editor** At the botanical garden, flower bulbs are planted each spring. The table shows the number of bulbs planted in each color (Lesson 1)

Color	Number
Y ellow	280
Red	245
Purple	393

If each flowerbed can hold 36 bulbs, how many flowerbeds will be completely filled with bulbs?

25

3. **Equation Editor** Divide 0.008 ÷ 0.25. (Lesson 2)

0.032

4. **Open Response** Mariam is making two kinds of paper lanterns. One type of lantern requires 0.75 square foot of construction paper, while the other requires 1.15 square feet. After making 5 of each type of lantern, Mariam has 12.75 square feet of leftover paper. (Lesson 2)

A. How many square feet of paper did Mariam use when making the 10 lanterns? Explain how you found this answer.

9.5 square feet; Sample answer: I multiplied 0.75 by 5 and 1. 15 by 5, added the products, and found the total paper used.

B. How many square feet of paper did Mariam begin with? Describe your reasoning.

22.25 square feet; Sample answer: I added the total amount of paper used, 9.5 square feet to the amount of leftover paper , 12.75 to find the amount of paper Mariam began with, 22.25 square feet.

5. **Multiple Choice** What number multiplied by $\frac{7}{9}$ has a product of 1? (Lesson 3)

Ⓐ $\frac{7}{9}$

Ⓑ $\frac{9}{9}$

Ⓒ 1

Ⓓ $\frac{9}{7}$

6. **Open Response** Divide $7 \div \frac{3}{5}$. (Lesson 3)

$11\frac{2}{3}$

Module 3 • Compute with Multi-Digit Numbers and Fractions **189**

7 . Equation Editor The table shows the ingredients needed to make one serving of marinade. Kat has 3 cups of soy sauce. She made the greatest number of servings possible. (Lesson 3)

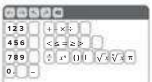
Ingredients	Amount
Ginger	$\frac{1}{8}$ T
Soy sauce	$\frac{5}{6}$ c
Garlic	$\frac{1}{4}$ c

A. How many whole servings of marinade will the 3 cups of soy sauce make?

3

B. How many cups of soy sauce will be left over?

$\frac{1}{2}$



8. Multiple Choice T ony is making chicken enchiladas. He needs $\frac{1}{8}$ jar of sauce for each enchilada. How many enchiladas can T ony make with $\frac{5}{6}$ jar of sauce? (Lesson 4)

- (A) 5 enchiladas
- (B) 6 enchiladas
- (C) 7 enchiladas
- (D) 8 enchiladas

9. Open Response Divide $\frac{2}{3} \div \frac{3}{4}$. (Lesson 4)

$\frac{8}{9}$

10. Multiselect A builder is dividing a hectare (about $2\frac{1}{2}$ acres of land) into $\frac{1}{3}$-acre lots to build houses. Which expression(s) can be used to find how many lots the builder will have to build on? Select all that apply. (Lesson 5)

- ☐ $\frac{5}{2} \div \frac{3}{1}$
- ☐ $\frac{2}{5} \div \frac{3}{1}$
- ✓ $\frac{5}{2} \times \frac{3}{1}$
- ☐ $\frac{5}{2} \times \frac{1}{3}$
- ☐ $\frac{2}{5} \times \frac{3}{1}$
- ✓ $\frac{5}{2} \div \frac{1}{3}$

11. Open Response Three-fifth pound of pasta is enough to feed 6 people. (Lesson 5)

A. Write a division equation to find the number of pounds in each serving.

$\frac{3}{5} \div 6 =$

B. How many pounds are in each serving?

$\frac{3}{5} \div 6 = \frac{3}{5} \times \frac{1}{6} = \frac{}{30} \frac{3}{10}$, or $\frac{1}{}$ pound

12. Multiple Choice A restaurant has a $\frac{3}{4}$-full pan of lasagna. If the cost is $20 per $\frac{1}{3}$ pan, how much will the restaurant charge for the $\frac{3}{4}$-full pan of lasagna? (Lesson 5)

- (A) $20
- (B) $45
- (C) $60
- (D) $125

13. Open Response Find the quotient of $13 \div 4\frac{7}{8}$ written in simplest form. (Lesson 5)

$2\frac{2}{3}$

190 Module 3 • Compute with Multi-Digit Numbers and Fractions

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How are integers and rational numbers related to the coordinate plane? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

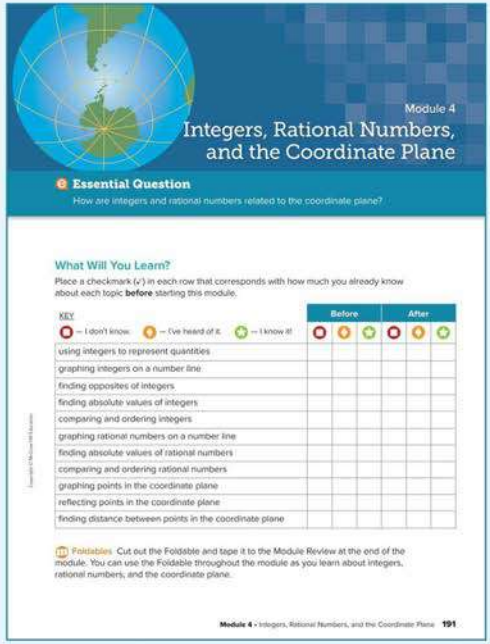
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of latitude and longitude to introduce the idea of integers, rational numbers, and the coordinate plane. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 4

Integers, Rational Numbers, and the Coordinate Plane

Essential Question

How are integers and rational numbers related to the coordinate plane?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY	Before			After		
— I don't know — I've heard of it — I know it!						
using integers to represent quantities						
graphing integers on a number line						
finding opposites of integers						
finding absolute values of integers						
comparing and ordering integers						
graphing rational numbers on a number line						
finding absolute values of rational numbers						
comparing and ordering rational numbers						
graphing points in the coordinate plane						
reflecting points in the coordinate plane						
finding distance between points in the coordinate plane						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about integers, rational numbers, and the coordinate plane.

Module 4 • Integers, Rational Numbers, and the Coordinate Plane 191

Interactive Presentation

Module 4

Integers, Rational Numbers, and the Coordinate Plane

Module Goal

Graph integers and rational numbers on number lines and on the coordinate plane.

Focus

Domain: The Number System

Major Cluster(s): **6.NS.C** Apply and extend previous understandings of numbers to the system of rational numbers.

Standards for Mathematical Content:

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

Also addresses 6.NS.C.5 and 6.NS.C.8.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- compare and order a set of whole numbers
- graph whole numbers on the number line
- graph points with whole-number coordinates in the first quadrant of the coordinate plane

Use the Module Pretest to diagnose students' readiness for this module. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students computed with multi-digit numbers and fractions.
6.NS.A.1, 6.NS.B.2, 6.NS.B.3

Now
Students graph integer and rational-valued points on number lines and the coordinate plane.
6.NS.C.5, 6.NS.C.6, 6.NS.C.7, 6.NS.C.8

Next
Students will perform operations with integers.
7.NS.A.1, 7.NS.A.2, 7.NS.A.3

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of whole numbers and number lines to develop *understanding* of integers, rational numbers, and the coordinate plane. They use this understanding to build *fluency* with representations of integers and absolute value, comparing and ordering rational numbers, and graphing points and finding distance on the coordinate plane. They also *apply* their understanding of integers, rational numbers, and the coordinate plane to solve real-world problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

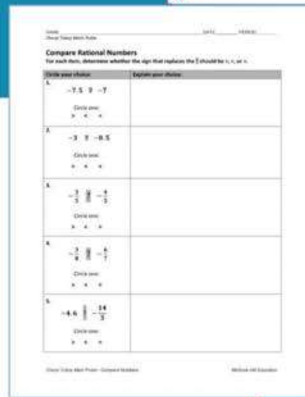
Lesson		Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video			**1**	**0.5**
4-1	Represent Integers	6.NS.C.5, 6.NS.C.6, 6.NS.C.6.C	2	1
4-2	Opposites and Absolute Value	6.NS.C.5, 6.NS.C.6, 6.NS.C.6.A, 6.NS.C.7, 6.NS.C.7.C	2	1
4-3	Compare and Order Integers	6.NS.C.7, 6.NS.C.7.A–D	2	1
4-4	Rational Numbers	6.NS.C.6, 6.NS.C.6.C, 6.NS.C.7, 6.NS.C.7.A, 6.NS.C.7.C	2	1
Put It All Together 1: Lessons 4-1, 4-3, and 4-4			**0.5**	**0.25**
4-5	The Coordinate Plane	6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8	3	1.5
4-6	Graph Reflections of Points	6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8	3	1.5
4-7	Absolute Value and Distance	6.NS.C.8	3	1.5
Put It All Together 2: Lessons 4-5, 4-6, and 4-7			**0.5**	**0.25**
Module Review			**1**	**0.5**
Module Assessment			**1**	**0.5**
		Total Days	**21**	**10.5**

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Compare Rational Numbers

Module Resource


Compare Rational Numbers

Correct Answers: 1. <; **2.** <; **3.** >; **4.** >; **5.** >

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students determine the correct inequality or equals sign to complete each statement.

Targeted Concept The magnitude of two negative quantities can be compared by reasoning about the distances from zero based on their positions on a number line, or by expressing both in decimal or fraction form.

Targeted Misconceptions

- Students may ignore the negative signs and apply positive number comparisons.
- Students may incorrectly interpret the relative position of numbers on a number line.

Assign the probe after Lesson 4.

Collect and Assess Student Work

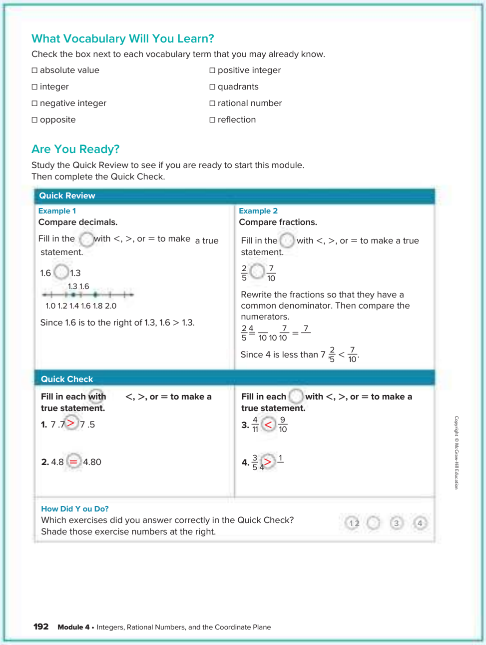
If the student selects...	**Then** the student likely...
1. > **2.** > **3.** < **4.** < **5.** >	ignores the negative number signs. **Example:** The student chooses all or most of these answers, and gives explanations that did not include references to negative numbers.
1. > **2.** > **3.** < **4.** < **5.** >	incorrectly interprets the relative position of the numbers on a number line. **Example:** The student chooses all or most of these answers, with explanations referring to a greater distance from zero (absolute value) as the greater number.
5. =	bases their comparison after rounding the fraction.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Integers and Rational Numbers
- Lesson 4, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.


What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ absolute value	☐ positive integer
☐ integer	☐ quadrants
☐ negative integer	☐ rational number
☐ opposite	☐ reflection

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Compare decimals.
Fill in the ◯ with <, >, or = to make a true statement.
1.6 ◯ 1.3
1.3 1.6
1.0 1.2 1.4 1.6 1.8 2.0
Since 1.6 is to the right of 1.3, 1.6 > 1.3.

Example 2
Compare fractions.
Fill in the ◯ with <, >, or = to make a true statement.
$\frac{2}{5}$ ◯ $\frac{7}{10}$
Rewrite the fractions so that they have a common denominator. Then compare the numerators.
$\frac{2}{5} = \frac{4}{10}$ $\frac{7}{10} = \frac{7}{10}$
Since 4 is less than 7, $\frac{2}{5} < \frac{7}{10}$.

Quick Check

Fill in each ◯ with <, >, or = to make a true statement.
1. 7.7 > 7.5
2. 4.8 = 4.80

Fill in each ◯ with <, >, or = to make a true statement.
3. $\frac{4}{11} < \frac{9}{10}$
4. $\frac{3}{5} > \frac{1}{4}$

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.
1 2 3 4

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define Absolute value is the distance between a number and zero on a number line.

Example
The absolute value of –3 is 3.

Ask What is |–8|? 8

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- graphing whole numbers on a number line
- comparing and ordering whole numbers
- writing fractions as decimals
- graphing ordered pairs in the first quadrant

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Integers and Rational Numbers** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Regular Reflection

When students are asked to regularly explain their thinking about a strategy they used to solve a problem, they are engaging in thought organization, concise consolidation of knowledge, and deductive and inductive reasoning.

How Can I Apply It?

Use the **Think About It!** and **Talk About It!** questions throughout each lesson to encourage students to reflect about what they just learned, or what they might do next.

Throughout the lesson, **Pause and Reflect** questions are included at point-of-use in the *Interactive Student Edition*. Encourage students to not skip over these questions, but to actually *pause* and *reflect* on the concept(s) they just learned and what questions they still might have.

Have students complete the **Exit Tickets** at the end of each lesson to reflect on their learning about the topics covered in each lesson. Have students share their reflections with a partner or in small groups.

Learn Use Integers to Represent Quantities

Objective

Students will understand what an integer is, how integers can represent real-world quantities, and where integers are located on the number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to think about where they have seen positive and negative integers represented on a vertical number line.

Teaching Notes

SLIDE 1

Prior to having students select the *Negative Integers* button or the *Positive Integers* button, you may wish to have students make a prediction as to where they think the integers will lie based on their relationship to zero on the number line. Have students generate other examples of positive and negative integers, other than the ones shown.

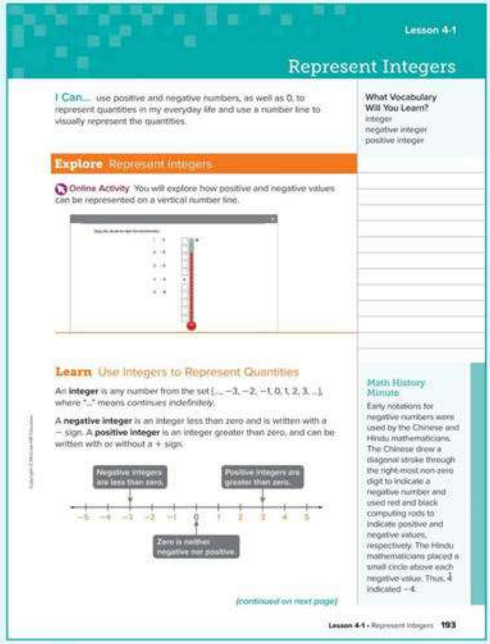
Point out that not all number lines are horizontal. You may wish to draw a vertical number line on the board and ask students where the integers 3 and −3 will be located on the number line in relationship to zero. When the number line is horizontal, negative integers are to the left of zero and positive integers are to the right of zero. When the number line is vertical, negative integers are below zero and positive integers are above zero.

(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of using integers to represent real-world quantities, have them work with a partner to generate examples of how integers are used in everyday life. They should generate at least three different examples. For each example, have them explain what a negative integer would represent, what a positive integer would represent, and explain the meaning of zero. One example response is shown. Sample answer: The elevation of a hiker descending into a canyon can be represented by a negative integer. The elevation of a hiker ascending a hill or mountain can be represented by a positive integer. The meaning of zero is represented by sea level.



Lesson 4-1

Represent Integers

I Can... use positive and negative numbers, as well as 0, to represent quantities in my everyday life and use a number line to visually represent the quantities.

What Vocabulary Will You Learn?
integer
negative integer
positive integer

Explore Represent Integers

Online Activity You will explore how positive and negative values can be represented on a vertical number line.

Learn Use Integers to Represent Quantities

An **integer** is any number from the set {..., −3, −2, −1, 0, 1, 2, 3, ...}, where "..." means continues indefinitely.

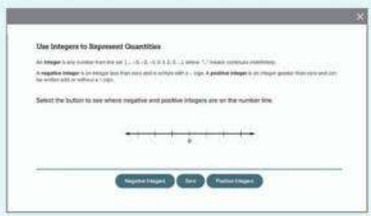
A **negative integer** is an integer less than zero and is written with a − sign. A **positive integer** is an integer greater than zero, and can be written with or without a + sign.

Math History Minute
Early notations for negative numbers were used by the Chinese and Hindu mathematicians. The Chinese drew a diagonal stroke through the right-most non-zero digit to indicate a negative number and used red and black computing rods to indicate positive and negative values, respectively. The Hindu mathematicians placed a small circle above each negative value. Thus, $\dot{4}$ indicated −4.

(continued on next page)

Lesson 4-1 • Represent Integers 193

Interactive Presentation



Learn, Use Integers to Represent Quantities, Slide 1 of 3

CLICK

On Slide 1, students select buttons to show the location of integers on the number line.

WATCH

On Slide 2, students watch an animation that explains how positive and negative integers are used to describe temperatures.

Lesson 4-1

Represent Integers

6.NS.C.5, 6.NS.C.6

LESSON GOAL

Students will use integers on a number line to represent quantities.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Represent Integers

Learn: Use Integers to Represent Quantities
Example 1: Use Integers to Represent Quantities
Learn: Graph Integers on a Number Line
Example 2: Graph Integers on a Number Line

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 20 of the *Language Development Handbook* to help your students build mathematical language related to integers.

ELL You can use the tips and suggestions on page T20 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.C** by graphing integers on a number line to represent quantities.
Standards for Mathematical Content: **6.NS.C.5, 6.NS.C.6, 6.NS.C.6.C**
Standards for Mathematical Practice: **MP2, MP3, MP5**

Coherence

Vertical Alignment

Previous
Students divided whole and mixed numbers.
6.NS.A.1

Now
Students graph integers on a number line to represent quantities.
6.NS.C.5, 6.NS.C.6

Next
Students will find the opposites of integers and use opposites to understand absolute value.
6.NS.C.5, 6.NS.C.6, 6.NS.C.7

Rigor

The Three Pillars of Rigor

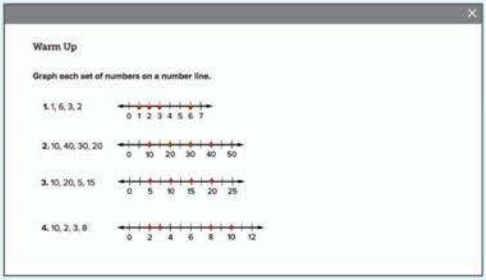
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of number lines (gained in prior grades) to begin to develop *understanding* of integers. They use this understanding to build *fluency* with writing integers, explaining the meaning of zero in a given situation, and graphing sets of integers on horizontal and vertical number lines.

Mathematical Background

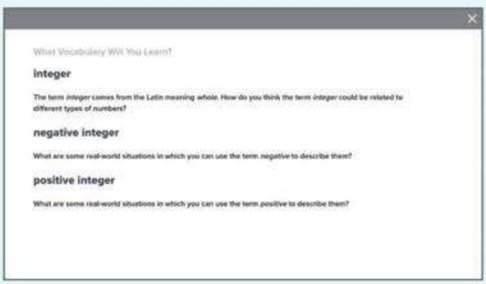
The set of *integers* is {..., −3, −2, −1, 0, 1, 2, 3, ...}. *Negative integers* are less than zero and *positive integers* are greater than zero. To graph an integer on a number line, place a dot on the number line at its location. Negative numbers are located to the left of zero, and positive integers are located to the right of zero on a horizontal number line. On a vertical number line, negative integers are located below zero, and positive integers are located above zero.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- graphing whole numbers on a number line (Exercises 1–4)
- solving word problems involving graphing whole numbers on a number line (Exercise 5)

Answers

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about checking account actions as a representation of integers.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *integer* comes from the Latin meaning *whole*. How do you think the term *integer* could be related to different types of numbers? Sample answer: The term integer might be used to describe numbers that are not fractions, but numbers that are whole.
- What are some real-world situations in which you can use the term *negative* to describe them? Sample answers: negative bank account balances, negative charges on electrons, a negative attitude
- What are some real-world situations in which you can use the term *positive* to describe them? Sample answers: positive bank account balances, a positive outlook on life or a positive attitude, being 100% sure of something

Explore Represent Integers

Objective

Students will explore how integers can be used to represent quantities.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the thermometer and different temperatures, both above and below zero. Students will explore positive and negative temperatures using a drag and drop activity. Students will label temperatures on the thermometer with positive and negative values.

Inquiry Question

How can positive and negative values be represented? Sample answer: I can represent positive and negative values using positive and negative signs with numbers or on a number line.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

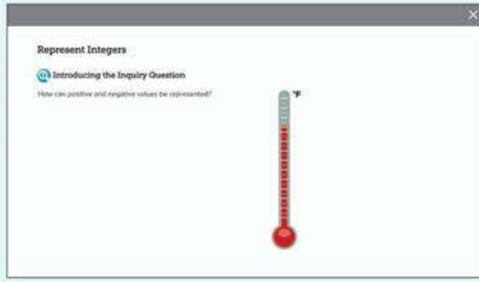
SLIDE 4

Mathematical Discourse

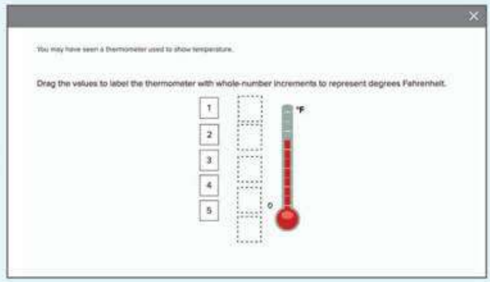
What negative number is the same distance from 0 as the number 4? −4 is the same distance from 0 as 4.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 7

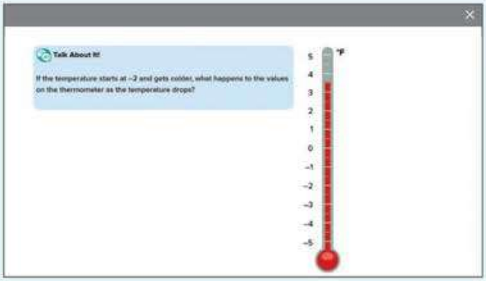


Explore, Slide 4 of 7

DRAG & DROP

On Slides 2 and 4, students drag to label thermometers.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Represent Integers *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to abstract the real-world situation involving temperature, in order to represent temperatures that are both above and below 0, using positive and negative values on the thermometer.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

If the temperature starts at –2 and gets colder, what happens to the values on the thermometer as the temperature drops? Sample answer: The numbers below zero seem to increase on the thermometer as the temperature gets colder.

Your Notes

Go Online Watch the animation to see how integers are used in real life.

Suppose Anabeth is traveling to different parts of the country. She logs the temperature in each location. When Anabeth was in Miami, Florida, the temperature was 80 degrees. That same week, she traveled to Caribou, Maine, where it was −10 degrees.

How can Anabeth represent the positive and negative values in her temperature log?

Talk About It! Give another example of when using a vertical number line is useful. Explain your reasoning.

Sample answer: It is useful when graphing elevations because on a vertical number line, 0 represents sea level. Above sea level would be represented by positive integers and below sea level would be represented by negative integers.

Think About It! What does the word loss mean?

See students' responses.

Example 1 Use Integers to Represent Quantities

A football team has a 10-yard loss in one play.

Write an integer to represent the situation. Explain the meaning of 0 in the situation.

Part A Write an integer to represent the situation.

Because the situation represents a loss, the integer is negative.

The integer used to represent the situation is −10.

Part B Explain the meaning of zero in this situation.

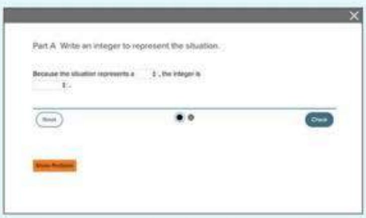
In a football play, the integer 0 represents no yards gained or lost.

Talk About It! Describe another real-world situation that can be represented by −10. Explain the meaning of zero in that situation.

Sample answer: a withdrawal of $10 from a bank account; In this situation, the meaning of zero is neither withdrawing nor depositing money in the account.

194 Module 4 • Integers, Rational Numbers, and the Coordinate Plane

Interactive Presentation



Example 1, Use Integers to Represent Quantities, Slide 2 of 5

CLICK

On Slide 2 of Example 1, students select from a drop-down menu to write an integer to represent the situation.

CLICK

On Slide 3 of Example 1, students select from a drop-down menu to describe the meaning of 0 in the situation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Use Integers to Represent Quantities *(continued)*

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how integers are used to describe temperatures.

Talk About It!

SLIDE 3

Mathematical Discourse

Give another example of when using a vertical number line is useful. Explain. Sample answer: It is useful when graphing elevations because on a vertical number line, 0 represents sea level. Above sea level would be represented by positive integers and below sea level would be represented by negative integers.

Example 1 Use Integers to Represent Quantities

Objective

Students will write an integer to represent a real-world quantity and explain the meaning of zero in the situation.

Questions for Mathematical Discourse

SLIDE 2

AL If you were to lose something, would that item be added to your collection, or subtracted? subtracted

OL Is the integer that represents a *loss of 10 yards* positive or negative? Explain. negative; Sample answer: A *loss* represents something that is subtracted, or taken away.

BL Describe what a positive integer would represent in this situation. Sample answer: If the football team had completed a 10-yard play, the integer would be positive.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Graph Integers on a Number Line

Objective

Students will learn how to graph a set of integers on a horizontal or vertical number line.

Teaching Notes

SLIDE 1

Point out that number lines can be horizontal or vertical. When graphing a set of integers, students should pay careful attention to how the sign of each integer indicates the location of that integer in relation to zero. On a horizontal number line, numbers to the left of zero are negative and numbers to the right of zero are positive. On a vertical number line, numbers below zero are negative and numbers above zero are positive. Prior to students selecting to graph each integer in the set {2, –3, 0}, have them first make a prediction as to the location of the integer in relationship to zero on each number line.

Check

The elevation of Death Valley National Park is the lowest in North America at 282 feet below sea level.

Part A

Write an integer to represent the situation. **–282**

Part B

Explain the meaning of zero in this situation.

In this situation, the integer 0 represents sea level.

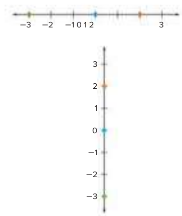
Go Online You can complete an Extra Example online.

Learn Graph Integers on a Number Line

Integers and sets of integers can be graphed on a number line. To graph an integer on a number line, place a dot on the number line at its location. Positive numbers are graphed to the right of zero on a horizontal number line, or above zero on a vertical number line. Negative numbers are graphed to the left of zero on a horizontal number line, or below zero on a vertical number line.

A set of integers is written using braces, such as {2, –3, 0}.

The set of integers {2, –3, 0} is graphed on each number line.



Talk About It!

Compare the horizontal and vertical number lines.

Sample answer: Positive numbers are graphed to the right of zero on a horizontal number line and above zero on a vertical number line. Negative numbers are graphed to the left of zero on a horizontal number line and below zero on a vertical number line.

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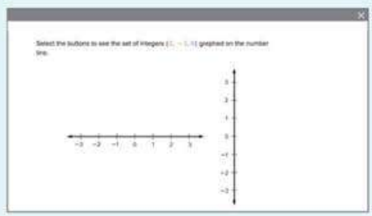
Interactive Presentation



Learn, Graph Integers on a Number Line, Slide 1 of 2

CLICK

On Slide 1, students select buttons to view a set of integers graphed on each number line.

DIFFERENTIATE

Reteaching Activity AL

To help students understand how to graph integers on a number line, explain that they can start by drawing a line and labeling the point 0. Have students determine if each of the following integers are to the left or to the right of 0 on a horizontal number line. Then have them determine if they are above 0 or below 0 on a vertical number line.

4 right; above

–9 left; below

–1 left; below

2 right; above

–5 left; below

6 right; above

3 right; above

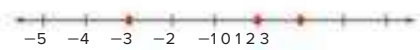
Example 2 Graph Integers on a Number Line

Graph the set of integers {−4, 2, −1} on the number line.

Place a dot at −4, 2, and −1.

Check

Graph the set of integers {−3, 1, 0} on a number line.



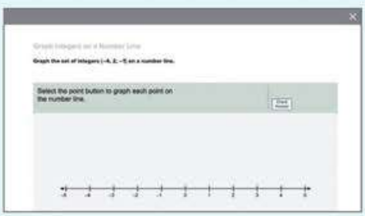
Go Online You can complete an Extra Example online.

Pause and Reflect

How well do you understand the concepts from today's lesson? What questions do you still have? How can you get those questions answered?

See students' observations.

Interactive Presentation



Example 2, Graph Integers on a Number Line, Slide 1 of 2

eTOOLS

On Slide 1, students use the Number Line eTool to graph a set of integers on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Graph Integers on a Number Line

Objective

Students will graph a set of integers on a number line.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use the Number Line eTool to graph the set of integers on a number line. Remind students to pay careful attention to 0 and to how each integer should be graphed in relationship to 0 on the number line.

Questions for Mathematical Discourse

SLIDE 1

AL How can you describe numbers to the left of 0 on a number line? numbers to the right of 0? Sample answer: Numbers to the left of 0 on a number line are negative. Numbers to the right of 0 on a number line are positive.

OL Why is −4 farther away from 0 than −1? Sample answer: −4 is 4 units to the left of 0, and −1 is 1 unit to the left of 0; 4 > 1

BL Which number in the set of integers is farthest away from 0? closest to 0? −4 is farthest away from 0; −1 is closest to 0

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Exit Ticket

Refer to the Exit Ticket slide. Phoebe deposits $225.00 into her savings account when she gets paid; then she withdraws $35.00 to see a movie with her friends. Describe the two situations using the words *positive* and *negative*. Sample answer: When Phoebe deposits $225.00 into her savings account, it will show up as a positive credit to her account because she is adding money to the account. When she withdraws $35.00, it will show up as a debit, which is a negative credit to her account. It shows up as a debit, because she is subtracting money from the account.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- AL Practice Form B
- OL Practice Form A
- BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	write an integer to represent a quantity and explain the meaning of zero in the situation	1–6
1	graph a set of integers on a number line	7–12
2	extend concepts learned in class to apply them in new contexts	13–14
3	solve application problems involving graphing integers on a number line	15–16
3	higher-order and critical thinking skills	17–20

Common Misconception

Students sometimes have difficultly determining whether or not a situation should be represented by a positive or a negative integer. As you complete the lesson, have students make a list of words that are commonly used to indicate positive and negative integers. Students can refer to this list as they complete Exercises 1–6. The list could include words such as *loss*, *owed*, *spent*, *below*, *under*, *gain*, *earned*, *saved*, *above*, and *rise*.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Write an integer to represent each situation. Explain the meaning of zero in each situation. (Example 1)

1. Since his last vet appointment, a cat lost 2 ounces.
   −2; The integer 0 represents no ounces gained or lost.

2. On first down, the football team gained 7 yards.
   7; The integer 0 represents no yards gained or lost.

3. Abigail withdrew $15 from her checking account.
   −15; The integer 0 represents no money withdrawn or deposited.

4. By noon, the temperature had risen 5 degrees Fahrenheit.
   5; The integer 0 represents no increase or decrease in temperature.

5. For the month of January, the amount of snowfall was 3 inches above average.
   3; The integer 0 represents average snowfall.

6. A dolphin is 20 feet below sea level.
   −20; The integer 0 represents sea level.

Graph each set of integers on a number line. (Example 2)

7. {−2, 0, 4}
   −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

8. {5, −5, −6}
   −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

9. {−8, −4, 1}
   −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3

10. {−7 , 3, 5}
    −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5

11. {7 ,−3, −1}
    −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

12. {−1, 0, 1}
    −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

Test Practice

13. The low temperatures for three consecutive days were −5°F, 3°F, and 4°F. Graph this set of integers on a number line.
    −5 −4 −3 −2 −1 0 1 2 3 4 5

14. **Multiple Choice** Salton City, California is located 38 meters below sea level. What is a possible elevation for Salton City?
    Ⓐ 380 m
    Ⓑ 38 m
    Ⓒ 0 m
    Ⓓ −38 m

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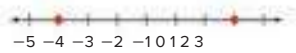
Interactive Presentation

Exit Ticket

Apply *indicates multi-step problem

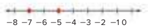
*15. Rodney is performing a science experiment. The table shows the temperature of two liquids he is using. Graph the integers that represent the temperatures on a number line. Which beaker's liquid is closer to 0°C? Explain.

Beaker	Temperature
A	−4°C
B	2°C



Beaker B; Sample answer: Beaker B is 2 units away from 0 on the number line, while Beaker A is 4 units from 0 on the number line. 4 > 2

*16. Sydney owes her mother \$5 and her brother owes her mother \$7 . Graph the integers that represent the amount they owe their mother as a negative integer on a number line. How much more will her brother have to repay their mother than Sydney? Explain.



\$2; Sample answer: Sydney's debt is 5 units from 0. Her brother's debt is 7 units from 0. This is 2 more units. So, he will have to pay \$2 more.

Higher-Order Thinking Problems

17. **MP Use Math Tools** Explain how to find the distance between 1 and −3 on a number line.

Sample answer: Graph 1 and −3 on a number line. Then count the units between each integer and zero. There is 1 unit between 0 and 1. There are 3 units between 0 and −3. So, 1 unit + 3 units = 4 units.

18. At midnight, the outside temperature was 0°F.

a. By 6:00 A.M., the temperature had dropped 4°F, and then the temperature raised 10°F by noon. What is the temperature at noon?

6°F

b. What represents zero in this situation? Explain.

Sample answer: Zero represents 0°F.

19. **Create** Describe a real-world situation that can be represented by a negative integer. Then write the integer.

Sample answer: Riley lost 10 points playing a trivia game; −10

20. **MP Justify Conclusions** Craig has \$28 in his checking account. He wants to make a withdrawal of \$30. Will his checking account balance be represented by a positive or negative integer after the withdrawal? Justify your conclusion.

negative; Sample answer: A withdrawal of \$28 would result in a balance of \$0. Since the withdrawal of \$30 is greater than \$28, the balance will be less than zero and would be represented with a negative integer.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically In Exercise 17, students explain how to use a number line to find the distance between two integers.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 20, students explain if the amount in a bank account will be represented by a positive or negative integer and why.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Interview a student.

Use with Exercise 18 Have pairs of students interview each other as they complete this problem. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 18 might be "What is the best method to use to find the new temperature?"

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1–5 odd, 13, 15, 17–20
- ALEKS Plotting and Comparing Signed Numbers

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–12, 15, 17, 18
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Plotting and Comparing Signed Numbers

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Signed Numbers

Learn Find Opposites

Objective

Students will understand what the opposite of an integer is, and where it is located on the number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to use reasoning to make sense of why 0 is the opposite of 0.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language in their explanations.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Explain why 0 is its own opposite. Sample answer: The opposite of a number *a* is the number *b* that is the same distance from zero as the number *a*, but on the opposite side of the number line from zero. Since 0 is 0 units away from 0, and 0 is neither positive nor negative, 0 is its own opposite.

DIFFERENTIATE

Reteaching Activity AL

To help students that may be struggling to understand how to identify opposites, explain that they should first calculate the distance to zero and note the direction of the integer. The opposite of an integer is the same distance from zero but in the opposite direction. For each of the following integers, have students identify the opposite's direction and distance from 0.

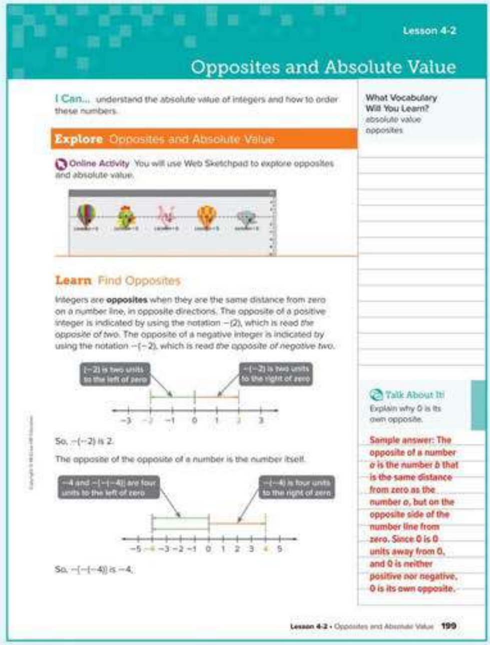
2 The opposite is 2 units to the left of 0.

–3 The opposite is 3 units to the right of 0.

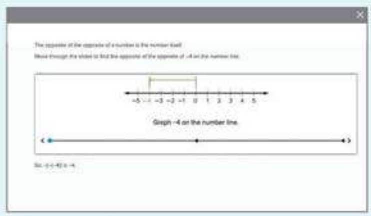
9 The opposite is 9 units to the left of 0.

–7 The opposite is 7 units to the right of 0.

–2 The opposite is 2 units to the right of 0.



Interactive Presentation



Learn, Find Opposites, Slide 2 of 3

CLICK

On Slide 1, students view examples of opposites on the number line.

CLICK

On Slide 2, students move through the slides to use a number line to see the opposite of the opposite of –4.

Lesson 4-2

Opposites and Absolute Value

6.NS.C.5, 6.NS.C.6, 6.NS.C.7

LESSON GOAL

Students will find the opposites of integers and use opposites to understand absolute value.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Opposites and Absolute Value

Learn: Find Opposites

Example 1: Use a Number Line to Find Opposites of Integers

Example 2: Find Opposites of Integers Using Symbols

Example 3: Find Opposites of Opposites of Integers

Learn: Absolute Value of Integers

Example 4: Find the Absolute Value of Integers

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

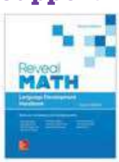
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 21 of the *Language Development Handbook* to help your students build mathematical language related to opposites and absolute value.



ELL You can use the tips and suggestions on page T21 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System

Major Cluster(s): In this lesson, students address major cluster **6.NS.C** by finding the opposites of integers and using opposites to understand absolute value.

Standards for Mathematical Content: **6.NS.C.5, 6.NS.C.6, 6.NS.C.6.A, 6.NS.C.7, 6.NS.C.7.C**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP5, MP6, MP8**

Coherence

Vertical Alignment

Previous

Students graphed integers on a number line to represent data.
6.NS.C.5, 6.NS.C.6

Now

Students find the opposites of integers and use opposites to understand absolute value.
6.NS.C.5, 6.NS.C.6, 6.NS.C.7

Next

Students will compare and order integers on a number line.
6.NS.C.7

Rigor

The Three Pillars of Rigor

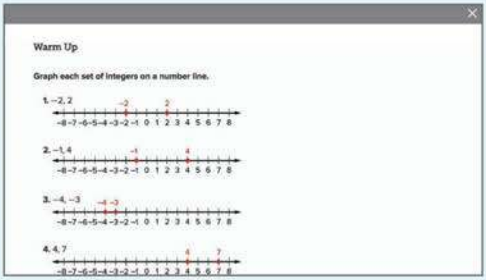
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of graphing integers on a number line to develop *understanding* of opposites of integers and absolute value. They use this understanding to build *fluency* with writing the opposite of an integer, writing the opposite of the opposite of an integer, and finding the absolute value of an integer. They also *apply* their understanding of opposites and absolute value to solve real-world problems.

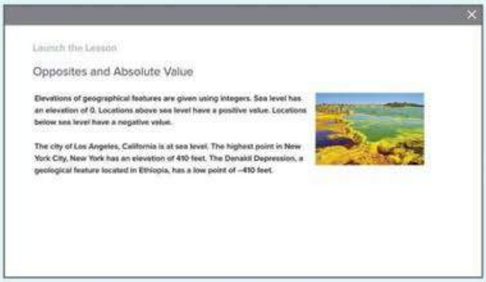
Mathematical Background

The *opposite* of a number is the number that is the same distance from zero on a number line. The *absolute value* of a number is the distance between the number and zero on a number line. Numbers that are the same distance from zero on a number line have the same absolute value.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- graphing integers on a number line (Exercises 1–5)

Answers

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about elevation as a representation of absolute value.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *absolute* comes from the Latin *absolutus*, which means *unrestricted*. Where might you have heard or seen the term absolute in everyday life? Sample answers: Someone who is 100% sure of a decision might say they are absolutely sure, something that is 100% true might be described as the absolute truth
- Give some examples of opposites from everyday life. Sample answers: left and right, up and down, in and out, stop and go

Explore Oppositesand Absolute Value

Objective

Students will use Web Sketchpad to explore opposites and absolute value.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with scenarios of balloons moving in specified directions. They will use integers to describe their distances and locations.

Inquiry Question

How can you use integers to describe direction and distance? Sample answer: Positive and negative integers can be used to describe direction. Positive integers can be used to describe distance.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

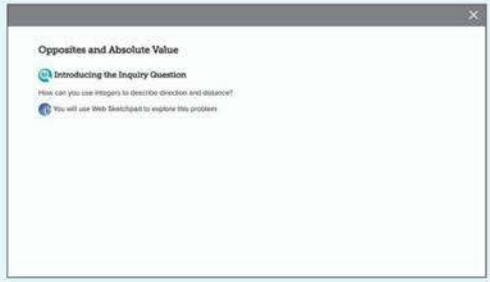
SLIDE 4

Mathematical Discourse

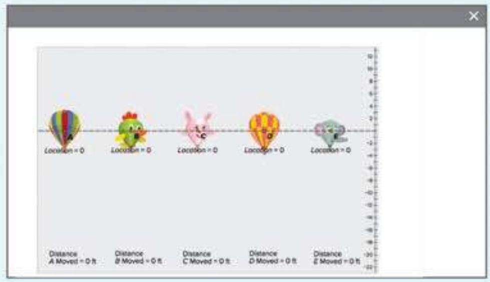
The integers, 8 and −8, used to represent the height of Balloon A and the new height of Balloon C, are called *opposites*. Think about the location of these integers on a number line to explain why these values are opposites. Sample answer: The values are the same distance from 0 on a number line. They are in the same position, but on opposite sides of 0.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 7



Explore, Slide 2 of 7

WEB SKETCHPAD

On Slide 2, students use Web Sketchpad to explore how integers can be used to describe direction and distance.

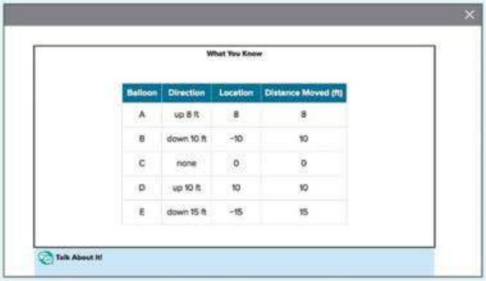
TYPE

On Slide 2, students complete a table to compare the vertical distance each balloon moved from its starting point.

TYPE

On Slide 3, students type to indicate how many feet Balloons B and D move.

Interactive Presentation



Balloon	Direction	Location	Distance Moved (ft)
A	up 8 ft	8	8
B	down 10 ft	−10	10
C	none	0	0
D	up 10 ft	10	10
E	down 15 ft	−15	15

Explore, Slide 5 of 7

TYPE

On Slide 6, students make a conjecture about the distance from zero for an integer and its opposite.

TYPE

On Slide 7, students respond to the Inquiry Question and can view a sample answer.

Explore Opposites and Absolute Value *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use Web Sketchpad to model the movements of the balloons, in order to explore how they can use integers to understand direction and distance.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

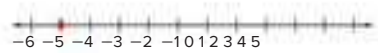
What do you notice about the values in the *Location* column and the values in the *Distance Moved* column for each balloon? Sample answer: The values in the *Distance Moved* column are all positive. The values in the *Location* column are either positive or negative, depending on whether the balloon moved up or down.

Your Notes

Example 1 Use a Number Line to Find Opposites of Integers

Find −(−5).

Graph −5 on the number line.



The point graphed at −5 is 5 units to the left of 0. The point that is the same number of units to the right of 0 is 5.

So, the opposite of −5 is 5.

Check

Find −(−21). 21

Show your work here

Go Online Y ou can complete an Extra Example online.

Example 2 Find Opposites of Integers Using Symbols

Asia and La T oya are building a sandcastle and digging a moat around the sandcastle. They would like the moat to be as deep as the sandcastle is tall. The sandcastle is 17 inches tall.

What integer represents the depth of the moat? How does this integer compare to the height of the sandcastle?

The depth of the moat can be expressed as the integer that is the opposite of 17 . The opposite of a positive is negative.

So, the integer that represents the depth of the moat is −(17) or −17.

The integers representing the height of the sandcastle and the depth of the moat are opposites.

Check

Josh is planting a flower that is 6 inches tall. He wants the hole he is digging to be as deep as the flower is tall. What integer represents the depth of the hole? How does this compare to the height of the flower?

Show your work here

−6; Sample answer: This is the opposite of the height of the plant.

Go Online Y ou can complete an Extra Example online.

Talk About It! Can all positive integers be written with or without the + sign? Can all negative integers be written with or without the − sign? Explain.

Sample answer: All positive integers can be written with or without the + sign, but all negative integers must be written with the − sign, in order to distinguish them from positive integers.

Interactive Presentation

Example 1, Use a Number Line to Find Opposites of Integers, Slide 1 of 2

eTOOLS

In Slide 1 of Example 1, students use the Number Line eTool to graph a point on a number line.

TYPE

On Slide 2 of Example 2, students type to indicate the opposite of 17.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Use a Number Line to Find Opposites of Integers

Objective

Students will find the opposite of an integer by using a number line.

Questions for Mathematical Discourse

SLIDE 1

AL Why is −5 to the left of 0? Negative numbers are found to the left of 0 on a horizontal number line.

OL Why are −5 and 5 opposites? Sample answer: −5 and 5 are opposites, because they are the same number of units away from 0, but on opposite sides of 0.

BL What is the opposite of the opposite of −5? −5

Example 2 Find Opposites of Integers Using Symbols

Objective

Students will find the opposite of an integer by using symbols.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the relationship between the height of the sandcastle and the depth of the moat. Students should understand why the integers representing these quantities are opposites.

Questions for Mathematical Discourse

SLIDE 2

AL What integer represents the height of the sandcastle? 17

OL Why is the integer that represents the depth of the moat the opposite of 17? The depth of a moat will be below the level of the ground, the same distance as the height of the sandcastle.

BL If the depth of the moat was half of the height of the sandcastle, what negative number would represent the depth of the moat? −8.5

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 3 Find Opposites of Opposites of Integers

Objective

Students will find the opposite of the opposite of an integer.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to compare the values and explain why the opposite of the opposite of a number is the original number. Students should explain the meaning of the symbols in the context of the problem.

8 Look for and Express Regularity in Repeated Reasoning Have students use patterns to make a conjecture about the relationship between the number of negative signs in an expression involving an integer and the integer itself. For example, [–(–3)] is the opposite of 3, but–(–3) is equivalent to 3.

Questions for Mathematical Discourse

SLIDE 2

AL How can you read the expression? the opposite of the opposite of −3

OL How can you simplify the problem into smaller steps that are easier to solve? Sample answer: First find the opposite of −3. Then find the opposite of that number.

BL How does −[−(−3)] compare to 3? They are opposites.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Absolute Value of Integers

Objective

Students will understand that the absolute value of an integer is the distance the integer is from zero on the number line.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

Why is the absolute value of a number never negative? Sample answer: The absolute value of a number refers to its distance from 0 on a number line. Distance cannot be a negative number.

Example 3 Find Opposites of Opposites of Integers

Find −[−(−3)].

−[−(−3)]

3 — The opposite of −3 is 3.

−3 — The opposite of 3 is −3.

So, the opposite of the opposite of −3 is −3.

Check

Find −[−(−11)]. −11

Show your work here

Go Online You can complete an Extra Example online.

Talk About It! Compare the opposite of the opposite of a number to the original number.

Sample answer: The opposite of the opposite of the number is the number itself.

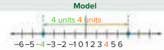
Learn Absolute Value of Integers

The integers 4 and −4 are each 4 units from 0, even though they are on opposite sides of 0. Numbers that are the same distance from zero on a number line have the same **absolute value**.

Words

The absolute value of a number is the distance between the number and zero on the number line.

Model



Symbols

The | | symbol around a number means the absolute value of that number.

$|4| = 4$ The absolute value of 4 is 4.

$|-4| = 4$ The absolute value of −4 is 4.

Talk About It! Why is the absolute value of a number never negative?

Sample answer: The absolute value of a number refers to its distance from 0 on a number line. Distance cannot be a negative number.

Lesson 4-2 • Opposites and Absolute Value 201

Interactive Presentation

Learn, Absolute Value of Integers, Slide 1 of 2

CLICK On Slide 2 of Example 3, students move through the steps to find –[–(–3)].

FLASHCARDS On Slide 1 of the Learn, students use Flashcards to view multiple representations of absolute value.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find the Absolute Value of Integers

Think About It!
Is the location represented by a positive or negative integer?
negative integer

A cave explorer started at sea level and descended in a cave. Her location, in relationship to her starting point, can be represented by −150 feet.

How many feet did the cave explorer travel?

To find how many feet the cave explorer traveled, you need to find |−150|.

To find the absolute value, find the distance between the number and zero on a number line.

Graph −150 on the number line.

How many units from 0 is −150? 150 units

So, the cave explorer traveled |−150| or 150 feet.

Talk About It!
What other number has the same absolute value as −150? Explain your reasoning.
150; Sample answer: Both numbers are 150 units from 0 on a number line.

Check

Yixi dropped a coin in a wishing well. The top of the well can be represented by 0 feet. The location of the coin can be represented by −32 feet. How many feet did the coin fall? 32

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How are opposites related to absolute value? Why do you think these concepts are covered in the same lesson?

Record your observations here

See students' observations.

Interactive Presentation

Example 4, Find the Absolute Value of Integers, Slide 2 of 4

CLICK

On Slide 2, students select from a drop-down menu to specify how to find absolute value.

TYPE

On Slide 2, students determine how many units –150 is from 0.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find the Absolute Value of Integers

Objective

Students will find the absolute value of an integer to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the relationship between the integer that represents the cave explorer's final location, and the distance traveled. Students should use reasoning to determine that the absolute value of the integer gives the distance traveled.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use reasoning to determine what other number has the same absolute value as the integer given in this example.

Questions for Mathematical Discourse

SLIDE 2

AL The explorer's final location can be represented by the integer −150. Is her final location above or below her starting point? below her starting point

OL Explain why you need to find the absolute value of −150 to solve this problem. Sample answer: The absolute value of −150 will give the distance the cave explorer traveled.

OL Why is the distance not −150 feet? Sample answer: Distance can never be negative.

BL If she traveled back up to her starting point, what will be her total distance traveled? What integer now represents her location? She traveled a total distance of 150 + 150, or 300 feet, but the integer representing her location in relationship to her starting point is 0.

Go Online

- Find additional teaching notes and the *Talk About It!* question.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Students may have heard the phrase *absolute certainty* in everyday life. In this context, the term *absolute* means *unchanging* or *universal*. Have students discuss how the term *absolute value* in math is related to the concept of something that is *unchanging* or *universal*. For example, the integers 3 and −3 both have the same absolute value of 3. The distance each integer is from zero is *unchanged*, even though the integers are on opposite sides of zero. Since the distance from zero is unchanged, the absolute value of the integers is the same.

Exit Ticket

Refer to the Exit Ticket slide. Write a few sentences comparing and contrasting the elevations of New York City and the Danakil Depression, and how far away each elevation is from sea level. Use the terms *opposite* and *absolute* value in your explanation. Sample answer: The elevations of New York City and the Danakil Depression are opposites of each other, because the integers 410 and −410 are the same distance from 0, but on opposite sides of 0 on the number line. The two elevations are the same distance from sea level because the absolute values of 410 and −410 are both equal to 410.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use a number line to find the opposite of an integer	1–3
2	use symbols to find the opposite of an integer	4, 5
1	find the opposite or the opposite of the opposite of an integer	6–11
2	find the absolute value of an integer to solve a real-world problem	12, 13
2	extend concepts learned in class to apply them in new contexts	14
3	solve application problems involving ordering using absolute value	15, 16
3	higher-order and critical thinking skills	17–20

Common Misconception

Students may confuse the definitions of opposites and absolute value. Remind students that *opposites* are on *opposite* sides of zero on the number line. There will always be a positive number and a negative number in a pair of opposites. The *absolute value* of a number is the *distance* that a number is from zero on a number line. Since distance is always positive, the absolute value of a number is always positive.

Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

Find the opposite of each integer. (Example 1)

1. −3 3
2. 2 −2
3. 6 −6

4. Chad is planting a plant that is 4 inches tall. He wants the hole he is digging to be as deep as the plant is tall. What integer represents the location of the bottom of the hole? How does this compare to the height of the plant? (Example 2)
−4; Sample answer: This is the opposite of the height of the plant.

5. A hill on a dirt bike course is 5 feet tall. The valley below the hill is as deep as the hill is tall. What integer represents the location of the bottom of the valley? How does this compare to the height of the hill? (Example 2)
−5; Sample answer: This is the opposite of the height of the hill.

Find each value. (Examples 2 and 3)

6. −(−15) = 15
7. −(−11) = 11
8. −[−(−7)] = −7
9. −[−(−1)] = −1
10. −[−(55)] = 55
11. −[−(100)] = 100

12. A mountain climber started at sea level and descended down a cliff. Her location can be represented by −75 feet. How many feet did the mountain climber travel? (Example 4)
75 feet

13. The temperature was −5°F when Tiffany woke up in the morning. By noon, the temperature was 0°F. How many degrees did the temperature change? (Example 4)
5 degrees

Test Practice

14. **Multiselect** Which of the following represent opposites?
- [x] −4 and 4
- [x] −1 and 1
- [ ] −2 and −1
- [ ] 0 and 1
- [ ] −7 and −8
- [x] 10 and −10

Lesson 4-2 • Opposites and Absolute Value 203

Interactive Presentation

Exit Ticket

Apply *indicates multi-step problem

*15. The table shows the minimum and maximum elevations, relative to sea level, of several hiking trails. Which hiking trail has the least change in elevation, related to sea level? Explain how you solved.

Trail	Minimum Elevation (ft)	Maximum Elevation (ft)
Eastern Point	−85	78
Northern Star	−150	34
Southern Moon	−62	48

Southern Moon; Sample answer: I found the absolute value of each minimum elevation and added the maximum elevation for each trail. The change in elevation for Southern Moon is 62 + 48, or 110, which is the least change of the three trails.

*16. The table shows the lowest and highest record temperatures for three cities. Which city had the greatest change in record temperature? Explain how you solved.

City	Lowest Temperature (F)	Highest Temperature (°F)
Boston	−30	104
Las Vegas	8	118
Pittsburgh	22	103

Boston; Sample answer: I found the absolute value of each lowest temperature and added the highest temperature for each city. The change in temperature for Boston is 30 + 104, or 134, which is the greatest change of the three cities.

Higher-Order Thinking Problems

17. MP **Reason Inductively** Determine if the following statement is *true* or *false*. Explain your reasoning.

The absolute value of a negative integer is always a negative integer.

false; Sample answer: Absolute value is a measure of distance and distance can never be negative.

18. MP **Find the Error** Judith states that $-|14| = 14$ because the absolute value can never be negative. Find her mistake and correct it.

Sample answer: $-|14|$ means *the opposite of the absolute value of 14*. Judith is correct that the absolute value can never be negative, but the opposite of the absolute value will always be negative (unless it is 0). The correct answer is $-|14| = -14$.

19. MP **Justify Conclusions** A student states that $-x$ is always equal to a negative integer. Is the student correct? Justify your reasoning.

no; Sample answer: If x is a positive integer such as 1, then the result is −1. If x is a negative integer such as −1, then the result is 1.

20. MP **Persevere with Problems** Identify integers for x and y that make the following statement true.

$x > y$ and $|x| < |y|$

Sample answer: $x = 5$ and $y = -7$

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 17, students determine if a statement is true or false and justify their reasoning.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students explain why another student's solution is incorrect and then correct the solution.

In Exercise 19, students analyze another student's statement to determine if it is correct.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 20, students use multiple steps to find integers that satisfy multiple criteria.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Listen and ask clarifying questions.

Use with Exercises 15–16 Have students work in pairs. Have students individually read Exercise 15 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 16.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 4, 12, 14, 16–20
- ALEKS Plotting and Comparing Signed Numbers

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–14, 18, 19
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Plotting and Comparing Signed Numbers

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Signed Numbers

Learn Compare Integers

Objective

Students will understand that they can compare two integers by reasoning about their signs and locations on a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the quantities –2 and –3, in order to reason that while $-2 > -3$, the absolute value of –2 is less than the absolute value of –3. This is true for all negative numbers. Encourage students to explain why, using their understanding of distance between a negative number and zero.

Teaching Notes

SLIDE 1

To compare two integers, have students first look at the signs of the two integers. Be sure they understand that, if the signs of two integers are different, a positive integer will always be greater than a negative integer.

SLIDE 2

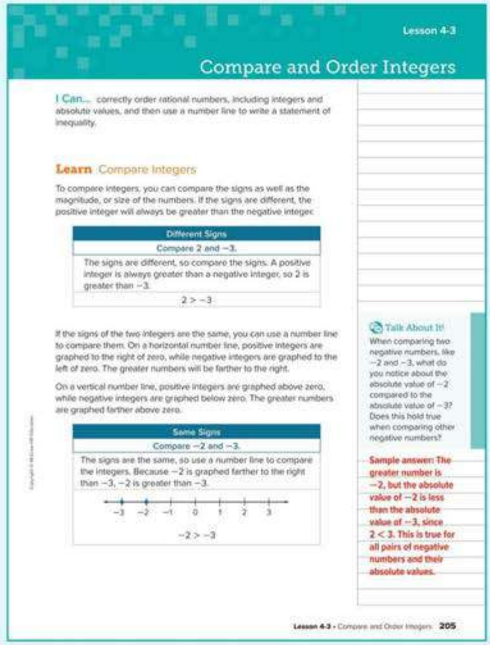
If the signs of two integers are the same, students can graph the integers on a number line to compare their magnitudes. When two numbers are graphed on a number line, the greater number is to the right of the lesser number. Students can also use reasoning to compare the numbers without physically graphing them on a number line. Have them imagine a number line in their minds. If both integers are positive, the number farther away from zero is greater. For example, $3 > 2$, because 3 is farther away from 0 than 2. If both integers are negative, the number closer to zero is greater. For example $-2 > -3$, because –2 is closer to 0 than –3.

Talk About It!

SLIDE 3

Mathematical Discourse

When comparing two negative numbers, like –2 and –3, what do you notice about the absolute value of –2 compared to the absolute value of –3? Does this hold true when comparing other negative numbers? Sample answer: The greater number is –2, but the absolute value of –2 is less than the absolute value of –3, since $2 < 3$. This is true for all pairs of negative numbers and their absolute values.



Lesson 4-3

Compare and Order Integers

I Can... correctly order rational numbers, including integers and absolute values, and then use a number line to write a statement of inequality.

Learn Compare Integers

To compare integers, you can compare the signs as well as the magnitude, or size of the numbers. If the signs are different, the positive integer will always be greater than the negative integer.

Different Signs
Compare 2 and −3.
The signs are different, so compare the signs. A positive integer is always greater than a negative integer, so 2 is greater than −3.
$2 > -3$

If the signs of the two integers are the same, you can use a number line to compare them. On a horizontal number line, positive integers are graphed to the right of zero, while negative integers are graphed to the left of zero. The greater numbers will be farther to the right.

On a vertical number line, positive integers are graphed above zero, while negative integers are graphed below zero. The greater numbers are graphed farther above zero.

Same Signs
Compare −2 and −3.
The signs are the same, so use a number line to compare the integers. Because −2 is graphed farther to the right than −3, −2 is greater than −3.
−3 −2 −1 0 1 2 3
$-2 > -3$

Talk About It!

When comparing two negative numbers, like −2 and −3, what do you notice about the absolute value of −2 compared to the absolute value of −3? Does this hold true when comparing other negative numbers?

Sample answer: The greater number is −2, but the absolute value of −2 is less than the absolute value of −3, since $2 < 3$. This is true for all pairs of negative numbers and their absolute values.

Lesson 4-3 • Compare and Order Integers 205

Interactive Presentation

Learn, Compare Integers, Slide 1 of 3

CLICK

On Slide 1, students move through the slides to compare integers with different signs.

CLICK

On Slide 2, students move through the Slides to compare integers with the same sign.

Lesson 4-3

Compare and Order Integers

6.NS.C.7

LESSON GOAL

Students will compare and order integers using a number line.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Compare Integers
Example 1: Compare Two Integers
Learn: Order Sets of Integers
Example 2: Order Sets of Integers
Learn: Distinguish Absolute Value from Order
Example 3: Comparisons with Absolute Value
Apply: Chemistry

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 22 of the *Language Development Handbook* to help your students build mathematical language related to comparing and ordering integers.

ELL You can use the tips and suggestions on page T22 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min: 1 day
45 min: 2 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.C** by comparing and ordering integers and using their absolute values to solve problems.
Standards for Mathematical Content: **6.NS.C.7, 6.NS.C.7.A, 6.NS.C.7.B, 6.NS.C.7.C, 6.NS.C.7.D,** *Also addresses 6.NS.C.6, 6.NS.C.6.C*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the opposites of integers and used opposites to understand absolute value.
6.NS.C.5, 6.NS.C.6, 6.NS.C.7

Now
Students compare and order integers on a number line.
6.NS.C.7

Next
Students will reason about rational numbers on a number line.
6.NS.C.6, 6.NS.C.7

Rigor

The Three Pillars of Rigor

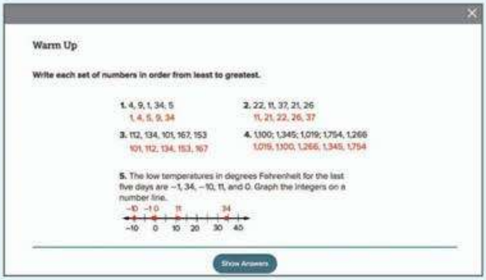
1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of integers by using the processes of comparing and ordering. They learn to write inequalities to build *fluency* with ordering sets of integers, and distinguish between comparisons of absolute value and comparisons about order. They *apply* their understanding of comparing and ordering integers to solve real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- comparing and ordering whole numbers (Exercises 1–4)
- graphing integers on a number line (Exercise 5)

Answers

1. 1, 4, 5, 9, 34
2. 11, 21, 22, 26, 37
3. 101, 112, 134, 153, 167
4. 1,019; 1,100; 1,266; 1,345; 1,754
5. See Warm Up slide online for correct answer.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about ordering a golfer's scores.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What is an example of the *absolute value* of a number? Sample answer: |−21|; The absolute value of −21 is 21.
- What is an example and non-example of an *integer*? Sample answer: An example of an integer is −4. A non-example of an integer is $\frac{1}{16}$.

asiseeit/E+/Getty Images

Your Notes

Think About It!
How can you compare two negative numbers?

See students' responses.

Talk About It!
What is another way to write an inequality comparing −3 and −5? Explain why this inequality is also true.

Sample answer: −5 < −3; Both inequalities show that −5 has a lesser value than −3.

Example 1 Compare Two Integers

Justin has a score of −5 on the Trueville Trivia Game. Desiree's score is −3.

Write an inequality to compare the scores. Then explain the meaning of the inequality.

Part A Write an inequality.

Graph the integers on the number line.

−5 −4 −3 −2 −1 0 1 2 3 4 5

Compare. Which number is farther to the right on the number line? −3

The inequality is −3 > −5.

Part B Explain the meaning of the inequality.

Since −3 > −5, Desiree has a greater score in the trivia game.

Check

Andrew and his father are hiking near Tackle Box Canyon. Their current elevation, in relation to sea level, is −38 feet. Tackle Box Canyon has an elevation of −83 feet.

Part A Write an inequality to compare the elevations.

−38 > −83

Part B Explain the meaning of the inequality.

Show your work here

Sample answer: Andrew and his father's current elevation is closer to sea level than Tackle Box Canyon.

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 1, Compare Two Integers, Slide 2 of 5

eTOOLS

On Slide 2, students use the Number Line eTool to graph integers on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Compare Two Integers

Objective

Students will write an inequality to compare two integers and explain the meaning of the inequality.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to manipulate the symbols in the inequality in order to write a different, yet equivalent inequality.

5 Use Appropriate Tools Strategically Students will use the Number Line eTool to graph the integers on the number line in order to visually compare them.

Questions for Mathematical Discourse

SLIDE 2

AL Which integer is farther to the right on the number line? What does this mean? −3 is farther to the right; This means that −3 > −5.

OL What other strategy, besides a number line, can you use to compare the integers? Sample answer: Since both integers are negative, the integer with the greater absolute value, −5, will be the lesser integer.

OL A classmate wrote the inequality −5 < −3. Is this inequality correct? Explain. yes; Sample answer: The inequality reads *−5 is less than −3*, which means the same as *−3 is greater than −5*.

BL How far away from −3 is −5? How can you determine this? 2 units; Sample answer: Count the number of units it takes to travel from −3 to −5.

SLIDE 3

AL What does the inequality −3 > −5 mean? The inequality means that −3 is greater than −5.

OL How is it possible that a negative score can be declared the winner? Sample answer: The rules of the game indicate that the greater score is the winner, and −3 > −5.

BL If Desiree's score increased by 2 and Justin's score decreased by 3, who will win the game? Explain. Desiree; Sample answer: Desiree's new score is −1 and Justin's new score is −8. Since −1 > −8, Desiree will win the game.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Order Sets of Integers

Objective

Students will understand that a number line can be used to order a set of integers.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use clear and precise mathematical language in order to explain the usefulness of a number line when comparing a large set of integers.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how a number line can help order integers from least to greatest.

Talk About It!

SLIDE 2

Mathematical Discourse

How does a number line help to organize a set of integers? Sample answer: The number line organizes the integers in order, so that I don't have to compare pairs of integers and remember in what order to place them.

Example 2 Order Sets of Integers

Objective

Students will order a set of integers.

Questions for Mathematical Discourse

SLIDE 1

AL After graphing the values on the number line, how can you tell what the least value is? The integer −105 is the farthest to the left of 0, so it is the least integer.

OL Explain why it makes sense that the greatest integer is still negative. Sample answer: Integers that are greater than other integers are not always positive integers. If the data set only contains negative integers, such as this one, then the greatest integer will be negative.

BL Which elevation is the closest to sea level? −15 (Australia)

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Order Sets of Integers

Y ou can use a number line to order a set of integers from least to greatest or from greatest to least.

Go Online Watch the animation to see how you can use a number line to order a set of integers.

The animation shows how to graph the set of integers {−8, 3, −1, 0, 6} on a number line.

From left to right, the integers from least to greatest are {−8, −1, 0, 3, 6}.

From right to left, the integers from greatest to least are {6, 3, 0, −1, −8}.

Talk About It!
How does a number line help to organize a set of integers?

Sample answer: The number line organizes the integers in order so that I don't have to compare pairs of integers and remember in what order to place them.

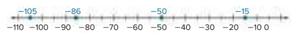
Example 2 Order Sets of Integers

The table shows the lowest accessible elevations for several continents.

Continent	Lowest Elevation (m)
Antarctica	−50
Australia	−15
North America	−86
South America	−105

Order the continents from least to greatest according to their lowest elevation.

Graph the integers on a number line.



Which continent has the least accessible elevation?
South America

Which continent has the greatest accessible elevation?
Australia

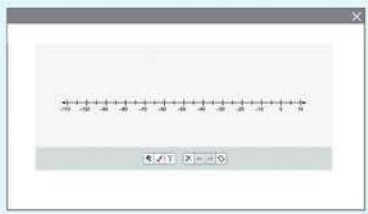
So, the continents written in order from least to greatest elevation are South America, North America, Antarctica, and Australia.

Talk About It!
The lowest elevation in Asia is near the Dead Sea at −423 meters. The lowest elevation in Africa is near Lake Assal at −157 meters. How would adding these values to the data set change the number line and the order of the elevations?

Sample answer: By adding two numbers that are less than the least value, I need to extend the number line to −430 to include the values −157 and −423. The order of the numbers would be −423, −157 ,−105, −86, −50, −15.

Lesson 4-3 • Compare and Order Integers 207

Interactive Presentation



Example 2, Order Sets of Integers, Slide 1 of 3

WATCH

On Slide 1 of the Learn, students watch an animation that illustrates how to use a number line to order sets of integers.

eTOOLS

On Slide 1 of Example 2, students use the Number Line eTool to graph integers on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The table shows Kesha's cell phone use over the last four months. Positive values indicate the number of minutes she had remaining, and negative values indicate the number of minutes she went over. Arrange the months from fewest to most minutes remaining at the end of each month. **February, May, April, March**

Month	Number of Minutes Over/Under
February	−156
March	12
April	0
May	−45

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you struggle with any of the concepts in this Check? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

Record your observations here

See students' observations.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of comparing and ordering integers, have them work with a partner to create two different sets of integers that are not in numerical order. There should be a mix of both positive and negative integers, and at least 4 integers in each set. Have them label one set as Set A, and the other set as Set B. Then have them trade sets with another pair of students. Each pair should order the integers in Set A from least to greatest, and the integers in Set B from greatest to least. Have pairs of students check each other's work, and discuss and resolve any differences.

Learn Distinguish Absolute Value from Order

Objective

Students will understand how to distinguish between comparisons of absolute value and comparisons about order.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use precise terminology from their everyday lives to describe situations where negative and positive integers are used.

Teaching Notes

SLIDE 1

Be sure that students understand that since the absolute value of a number represents the distance the number is from zero, the absolute value increases the farther the number is from zero. As students move through the slides in the interactive tool, ask them to clarify their own understanding by generating numerical examples. For example, they might use the numbers 2 and 3 to illustrate that, as the value of a positive number increases, the absolute value increases. The number 3 is greater than 2 (because it is to the right of 2), and the absolute value of 3 is also greater than 2 (because 3 is farther away from 0 than 2). They might use the numbers −2 and −3 to illustrate that, as the value of a negative number decreases, the absolute value increases. The number −3 is less than −2 (because it is to the left of −2), but the absolute value of −3 is greater than the absolute value of −2 (because it is farther away from 0).

SLIDE 2

Have students further their understanding of this concept by discussing the real-world scenario presented. Ask students to compare the integers −25 and −30. They should note that −25 is greater than −30, because −25 is to the right of −30 on a number line. However, Kaito's depth was *less than* Ember's depth. Ask students to explain why. They should note that the depth of each diver is the distance from sea level (0 depth). Since Ember dove deeper than Kaito (farther away from sea level), her depth is *greater*, even though −30 is *less than* −25.

Talk About It!

SLIDE 3

Mathematical Discourse

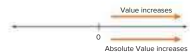
Some words imply a negative value, like depth. What other words imply the sign of the number? Sample answer: loss, gain, withdrawal, deposit, profit, debt

Learn Distinguish Absolute Value from Order

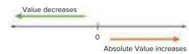
Y ou know how to order numbers when you see them on a horizontal number line. The values increase as they move to the right, and the values decrease as they move to the left.

What happens to the absolute value, or magnitude, of numbers as the values increase or decrease? Since absolute value is the distance a number is from zero, the absolute value increases the farther the number is from zero.

As a positive value increases, or moves farther from 0, its absolute value also increases.



As a negative value decreases, or moves farther from 0, its absolute value increases.



Suppose Kaito and Ember are scuba diving.

Kaito dove to 25 feet below sea level. This can be represented by the integer __−25__.

Ember dove to 30 feet below sea level. This can be represented by the integer __−30__. Who reached a greater depth?

Y ou know that−25 > −30, but this does not mean that Kaito's depth was greater. When determining who reached a greater depth, you need to consider the magnitude of the numbers, not just their placement on the number line.

The absolute value of a number takes into account the number's magnitude.

What is the absolute value of −30? __30__

What is the absolute value of −25? __25__

Which absolute value is greater? __30__

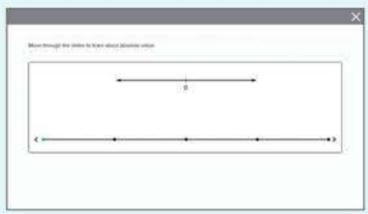
Since |−30| > |−25|, Ember's depth is greater.

Talk About It!
Some words imply a negative value, like depth. What other words imply the sign of the number?

Sample answer: loss, gain, withdrawal, deposit, profit, debt

Lesson 4-3 • Compare and Order Integers 209

Interactive Presentation

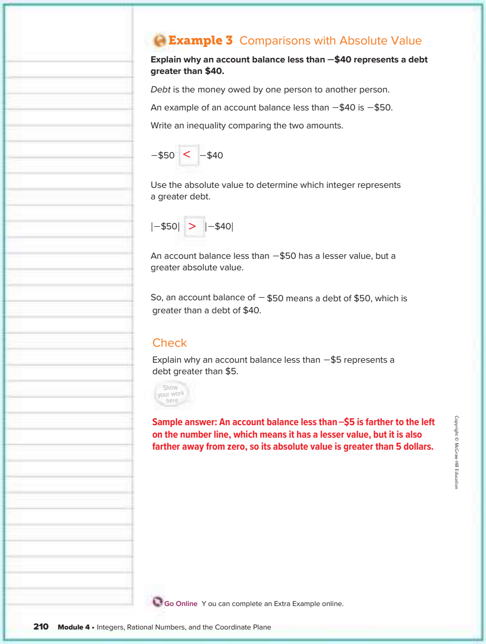


Learn, Distinguish Absolute Value from Order, Slide 1 of 3

CLICK On Slide 1, students use a visual model to distinguish absolute value from order.

CLICK On Slide 2, students distinguish absolute value from order in a real-world setting.

TYPE On Slide 2, students type to indicate absolute values of integers.



Example 3 Comparisons with Absolute Value

Explain why an account balance less than –$40 represents a debt greater than $40.

Debt is the money owed by one person to another person.

An example of an account balance less than –$40 is –$50.

Write an inequality comparing the two amounts.

–$50 < –$40

Use the absolute value to determine which integer represents a greater debt.

|–$50| > |–$40|

An account balance less than –$50 has a lesser value, but a greater absolute value.

So, an account balance of – $50 means a debt of $50, which is greater than a debt of $40.

Check

Explain why an account balance less than –$5 represents a debt greater than $5.

Show your work here

Sample answer: An account balance less than –$5 is farther to the left on the number line, which means it has a lesser value, but it is also farther away from zero, so its absolute value is greater than 5 dollars.

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Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3, Comparisons with Absolute Value, Slide 1 of 2

CLICK

On Slide 1, students distinguish absolute value from order in a real-world setting.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Comparisons with Absolute Value

Objective

Students will distinguish between comparisons of absolute value and comparisons about order.

Questions for Mathematical Discourse

SLIDE 1

AL What do you know about debt? Sample answer: Debt is when you owe money to someone else. The greater the debt, the more money you owe.

AL What are some possible account balances that are less than –$40? Sample answers: –$50, –$75, –$90

OL Explain why it makes sense that as a negative account balance decreases, the debt owed increases. Sample answer: Any negative account balance represents a debt owed. As that balance becomes more and more negative, the debt owed will increase.

BL Suppose the account balance is $40. Is it still true that an account balance less than $40 will represent a debt greater than $40? Explain. not necessarily; Sample answer: Since the account balance is positive, any balances that are still positive do not represent any debt. Only balances that are less than zero will represent debt.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Students may need support distinguishing comparisons of *absolute value* from comparisons about *order*. Write the following phrases on the board that refer to the quantities in Example 3. Have students generate three possible quantities for the first phrase. Sample phrases are shown. Discuss why each quantity is actually a *debt* that is *greater than $40*. *Debt* is something that someone *owes*. If a *debt* is *greater* than $40, the amount *owed* is *greater* than $40. This means the account balance will be *less* than –$40.

account balance less than –$40	debt greater than $40
–$45	debt of $45
–$52	debt of $52
–$67	debt of $67

Apply Chemistry

Objective

Students will come up with their own strategy to solve an application problem involving freezing points of substances.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How might thinking about 0°C help you?
- How will you organize the data to make it easier to compare?
- If you include the freezing point of nitric acid in the data, where is it located?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Chemistry

The table shows the freezing points in degrees Celsius for six substances. Nitric acid freezes at −42°C. Between the freezing points of which two substances is the freezing point of nitric acid?

Substance	Freezing Point (°Celsius)
Aniline	−6
Acetic Acid	17
Acetone	−95
Water	0
Carbon Dioxide	−78
Sea Water	−2

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

aniline and carbon dioxide; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How could you solve this problem another way?

See students' responses.

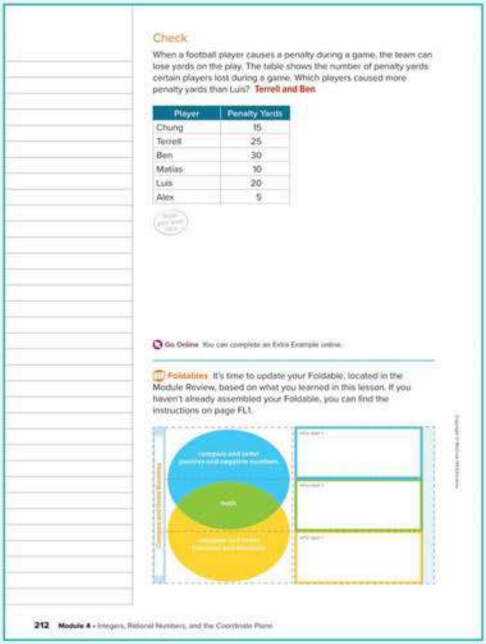
Lesson 4-3 • Compare and Order Integers 211

Interactive Presentation

Apply, Chemistry

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

When a football player causes a penalty during a game, the team can lose yards on the play. The table shows the number of penalty yards certain players lost during a game. Which players caused more penalty yards than Luis? Terrell and Ben

Player	Penalty Yards
Chung	15
Terrell	25
Ben	30
Matias	10
Luis	20
Alex	5

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

212 Module 4 • Integers, Rational Numbers, and the Coordinate Plane

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can describe the similarities and differences between comparing and ordering positive and negative integers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. If a group of golfer's scores are 2, −1, 3, 0, −1, −2, 1, and −3, what is the order of scores beginning with the winner? Write a mathematical argument that can be used to defend your solution. −3, −2, −1, −1, 0, 1, 2, 3; Sample answer: In golf the lesser score is the winning score. When graphed on a number line, the integers in order from least to greatest are −3, −2, −1, −1, 0, 1, 2, 3. Since the lesser number is the winning score, the winning score is −3 followed by −2 and so on.

Interactive Presentation

Exit Ticket

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 11–14
- ALEKS Plotting and Comparing Signed Numbers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 9, 12, 13
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Ordering and Estimation

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ordering and Estimation

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	write an inequality to compare two integers and explain the meaning of the inequality	1, 2
2	order a set of integers	3, 4
2	distinguish between comparisons of absolute value and comparisons about order	5, 6
2	extend concepts learned in class to apply them in new contexts	7, 8
3	solve application problems involving comparing and ordering integers	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Students may mistakenly order integers based on absolute value rather than numerical value or vice versa. Explain to students the importance of identifying what they need to find prior to jumping into an ordering attempt. In Exercise 3, students are asked to order the gases from least to greatest according to their freezing points. If a student ordered the integers from greatest to least, they may have compared the absolute value of the integers, rather than the actual value.

Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

1. After playing 18 holes of golf, John's score was −4 and Terry's score was −1. Write an inequality to compare the scores. Then explain the meaning of the inequality. (Example 1)

−4 < −1; Since −4 < −1, John has a lesser score than Terry.

2. The record low temperature for Buffalo, New York is −20°F. The record low temperature for Chicago, Illinois is −27°F. Write an inequality to compare the record low temperatures. Then explain the meaning of the inequality. (Example 1)

−27 < −20; Chicago's record low temperature is farther away from 0, so it is colder than Buffalo's record low temperature.

3. The table shows the freezing points for gases. Order the gases from least to greatest according to their freezing points. (Example 2)

Gas	Freezing Points (°C)
Argon	−189
Carbon Monoxide	−205
Ethane	−297
Helium	−272
Oxygen	−219
Sulfur Dioxide	−72

ethane, helium, oxygen, carbon monoxide, argon, sulfur dioxide

4. The table shows the scores for players in a trivia game after the first round. Order the players from least to greatest according to their scores. (Example 2)

Player	Score
Ace	−11
Diana	3
Jace	−3
Oneida	−7
Nolan	5
Rachel	1

Ace, Oneida, Jace, Rachel, Diana, Nolan

5. Explain why an elevation less than −5 feet represents a distance from sea level greater than 5 feet. (Example 3)

Sample answer: An elevation less than −5 feet is −10 feet. This means the distance is 10 feet from sea level, which is greater than a distance of 5 feet from sea level.

6. Explain why a balance of less than −$10 represents a debt greater than $10. (Example 3)

Sample answer: A balance less than −$10 is −$15, which means a debt of $15. This is greater than a debt of $10.

7. In a golf match, Jesse scored 5 over par, Neil scored 3 under par, Felipe scored 2 over par, and Dawson scored an even par. Order the players from least to greatest score.

Neil, Dawson, Felipe, Jesse

Test Practice

8. Table Item Order the integers from least to greatest.

9, −8, −2, 4, −9

least				greatest
−9	**−8**	**−2**	**4**	**9**

Apply *indicates multi-step problem

*9. The table shows the lowest elevations for several countries. The lowest elevation in the United States is −86 meters. Between the elevations of which two countries is the elevation for the United States?

Morocco and Argentina

Country	Lowest Elevation (m)
Argentina	−105
China	−154
Egypt	−133
Ethiopia	−125
Libya	−47
Morocco	−55

*10. A group of students participated in a small business challenge. The table shows results for the students' budgets. The student with the greatest amount under budget wins the challenge. In what place did Dave finish?

4th

Student Budget	
Casey	$2 under
Dave	even
Lily	$5 over
Luke	$4 over
Mike	$1 under
Tyrone	$6 under

Higher-Order Thinking Problems

11. **Create** Write a real-world situation that compares two negative integers. Then represent the situation with an inequality.

Sample answer: On Saturday the high temperature was −1°F. On Sunday the high temperature was −3°F; $-1 > -3$

12. **MP Justify Conclusions** A student said −5 is less than −4 and |−5| is less than |−4|. Is the student correct? Justify your reasoning.

no; Sample answer: Since −5 is to the left of −4 on a number line, −5 is less than −4. However, the absolute value of −5 is 5 and the absolute value of −4 is 4 and 5 is greater than 4.

13. Order {−2.5, 4, 23, −1, 5, −3, 0.66} from least to greatest.

−3, −2.5, −1, 0.66, 4, 5, 23

14. **MP Identify Structure** Suppose $y = 2$. Identify all the integers for x that make $|x| < |y|$ a true statement.

−1, 0, 1

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students determine if a student's reasoning is correct and justify their reasoning.

7 Look for and Make Use of Structure In Exercise 14, students use the structure of an inequality to solve it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 9 Have students work together to prepare a brief demonstration that illustrates why this problem might require multiple steps to solve. For example, before they can identify where the lowest elevation for the United States falls, they have to order all of the elevations from least to greatest. Have each pair or group of students present their response to the class.

Clearly and precisely explain.

Use with Exercise 12 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as showing the values on a number line to compare.

Learn Rational Numbers

Objective

Students will understand what a rational number is, and how it includes the sets of natural numbers, whole numbers, and integers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the quantity −3.77 to construct an argument for why the number is a rational number.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Is −3.77 a rational number? Explain your reasoning. yes; Sample answer: −3.77 can be written as the fraction $-\frac{377}{100}$.

Learn Graph Rational Numbers on a Number Line

Objective

Students will understand that rational numbers are points on the number line, and how to use a number line to represent them.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to reason about each integer's relationship to 0 on the number line so that they can compare its location on a vertical number line with its location on a horizontal number line.

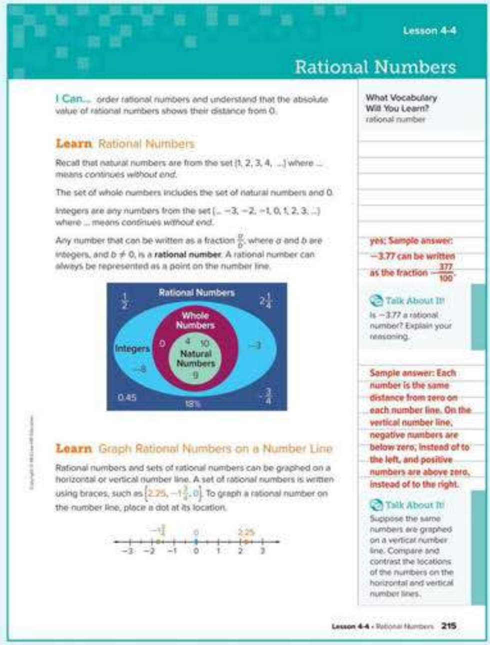
Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Suppose the same numbers are graphed on a vertical number line. Compare and contrast the locations of the numbers on the horizontal and vertical number lines. Sample answer: Each number is the same distance from zero on each number line. On the vertical number line, negative numbers are below zero, instead of to the left, and positive numbers are above zero, instead of to the right.


Lesson 4-4

Rational Numbers

I Can... order rational numbers and understand that the absolute value of rational numbers shows their distance from 0.

What Vocabulary Will You Learn?
rational number

Learn Rational Numbers

Recall that natural numbers are from the set {1, 2, 3, 4, ...} where ... means continues without end.

The set of whole numbers includes the set of natural numbers and 0.

Integers are any numbers from the set {... −3, −2, −1, 0, 1, 2, 3, ...} where ... means continues without end.

Any number that can be written as a fraction $\frac{a}{b}$, where a and b are integers, and $b \neq 0$, is a **rational number**. A rational number can always be represented as a point on the number line.

yes; Sample answer: −3.77 can be written as the fraction $-\frac{377}{100}$.

Talk About It! Is −3.77 a rational number? Explain your reasoning.

Learn Graph Rational Numbers on a Number Line

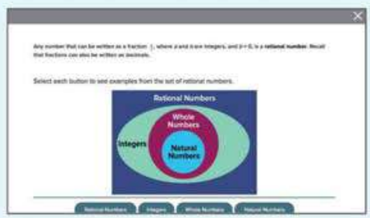
Rational numbers and sets of rational numbers can be graphed on a horizontal or vertical number line. A set of rational numbers is written using braces, such as $\{2.25, -1\frac{3}{4}, 0\}$. To graph a rational number on the number line, place a dot at its location.

Sample answer: Each number is the same distance from zero on each number line. On the vertical number line, negative numbers are below zero, instead of to the left, and positive numbers are above zero, instead of to the right.

Talk About It! Suppose the same numbers are graphed on a vertical number line. Compare and contrast the locations of the numbers on the horizontal and vertical number lines.

Lesson 4-4 • Rational Numbers 215

Interactive Presentation



Learn, Rational Numbers, Slide 1 of 2

CLICK

On Slide 1 of Learn, Rational Numbers, students view examples from the set of rational numbers.

CLICK

On Slide 1 of Learn, Graph Rational Numbers, students move through the slides to learn how to graph rational numbers.

Lesson 4-4

Rational Numbers

6.NS.C.6, 6.NS.C.7

LESSON GOAL

Students will reason about rational numbers using a number line.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Rational Numbers
Learn: Graph Rational Numbers on a Number Line
Example 1: Graph Sets of Rational Numbers
Learn: Absolute Value of Rational Numbers
Example 2: Find Absolute Value of Rational Numbers
Learn: Compare Rational Numbers
Example 3: Compare Rational Numbers
Learn: Order Rational Numbers
Example 4: Order Sets of Rational Numbers
Apply: Gardening

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 23 of the *Language Development Handbook* to help your students build mathematical language related to rational numbers.

ELL You can use the tips and suggestions on page T23 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.C** by comparing and ordering rational numbers.
Standards for Mathematical Content: **6.NS.C.6, 6.NS.C.6.C, 6.NS.C.7, 6.NS.C.7.A, 6.NS.C.7.C,** *Also addresses 6.NS.C.7.B*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students used a number line to compare and order integers.
6.NS.C.7

Now
Students reason about rational numbers using a number line.
6.NS.C.6, 6.NS.C.7

Next
Students will identify ordered pairs, points, and quadrants and graph ordered pairs in the coordinate plane.
6.NS.C.6, 6.NS.C.8

Rigor

The Three Pillars of Rigor

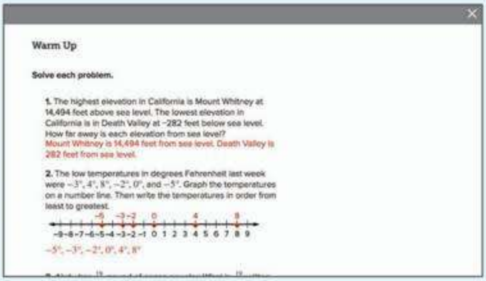
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of natural and whole numbers and integers, to develop *understanding* of rational numbers. They learn to graph rational numbers on a number line and write inequalities to build *fluency* with comparing and ordering rational numbers. They *apply* their understanding of rational numbers to solve real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding the absolute value of integers (Exercise 1)
- ordering and graphing integers on number line (Exercise 2)
- writing fractions as decimals (Exercise 3)

Answers

1. Mount Whitney is 14,494 feet from sea level. Death Valley is 282 feet from sea level.
2. See Warm Up slide online for correct answer.
3. 0.95

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about comparing and ordering elevations.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

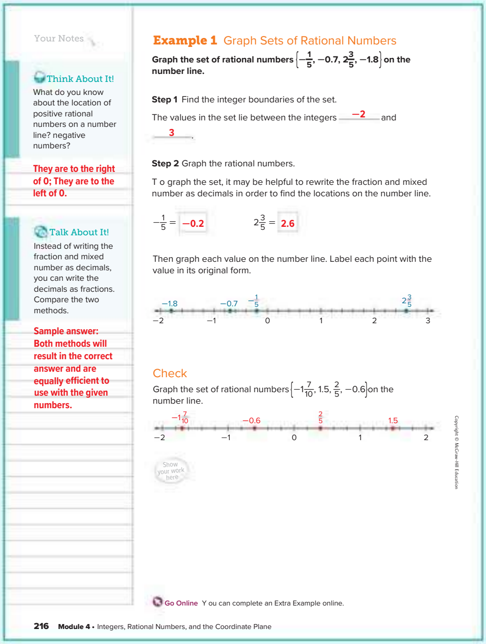
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The term *rational* uses the root word *ratio*. What is a *ratio*? Make a conjecture as to what you think a *rational number* might be? Sample answer: A ratio is a comparison of two numbers, which can be written as a fraction. A rational number might be a number that can be written as a ratio, or a fraction.



Your Notes

Think About It!
What do you know about the location of positive rational numbers on a number line? negative numbers?

They are to the right of 0; They are to the left of 0.

Talk About It!
Instead of writing the fraction and mixed number as decimals, you can write the decimals as fractions. Compare the two methods.

Sample answer: Both methods will result in the correct answer and are equally efficient to use with the given numbers.

Example 1 Graph Sets of Rational Numbers

Graph the set of rational numbers $\left\{-\frac{1}{5}, -0.7, 2\frac{3}{5}, -1.8\right\}$ on the number line.

Step 1 Find the integer boundaries of the set.

The values in the set lie between the integers **−2** and **3**.

Step 2 Graph the rational numbers.

T o graph the set, it may be helpful to rewrite the fraction and mixed number as decimals in order to find the locations on the number line.

$-\frac{1}{5} =$ **−0.2** $\qquad 2\frac{3}{5} =$ **2.6**

Then graph each value on the number line. Label each point with the value in its original form.

Check

Graph the set of rational numbers $\left\{-1\frac{7}{10}, 1.5, \frac{2}{5}, -0.6\right\}$ on the number line.

Show your work here

Go Online Y ou can complete an Extra Example online.

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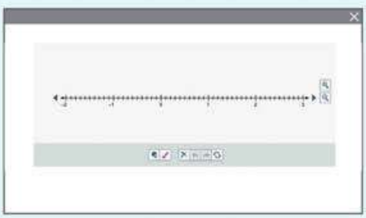
Interactive Presentation



Example 1, Graph Sets of Rational Numbers, Slide 3 of 5

DRAG & DROP

On Slide 2, students drag to indicate the integer boundaries.

eTOOLS

On Slide 3, students use the Number Line eTool to graph the numbers on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Graph Sets of Rational Numbers

Objective

Students will graph a set of rational numbers on a number line.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
As students discuss the *Talk About It!* question on Slide 4, they should be able to explain the similarities and differences between the two methods for graphing rational numbers on a number line.

5 Use Appropriate Tools Strategically Students will use the Number Line eTool to graph the set of rational numbers on the number line.

Questions for Mathematical Discourse

SLIDE 2

AL What does it mean to say that 3 is the upper limit? Sample answer: There is no number in the set that is greater than 3.

OL Why is it important to establish upper and lower limits? Sample answer: Establishing upper and lower limits will help when graphing the set. If I graph a number beyond the limits, it will be a sign that I need to re-evaluate the position of the number.

BL If 0.5 was added to the set of numbers, would you need to change the upper limit? What if 3.1is added? Explain. Sample answer: If 0.5 is added to the set, I would not need to change the upper limit, because $0.5 < 3$. However, if 3.1 is added to the set, I would need to change the upper limit to be the integer 4, since $3.1 > 3$, but $3.1 < 4$.

SLIDE 3

AL How many numbers will be graphed to the left of 0? to the right of 0? Three numbers will be graphed to the left of 0 and 1 number will be graphed to the right of 0.

OL Why will $-\frac{1}{5}$ be graphed between 0 and −1? Sample answer: $-\frac{1}{5} = -0.2$, and -0.2 is between −1 and 0, but much closer to 0.

BL If $-\frac{1}{4}$ is added to the set, between which two other numbers of the set should you graph it? $-\frac{1}{4} = -0.25$, so graph $-\frac{1}{4}$ between $-\frac{1}{5}$ and -0.7.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Absolute Value of Rational Numbers

Objective

Students will understand that the absolute value of a rational number is the distance the number is from zero on the number line.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to adhere to the definitions of *absolute value* and *opposites* in order to explain why they do not represent the same concept. You may wish to ask students why the absolute value of −2.5 is the same as the opposite of −2.5, but the absolute value of 2.5 is not the same as the opposite of 2.5.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Why is the absolute value of a number not the same as the opposite of a number? Sample answer: Absolute value refers to distance, and distance cannot be negative. Opposites are the same distance from 0, but are on the opposite sides of the number line, so they can be negative.

Example 2 Find Absolute Value of Rational Numbers

Objective

Students will find the absolute value of a rational number.

Questions for Mathematical Discourse

SLIDE 1

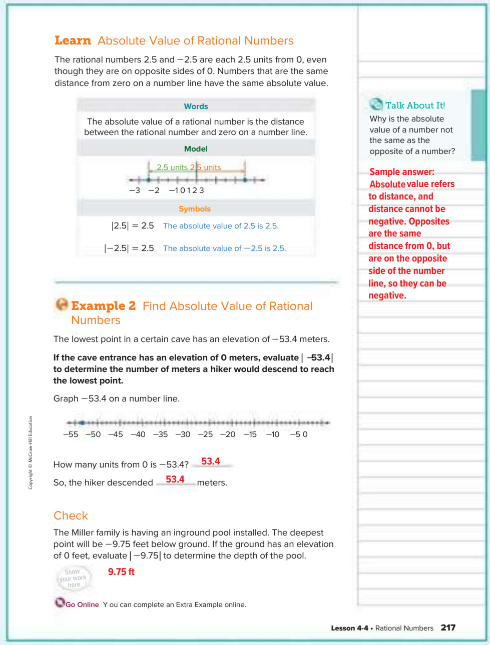
AL Why is the elevation negative? It is negative because the hiker is descending to an elevation that is lower than the entrance which is at an elevation of 0 feet.

OL Why can you use absolute value to find how many feet the hiker descended? The absolute value gives the distance the hiker descended to the lowest point of the cave, since distance cannot be negative.

BL What is −(−|−53.4|)? 53.4

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Absolute Value of Rational Numbers

The rational numbers 2.5 and −2.5 are each 2.5 units from 0, even though they are on opposite sides of 0. Numbers that are the same distance from zero on a number line have the same absolute value.

Words
The absolute value of a rational number is the distance between the rational number and zero on a number line.
Model
2.5 units 2.5 units; −3 −2 −1 0 1 2 3
Symbols
$|2.5| = 2.5$ The absolute value of 2.5 is 2.5.
$|-2.5| = 2.5$ The absolute value of −2.5 is 2.5.

Talk About It!

Why is the absolute value of a number not the same as the opposite of a number?

Sample answer: Absolute value refers to distance, and distance cannot be negative. Opposites are the same distance from 0, but are on the opposite side of the number line, so they can be negative.

Example 2 Find Absolute Value of Rational Numbers

The lowest point in a certain cave has an elevation of −53.4 meters.

If the cave entrance has an elevation of 0 meters, evaluate |−53.4| to determine the number of meters a hiker would descend to reach the lowest point.

Graph −53.4 on a number line.

−55 −50 −45 −40 −35 −30 −25 −20 −15 −10 −5 0

How many units from 0 is −53.4? 53.4

So, the hiker descended 53.4 meters.

Check

The Miller family is having an inground pool installed. The deepest point will be −9.75 feet below ground. If the ground has an elevation of 0 feet, evaluate |−9.75| to determine the depth of the pool.

Show your work here 9.75 ft

Go Online You can complete an Extra Example online.

Interactive Presentation

Learn, Absolute Value of Rational Numbers, Slide 1 of 2

FLASHCARDS

On Slide 1 of the Learn, students use Flashcards to view multiple representations of absolute value.

eTOOLS

On Slide 1 of Example 2, students use the Number Line eTool to graph a number on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Talk About It!
How can you use what you know about the signs of the rational numbers to quickly compare them?

Sample answer: If the signs are different, I know that a positive rational number is greater than a negative rational number. If they are the same, it might be helpful to compare the magnitude using a number line.

Learn Compare Rational Numbers

To compare two rational numbers, you can compare the signs as well as the magnitude, or size of the numbers.

If the signs are different, the positive rational number will always be greater than the negative rational number.

Different Signs
Compare 1.5 and −1.2.
The signs are different, so compare the signs. A positive rational number is always greater than a negative rational number.
1.5 > −1.2

If the signs of the two rational numbers are the same, you can graph the numbers on a number line to compare them. If the numbers are written in different forms, it may help to graph the numbers if they are both written as decimals or both written as fractions. Greater numbers are graphed farther to the right on the number line.

Same Signs
Compare −1.5 and −1.2.
The signs are the same, so use a number line to compare the numbers.

−1.5 < −1.2

Pause and Reflect

Are you ready to move on to the Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

See students' observations.

Interactive Presentation

Learn, Compare Rational Numbers, Slide 2 of 3

CLICK

On Slides 1 and 2, students move through slides to compare integers.

Learn Compare Rational Numbers

Objective

Students will understand that they can compare two rational numbers by reasoning about their signs and locations on a number line.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them

As students discuss the *Talk About It!* question on Slide 3, they should be able to apply what they learned about comparing integers to comparing rational numbers.

Teaching Notes

SLIDE 1

To compare two rational numbers, have students first look at the signs of the two numbers. Be sure they understand that, if the signs of two numbers are different, a positive rational number will always be greater than a negative rational number.

SLIDE 2

If the signs of two rational numbers are the same, students can graph the integers on a number line to compare their magnitudes. When two numbers are graphed on a number line, the greater number is to the right of the lesser number. Students can also use reasoning to compare the numbers without physically graphing them on a number line. Have them imagine a number line in their minds. If both numbers are positive, the number farther away from zero is greater. For example, 1.5 > 1.2, because 1.5 is farther away from 0 than 1.2. If both numbers are negative, the number closer to zero is greater. For example −1.2 > −1.5, because −1.2 is closer to 0 than −1.5.

Talk About It!

SLIDE 3

Mathematical Discourse

How can you use what you know about the signs of the rational numbers to quickly compare them? Sample answer: If the signs are different, I know that a positive rational number is greater than a negative rational number. If they are the same, it might be helpful to compare the magnitude using a number line.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of comparing and ordering rational numbers, have them work with a partner to create two different sets of rational numbers that are not in numerical order. There should be a mix of both positive and negative numbers, a mix of fractions, decimals, and integers, and at least 4 numbers in each set. Have them label one set as Set A, and the other set as Set B. Then have them trade sets with another pair of students. Each pair should order the integers in Set A from least to greatest, and the integers in Set B from greatest to least. Have pairs of students check each other's work, and discuss and resolve any differences.

Example 3 Compare Rational Numbers

Objective

Students will write an inequality to compare two rational numbers.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use the Number Line eTool to graph the values on the number line, in order to visually compare them.

6 Attend to Precision Students should accurately and efficiently write the numbers in the same form in order to graph them, and compare them by paying special attention to the fact that both numbers are negative.

As students discuss the *Talk About It!* question on Slide 4, encourage them to use clear and precise mathematical language as they explain how to compare the numbers without graphing them.

Questions for Mathematical Discourse

SLIDE 2

AL How can the fraction be written as a decimal? Sample answer: Multiply the numerator and denominator by 4, so that the denominator is 100. Then write as a decimal.

OL How can you use reasoning to compare $-\frac{12}{25}$ and -0.51? Sample answer: $-\frac{12}{25}$ is a little greater than $-\frac{1}{2}$, since half of 25 is 12.5. -0.51 is a little less than $-\frac{1}{2}$, since $-\frac{1}{2} = -0.5$. So, $-\frac{12}{25} > -0.51$.

BL Generate a negative rational number that is greater than either of these two numbers. Sample answer: $-\frac{1}{4}$

SLIDE 3

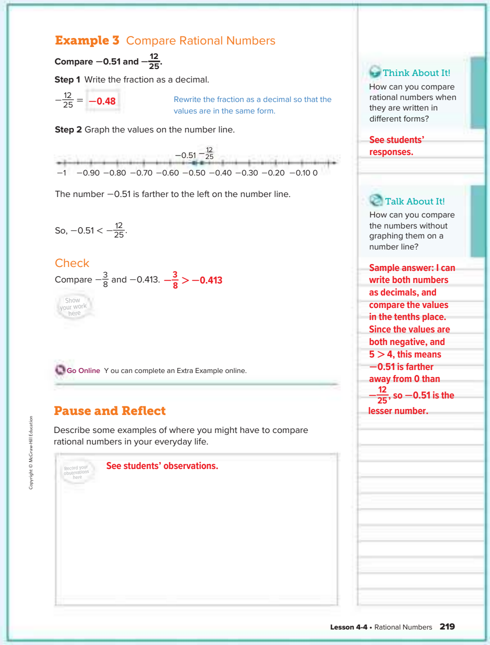
AL When comparing two numbers, is the number farther to the left on a number line always the lesser number? Explain. yes; Sample answer: This is why we use number lines to compare. Numbers to the left are always less than numbers to the right.

OL When comparing two numbers, is the number closer to 0 always the lesser number? Explain. no; Sample answer: When comparing two positive numbers, the number closer to 0 is the lesser number. When comparing two negative numbers, the number closer to 0 is actually the greater number. When comparing a positive and a negative number, the negative number is the lesser number, regardless of which number is closer to 0.

BL Generate another number that is between -0.51 and $-\frac{12}{25}$. Sample answer: -0.49

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Compare Rational Numbers

Compare -0.51 and $-\frac{12}{25}$.

Step 1 Write the fraction as a decimal.

$-\frac{12}{25} = -0.48$

Rewrite the fraction as a decimal so that the values are in the same form.

Step 2 Graph the values on the number line.

-0.51 $-\frac{12}{25}$

-1 -0.90 -0.80 -0.70 -0.60 -0.50 -0.40 -0.30 -0.20 -0.10 0

The number -0.51 is farther to the left on the number line.

So, $-0.51 < -\frac{12}{25}$.

Check

Compare $-\frac{3}{8}$ and -0.413. $-\frac{3}{8} > -0.413$

Show your work here

Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Describe some examples of where you might have to compare rational numbers in your everyday life.

Record your observations here

See students' observations.

Think About It!
How can you compare rational numbers when they are written in different forms?

See students' responses.

Talk About It!
How can you compare the numbers without graphing them on a number line?

Sample answer: I can write both numbers as decimals, and compare the values in the tenths place. Since the values are both negative, and $5 > 4$, this means -0.51 is farther away from 0 than $-\frac{12}{25}$, so -0.51 is the lesser number.

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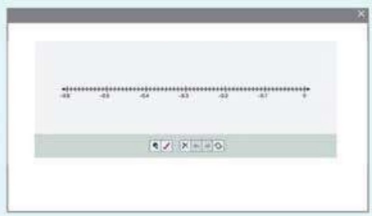
Lesson 4-4 • Rational Numbers 219

Interactive Presentation



Example 3, Compare Rational Numbers, Slide 3 of 5

eTOOLS

On Slide 3, students use the Number Line eTool to graph rational numbers on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Talk About It!
How does place value help you order the set of numbers $\left\{\frac{1}{4}, -0.375, -\frac{17}{50}, 0.3\right\}$?

Sample answer: I can rewrite all of the rational numbers as decimals. Then I can compare the values in the tenths and hundredths place to order the numbers.

Think About It!
How can you order rational numbers when they are written in different forms?

See students' responses.

Talk About It!
How does a number line help you visualize the order of rational numbers?

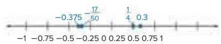
Sample answer: By placing the numbers on the number line, I can quickly see the least number is farthest to the left, and the greatest number is farthest to the right.

Learn Order Rational Numbers

T o order rational numbers, follow these steps:

1. Write each number in the same form. Since there may be different denominators in the fractions, it may be easier to write all of the numbers as decimals.
2. Use the signs of the numbers, place value, or a number line to compare the numbers.
3. Order the values from least to greatest or greatest to least.

T o order the set of numbers $\left\{\frac{1}{4}, -0.375, -\frac{17}{50}, 0.3\right\}$, graph each number on a number line. The least value is farthest to the left and the greatest value is farthest to the right.



So, the set of numbers in order from least to greatest is $\left\{-0.375, -\frac{17}{50}, \frac{1}{4}, 0.3\right\}$ and from greatest to least is $\left\{0.3, \frac{1}{4}, -\frac{17}{50}, -0.375\right\}$.

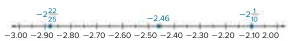
Example 4 Order Sets of Rational Numbers

Order the set $\left\{-2.46, -2\frac{22}{25}, -2\frac{1}{10}\right\}$ from least to greatest.

Step 1 Write the mixed numbers as decimals.

$-2.46 = -2.46$ $\quad -2\frac{22}{25} =$ **−2.88** $\quad -2\frac{1}{10} =$ **−2.1**

Step 2 Graph the numbers on a number line.



So, the set of numbers in order from least to greatest is $-2\frac{22}{25}$, −2.46, $-2\frac{1}{10}$.

Check

Order the set $\left\{2.12, -2.1, 2\frac{1}{10}, -2\frac{1}{5}\right\}$ from least to greatest.

Show your work here

$\left\{-2\frac{1}{5}, -2.1, 2\frac{1}{10}, 2.12\right\}$

Go Online Y ou can complete an Extra Example online.

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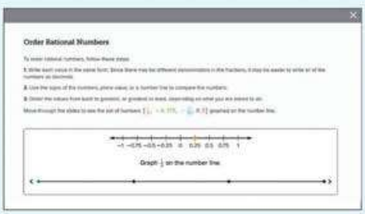
Interactive Presentation



Learn, Order Rational Numbers, Slide 1 of 2

CLICK

On Slide 1 of the Learn, students move through the slides to graph a set of rational numbers on the number line.

eTOOLS

On Slide 3 of Example 4, students use the Number Line eTool to graph rational numbers on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Order Rational Numbers

Objective

Students will understand how a number line can be used to order a set of rational numbers.

Go Online

- Find additional teaching notes.
- Teaching the Mathematical Practices
- Sample answer for the *Talk About It!* question on Slide 2.

Example 4 Order Sets of Rational Numbers

Objective

Students will order a set of rational numbers.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you write the mixed numbers as decimals? Sample answer: The mixed numbers can be written as decimals, so that the numbers are easier to graph and compare.

OL Can you compare the numbers without graphing them on the number line? Explain. yes; Sample answer: Since the numbers are all negative, I can compare place value. Since $1 < 4 < 8$ in the tenths place, I know that −2.88 is the least number, because it is the farthest away from zero. The next least number is −2.46, and the greatest number is −2.1.

BL If one of the numbers was positive, is it enough to only compare the digits in the tenths place? Explain. no; Sample answer: If one of the numbers was positive, I know that is the greatest number since it is farthest to the right on the number line. Then I can compare the tenths digits of the other two numbers, since they are both negative.

SLIDE 3

AL What is true about all of the numbers? They are all negative.

OL Which number is the greatest? How do you know? $-2\frac{1}{10}$; It's the number that is closest to 0, and all the numbers are negative.

BL Which number has the greatest magnitude? Explain. $-2\frac{22}{25}$; It has the greatest absolute value.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Gardening

Objective

Students will come up with their own strategy to solve an application problem involving comparisons to the record weight of a pumpkin.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

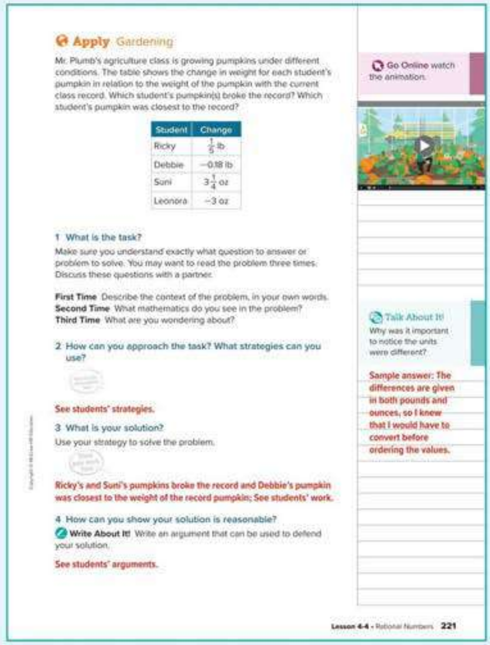
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you write the numbers in the same form?
- What do you notice about the units?
- Whose pumpkin had a change in weight that was closest to 0?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Gardening

Mr. Plumb's agriculture class is growing pumpkins under different conditions. The table shows the change in weight for each student's pumpkin in relation to the weight of the pumpkin with the current class record. Which student's pumpkin(s) broke the record? Which student's pumpkin was closest to the record?

Student	Change
Ricky	$\frac{1}{5}$ lb
Debbie	−0.18 lb
Suni	$3\frac{1}{4}$ oz
Leonora	−3 oz

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Ricky's and Suni's pumpkins broke the record and Debbie's pumpkin was closest to the weight of the record pumpkin; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online watch the animation.

Talk About It!

Why was it important to notice the units were different?

Sample answer: The differences are given in both pounds and ounces, so I knew that I would have to convert before ordering the values.

Lesson 4-4 • Rational Numbers 221

Interactive Presentation

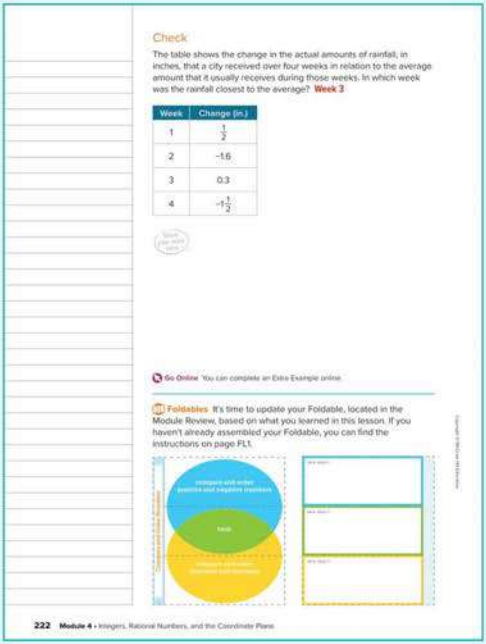
Apply, Gardening

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

The table shows the change in the actual amounts of rainfall, in inches, that a city received over four weeks in relation to the average amount that it usually receives during those weeks. In which week was the rainfall closest to the average? Week 3

Week	Change (in.)
1	$\frac{1}{2}$
2	-1.6
3	0.3
4	$-1\frac{1}{2}$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can add descriptions of the similarities and differences between comparing and ordering different types of rational numbers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Order the locations in the table from the least elevation to the greatest elevation. Write a mathematical argument that can be used to defend your solution. Bentley Subglacial Trench, Dead Sea, Lake Assal, Death Valley, Valdes Peninsula, Caspian Sea, Lake Eyre; Sample answer: When the numbers are all graphed on a number line, the points in order from left to right indicate the elevations in order from least to greatest.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 3, 11, 13, 15–17
- Extension: Extension Resources
- ALEKS Plotting and Comparing Signed Numbers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–10, 13, 16
- Extension: Extension Resources
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Plotting and Ordering Fractions

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Ordering Fractions

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

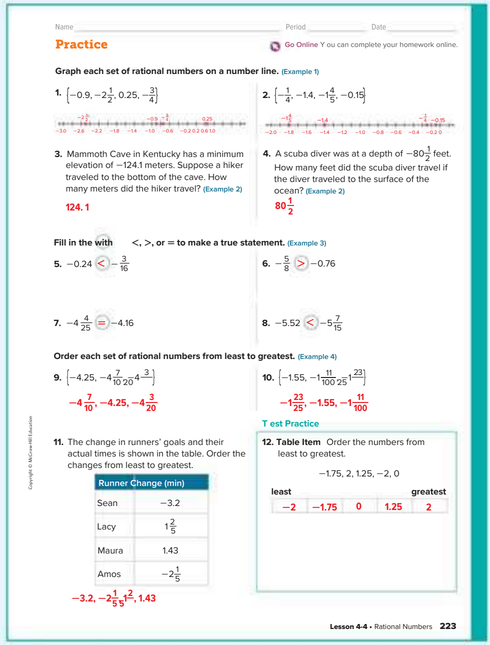
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	graph a set of rational numbers on a number line	1, 2
2	find the absolute value of a rational number	3, 4
1	write an inequality to compare two rational numbers	5–8
1	order a set of rational numbers	9, 10
2	extend concepts learned in class to apply them in new contexts	11, 12
3	solve application problems involving comparing and ordering rational numbers	13, 14
3	higher-order and critical thinking skills	15–17

Common Misconception

Students may have trouble understanding negative mixed numbers. In Exercise 7, students may understand that -4 indicates four units to the left of 0 on a number line. However, they might not recognize that the fractional portion of the number indicates an additional $\frac{4}{25}$ unit to the left of -4 on the number line. Students may mistakenly always consider the fractional part as positive rather than negative. The rational number $-4\frac{4}{25}$ can be thought of as $-4 + \left(-\frac{4}{25}\right)$ not $-4 + \frac{4}{25}$.



Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

Graph each set of rational numbers on a number line. (Example 1)

1. $\left\{-0.9, -2\frac{1}{2}, 0.25, -\frac{3}{4}\right\}$

2. $\left\{-\frac{1}{4}, -1.4, -1\frac{4}{5}, -0.15\right\}$

3. Mammoth Cave in Kentucky has a minimum elevation of −124.1 meters. Suppose a hiker traveled to the bottom of the cave. How many meters did the hiker travel? (Example 2)

124. 1

4. A scuba diver was at a depth of $-80\frac{1}{2}$ feet. How many feet did the scuba diver travel if the diver traveled to the surface of the ocean? (Example 2)

$80\frac{1}{2}$

Fill in the with <, >, or = to make a true statement. (Example 3)

5. $-0.24 < -\frac{3}{16}$

6. $-\frac{5}{8} > -0.76$

7. $-4\frac{4}{25} = -4.16$

8. $-5.52 < -5\frac{7}{15}$

Order each set of rational numbers from least to greatest. (Example 4)

9. $\left\{-4.25, -4\frac{7}{10}, -4\frac{3}{20}\right\}$

$-4\frac{7}{10}, -4.25, -4\frac{3}{20}$

10. $\left\{-1.55, -1\frac{11}{100}, -1\frac{23}{25}\right\}$

$-1\frac{23}{25}, -1.55, -1\frac{11}{100}$

11. The change in runners' goals and their actual times is shown in the table. Order the changes from least to greatest.

Runner	Change (min)
Sean	−3.2
Lacy	$1\frac{2}{5}$
Maura	1.43
Amos	$-2\frac{1}{5}$

$-3.2, -2\frac{1}{5}, 1\frac{2}{5}, 1.43$

T est Practice

12. Table Item Order the numbers from least to greatest.

−1.75, 2, 1.25, −2, 0

least				greatest
−2	−1.75	0	1.25	2

Lesson 4-4 • Rational Numbers 223

Apply *indicates multi-step problem

*13. Saeng wants to run the 100-meter-dash in a certain number of seconds. The table shows the change in times from her goal and her actual times for five races. Between which two race numbers is Saeng's third race?

Race	Change in Time from Goal (s)
1	-1.2
2	$+1\frac{1}{10}$
3	$-1\frac{1}{4}$
4	-1.4
5	$+1\frac{1}{2}$

Race 4 and Race 1

*14. In science class, students are growing plants. The table shows the change in the heights between the heights of some students' plants and the height of last year's tallest plant. Order the changes from least to greatest.

Student	Change
Ellen	$-2\frac{3}{4}$ in.
Juan	$\frac{1}{4}$ ft
Patty	3.1 in.
Sonny	$-\frac{1}{5}$ ft

$-2\frac{3}{4}$ in., $-\frac{1}{5}$ ft, $\frac{1}{4}$ ft, 3.1 in.

Higher-Order Thinking Problems

15. **Create** Write about a real-world situation in which you compare two negative rational numbers. Then write an inequality comparing the two numbers.

Sample answer: Ming's account balance is −\$10.50. Her brother's account balance is −\$15.50. Compare their balances; $-\$10.50 > -\$15.50$

16. MP **Justify Conclusions** A student said $-2\frac{1}{4}$ is less than -2.2 and $\left|-2\frac{1}{4}\right|$ is less than $|-2.2|$. Is the student correct? Justify your reasoning.

no; Sample answer: $-2\frac{1}{4} = -2.25$, so, it is to the left of -2.2 on a number line. The absolute value of $-2\frac{1}{4}$ is $2\frac{1}{4}$ or 2.25 and the absolute value of -2.2 is 2.2, which is less than 2.25 or $2\frac{1}{4}$.

17. MP **Reason Inductively** Determine whether the following statement is *always*, *sometimes*, or *never* true. Justify your reasoning.

If x and y are both less than 0 and $x < y$, then $-x > -y$.

always; Sample answer: The lesser the number, the closer it is to 0; therefore, it's opposite is also closer to 0. $x = -3$, $y = -2$

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students analyze another student's statement to determine whether or not it is true.

2 Reason Abstractly and Quantitatively In Exercise 17, students determine if an inequality is true for some, all, or no values of two variables.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create Your Own Problem

Use with Exercises 13–14 Have students work in groups of 3–4 to solve the problem in Exercise 13. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 14.

Make sense of the problem.

Use with Exercise 16 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise said that $\left|-2\frac{1}{4}\right|$ is less than $|-2.2|$. Have each pair or group of students present their explanations to the class.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn The Coordinate Plane

Objective

Students will understand how to determine the sign of the *x*- and *y*-coordinates for ordered pairs graphed within the four quadrants of the coordinate plane.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the coordinates of the point $\left(\frac{2}{3}, -7\right)$ and their signs in order to determine the location of the point.

Teaching Notes

SLIDE 1

Be sure students understand the structure of the coordinate plane, and how it is formed by the intersection of two perpendicular number lines (the *x*- and *y*-axes), creating four quadrants. Quadrants are named using Roman Numerals.

SLIDE 2

Have students explore the interactive tool in order to understand the patterns for the signs of the *x*- and *y*-coordinates that are located in each quadrant, and along each axis. For example, an ordered pair with a negative *x*-coordinate and a *y*-coordinate of zero will be located on the *x*-axis to the left of the origin. You may wish to have students generate other examples and use the interactive tool to locate those points.

SLIDE 3

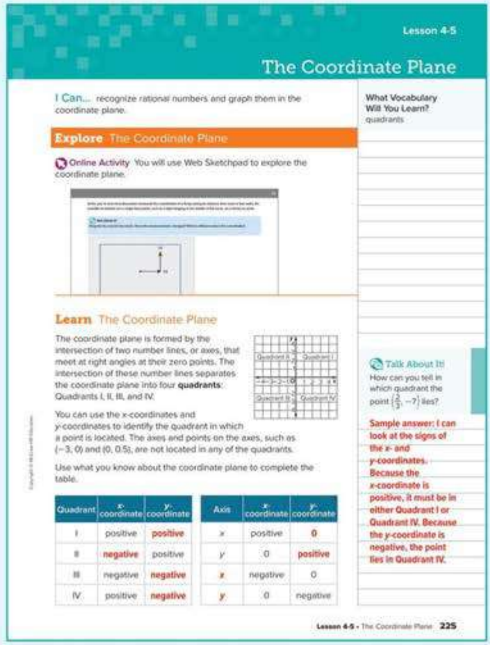
Have students complete the tables that identify the signs of the *x*- and *y*-coordinates corresponding to each quadrant or axis. Some students mistakenly think that the *x*-coordinate is zero for any point along the *x*-axis. Encourage them to use reasoning about what each coordinate indicates in an ordered pair. The *x*-coordinate indicates the distance and direction an ordered pair is from the origin, along the *x*-axis. If the *x*-coordinate is zero, then the point lies somewhere along the *y*-axis (or origin).

Talk About It!

SLIDE 4

Mathematical Discourse

How can you tell in which quadrant the point $\left(\frac{2}{3}, -7\right)$ lies?
Sample answer: I can look at the signs of the *x*- and *y*-coordinates. Because the *x*-coordinate is positive, it must be in either Quadrant I or Quadrant IV. Because the *y*-coordinate is negative, the point lies in Quadrant IV.


Lesson 4-5

The Coordinate Plane

I Can... recognize rational numbers and graph them in the coordinate plane.

What Vocabulary Will You Learn? quadrants

Explore The Coordinate Plane

Online Activity You will use Web Sketchpad to explore the coordinate plane.

Learn The Coordinate Plane

The coordinate plane is formed by the intersection of two number lines, or axes, that meet at right angles at their zero points. The intersection of these number lines separates the coordinate plane into four **quadrants**: Quadrants I, II, III, and IV.

You can use the *x*-coordinates and *y*-coordinates to identify the quadrant in which a point is located. The axes and points on the axes, such as (−3, 0) and (0, 0.5), are not located in any of the quadrants.

Use what you know about the coordinate plane to complete the table.

Quadrant	*x*-coordinate	*y*-coordinate
I	positive	positive
II	negative	positive
III	negative	negative
IV	positive	negative

Axis	*x*-coordinate	*y*-coordinate
x	positive	0
y	0	positive
x	negative	0
y	0	negative

Talk About It! How can you tell in which quadrant the point $\left(\frac{2}{3}, -7\right)$ lies?

Sample answer: I can look at the signs of the *x*- and *y*-coordinates. Because the *x*-coordinate is positive, it must be in either Quadrant I or Quadrant IV. Because the *y*-coordinate is negative, the point lies in Quadrant IV.

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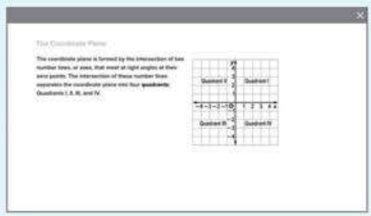
Interactive Presentation



Learn, The Coordinate Plane, Slide 1 of 4

CLICK

On Slide 2, students select markers to understand the patterns for the signs of *x*- and *y*-coordinates.

CLICK

On Slide 3, students indicate the signs of the coordinates for each quadrant or axis.

Lesson 4-5

The Coordinate Plane

6.NS.C.6, 6.NS.C.8

LESSON GOAL

Students will identify ordered pairs, points, and quadrants and graph ordered pairs on the coordinate plane.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: The Coordinate Plane

Learn: The Coordinate Plane
Example 1: Identify the Quadrant
Example 2: Identify the Axis
Learn: Identify Ordered Pairs
Example 3: Identify Ordered Pairs
Learn: Identify Points
Example 4: Identify Points
Learn: Graph Ordered Pairs
Example 5: Graph Ordered Pairs
Apply: Maps

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 24 of the *Language Development Handbook* to help your students build mathematical language related to the coordinate plane.

ELL You can use the tips and suggestions on page T24 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.C** by identifying ordered pairs, points, and quadrants, and graphing ordered pairs on the coordinate plane.
Standards for Mathematical Content: **6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students reasoned about rational numbers using a number line.
6.NS.C.6, 6.NS.C.7

Now
Students identify ordered pairs, points, and quadrants and graph ordered pairs on the coordinate plane.
6.NS.C.6, 6.NS.C.8

Next
Students will graph reflections of points on the coordinate plane.
6.NS.C.6, 6.NS.C.8

Rigor

The Three Pillars of Rigor

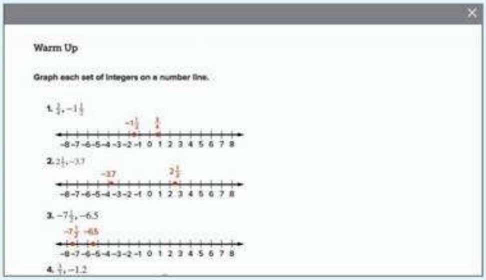
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of graphing points on a number line to develop *understanding* of the coordinate plane. They learn to identify ordered pairs, points, and quadrants, to build *fluency* with writing ordered pairs and graphing them on the coordinate plane.

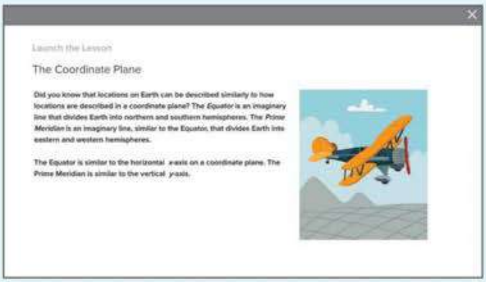
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- graphing rational numbers on a number line (Exercises 1–5)

Answers

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about locations on Earth being related to a coordinate plane.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Can you think of any other words with the prefix quad-? What does the prefix *quad*- mean? Sample answer: quadrilateral, quadruplet; The prefix *quad*- means four.

Explore The Coordinate Plane

Objective

Students will use Web Sketchpad to explore the coordinate plane.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations

Summary of Activity

Students will be presented with a model of a fly on a ceiling that has the horizontal and vertical distance marked. Students will then further explore the idea of *x* and *y* coordinates and how they are related to the position of the fly on the model. Students will use their observations to make conjectures about the values of the *x*- and *y*-coordinates in each quadrant.

Inquiry Question

How are integers and rational numbers used in the coordinate plane? Sample answer: Integers and rational numbers are used as *x*- and *y*-coordinates to locate positions on a coordinate plane.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

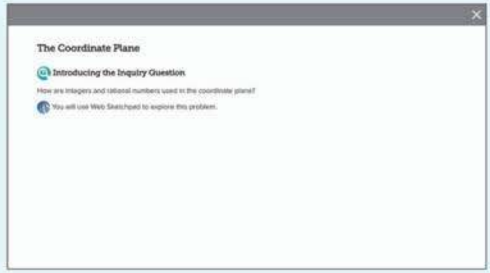
SLIDE 5

Mathematical Discourse

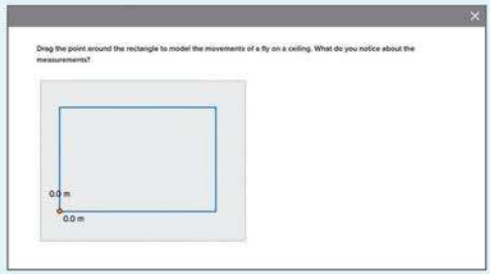
How can you determine the dimensions of the ceiling from the coordinates of the four corners? Sample answer: I can use the greatest *x*-value, 6, to find the length, and the greatest *y*-value, 4, to find the width. So, the ceiling is 6 by 4 (or 4 by 6).

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 11



Explore, Slide 3 of 11

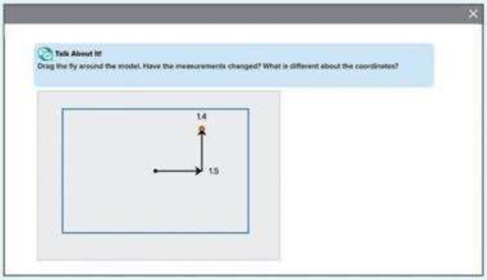
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how integers and rational numbers are used in the coordinate plane.

TYPE

On Slide 4, students complete a table to identify the coordinates of the four corners.

Interactive Presentation



Explore, Slide 7 of 11

CLICK

On Slide 8, students identify the point represented by the given coordinates.

CLICK

On Slide 10, students identify the figure they graphed in the coordinate plane.

CHECK

On Slide 11, students respond to the Inquiry Question and view a sample answer.

Explore The Coordinate Plane *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and examine the coordinates of the fly on the ceiling, which represents the coordinate plane. Encourage students to notice that there are four quadrants and to make observations about each of the quadrants.

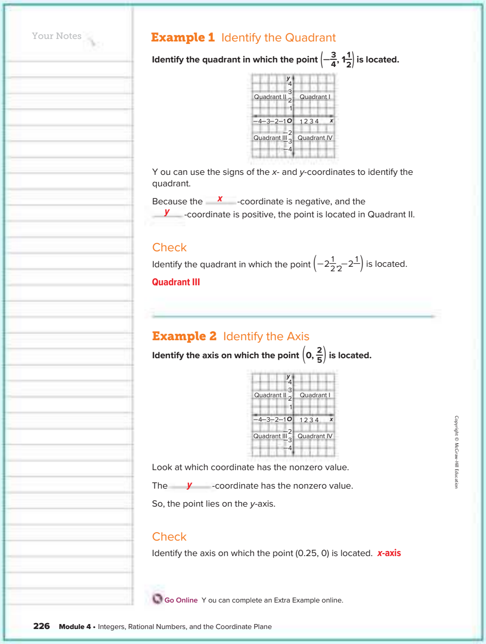
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 7 are shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Drag the fly around the model. Have the measurements changed? What is different about the coordinates? Sample answer: The ceiling is still 6 by 4. The *x*-coordinates go from −3 to 3 and the *y*-coordinates go from −2 to 2.



Your Notes

Example 1 Identify the Quadrant

Identify the quadrant in which the point $\left(-\frac{3}{4}, 1\frac{1}{2}\right)$ is located.

Y ou can use the signs of the *x*- and *y*-coordinates to identify the quadrant.

Because the ___*x*___-coordinate is negative, and the ___*y*___-coordinate is positive, the point is located in Quadrant II.

Check

Identify the quadrant in which the point $\left(-2\frac{1}{2}, -2\frac{1}{2}\right)$ is located.

Quadrant III

Example 2 Identify the Axis

Identify the axis on which the point $\left(0, \frac{2}{5}\right)$ is located.

Look at which coordinate has the nonzero value.

The ___*y*___-coordinate has the nonzero value.

So, the point lies on the *y*-axis.

Check

Identify the axis on which the point (0.25, 0) is located. ***x*-axis**

Go Online You can complete an Extra Example online.

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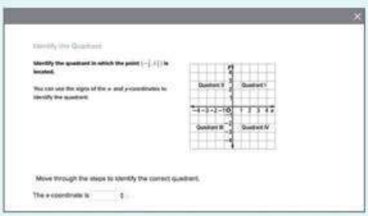
Interactive Presentation



Example 1, Identify the Quadrant, Slide 1 of 2

CLICK

On Slide 1 of Example 1, students move through the steps to identify the correct quadrant.

CLICK

On Slide 1 of Example 2, students select from drop-down menus to identify the axis on which a point lies.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Identify the Quadrant

Objective

Students will identify the quadrant of the coordinate plane in which a given point is located.

Questions for Mathematical Discourse

SLIDE 1

AL How many quadrants are in the coordinate plane? 4

AL What is the sign of each coordinate? The *x*-coordinate is negative. The *y*-coordinate is positive.

OL Since the *x*-coordinate is negative, which quadrants can you eliminate? Sample answer: Since the *x*-coordinate is negative, the point cannot be in Quadrant I or Quadrant IV.

BL How far away from the *x*-axis is the point? the *y*-axis? The point is $1\frac{1}{2}$ units from the *x*-axis and $\frac{3}{4}$ unit from the *y*-axis.

Example 2 Identify the Axis

Objective

Students will identify the axis on which a given point is located.

Questions for Mathematical Discourse

SLIDE 1

AL What do you notice about the given point? Sample answer: The *x*-coordinate is 0.

OL Explain why *any* point with an *x*-coordinate of 0 is located on the *y*-axis. Sample answer: When the *x*-coordinate is zero, the horizontal distance the point is from 0 along the horizontal *x*-axis is zero. This means that the point lies on the *y*-axis.

OL There is one point with an *x*-coordinate of 0 that is located on the *x*-axis. Identify that point. the origin, (0, 0)

BL What are the coordinates of the point whose *x*-coordinate remains unchanged, but whose *y*-coordinate is the opposite of the given *y*-coordinate? $\left(0, -\frac{2}{5}\right)$

Go Online

- Find teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Identify Ordered Pairs

Objective

Students will learn how to identify an ordered pair that represents a point graphed on the coordinate plane.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to pay attention to the order in which the coordinates of an ordered pair must be written.

Teaching Notes

SLIDE 1

Be sure students understand that the ordered pairs are written in the form (*x*, *y*), so to find the first coordinate in the ordered pair, they must identify the direction and distance the point is from the origin along the horizontal *x*-axis. Then they can identify the direction and distance the point is from the origin along the vertical *y*-axis.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to identify ordered pairs of points graphed on the coordinate plane.

Talk About It!

SLIDE 2

Mathematical Discourse

When identifying an ordered pair that represents a graphed point, why is it important to count the *horizontal* movement from the origin to that point first? Sample answer: Ordered pairs are defined so that the *x*-coordinate is written first, followed by the *y*-coordinate. Counting the horizontal distance first ensures that I will write the coordinates in the correct order.

DIFFERENTIATE

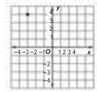
Language Development Activity ELL

Students should be familiar with ordered pairs from Grade 5. They may need support in distinguishing the vocabulary terms *coordinates*, *ordered pair*, and *point*. Have students use a blank coordinate grid to plot and label point *P* at (1, 5). The *point* is *point P*. The *ordered pair* that represents *point P* is (1, 5). Discuss that the term *ordered pair* denotes a *pair* of numbers in a particular *order*. Students can use this understanding to remember what an *ordered pair* is. The *ordered pair* contains two *coordinates*, an *x-coordinate* and a *y-coordinate*.

Learn Identify Ordered Pairs

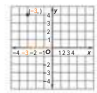
Go Online Watch the animation to learn how to identify ordered pairs of points graphed on the coordinate plane.

T o identify the ordered pair graphed on the coordinate plane, start at the origin.



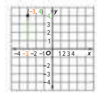
First, move horizontally along the *x*-axis, counting the units.

The *x*-coordinate of the point is −3.



Next, move vertically toward the point, counting the units.

The *y*-coordinate of the point is 4.



So, the ordered pair for the point is (−3, 4).

Talk About It!

When identifying an ordered pair that represents a graphed point, why is it important to count the *horizontal* movement from the origin to that point first?

Sample answer: Ordered pairs are defined so that the *x*-coordinate is written first, followed by the *y*-coordinate. Counting the horizontal distance first ensures that I will write the coordinates in the correct order.

Pause and Reflect

Are you ready to move on to the Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

Record your observations here

See students' observations.

Interactive Presentation

Learn, Identify Ordered Pairs, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that illustrates how to identify ordered pairs of points graphed on the coordinate plane.

Think About It!
In which quadrant does point *D* lie?

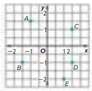
IV

Talk About It!
Why is the ordered pair $\left(-1, 1\frac{1}{2}\right)$ incorrect for naming point *D*?

Sample answer: The coordinates are in the wrong order. The coordinates must be written *x* first, then *y*.

Example 3 Identify Ordered Pairs

Identify the ordered pair that names point *D*.



Start at the origin.

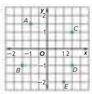
Move $1\frac{1}{2}$ units right on the *x*-axis until you reach the vertical line that intersects with point *D*. The *x*-coordinate of point *D* is $1\frac{1}{2}$.

Move down 1 unit to reach point *D*. The *y*-coordinate of point *D* is −1.

So, the ordered pair that names point *D* is $\left(1\frac{1}{2}, -1\right)$.

Check

Identify the ordered pair that names point *B*. $\left(-1\frac{1}{2}, -1\right)$



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Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3, Identify Ordered Pairs, Slide 2 of 4

CLICK

On Slide 2, students select from drop-down menus to determine the *x*-coordinate.

TYPE

On Slide 2, students determine the *y*-coordinate.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Identify Ordered Pairs

Objective

Students will identify an ordered pair that represents a point graphed on the coordinate plane.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to construct a plausible argument to explain why the given ordered pair is incorrect when naming point *D*.

6 Attend to Precision Encourage students to accurately and efficiently determine the quadrant in which the point lies, and pay attention to the order in which the coordinates must be written, as well as the sign of each coordinate.

Questions for Mathematical Discourse

SLIDE 2

AL Will the *x*-coordinate be positive or negative? the *y*-coordinate? Explain. The *x*-coordinate will be positive because the point is to the right of the *y*-axis. The *y*-coordinate will be negative because the point is below the *x*-axis.

OL Why is it helpful to identify the quadrant in which the point lies? Sample answer: Since point *D* lies in Quadrant IV, it means that the *x*-coordinate will be positive and the *y*-coordinate will be negative.

BL How far away is point *D* from the *x*-axis? the *y*-axis? Point *D* is 1 unit from the *x*-axis and $1\frac{1}{2}$ units from the *y*-axis.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Identify Points

Objective

Students will learn how to identify a point on the coordinate plane given an ordered pair.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to provide a clear explanation about how they can use the signs of the coordinates in each quadrant to quickly identify points.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the ordered pair (−2, 4) is shown, and ask students to discuss with a partner how they can rule out some of the points given in the coordinate plane. For example, some students may rule out point *R* because the *x*-coordinate that represents point *R* should be positive, since the point lies in Quadrant IV. You may wish to repeat the discussion after the ordered pair (4, −2) is shown.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to identify a point graphed on the coordinate plane given the ordered pair.

Talk About It!

SLIDE 2

Mathematical Discourse

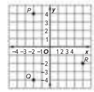
How can you use what you know about the signs of the coordinates in each quadrant to quickly identify the point? Sample answer: If the *x*- and *y*-values are both negative, for example, I know the point is located in Quadrant III, so I can quickly identify the point.

Learn Identify Points

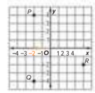
Go Online Watch the animation to learn how to identify points graphed in the coordinate plane, given the ordered pair.

Y ou can identify a point graphed on the coordinate plane using the *x*- and *y*-coordinates. The *x*-coordinate indicates how far left or right to move from the origin. The *y*-coordinate indicates how far up or down to move from the origin.

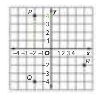
Identify the point graphed at (−2, 4).



Because the *x*-coordinate is negative, move left two units on the *x*-axis.



Because the *y*-coordinate is positive, move up four units.



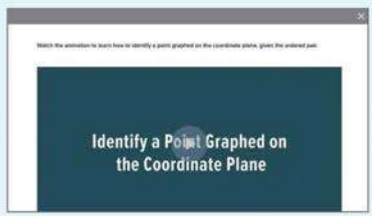
Point P is located at (−2, 4).

Talk About It! How can you use what you know about the signs of the coordinates in each quadrant to quickly identify the point?

Sample answer: If the *x*- and *y*-values are both negative, for example, I know the point is located in Quadrant III, so I can quickly identify the point.

Lesson 4-5 • The Coordinate Plane 229

Interactive Presentation



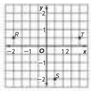
Learn, Identify Points, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that illustrates how to identify a point graphed on the coordinate plane given the ordered pair.

Example 4 Identify Points

Identify the point located at $\left(-2, \frac{1}{2}\right)$.



Start at the origin.

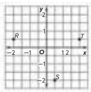
Because the x-coordinate is negative, move __2__ units left on the __x__-axis.

Move up __$\frac{1}{2}$__ unit because the y-coordinate is positive.

So, point R is located at $\left(-2, \frac{1}{2}\right)$.

Check

Identify the point located at $\left(\frac{1}{2}, -2\right)$. **Point S**



Go Online You can complete an Extra Example online.

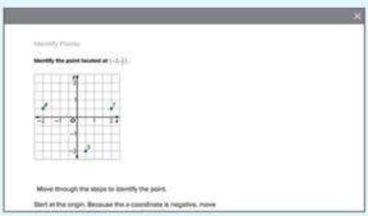
Pause and Reflect

How does what you already know about graphing integers on a number line help you with identifying points on the coordinate plane?

Record your observations here

See students' observations.

Interactive Presentation



Example 4, Identify Points, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to identify the correct point.

CLICK

On Slide 1, students indicate how to move vertically on the coordinate plane.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Identify Points

Objective

Students will identify a point on the coordinate plane given an ordered pair.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to pay careful attention to the sign of each coordinate in order to determine which point correctly names the given ordered pair.

Questions for Mathematical Discourse

SLIDE 1

AL What does the sign of the x-coordinate tell you about the point? The x-coordinate tells me that the point is left of the origin.

OL Why is point R the only point that could have this ordered pair? Sample answer: Both points S and T will have x-coordinates that are positive, because they are located to the right of the origin.

BL If the point $(-2, -2)$ was graphed, how many units below point R will this point be? The point $(-2, -2)$ will be 2.5 units below point R.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Graph Ordered Pairs

Objective

Students will learn how to graph ordered pairs with rational number coordinates in the coordinate plane.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the ordered pair (−4, 3) is shown, and ask students to discuss with a partner how they would graph that point in the coordinate plane. Make sure they can clearly explain what process they would use, and why. Then have them continue the animation to compare their process with the one shown. Ask them how they can use their understanding of the signs of coordinates to verify they graphed the point in the correct quadrant.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to graph an ordered pair on the coordinate plane.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of graphing ordered pairs, have them identify and graph an ordered pair to represent the final position of the runner in the following real-world situation.

Calida starts her run at the intersection of Main Street and Lafayette Avenue. Main Street runs north and south, and Lafayette Avenue runs east and west. The streets in her city can be modeled using a coordinate grid with her starting position at the origin. On her route, Calida runs 5 blocks west, then turns and runs 6 blocks north. She then runs 2 blocks east, then turns and runs 10 blocks south where she stops. What ordered pair can be used to represent her stopping position if her beginning position was (0, 0) on the grid? How far west of Main Street is Calida when she stops?
(−3, −4); 3 blocks

Learn Graph Ordered Pairs

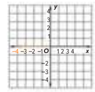
To graph an ordered pair, place a dot at the point that corresponds to the coordinates.

Go Online Watch the animation to see how to graph ordered pairs.

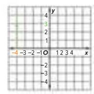
You can graph a point on the coordinate plane using the *x*- and *y*-coordinates.

Graph *A*(−4, 3). The *x*-coordinate is −4. The *y*-coordinate is 3.

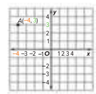
Because the *x*-coordinate is negative, move left four units on the *x*-axis from the origin.



Because the *y*-coordinate is positive, move up three units.



Graph point *A* by placing a dot at (−4, 3).



Interactive Presentation

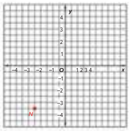
Learn, Graph Ordered Pairs

WATCH

On Slide 1, students watch an animation that explains how to graph ordered pairs.

Example 5 Graph Ordered Pairs

Graph $N\left(-2\frac{1}{2}, -3\frac{1}{2}\right)$.



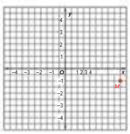
Start at the origin.

The x-coordinate is negative, so move $2\frac{1}{2}$ units left along the x-axis.

Next, since the y-coordinate is negative, move $3\frac{1}{2}$ units down. Place a dot at this location.

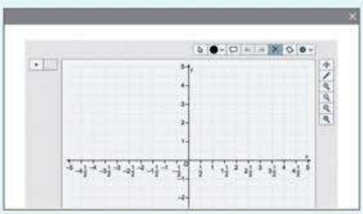
Check

Graph $M(4.5, -1)$.



Go Online You can complete an Extra Example online.

Interactive Presentation



Example 5, Graph Ordered Pairs, Slide 1 of 2

eTOOLS

On Slide 1, students use the Graphing eTool to graph an ordered pair on the coordinate plane.

CLICK

On Slide 1, students indicate how to move on the coordinate plane based on the ordered pair.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Graph Ordered Pairs

Objective

Students will graph ordered pairs with rational number coordinates on the coordinate plane.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use the Graphing eTool to graph the point.

6 Attend to Precision Encourage students to pay careful attention to the sign of each coordinate and how the ordered pair indicates movement on the coordinate plane.

Questions for Mathematical Discourse

SLIDE 1

AL In which quadrant will the point be located? Quadrant III

OL What does the x-coordinate tell you to do? Sample answer: Starting at the origin, move $2\frac{1}{2}$ units to the left.

OL What does the y-coordinate tell you to do? Sample answer: From my current location, move $3\frac{1}{2}$ units down, and place a dot.

BL How far is the point from the x-axis? the y-axis? $3\frac{1}{2}$ units; $2\frac{1}{2}$ units

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Maps

Objective

Students will come up with their own strategy to solve an application problem involving locations of places on a map of a town.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- To locate the library, in which direction should you go on the x-axis? the y-axis?
- If the dry cleaner is west of the library, should you go left or right? How many units?
- If the dry cleaner is north of the library, should you go up or down? How many units?

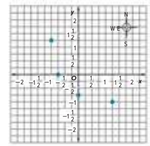
Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Maps

The table shows the locations for several different places around town. The grid shows a map of the town, and each square on the grid represents one city block. Ben needs to go to the dry cleaner, which is 3 blocks west and 5 blocks north of the library. Where on the grid should he go?

Place	Location
Bank	$\left(1\frac{1}{4}, -1\right)$
Grocery	$\left(-\frac{3}{4}, 0\right)$
Library	$\left(0, -\frac{3}{4}\right)$
Post Office	$\left(-1, 1\frac{1}{4}\right)$



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$\left(-\frac{3}{4}, \frac{1}{2}\right)$; See students' work.

4 How can you show your solution is reasonable?

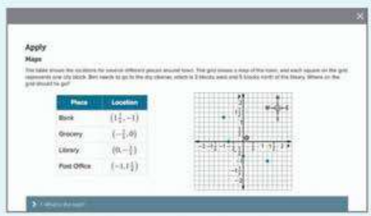
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Why was the location of the library important?

Sample answer: Before I could find the ordered pair for the dry cleaner, I first had to locate the library on the coordinate plane, using the ordered pair given in the table.

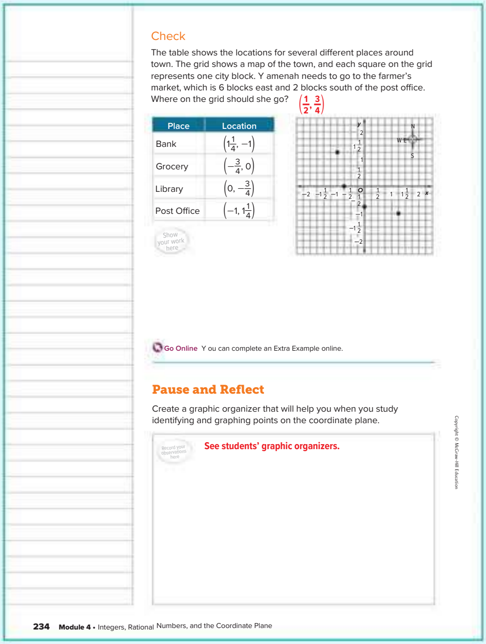
Interactive Presentation



Apply, Maps

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows the locations for several different places around town. The grid shows a map of the town, and each square on the grid represents one city block. Y amenah needs to go to the farmer's market, which is 6 blocks east and 2 blocks south of the post office. Where on the grid should she go? $\left(\frac{1}{2}, \frac{3}{4}\right)$

Place	Location
Bank	$\left(1\frac{1}{4}, -1\right)$
Grocery	$\left(-\frac{3}{4}, 0\right)$
Library	$\left(0, -\frac{3}{4}\right)$
Post Office	$\left(-1, 1\frac{1}{4}\right)$

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that will help you when you study identifying and graphing points on the coordinate plane.

See students' graphic organizers.

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Interactive Presentation

Exit Ticket

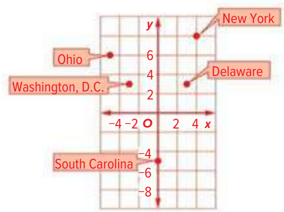
Essential Question Follow-Up

How are integers and rational numbers related to the coordinate plane?

In Lesson 1, students learned how to represent integers on a number line. In Lesson 4, students learned how to represent rational numbers on a number line. In this lesson, students learned how to represent ordered pairs of integers and rational numbers on the coordinate plane. Encourage them to work with a partner to describe how their understanding of graphing these numbers on a number line can help them understand how to graph ordered pairs with integer and rational number coordinates on the coordinate plane.

Exit Ticket

Refer to the Exit Ticket slide. Plot and label the locations on a coordinate plane.



ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 15–19
- ALEKS Ordered Pairs

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–13, 15–17
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS Plotting and Comparing Signed Numbers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Signed Numbers

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

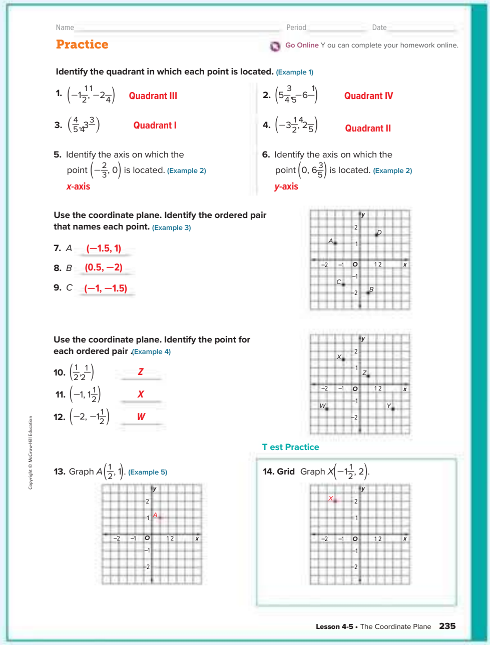
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify the quadrant of the coordinate plane in which a given point is located	1–4
1	identify the axis on which a point is located	5, 6
1	identify an ordered pair that represents a point graphed on the coordinate plane	7–9
1	identify a point on the coordinate plane given an ordered pair	10–12
1	graph ordered pairs with rational number coordinates on the coordinate plane	13
2	extend concepts learned in class to apply them in new contexts	14
3	solve application problems involving writing an ordered pair	15
3	higher-order and critical thinking skills	16–19

Common Misconception

In Exercise 7–9, students might write the incorrect ordered pairs for each point because they did not refer to the unit labels on the x-and y-axes. If students give the ordered pair (−3, 2) for point A, (1, −4) for point B, and (−2, −3) for point C, have students take a second look at the unit labels. If the units are not labeled, it indicates that the units are in increments of 1. If there are numbers on the axes, then the units might not be in increments of 1. Inform students that with any graph, it is important to look at the unit labels to determine whether the units are in increments of 1.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Identify the quadrant in which each point is located. (Example 1)

1. $\left(-1\frac{1}{2}, -2\frac{1}{4}\right)$ Quadrant III
2. $\left(5\frac{3}{4}, -6\frac{1}{5}\right)$ Quadrant IV
3. $\left(\frac{4}{5}, 3\frac{3}{4}\right)$ Quadrant I
4. $\left(-3\frac{1}{2}, 2\frac{4}{5}\right)$ Quadrant II
5. Identify the axis on which the point $\left(-\frac{2}{3}, 0\right)$ is located. (Example 2) *x*-axis
6. Identify the axis on which the point $\left(0, 6\frac{3}{5}\right)$ is located. (Example 2) *y*-axis

Use the coordinate plane. Identify the ordered pair that names each point. (Example 3)

7. *A* (−1.5, 1)
8. *B* (0.5, −2)
9. *C* (−1, −1.5)

Use the coordinate plane. Identify the point for each ordered pair. (Example 4)

10. $\left(\frac{1}{2}, \frac{1}{2}\right)$ *Z*
11. $\left(-1, 1\frac{1}{2}\right)$ *X*
12. $\left(-2, -1\frac{1}{2}\right)$ *W*

13. Graph $A\left(\frac{1}{2}, 1\right)$. (Example 5)

Test Practice

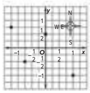
14. **Grid** Graph $X\left(-1\frac{1}{2}, 2\right)$.

Lesson 4-5 • The Coordinate Plane 235

Apply *indicates multi-step problem

*15. The table shows the locations for several different places around a small city. The grid shows a map of the city, and each square on the grid represents one city block. Shannon needs to go to the library that is 2 blocks east and 3 blocks south of the bakery. Where on the grid should she go? $\left(-\frac{1}{4}, -1\frac{1}{4}\right)$

Place	Location
Bakery	$\left(-\frac{3}{4}, -\frac{1}{2}\right)$
Courthouse	$\left(0, \frac{1}{2}\right)$
Restaurant (1,	−1)
T own Hall	$\left(-1\frac{1}{4}, \frac{3}{4}\right)$



Higher-Order Thinking Problems

16. **Identify Structure** If the point (a, b) is located in Quadrant I, in which Quadrant is the point $(a, -b)$ located?

Quadrant IV

17 . **Identify Structure** If the point $(-m, n)$ is located in Quadrant I, what must be true about the value of m? the value of n?

***m* is a negative number; *n* is a positive number**

18. **Reason Inductively** Determine if the following statement is *true* or *false*. Explain your reasoning.

A point can be represented by more than one ordered pair.

false; Sample answer: A point is defined by only one ordered pair: an *x*-coordinate that corresponds to a number on the *x*-axis and a *y*-coordinate that corresponds to a number on the *y*-axis.

19. **Find the Error** A student stated that if the point $(-a, b)$ is located in Quadrant I, then the point (a, b) is located in Quadrant IV . Find the student's mistake and correct it.

Sample answer: The student did not consider that *b* is positive, and therefore would be in either Quadrant I or II. The correct answer is Quadrant II.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure
In Exercise 16, students examine the coordinates of points in the first and third quadrants.

In Exercise 17, students examine the coordinates of points in the second and fourth quadrants.

2 Reason Abstractly and Quantitatively In Exercise 18, students determine if a point can be represented by more than one ordered pair.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 19, students find and correct a student's mistake.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercise 15 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement might be "The point representing the location of the bakery is located in Quadrant III." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Create your own higher-order thinking problem.
Use with Exercises 16–19 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Learn Reflections of Points

Objective

Students will understand that when two ordered pairs differ only by signs, the points are reflections of each other across one or both axes.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the coordinates of *P′* after a reflection across the *x*-axis, and how they relate to the coordinates of the original point *P*.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 1

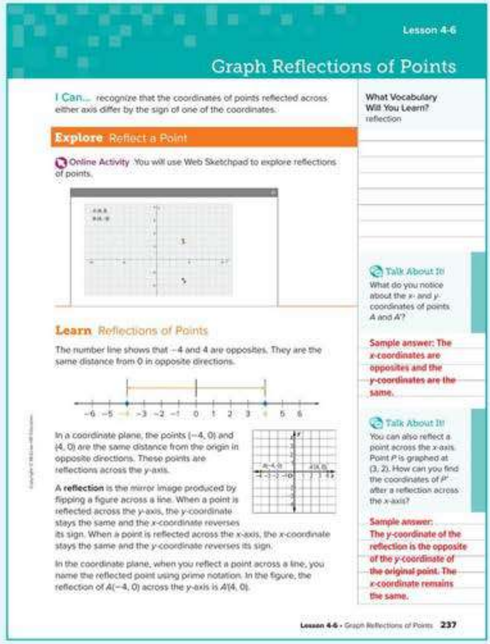
Mathematical Discourse

What do you notice about the *x*- and *y*-coordinates of points *A* and *A′*? Sample answer: The *x*-coordinates are opposites and the *y*-coordinates are the same.

SLIDE 3

Mathematical Discourse

You can also reflect a point across the *x*-axis. Point *P* is graphed at (3, 2). How can you find the coordinates of *P′* after a reflection across the *x*-axis? Sample answer: The *y*-coordinate of the reflection is the opposite of the *y*-coordinate of the original point. The *x*-coordinate remains the same.


Lesson 4-6

Graph Reflections of Points

I Can... recognize that the coordinates of points reflected across either axis differ by the sign of one of the coordinates.

What Vocabulary Will You Learn?
reflection

Explore Reflect a Point

Online Activity You will use Web Sketchpad to explore reflections of points.

Talk About It!
What do you notice about the *x*- and *y*-coordinates of points *A* and *A′*?

Sample answer: The *x*-coordinates are opposites and the *y*-coordinates are the same.

Learn Reflections of Points

The number line shows that −4 and 4 are opposites. They are the same distance from 0 in opposite directions.

In a coordinate plane, the points (−4, 0) and (4, 0) are the same distance from the origin in opposite directions. These points are reflections across the *y*-axis.

A **reflection** is the mirror image produced by flipping a figure across a line. When a point is reflected across the *y*-axis, the *y*-coordinate stays the same and the *x*-coordinate reverses its sign. When a point is reflected across the *x*-axis, the *x*-coordinate stays the same and the *y*-coordinate reverses its sign.

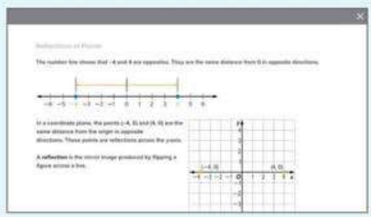
In the coordinate plane, when you reflect a point across a line, you name the reflected point using prime notation. In the figure, the reflection of *A*(−4, 0) across the *y*-axis is *A′*(4, 0).

Talk About It!
You can also reflect a point across the *x*-axis. Point *P* is graphed at (3, 2). How can you find the coordinates of *P′* after a reflection across the *x*-axis?

Sample answer: The y-coordinate of the reflection is the opposite of the y-coordinate of the original point. The x-coordinate remains the same.

Lesson 4-6 • Graph Reflections of Points 237

Interactive Presentation



Learn, Reflections of Points, Slide 1 of 3

DIFFERENTIATE

Reteaching Activity AL

For students that may be struggling to understand how points are reflected across axes, explain to them that they can think about the axis of reflection as the line that the coordinate plane can be folded over to match up a point and its reflection. Students may benefit from drawing a coordinate plane on paper and making a fold along the axis of reflection to identify the point of reflection. Have students use this method to reflect each of the following points about the axis specified.

(1, 3); *x*-axis (1, −3)

(−4, 2); *y*-axis (4, 2)

(−7, −3); *y*-axis (7, −3)

(8, −5); *x*-axis (8, 5)

Lesson 4-6

Graph Reflections of Points

6.NS.C.6, 6.NS.C.8

LESSON GOAL

Students will graph reflections of points within the coordinate plane.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Reflect a Point

Learn: Reflections of Points

Example 1: Identify Reflections of Points Across the *x*-axis

Example 2: Identify Reflections of Points Across the *y*-axis

Example 3: Identify the Axis of Reflection

Apply: Geography

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Translations in the Coordinate Plane		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 25 of the *Language Development Handbook* to help your students build mathematical language related to reflections of points in the coordinate plane.

ELL You can use the tips and suggestions on page T25 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: The Number System

Major Cluster(s): In this lesson, students address major cluster **6.NS.C** by graphing reflections of points within the coordinate plane.

Standards for Mathematical Content: **6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8,** *Also addresses 6.NS.C.6.A*

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous

Students identified ordered pairs, points, and quadrants and graphed ordered pairs on the coordinate plane.
6.NS.C.6, 6.NS.C.8

Now

Students graph reflections of points within the coordinate plane.
6.NS.C.6, 6.NS.C.8

Next

Students will use absolute value to find distance on the coordinate plane.
6.NS.C.8

Rigor

The Three Pillars of Rigor

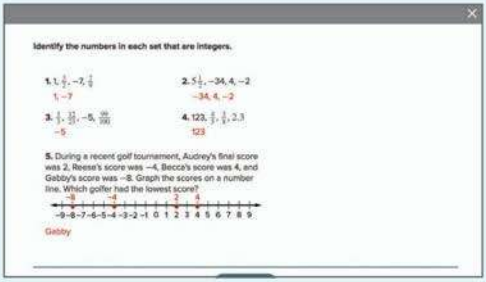
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of the coordinate plane as they explore reflections of points within the plane. They build *fluency* with using prime notation when writing ordered pairs of reflected points and identifying the axis of reflection of a given point. They *apply* their understanding of reflections of points to solve real-world problems.

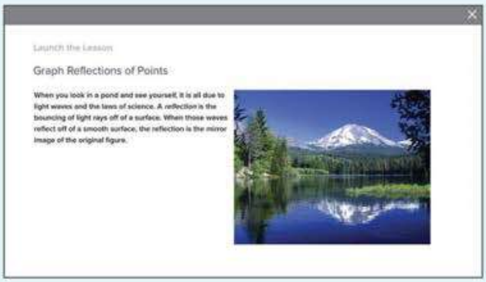
Mathematical Background

A *reflection* is the mirror image produced by flipping a figure across a line. The reflection of the point (x, y) across the x-axis is $(x, -y)$. The reflection of the point (x, y) across the y-axis is $(-x, y)$.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding integers (Exercises 1–4)
- graphing integers on a number line (Exercise 5)

Answers

1. 1, −7
2. −34, 4, −2
3. −5
4. 123
5. See Warm Up slide online for correct answer.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about reflections in the world around us.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Describe how you have used the words *reflect* or *reflection* in everyday life. Sample answer: I see my reflection in the mirror; A shiny surface reflects sunlight.

Explore Reflect a Point

Objective

Students will use Web Sketchpad to explore reflections of points.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the Talk About It! questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations

Summary of Activity

Students will be presented with a graph with points *A* and *B*. Students will explore the relationship between point *A* and point *B*. Students will then be presented with a graph with points *A* and *C*. Students will compare the similarities and differences between the two graphs.

Inquiry Question

What happens to the coordinates of a point when a point is reflected across an axis? Sample answer: When the point is reflected across the *x*-axis, the *x*-coordinate stays the same and the *y*-coordinates are opposites, except when the point is on the axis. When the point is reflected across the *y*-axis, the *y*-coordinate stays the same and the *x*-coordinates are opposites, except when the point is on the axis.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

What are the similarities and differences in the coordinates of the points? Sample answer: The *x*-coordinates are the same, but the *y*-coordinates are opposites, unless the point is on the *x*-axis.

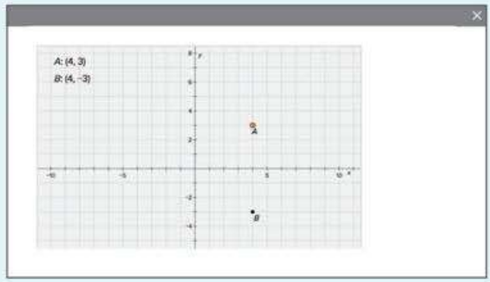
What do you notice about their location in the coordinate plane? Sample answer: The points are always on the same side of the *y*-axis, but opposite sides of the *x*-axis, unless the point is on the *x*-axis.

Where can you drag the points so their coordinates are the same? any point on the *x*-axis

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 2 of 6

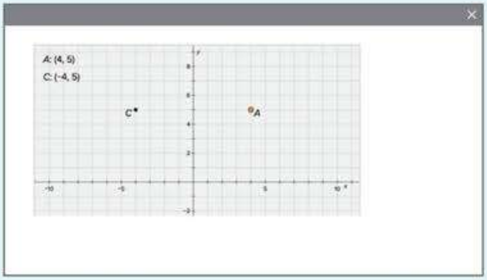
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore what happens to the coordinates of a point when it is reflected across an axis.

TYPE

On Slide 3, students explain what they think represents the mirror given that two points are mirror images of one another.

Interactive Presentation



Explore, Slide 4 of 6

TYPE

On Slide 5, students explain what they think represents the mirror given that two points are mirror images of one another.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Reflect a Point *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and deepen their understanding of reflections of points. Encourage students to examine the sketches and compare the coordinates of the points and their locations on the coordinate plane.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What are the similarities and differences in the coordinates of the points? Sample answer: The *y*-coordinates are always the same, and the *x*-coordinates are always opposites, unless the point is on the *y*-axis.

What do you notice about their location in the coordinate plane? Sample answer: The points are always on the same side of the *x*-axis, but opposite sides of the *y*-axis, unless the point is on the *y*-axis.

Where can you drag the points so their coordinates are the same? any point on the *y*-axis

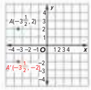
Your Notes

Think About It!
In what quadrant is point *A* located? In what quadrant will the reflection of point *A* across the *x*-axis be located?

II; III

Example 1 Identify Reflections of Points Across the *x*-axis

Write the ordered pair that is a reflection of $A\left(-3\frac{1}{2}, 2\right)$ across the *x*-axis.



Find the point on the coordinate plane that is the same distance from the *x*-axis as the original point. Graph the point on the coordinate plane and label it.

When a point is reflected across the *x*-axis, the ___*x*___-coordinate stays the same and the ___*y*___-coordinates are opposites.

So, the coordinates of the reflection of $A\left(-3\frac{1}{2}, 2\right)$ across the *x*-axis are $\left(-3\frac{1}{2}\ -2\right)$.

Talk About It!
How do you know, without graphing, that the point $A'\left(-3\frac{1}{2}, -2\right)$ is the reflection of the point $A\left(-3\frac{1}{2}, 2\right)$ across the *x*-axis?

Sample answer: The coordinates of the point and the coordinates of its reflection have the same *x*-coordinate and opposite *y*-coordinates. Therefore, the reflection is across the *x*-axis.

Check

Write the ordered pair that is a reflection of $Q\left(1\frac{1}{2}\ 2\frac{1}{4}\right)$ across the *x*-axis.

$\left(1\frac{1}{2}, -2\frac{1}{4}\right)$

Show your work here

Go Online Y ou can complete an Extra Example online.

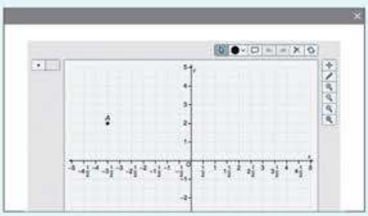
Pause and Reflect

Did you make any errors when completing the Check exercise? What can you do to make sure you don't repeat that error in the future?

Record your observations here

See students' observations.

Interactive Presentation



Example 1, Identify Reflections of Points Across the *x*-Axis, Slide 2 of 4

eTOOLS

On Slide 2, students use the Graphing eTool to graph a point on the coordinate plane.

TYPE

On Slide 2, students enter the coordinates of the reflection of a point.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Identify Reflections of Points Across the *x*-axis

Objective

Students will write an ordered pair to represent the reflection of a given point across the *x*-axis.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use the Graphing eTool to graph and locate the point on the coordinate plane that is the same distance away from the *x*-axis as the original point.

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to study the structure of each ordered pair, noting that the *x*-coordinates are the same and the *y*-coordinates are opposites. This will tell them that point *A′* is the reflection of point *A* across the *x*-axis.

Questions for Mathematical Discourse

SLIDE 2

AL Which axis is the point being reflected across? *x*-axis

OL What do you know about points being reflected across the *x*-axis? Sample answer: When a point is reflected across the *x*-axis the *x*-coordinate stays the same, and the *y*-coordinates are opposites.

BL How many units away is point *A* from point *A′*? 4 units

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Identify Reflections of Points Across the *y*-axis

Objective

Students will write an ordered pair to represent the reflection of a given point across the *y*-axis.

Questions for Mathematical Discourse

SLIDE 1

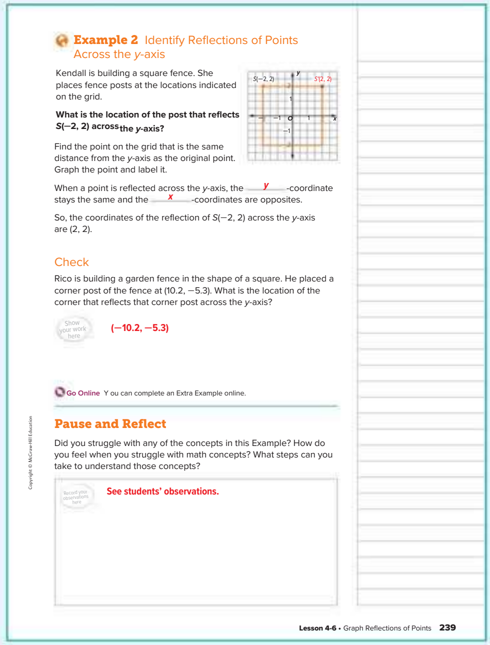
AL In what quadrant will the reflected point be located? Quadrant I

OL What do you know about reflections across the *y*-axis? Sample answer: When a point is reflected across the *y*-axis, the *y*-coordinate stays the same, and the *x*-coordinates are opposites.

BL Reflect the point (—1, 1) across the *y*-axis and then move the reflected point 2 units down. How many units away is the final location of the point from the *x*-axis? 1 unit

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Identify Reflections of Points Across the *y*-axis

Kendall is building a square fence. She places fence posts at the locations indicated on the grid.

What is the location of the post that reflects *S*(−2, 2) across the *y*-axis?

Find the point on the grid that is the same distance from the *y*-axis as the original point. Graph the point and label it.

When a point is reflected across the *y*-axis, the ___*y*___-coordinate stays the same and the ___*x*___-coordinates are opposites.

So, the coordinates of the reflection of *S*(−2, 2) across the *y*-axis are (2, 2).

Check

Rico is building a garden fence in the shape of a square. He placed a corner post of the fence at (10.2, −5.3). What is the location of the corner that reflects that corner post across the *y*-axis?

Show your work here

(−10.2, −5.3)

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you struggle with any of the concepts in this Example? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

Record your observations here

See students' observations.

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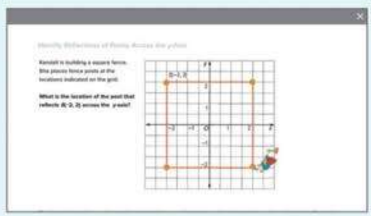
Lesson 4-6 • Graph Reflections of Points 239

Interactive Presentation



Example 2, Identify Reflections of Points Across the *y*-axis, Slide 1 of 2

TYPE

On Slide 1, students enter the coordinates of the reflection of a point.

CHECK

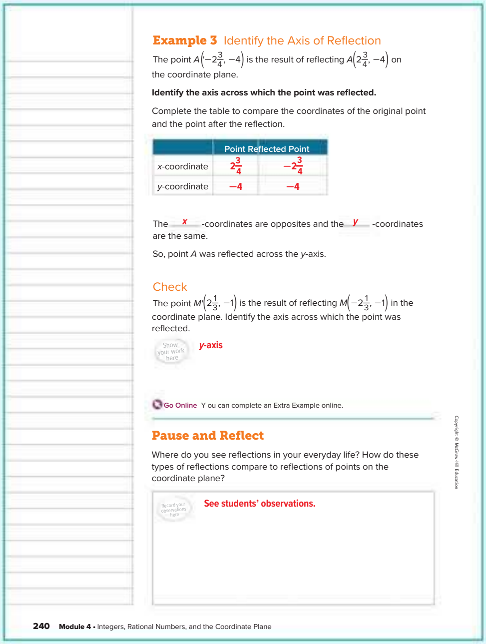
Students complete the Check exercise online to determine if they are ready to move on.

DIFFERENTIATE

Language Development Activity ELL

Students should be familiar with *reflections* in their everyday life. Discuss with students how they have seen their reflection in a mirror, or the reflection of the sky in a lake. When they see their reflection in the mirror, the distance their reflection is from the mirror is the same distance they are from the mirror. If they walk closer to the mirror, their reflection will become closer. Because a *reflection* is a mirror image, in mathematics, the *reflection* of a point is the mirror image of that point produced by flipping that point over a *line*, called the *line of reflection*. Have students work with a partner to create a table like the one shown. They should generate three points and list their ordered pairs in the table. Have them trade tables with another pair of students. Each pair should reflect the points across the *x*-axis and *y*-axis and record the resulting ordered pairs.

Point	Reflection across *x*-axis	Reflection across *y*-axis



Example 3 Identify the Axis of Reflection

The point $A\left(-2\frac{3}{4}, -4\right)$ is the result of reflecting $A\left(2\frac{3}{4}, -4\right)$ on the coordinate plane.

Identify the axis across which the point was reflected.

Complete the table to compare the coordinates of the original point and the point after the reflection.

	Point	Reflected Point
x-coordinate	$2\frac{3}{4}$	$-2\frac{3}{4}$
y-coordinate	−4	−4

The *x* -coordinates are opposites and the *y* -coordinates are the same.

So, point *A* was reflected across the *y*-axis.

Check

The point $M\left(2\frac{1}{3}, -1\right)$ is the result of reflecting $M\left(-2\frac{1}{3}, -1\right)$ in the coordinate plane. Identify the axis across which the point was reflected.

Show your work here

y-axis

Go Online You can complete an Extra Example online.

Pause and Reflect

Where do you see reflections in your everyday life? How do these types of reflections compare to reflections of points on the coordinate plane?

Record your observations here

See students' observations.

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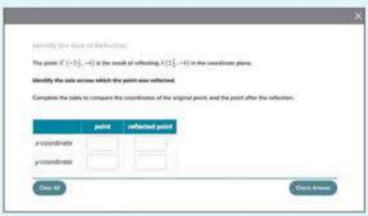
240 Module 4 • Integers, Rational Numbers, and the Coordinate Plane

Interactive Presentation



Example 3, Identify the Axis of Reflection, Slide 1 of 2

TYPE

On Slide 1, students complete a table to compare the coordinates of the original point and the point after the reflection.

CLICK

On Slide 1, students identify the axis of reflection.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Identify the Axis of Reflection

Objective

Students will identify the axis of reflection for a point graphed on the coordinate plane.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to identify the axis of reflection accurately and precisely, by paying careful attention to the signs of the coordinates of the original point and its reflection.

Questions for Mathematical Discourse

SLIDE 1

AL What do you need to find? the axis of reflection

OL What do you know about the coordinates of points when they are reflected across the *x*-axis? the *y*-axis? How can this help you determine the axis of reflection? Sample answer: When a point is reflected across the *x*-axis, the *x*-coordinate stays the same, and the *y*-coordinates are opposites. When a point is reflected across the *y*-axis, the *y*-coordinate stays the same, and the *x*-coordinates are opposites. In this case, the *y*-coordinate stays the same and the *x*-coordinates are opposites, so the axis of reflection is the *y*-axis.

BL What is the horizontal distance between the original point and its reflection? $2\frac{3}{4} + 2\frac{3}{4}$, or $5\frac{1}{2}$ units

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Geography

Objective

Students will come up with their own strategy to solve an application problem involving the locations of objects in a neighborhood park.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

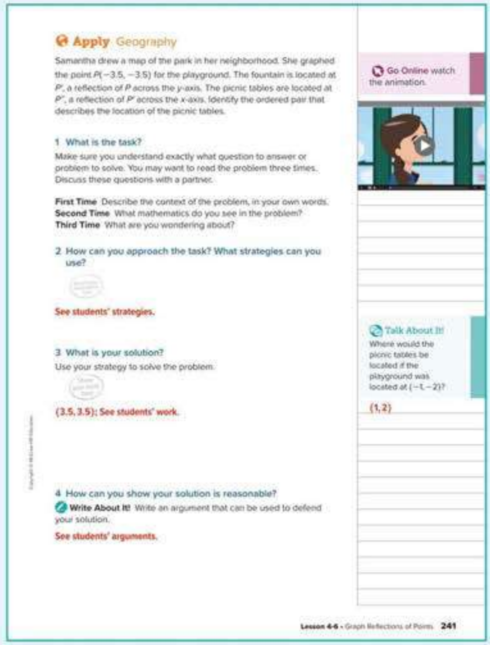
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What object should you find the location of first?
- What do you notice about the location of the fountain?
- How will you use the location of the fountain to determine the location of the picnic tables?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Geography

Samantha drew a map of the park in her neighborhood. She graphed the point $P(-3.5, -3.5)$ for the playground. The fountain is located at P', a reflection of P across the y-axis. The picnic tables are located at P'', a reflection of P' across the x-axis. Identify the ordered pair that describes the location of the picnic tables.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

(3.5, 3.5); See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online watch the animation.

Talk About It! Where would the picnic tables be located if the playground was located at $(-1, -2)$?

(1, 2)

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Interactive Presentation

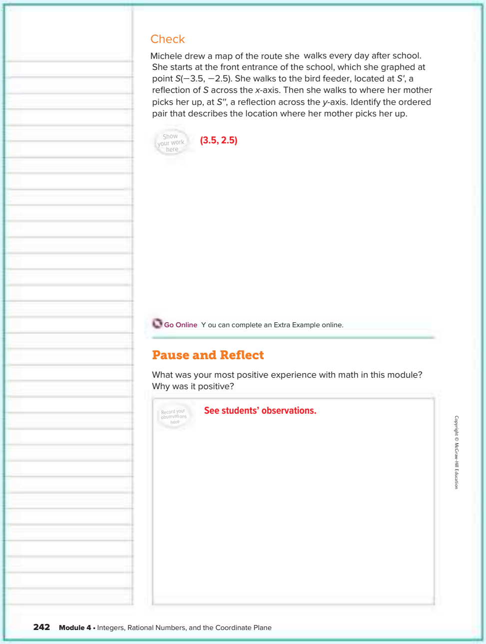
Apply, Geography

WATCH

On Slide 1, students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Michele drew a map of the route she walks every day after school. She starts at the front entrance of the school, which she graphed at point $S(-3.5, -2.5)$. She walks to the bird feeder, located at S', a reflection of S across the x-axis. Then she walks to where her mother picks her up, at S'', a reflection across the y-axis. Identify the ordered pair that describes the location where her mother picks her up.

Show your work here

(3.5, 2.5)

Go Online You can complete an Extra Example online.

Pause and Reflect

What was your most positive experience with math in this module? Why was it positive?

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. If the y-axis passes through points A and B, and the y-coordinate for point A is 5, what ordered pairs represent each point? point A: (0, 5); point B: (0, −5)

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 9, 11, 13–16
- Extension: Translations in the Coordinate Plane
- ALEKS Ordered Pairs

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 11, 13, 15
- Extension: Translations in the Coordinate Plane
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Plotting and Comparing Signed Numbers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Signed Numbers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	write an ordered pair to represent the reflection of a given point across the *x*-axis	1–4
2	write an ordered pair to represent the reflection of a given point across the *y*-axis	5, 6
1	identify the axis of reflection for a point graphed on the coordinate plane	7, 8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving writing ordered pairs to represent reflections	11, 12
3	higher-order and critical thinking skills	13–16

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Write the ordered pair that is a reflection of each point across the *x*-axis. (Example 1)

1. $A\left(-2\frac{3}{4}, 1\right)$ $\left(-2\frac{3}{4}, -1\right)$

2. $B\left(1\frac{1}{4}, -\frac{1}{2}\right)$ $\left(1\frac{1}{4}, \frac{1}{2}\right)$

3. $C\left(-4, -2\frac{1}{2}\right)$ $\left(-4, 2\frac{1}{2}\right)$

4. $D\left(\frac{3}{4}, 3\right)$ $\left(\frac{3}{4}, -3\right)$

5. Aika is building a square garden. She places a garden post at (3.5, 3.5). What is the location of the corner that reflects (3.5, 3.5) across the *y*-axis? (Example 2)

(−3.5, 3.5)

6. A farmer is installing a chicken pen in the shape of a square. He placed a corner of the enclosure at (−5.25, −5.25). What is the location of the corner that reflects (−5.25, −5.25) across the *y*-axis? (Example 2)

(5.25, −5.25)

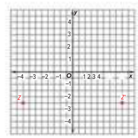
7. The point $C'(-4, -2)$ is the result of reflecting $C(4, -2)$ in the coordinate plane. Identify the axis across which the point was reflected. (Example 3)

***y*-axis**

8. The point $B'\left(-5\frac{1}{4}, -3\frac{1}{2}\right)$ is the result of reflecting $B\left(-5\frac{1}{4}, 3\frac{1}{2}\right)$ in the coordinate plane. Identify the axis across which the point was reflected. (Example 3)

***x*-axis**

9. Graph point $Z(-4, -2.5)$ on the coordinate plane. Then graph its reflection across the *y*-axis.



Test Practice

10. Multiple Choice Which ordered pair represents a reflection of point $Y\left(1\frac{3}{4}, -4\right)$ across the *x*-axis?

Ⓐ $\left(-4, 1\frac{3}{4}\right)$

Ⓑ $\left(1\frac{3}{4}, -4\right)$

Ⓒ $\left(1\frac{3}{4}, 4\right)$

Ⓓ $\left(-1\frac{3}{4}, 4\right)$

Apply *indicates multi-step problem

*11. T rey drew a map of the summer camp he is staying at this summer . He graphed the point $D(-4.5, 4.5)$ for the dining hall. The flag pole is located at D', a reflection of D across the y-axis. The campfire is located at D'', a reflection of D' across the x-axis. Identify the ordered pair that describes the location of the campfire.

(4.5, −4.5)

*12. Liv drew a map of her favorite park. She graphed the point $S\left(2\frac{1}{2}, -2\right)$ for the swings. The picnic tables are located at S', a reflection of S across the x-axis. The lake is located at S'', a reflection of S' across the y-axis. Identify the ordered pair that describes the location of the lake.

$\left(-2\frac{1}{2}, 2\right)$

Higher-Order Thinking Problems

13. MP **Find the Error** A student was finding the ordered pair for point $Y(1.5, -2)$ after its reflection across the x-axis. Find the student's mistake and correct it.

$Y(1.5, -2) \rightarrow Y'(-1.5, -2)$

Sample answer: The student wrote the ordered pair for a reflection across the *y*-axis, not the *x*-axis. The correct ordered pair for point *Y'* is (1.5, 2).

14. MP **Persevere with Problems** Determine whether the statement is *always*, *sometimes*, or *never* true. Justify your response.

When a point is reflected across the x-axis, the new point has a negative y-coordinate.

sometimes; Sample answer: The *y*-coordinate of the new point will be negative if the *y*-coordinate of the original point was positive.

15. Identify the coordinates of a point located in Quadrant III. Reflect the point across the y-axis. Then give the coordinates of the reflected point.

Sample answer: *A*(−1, −1); *A'*(1, −1)

16. MP **Reason Inductively** A point is located on the y-axis. It is reflected across the x-axis. What do you know about the x- and y-coordinates of the reflected point?

Sample answer: The *x*-coordinate is equal to 0 since the point lies on the *y*-axis. The *y*-coordinate is equal to −*y* (the opposite of *y*) since the point was reflected across the *x*-axis.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students find and correct a student's mistake.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 14, students determine if a statement is always, sometimes, or never true.

2 Reason Abstractly and Quantitatively In Exercise 16, students explain what they know about a point located on the y-axis that is reflected across the x-axis.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 11–12 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Explore the truth of statements created by others.
Use with Exercises 13–16 Have students work in pairs. After completing the exercises, have students write two true statements about reflections of points and one false statement. An example of a true statement might be, "When a point is reflected across the x-axis, the x-coordinate remains the same." An example of a false statement might be, "When a point is reflected across the y-axis, both coordinates remain the same." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. For each false statement, have them correct the statement. Have them discuss and resolve any differences.

Learn Find Horizontal Distance

Objective

Students will learn how to find the horizontal distance between two points with the same *y*-coordinate.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, encourage them to construct a plausible argument for why the distance cannot be negative, even if both points are in Quadrant III.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find the horizontal distance between two points with the same *y*-coordinate by using the absolute values of the *x*-coordinates.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the first set of points are shown, (−5, −4) and (−1, −4). Ask students how they can determine the distance between the points. Some students will simply count the units to find that the distance is 4 units. Be sure that students understand how this is related to using absolute value to find the distance between the two points. You may wish to continue the same discussion after the second two points are shown, (−4, 2) and (1, 2).

Talk About It!

SLIDE 2

Mathematical Discourse

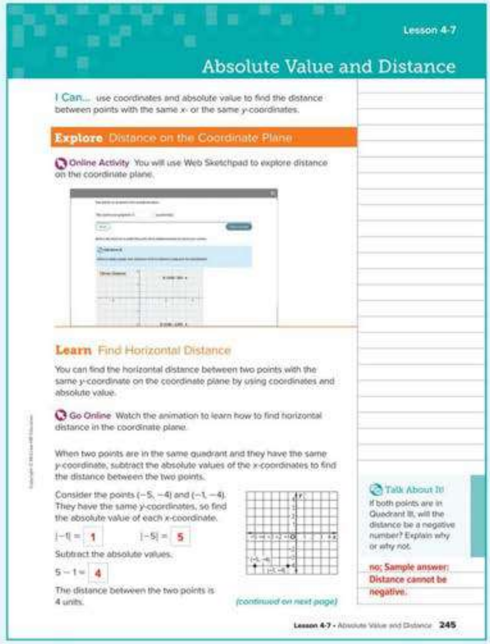
If both points are in Quadrant III, will the distance be a negative number? Explain why or why not. no; Sample answer: Distance cannot be negative.

DIFFERENTIATE

Reteaching Activity AL

For students that may be struggling to understand how to find the horizontal distance between two points, explain that the absolute value of each coordinate is used to determine the distance of one point to the corresponding axis. For each of the following ordered pairs, have students identify the horizontal distance from the point to the *y*-axis, found using the absolute value of the *x*-coordinate. Students may benefit from actually graphing the point to determine the distance.

(−3, 4) 3 units	(6, −1) 6 units	(−5, 2) 5 units
(7, 9) 7 units	(−2, −8) 2 units	(4, −7) 4 units



Lesson 4-7

Absolute Value and Distance

I Can... use coordinates and absolute value to find the distance between points with the same *x*- or the same *y*-coordinates.

Explore Distance on the Coordinate Plane

Online Activity You will use Web Sketchpad to explore distance on the coordinate plane.

Learn Find Horizontal Distance

You can find the horizontal distance between two points with the same *y*-coordinate on the coordinate plane by using coordinates and absolute value.

Go Online Watch the animation to learn how to find horizontal distance in the coordinate plane.

When two points are in the same quadrant and they have the same *y*-coordinate, subtract the absolute values of the *x*-coordinates to find the distance between the two points.

Consider the points (−5, −4) and (−1, −4). They have the same *y*-coordinates, so find the absolute value of each *x*-coordinate.

$|-1| = 1$ $|-5| = 5$

Subtract the absolute values.

$5 - 1 = 4$

The distance between the two points is 4 units.

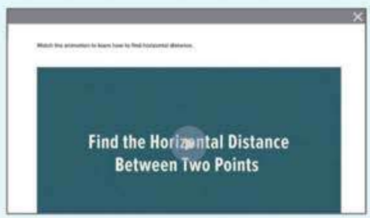
(continued on next page)

Talk About It! If both points are in Quadrant III, will the distance be a negative number? Explain why or why not.

no; Sample answer: Distance cannot be negative.

Lesson 4-7 • Absolute Value and Distance 245

Interactive Presentation



Learn, Find Horizontal Distance, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that explains how to find the horizontal distance between two pointswith the same *y*-coordinate.

LESSON GOAL

Students will use absolute value to find the distance between points on the coordinate plane.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Distance on the Coordinate Plane

Learn: Find Horizontal Distance
Example 1: Find Horizontal Distance in the Same Quadrant
Example 2: Find Horizontal Distance in Different Quadrants
Learn: Find Vertical Distance
Example 3: Find Vertical Distance in the Same Quadrant
Example 4: Find Vertical Distance in Different Quadrants
Apply: Distance

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

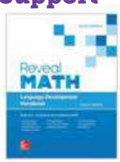
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Arrive**MATH** Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 26 of the *Language Development Handbook* to help your students build mathematical language related to absolute value and distance.



ELL You can use the tips and suggestions on page T26 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **6.NS.C** by using the absolute value of integers to find the distance between points on a coordinate plane.
Standards for Mathematical Content: **6.NS.C.8,** *Also addresses 6.NS.C.6, 6.NS.C.7.C*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students graphed reflections of points within the coordinate plane.
6.NS.C.6, 6.NS.C.8

Now
Students use absolute value to find the distance between points on the coordinate plane.
6.NS.C.8

Next
Students will solve problems involving adding integers and rational numbers.
7.NS.A.1

Rigor

The Three Pillars of Rigor

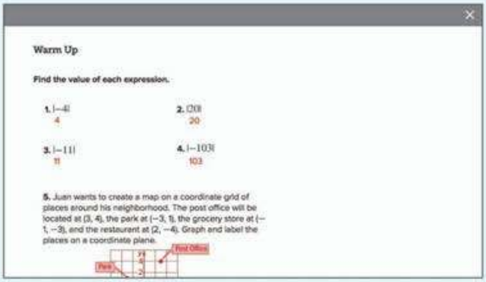
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their *understanding* of absolute value to find distance between points on the coordinate plane. They build *fluency* with finding distance in the same and different quadrants of the coordinate plane. They *apply* their understanding of distance on the coordinate plane to solve real-world problems.

Mathematical Background

If two points in the coordinate plane have the same y-coordinate, and lie in the same quadrant, the distance between the two points is the difference of the absolute values of the x-coordinates. If the points lie in different quadrants, the distance is the sum of the absolute values of the x-coordinates. Similarly, if the points have the same x-coordinate, and lie in the same quadrant, the distance between the points is the difference of the absolute values of the y-coordinates. If the points lie in different quadrants, the distance is the sum of the absolute values of the y-coordinates.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding absolute value of integers (Exercises 1–4)
- graphing ordered pairs in all four quadrants (Exercise 5)

Answers

1. 4
2. 20
3. 11
4. 103
5. See Warm Up slide online for correct answer.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about planning a sightseeing trip around Washington, D.C.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Describe in your own words what the absolute value of a number means. Sample answer: The absolute value of a number is the distance that number is from 0 on a number line.
- Name three parts of the *coordinate plane*. Sample answer: origin, x-axis, y-axis
- Describe in your own words what *quadrant* means. Sample answer: The coordinate plane is divided into four quadrants, or sections.

Explore Distance on the Coordinate Plane

Objective

Students will use Web Sketchpad to explore distance on the coordinate plane.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the Talk About It! questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations

Summary of Activity

Students will be presented with points plotted in a coordinate grid. Throughout this activity, students will explore various ways to find the distance between two points. Students will use their observations to make conjectures about how to accurately find the distance between points with the same *x*- or *y*-coordinates, graphed on the coordinate plane.

Inquiry Question

How can you use absolute value to find distance on the coordinate plane? Sample answer: Absolute value makes it possible to add or subtract the coordinates to find the distance between points.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

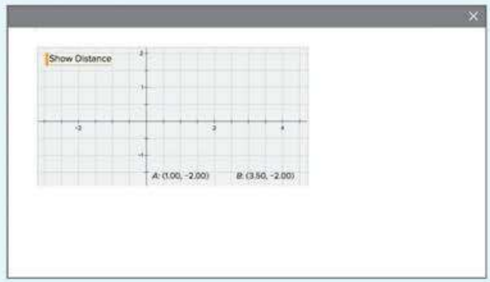
Mathematical Discourse

Describe a method you could use to find the distance between the two points. What is the distance in units? Sample answer: I can count the number of unit squares between the points. Because the scale is 0.5, the distance is 2.5 units.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 9



Explore, Slide 2 of 9

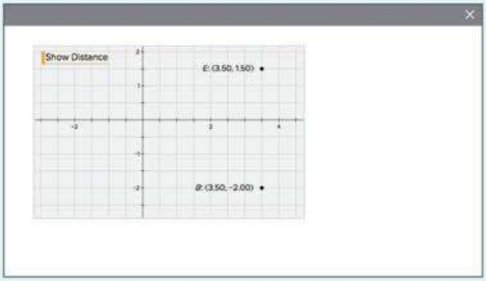
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how absolute value can be used to find the distance between points on the coordinate plane.

CLICK

On Slides 2 and 4, students identify the quadrant in which the two points are graphed.

Interactive Presentation



Explore, Slide 7 of 9

CLICK

On Slides 6 and 7, students identify the quadrant(s) in which the two points are graphed.

TYPE

On Slide 8, students write a rule to find distance on the coordinate plane when the points are in the same quadrant and in different quadrants.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Distance on the Coordinate Plane *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to examine and deepen their understanding of the distance between two points in a coordinate plane and how this distance might be related to the absolute value of the coordinates.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore the distance between two points located in the coordinate plane.

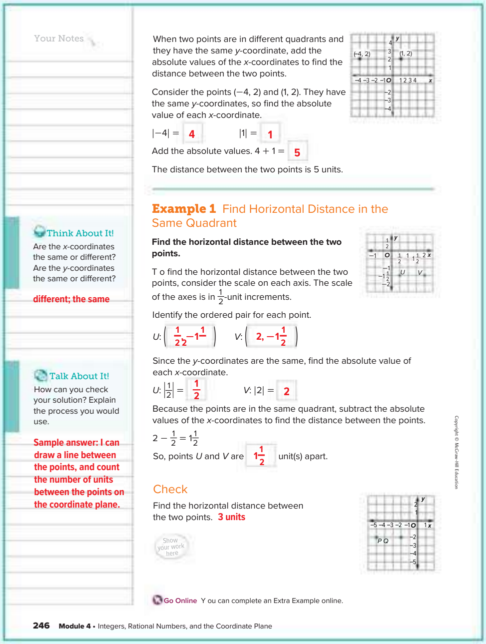
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Without using a graph, how could you find the distance using only the coordinates? Sample answer: Points *E* and *B* have the same *x*-coordinates but different *y*-coordinates. I can find the absolute value of the *y*-coordinates and add to find the distance.



Your Notes

When two points are in different quadrants and they have the same *y*-coordinate, add the absolute values of the *x*-coordinates to find the distance between the two points.

Consider the points (−4, 2) and (1, 2). They have the same *y*-coordinates, so find the absolute value of each *x*-coordinate.

$|-4| =$ 4 $|1| =$ 1

Add the absolute values. $4 + 1 =$ 5

The distance between the two points is 5 units.

Example 1 Find Horizontal Distance in the Same Quadrant

Find the horizontal distance between the two points.

T o find the horizontal distance between the two points, consider the scale on each axis. The scale of the axes is in $\frac{1}{2}$-unit increments.

Identify the ordered pair for each point.

U: $\left(\frac{1}{2}, -1\frac{1}{2}\right)$ *V*: $\left(2, -1\frac{1}{2}\right)$

Since the *y*-coordinates are the same, find the absolute value of each *x*-coordinate.

U: $\left|\frac{1}{2}\right| = \frac{1}{2}$ *V*: $|2| = 2$

Because the points are in the same quadrant, subtract the absolute values of the *x*-coordinates to find the distance between the points.

$2 - \frac{1}{2} = 1\frac{1}{2}$

So, points *U* and *V* are $1\frac{1}{2}$ unit(s) apart.

Think About It! Are the *x*-coordinates the same or different? Are the *y*-coordinates the same or different?

different; the same

Talk About It! How can you check your solution? Explain the process you would use.

Sample answer: I can draw a line between the points, and count the number of units between the points on the coordinate plane.

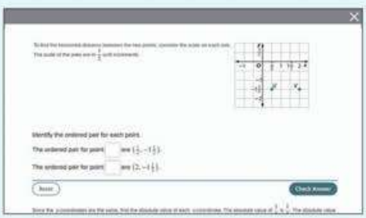
Check

Find the horizontal distance between the two points. 3 units

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Find Horizontal Distance in the Same Quadrant, Slide 2 of 4

TYPE

On Slide 2, students identify the ordered pair for each point.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find Horizontal Distance in the Same Quadrant

Objective

Students will find the horizontal distance between two points in the same quadrant on the coordinate plane.

MP Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them As students discuss the *Talk About It!* question on Slide 3, encourage them to think of another strategy they could have used to find the distance between the two points, and to use that strategy to check their solution.

6 Attend to Precision Encourage students to accurately and efficiently find the coordinates of each point, the correct absolute value of each *x*-coordinate, and the distance between the two points.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the horizontal distance between the two points

OL Why is it important to use the absolute value of each *x*-coordinate? It is important to use the absolute value of each *x*-coordinate because in some situations, the *x*-coordinate(s) may be negative. Distance is always positive.

OL In this specific example, do you need to find the absolute value first? Explain. no; Sample answer: The *x*-coordinates of both points were already positive. I could have just subtracted the lesser value from the greater.

BL If point *U* was instead located in Quadrant III with the same *y*-coordinate of point *V*, describe how you could find the distance between points *U* and *V*. Sample answer: I can find the absolute value of the *x*-coordinates and add them, instead of subtracting them, since they are on opposite sides of the *y*-axis.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find Horizontal Distance in Different Quadrants

Objective

Students will find the horizontal distance between two points in different quadrants on the coordinate plane.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the fact that when two points are in different quadrants, the absolute values must be added together instead of subtracted. Encourage students to explain why, in order to solidify their understanding.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language about which operation to use (addition or subtraction) when the points are located in the same or different quadrants.

Questions for Mathematical Discourse

SLIDE 2

AL Why is it important to identify how many units each square represents? in order to find the correct distance

OL What might happen if you do not find the absolute value of each *x*-coordinate? Sample answer: I might find an incorrect distance. Point *E*'s *x*-coordinate is negative, so adding a negative value to the value of Point *F*'s *x*-coordinate will result in an incorrect distance.

OL Explain why you add the absolute values in this example, instead of subtracting them. Sample answer: The points are located in different quadrants, so they are on opposite sides of the *y*-axis. I need to add the distance from one point to the *y*-axis to the distance from the other point to the *y*-axis.

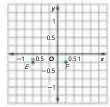
BL Could you use this method to find the distance between two points that have different *y*-coordinates? Explain. no; Sample answer: I am finding horizontal distance between two points with the same *y*-coordinate. If the *y*-coordinates are not the same, I will need to use a different method.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find Horizontal Distance in Different Quadrants

Find the horizontal distance between the two points.



To find the horizontal distance between the two points, consider the scale on each axis. The scale of the axes are in 0.25-unit increments.

Identify the ordered pair for each point.

E: (−0.75, −0.25) *F*: (0.25, −0.25)

Since the *y*-coordinates are the same. Find the absolute value of each *x*-coordinate.

E: $|-0.75| =$ 0.75 *F*: $|0.25| =$ 0.25

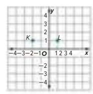
Because the points are in different quadrants, add the absolute values of the *x*-coordinates to find the distance between the points.

$0.75 + 0.25 = 1$

So, points *U* and *V* are 1 unit(s) apart.

Check

Find the horizontal distance between the two points. 3 units



Show your work here

Go Online You can complete an Extra Example online.

Think About It! Are the points in the same quadrant? How will that affect how you find the distance?

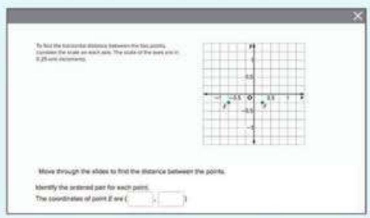
See students' responses.

Talk About It! Use the graph to explain why the absolute values of the *x*-coordinates are added when the points are in different quadrants.

Sample answer: The absolute value of the *x*-coordinate gives the distance the point is from the *y*-axis. The distances from each point to the *y*-axis do not overlap, like they do when the points are in the same quadrant. Therefore, the distances have to be added together.

Lesson 4-7 • Absolute Value and Distance 247

Interactive Presentation

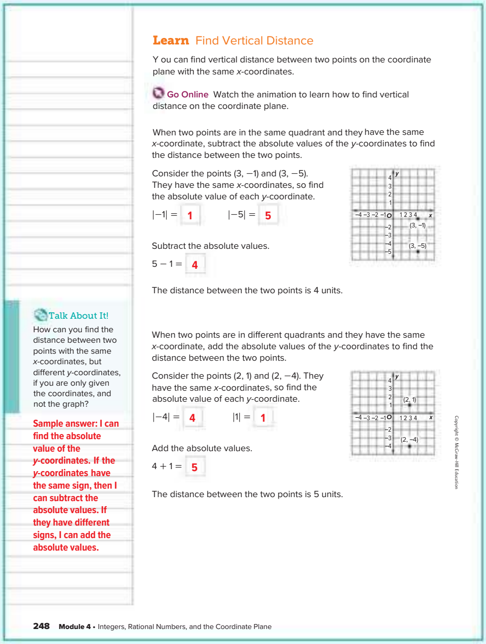


Example 2, Find Horizontal Distance in Different Quadrants, Slide 2 of 4

CLICK On Slide 2, students move through the slides to find the distance between the points.

TYPE On Slide 2, students determine the absolute values.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Learn Find Vertical Distance

Y ou can find vertical distance between two points on the coordinate plane with the same *x*-coordinates.

Go Online Watch the animation to learn how to find vertical distance on the coordinate plane.

When two points are in the same quadrant and they have the same *x*-coordinate, subtract the absolute values of the *y*-coordinates to find the distance between the two points.

Consider the points (3, −1) and (3, −5). They have the same *x*-coordinates, so find the absolute value of each *y*-coordinate.

$|-1| =$ 1 $|-5| =$ 5

Subtract the absolute values.

$5 - 1 =$ 4

The distance between the two points is 4 units.

When two points are in different quadrants and they have the same *x*-coordinate, add the absolute values of the *y*-coordinates to find the distance between the two points.

Consider the points (2, 1) and (2, −4). They have the same *x*-coordinates, so find the absolute value of each *y*-coordinate.

$|-4| =$ 4 $|1| =$ 1

Add the absolute values.

$4 + 1 =$ 5

The distance between the two points is 5 units.

Talk About It!

How can you find the distance between two points with the same *x*-coordinates, but different *y*-coordinates, if you are only given the coordinates, and not the graph?

Sample answer: I can find the absolute value of the *y*-coordinates. If the *y*-coordinates have the same sign, then I can subtract the absolute values. If they have different signs, I can add the absolute values.

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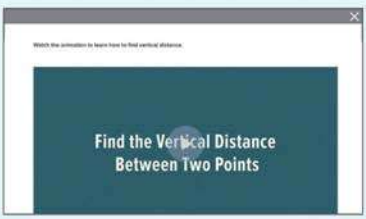
Interactive Presentation



Learn, Find Vertical Distance, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that explains how to find the vertical distance between two points with the same *x*-coordinate.

Learn Find Vertical Distance

Objective

Students will learn how to find the vertical distance between two points with the same *x*-coordinate.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to explain how they could study the structure of two ordered pairs (with the same *x*-coordinate), without looking at the graph, in order to determine how to find the distance between the points.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the first set of points are shown, (3, −1) and (3, −5). Ask students how they can determine the distance between the points. Some students will simply count the units to find that the distance is 4 units. Be sure that students understand how this is related to using absolute value to find the distance between the two points. You may wish to continue the same discussion after the second two points are shown, (2, 1) and (2, −4).

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find the vertical distance between two points on a coordinate plane that have the same *x*-coordinates.

Talk About It!

SLIDE 2

Mathematical Discourse

How can you find the distance between two points with the same *x*-coordinates, but different *y*-coordinates, if you are only given the coordinates, and not the graph? Sample answer: I can find the absolute value of the *y*-coordinates. If the *y*-coordinates have the same signs, then I can subtract the absolute values. If they are different signs, I can add the absolute values.

Example 3 Find Vertical Distance in the Same Quadrant

Objective

Students will find the vertical distance between two points in the same quadrant on the coordinate plane.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them As students discuss the *Talk About It!* question on Slide 3, encourage them to think of another strategy they could have used to find the distance between the two points, and to use that strategy to check their solution.

6 Attend to Precision Encourage students to accurately and efficiently determine that the points are in the same quadrant, find the correct absolute value of each *y*-coordinate, and determine that they need to subtract the absolute values to find the distance.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the signs of the coordinates of the points? Each coordinate of each point is negative.

AL Compare the *x*-coordinates. What do you notice? They are the same.

OL Why is it important to find the absolute values of the *y*-coordinates? Sample answer: The points are in Quadrant III, and the coordinates are negative. If I do not find the absolute value before adding, I will be subtracting negative values to find the distance. Distance is always positive.

OL Explain why you subtract the absolute values in this example, instead of adding them. Sample answer: The points are located in the same quadrant, so they are on the same side of the *x*-axis. I need to subtract the distance from one point to the *x*-axis from the distance from the other point to the *x*-axis, because the distances overlap.

BL If point *D* was located at (−1, 3), how would this affect the process you would use to find the distance? Points *C* and *D* will be in different quadrants, so I will need to add the absolute values of the *y*-coordinates to find the distance.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Find Vertical Distance in the Same Quadrant

Find the vertical distance between the points $D\left(-1, -\frac{1}{2}\right)$ and $C(-1, -2)$.

The *x*-coordinates are negative and the *y*-coordinates are negative.

This means the points are in Quadrant III.

The *x*-coordinates are the same. T o find the distance each point is from the *x*-axis, find the absolute value of each *y*-coordinate.

$C: |-2| = 2$ $D: \left|-\frac{1}{2}\right| = \frac{1}{2}$

Because the points are in the same quadrant, subtract the absolute values of the *y*-coordinates to find the distance between the points.

$2 - \frac{1}{2} = 1\frac{1}{2}$

So, points *C* and *D* are $1\frac{1}{2}$ unit(s) apart.

Check

Find the vertical distance between the points $A\left(-\frac{1}{3}, -\frac{2}{3}\right)$ and $B\left(-\frac{1}{3}, -1\frac{1}{3}\right)$. $\frac{2}{3}$ unit

Show your work here

Go Online You can complete an Extra Example online.

Think About It! Are the *x*-coordinates the same or different? Are the *y*-coordinates the same or different?

the same; different

Talk About It! How can you check your solution? Explain a process you could use.

Sample answer: I can graph the points on a coordinate plane. Then I can count the units between the points, paying careful attention to the scale on the axes.

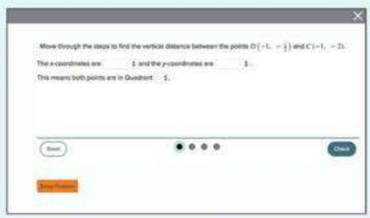
Pause and Reflect

Did you struggle with any of the concepts in this Example and Check? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

Record your observations here

See students' observations.

Interactive Presentation



Example 3, Find Vertical Distance in the Same Quadrant, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the vertical distance between two points.

CLICK

On Slide 2, students identify the quadrant in which the points lie.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
Are the points in the same quadrant? How will that affect how you find the distance?

See students' responses.

Talk About It!
Compare and contrast finding vertical and horizontal distance between two points in the coordinate plane.

Sample answer: To find vertical distance, the points must have the same *x*-coordinates. To find horizontal distance, the points must have the same *y*-coordinates. If the points are in the same quadrant, subtract the absolute values. If the points are in different quadrants, add the absolute values.

Example 4 Find Vertical Distance in Different Quadrants

Find the vertical distance between points $S(1, 0.5)$ and $T(1, -0.5)$.

The *x*-coordinates have the same signs.

The *y*-coordinates have different signs.

This means the points are in different quadrants.

The *x*-coordinates are the same. T o find the distance each point is from the *x*-axis, find the absolute value of each *y*-coordinate.

S: $|0.5| =$ 0.5 $\quad$ T: $|-0.5| =$ 0.5

Because the points are in different quadrants, add the absolute values of the *y*-coordinates to find the distance between the points.

$0.5 + 0.5 = 1$

So, points S and T are 1 unit(s) apart.

Check

Find the vertical distance between points $E(0.5, 1.5)$ and $F(0.5, -2)$.

Show your work here

3.5 units

Go Online Y ou can complete an Extra Example online.

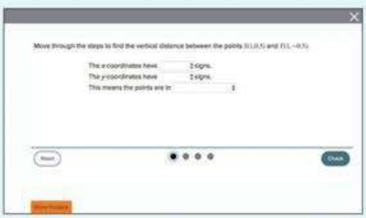
Pause and Reflect

Could you use the methods described in this lesson to find the distance between two points on a number line? Explain your reasoning.

Record your observations here

See students' observations.

Interactive Presentation



Example 4, Find Vertical Distance in Different Quadrants, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the vertical distance between two points.

TYPE

On Slide 2, students determine the absolute values.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find Vertical Distance in Different Quadrants

Objective

Students will find the vertical distance between two points in different quadrants on the coordinate plane.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to accurately and efficiently determine that the points are in different quadrants, find the correct absolute value of each *y*-coordinate, and determine that they need to add the absolute values to find the distance.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language in their explanations.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know the points are in different quadrants? The *x*-coordinates are the same, and the *y*-coordinates have different signs. This means they are not in the same quadrant.

OL Since the points are in different quadrants, do you add or subtract the absolute values? Explain. add; Sample answer: The distances from each point to the *x*-axis do not overlap, so I can add them.

BL How could you change the coordinates of point S in order for you to subtract the absolute values? Sample answer: Point S would need to be in the same quadrant as Point T. If Point S had coordinates $(1, -1)$, then I could subtract the absolute values.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Distance

Objective

Students will come up with their own strategy to solve an application problem involving determining which friend has a farther distance to travel to a park.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

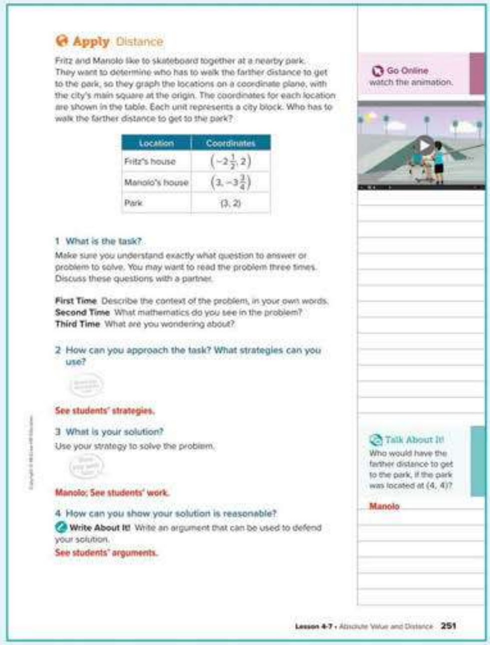
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Are the locations in the same or different quadrants?
- Will graphing the points on a coordinate plane help you? Why or why not?
- What do you notice about the x- and y-coordinates for each location?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Distance

Fritz and Manolo like to skateboard together at a nearby park. They want to determine who has to walk the farther distance to get to the park, so they graph the locations on a coordinate plane, with the city's main square at the origin. The coordinates for each location are shown in the table. Each unit represents a city block. Who has to walk the farther distance to get to the park?

Location	Coordinates
Fritz's house	$\left(-2\frac{1}{2}, 2\right)$
Manolo's house	$\left(3, -3\frac{3}{4}\right)$
Park	(3, 2)

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Manolo; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online watch the animation.

Talk About It! Who would have the farther distance to get to the park, if the park was located at (4, 4)?

Manolo

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Interactive Presentation

Apply, Distance

WATCH

On Slide 1, students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Fernando has a dog-walking job and will walk the dogs from his house to one of the two parks shown. He wants to go to the park that will give the dogs a longer walk. T o which park should he go?

Location	Coordinates
Fernando's house	$\left(-2\frac{1}{2}, 2\frac{1}{4}\right)$
Cobblestone Dog Park	$\left(1\frac{3}{4}, 2\frac{1}{4}\right)$
Blue Limestone Park	$\left(-2\frac{1}{2}, -1\frac{1}{2}\right)$

Cobblestone Dog Park

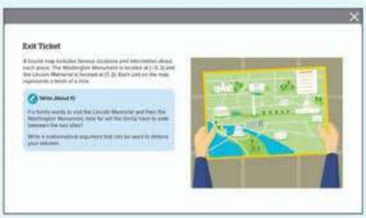
Show your work here

Go Online You can complete an Extra Example online.

Exit Ticket

Refer to the Exit Ticket slide. If a family wants to visit the Lincoln Memorial and then the Washington Monument, how far will the family have to walk between the two sites? Write a mathematical argument that can be used to defend your solution. 1 mile; Sample answer: The x-coordinates are the same and the y-coordinates have different signs, so the locations are not in the same quadrant. The locations are $|3| + |7|$ or 10 units apart. Since each unit represents a tenth of a mile, the locations are 10×0.1 mile or 1 mile apart.

Interactive Presentation



Exit Ticket

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11–15
- ALEKS Ordered Pairs

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 11–13
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Plotting and Comparing Signed Numbers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Signed Numbers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

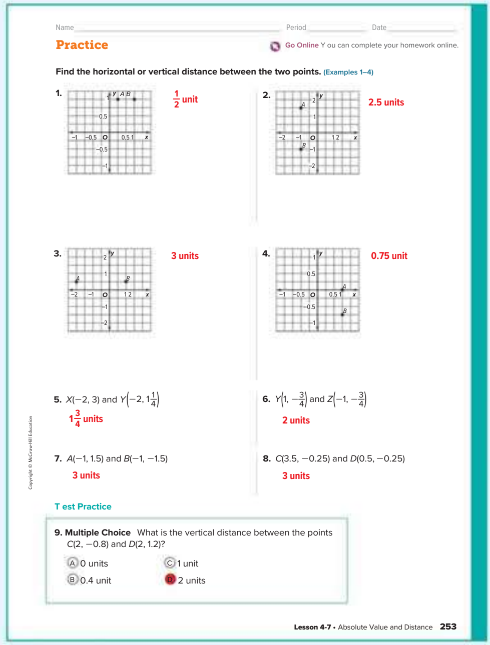
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the horizontal or vertical distance between two points	1–8
2	extend concepts learned in class to apply them in new contexts	9
3	solve application problems involving finding horizontal or vertical distance between points	10, 11
3	higher-order and critical thinking skills	12–15

Common Misconception

Students may have trouble finding the distance between two points on the coordinate plane when the units on the *x*- and *y*-axes are not 1 unit. In Exercises 1–4, the units are less than 1, so students may incorrectly identify the ordered pair for a given point. For example, in Exercise 1, students may identify the points as *A*(1, 4) and *B*(3, 4) instead of *A*(0.5, 1) and *B*(1.5, 1) This would give students the incorrect distance of 2 units instead of 1 unit. Remind students that they need to examine the units on every coordinate plane to make sure that they identify each ordered pair correctly.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find the horizontal or vertical distance between the two points. (Examples 1–4)

1. $\frac{1}{2}$ unit

2. 2.5 units

3. 3 units

4. 0.75 unit

5. $X(-2, 3)$ and $Y\left(-2, 1\frac{1}{4}\right)$ — $1\frac{3}{4}$ units

6. $Y\left(1, -\frac{3}{4}\right)$ and $Z\left(-1, -\frac{3}{4}\right)$ — 2 units

7. $A(-1, 1.5)$ and $B(-1, -1.5)$ — 3 units

8. $C(3.5, -0.25)$ and $D(0.5, -0.25)$ — 3 units

Test Practice

9. **Multiple Choice** What is the vertical distance between the points $C(2, -0.8)$ and $D(2, 1.2)$?

Ⓐ 0 units
Ⓑ 0.4 unit
Ⓒ 1 unit
Ⓓ 2 units

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Apply *indicates multi-step problem

*10. There are two parks near Kennedy's house. She wants to go the park closer to her house. T o which park should Kennedy go?

Maple Avenue Park

Location	Coordinates
Maple Avenue Park	$\left(2, 1\frac{1}{2}\right)$
Oak Woods Park	$\left(-\frac{1}{2}, -\frac{3}{4}\right)$
Kennedy's House	$\left(2, -\frac{3}{4}\right)$

*11. James and Amber walk their dogs together at a nearby dog park. They want to determine who has to walk a farther distance to get to the dog park, so they graph the locations on a coordinate plane, with the town square at the origin. Each whole unit represents a city block. James's house is located at the point (–1.5, 4). Amber's house is located at the point (2, 0.25). The dog park is located at the point (2, 4). Who has to walk the farther distance to get to the dog park?

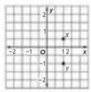
Amber

Higher-Order Thinking Problems

12. Explain how to find the distance between the points $A(-2, 2)$ and $B(-2, -2)$.

Sample answer: Graph the points on the coordinate plane. Then count the vertical units between the points. There are 4 units between the points.

13. **MP Find the Error** A student said that the vertical distance between the two points graphed is 3 units. Find the student's mistake and correct it.



Sample answer: The student did not use the scale on the *y*-axis. The scale is 0.5 unit. So, the actual distance is 1.5 units.

14. Give the coordinates for two points that have a vertical distance between them of 1.5 units.

Sample answer: $X(-1, 0)$ and $Y(-1, 1.5)$

15. Y ara said that the vertical distance between two points is –1.5 units. How do you know that Y ara's answer is incorrect?

Sample answer: Distance cannot be negative. Y ou have to find the absolute value of each coordinate and subtract the lesser number from the greater number.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students explain why another student's solution is incorrect and then correct the solution.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 11 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.

Use with Exercises 14–15 Have students work in groups of 3–4 to solve the problem in Exercise 14. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 15.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of comparing and ordering rational number sets. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

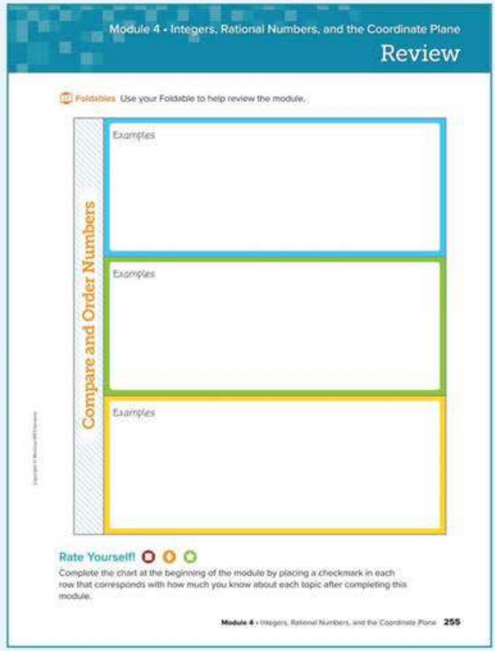
Assessment Resources

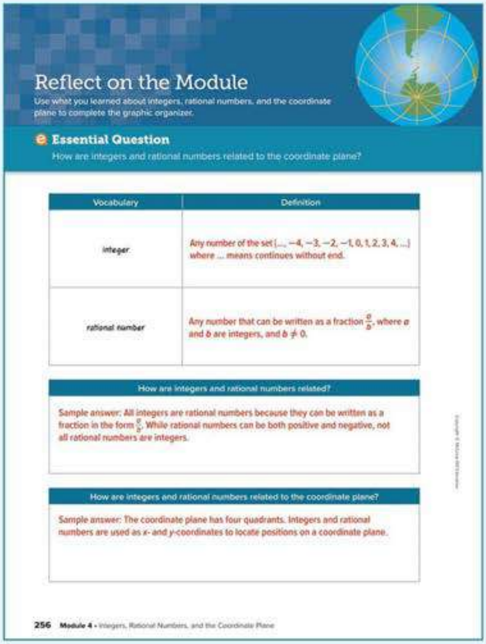
Put It All Together 1: Lessons 4-1, 4-3, and 4-4
Put It All Together 2: Lessons 4-5, 4-6, and 4-7
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **The Number System**.

- Integers, Absolute Value, and Opposites
- The Coordinate Plane
- Plotting Points on a Coordinate Plane
- Graphing Ordered Pairs in an Application





Reflect on the Module

Use what you learned about integers, rational numbers, and the coordinate plane to complete the graphic organizer.

Essential Question

How are integers and rational numbers related to the coordinate plane?

Vocabulary	Definition
integer	Any number of the set {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...} where ... means continues without end.
rational number	Any number that can be written as a fraction $\frac{a}{b}$, where a and b are integers, and $b \neq 0$.

How are integers and rational numbers related?
Sample answer: All integers are rational numbers because they can be written as a fraction in the form $\frac{a}{b}$. While rational numbers can be both positive and negative, not all rational numbers are integers.

How are integers and rational numbers related to the coordinate plane?
Sample answer: The coordinate plane has four quadrants. Integers and rational numbers are used as x- and y-coordinates to locate positions on a coordinate plane.

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How are integers and rational numbers related to the coordinate plane? See students' graphic organizers.

Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–12 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	8
Multiselect	Multiple answers may be correct. Students must select all correct answers.	10
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	3, 6, 12
Table Item	Students complete a table by correctly classifying the information.	4, 9
Grid	Students create a graph on an online coordinate plane or number line.	2, 11
Open Response	Students construct their own response in the area provided.	1, 5, 7

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.NS.C.5	4-1, 4-2	1–3
6.NS.C.6	4-1, 4-2, 4-4, 4-5, 4-6	1–3, 8–11
6.NS.C.6.A	4-2	3
6.NS.C.6.B	4-5, 4-6	8–11
6.NS.C.6.C	4-1, 4-4, 4-5, 4-6	1, 2, 8–11
6.NS.C.7	4-2, 4-3, 4-4	4–7
6.NS.C.7.A	4-3, 4-4	4, 5
6.NS.C.7.B	4-3	4, 5
6.NS.C.7.C	4-2, 4-3, 4-4	6, 7
6.NS.C.7.D	4-3	4, 5
6.NS.C.8	4-5, 4-6, 4-7	8–12

Name ______ Period ______ Date ______

Test Practice

1. Open Response While riding one of the rides at the localamusement park, Zachary lost \$5 from his pocket. (Lesson 1)

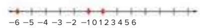
A. Write an integer to represent this situation. Explain.

−5; Because the situation represents a loss, the integer is negative.

B. Explain the meaning of zero in this situation.

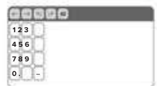
Zero represents no money gained or lost.

2. Grid Graph the set of integers {−6, −1, 0} on the number line. (Lesson 1)



3. Equation Editor Find −(−14). (Lesson 2)

14



4. Table Item Indicate whether each inequality is true or false. (Lesson 3)

	True	False
−7 < −9		X
5 > −1	X	
−12 < −10	X	

5. Open Response The table shows the boiling points, to the nearest degree Celsius, for six substances. Carbon dioxide boils at −79°C. Between which two substances is the boiling point of carbon dioxide? (Lesson 3)

Substance	Boiling Point (°C)
Ammonia	−36
Benzene	80.4
Acetylene	−84
Ethanol	79
Fluorine	−187
Water	100

acetylene and ammonia

6. Equation Editor Evaluate |−6.2|. (Lesson 4)

6.2

7 . Open Response During the overnight hours, the temperature in Juneau fell from 0°F to −12°F. How many degrees did the temperature fall? (Lesson 4)

12°

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8. Multiple Choice Identify the quadrant in which the point $\left(\frac{2}{3}, -1\frac{1}{5}\right)$ is located. (Lesson 5)

- A. Quadrant I
- B. Quadrant II
- C. Quadrant III
- D. Quadrant IV ●

9. Table Item Indicate the axis on which each of the points lie. (Lesson 5)

	x-axis	*y*-axis
(−4, 0)	X	
(0, 9)		X
(0, −6)		X

10. Multiselect Consider the point $A\left(-2\frac{1}{4}, -3\right)$. Which of the following statements are true regarding the reflection of this point? Select all that apply. (Lesson 6)

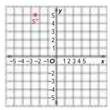
- ☐ When this point is reflected across the *x*-axis, the *x*-coordinate is the opposite and the *y*-coordinate stays the same.
- ☐ The reflection of $A\left(-2\frac{1}{4}, -3\right)$ across the *x*-axis can be represented by $A\left(2\frac{1}{4}, -3\right)$.
- ✓ When this point is reflected across the *x*-axis, the *x*-coordinate stays the same and the *y*-coordinate is the opposite.
- ✓ The reflection of $A\left(-2\frac{1}{4}, -3\right)$ across the *y*-axis can be represented by $A\left(2\frac{1}{4}, -3\right)$.
- ✓ When this point is reflected across the *y*-axis, the *x*-coordinate is the opposite and the *y*-coordinate stays the same.

11. Grid Derrius drew a map of the community playground. He graphed the point $S\left(2\frac{1}{2}, -5\right)$ for the slide. The swings are located at *S′*, a reflection across the *x*-axis. The restrooms are located at *S″*, a reflection across the *y*-axis. (Lesson 6)

A. Identify the ordered pair that describes the location of the restrooms.

$S''\left(-2\frac{1}{2}, 5\right)$

B. Plot and label the point *S″* on the coordinate plane.



12. Equation Editor What number of units describes the vertical distance between the points *X*(3, 4.5) and *Y*(3, −1)? (Lesson 7)

5.5

258 Module 4 • Integers, Rational Numbers, and the Coordinate Plane

Foldables Study Organizers

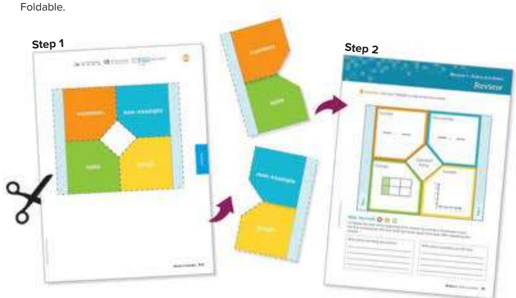
Foldables Study Organizers

What Are Foldables and How Do I Create Them?

Foldables are three-dimensional graphic organizers that help you create study guides for each module in your book.

Step 1 Go to the back of your book to find the Foldable for the module you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Module Review at the end of the module you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.



How Will I Know When to Use My Foldable?

You will be directed to work on your Foldable at the end of selected lessons. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the module test.

Foldables

How Do I Complete My Foldable?

No two Foldables in your book will look alike. However , some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

Instructions and What They Mean

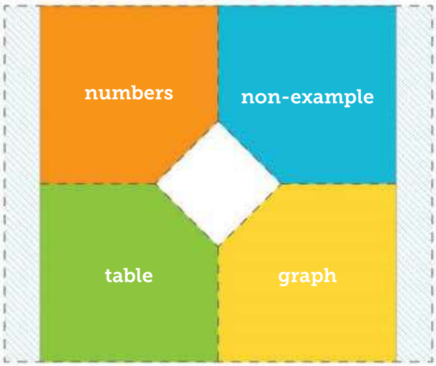
Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. Y ou may use one already in the text or you can make up your own.
Example	Write an example about the concept. Y ou may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. Y ou may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. Dinah is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

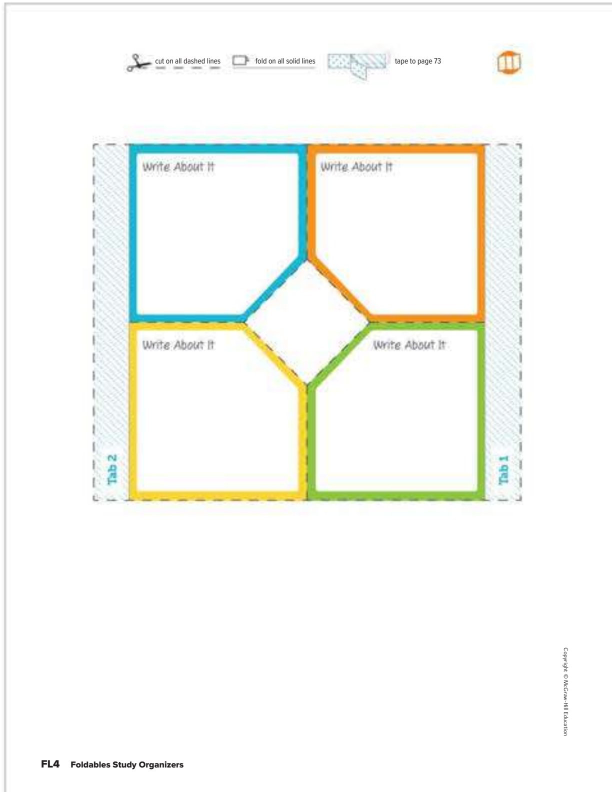
FL2 Foldables Study Organizers

cut on all dashed lines fold on all solid lines tape to page 73



Foldables

Foldables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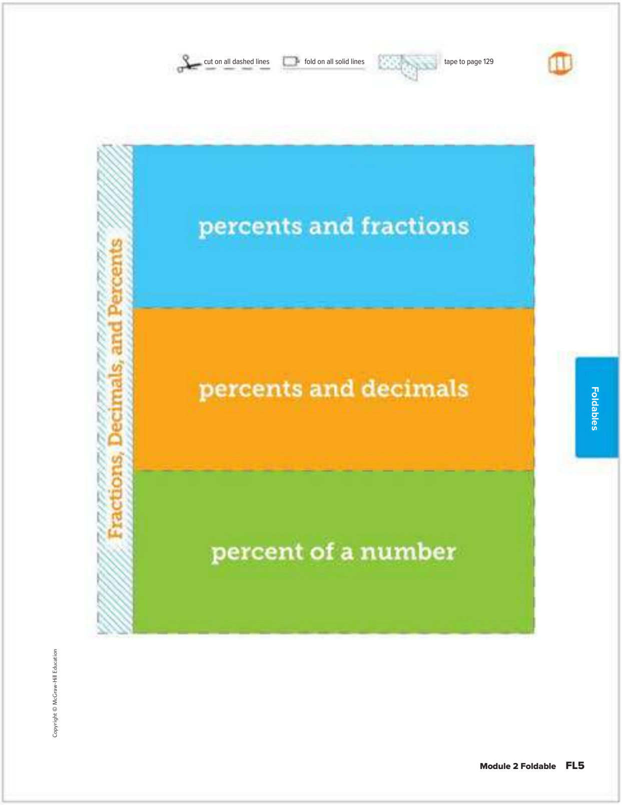
tape to page 73
Write About It
Write About It
Write About It
Write About It
Tab 2
Tab 1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Fractions, Decimals, and Percents
percents and fractions
percents and decimals
percent of a number
Foldables
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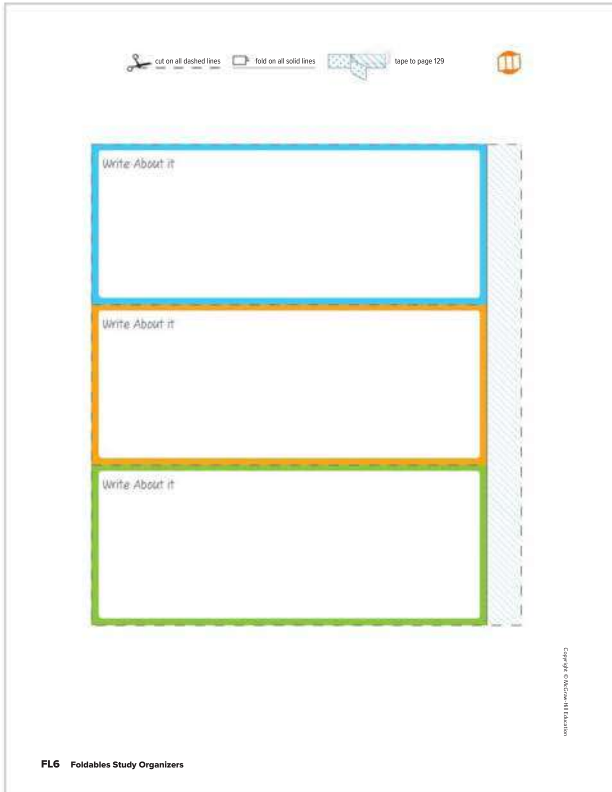
Module 2 Foldable FL5

Foldables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Write About it
Write About it
Write About it
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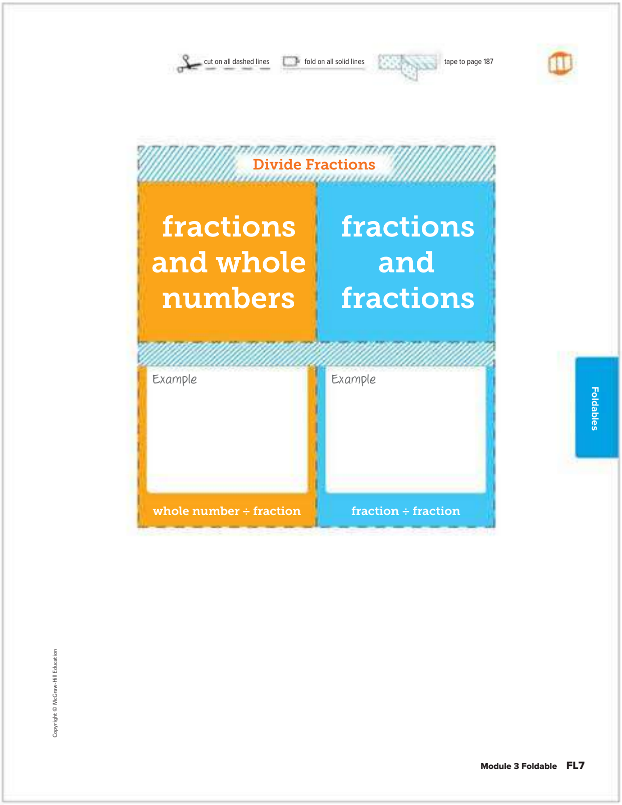
FL6 Foldables Study Organizers



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fold on all solid lines
tape to page 187

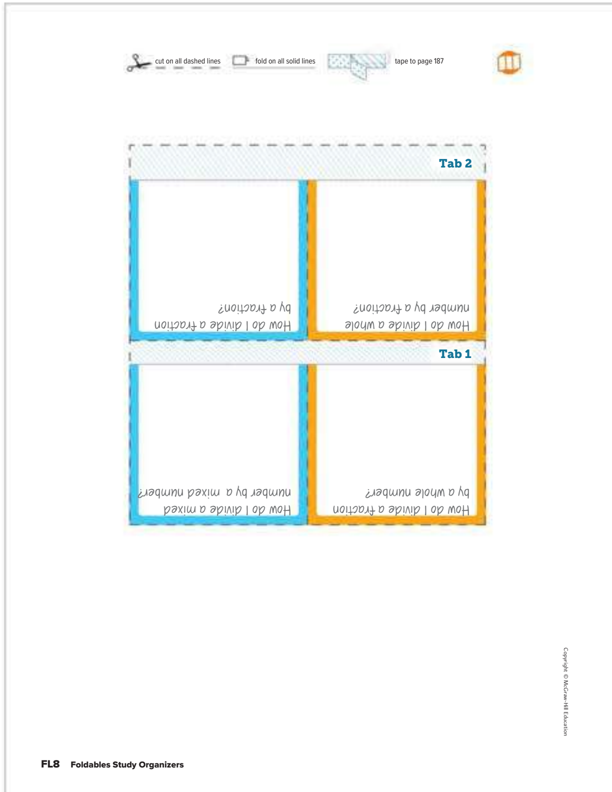
Divide Fractions

fractions and whole numbers	fractions and fractions
Example	Example
whole number ÷ fraction	fraction ÷ fraction

Foldables

Module 3 Foldable FL7

Foldables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Tab 2

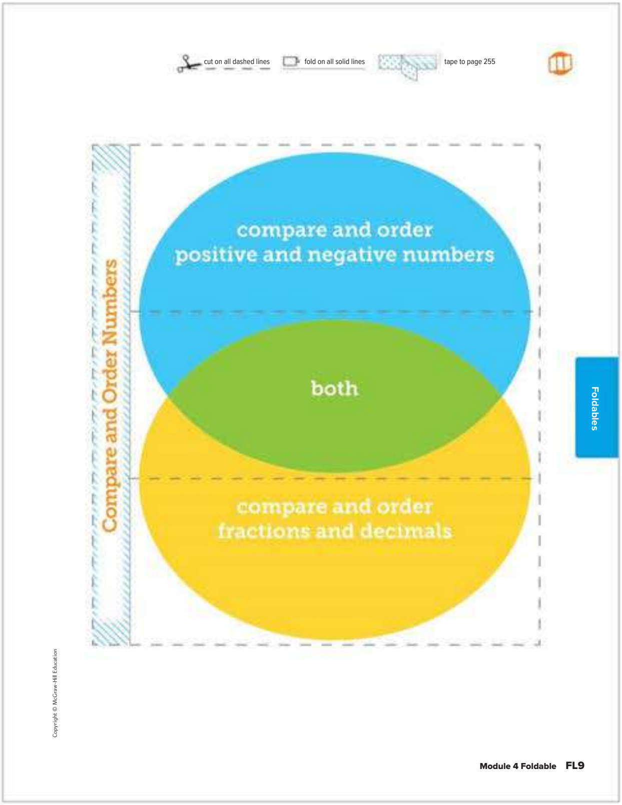
How do I divide a fraction by a fraction?

How do I divide a whole number by a fraction?

Tab 1

How do I divide a mixed number by a mixed number?

How do I divide a fraction by a whole number?


cut on all dashed lines
fold on all solid lines
tape to page 255
Compare and Order Numbers
compare and order positive and negative numbers
both
compare and order fractions and decimals
Foldables
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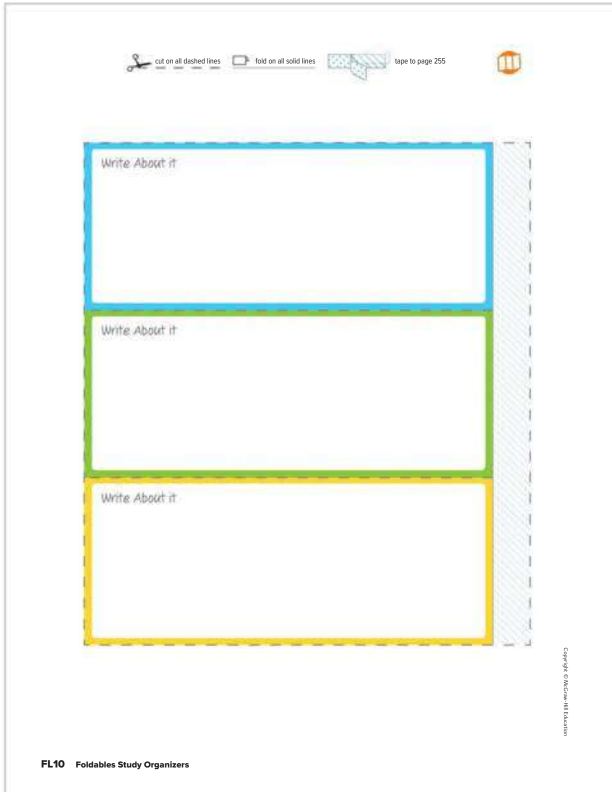
Module 4 Foldable FL9

Foldables


cut on all dashed lines
fold on all solid lines
tape to page 255
Write About it
Write About it
Write About it
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FL10 Foldables Study Organizers

Glossary

The Multilingual eGlossary contains words and definitions in the following 14 languages:

Arabic	**English**	**Hmong**	**Russian**	**Urdu**
Bengali	**French**	**Korean**	**Spanish**	**Vietnamese**
Brazilian Portuguese	**Haitian Creole**	**Mandarin**	**Tagalog**	

English	Español
A	
absolute value (Lesson 4-2) The distance between a number and zero on a number line.	**valor absoluto** Distancia entre un número y cero en la recta numérica.
Addition Property of Equality (Lesson 6-8) If you add the same number to each side of an equation, the two sides remain equal.	**propiedad de adición de la igualdad** Si sumas el mismo número a ambos lados de una ecuación, los dos lados permanecen iguales.
algebra (Lesson 5-3) A mathematical language of symbols, including variables.	**álgebra** Lenguaje matemático que usa símbolos, incluyendo variables.
algebraic expression (Lesson 5-3) A combination of variables, numbers, and at least one operation.	**expresión algebraica** Combinación de variables, números y, por lo menos, una operación.
analyze (Lesson 10-1) To use observations to describe and compare data.	**analizar** Usar observaciones para describir y comparar datos.
area (Lesson 8-1) The measure of the interior surface of a two-dimensional figure.	**área** La medida de la superficie interior d una figura bidimensional.
Associative Property (Lesson 5-7) The way in which numbers are grouped does not change the sum or product.	**propiedad asociativa** La forma en que se agrupan tres números al sumarlos o multiplicarlos no altera su suma o producto.
average (Lesson 10-3) The sum of two or more quantities divided by the number of quantities; the mean.	**promedio** La suma de dos o más cantidades dividida entre el número de cantidades; la media.
B	
base (Lesson 8-1) Any side of a parallelogram or any side of a triangle.	**base** Cualquier lado de un paralelogramo o cualquier lado de un triángulo.
base (Lesson 9-1) One of the two parallel congruent faces of a prism.	**base** Una de las dos caras paralelas congruentes de un prisma.

base (Lesson 5-1) In a power, the number used as a factor. In 10^3, the base is 10. That is, $10^3 = 10 \times 10 \times 10$.	**base** En una potencia, el número usado como factor. En 10^3, la base es 10. Es decir, $10^3 = 10 \times 10 \times 10$.
bases (Lesson 8-3) The bases of a trapezoid are the two parallel sides.	**bases** Las bases de un trapecio son los dos lados paralelos.
benchmark percent (Lesson 2-5) A common percent used when estimating part of a whole.	**porcentaje de referencia** Porcentaje común utilizado para estimar parte de un todo.
box plot (Lesson 10-4) A diagram that is constructed using five values.	**diagrama de caja** Diagrama que se construye usando cinco valores.

C

cluster (Lesson 10-7) Data that are grouped closely together.	**agrupamiento** Conjunto de datos que se agrupan.
coefficient (Lesson 5-3) The numerical factor of a term that contains a variable.	**coeficiente** El factor numérico de un término que contiene una variable.
common factor (Lesson 5-5) A number that is a factor of two or more numbers.	**factor común** Un número que es un factor de dos o más números.
Commutative Property (Lesson 5-7) The order in which numbers are added or multiplied does not change the sum or product.	**propiedad commutativa** La forma en que se suman o multiplican dos números no altera su suma o producto.
congruent (Lesson 8-2) Having the same measure.	**congruente** Ques tienen la misma medida.
congruent figures (Lesson 8-2) Figures that have the same size and same shape; corresponding sides and angles have equal measures.	**figuras congruentes** Figuras que tienen el mismo tamaño y la misma forma; los lados y los ángulos correspondientes con igual medida.
constant (Lesson 5-3) A term without a variable.	**constante** Un término sin una variable.
coordinate plane (Lesson 1-3) A plane in which a horizontal number line and a vertical number line intersect at their zero points.	**plano de coordenadas** Plano en que una recta numérica horizontal y una recta numérica vertical se intersecan en sus puntos cero.
cubic units (Lesson 9-1) Used to measure volume. Tells the number of cubes of a given size it will take to fill a three-dimensional figure.	**unidades cúbicas** Se usan para medir el volumen. Indican el número de cubos de cierto tamaño que se necesitan para llenar una figura tridimensional.

D

data (Lesson 10-1) Information, often numerical, which is gathered for statistical purposes.	**datos** Información, con frecuencia numérica, que se recoge con fines estadísticos.
defining the variable (Lesson 5-3) Choosing a variable and deciding what the variable represents.	**definir la variable** Elegir una variable y decidir lo que representa.

dependent variable (Lesson 7-1) The variable in a relation with a value that depends on the value of the independent variable.	**variable dependiente** La variable en una relación cuyo valor depende del valor de la variable independiente.
distribution (Lesson 10-7) The arrangement of data values.	**distribución** El arreglo de valores de datos.
Distributive Property (Lesson 5-6) To multiply a sum by a number, multiply each addend by the number outside the parentheses.	**propiedad distributiva** Para multiplicar una suma por un número, multiplica cada sumando por el número fuera de los paréntesis.
dividend (Lesson 3-1) The number that is divided in a division problem.	**dividendo** El número que se divide en un problema de división.
Division Property of Equality (Lesson 6-4) If you divide each side of an equation by the same nonzero number, the two sides remain equal.	**propiedad de igualdad de la división** Si divides ambos lados de una ecuación entre el mismo número no nulo, los lados permanecen iguales.
divisor (Lesson 3-1) The number used to divide another number in a division problem.	**divisor** El número utilizado para dividir otro número en un problema de división.
double number line (Lesson 1-2) A double number line consists of two number lines, in which the coordinated quantities are equivalent ratios.	**línea doble** Una línea numérica doble consta de dos líneas numéricas, en las cuales las cantidades coordinadas son proporciones equivalentes.
dot plot (Lesson 10-2) A diagram that shows the frequency of data on a number line. Also known as a line plot.	**diagrama de puntos** Diagrama que muestra la frecuencia de los datos sobre una recta numérica.

E

equals sign (Lesson 6-1) A symbol of equality, =.	**signo de igualdad** Símbolo que indica igualdad, =.
equation (Lesson 6-1) A mathematical sentence showing two expressions are equal. An equation contains an equals sign, =.	**ecuación** Enunciado matemático que muestra que dos expresiones son iguales. Una ecuación contiene el signo de igualdad, =.
equivalent expressions (Lesson 5-7) Expressions that have the same value, regardless of the values of the variable(s).	**expresiones equivalentes** Expresiones que poseen el mismo valor, sin importer los valores de la(s) variable(s).
equivalent ratios (Lesson 1-2) Ratios that express the same relationship between two quantities.	**razones equivalentes** Razones que expresan la misma relación entre dos cantidades.
evaluate (Lesson 5-2) To find the value of an algebraic expression by replacing variables with numbers.	**evaluar** Calcular el valor de una expresión algebraica sustituyendo las variables por número.
exponent (Lesson 5-1) In a power, the number that tells how many times the base is used as a factor. In 5^3, the exponent is 3. That is, $5^3 = 5 \times 5 \times 5$.	**exponente** En una potencia, el número que indica las veces que la base se usa como factor. En 5^3, el exponente es 3. Es decir, $5^3 = 5 \times 5 \times 5$.

F

English	Español
face (Lesson 9-1) A flat surface of a prism or pyramid.	cara Una superficie plana de un prisma o pirámide.
factoring the expression (Lesson 5-6) The process of writing numeric or algebraic expressions as a product of their factors.	factorizar la expresión El proceso de escribir expresiones numéricas o algebraicas como el producto de sus factores.
first quartile (Lesson 10-4) The first quartile is the median of the data values less than the median.	primer cuartil El primer cuartil es la mediana de los valores menores que la mediana.

G

English	Español
gap (Lesson 10-7) An empty space or interval in a set of data.	laguna Espacio o intervalo vacío en un conjunto de datos.
graph (Lesson 1-3) To place a dot on a number line, or on the coordinate plane at a point named by an ordered pair.	graficar Colocar una marca puntual en una línea numérica, o en el plano de coordenadas en el punto que corresponde a un par ordenado.
greatest common factor (GCF) (Lesson 5-5) The greatest of the common factors of two or more numbers.	máximo común divisor (MCD) El mayor de los factores comunes de dos o más números.
guess, check, and revise strategy (Lesson 6-1) A strategy used to solve a problem which involves narrowing in on the correct answer using educated guesses.	adivinar, comprobar y revisar la estrategia Una estrategia utilizada para resolver un problema que implica el estrechamiento en la respuesta correcta usando conjeturas educadas.

H

English	Español
height (Lesson 8-1) The height of a parallelogram is the perpendicular distance between the base and its opposite side.	altura La altura de un paralelogramo es la distancia perpendicular entre la base y su lado opuesto.
height (Lesson 8-2) The height of a triangle is the perpendicular distance from the base to the opposite vertex.	altura La altura de un triángulo es la distancia perpendicular de la base al vértice opuesto.
height (Lesson 8-3) The height of a trapezoid is the perpendicular distance between the two bases.	altura La altura de un trapecio es la distancia perpendicular entre las dos bases.
histogram (Lesson 10-2) A type of bar graph used to display numerical data that have been organized into equal intervals.	histograma Tipo de gráfica de barras que se usa para exhibir datos que se han organizado en intervalos iguales.

I

English	Español
Identity Properties (Lesson 5-7) Properties that state that the sum of any number and 0 equals the number and that the product of any number and 1 equals the number.	propiedades de identidad Propiedades que establecen que la suma de cualquier número y 0 es igual al número y que el producto de cualquier número y 1 es igual al número.
independent variable (Lesson 7-1) The variable in a relationship with a value that is subject to choice.	variable independiente Variable en una relación cuyo valor está sujeto a elección.
inequality (Lesson 6-6) A mathematical sentence indicating that two quantities are not equal.	desigualdad Enunciado matemático que indica que dos cantidades no son iguales.
integer (Lesson 4-1) Any number from the set $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ where … means *continues without end.*	entero Cualquier número del conjunto $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ donde … significa que *continúa sin fin.*
interquartile range (IQR) (Lesson 10-4) A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set.	rango intercuartil (RIQ) El rango intercuartil, una medida de la variación en un conjunto de datos numéricos, es la distancia entre el primer y el tercer cuartil del conjunto de datos.
interval (Lesson 10-2) The difference between successive values on a scale.	intervalo La diferencia entre valores sucesivos de una escala.
inverse operations (Lesson 6-2) Operations which undo each other. For example, addition and subtraction are inverse operations.	operaciones inversas Operaciones que se anulan mutuamente. La adición y la sustracción son operaciones inversas.
Inverse Property of Multiplication (Lesson 3-3) A property that states that the product of a number and its multiplicative inverse is 1.	propiedad inversa de la multiplicación Una propiedad que indica que el producto de un número y su inverso multiplicativo es 1.

L

English	Español
lateral face (Lesson 9-4) Any face that is not a base.	cara lateral Cualquier superficie plana que no sea la base.
least common multiple (LCM) (Lesson 5-5) The smallest whole number greater than 0 that is a common multiple of each of two or more numbers.	mínimo común múltiplo (mcm) El menor número entero, mayor que 0, múltiplo común de dos o más números.
like terms (Lesson 5-3) Terms that contain the same variable(s) to the same power.	términos semejantes Términos que contienen la misma variable o variables elevadas a la misma potencia.

M

English	Español
mean (Lesson 10-3) The sum of the numbers in a set of data divided by the number of pieces of data.	media La suma de los números en un conjunto de datos dividida entre el número total de datos.

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Glossary · Glosario

mean absolute deviation (MAD) (Lesson 10-5) A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.

desviación media absoluta (DMA) Una medida de variación en un conjunto de datos numéricos que se calcula sumando las distancias entre el valor de cada dato y la media, y luego dividiendo entre el número de valores.

measures of center (Lesson 10-3) Numbers that are used to describe the center of a data set. These measures include the mean and median.

medidas del centro Numéros que se usan para describir el centro de un conjunto de datos. Estas medidas incluyen la media, la mediana y la moda.

measures of variation (Lesson 10-4) A measure that is used to describe the variability, or spread, of a data set.

medidas de variación Medida que se utiliza para describir la variabilidad o la dispersión de un conjunto de datos.

median (Lesson 10-3) A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list, or the mean of the two central values, if the list contains an even number of values.

mediana Una medida del centro en un conjunto de datos numéricos. La mediana de una lista de valores es el valor que aparece en el centro de una versión ordenada de la lista, o la media de los dos valores centrales si la lista contiene un número par de valores.

Multiplication Property of Equality (Lesson 6-5) If you multiply each side of an equation by the same nonzero number, the two sides remain equal.

propiedad de multiplicación de la igualdad Si multiplicas ambos lados de una ecuación por el mismo número no nulo, lo lados permanecen iguales.

multiplicative inverses (Lesson 3-3) Any two numbers that have a product of 1.

inversos multiplicativos Cualquier dos números que tengan un producto de 1.

N

negative integer (Lesson 4-1) A number that is less than zero. It is written with a − sign.

entero negativo Número que es menor que cero y se escribe con el signo −.

net (Lesson 9-2) A two-dimensional figure that can be used to build a three-dimensional figure.

red Figura bidimensional que sirve para hacer una figura tridimensional.

numerical expression (Lesson 5-2) A combination of numbers and operations.

expresión numérica Una combinación de números y operaciones.

O

opposites (Lesson 4-2) Two integers are opposites if they are represented on the number line by points that are the same distance from zero, but on opposite sides of zero. The sum of two opposites is zero.

opuestos Dos enteros son opuestos si, en la recta numérica, están representados por puntos que equidistan de cero, pero en direcciones opuestas. La suma de dos opuestos es cero.

order of operations (Lesson 5-2) The rules that tell which operation to perform first when more than one operation is used.

1. Simplify the expessions inside grouping symbols.
2. Find the value of all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

orden de las operaciones Reglas que establecen cuál operación debes realizar primero, cuando hay más de una operación involucrada.

1. Ejecuta todas las operaciones dentro de los símbolos de agrupamiento.
2. Evalúa todas las potencias.
3. Multiplica y divide en orden de izquierda a derecha.
4. Suma y resta en orden de izquierda a derecha.

ordered pair (Lesson 1-3) A pair of numbers used to locate a point on the coordinate plane. The ordered pair is written in the form (*x*-coordinate, *y*-coordinate).

par ordenado Par de números que se utiliza para ubicar un punto en un plano de coordenadas. Se escribe de la forma (coordenada *x*, coordenada *y*).

origin (Lesson 1-3) The point of intersection of the *x*-axis and *y*-axis on a coordinate plane.

origen Punto de intersección de los ejes axiales en un plano de coordenadas.

outlier (Lesson 10-6) A value that is much greater than or much less than the other values in a set of data.

valor atípico Dato que se encuentra muy separado de los otros valores en un conjunto de datos.

P

parallelogram (Lesson 8-1) A quadrilateral with opposite sides parallel and opposite sides congruent.

paralelogramo Cuadrilátero cuyos lados opuestos son paralelos y congruentes.

part-to-part ratio (Lesson 1-1) A ratio that compares one part of a group to another part of the same group.

proporción de parte a parte Una proporción que compara una parte de un grupo con otra parte del mismo grupo.

part-to-whole ratio (Lesson 1-1) A ratio that compares one part of a group to the whole group.

proporción de parte a total Una proporción que compara una parte de un grupo con todo el grupo.

peak (Lesson 10-7) The most frequently occurring value in a line plot.

pico El valor que ocurre con más frecuencia en un diagrama de puntos.

percent (Lesson 2-1) A ratio, or rate, that compares a number to 100.

por ciento Una relación, o tasa, que compara un número a 100.

positive integer (Lesson 4-1) A number that is greater than zero. It can be written with or without a + sign.

entero positivo Número que es mayor que cero y se puede escribir con o sin el signo +.

powers (Lesson 5-1) A number expressed using an exponent. The power 3^2 is read *three to the second power*, or *three squared.*

potncias Números que se expresan usando exponentes. La potencia 3^2 se lee *tres a la segunda potencia* o *tres al cuadrado.*

prism (Lesson 9-1) A three-dimensional figure with at least three rectangular lateral faces and top and bottom faces parallel.

prisma Figura tridimensional que tiene por lo menos tres caras laterales rectangulares y caras paralelas superior e inferior.

pyramid (Lesson 9-4) A three-dimensional figure with at least three triangular sides that meet at a common vertex and only one base that is a polygon.	**pirámide** Una figura de tres dimensiones con que es en un un polígono y tres o mas caras triangulares que se encuentran en un vértice común.

Q

quadrants (Lesson 4-5) The four regions in a coordinate plane separated by the x-axis and y-axis.	**cuadrantes** Las cuatro regiones de un plano de coordenadas separadas por el eje x y el eje y.
quartiles (Lesson 10-4) Values that divide a data set into four equal parts.	**cuartiles** Valores que dividen un conjunto de datos en cuatro partes iguales.
quotient (Lesson 3-1) The result when one number is divided by another.	**cociente** El resultado cuando un número es dividido por otro.

R

range (Lesson 10-4) The difference between the greatest number and the least number in a set of data.	**rango** La diferencia entre el número mayor y el número menor en un conjunto de datos.
rate (Lesson 1-7) A special kind of ratio in which the units are different.	**tasa** Un tipo especial de relación en el que las unidades son diferentes.
ratio (Lesson 1-1) A comparison between two quantities, in which for every a units of one quantity, there are b units of another quantity.	**razón** Una comparación entre dos cantidades, en la que por cada a unidades de una cantidad, hay unidades b de otra cantidad.
ratio table (Lesson 1-2) A collection of equivalent ratios that are organized in a table.	**table de razones** Una colección de proporciones equivalentes que se organizan en una tabla.
rational number (Lesson 4-4) A number that can be written as a fraction.	**número racional** Número que se puede expresar como fracción.
reciprocals (Lesson 3-3) Any two numbers that have a product of 1. Since $\frac{5}{6} \times \frac{6}{5} = 1$, $\frac{5}{6}$ and $\frac{6}{5}$ are reciprocals.	**recíproco** Cualquier par de números cuyo producto es 1. Como $\frac{5}{6} \times \frac{6}{5} = 1$, $\frac{5}{6}$ y $\frac{6}{5}$ son recíprocos.
rectangular prism (Lesson 9-1) A prism that has rectangular bases.	**prisma rectangular** Una prisma que tiene bases rectangulares.
reflection (Lesson 4-6) The mirror image produced by flipping a figure over a line.	**reflexión** Transformación en la cual una figura se voltea sobre una recta. También se conoce como simetría de espejo.
regular polygon (Lesson 8-4) A polygon with all congruent sides and all congruent angles.	**polígono regular** Un polígono con todos los lados congruentes y todos los ángulos congruentes.

S

scaling (Lesson 1-2) The process of multiplying each quantity in a ratio by the same number to obtain equivalent ratios.	**homotecia** El proceso de multiplicar cada cantidad en una proporción por el mismo número para obtener relaciones equivalentes.
second quartile (Lesson 10-4) Another name for the median, or the center of a set of numerical data.	**segundo cuartil** Otro nombre para la mediana, o el centro de un conjunto de datos numéricos.
simplest form (Lesson 5-4) The status of an expression when it has no like terms and no parentheses.	**forma más simple** El estado de una expresión cuando no tiene términos iguales y no hay paréntesis.
slant height (Lesson 9-4) The height of each lateral face of a pyramid.	**altura oblicua** Altura de cada cara lateral de un pirámide.
solution (Lesson 6-1) The value of a variable that makes an equation true.	**solución** Valor de la variable de una ecuación que hace verdadera la ecuación.
solve (Lesson 6-1) To replace a variable with a value that results in a true sentence.	**resolver** Reemplazar una variable con un valor que resulte en un enunciado verdadero.
statistical question (Lesson 10-1) A question that anticipates and accounts for a variety of answers.	**cuestión estadística** Una pregunta que se anticipa y da cuenta de una variedad de respuestas.
statistics (Lesson 10-1) Collecting, organizing, and interpreting data.	**estadística** Recopilar, ordenar e interpretar datos.
Subtraction Property of Equality (Lesson 6-2) If you subtract the same number from each side of an equation, the two sides remain equal.	**propiedad de sustracción de la igualdad** Si sustraes el mismo número de ambos lados de una ecuación, los dos lados permanecen iguales.
surface area (Lesson 9-2) The sum of the areas of all the surfaces (faces) of a three-dimensional figure.	**área de superficie** La suma de las áreas de todas las superficies (caras) de una figura tridimensional.
survey (Lesson 10-1) A question or set of questions designed to collect data about a specific group of people, or population.	**encuesta** Pregunta o conjunto de preguntas diseñadas para recoger datos sobre un grupo específico de personas o población.
symmetric distribution (Lesson 10-7) Data that are evenly distributed.	**distribución simétrica** Datos que están distribuidos.

T

term (Lesson 5-3) Each part of an algebraic expression separated by a plus or minus sign.	**término** Cada parte de un expresión algebraica separada por un signo más o un signo menos.
third quartile (Lesson 10-4) The third quartile is the median of the data values greater than the median.	**tercer cuartil** El tercer cuartil es la mediana de los valores mayores que la mediana.

Glossary · Glosario

three-dimensional figure (Lesson 9-1) A figure with length, width, and height.

figura tridimensional Una figura que tiene largo, ancho y alto.

trapezoid (Lesson 8-3) A quadrilateral with one pair of parallel sides.

trapecio Cuadrilátero con un único par de lados paralelos.

triangular prism (Lesson 9-3) A prism that has triangular bases.

prisma triangular Prisma con bases triangulares.

U

unit price (Lesson 1-7) The cost per unit of an item.

precio unitario El costo por unidad de un artículo.

unit rate (Lesson 1-7) A rate in which the first quantity is compared to 1 unit of the second quantity.

tasa unitaria Una tasa en la que la primera cantidad se compara con 1 unidad de la segunda cantidad.

unit ratio (Lesson 1-6) A ratio in which the first quantity is compared to 1 unit of the second quantity.

razón unitaria Una relación en la que la primera cantidad se compara con 1 unidad de la segunda cantidad.

V

variable (Lesson 5-3) A symbol, usually a letter, used to represent a number.

variable Un símbolo, por lo general, una letra, que se usa para representar un número.

volume (Lesson 9-1) The amount of space inside a three-dimensional figure. Volume is measured in cubic units.

volumen Cantidad de espacio dentro de una figura tridimensional. El volumen se mide en unidades cúbicas.

X

x-axis (Lesson 1-3) The horizontal line of the two perpendicular number lines in a coordinate plane.

eje *x* La recta horizontal de las dos rectas numéricas perpendiculares en un plano de coordenadas.

x-coordinate (Lesson 1-3) The first number of an ordered pair. The *x*-coordinate corresponds to a number on the *x*-axis.

coordenada *x* El primer número de un par ordenado, el cual corresponde a un número en el eje *x*.

Y

y-axis (Lesson 1-3) The vertical line of the two perpendicular number lines in a coordinate plane.

eje *y* La recta vertical de las dos rectas numéricas perpendiculares en un plano de coordenadas.

y-coordinate (Lesson 1-3) The second number of an ordered pair. The *y*-coordinate corresponds to a number on the *y*-axis.

coordenada *y* El segundo número de un par ordenado, el cual corresponde a un número en el eje *y*.

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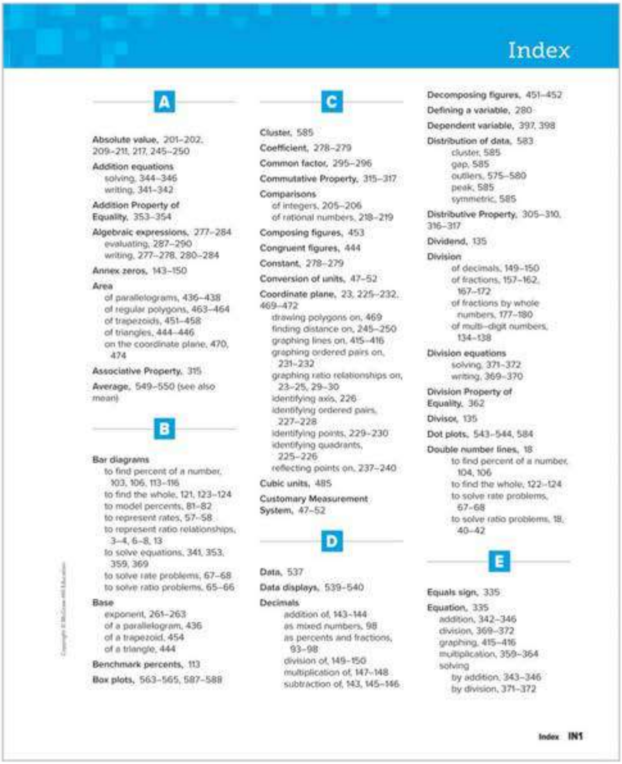

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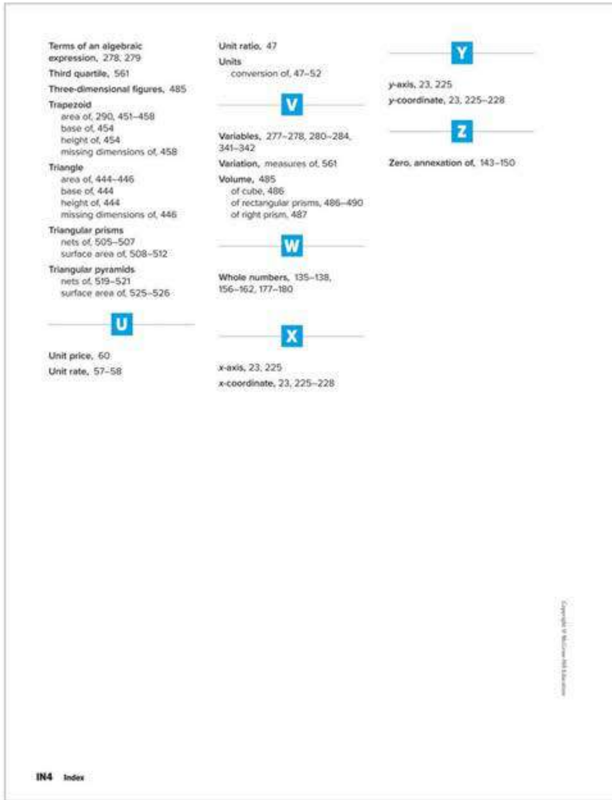
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Selected Answers

Lesson 1-1 Understand Ratios, Practice Pages 11–12

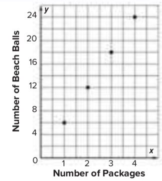
1. no; Sample answer: Suri's ratio is 6 : 4 and Martha's is 5 : 3. **3.** 6 cups **5.** 10 chocolate doughnuts **7.** 36 players **9.** 8 containers; Sample answer: She has 2 cups or 16 fluid ounces of liquid starch. She will make 16 ÷ 4 or 4 batches of slime. Each batch makes 4 × 3 or 12 fluid ounces, so she will make a total of 48 fluid ounces of slime. If each container holds 6 fluid ounces, she needs 48 ÷ 6 or 8 containers. **11.** 4 : 24; Sample answer: If 4 students bike to school, then 28 − 4 or 24 students do not bike to school. The ratio is 4 : 24. **13.** $\frac{3.14}{1}$ or 3.14

Lesson 1-2 Tables of Equivalent Ratios, Practice Pages 21–22

1. 30 snow cones **3.** 83 skips **5.** 98 minutes **7.** 25 pencils **9.** 20 biscuits **11.** no; Sample answer: If 5 goats and 5 chickens are added, there would be 26 goats and 40 chickens on the farm, with a goat-to-chicken ratio of 13 : 20. The ratio of goats to chickens was originally 3 : 5, which is not equivalent to 13 : 20. **13.** Sample answer: Seth's bouquet has 21 flowers with 15 roses. Keith's bouquet has 35 flowers with 25 roses. Are the ratios of roses to flowers the same? Yes, they both scale to 5 roses to 7 flowers.

Lesson 1-3 Graphs of Equivalent Ratios, Practice Pages 27–28

1. (1, 6), (2, 12), (3, 18), (4, 24); Sample answer: The points appear to be in a straight line. Each point is 6 units up from and 1 unit to the right of the previous point. This means that the number of beach balls increases by 6 as the number of packages increases by 1.



3. Sample answer: The ratio of photos to pages for Lexi's scrapbook is 4 : 1. The ratio of photos to pages for Audrey's scrapbook is 6 : 1. Audrey uses more photos per page than Lexi. **5.** dimes to dollars; Sample answer: The ratio of dimes to dollars is 10 : 1 and the ratio of quarters to dollars is 4 : 1. Since 10 is greater than 4, the ratio of dimes to dollars will have a steeper line. **7.** yes; Sample answer: A bracelet could have a length of 10.5 inches and 42 beads.

Lesson 1-4 Compare Ratio Relationships, Practice Pages 35–36

1. Brand B; Sample answer: When all three ratio relationships are graphed on the same graph, the graph for Brand B is the steepest. This means that Brand B has the greatest ratio of raisins to ounces of cereal. **3.** white bread **5.** Miguel **7.** Sample answer: Three packages of hot dogs cost $9.50. The relationship was displayed in words because it's easier and faster for people to understand while shopping.

Selected Answers

Lesson 1-5 Solve Ratio Problems, Practice Pages 45–46

1. 640 students **3.** 15 baskets **5.** 225 students **7.** 480 students **9.** $1,015 **11.** false; Sample answer: For the ratios to be equivalent, they must be equivalent fractions. So, the numerator of the second fraction must also be greater than the denominator. Otherwise, the ratios are not equivalent. **13.** 8 people; Sample answer: Using equivalent ratios, $\frac{20}{140} = \frac{?}{540}$. So, 72 people in a group of 504, would play tennis. Using equivalent ratios, $\frac{1}{9} = \frac{?}{72}$. So, 8 people out of those 72 would have a tennis coach.

Lesson 1-6 Convert Customary Measurement Units, Practice Pages 55–56

1. 144 fluid ounces **3.** 12 cups **5.** $1\frac{1}{2}$ tons **7.** 50 gallons **9.** 250 quarts **11.** $15.75 **13.** Sample answer: First, convert 20 miles to feet. There are 5,280 × 20 or 105,600 feet in 20 miles. Then convert one hour to seconds. There are 60 × 60 or 3,600 seconds in one hour. So, $\frac{105{,}600 \text{ ft}}{3{,}600 \text{ s}} \approx \frac{29.3 \text{ ft}}{1 \text{ s}}$ or about 29.3 feet per second. **15.** Sample answer: I can use the equivalent ratios $\frac{1 \text{ km}}{1{,}000 \text{ m}} = \frac{2.2 \text{ km}}{? \text{ m}}$ to find that 2.2 kilometers is equal to 2,200 meters. I can then use the equivalent ratios $\frac{1 \text{ m}}{100 \text{ cm}} = \frac{2{,}200 \text{ m}}{? \text{ cm}}$ to convert meters to centimeters. So, 2.2 kilometers is equal to 100 × 2,200 or 220,000 centimeters.

Lesson 1-7 Understand Rates and Unit Rates, Practice Pages 63–64

1. 0.4 km per min **3.** 3 beats per second **5.** 25 game tickets for $10 **7.** 6-pack of Student Tickets **9.** Party R Us; $0.25 less **11.** Sample answer: 1 bagel for $0.50 **13.** 1 min; Sample answer: There are 60 minutes in 1 hour, so 1 mile per minute is equivalent to 60 miles per hour.

Lesson 1-8 Solve Rate Problems, Practice Pages 71–72

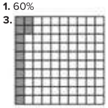
1. $9 **3.** $7.50 **5.** 390 donuts **7.** 36 minutes **9.** yes; Sample answer: 2 hours = 120 minutes; Billie bikes at the rate of $\frac{45 \text{ min}}{9 \text{ mi}}$ or $\frac{5 \text{ min}}{1 \text{ mi}}$ and $\frac{5 \text{ min}}{1 \text{ mi}} = \frac{120 \text{ min}}{24 \text{ mi}}$. **11.** 48 mandarin oranges

Module 1 Review Pages 75–76

1. 18 **3.** B **5.** 65 miles per hour; 3 questions for each lesson **7.** no; Sample answer: Since the rates do not have the same unit rate, they are not equivalent. **9.** 25 students **11a.** rate of speed downstream = 15 mph; rate of speed upstream = 10 mph; The rate of speed downstream was faster than the rate of speed upstream. **11b.** 5 miles per hour

Lesson 2-1 Understand Percents, Practice Pages 83–84

1. 60%
3.

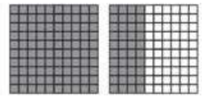


5. 90%
7.

9. yes; Sample answer: Each section of the model represents 20%. The 3 sections not shaded represent the percentage of students who did not vote for the tiger. So, 20% × 3 = 60% and 60% is greater than 50%. **11.** 20%; Sample answer: Each section represents 4%. Since 5 sections are shaded, 5 × 4% = 20%. **13.** yes; Sample answer: To model 110%, use two bar diagrams each divided into 10 equal sections. Shade one bar diagram entirely to represent 100% and then shade the remaining 10% in the second bar diagram.

Lesson 2-2 Percents Greater Than 100% and Less Than 1%, Practice Pages 91–92

1. 136% **3.** 0.75%
5. 140%



7. 0.0085 **9.** 30 mph **11.** Sample answer: The student modeled 2%, not 0.2%. To model 0.2%, only $\frac{1}{5}$ of one square should be shaded.

Lesson 2-3 Relate Fractions, Decimals, and Percents, Practice Pages 101–102

1. $\frac{9}{20}$, 0.45 **3.** $\frac{4}{5}$, 0.8 **5.** 175%, 1.75 **7.** 89%, $\frac{89}{100}$ **9.** 65%, $\frac{13}{20}$ **11.** $\frac{3}{10}$, 0.3 **13.** 0.85; $\frac{85}{100}$; $\frac{17}{20}$ **15.** $\frac{7}{10}$ **17.** no; Sample answer: 0.22 + 0.24 = 0.46 and 0.46 = 46%. Since 46% < 50%, chocolate milk and lemonade did not receive more than 50% of the votes. **19.** Sample answer: The percent will be less than 100% if the numerator is less than the denominator. The percent will equal 100% if the numerator and the denominator are equal. The percent will be greater than 100% if the numerator is greater than the denominator.

Lesson 2-4 Find the Percent of a Number, Practice Pages 111–112

1. 48 students **3.** 36 **5.** 8 **7.** 66 **9.** 0.525 **11.** 0.9 **13.** 43 students **15.** $103.08 **17.** Sample answer: 40% can be represented as 10% + 10% + 10% + 10%. 10% of 150 is 15. 15 + 15 + 15 + 15 = 60, So, 40% of 150 is 60.

Lesson 2-5 Estimate the Percent of a Number, Practice Pages 119–120

1. Sample answer: 30; 50% of 60 = 30 **3.** Sample answer: 80; 40% of 200 = 80 **5.** Sample answer: 20; 20% of 100 = 20 **7.** Sample answer: about $10; 25% of 40 = 10 **9.** Sample answer: about 225 customers; 75% of 300 = 225 **11.** Sample answer: about 125 students; 25% of 500 = 125 **13.** about $55 **15.** about 14,250 people **17.** Sample answer: First, round 39% to 40% and $197 to $200. Next, find 10% of $200, which is $20. Last, multiply $20 by 4 to find 40% of 200, or $80.

Lesson 2-6 Find the Whole, Practice Pages 127–128

1. 25 members **3.** $25 **5.** 400 pictures **7.** 500 minutes **9.** 300 lunches; $1,050 **11.** no; Sample answer: A percent compares the part to the whole. In this case, the only known value is the part. To compare percents, the whole, the total number of sixth grade students and the total number of seventh grade students, must be known. **13.** Sample answer: James's soccer team won 68% of the games they played. If they won 17 games, how many did they play? 25 games

Module 2 Review Pages 129–130

1. B **3.** 110% **5.** 28%; $\frac{28}{100}$, $\frac{14}{50}$, $\frac{7}{25}$ **7.** 80 shots **9.** 27 students **11a.** 1,500 items **11b.** $16,425

Lesson 3-1 Divide Multi-Digit Whole Numbers, Practice Pages 141–142

1. 3,472 **3.** 36 **5.** 222.25 **7.** 28.125 **9.** 36.10625 **11.** 134 **13.** 24 bags **15.** 1,020 **17.** Sample answer: Check your answer by multiplying the quotient by the divisor. Compare this answer to the dividend. They should be equal.

Lesson 3-2 Compute With Multi-Digit Decimals, Practice Pages 153–154

1. 49.892 **3.** 80.027 **5.** 0.031 **7.** 2,042.125 **9.** 8.52 mi **11.** $1.51 **13.** Sample answer: Since the decimal 0.95 is less than 1, the product of 5.5 × 0.95 must be less than 5.5 × 1 or 5.5. **15.** Sample answer: If you add the whole numbers, the sum is 40. The sum of the decimals will be added to 40 which will make the sum greater than 40.

Lesson 3-3 Divide Whole Numbers by Fractions, Practice Pages 165–166

1. 2 **3.** $\frac{1}{8}$ **5.** $\frac{10}{7}$ **7.** 10 **9.** $11\frac{1}{5}$; Marie can make $11\frac{1}{5}$ scarves or 11 whole scarves. **11.** 27 **13.** $\frac{1}{3}$ cup **15.** yes; Sample answer: $20 \div \frac{1}{3} = \frac{20}{1} \times \frac{3}{1}$ or 60, which is greater than 55, so Zach will have enough sandwich pieces. **17.** 4; Sample answer: The reciprocal of 4 is $\frac{1}{4}$, which is equal to 0.25, and 0.2 < 0.25 < 0.3.

Lesson 3-4 Divide Fractions by Fractions, Practice Pages 275–276

1. 2 **3.** 6 **5.** $\frac{7}{8} \div \frac{1}{4} = 3\frac{1}{2}$; Chelsea can make 3 batches of icing. **7.** $2\frac{2}{5}$ **9.** 1 more bookmark **11.** yes; Sample answer: $\frac{9}{10} \div \frac{1}{3} = \frac{9}{10} \times \frac{3}{1} = \frac{27}{10} = 2\frac{7}{10}$. He only needs 2 flags. So, he has enough. **13.** Sample answer: $\frac{7}{8} \div \frac{7}{8} = \frac{7}{8} \times \frac{8}{7} = \frac{56}{56} = 1$

Lesson 3-5 Divide with Whole and Mixed Numbers, Practice Pages 285–286

1. $\frac{1}{2} \div 6 = \frac{1}{12}$; $\frac{1}{12}$ yd **3.** $\frac{7}{10}$ **5.** $\frac{7}{9}$ **7.** $\frac{9}{4}$ or $2\frac{1}{4}$ **9.** 3 pairs **11.** $6\frac{1}{4}$ times greater **13.** Sample answer: A bag contains $22\frac{1}{2}$ cups of flour. A recipe for pancakes uses $1\frac{1}{4}$ cups of flour. How many batches of pancakes can be made with one bag of flour? 18 batches

15. less than; Sample answer: $\frac{9}{10} \div 3$ is divided into more parts than $\frac{9}{10} \div 2$. Since it is divided into more parts, each part represents a lesser amount. So, $\frac{9}{10} \div 2 > \frac{9}{10} \div 3$.

Module 3 Review Pages 289–290

1. 139.5 acres; Divide 8,370 by 60 to find that each farm is 139.5 acres. **3.** 0.032 **5.** D **7a.** 3 **7b.** $\frac{3}{5}$ **9.** $\frac{8}{9}$ **11a.** $\frac{3}{5} \div 6$ **11b.** $\frac{3}{5} \div 6 = \frac{3}{5} \times \frac{1}{6} = \frac{3}{30}$, or $\frac{1}{10}$ pound **13.** $2\frac{2}{3}$

Lesson 4-1 Represent Integers, Practice Pages 297–298

1. –2; The integer 0 represents no ounces gained or lost. **3.** –15; The integer 0 represents no money withdrawn or deposited. **5.** 3; The integer 0 represents average snowfall.

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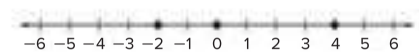

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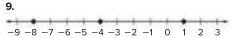

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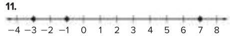

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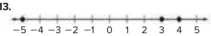
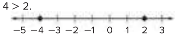


15. Beaker B; Sample answer: Beaker B is 2 units away from 0 on the number line, while Beaker A is 4 units from 0 on the number line. 4 > 2.



17. Sample answer: Graph 1 and −3 on a number line. Then count the units between each integer and zero. There is 1 unit between 0 and 1. There are 3 units between 0 and −3. So, 1 unit + 3 units = 4 units. **19.** Sample answer: Riley lost 10 points playing a trivia game; −10.

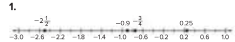
Lesson 4-2 Opposites and Absolute Value, Practice Pages 203–204

1. 3 **3.** −6 **5.** −5; Sample answer: This is the opposite of the height of the hill. **7.** 11 **9.** −1 **11.** 100 **13.** 5 degrees **15.** Southern Moon; Sample answer: I found the absolute value of each minimum elevation and added the maximum elevation for each trail. The change in elevation for Southern Moon is 62 + 48, or 110, which is the least change of the three trails. **17.** false; Sample answer: Absolute value is a measure of distance and distance can never be negative. **19.** no; Sample answer: If *x* is a positive integer such as 1, then the result is −1. If *x* is a negative integer such as −1, then the result is 1.

Lesson 4-3 Compare and Order Integers, Practice Pages 213–214

1. −4 < –1; Since −4 < –1, John has a lesser score than Terry. **3.** ethane, helium, oxygen, carbon monoxide, argon, sulfur dioxide **5.** Sample answer: An elevation less than –5 feet is –10 feet. This means the distance is 10 feet from sea level, which is greater than a distance of 5 feet from sea level. **7.** Neil, Dawson, Felipe, Jesse **9.** Morocco and Argentina **11.** Sample answer: On Saturday the high temperature was –1°F. On Sunday the high temperature was −3°F; −1 > −3 **13.** −3, −2.5, −1, 0.66, 4, 5, 23

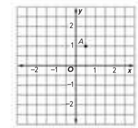
Lesson 4-4 Rational Numbers, Practice Pages 223–224

1.


3. 124.1 **5.** < **7.** = **9.** $-4\frac{7}{10}$, −4.25, $-4\frac{3}{20}$ **11.** −3.2, $-2\frac{1}{5}$, $1\frac{2}{5}$, 1.43 **13.** Race 4 and Race 1 **15.** Sample answer: Ming's account balance is −$10.50. Her brother's account balance is −$15.50. Compare their balances; −$10.50 > −$15.50 **17.** always; Sample answer: The lesser the number, the closer it is to 0; therefore, it's opposite is also closer to 0. *x* = −3, *y* = −2

Lesson 4-5 The Coordinate Plane, Practice Pages 235–236

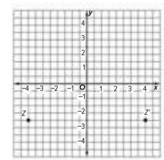
1. Quadrant III **3.** Quadrant I **5.** *x*-axis **7.** (−1.5, 1) **9.** (−1, −1.5) **11.** *X*

13.


15. $\left(-\frac{1}{4}, -1\frac{11}{4}\right)$ **17.** *m* is a negative number; *n* is a positive number **19.** Sample answer: The student did not consider that *b* is positive, and therefore would be in either Quadrant I or II. The correct answer is Quadrant II.

Lesson 4-6 Graph Reflections of Points, Practice Pages 243–244

1. $\left(-2\frac{3}{4}, -1\right)$ **3.** $\left(-4, 2\frac{1}{2}\right)$ **5.** (−3.5, 3.5) **7.** *y*-axis

9.


11. (4.5, −4.5) **13.** Sample answer: The student wrote the ordered pair for a reflection across the *y*-axis, not the *x*-axis. The correct ordered pair for point *Y*′ is (1.5, 2). **15.** Sample answer: *A*(−1, −1); *A*′(1, −1)

Lesson 4-7 Absolute Value and Distance, Practice Pages 253–254

1. $\frac{1}{2}$ unit **3.** 3 units **5.** $1\frac{3}{4}$ units **7.** 3 units **9.** D **11.** Amber **13.** Sample answer: The student did not use the scale on the *y*-axis. The scale is 0.5 unit. So, the actual distance is 1.5 units. **15.** Sample answer: Distance cannot be negative. You have to find the absolute value of each coordinate.

Module 4 Review Pages 257–258

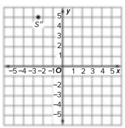
1a. –5; Because the situation represents a loss, the integer is negative. **1b.** Zero represents no money gained or lost. **3.** 14 **5.** acetylene and ammonia **7.** 12°

9.

	x-axis	*y*-axis
(–4, 0)	x	
(0, 9)		x
(0, –6)		x

11a. $S''\left(-2\frac{1}{2}, 5\right)$

11b.



SA6 Selected Answers

Mathematics Reference Sheet

Formulas

Perimeter	Square	$P = 4s$	Rectangle	$P = 2\ell + 2w$ or $P = 2(\ell + w)$
Area	Square	$A = s^2$	Rectangle	$A = \ell w$
	Parallelogram	$A = bh$	Triangle	$A = \frac{1}{2}bh$
	Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$		
Volume	Cube	$V = s^3$	Prism	$V = \ell wh$ or Bh
Temperature	Fahrenheit to Celsius	$C = \frac{5}{9}(F - 32)$	Celsius to Fahrenheit	$F = \frac{9}{5}C + 32$

Measurement Conversions

Length	1 kilometer (km) = 1,000 meters (m) 1 meter (m)= 100 centimeters (cm) 1 centimeter = 10 millimeters (mm)	1 foot (ft) = 12 inches (in.) 1 yard (yd) = 3 feet or 36 inches 1 mile (mi) = 1,760 yards or 5,280 feet
Volume and Capacity	1 liter (L) = 1,000 milliliters (mL) 1 kiloliter (kL) = 1,000 liters	1 cup (c) = 8 fluid ounces (fl oz) 1 pint (pt) = 2 cups 1 quart (qt) = 2 pints 1 gallon (gal) = 4 quarts
Weight and Mass	1 kilogram (kg) = 1,000 grams (g) 1 gram = 1,000 milligrams (mg) 1 metric ton = 1,000 kilograms	1 pound (lb) = 16 ounces (oz) 1 ton (T) = 2,000 pounds
Time	1 minute (min) = 60 seconds (s) 1 hour (h) = 60 minutes 1 day (d) = 24 hours	1 week (wk) = 7 days 1 year (yr) = 12 months (mo) or 52 weeks or 365 days 1 leap year = 366 days
Metric to Customary	1 meter ≈ 39.37 inches 1 kilometer ≈ 0.62 mile 1 centimeter ≈ 0.39 inch	1 kilogram ≈ 2.2 pounds 1 gram ≈ 0.035 ounce 1 liter ≈ 1.057 quarts

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ISBN: 978-0-07-899175-2 *(Reveal Math, Course 1, Teacher Edition, Volume 2)*
MHID: 0-07-899175-7 *(Teacher Edition, Volume 2)*
ISBN: 978-0-07-899714-3 *(Interactive Student Edition, Volume 2)*
MHID: 0-07-899714-3 *(Interactive Student Edition, Volume 2)*

Printed in the United States of America.

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Contents in Brief

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3 Compute with Multi-Digit Numbers and Fractions

4 Integers, Rational Numbers, and the Coordinate Plane

5 Numerical and Algebraic Expressions

6 Equations and Inequalities

7 Relationships Between Two Variables

8 Area

9 Volume and Surface Area

10 Statistical Measures and Displays

Reveal Math™ Guiding Principles

Academic research and the science of learning provide the foundation for this powerful K–12 math program designed to help reveal the mathematician in every student.

Reveal Math is built on a solid foundation of **RESEARCH** that shaped the **PEDAGOGY** of the program.

Reveal Math used findings from research on teaching and learning mathematics to develop its instructional model. Based on analyses of research findings, these areas form the foundational structure of the program:

- Rigor
- Productive Struggle
- Formative Assessment
- Rich Tasks
- Mathematical Discourse
- Collaborative Learning

Instructional Model

1 Launch

WARM UP

During the **Warm Up,** students complete exercises to activate prior knowledge and review prerequisite concepts and skills.

LAUNCH THE LESSON

In **Launch the Lesson**, students view a real-world scenario and image to pique their interest in the lesson content. They are introduced to questions that they will be able to answer at the end of the lesson.

EXPLORE

During the **Explore** activity, students work in partners or small groups to explore a rich, real-world or mathematical problem related to the lesson content.

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY

Reveal the full potential in every student!

2 Explore and Develop

LEARN

In the **Learn** section, students gain the foundational knowledge needed to actively work through upcoming Examples.

EXAMPLES & CHECK

Students work through **Examples** related to the key concepts and engage in mathematical discourse.

Students complete a **Check** after each Example as a quick formative assessment to help teachers adjust instruction as needed.

3 Reflect and Practice

EXIT TICKET

The **Exit Ticket** gives students an opportunity to convey their understanding of the lesson concepts.

PRACTICE

Students complete **Practice** exercises individually or collaboratively to solidify their understanding of lesson concepts or build proficiency with lesson skills.

Reveal Math Key Areas of Focus

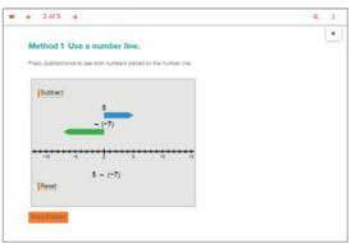
Reveal Math has a strong focus on rigor—especially the development of conceptual understanding—an emphasis on student mindset, and ongoing formative assessment feedback loops.

Rigor

Reveal Math has been thoughtfully designed to incorporate a balance of the three elements of rigor: conceptual understanding, procedural skills and fluency, and application.

Conceptual Understanding

Explore activities give all students an opportunity to work collaboratively and discuss their thinking as they build conceptual understanding of new topics. In the Explore activity to the left, students use algebra tiles to gain an understanding of operations with positive and negative integers.



Procedural Skills and Fluency

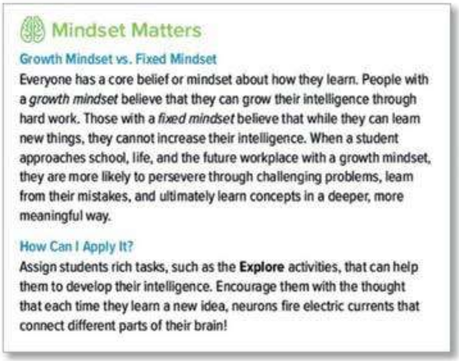
As students move through the lesson, they will use different strategies and tools to build procedural fluency. In the **Example** shown, students use **Web Sketchpad**® to develop proficiency with integer operations.

Application

Real-world examples and practice problems are opportunities for students to apply their learning to new situations. In the real-world example to the left, students apply their understanding of percents to solve a percent error problem.

Student Mindset

Mindset Matters tips located in each module provide specific examples of how *Reveal Math* content can be used to promote a growth mindset in all students. Another feature focused on promoting a growth mindset is **Ignite! Activities** developed by Dr. Raj Shah to spark student curiosity about why the math works. An **Ignite!** delivers problem sets that are flexible enough so that students with varying background knowledge can engage with the content and motivates them to ask questions, solve complex problems, and develop a can-do attitude toward math.



Mindset Matters

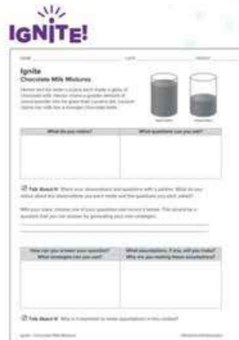
Growth Mindset vs. Fixed Mindset

Everyone has a core belief or mindset about how they learn. People with a *growth mindset* believe that they can grow their intelligence through hard work. Those with a *fixed mindset* believe that while they can learn new things, they cannot increase their intelligence. When a student approaches school, life, and the future workplace with a growth mindset, they are more likely to persevere through challenging problems, learn from their mistakes, and ultimately learn concepts in a deeper, more meaningful way.

How Can I Apply It?

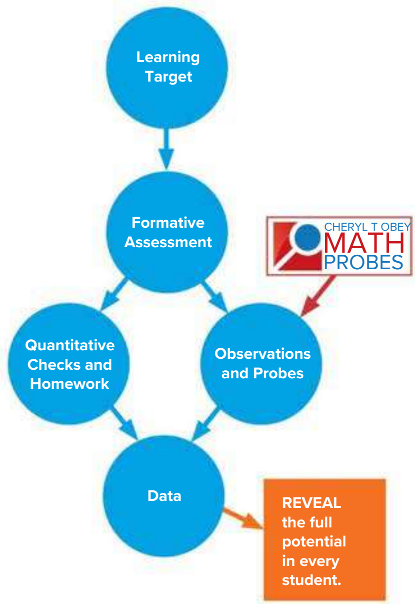
Assign students rich tasks, such as the **Explore** activities, that can help them to develop their intelligence. Encourage them with the thought that each time they learn a new idea, neurons fire electric currents that connect different parts of their brain!

Teacher Edition Mindset Tip



Student Ignite! Activity

Formative Assessment



The key to reaching all learners is to adjust instruction based on each student's understanding. *Reveal Math* offers powerful formative assessment tools that help teachers to efficiently and effectively differentiate instruction for all students.

Math Probes

Each module includes a **Cheryl Tobey Formative Assessment Math Probe** that is focused on addressing student misconceptions about key math topics. Students can complete these probes at the beginning, middle, or end of a module. The teacher support includes a list of recommended differentiated resources that teachers assign based on students' responses.

Example Checks

Each example is followed by a formative assessment **Check** that students complete on their own that allows teachers to gauge students' understanding of the concept or skill presented. When students complete the Check, the teacher receives resource recommendations, which can be assigned to all students.

A Powerful Blended Learning Experience

The *Reveal Math* blended learning experience was designed to include purposeful print and digital components focused on sparking student curiosity and providing teachers with flexible implementation options.

Reveal Math has been thoughtfully developed to provide a rich learning experience no matter where a district, school, or classroom falls on the digital spectrum. All of the instructional content can be projected or can be accessed via desktop, laptop, or tablet.

Lesson

1 Launch

WARM UP

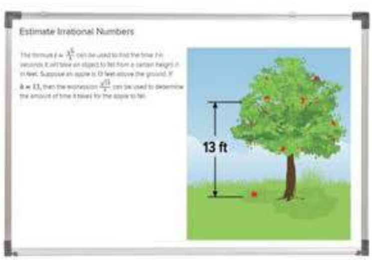
The **Warm Up** exercise can be projected on an interactive whiteboard.

LAUNCH THE LESSON

Launch the Lesson can be projected or assigned to students to access on their own devices.

EXPLORE

The **Explore** Activity can be projected while students record their observations in the Interactive Student Edition or can be assigned for students to complete on individual devices.



Launch the Lesson

Explore

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY
INTERACTIVE PRESENTATION
PRINT INTERACTIVE STUDENT EDITION

2 Explore and Develop

3 Reflect and Practice

LEARN

As students are introduced to the key lesson concepts, they can progress through the **Learn** by recording notes in their Interactive Student Edition or on their own devices.

EXAMPLES & CHECK

In their Interactive Student Edition or on an individual device, students work through one or more **Examples** related to key lesson concepts.

A **Check** follows each Example in either the Interactive Student Edition or on each student device.

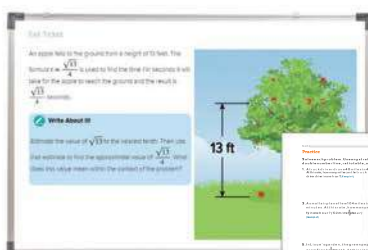
EXIT TICKET

The **Exit Ticket** is projected or accessed via student devices to provide students with lesson closure and an opportunity to revisit the lesson concepts.

PRACTICE

Assign students **Practice** problems from their Interactive Student Edition or create a digital assignment for them to work on their device in class or at home to solidify lesson concepts.

Aligned Digital Lesson Presentation to Interactive Student Edition



Exit Ticket

Practice

Supporting All Learners

The *Reveal Math* program was designed so that all students have access to:

- rich tasks that promote productive struggle,
- opportunities to develop proficiency with the habits of mind and thinking strategies of mathematicians, and
- prompts to promote mathematical discourse and build academic language.

Resources for Differentiating Instruction

When needed, resources are available to differentiate math instruction for students who may need to see a concept in a different way, practice prerequisite skills, or are ready to extend their learning.

AL

Approaching Level Resources

- Remediation Activities
- Extra Examples
- *Arrive Math* Take Another Look Mini Lessons

BL

Beyond Level Resources

- Beyond Level Differentiated Activities
- Extension Activities

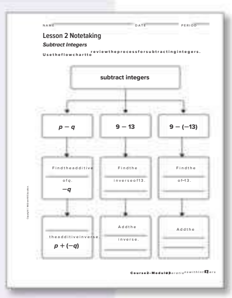
Resources for English Language Learners

Reveal Math also includes student and teacher resources to support students who are simultaneously learning grade-level math and building their English proficiency. Appropriate, research-based language scaffolds are also provided to support students as they engage in rigorous mathematical tasks and discussions.

ELL

English Language Learners

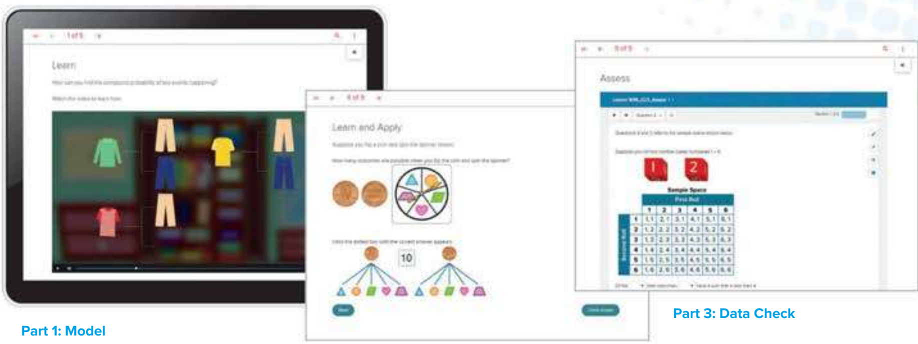
- Spanish Interactive Student Edition
- Spanish Personal Tutors
- Math Language-Building Activities
- Language Scaffolds
- *Think About It!* and *Talk About It!* Prompts
- Multilingual eGlossary
- Audio
- Graphic Organizers
- Web Sketchpad, Desmos, and eTools



Embedded Reteach Support
Arrive Math Booster Mini-Lessons

Reveal Math ensures a seamless connection for students who need extra topic support with embedded *Arrive Math Booster* mini-lessons. These mini-lessons, called *Take Another Look*, have been included in *Reveal Math* to provide students direct support related to the lesson objective.

- Teacher-assigned option based on Example Check results
- Digital, student-driven lesson
- Gradual release experience in three parts



Part 1: Model

Part 2: Interactive Practice

Part 3: Data Check

Arrive MATH BOOSTER

Complement *Reveal Math* with the K-8 *Arrive Math Booster* supplemental intervention to equip teachers with all the resources they need to supplement their instruction and meet the needs of all learners.

Digital mini-lessons

Utilize over 1,160 *Take Another Look* digital mini-lessons for every skill within the K-8 standards.

Hands-On Lesson

Complement the *Take Another Look* lessons with concrete modeling support using hands on, teacher-led activities.

Games

Engage students through exciting math games to become fluent in critical math skills.

Reveal Student Readiness with Individualized Learning Tools

Reveal Math incorporates innovative, technology-based tools that are designed to extend the teachers' reach in the classroom to help address a wide range of knowledge gaps, set and align academic goals, and meet student individualized learning needs.

LEARNSMART®

Topic Mastery

With embedded **LearnSmart**,® students have a built-in study partner for topic practice and review to prepare for multi-module, or mid-year tests.

LearnSmart's revolutionary adaptive technology measures students' awareness of their own learning, time on topic, answer accuracy, and suggests alternative resources to support student learning, confidence, and topic mastery.

ALEKS®

Individualized Learning Pathways

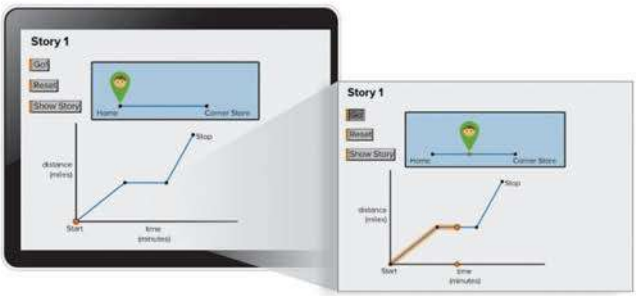
Learners of all levels benefit from the use of **ALEKS'** adaptive, online math technology designed to pinpoint what each student knows, does not know, and most importantly, what each student is ready to learn.

When paired with *Reveal Math,* **ALEKS** is a powerful tool designed to provide integrated instructionally actionable data enabling teachers to utilize *Reveal Math* resources for individual students, groups, or the entire classroom.

Activity Report

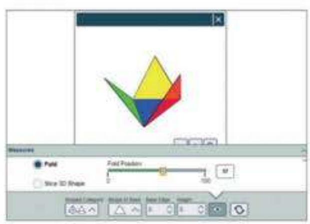
Powerful Tools for Modeling Mathematics

Reveal Math has been designed with purposeful, embedded digital tools to increase student engagement and provide unique modeling opportunities.



Web Sketchpad® Activities

The leading dynamic mathematics visualization software has now been integrated with **Web Sketchpad Activities** at point of use within *Reveal Math*. Student exploration (and practice) using **Web Sketchpad** encourages problem solving and visualization of abstract math concepts.



desmos

The powerful **Desmos** graphing calculator is available in *Reveal Math* for students to explore, model, and apply math to the real-world.

eTools

By using a wide-variety of digital **eTools** embedded within *Reveal Math,* students gain additional hands-on experience while they learn and teachers have the option to create problem-based learning opportunities.

Technology-Enhanced Items

Embedded within the digital lesson, technology-enhanced items—such as drag-and-drop, flashcard flips, or diagram completion—are strategically placed to give students the practice with common computer functions needed to master computer-based testing.

- TYPE
- SWIPE
- DRAG & DROP
- FLASHCARDS
- eTOOLS
- MULTI-SELECT
- WATCH
- EXPAND

Assessment Tools to Reveal Student Progress and Success

Reveal Math provides a comprehensive array of assessment tools to measure student understanding and progress. The digital assessment tools include next generation assessment items, such as multiple-response, selected-response, and technology-enhanced items.

Assessment

Reveal Math provides embedded, regular formative checkpoints to monitor student learning and provide feedback that can be used to modify instruction and help direct student learning using reports and recommendations based on resulting scores.

Summative assessments built in *Reveal Math* evaluate student learning at the module conclusion by comparing it against the state standards covered.

Formative Assessment Resources

- Cheryl Tobey Formative Assessment Math Probes
- Checks
- Exit Tickets
- Put It All Together

Summative Assessment Resources

- Module Tests
- Performance Tasks
- Benchmark Tests
- End-of-Course Tests
- LearnSmart

Or **Build Your Own** assessments focused on standards or objectives. Access to banks of questions, including those with tech-enhanced capabilities, enable a wide range of options to mirror high-stakes assessment formats.

Reporting

Clear, instructionally actionable data will be a click away with the *Reveal Math* Reporting Dashboard.

Activity Report Real-time class and student reporting of activities completed by the class. Includes average score, submission rate, and skills covered for the class and each student.

- **Item Analysis Report** Review a detailed analysis of response rates and patterns, answers, and question types in a class snapshot or by student.
- **Standards Report** Performance data by class or individual student is aggregated by standards, skills, or objectives linked to the related activities completed.

Activity Report

Professional Development Support for Continuous Learning

McGraw-Hill Education supports lifelong learning and demonstrates commitment to teachers with a built-in professional learning environment designed for support during planning or extended learning opportunities.

What You Will Find

- Best-practices resources
- Implementation support
- Teaching Strategies
- Classroom Videos
- Math Misconception Videos
- Content and Pedagogy Videos
- Content Progression Information

Why Professional Development is so Important

- Research-based understanding of student learning
- Improved student performance
- Evidence-based instructional best-practices
- Collaborative content strategy planning
- Extended knowledge of program how-to's

Reveal Math Expert Advisors

Cathy Seeley, Ed.D.

Austin, Texas

Mathematics educator, speaker, and writer, former Senior Fellow at the Charles A. Dana Center at The University of Texas at Austin, past President of NCTM, former Director of K-12 Mathematics for the State of Texas

Areas of expertise:
Mathematics Teaching, Equity, Assessment, STEM Learning, Informal Learning, Upside-Down Teaching, Productive Struggling, Mathematical Practices, Mathematical Habits of Mind, Family and Community Outreach, Mathematics Education Policy, Advocacy

"We want students to believe deeply that mathematics makes sense–in generating answers to problems, discussing their thinking and other students' thinking, and learning new material."

—Seeley, 2016, Making Sense of Math

Cheryl R. Tobey, M.Ed.

Gardiner, Maine

Senior Mathematics Associate at Education Development Center (EDC)

Areas of expertise:
Formative assessment and professional development for mathematics teachers; tools and strategies to uncovering misconceptions

"Misunderstandings and partial understandings develop as a normal part of learning mathematics. Our job as educators is to minimize the chances of students' harboring misconceptions by knowing the potential difficulties students are likely to encounter, using assessments to elicit misconceptions and implementing instruction designed to build new and accurate mathematical ideas."

— Tobey, et al 2007, 2009, 2010, 2013, 2104, Uncovering Student Thinking Series

Nevels Nevels, Ph.D.

Saint Louis, Missouri

PK-12 Mathematics Curriculum Coordinator for Hazelwood School District

Areas of expertise:
Mathematics Teacher Education; Student Agency & Identity; Socio-Cultural Perspective in Mathematics Learning

"A school building is one setting for learning mathematics. It is understood that all children should be expected to learn meaningful mathematics within its walls. Additionally, teachers should be expected to learn within the walls of this same building. More poignantly, I posit that if teachers are not learning mathematics in their school building, then it is not a school."

—Nevels, 2018

Raj Shah, Ph.D.

Columbus, Ohio

Founder of Math Plus Academy, a STEM enrichment program and founding member of The Global Math Project

Areas of expertise:
Sparking student curiosity, promoting productive struggle, and creating math experiences that kids love

"As teachers, it's imperative that we start every lesson by getting students to ask more questions because curiosity is the fuel that drives engagement, deeper learning and perseverance."

—Shah, 2017

Walter Secada, Ph.D.

Coral Gables, Florida

Professor of Teaching and Learning at the University of Miami

Areas of expertise:
Improving education for English language learners, equity in education, mathematics education, bilingual education, school restructuring, professional development of teachers, student engagement, Hispanic dropout and prevention, and reform

"The best lessons take place when teachers have thought about how their individual English language learners will respond not just to the mathematical content of that lesson, but also to its language demands and mathematical practices."

—Secada, 2018

Ryan Baker, Ph.D.

Philadelphia, Pennsylvania

Associate Professor and Director of Penn Center for Learning Analytics at the University of Pennsylvania

Areas of expertise:
Interactions between students and educational software; data mining and learning analytics to understand student learning

"The ultimate goal of the field of Artificial Intelligence in Education is not to promote artificial intelligence, but to promote education... systems that are designed intelligently, and that leverage teachers' intelligence. Modern online learning systems used at scale are leveraging human intelligence to improve their design, and they're bringing human beings into the decision-making loop and trying to inform them."

—Baker, 2016

Chris Dede, Ph.D.

Cambridge, Massachusetts

Timothy E. Wirth Professor in Learning Technologies at Harvard Graduate School of Education

Areas of expertise:
Provides leadership in educational innovation; educational improvements using technology

"People are very diverse in how they prefer to learn. Good instruction is like an ecosystem that has many niches for alternative types of learning: lectures, games, engaging video-based animations, readings, etc. Learners then can navigate to the niche that best fulfills their current needs."

—Dede, 2017

Dinah Zike, M.Ed.

Comfort, Texas

President of Dinah.com in San Antonio, Texas and Dinah Zike Academy

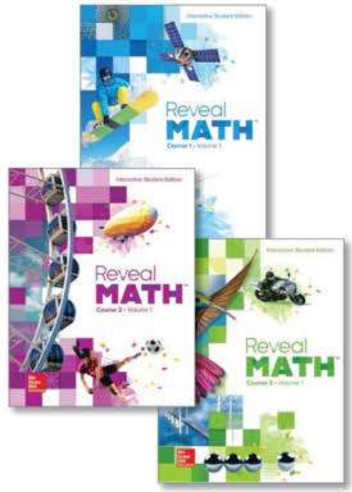
Areas of expertise:
Developing educational materials that include three-dimensional graphic organizers; interactive notebook activities for differentiation; and kinesthetic, cross-curricular manipulatives

"It is education's responsibility to meet the unique needs of students, and not the students' responsibility to meet education's need for uniformity."

—Zike, 2017, InRIGORating Math Notebooks

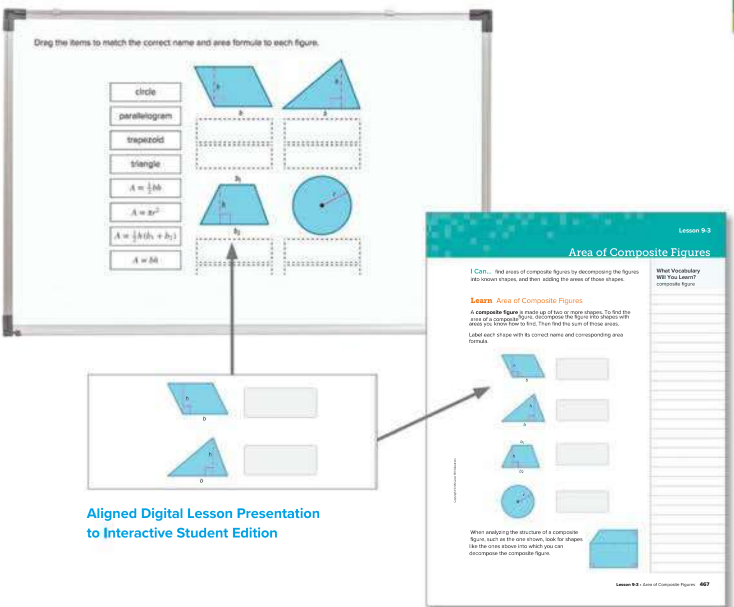
Reveal Everything Needed for Effective Instruction

Reveal Math provides both print and innovative, technology-based tools designed to address a wide range of classrooms. No matter whether you're in a 1:1 district, or have a classroom projector, *Reveal Math* provides you with the resources you need for a rich learning experience.



Blended Classrooms

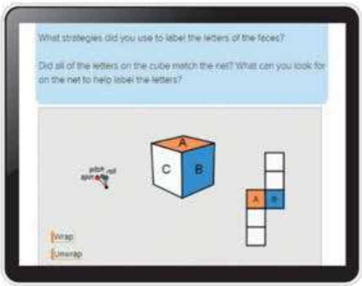
Focused on projection of the **Interactive Presentation**, students follow along taking notes and working through problems in their Interactive Student Edition during class time. Also included in the Interactive Student Edition is a glossary, **Foldables**® at point of use and in the back of the book, selected answers, and a reference sheet.



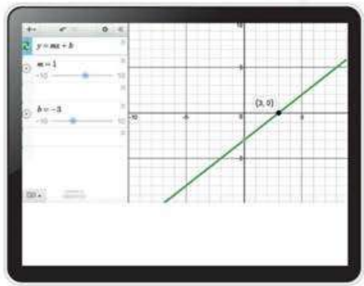
Aligned Digital Lesson Presentation to Interactive Student Edition

Digital Classrooms

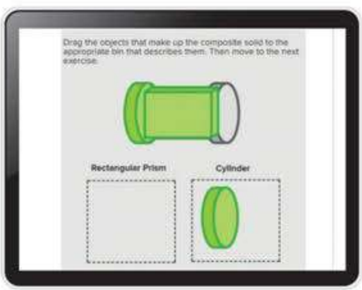
Projection is a focal point for key areas of the course with students interacting with the lesson using their own devices. Each student can access teacher-assigned sections of the lessons for **Explore** activities, **Learn** sections and **Examples**. Point of use videos, animations, as well as interactive content enable students to experience math in interesting and impactful ways.



Web Sketchpad

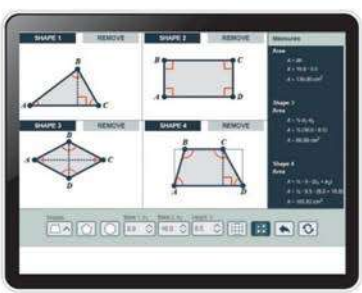


Desmos



Drag-and-Drop

Videos and Animations

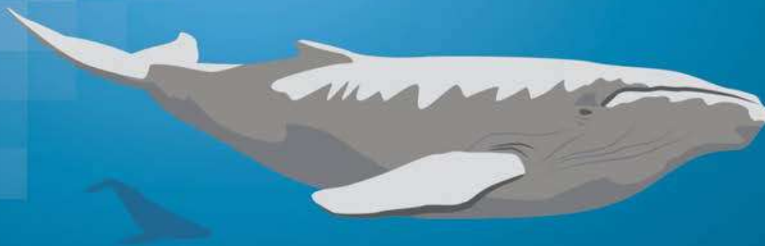


eTools

In *Reveal Math*, R is for—

- Research
- Rigor
- Relevant Connections

Are you… READY to start?



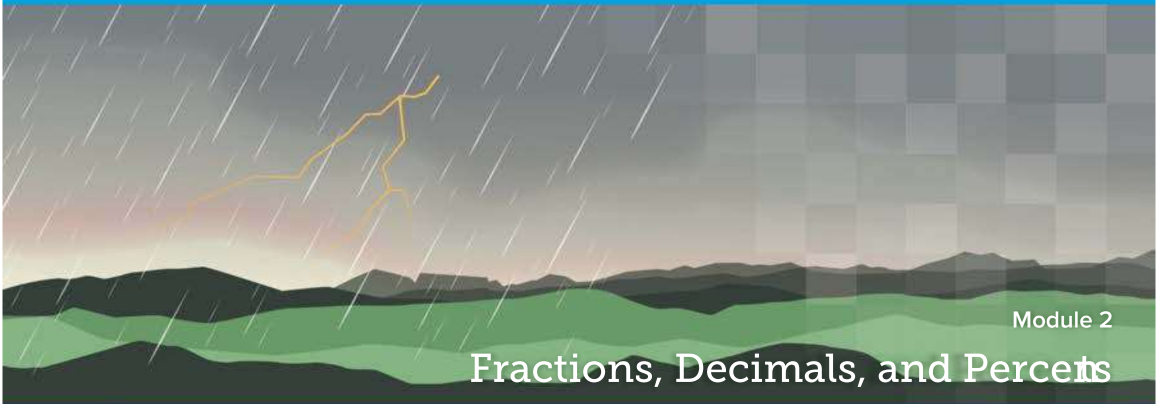
Module 1

Ratios and Rates

Essential Question

How can you describe how two quantities are related?

				Content Standards
		What Will You Learn?	1	
Lesson	1-1	Understand Ratios	3	6.RP.A.1
		Explore Compare Two Quantities		
	1-2	Tables of Equivalent Ratios	13	6.R.A.3, 6.RP.A.3.A
		Explore Compare Equivalent Ratios		
	1-3	Graphs of Equivalent Ratios	23	6.RP.A.3, 6.RP.A.3.A
	1-4	Compare Ratio Relationships	29	6.RP.A.3, 6.RP.A.3.A
	1-5	Solve Ratio Problems	37	6.RP.A.3
	1-6	Convert Customary Measurement Units	47	6.RP.A.3, 6.RP.A.3.D
	1-7	Understand Rates and Unit Rates	57	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.A, 6.RP.A.3.B
		Explore Compare Quantities with Different Units		
	1-8	Solve Rate Problems	65	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.B
		Module 1 Review	73	



Module 2

Fractions, Decimals, and Percents

Essential Question

How can you use fractions, decimals, and percents to solve everyday problems?

			Content Standards
		What Will You Learn? 77	
Lesson	2-1	Understand Percents 79	Foundational for 6.RP.A.3, 6.RP.A.3.C
	2-2	Percents Greater Than 100% and Less Than 1% 85	Foundational for 6.RP.A.3, 6.RP.A.3.C
	2-3	Relate Fractions, Decimals, and Percents .... 93 Explore Percents and Ratios	Foundational for 6.RP.A.3, 6.RP.A.3.C
	2-4	Find the Percent of a Number .... 103 Explore Percent of a Number	6.RP.A.3, 6.RP.A.3.C
	2-5	Estimate the Percent of a Number 113	6.RP.A.3, 6.RP.A.3.C
	2-6	Find the Whole 121	6.RP.A.3, 6.RP.A.3.C
		Module 2 Review 129	

Module 3

Compute with Multi-Digit Numbers and Fractions

Essential Question

How are operations with fractions and decimals related to operations with whole numbers?

				Content Standards
		What Will You Learn?	**133**	
Lesson	**3-1**	**Divide Multi-Digit Whole Numbers**	**135**	6.NS.B.2
	3-2	**Compute With Multi-Digit Decimals**	**143**	6.NS.B.3
	3-3	**Divide Whole Numbers by Fractions** Explore Divide Whole Numbers by Fractions	**155**	6.NS.A.1
	3-4	**Divide Fractions by Fractions**	**167**	6.NS.A.1
	3-5	**Divide with Whole and Mixed Numbers** Explore Divide Fractions by Whole Numbers	**177**	6.NS.A.1
		Module 3 Review	**187**	

Module 4

Integers, Rational Numbers, and the Coordinate Plane

Essential Question

How are integers and rational numbers related to the coordinate plane?

			Content Standards
		What Will You Learn? 191	
Lesson	4-1	Represent Integers .... 193 Explore Represent Integers	6.NS.C.5, 6.NS.C.6, 6.NS.C.6.C
	4-2	Opposites and Absolute Value .... 199 Explore Opposites and Absolute Value	6.NS.C.5, 6.NS.C.6, 6.NS.C.6.A, 6.NS.C.7, 6.NS.C.7.C
	4-3	Compare and Order Integers 205	6.NS.C.7, 6.NS.C.7.A, 6.NS.C.7.B, 6.NS.C.7.C, 6.NS.C.7.D
	4-4	Rational Numbers 215	6.NS.C.6, 6.NS.C.6.C, 6.NS.C.7, 6.NS.C.7.A, 6.NS.C.7.C
	4-5	The Coordinate Plane .... 225 Explore The Coordinate Plane	6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8
	4-6	Graph Reflections of Points .... 237 Explore Reflect a Point	6.NS.C.6, 6.NS.C.6.B, 6.NS.C.6.C, 6.NS.C.8
	4-7	Absolute Value and Distance .... 245 Explore Distance on the Coordinate Plane	6.NS.C.8
		Module 4 Review 255	

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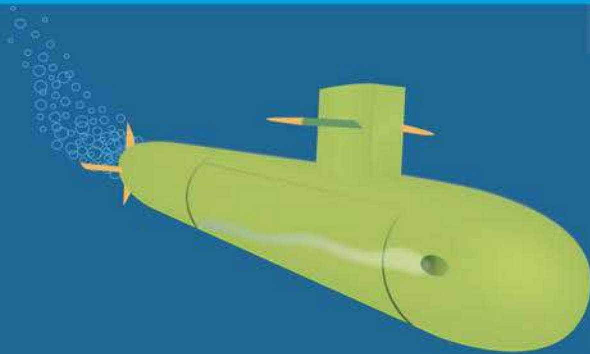
Module 5

Numerical and Algebraic Expressions

Essential Question

How can we communicate algebraic relationships with mathematical symbols?

			Content Standards
		What Will You Learn? **259**	
Lesson	**5-1**	**Powers and Exponents** **261**	6.EE.A.1
	5-2	**Numerical Expressions** **269**	6.EE.A.1
	5-3	**Write Algebraic Expressions** **277**	6.EE.A.2, 6.EE.A.2.A
		Explore Write Algebraic Expressions	6.EE.A.2.B, 6.EE.B.6
	5-4	**Evaluate Algebraic Expressions** **287**	6.EE.A.2, 6.EE.A.2.C,
		Explore Algebraic Expressions	6.EE.B.6
	5-5	**Factors and Multiples** **295**	6.NS.B.4
		Explore Greatest Common Factor	
		Explore Least Common Multiple	
	5-6	**Use the Distributive Property** **305**	6.NS.B.4, 6.EE.A.3
		Explore Use Algebra Tiles to Model the Distributive Property	
	5-7	**Equivalent Algebraic Expressions** **315**	6.EE.A.3, 6.EE.A.4
		Explore Properties and Equivalent Expressions	
		Module 5 Review **329**	



Module 6

Equations and Inequalities

Essential Question

How are the solutions of equations and inequalities different?

		What Will You Learn?	**333**	**Content Standards**
Lesson	**6-1**	**Use Substitution to Solve One-Step Equations**	**335**	6.EE.B.5
	6-2	**One-Step Addition Equations**	**341**	6.EE.B.6, 6.EE.B.7
		Explore Use Bar Diagrams to Write Addition Equations		
		Explore One-Step Addition Equations		
	6-3	**One-Step Subtraction Equations**	**351**	6.EE.B.6, 6.EE.B.7
		Explore Use Bar Diagrams to Write Subtraction Equations		
	6-4	**One-Step Multiplication Equations**	**359**	6.EE.B.6, 6.EE.B.7
		Explore Use Bar Diagrams to Write Multiplication Equations		
	6-5	**One-Step Division Equations**	**369**	6.EE.B.6, 6.EE.B.7
		Explore Use Bar Diagrams to Write Division Equations		
	6-6	**Inequalities**	**377**	6.EE.B.5, 6.EE.B.8
		Explore Inequalities		
		Module 6 Review	**391**	

Module 7

Relationships Between Two Variables

Essential Question

What are the ways in which a relationship between two variables can be displayed?

			Content Standards
		What Will You Learn? **395**	
Lesson	**7-1**	**Relationships Between Two Variables** .... **397** Explore Relationships Between Two Variables	6.EE.C.9
	7-2	**Write Equations to Represent Relationships Represented in Tables** .... **405** Explore Relationships with Rules that Require Two Steps	6.EE.C.9
	7-3	**Graphs of Relationships** **415**	6.EE.C.9
	7-4	**Multiple Representations** **423**	6.EE.C.9
		Module 7 Review **429**	

Module 8

Area

Essential Question

How are the areas of triangles and rectangles used to find the areas of other polygons?

			Content Standards
		What Will You Learn? **433**	
Lesson	**8-1**	**Area of Parallelograms** .... **435** Explore Area of Parallelograms	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C
	8-2	**Area of Triangles** .... **443** Explore Parallelograms and Area of Triangles Explore Area of Triangles	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C
	8-3	**Area of Trapezoids** **451**	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C
	8-4	**Area of Regular Polygons** .... **463** Explore Area of Regular Polygons	6.G.A.1
	8-5	**Polygons on the Coordinate Plane** .... **469** Explore Explore the Coordinate Plane	6.G.A.3
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Module 9

Volume and Surface Area

Essential Question

How can you describe the size of a three-dimensional figure?

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Lesson	9-1	Volume of Rectangular Prisms	485	6.G.A.2
	9-2	Surface Area of Rectangular Prisms Explore Cube Nets	495	6.G.A.4
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	9-4	Surface Area of Pyramids	517	6.G.A.4
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Module 10

Statistical Measures and Displays

Essential Question

Why is data collected and analyzed and how can it be displayed?

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Lesson	**10-1**	**Statistical Questions**	**537**	6.SP.A.1
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		Explore Mean		
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		Explore Mean, Median, and Outliers		
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Mathematical Overview for *Reveal Math*, Course 1

Reveal Math, Course 1, focuses on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Key Mathematical Understandings*, Grade 6

Ratios and Proportional Relationships (Domain 6.RP)

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System (Domain 6.NS)

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations (Domain 6.EE)

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry (Domain 6.G)

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability (Domain 6.SP)

- Develop understanding of statistical variability.
- Summarize and describe distributions.

**From the Common Core State Standards for Mathematics*

Standards for Mathematical Content, Grade 6

This correlation shows the alignment of *Reveal Math*, Course 1 to the Standards for Mathematical Content, Grade 6, from the Common Core State Standards for Mathematics. **Primary references are bold.** *Supporting references are italicized.*

Standards for Mathematical Content		Lesson(s)
6.RP **Ratios and Proportional Relationships**		
Understand ratio concepts and use ratio reasoning to solve problems. (Major Cluster)		
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**1-1**, *1-5*, *1-6*, *10-7*
6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] [1]Expectations for unit rates in this grade are limited to non-complex fractions.	**1-7**, **1-8**
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	**1-2**, **1-3**, **1-4**, **1-5**, **1-6**, **1-7**, **1-8**, **2-4**, **2-5**, **2-6**, *10-7*
	6.RP.A.3.A Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	**1-2**, **1-3**, **1-4**, **1-7**, *7-3*, *7-4*
	6.RP.A.3.B Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*	**1-7**, **1-8**
	6.RP.A.3.C Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	**2-4**, **2-5**, **2-6**
	6.RP.A.3.D Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	**1-6**

Standards for Mathematical Content		Lesson(s)
6.NS The Number System		
Apply and extend previous understandings of multiplication and division to divide fractions by fractions. (Major Cluster)		
6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**3-3**, **3-4**, **3-5**
Compute fluently with multi-digit numbers and find common factors and multiples. (Additional Cluster)		
6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.	**3-1**
6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**3-2**
6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*	**5-5**, **5-6**
Apply and extend previous understandings of numbers to the system of rational numbers. (Major Cluster)		
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**4-1**, **4-2**
6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	**4-1**, **4-2**, *4-3*, **4-4**, **4-5**, **4-6**, *4-7*, *6-6*, *7-3*, *7-4*
	6.NS.C.6.A Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., −(−3) = 3, and that 0 is its own opposite.	**4-2**, *4-6*
	6.NS.C.6.B Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	**4-5**, **4-6**
	6.NS.C.6.C Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	**4-1**, *4-3*, **4-4**, **4-5**, **4-6**, *6-6*, *7-3*, *7-4*

Standards for Mathematical Content		Lesson(s)
6.NS.C.7	Understand ordering and absolute value of rational numbers.	**4-2**, **4-3**, **4-4**, *4-7*
	6.NS.C.7.A Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right.*	**4-3**, **4-4**
	6.NS.C.7.B Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3°C > -7°C$ *to express the fact that* $-3°C$ *is warmer than* $-7°C$.	**4-3**, *4-4*
	6.NS.C.7.C Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* -30 *dollars, write* $\lvert -30 \rvert = 30$ *to describe the size of the debt in dollars.*	**4-2**, **4-3**, **4-4**, *4-7*
	6.NS.C.7.D Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* -30 *dollars represents a debt greater than 30 dollars.*	**4-3**
6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**4-5**, **4-6**, **4-7**
6.EE Expressions and Equations		
Apply and extend previous understandings of arithmetic to algebraic expressions. (Major Cluster)		
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	**5-1**, **5-2**
6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers.	**5-2**, **5-3**, **5-4**, *5-7*, *7-1*, **8-1**, **8-2**, **8-3**
	6.EE.A.2.A Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* $5 - y$.	**5-3**
	6.EE.A.2.B Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.*	**5-3**, *5-6*
	6.EE.A.2.C Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.	*5-2*, **5-4**, *7-1*, **8-1**, **8-2**, **8-3**
6.EE.A.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$.	**5-6**, **5-7**

Standards for Mathematical Content		Lesson(s)
6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*	**5-7**
Reason about and solve one-variable equations and inequalities. (Major Cluster)		
6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**6-1**, **6-6**
6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**5-3**, **5-4**, *6-1*, **6-2**, **6-3**, **6-4**, **6-5**, *6-6*, *7-2*, *7-3*, *7-4*, *9-1*, *10-3*
6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	**6-2**, **6-3**, **6-4**, **6-5**, *7-2*, *7-3*, *7-4*
6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**6-6**
Represent and analyze quantitative relationships between dependent and independent variables. (Major Cluster)		
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*	**7-1**, **7-2**, **7-3**, **7-4**

Standards for Mathematical Content		Lesson(s)
6.G Geometry		
Solve real-world and mathematical problems involving area, surface area, and volume. (Supporting Cluster)		
6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**8-1**, **8-2**, **8-3**, **8-4**, *8-5*
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**9-1**
6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**8-5**
6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**9-2**, **9-3**, **9-4**

Standards for Mathematical Content		Lesson(s)
6.SP Statistics and Probability		
Develop understanding of statistical variability. (Additional Cluster)		
6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**10-1**
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**10-4, 10-7**
6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	**10-3, 10-4, 10-5, 10-6, 10-7**
Summarize and describe distributions. (Additional Cluster)		
6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**10-2, 10-3, 10-4, 10-6, 10-7**
6.SP.B.5	Summarize numerical data sets in relation to their context, such as by:	*10-1*, **10-2, 10-3, 10-4, 10-5, 10-6, 10-7**
	6.SP.B.5.A Reporting the number of observations.	*10-1*, **10-2, 10-3, 10-5, 10-7**
	6.SP.B.5.B Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.	**10-3, 10-5, 10-7**
	6.SP.B.5.C Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	**10-3, 10-4, 10-5, 10-6, 10-7**
	6.SP.B.5.D Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**10-6, 10-7**

Standards for Mathematical Practice, Grade 6

This correlation shows the alignment of *Reveal Math*, Course 1 to the Standards for Mathematical Practice, from the Common Core State Standards.

Standards for Mathematical Practice		Lesson(s)
MP1	**Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	A strong problem-solving strand is present throughout the program with an emphasis on having students explain to themselves and others the meanings of problems and plan their solution strategies. Look for the **Apply** problems and exercises labeled as **Persevere with Problems**. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-2, Apply • Lesson 3-1, Practice Exercise 15 • Lesson 3-3, Apply • Lesson 8-1, Apply • Lesson 9-1, Apply
MP2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Students are routinely asked to make sense of quantities and their relationships, and attend to the meaning of quantities as opposed to just computing with them. Students are often asked to decontextualize a real-world problem by representing it symbolically as an expression, equation, or inequality. Look for lessons addressing these algebraic topics and the exercises labeled as **Reason Abstractly**. Many ***Talk About It!*** question prompts ask students to reason about relationships between quantities. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-6, Example 1 • Lesson 5-3, Examples 2, 4, 5 • Lesson 6-2, Example 1 • Lesson 7-1, Example 2 • Lesson 7-3, Learn *Write an Equation from a Graph*

Standards for Mathematical Practice		Lesson(s)
MP3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	Students are required to justify their reasoning and to find the errors in another student's reasoning or work. Look for the **Apply** problems (Step 4) and the exercises labeled as **Make a Conjecture**, **Find the Error**, **Use a Counterexample**, **Make an Argument**, or **Justify Conclusions.** Many ***Talk About It!*** question prompts ask students to justify conclusions and/or critique another student's reasoning. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 2-3, Practice Exercises 16-17 • Lesson 8-2, Practice Exercises 11, 14 • Lesson 9-1, Practice Exercise 9 • Lesson 9-4, Example 2, *Talk About It!*
MP4	**Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Students apply the mathematics they know to solve real-world problems by using mathematical modeling. In the **Apply** problems, students determine their own strategy to solve application problems by choosing mathematical models to aid them. Look also for the exercises labeled as **Model with Mathematics.** In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 6-2, Example 1 • Lesson 6-4, Apply • Lesson 6-5, Apply • Lesson 7-2, Examples 1–2 • Lesson 7-2, Apply

Standards for Mathematical Practice		Lesson(s)
MP5	**Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	In addition to traditional tools such as estimation, mental math, or measurement tools, students are encouraged to use digital tools, such as Web Sketchpad, eTools, etc. to help solve problems. Students are routinely asked to compare and contrast methods, tools, and representations and note when one tool might be more advantageous to use than another. Look for selected *Talk About It!* prompts and exercises labeled as **Use Math Tools.** Many **Explore** activities ask students to select and use appropriate tools as they progress through the activities. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-4, Learn *Use Graphs to Compare Ratio Relationships* • Lesson 1-5, Learn *Use Double Number Lines and Equivalent Ratios to Solve Ratio Problems* • Lesson 1-6, Learn *Convert Larger Units to Smaller Units* • Lesson 3-3, Examples 4-5 • Lesson 5-3, Explore activity *Write Algebraic Expressions*
MP6	**Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Students are routinely required to communicate precisely to partners, the teacher, or the entire class by using precise definitions and mathematical vocabulary. Look for the exercises labeled as **Be Precise.** Many ***Talk About It!*** question prompts ask students to clearly and precisely explain their reasoning. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 3-1, Learn *Divide Multi-Digit Numbers* • Lesson 4-4, Learn *Absolute Value of Rational Numbers, Talk About It!* • Lesson 6-2, Learn *Write Addition Equations, Talk About It!*

	Standards for Mathematical Practice	Lesson(s)
MP7	**Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Students are routinely encouraged to look for patterns or structure present in problem situations. For example, students look for structure present in algebraic expressions and use the structure of three-dimensional figures to create nets Look for the exercises labeled as **Identify Structure.** Many ***Talk About It!*** question prompts ask students to study the structure of expressions and figures. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 4-6, Example 1, *Talk About It!* • Lesson 4-7, Learn *Find Vertical Distance, Talk About It!* • Lesson 5-3, Learn *Structure of Algebraic Equations, Talk About It!* • Lesson 5-3, Example 1 • Lesson 6-1, Learn *Equations, Talk About It!* • Lesson 9-2, Learn *Make a Net to Represent a Rectangular Prism, Talk About It!* • Lesson 9-3, Example 2
MP8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Students are encouraged to look for repeated calculations that lead them to sound mathematical conclusions. For example, students notice that division ends when a remainder is zero. Look for the exercises labeled as **Identify Repeated Reasoning.** Several ***Talk About It!*** question prompts ask students to look for repeated calculations. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 3-1, Example 2, *Talk About It!* • Lesson 4-2, Example 3 • Lesson 6-2, Explore activity *One-Step Addition Equations*

![IGNITE!]

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can we communicate algebraic relationships with mathematical symbols? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

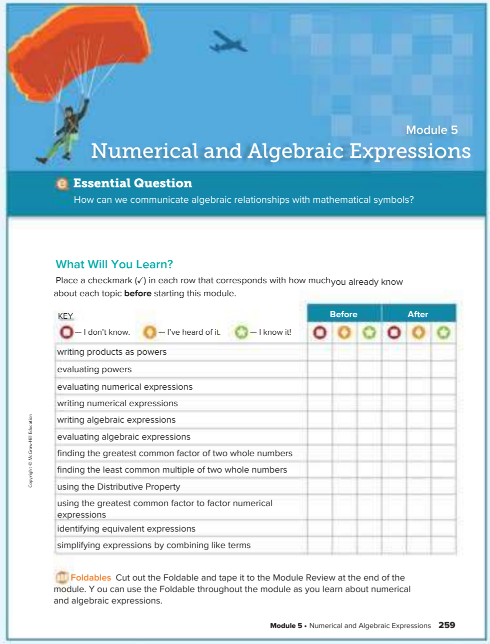
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of objects in freefall and the cost of attending a hockey game to introduce the idea of numerical and algebraic expressions. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 5

Numerical and Algebraic Expressions

Essential Question

How can we communicate algebraic relationships with mathematical symbols?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know	I've heard of it	I know it!	I don't know	I've heard of it	I know it!
writing products as powers						
evaluating powers						
evaluating numerical expressions						
writing numerical expressions						
writing algebraic expressions						
evaluating algebraic expressions						
finding the greatest common factor of two whole numbers						
finding the least common multiple of two whole numbers						
using the Distributive Property						
using the greatest common factor to factor numerical expressions						
identifying equivalent expressions						
simplifying expressions by combining like terms						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. Y ou can use the Foldable throughout the module as you learn about numerical and algebraic expressions.

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Module 5 • Numerical and Algebraic Expressions 259

Interactive Presentation

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Module 5
Numerical and Algebraic Expressions

Module Goal
Write and evaluate numerical and algebraic expressions.

Focus
Domain: Expressions and Equations
Major Cluster(s):
6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.
6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.B Reason about and solve one-variable equations and inequalities.
Standards for Mathematical Content:
6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.
6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.
Also addresses 6.NS.B.4, 6.EE.A.2.A, 6.EE.A.2.B, 6.EE.A.2.C, 6.EE.A.3, 6.EE.A.4, and 6.EE.B.6.
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover
Students need to have a thorough understanding of the prerequisite skills required for this module.

- fluently add, subtract, multiply, and divide positive rational numbers

Use the Module Pretest to diagnose students' readiness for this module. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence
Vertical Alignment

Previous
Students wrote and interpreted numerical expressions.
5.OA.A.1, 5.OA.A.2

Now
Students write and evaluate numerical and algebraic expressions.
6.NS.B.4, 6.EE.A.1, 6.EE.A.2, 6.EE.A.2.A, 6.EE.A.2.B, 6.EE.A.2.C, 6.EE.A.3, 6.EE.A.4, 6.EE.B.6

Next
Students will write and solve one-step equations and inequalities.
6.EE.B.5, 6.EE.B.6, 6.EE.B.7, 6.EE.B.8

Rigor
The Three Pillars of Rigor
In this module, students draw on their knowledge of the four basic operations to develop *understanding* of numerical and algebraic expressions. They use this understanding to build *fluency* with using powers and exponents, order of operations, and mathematical properties, as well as evaluating multi-step algebraic expressions and generating and simplifying equivalent algebraic expressions. They also *apply* their understanding of numerical and algebraic expressions to solve real-world problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

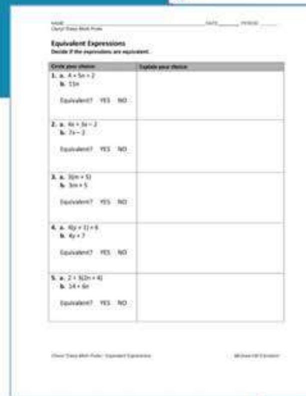
Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
5-1	Powers and Exponents	6.EE.A.1	2	1
5-2	Numerical Expressions	6.EE.A.1, *Also addresses 6.EE.A.2.C*	2	1
5-3	Write Algebraic Expressions	6.EE.A.2, 6.EE.A.2.A, 6.EE.A.2.B, 6.EE.B.6	2	1
5-4	Evaluate Algebraic Expressions	6.EE.A.2, 6.EE.A.2.C, 6.EE.B.6	3	1.5
Put It All Together 1: Lessons 5-1, 5-2, 5-3, and 5-4			0.5	0.25
5-5	Factors and Multiples	6.NS.B.4	2	1
5-6	Use the Distributive Property	6.NS.B.4, 6.EE.A.3, *Also addresses 6.EE.A.2.B*	3	1.5
5-7	Equivalent Algebraic Expressions	6.EE.A.3, 6.EE.A.4, *Also addresses 6.EE.A.2*	3	1.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	**20.5**	**10.25**

Module Resource

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Equivalent Expressions



Correct Answers: 1. No; **2.** Yes; **3.** No; **4.** No; **5.** Yes

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine if the expressions in each pair of expressions are equivalent.

Targeted Concept Expressions can look different but still be equivalent. Strategies such as combining like terms and distribution can be used to determine whether expressions are equivalent.

Targeted Misconceptions

- Students may incorrectly apply the Distributive Property.
- Students may incorrectly attempt to combine unlike terms.

Assign the probe after Lesson 7.

Collect and Assess Student Work

If the student selects...	**Then** the student likely...
3. Yes **4.** Yes **5.** No	multiplies the terms outside of the parentheses by only the first term in the expression inside the parentheses. **Example:** For Exercise 3, the student multiplies 3 by *m* but does not multiply 3 by 5.
1. Yes **2.** No **3.** No	combines all terms instead of only combining like terms. **Example:** For Exercise 2, the student may simplify the first expression as 9*x* by adding all of the terms together, or as 5*x* by subtracting 2 from 7*x*. **Example:** For Exercise 3, the student may simplify the first expression as 8*m* or 18*m*.
Other various patterns	incorrectly simplifies by combining terms incorrectly and/or incorrectly applying the Distributive Property.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Equations and Inequalities
- Lesson 6, Examples 1–6

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ algebra	☐ Distributive Property	☐ like terms
☐ algebraic expression	☐ equivalent expressions	☐ numerical expression
☐ Associative Property	☐ evaluate	☐ order of operations
☐ base	☐ exponent	☐ power
☐ coefficient	☐ factoring the expression	☐ simplest form
☐ Commutative Property	☐ greatest common factor	☐ term
☐ constant	☐ Identity Property	☐ variable
☐ defining the variable	☐ least common multiple	

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Multiply repeated factors.
Multiply $5 \times 5 \times 5 \times 5$.
The number 5 is used as a factor four times.
$5 \times 5 \times 5 \times 5 = 625$

Example 2
Subtract fractions and mixed numbers.
Find $3\frac{7}{8} - 1\frac{1}{2}$.
$3\frac{7}{8} - 1\frac{1}{2}$
$= 3\frac{7}{8} - 1\frac{4}{8}$ Rewrite using the LCD, 8.
$= 2\frac{3}{8}$ Subtract.

Quick Check

1. Multiply $7 \times 7 \times 7$. 343
2. Multiply $2 \times 2 \times 2 \times 2 \times 2$. 32
3. Find $\frac{4}{5} - \frac{1}{2}$. $\frac{3}{10}$
4. Find $3\frac{1}{10} - 2\frac{5}{6}$. $\frac{4}{15}$

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

1 2 3 4

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define The **Commutative Property** states that the order in which numbers are added or multiplied does not change the sum or product.

Example $5 \times 8 = 40$; $8 \times 5 = 40$

Ask If Clint wanted to organize 30 chairs into 5 or 6 rows, how would you express that as two multiplication sentences, that illustrate the Commutative Property? $5 \times 6 = 30$; or $6 \times 5 = 30$

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- multiplying whole numbers, fractions and decimals
- finding prime factors

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Equations and Inequalities** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

"Not Yet" Doesn't Mean "Never"

Students with a growth mindset understand that just because they haven't yet found a solution, that does not mean they won't find one with additional effort and reasoning. It can take time and continued effort to reason through different strategies that can be used to solve a problem.

How Can I Apply It?

Assign students the **Formative Assessment Math Probes** that are available for each module. Have them complete the probe before starting the module, and then again at the specified lesson within the module, or at the end of the module so that they can see their progress.

Learn Products as Powers

Objective

Students will learn how to write products of the same factor as powers using whole-number exponents.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to revisit the definitions of *exponent* and *base*, and to make sense of the terms *factor* and *power* when accurately describing the difference between exponents and bases.

7 Look for and Make Use of Structure Students should analyze the structure of each part represented and label it using the given vocabulary terms in order to complete the activity.

Teaching Notes

SLIDE 1

Students will learn the definitions of *exponent*, *power*, and *base*. Play the animation for the class. Students will learn how to write a power using a base and an exponent and how to label each part of the expression, including expressions with multiple bases and exponents.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation shows how to write an expression as a power.

Talk About It!

SLIDE 3

Mathematical Discourse

Explain the difference between a base and an exponent. Sample answer: A base is the factor used in evaluating a power, while an exponent indicates the number of times that base is used as a factor.

Lesson 5-1

Powers and Exponents

I Can... write a product of whole numbers, fractions, or decimals as a power and write a power as a product of factors.

What Vocabulary Will You Learn?
base
exponent
power

Learn Products as Powers

A product of like factors can be written in exponential form using an exponent and a base. A number expressed using an exponent is called a **power**. The **base** is the number used as a factor. The **exponent** tells how many times a base is used as a factor.

Go Online Watch the animation to see how an expression involving repeated factors can be written as a power.

The animation explains how to write the product of the repeated factor, 3, as a power.

4 factors

$3 \times 3 \times 3 \times 3 = 3^4$

Use an exponent to express the number of times 3 is used as a factor.

base

$3 \times 3 \times 3 \times 3 = 3^4$ ← exponent

The base is the common factor that is being multiplied, and the exponent tells how many times the base is used as a factor.

Label each part of the equation using the words below.

factors exponent base power

power

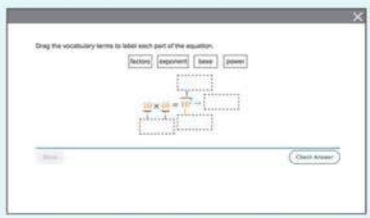
$10 \times 10 = 10^2$ ← exponent

factors base

Talk About It! Explain the difference between a base and an exponent.

Sample answer: A base is the factor used in evaluating a power, while an exponent indicates the number of times that base is used as a factor.

Interactive Presentation



Learn, Products as Powers, Slide 2 of 3

WATCH

On Slide 1, students watch an animation that shows how to write an expression as a power.

DRAG & DROP

On Slide 2, students drag the vocabulary terms to label each part of the equation.

Lesson 5-1

Powers and Exponents

6.EE.A.1

LESSON GOAL

Students will write and evaluate powers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Products as Powers
Example 1: Write Products as Powers
Example 2: Write Products as Powers
Learn: Powers as Products
Example 3: Evaluate Powers
Example 4: Evaluate Powers
Example 5: Evaluate Powers
Apply: Biology

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

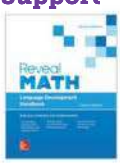
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Negative Exponents		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 27 of the *Language Development Handbook* to help your students build mathematical language related to powers and exponents.



ELL You can use the tips and suggestions on page T27 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.A** by writing and evaluating powers.
Standards for Mathematical Content: **6.EE.A. 1**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students wrote simple expressions that record calculations with numbers, and interpreted numerical expressions without evaluating them.
5.OA.A.2

Now
Students write and evaluate powers.
6.EE.A.1

Next
Students will write and evaluate numerical expressions.
6.EE.A.1

Rigor

The Three Pillars of Rigor

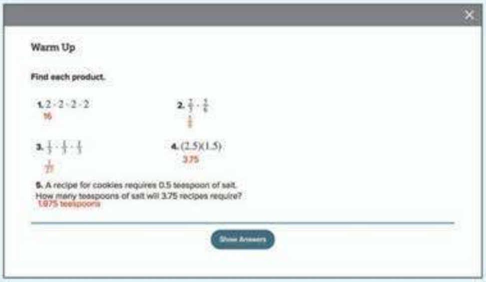
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of products to begin to develop *understanding* of powers and exponents. They use this understanding to build *fluency* with writing products involving rational numbers as powers using whole-number exponents. They also build fluency with writing powers as products with whole number, fractional, and decimal factors. They *apply* their understanding of powers and exponents to solve real-world problems.

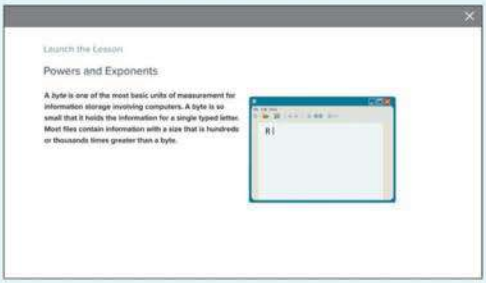
Mathematical Background

A *power* is an expression involving a base and an exponent. In a power, the *exponent* tells how many times the *base* is used as a factor. Exponents are to multiplication as multiplication is to addition. In other words, while multiplication is repeated addition, exponentiation is repeated multiplication. One of the advantages of using exponents is that they allow us to write very big numbers very compactly.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying whole numbers (Exercise 1)
- multiplying fractions (Exercises 2–3)
- multiplying decimals (Exercises 4–5)

Answers

1. 16

2. $\frac{5}{9}$

3. $\frac{1}{27}$

4. 3.75

5. 1.875 teaspoons

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the byte as one basic unit of measurement for information storage involving computers.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

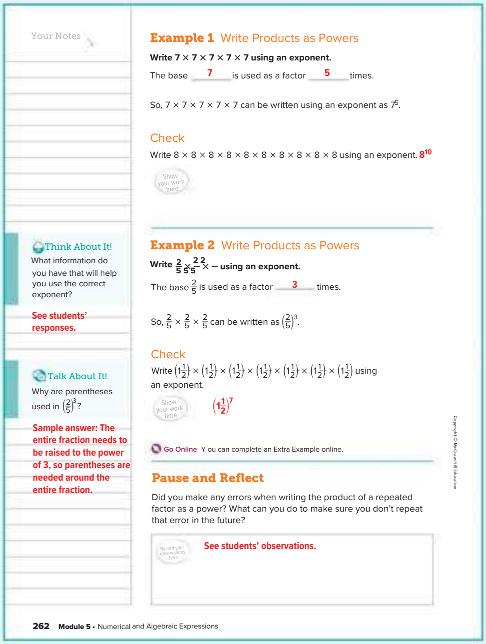
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- In what other area(s) of math have you heard of the term *base*? Sample answers: the base of a parallelogram, the base of a triangle, the base of a rectangular prism
- In what other areas of math, or everyday life, have you seen *exponents* used? Sample answer: The formulas for the area of a square and volume of a cube use exponents. The units for area are in square units, such as square feet or square inches.
- What does *power* mean in everyday life? Sample answer: strength, authority, influence



Your Notes

Example 1 Write Products as Powers

Write 7 × 7 × 7 × 7 × 7 using an exponent.

The base 7 is used as a factor 5 times.

So, 7 × 7 × 7 × 7 × 7 can be written using an exponent as 7^5.

Check

Write 8 × 8 × 8 × 8 × 8 × 8 × 8 × 8 × 8 × 8 using an exponent. 8^{10}

Show your work here

Think About It!

What information do you have that will help you use the correct exponent?

See students' responses.

Example 2 Write Products as Powers

Write $\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$ using an exponent.

The base $\frac{2}{5}$ is used as a factor 3 times.

So, $\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$ can be written as $\left(\frac{2}{5}\right)^3$.

Check

Write $\left(1\frac{1}{2}\right) \times \left(1\frac{1}{2}\right) \times \left(1\frac{1}{2}\right) \times \left(1\frac{1}{2}\right) \times \left(1\frac{1}{2}\right) \times \left(1\frac{1}{2}\right) \times \left(1\frac{1}{2}\right)$ using an exponent.

Show your work here

$\left(1\frac{1}{2}\right)^7$

Talk About It!

Why are parentheses used in $\left(\frac{2}{5}\right)^3$?

Sample answer: The entire fraction needs to be raised to the power of 3, so parentheses are needed around the entire fraction.

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you make any errors when writing the product of a repeated factor as a power? What can you do to make sure you don't repeat that error in the future?

Record your observations here

See students' observations.

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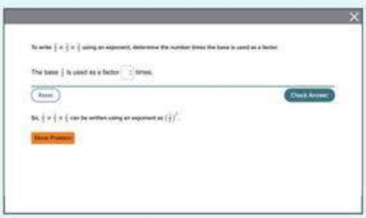
Interactive Presentation



Example 2, Write Products as Powers, Slide 2 of 4

CLICK

On Slide 2 of Example 2, students identify the number of times the base is used as a factor.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Write Products as Powers

Objective

Students will write products as powers using whole-number exponents.

Questions for Mathematical Discourse

SLIDE 1

AL What is the repeated factor in this expression? 7

AL How many times is the base used as a factor? 5

OL Explain the difference between the expressions 7^5 and 5^7. 7^5 is 7 × 7 × 7 × 7 × 7, and 5^7 is 5 × 5 × 5 × 5 × 5 × 5 × 5.

BL How many times greater is 7^6 than 7^5? Explain. 7 times greater; Sample answer: $7^6 = 7 \times 7 \times 7 \times 7 \times 7 \times 7$ and $7^5 = 7 \times 7 \times 7 \times 7 \times 7$; There is one more factor of 7 in 7^6 than in 7^5.

Example 2 Write Products as Powers

Objective

Students will write products involving rational numbers as powers using whole-number exponents.

Questions for Mathematical Discourse

SLIDE 2

AL What does the term *factor* mean? Sample answer: A factor is multiplied by another factor (or factors) to obtain a product.

OL Why is $\frac{2}{5}$ the base in this example? Sample answer: The base is the factor that is being multiplied by itself a certain number of times. In this example, that number is $\frac{2}{5}$.

OL The base is a fraction. Does this affect the process you use to find the exponent? Sample answer: No, I still count the number of times the base is used as a fraction, and then write the base with that exponent.

BL A classmate wrote the power as $\frac{2^3}{5}$. Why is this incorrect? Sample answer: The base is the entire fraction, $\frac{2}{5}$, not just the numerator, 2.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Powers as Products

Objective

Students will learn how to evaluate powers with whole-number factors.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to study the structure of the expression in order to determine what some possible missteps might be in evaluating it.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

What are some mistakes that could be made when evaluating 5^3? Sample answer: I might find 5×3 by mistake, rather than finding $5 \times 5 \times 5$. I might also confuse the base and the exponent to arrive at the product of $3 \times 3 \times 3 \times 3 \times 3$, or 243.

Example 3 Evaluate Powers

Objective

Students will evaluate powers with whole-number factors.

Questions for Mathematical Discourse

SLIDE 2

AL What is the base, and what is the exponent? The base is 4, and the exponent is 5.

OL Why is the 4 repeated five times as a factor? Sample answer: The exponent tells us how many times the factor is repeated, and in this case the exponent is 5.

BL How many times greater is 4^5 than 4^3? Explain. 16 times greater; Sample answer: $4^5 = 4 \times 4 \times 4 \times 4 \times 4$ and $4^3 = 4 \times 4 \times 4$; There are two more factors of 4 in 4^5 than in 4^3.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Powers as Products

To write powers as products, determine the base and the exponent. The base of 3^2 is 3 and the exponent is 2. To read powers, consider the exponent. The power 3^2 is read as *three to the second power* or *three squared*, the power 3^3 is read *three to the third power* or *three cubed*, and 3^5 is read *three to the fifth power*.

To evaluate powers, find the value of the power after multiplying.

Complete the table for the first four powers of 5.

Powers			
Power	Words	Factors	Value
5^1	5 to the first power	5	5
5^2	5 to the second power	5×5	25
5^3	5 to the third power	$5 \times 5 \times 5$	125
5^4	5 to the fourth power	$5 \times 5 \times 5 \times 5$	625

Talk About It!
What are some mistakes that could be made when evaluating 5^3?

Sample answer: I might find 5×3 by mistake rather than $5 \times 5 \times 5$. I might also confuse the base and the exponent to arrive at the product of $3 \times 3 \times 3 \times 3 \times 3$ or 243.

Example 3 Evaluate Powers

Evaluate 4^5.

$4^5 = 4 \times 4 \times 4 \times 4 \times 4$ Write 4^5 as a product.

$= 1{,}024$ Simplify.

So, 4^5 is 1,024.

Check

Evaluate 8^4. 4,096

Show your work here

Talk About It!
A friend evaluates the expression 4^5 and arrives at a value of 20 for the solution. Describe the mistake.

Sample answer: They multiplied the base and the exponent. They should have used the base, 4, as a factor five times to arrive at a product of 1,024.

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 3, Evaluate Powers, Slide 2 of 4

CLICK On Slide 2 of Example 3, students move through the steps to evaluate the expression.

TYPE On Slide 2 of Example 3, students determine the value of the power.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
What does the exponent of 4 mean?
$\frac{1}{3}$ is used as a factor 4 times.

Talk About It!
A friend evaluates the expression $\left(\frac{1}{3}\right)^4$ and arrives at a value of $\frac{1}{12}$. Describe the likely mistake.
Sample answer: They multiplied the exponent by the denominator of the base instead of using $\frac{1}{3}$ as a factor four times.

Example 4 Evaluate Powers

Evaluate $\left(\frac{1}{3}\right)^4$.

$\left(\frac{1}{3}\right)^4 = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$ Write $\left(\frac{1}{3}\right)^4$ as a product.

$= \frac{1}{81}$ Simplify.

So, $\left(\frac{1}{3}\right)^4$ is $\frac{1}{81}$.

Check

Evaluate $\left(\frac{2}{5}\right)^3$. $\frac{8}{125}$

Show your work here

Example 5 Evaluate Powers

Evaluate $(2.5)^3$.

$(2.5)^3 = (2.5) \times (2.5) \times (2.5)$ Write $(2.5)^3$ as a product.

$= 15.625$ Simplify.

So, $(2.5)^3$ is 15.625.

Check

Evaluate $(0.2)^4$. 0.0016

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 4, Evaluate Powers, Slide 2 of 4

CLICK

On Slide 2 of Example 4, students move through the steps to evaluate the expression.

TYPE

On Slide 2 of Example 4, students determine the value of the power.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 4 Evaluate Powers

Objective

Students will evaluate powers with factors that are fractions.

Questions for Mathematical Discourse

SLIDE 2

AL What is the base in this expression? $\frac{1}{3}$

OL Why is the numerator 1 and not 4 after multiplying? Sample answer: The exponent of 4 does not mean that 1 is multiplied by 4. It means that 4 ones are multiplied.

BL Why is the value of $\left(\frac{1}{3}\right)^4$ less than the base of $\frac{1}{3}$? Sample answer: When any number is multiplied by a fraction between 0 and 1, the product is less than the number. In this problem, $\frac{1}{3}$ is multiplied by itself (a fraction between 0 and 1) four times, so the product is less than $\frac{1}{3}$.

Example 5 Evaluate Powers

Objective

Students will evaluate powers with factors that are decimals.

Questions for Mathematical Discourse

SLIDE 1

AL What is the base in this expression? 2.5

AL How many times is the base used as a factor? 3

OL How does the value of $(2.5)^3$ compare to the base? $(2.5)^3$ is greater than 2.5.

OL How can you estimate the value of $(2.5)^3$, without calculating it? Sample answer: $2.5 < 3$, and $3 \times 3 \times 3 = 27$, so $(2.5)^3$ will be less than 27. I also know that $2.5 > 2$, and $2 \times 2 \times 2 = 8$, so $(2.5)^3$ will be greater than 8.

BL How does the value of $(2.5)^3$ compare to the value of $(2.5)^2$? Sample answer: The value of $(2.5)^3$ is 2.5 times greater than $(2.5)^2$ because $(2.5)^3$ has one more factor of 2.5 than $(2.5)^2$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Biology

Objective

Students will come up with their own strategy to solve an application problem involving the amount of bacteria in a petri dish.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is a petri dish?
- How many total bacteria are there after 5 hours? 10 hours?
- How can you use the table to find the amount of bacteria after 30 hours?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Biology

Delmar is studying the growth rate of a specific type of bacteria. He places 3 cells in a Petri dish and records the number of bacteria over time. He records the results over 20 hours in the table shown and notices a pattern. At this rate, how many bacteria are expected to be present in the Petri dish after 30 hours?

Number of Hours	Number of Bacteria
5	3×3
10	$3 \times 3 \times 3$
15	$3 \times 3 \times 3 \times 3$
20	$3 \times 3 \times 3 \times 3 \times 3$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

2, 187 bacteria cells; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

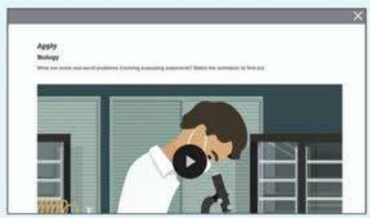
Go Online watch the animation.

Talk About It!

Suppose Delmar originally placed 4 cells in the Petri dish. Could you use the same method to determine the total cells after 30 hours? Explain.

no; Sample answer: Y ou don't know the rate at which the cells grow so you cannot use the same method.

Interactive Presentation



Apply, Biology

WATCH

On Slide 1, students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Faith is turning 12 this year. She asks her parents to give her $2 on her birthday and to double that amount for her next birthday. If she continues with this pattern, how much money will Faith receive on her 20th birthday? $512

Birthday	Amount ($)
12th	2
13th	2×2
14th	$2 \times 2 \times 2$
15th	$2 \times 2 \times 2 \times 2$

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How well do you understand the process of evaluating powers? What questions do you still have? How can you get those questions answered?

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. How many possible values are there for a byte? Include the expression used to calculate your answer. Write a mathematical argument that can be used to defend your solution. Sample answer: $2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$ different values

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of powers, have them make a conjecture about the value of a number raised to the zero power using the following steps.

1. Find the value of 3^4, 3^3, 3^2, and 3^1. 81; 27; 9; 3

2. What do you notice about the values of the expressions? Sample answer: To obtain the value of the each expression, you divide the previous expression by 3.

3. Based on this, make a conjecture about how would you find the value of 3^0. Sample answer: I would divide the value of 3^1 by 3.

4. What is the value of the expression 3^0? 1

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 17, 19–23
- Extension: Negative Exponents
- ALEKS Exponents and Order of Operations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–16, 19, 21, 23
- Extension: Negative Exponents
- Personal Tutor
- Extra Examples 1–5
- ALEKS Exponents and Order of Operations

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Exponents and Order of Operations

Practice and Homework

The Independent Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

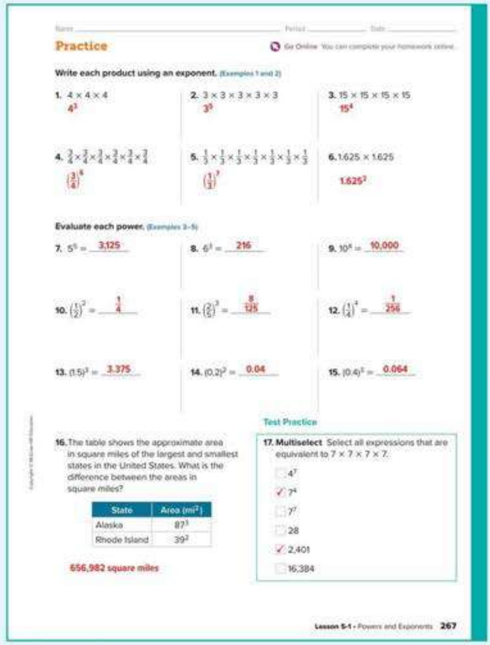
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	write products as powers	1–6
1	evaluate powers	7–15
2	extend concepts learned in class to apply them in new contexts	16, 17
3	solve application problems that involve powers and exponents	18, 19
3	higher-order and critical thinking skills	20–23

Common Misconception

Some students may incorrectly evaluate powers. Remind students that powers can be written as products. Stress that 3^3 is equivalent to $3 \times 3 \times 3$, not 3×3. Encourage students to write a power as a product before evaluating.



Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Write each product using an exponent. (Examples 1 and 2)

1. $4 \times 4 \times 4$ — 4^3
2. $3 \times 3 \times 3 \times 3 \times 3$ — 3^5
3. $15 \times 15 \times 15 \times 15$ — 15^4
4. $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ — $\left(\frac{3}{4}\right)^6$
5. $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$ — $\left(\frac{1}{3}\right)^7$
6. 1.625×1.625 — 1.625^2

Evaluate each power. (Examples 3–5)

7. $5^5 =$ 3,125
8. $6^3 =$ 216
9. $10^4 =$ 10,000
10. $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$
11. $\left(\frac{2}{5}\right)^3 = \frac{8}{125}$
12. $\left(\frac{1}{4}\right)^4 = \frac{1}{256}$
13. $(1.5)^3 =$ 3.375
14. $(0.2)^2 =$ 0.04
15. $(0.4)^3 =$ 0.064

16. The table shows the approximate area in square miles of the largest and smallest states in the United States. What is the difference between the areas in square miles?

State	Area (mi^2)
Alaska	87^3
Rhode Island	39^2

656,982 square miles

Test Practice

17. **Multiselect** Select all expressions that are equivalent to $7 \times 7 \times 7 \times 7$.

- [ ] 4^7
- [x] 7^4
- [ ] 7^7
- [ ] 28
- [x] 2,401
- [ ] 16,384

Lesson 5-1 • Powers and Exponents 267

Apply *indicates multi-step problem

*18. Willa is studying the growth rate of a specific type of organism called a ciliate. She places 2 cells in a dish and records the number of cells over time. The table shows her results. If the pattern continues, how many cells will be in the dish after 12 hours?

Number of Hours	Number of Cells
2	2×2
4	$2 \times 2 \times 2$
6	$2 \times 2 \times 2 \times 2$
8	$2 \times 2 \times 2 \times 2 \times 2$

128 cells

*19. Christiano is performing a science experiment and studying the growth rate of a certain type of onion root cell under different conditions. He places a cell in a dish and records the number of cells each day. Based on the pattern shown in the table, predict the number of cells in the dish after 5 days.

Number of Days	Number of Cells
1	4
2	4×4
3	$4 \times 4 \times 4$

1,024 cells

Higher-Order Thinking Problems

20. Write a power whose value is greater than 500 but less than 1,000.

Sample answer: 9^3

21. MP **Find the Error** A student was evaluating 2^3. Find the student's mistake and correct it.

$$2^3 = 3 \times 3$$
$$= 9$$

Sample answer: The student used the exponent as the base. The base should be 2 and the exponent is 3. The power evaluated should be $2 \times 2 \times 2 = 8$.

22. MP **Reason Inductively** Suppose the world population is about 8 billion. Is 8 billion closer to 10^{10} or 10^{11}? Explain.

10^{10}; Sample answer: 10^{10} is equal to 10,000,000,000 and 10^{11} is equal to 100,000,000,000. 10,000,000,000 is much closer to 8,000,000,000 than 100,000,000,000.

23. MP **Be Precise** Explain how exponential form is similar to multiplication being the process of repeated addition.

Sample answer: Exponential form is repeated multiplication of a common factor.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 21, students will find the mistake made in the power that has been evaluated. Encourage students to identify the error and how to correct it.

2 Reason Abstractly and Quantitatively In Exercise 22, students will reason which power is closer to 8 billion. Encourage students to use reasoning to explain their answer.

6 Attend to Precision In Exercise 23, students will explain how exponential form is similar to multiplication being the process of repeated addition.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 18–19 Have students work in pairs. Have students individually read Exercise 18 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 19.

Be sure everyone understands.

Use with Exercises 21–22 Have students work in groups of 3–4 to solve the problem in Exercise 21. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 22.

Learn Numerical Expressions

Objective

Students will understand that the order of operations can be used to evaluate numerical expressions.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, encourage them to create a plausible argument for why rules are needed to evaluate expressions, using the given expression as an example.

Go Online

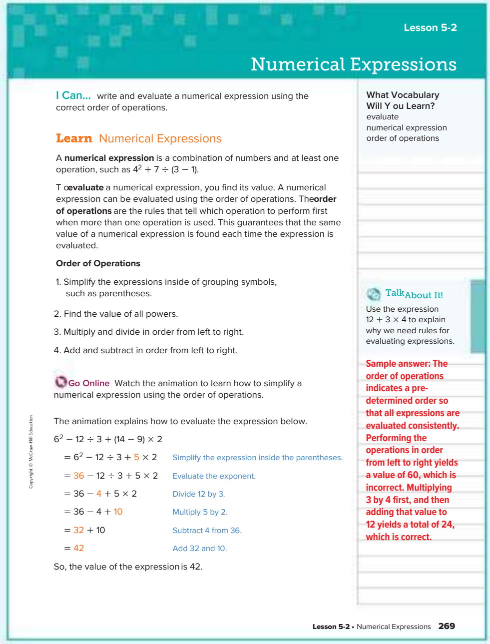
- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to use the order of operations to simplify a numerical expression.

Talk About It!

SLIDE 2

Mathematical Discourse

Use the expression 12 ÷ 3 × 4 to explain why multiplication and division must be performed in order from left to right. Sample answer: The order of operations indicates a predetermined order so that all expressions are evaluated consistently. Performing the operations in order from left to right yields a value of 60, which is incorrect. Multiplying 3 by 4 first, and then adding that value to 12 yields a total of 24, which is correct.


Lesson 5-2

Numerical Expressions

I Can... write and evaluate a numerical expression using the correct order of operations.

Learn Numerical Expressions

A **numerical expression** is a combination of numbers and at least one operation, such as $4^2 + 7 \div (3 - 1)$.

T o**evaluate** a numerical expression, you find its value. A numerical expression can be evaluated using the order of operations. The**order of operations** are the rules that tell which operation to perform first when more than one operation is used. This guarantees that the same value of a numerical expression is found each time the expression is evaluated.

Order of Operations

1. Simplify the expressions inside of grouping symbols, such as parentheses.
2. Find the value of all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Go Online Watch the animation to learn how to simplify a numerical expression using the order of operations.

The animation explains how to evaluate the expression below.

$6^2 - 12 \div 3 + (14 - 9) \times 2$

$= 6^2 - 12 \div 3 + 5 \times 2$	Simplify the expression inside the parentheses.
$= 36 - 12 \div 3 + 5 \times 2$	Evaluate the exponent.
$= 36 - 4 + 5 \times 2$	Divide 12 by 3.
$= 36 - 4 + 10$	Multiply 5 by 2.
$= 32 + 10$	Subtract 4 from 36.
$= 42$	Add 32 and 10.

So, the value of the expression is 42.

What Vocabulary Will Y ou Learn?
evaluate
numerical expression
order of operations

Talk About It!
Use the expression 12 + 3 × 4 to explain why we need rules for evaluating expressions.

Sample answer: The order of operations indicates a pre-determined order so that all expressions are evaluated consistently. Performing the operations in order from left to right yields a value of 60, which is incorrect. Multiplying 3 by 4 first, and then adding that value to 12 yields a total of 24, which is correct.

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Lesson 5-2 • Numerical Expressions 269

Interactive Presentation

Learn, Numerical Expressions, Slide 1 of 2

WATCH

On Slide 1, students watch the animation to learn about using the order of operations to simplify a numerical expression.

DIFFERENTIATE

Reteaching Activity AL

If any of your students have difficulty remembering the order of operations, have them create a chart for assistance. The chart should include each of the steps of the order of operations and could even include examples illustrating each step. Have them work with a partner to create several numerical expressions with multiple operations. Then have each pair trade expressions with another pair. Each pair should use their chart to simplify the expressions. Have pairs exchange solutions, and discuss and resolve any differences.

Lesson 5-2

Numerical Expressions

6.EE.A.1

LESSON GOAL

Students will write and evaluate numerical expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Numerical Expressions
Example 1: Evaluate Numerical Expressions
Example 2: Evaluate Numerical Expressions
Learn: Write Numerical Expressions
Example 3: Write and Evaluate Numerical Expressions
Apply: Art Supplies

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

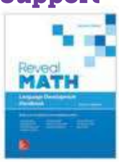
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Variables and Absolute Value		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 28 of the *Language Development Handbook* to help your students build mathematical language related to numerical expressions.



ELL You can use the tips and suggestions on page T28 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.A** by writing and evaluating numerical expressions.
Standards for Mathematical Content: **6.EE.A.1,** *Also addresses 6.EE.A.2.C*
Standards for Mathematical Practice: **MP1, MP3, MP4**

Coherence

Vertical Alignment

Previous
Students wrote and evaluated powers.
6.EE.A.1

Now
Students write and evaluate numerical expressions.
6.EE.A.1

Next
Students will write algebraic expressions.
6.EE.A.2.A, 6.EE.A.2.B, 6.EE.B.6

Rigor

The Three Pillars of Rigor

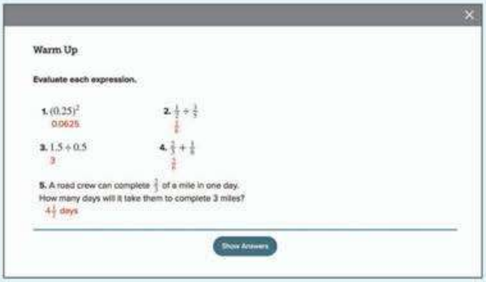
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of powers and exponents to develop *understanding* of numerical expressions. They learn to use the order of operations to build *fluency* with writing and evaluating numerical expressions. They also *apply* their understanding of numerical expressions to solve real-world problems.

Mathematical Background

A *numerical expression* is a mathematical expression involving numbers and one or more operations. The *order of operations* governs the precedence that certain operations have over others. When evaluating a numerical expression, first evaluate expressions inside grouping symbols. Powers have the next highest precedence. After powers, evaluate multiplication and division, followed by addition and subtraction. If two operations have the same precedence, e.g. multiplication and division, or even subtraction followed by another subtraction, evaluate those operations from left to right.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding exponents (Exercise 1)
- performing operations with positive rational numbers (Exercises 2–5)

Answers

1. 0.0625 **4.** $\frac{5}{6}$

2. $\frac{5}{6}$ **5.** $4\frac{1}{2}$ days

3. 3

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the cost of admission to a circus, expressed as a numerical expression.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Can you think of a mathematical term that is similar to the term *evaluate*? What does it mean? Sample answer: find; To find the solution of a problem.
- What is another use of the term *expression* in the English language? Sample answer: A facial expression is a way of using your face to show emotion; a verbal expression is a way of communicating something using words.
- How does the meaning of the term *order* and what you know about mathematical operations help you understand the meaning of the term *order of operations*? Sample answer: The term *order* implies a predetermined process or workflow for completing steps in a task. Some mathematical operations are addition, subtraction, multiplication, and division. The *order of operations* might mean that there is a predetermined workflow that I should use to perform mathematical operations.

Your Notes

Think About It!
What does the order of operations tell you to do first?

evaluate operations inside grouping symbols

Talk About It!
Explain why subtraction was performed before addition when the expression showed $100 - 90 + 2$.

Sample answer: The subtraction was performed before the addition because it occurred in the expression before addition. According to the order of operations, these should be performed in order from left to right.

Example 1 Evaluate Numerical Expressions

Evaluate $100 - 3^2 \times (6 + 4) + 2$.

$100 - 3^2 \times (6 + 4) + 2$	Write the expression.
$100 - 3^2 \times (6 + 4) + 2 = 100 - 3^2 \times 10 + 2$	Simplify parentheses.
$= 100 - 9 \times 10 + 2$	Evaluate the exponent.
$= 100 - 90 + 2$	Multiply.
$= 10 + 2$	Subtract.
$= 12$	Add.

So, the value of the expression is 12.

Check

Evaluate $[23 - (8 + 2^3)] \times 2 + 10$. 24

Show your work here

Example 2 Evaluate Numerical Expressions

Evaluate $5 + \left(8^2 \div \frac{2}{5}\right) \times 2$.

$5 + \left(8^2 \div \frac{2}{5}\right) \times 2$	Write the expression.
$5 + \left(8^2 \div \frac{2}{5}\right) \times 2 = 5 + \left(64 \div \frac{2}{5}\right) \times 2$	Evaluate the exponent inside of the parentheses.
$= 5 + 160 \times 2$	Simplify parentheses.
$= 5 + 320$	Multiply.
$= 325$	Simplify.

So, the value of the expression is 325.

Check

Evaluate $8.2 \times (2^4 - 3) + 8$. 114.6

Show your work here

Go Online Y ou can complete an Extra Example online.

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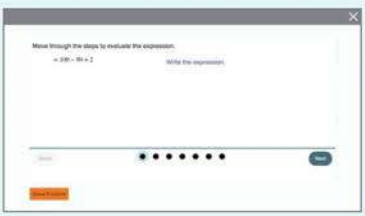
Interactive Presentation



Example 1, Evaluate Numerical Expressions, Slide 2 of 4

CLICK

On Slide 2 of Example 1, students move through the steps to evaluate the expression.

TYPE

On Slide 2 of Example 1, students determine the value of the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Evaluate Numerical Expressions

Objective

Students will evaluate numerical expressions with whole numbers.

Questions for Mathematical Discourse

SLIDE 2

AL What should you do first? Simplify the expression inside of the grouping symbols.

OL Describe the steps, in order, for how to evaluate this expression. Sample answer: Add 6 and 4. Find 3^2. Multiply 9 by 10. Then subtract 90 from 100. Finally, add 2.

OL A classmate wrote the expression as $100 - 9 \times 10 + 2$. Is this equivalent? Explain. yes; Sample answer: The classmate evaluated the power and the expression inside the parentheses.

BL Describe a mistake in evaluating the expression that might be made. Sample answer: A possible mistake is performing all of the operations in order from left to right, as in $100 - 9 = 91$; $91 \times 6 = 546$; $546 + 4 = 550$; $550 + 2 = 552$

Example 2 Evaluate Numerical Expressions

Objective

Students will evaluate numerical expressions with rational numbers.

Questions for Mathematical Discourse

SLIDE 1

AL Identify all of the operations to be performed in this expression. addition, evaluating a power, operations within the grouping symbols, division, multiplication

OL Why is evaluating the power the first thing you should do, even before the division? Sample answer: Evaluating an expression inside the parentheses comes first, and the power is inside the parentheses. So, the power must be evaluated before the division.

BL A classmate says that the parentheses in this problem are not necessary. Is this correct? Explain. yes; Sample answer: Even without the parentheses, the power will be evaluated first, followed by the division, the multiplication, and finally the addition.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Write Numerical Expressions

Objective

Students will learn how to write a numerical expression to model a real-world problem.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
As students discuss the *Talk About It!* question on Slide 2, encourage them to consider alternative ways that the expression can be written.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

How else can you represent the part of the expression written as (4×4)? Sample answer: This can also be written as a power, 4^2.

Example 3 Write and Evaluate Numerical Expressions

Objective

Students will write and evaluate a numerical expression that models a real-world problem.

Questions for Mathematical Discourse

SLIDE 2

AL What does 7.80 represent in the second part of the expression? What does the 2 represent? 7.80 represents the cost of one candle in dollars; 2 represents the fact that there are 2 candles

OL Is there another way you can write the expression representing the total cost of the lotions? Sample answer: 5×5

OL Are parentheses necessary around each part of the expression? Explain. no; Sample answer: The order of operations will indicate that the power is evaluated first and then the multiplication, so parentheses are not necessary.

BL What is another way to write the expression, without using multiplication? Sample answer: $5^2 + 7.80 + 7.80 + 2.49 + 2.49 + 2.49 + 2.49$

(continued on next page)

Learn Write Numerical Expressions

In a real-world situation where one or more operations occur, you can write an expression to represent the situation.

Suppose Mariana and her friends are buying snacks at a hockey game. **Hot dogs cost $4**, **boxes of popcorn cost $2**, and **drinks cost $2.50**. The expression below represents the total cost of **4 hot dogs**, **3 boxes of popcorn**, and **2 drinks**.

The different colored text represents each part of the expression.

hot dogs + **popcorn** + **drinks**

($4 × 4) + **($2 × 3)** + **($2.50 × 2)**

Talk About It!
How else can you represent the part of the expression written as (4×4)?

Sample answer: This can also be written as a power, 4^2.

Example 3 Write and Evaluate Numerical Expressions

Paula is shopping for the items shown in the table.

Item	lotion	candle	lip balm
Cost ($)	5.00	7.80	2.49

Write an expression to represent the total cost of 5 lotions, 2 candles, and 4 lip balms. Then find the total cost.

Part A Write an expression.

cost of lotions + cost of candles + cost of lip balms

$(5^2) + (2 \times 7.80) + (4 \times 2.49)$

Part B Find the total cost.

$(5^2) + (2 \times 7.80) + (4 \times 2.49) = 25 + 15.60 + 9.96$

$= 50.56$

So, the total cost is $ 50.56.

Talk About It!
In this situation, does the placement of the parentheses have an effect on the evaluation of the expression? Explain.

no; Sample answer: The order of operations says that exponents should be evaluated and multiplication performed before addition. In this situation, the parentheses do not affect the value of the simplified expression.

Lesson 5-2 • Numerical Expressions 271

Interactive Presentation

Example 3, Write and Evaluate Numerical Expressions, Slide 2 of 5

CLICK

On Slide 2 of Example 3, students select *Show Expressions* to view the parts of the expression.

TYPE

On Slide 3 of Example 3, students determine the total cost of the items.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Math History Minute

Mary G. Ross (1908–2008) is considered the first known Native American female mathematician and engineer . After her retirement, Ross was an advocate of women studying STEM fields (Science, T echnology, Engineering, and Mathematics). She earned a place in the Silicon Valley Engineering Council's Hall of Fame.

Check

Tickets to a play cost $10.50 for adult members of the theater, $19.95 for adult non-members, and $8 for students.

Part A

Suppose 4 non-members, 2 members, and 8 students are buying tickets for the play. Which expression could be used to find the total cost of the tickets?

- A $4(19.95 + 10.50 + 8)$
- B $(4 \times 19.95) + (2 \times 10.50) + 8^2$
- C $(2 \times 19.95) + (4 \times 10.50) + 8^2$
- D $(4 \times 19.95) + (2 \times 10.50) + 8^8$

Part B

What is the total cost of the tickets? **$164.80**

Show your work here

Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Describe some examples of when writing an expression can help you solve problems in everyday life. How does understanding the order of operations help you to evaluate those expressions?

Record your observations here

See students' observations.

Example 3 Write and Evaluate Numerical Expressions *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why is 25 written as 25.00? These amounts represent the cost in dollars and cents.

OL How can you use estimation to know if your answer is reasonable? Sample answer: The cost of 5 lotions is $25, the cost of the 2 candles is about $16, and the cost of the 4 lip balms is about $10. The total cost is about $51, which is close to my answer. So, my answer is reasonable.

BL Why are you able to use a power on the expression representing the cost of the lotions, but not for the candles or lip balm? The cost of each lotion is equal to the number of lotions purchased.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Art Supplies

Objective

Students will come up with their own strategy to solve an application problem involving art supplies.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What items might be in a kit of art supplies?
- How does the number of sketch pads in the large kit compare to the number in the medium kit?
- What operation(s) would you use to find the total number of crayons and sketch pads purchased?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Art Supplies

An art store sells different-sized art kits that include crayons and a sketch pad. The table shows the number of boxes of crayons and sketch pads in each kit. A school buys 30 small, 35 medium, and 10 large art kits. Then they return 11 medium art kits. How many boxes of crayons and sketch pads do they have in all?

Art Kit Size	Boxes of Crayons	Sketch Pads
Small	16	20
Medium	24	40
Large	68	100

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

1,736 boxes of crayons and 2,560 sketch pads; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How could you solve this problem another way?

See students' responses.

Interactive Presentation

Apply, Art Supplies

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

At a summer camp, you can buy clothing embroidered with the camp logo. Short sleeve T-shirts are $15 each, shorts are $18 each, and long sleeve T-shirts are $25 each. During the final week of summer, the clothing is marked down to $\frac{3}{4}$ the original price. You buy two short sleeve T-shirts, one pair of shorts, and three long sleeve T-shirts. What was the total cost of the purchase? $92.25

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Where did you encounter difficulty in this lesson, and how did you deal with it? Write down any questions you still have.

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. What is the total cost for 3 adults and 8 students? $112

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 13, 15–19
- Extension: Variables and Absolute Value
- ALEKS Exponents and Order of Operations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–12, 15–17
- Extension: Variables and Absolute Value
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Exponents and Order of Operations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Exponents and Order of Operations

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	evaluate numerical expressions with whole numbers	1–6
1	evaluate numerical expressions with rational numbers	7–10
1	write and evaluate numerical expressions that model real-world problems	11, 12
2	extend concepts learned in class to apply them in new contexts	13
3	solve application problems that involve numerical expressions	14, 15
3	higher-order and critical thinking skills	16–19

Common Misconception

Some students may incorrectly evaluate numerical expressions by not following the order of operations. Have students review the order of operations before evaluating each expression. Some students may find it beneficial to check off each step of the order of operations while evaluating.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Evaluate each expression. (Examples 1 and 2)

1. $64 \div (15 - 7) \times 2 - 9$ **7**

2. $9 + 8 \times 3 - (5 \times 2)$ **23**

3. $4 \times (5^2 - 12) - 6$ **46**

4. $78 - 2^4 \div (14 - 6) \times 2$ **74**

5. $9 + 7 \times (15 + 3) \div 3^2$ **23**

6. $13 + (4^3 \div 2) \times 5 - 17$ **156**

7. $4 + \left(6^2 \div \frac{1}{4}\right) \times 3$ **436**

8. $12 + \left(2^3 \div \frac{2}{3}\right) - 2$ **22**

9. $36 \div \left(3^2 \div \frac{3}{4}\right) - 2.4$ **$\frac{3}{5}$ or 0.6**

10. $80 \div \left(4^2 \div \frac{2}{5}\right) + 3.75$ **$5\frac{3}{4}$ or 5.75**

11. Mei is shopping for the items shown in the table. Write an expression to represent the total cost of 6 bubbles, 2 beach balls, and 3 sand buckets. Then find the total cost. (Example 3)

Item	bubbles	beach ball	sand buckets
Cost	$1.49	$2.00	$3.50

Sample expression:
$(6 \times 1.49) + (2^2) + (3 \times 3.50)$; $23.44

12. Roy and 2 friends are at a game center. Each person buys a hot dog for $3, fries for $2.49, and a drink for $2.50. They also have a coupon for $1 off each drink. Write an expression to represent the total cost. Then find the total cost. (Example 3)

Sample expression:
$3^2 + (3 \times 2.49) + 3 \times (2.50 - 1)$; $20.97

Test Practice

13. Multiselect Alice takes an art class after school once a week. At each class, she buys a bottle of juice for $1.25 and a bag of pretzels for $0.85. Select all expressions that represent the amount of money, in dollars, Alice spends after attending 8 art classes.

- [ ] 8(1.25)(0.85)
- [x] 8(1.25 + 0.85)
- [x] 8(1.25) + 8(0.85)
- [ ] 8 + 1.25 + 0.85
- [ ] (8 + 1.25) + (8 + 0.85)

Apply *indicates multi-step problem

*14. An art teacher is ordering colored pencils for the new school year. The table shows the number of colored pencils per box size. The teacher buys 24 small boxes, 12 medium boxes, and 5 large boxes. She had to return 3 small boxes due to defects. How many colored pencils does the teacher have in all?

Box Size	Number of Colored Pencils
Small	12
Medium	64
Large	100

1,520 colored pencils

*15. A bakery sells boxes of muffins in the sizes shown in the table. On Monday, the bakery sold 15 minis, 8 dozens, and 6 jumbos. However, 6 of the minis sold were free with a coupon. How many total muffins were paid for on Monday?

Size	Number of Muffins
Mini	6
Dozen	12
Jumbo	24

294 muffins

Higher-Order Thinking Problems

16. **MP Persevere with Problems** Refer to the expression $2 + 6 \div 2 + 4 \times 3$.

a. Place parentheses in the expression so that the value of the expression is 16.

$(2 + 6) \div 2 + 4 \times 3$

b. Place parentheses in the expression so that the value is *not* equal to 16. Then find the value of the new expression.

Sample answer: $2 + (6 \div 2) + 4 \times 3$; 17

17. **MP Find the Error** A student is evaluating the expression $42 + 6 \div 2$. Find the student's mistake and correct it.

$$42 + 6 \div 2 = 48 \div 2$$
$$= 24$$

Sample answer: The student did not follow the order of operations. The student added first before dividing. The division should have been performed first.
$42 + 6 \div 2 = 42 + 3$ **or 45**

18. Write an expression that contains parentheses, 5 numbers, two different operations, and has a value of 20.

Sample answer: $5 \times (4^2 \div 2) - \left(40 \times \frac{1}{2}\right)$

19. **Create** Write about a real-world situation that could be represented by a numerical expression. Then write and evaluate the expression.

Sample answer: Frankie and his two sisters each order a hamburger , a fruit cup, and a bottled water for lunch. A hamburger costs $3, a fruit cup cost $0.75, and a bottled water costs $1.25.; 3 + (3 × 0.75)+ (3 × 1.25); $15

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 16, students will place parentheses in the expression so that the value of the expression is 16 and then another number. Encourage students to place the parentheses strategically throughout the expression.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students will find the error in the evaluated expression. Encourage students to identify the error and then construct an explanation that fixes the error.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 18, students will write an expression with a value of 20. Encourage students to identify what the problem is asking and to construct the expression with all of these components.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 14 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 17 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that you should add before dividing. Have each pair or group of students present their explanations to the class.

Learn Structure of Algebraic Expressions

Objective

Students will learn about the structure of an algebraic expression and how to identify its parts.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to analyze the structure of each term and note that in the first term, x is squared, and in the second term, y is squared. Since each term has a different exponent of both x any y, they are not like terms.

Teaching Notes

SLIDE 1

Students will learn the definition of and various parts of an *algebraic expression*, as well as different ways of writing multiplication and division.

Talk About It!

SLIDE 1

Mathematical Discourse

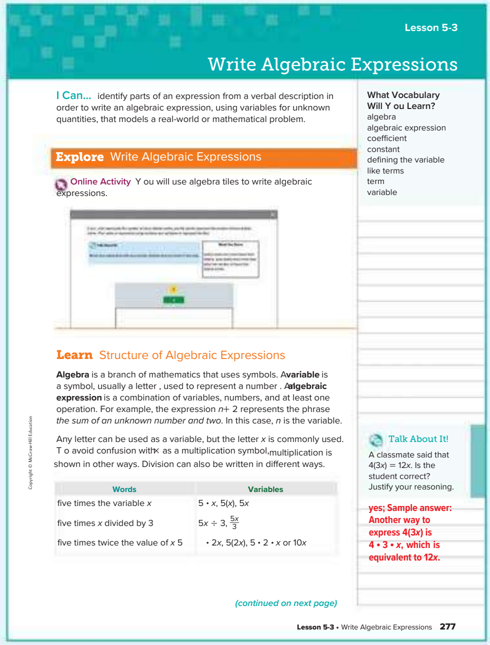
A classmate said that $4(3x) = 12x$. Is the student correct? Justify your reasoning. yes; Sample answer: Another way to express $4(3x)$ is $4 \cdot 3 \cdot x$, which is equivalent to $12x$.

(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

If any of your students have difficulty writing algebraic expressions, encourage them to mimic the sample expression, *5 times the variable x*, using different numbers. Students can use the structure of the given sample to write the different algebraic expressions. Allow students to quickly share their expressions with a classmate. Sample expressions could include $6x$, $11x$, $\frac{1}{3}x$, and so on.



Lesson 5-3

Write Algebraic Expressions

I Can... identify parts of an expression from a verbal description in order to write an algebraic expression, using variables for unknown quantities, that models a real-world or mathematical problem.

What Vocabulary Will Y ou Learn?
algebra
algebraic expression
coefficient
constant
defining the variable
like terms
term
variable

Explore Write Algebraic Expressions

Online Activity Y ou will use algebra tiles to write algebraic expressions.

Learn Structure of Algebraic Expressions

Algebra is a branch of mathematics that uses symbols. A **variable** is a symbol, usually a letter , used to represent a number . An **algebraic expression** is a combination of variables, numbers, and at least one operation. For example, the expression $n + 2$ represents the phrase *the sum of an unknown number and two*. In this case, n is the variable.

Any letter can be used as a variable, but the letter x is commonly used. T o avoid confusion with × as a multiplication symbol, multiplication is shown in other ways. Division can also be written in different ways.

Words	Variables
five times the variable x	$5 \cdot x$, $5(x)$, $5x$
five times x divided by 3	$5x \div 3$, $\frac{5x}{3}$
five times twice the value of x	$5 \cdot 2x$, $5(2x)$, $5 \cdot 2 \cdot x$ or $10x$

Talk About It! A classmate said that $4(3x) = 12x$. Is the student correct? Justify your reasoning.

yes; Sample answer: Another way to express $4(3x)$ is $4 \cdot 3 \cdot x$, which is equivalent to $12x$.

(continued on next page)

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Lesson 5-3 • Write Algebraic Expressions 277

Interactive Presentation

Learn, Structure of Algebraic Expressions, Slide 1 of 3

Lesson 5-3

Write Algebraic Expressions

6.EE.A.2, 6.EE.B.6

LESSON GOAL

Students will write algebraic expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Write Algebraic Expressions

Learn: Structure of Algebraic Expressions
Example 1: Identify Parts of Algebraic Expressions
Learn: Write One-Step Algebraic Expressions
Example 2: Write One-Step Algebraic Expressions
Example 3: Write One-Step Algebraic Expressions
Learn: Write Two-Step Algebraic Expressions
Example 4: Write Two-Step Algebraic Expressions
Example 5: Write Algebraic Expressions

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

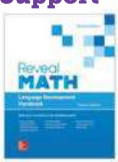
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 29 of the *Language Development Handbook* to help your students build mathematical language related to writing algebraic expressions.



ELL You can use the tips and suggestions on page T29 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major clusters **6.EE.A** and **6.EE.B** by writing algebraic expressions.
Standards for Mathematical Content: **6.EE.A.2, 6.EE.A.2.A, 6.EE.A.2.B, 6.EE.B.6**
Standards for Mathematical Practice: **MP1, MP2,MP3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students wrote and evaluated numerical expressions.
6.EE.A.1

Now
Students write algebraic expressions.
6.EE.A.2, 6.EE.A.2.A, 6.EE.A.2.B, 6.EE.B.6

Next
Students will evaluate algebraic expressions.
6.EE.A.2.C, 6.EE.B.6

Rigor

The Three Pillars of Rigor

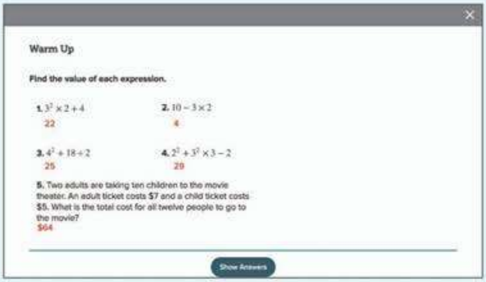
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of numerical expressions as they develop *understanding* of writing algebraic expressions. They come to understand the importance of defining the variable, as they build *fluency* with writing one and two-step algebraic expressions involving the four basic operations.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- evaluating numerical expressions (Exercises 1–5)

Answers

1. 22
2. 4
3. 25
4. 29
5. $64

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about variables, using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- *Algebra* is defined as a mathematical language of symbols including variables. How do you think you will use *algebra* as you progress through the lesson? Sample answer: I may use symbols (variables) to represent numbers or unknown information in an expression.
- Using the definition of a numerical expression that you previously learned, what do you think is an *algebraic expression*? Sample answer: numbers and variables that are combined with operations
- The term *efficient* comes from a Latin term meaning *to accomplish*. What does the prefix *co-* mean, and what might that mean for the term *coefficient?* Sample answer: *co-* means *with,* so *coefficient* might mean *to accomplish with.*
- What does it mean to drive at a *constant* speed? Sample answer: It means that the speed does not change.

Explore Write Algebraic Expressions

Objective

Students will use algebra tiles to explore writing algebraic expressions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with information about the number of hours worked by three people (Jose, Valerie, and Leticia). Throughout the activity, students will use algebra tiles to represent the hours of work for each of the three individuals. They will use their algebra tile representation to find an algebraic expression representing the total number of hours worked. The goal is to understand that expressions involving variables and numbers can be used to represent real-world situations.

Inquiry Question

How can you represent situations using symbols? Sample answer: I can use symbols to represent unknown values and operations to model the situation in a word problem.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

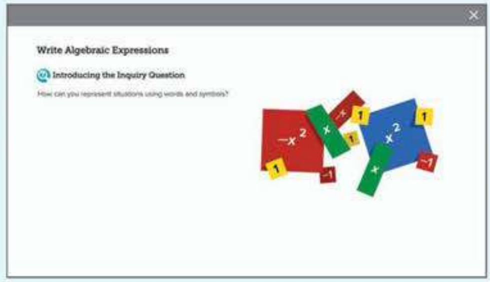
SLIDE 3

Mathematical Discourse

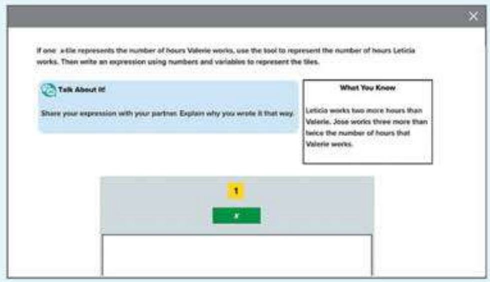
Which tile would you use to represent the number of hours that Valerie works? Explain your reasoning. Sample answer: I would use the x-tile to represent the number of hours worked. The number of hours that Valerie worked is unknown, so it should be represented by a variable.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8



Explore, Slide 4 of 8

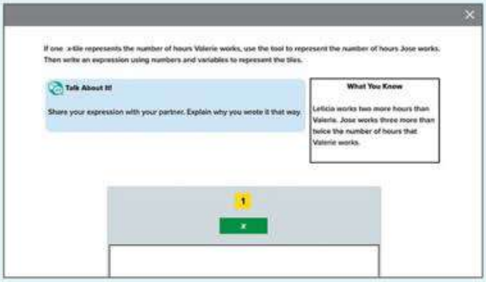
WATCH

On Slide 2, students watch a video that demonstrates how to use algebra tiles to represent algebraic expressions.

DRAG & DROP

On Slide 4, students drag algebra tiles to model a real-world scenario with an algebraic expression.

Interactive Presentation



Explore, Slide 5 of 8

DRAG & DROP

On Slide 5 and 6, students drag algebra tiles to model real-world scenarios.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Write Algebraic Expressions *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to identify the important quantities in the real-world problem and decontextualize them by representing them with algebra tiles.

5 Use Appropriate Tools Strategically Students will use the algebra tiles to represent the number of hours each person works.

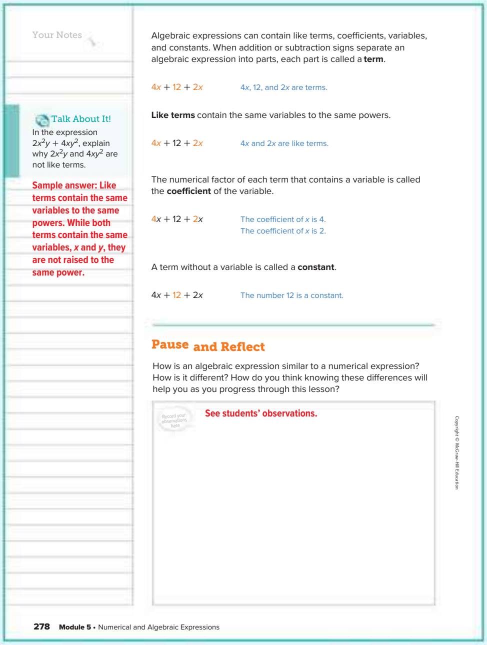
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Share your expression with your partner. Explain why you wrote it that way. Sample answer: I wrote the expression $2x + 3$ because Jose worked three more hours than twice the amount than Valerie who worked x hours.



Your Notes

Talk About It!
In the expression $2x^2y + 4xy^2$, explain why $2x^2y$ and $4xy^2$ are not like terms.

Sample answer: Like terms contain the same variables to the same powers. While both terms contain the same variables, *x* and *y*, they are not raised to the same power.

Algebraic expressions can contain like terms, coefficients, variables, and constants. When addition or subtraction signs separate an algebraic expression into parts, each part is called a **term**.

$4x + 12 + 2x$ — $4x$, 12, and $2x$ are terms.

Like terms contain the same variables to the same powers.

$4x + 12 + 2x$ — $4x$ and $2x$ are like terms.

The numerical factor of each term that contains a variable is called the **coefficient** of the variable.

$4x + 12 + 2x$ — The coefficient of *x* is 4. The coefficient of *x* is 2.

A term without a variable is called a **constant**.

$4x + 12 + 2x$ — The number 12 is a constant.

Pause and Reflect

How is an algebraic expression similar to a numerical expression? How is it different? How do you think knowing these differences will help you as you progress through this lesson?

Record your observations here

See students' observations.

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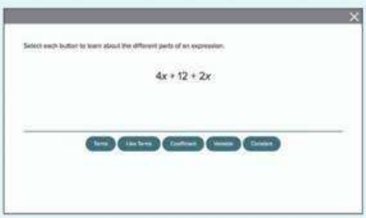
Interactive Presentation



Learn, Structure of Algebraic Expressions, Slide 2 of 3

CLICK

On Slide 2, students select each button to view the different parts of an algebraic expression.

Learn Structure of Algebraic Expressions *(continued)*

Teaching Notes

SLIDE 2

You may wish to have student volunteers come up to the board to select each button to reveal its meaning. Ask students to identify the difference between the vocabulary terms. For example, ask students to explain the difference between a coefficient and a constant, or the difference between a term and a variable.

Talk About It!

SLIDE 3

Mathematical Discourse

In the expression $2x^2y + 4xy^2$, explain why $2x^2y$ and $4xy^2$ are not like terms. Sample answer: Like terms contain the same variables to the same powers. While both terms contain the same variables, *x* and *y*, they are not raised to the same power.

DIFFERENTIATE

Language Development Activity ELL

To further students' understanding of the parts of an expression, have them create their own algebraic expressions that satisfy the given conditions.

- The expression should have at least four terms.
- At least two of the terms should be like terms.
- There should be at least two coefficients.
- There should be at least one constant.

Then have them trade expressions with another pair of students. Each pair should identify the terms, like terms, coefficients, variables, and constants in each other's expressions. Have pairs check each other's work.

Example 1 Identify Parts of Algebraic Expressions

Objective

Students will identify the parts of an algebraic expression.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* questions on Slide 3, encourage them to make sense of the terms in the expression in order to determine whether there are any constants in the expression that are like terms. Students should be able to use reasoning to determine that the coefficient of a term such as n is 1.

6 Attend to Precision Students should use the definitions of each term as they identify the corresponding parts of the expression.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the expression in order to determine the terms, like terms, coefficients, and constants.

Questions for Mathematical Discourse

SLIDE 2

AL What operations separate the terms? addition

OL Why doesn't multiplication separate the terms? Sample answer: The term $6n$ is one term, even though it is a product of 6 and n.

OL Why isn't 4 one of the like terms? Sample answer: 4 is the only constant and does not have the same variable part as the other like terms.

BL A classmate says that 7 is also a constant just like 4. Why is this not correct? Sample answer: Since 7 immediately precedes a variable n, as in $7n$, it is not a constant. It is a coefficient.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 1 Identify Parts of Algebraic Expressions

Identify the terms, like terms, coefficients, and constants in the expression $6n + 7n + 4 + 2n$.

Terms are parts of the expression that are separated by addition and subtraction, so the terms are:

$6n$, $7n$, 4, $2n$

Circle the like terms above.

Write the coefficients of the terms and the constants in the appropriate bin.

Coefficients	Constants
6 7 2	4

So, the terms are $6n$, $7n$, 4, and $2n$.

The like terms are $6n$, $7n$, and $2n$.

The coefficients are 6, 7, and 2.

The constant is 4.

Think About It! How would you begin identifying the parts of the expression? See students' responses.

Talk About It! Are there any like terms in this expression that are constants? Explain. no; Sample answer: There is only one constant, 4, in the expression.

Talk About It! Suppose that an additional term, n, is added to the end of the expression. What is the coefficient for n? Sample answer: The coefficient for the additional term n is 1, because n can be rewritten as $1n$.

Check

Identify the *terms*, *like terms*, *coefficients*, and *constants* in the expression $3x + 2 + 10 + 4x$.

terms: $3x$, 2, 10, $4x$

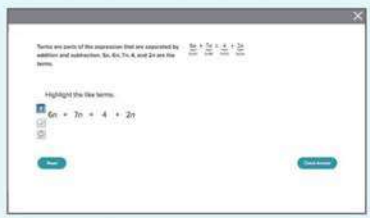
like terms: $3x$ and $4x$, 2 and 10

coefficients: 3, 4

constants: 2, 10

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Identify Parts of Algebraic Expressions, Slide 2 of 4

CLICK

On Slide 2, students highlight the like terms in an expression.

DRAG & DROP

On Slide 2, students drag the terms to the appropriate bin to identify them as a coefficient or a constant.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Write One-Step Algebraic Expressions

T o write verbal phrases as algebraic expressions, use the table below. When **defining the variable**, choose a variable and decide what it represents.

Words
Describe the mathematics of the problem.
Variable
Define a variable to represent the unknown quantity.
Expression
Translate the words into an algebraic expression.

In order to translate a situation into an expression, it is important to correctly identify operations that are described in words.

Write each phrase below the operation that it describes.

the product of — increased by — less than a number — the quotient of — the sum of

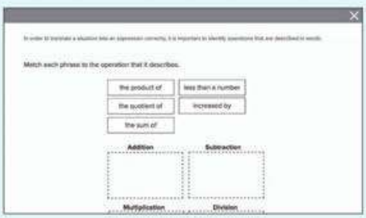
Addition	Subtraction
increased by the sum of	less than a number

Multiplication	Division
the product of	the quotient of

Talk About It!
Make a list of additional phrases that could be represented by mathematical operations. Share your list and explain how those phrases represent that operation.

See students' responses.

Interactive Presentation



Learn, Write One-Step Algebraic Expressions, Slide 2 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view the steps for writing algebraic expressions.

DRAG & DROP

On Slide 2, students match each phrase to the operation that it describes.

Learn Write One-Step Algebraic Expressions

Objective

Students will learn how to write one-step algebraic expressions.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to clearly explain why their chosen phrases represent certain operations.

Teaching Notes

SLIDE 1

Be sure students understand the importance of defining the variable when writing a verbal phrase as an algebraic expression. You may wish to have students create their own verbal expressions that involve one operation, such as *five degrees warmer than yesterday's temperature*. Have students choose a variable, such as x or t, and clearly explain what that variable represents (yesterday's temperature). Then have them write an expression to represent the verbal phrase, such as $x + 5$, $t + 5$, $5 + x$, or $5 + t$.

SLIDE 2

Students will learn some common phrases that describe each of the four operations. You may wish to have student volunteers come up to the board to drag each phrase to its appropriate bin. Ask students to identify the key word in each phrase that helps them match it to the appropriate operation.

Talk About It!

SLIDE 3

Mathematical Discourse

Make a list of additional phrases that could be represented by mathematical operations. Share your list and explain how those phrases represent that operation. See students' responses.

Example 2 Write One-Step Algebraic Expressions

Objective

Students will write one-step algebraic expressions involving addition or subtraction.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the given real-world phrase by representing the quantities symbolically with the correct algebraic expression. Students should make sure to define the variable before writing the expression.

Questions for Mathematical Discourse

SLIDE 1

AL What is the unknown quantity? The amount Anthony earned is unknown.

OL Why is the expression an addition expression? The phrase "more than" corresponds to addition.

OL Why is it important to define the variable? Sample answer: It is important because you need to know what the variables represent.

BL A classmate says that the expression should be $10d$. Why is this incorrect, and what would this expression represent? Sample answer: This expression would represent 10 times more than Anthony earned, not $10 more than Anthony earned.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Write One-Step Algebraic Expressions

Define a variable to represent the unknown in the phrase *ten dollars more than Anthony earned*. Then write the phase as an algebraic expression.

Words
ten dollars more than Anthony earned
Variable
Let d represent the number of dollars Anthony earned.
Expression
$d + 10$

So, the expression $d + 10$ can be used to model the phrase *ten dollars more than Anthony earned.*

Check

Define a variable to represent the unknown in the phrase *twelve dollars less than the original price*. Then write the phrase as an algebraic expression.

Sample answer:
Let p represent the original price; $p - 12$

Go Online You can complete an Extra Example online.

Pause and Reflect

Why is it important to define the variable when writing an algebraic expression? What possible errors might be made if the variable is not correctly defined?

Record your observations here

See students' observations.

Interactive Presentation

Example 2, Write One-Step Algebraic Expressions, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view the steps for writing the algebraic expression.

TYPE

On Slide 1, students write the algebraic expression.

CLICK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Write One-Step Algebraic Expressions

Define a variable to represent the unknown in the phrase *four and one-half times the number of gallons*. Then write the phrase as an algebraic expression.

Words
four and one-half times the number of gallons
Variable
Let g represent the number of gallons.
Expression
$4\frac{1}{2}g$ or $4.5g$

So, the expression $4\frac{1}{2}g$ or $4.5g$ can be written to model the phrase *four and one-half times the number of gallons*.

Talk About It!
The expression $4\frac{1}{2}g$ can also be written as $4.5g$. What is another way that the expression could be written and still be equivalent to the original expression?

Sample answer: $\frac{9}{2}g$

Check

Define a variable to represent the unknown in the phrase *six times more money than Eliot saved*. Then write the phrase as an algebraic expression.

Sample answer: Let m represent the amount of money Eliot saved; $6m$

Go Online You can complete an Extra Example online.

Learn Write Two-Step Algebraic Expressions

Two-step expressions contain two different operations. The table shows how to translate a verbal phrase into an algebraic expression.

Words
Describe the mathematics of the problem.
Variable
Define a variable to represent the unknown quantity.
Expression
Translate the words into an algebraic expression.

Talk About It!
How can you write an algebraic expression for the phrase *two more than three times a number*?

Sample answer: The expression $3x + 2$ can represent the phrase *two more than three times a number* algebraically.

Interactive Presentation

Example 3, Write One-Step Algebraic Expressions, Slide 2 of 4

FLASHCARDS

On Slide 2 of Example 3, students use Flashcards to view the steps for writing the algebraic expression.

TYPE

On Slide 2 of Example 3, students write the algebraic expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Write One-Step Algebraic Expressions

Objective

Students will write one-step algebraic expressions involving multiplication or division.

Questions for Mathematical Discourse

SLIDE 2

AL What is the operation in this problem? multiplication

AL What number is represented by *four and one-half*? the mixed number $4\frac{1}{2}$ or the decimal 4.5

OL Can you use any letter to represent the number of gallons? Explain. yes; Sample answer: As long as you define the variable, you can use any letter you want.

BL A classmate says that $4g + \frac{1}{2}g$ is also a correct expression. Is this correct? yes; Sample answer: $4\frac{1}{2}$ gallons is equivalent to 4 gallons plus $\frac{1}{2}$ of a gallon.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Write Two-Step Algebraic Expressions

Objective

Students will learn how to write two-step algebraic expressions.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

How can you write an algebraic expression for the phrase *two more than three times a number*? Sample answer: The expression $3x + 2$ can represent the phrase *two more than three times a number* algebraically.

Example 4 Write Two-Step Algebraic Expressions

Objective

Students will write two-step algebraic expressions.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to create a plausible argument for why the two expressions, $3p - 5$ and $5 - 3p$, are not equivalent.

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the given real-world phrase by representing the quantities symbolically with the correct algebraic expression. Students should make sure to define the variable before writing the expression.

Questions for Mathematical Discourse

SLIDE 2

AL What operation corresponds to *less than*? subtraction

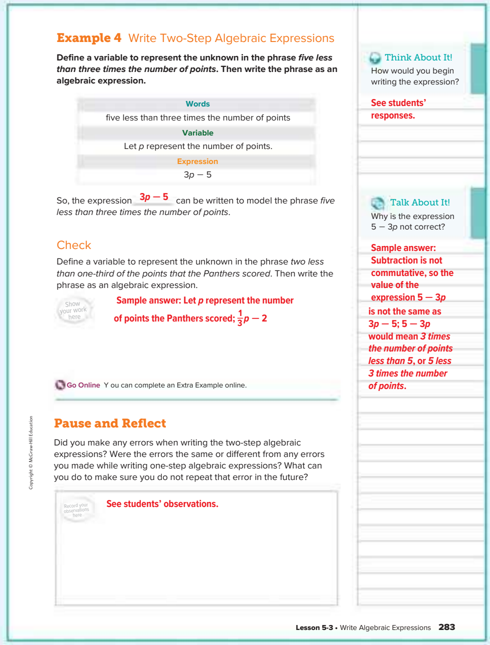
OL How do you know there are two operations in this expression? Sample answer: *Less than* corresponds to subtraction, but *times* corresponds to multiplication. These are the two operations.

OL How do you know that 5 is subtracted from $3p$, and not the other way around? Sample answer: The phrase says 5 less than a certain quantity, $3p$. This means that the quantity, $3p$, is the greater quantity.

BL How would you describe the expression $3(x - 5)$? Sample answer: *Three times the quantity five less than the number of points.*

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Write Two-Step Algebraic Expressions

Define a variable to represent the unknown in the phrase *five less than three times the number of points*. Then write the phrase as an algebraic expression.

Words
five less than three times the number of points
Variable
Let p represent the number of points.
Expression
$3p - 5$

So, the expression $3p - 5$ can be written to model the phrase *five less than three times the number of points.*

Think About It! How would you begin writing the expression? See students' responses.

Talk About It! Why is the expression $5 - 3p$ not correct? Sample answer: Subtraction is not commutative, so the value of the expression $5 - 3p$ is not the same as $3p - 5$; $5 - 3p$ would mean *3 times the number of points less than 5*, or *5 less 3 times the number of points.*

Check

Define a variable to represent the unknown in the phrase *two less than one-third of the points that the Panthers scored.* Then write the phrase as an algebraic expression.

Show your work here

Sample answer: Let p represent the number of points the Panthers scored; $\frac{1}{3}p - 2$

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you make any errors when writing the two-step algebraic expressions? Were the errors the same or different from any errors you made while writing one-step algebraic expressions? What can you do to make sure you do not repeat that error in the future?

Record your observations here

See students' observations.

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Interactive Presentation

Example 4, Write Two-Step Algebraic Expressions, Slide 2 of 4

FLASHCARDS

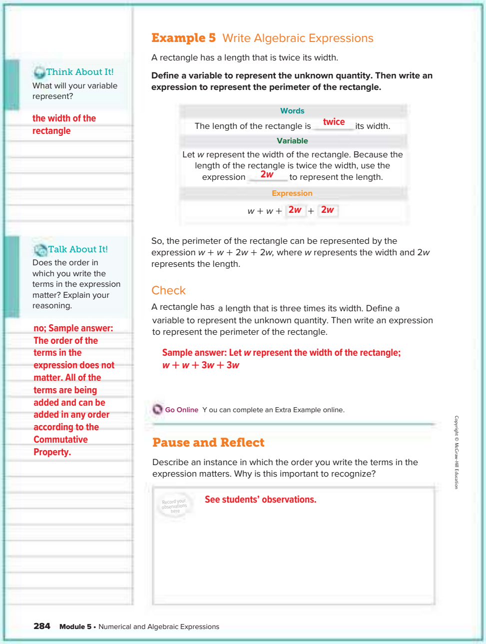
On Slide 2, students use Flashcards to view the steps for writing the algebraic expression.

TYPE

On Slide 2, students write the algebraic expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 5 Write Algebraic Expressions

A rectangle has a length that is twice its width.

Define a variable to represent the unknown quantity. Then write an expression to represent the perimeter of the rectangle.

Words

The length of the rectangle is **twice** its width.

Variable

Let w represent the width of the rectangle. Because the length of the rectangle is twice the width, use the expression **$2w$** to represent the length.

Expression

$w + w +$ **$2w$** $+$ **$2w$**

So, the perimeter of the rectangle can be represented by the expression $w + w + 2w + 2w$, where w represents the width and $2w$ represents the length.

Think About It! What will your variable represent? **the width of the rectangle**

Talk About It! Does the order in which you write the terms in the expression matter? Explain your reasoning. **no; Sample answer: The order of the terms in the expression does not matter. All of the terms are being added and can be added in any order according to the Commutative Property.**

Check

A rectangle has a length that is three times its width. Define a variable to represent the unknown quantity. Then write an expression to represent the perimeter of the rectangle.

Sample answer: Let w represent the width of the rectangle; $w + w + 3w + 3w$

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe an instance in which the order you write the terms in the expression matters. Why is this important to recognize?

See students' observations.

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Interactive Presentation

Example 5, Write Algebraic Expressions, Slide 2 of 4

FLASHCARDS

On Slide 2, students use Flashcards to view the steps for writing the algebraic expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Write Algebraic Expressions

Objective

Students will write algebraic expressions to represent the perimeter of a geometric figure.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the given real-world phrase by representing the quantities symbolically with a correct algebraic expression. Students should make sure to define the variable before writing the expression.

6 Attend to Precision While discussing the *Talk About It!* question on Slide 3, encourage students to use clear and precise mathematical language, such as the *Commutative Property*, when explaining their reasoning.

Questions for Mathematical Discourse

SLIDE 2

AL What does it mean for the length to be *twice* the width? Sample answer: The length is equal to the width multiplied by 2.

OL At this point in the problem, why does it seem like there are two variables? Sample answer: There are two unknowns: length and width.

BL How do you know whether the length or the width is greater? Sample answer: Since the length is twice the width, the length must be greater than the width.

BL Could you have written the length as ℓ and then written the width in terms of the length? Explain. yes; Sample answer: If the length is ℓ, then the width is half the length, or $\frac{1}{2}\ell$.

Go Online

- Find additional teaching notes, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Essential Question Follow-Up

How can we communicate algebraic relationships with mathematical symbols? In this lesson, students learned how to identify parts of algebraic expressions and write one- and two-step algebraic expressions from verbal descriptions. Encourage them to discuss with a partner the benefits of representing a verbal description as an algebraic expression. Some students may say that the algebraic expression is a succinct representation of the description, without using words.

Exit Ticket

Refer to the Exit Ticket slide. Represent the phrase *9 more than 5 times a number* with an algebraic expression. $5x + 9$

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

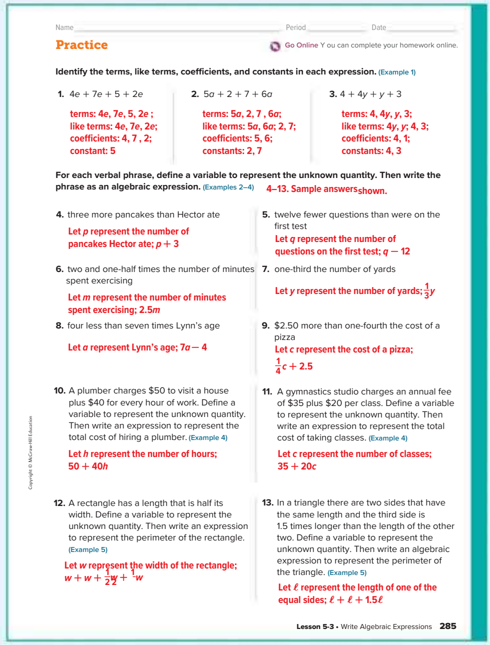
The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- AL Practice Form B
- OL Practice Form A
- BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify the parts of algebraic expressions	1–3
1	write one-step algebraic expressions involving addition or subtraction	4, 5
1	write one-step algebraic expressions involving multiplication or division	6, 7
1	write two-step algebraic expressions	8–11
1	write two-step algebraic expressions to represent the perimeter of a geometric figure	12, 13
2	extend concepts learned in class to apply them in new contexts	14
3	higher-order and critical thinking skills	15–18



Name Period Date

Practice

Go Online You can complete your homework online.

Identify the terms, like terms, coefficients, and constants in each expression. (Example 1)

1. $4e + 7e + 5 + 2e$
terms: $4e$, $7e$, 5, $2e$; like terms: $4e$, $7e$, $2e$; coefficients: 4, 7, 2; constant: 5

2. $5a + 2 + 7 + 6a$
terms: $5a$, 2, 7, $6a$; like terms: $5a$, $6a$; 2, 7; coefficients: 5, 6; constants: 2, 7

3. $4 + 4y + y + 3$
terms: 4, $4y$, y, 3; like terms: $4y$, y; 4, 3; coefficients: 4, 1; constants: 4, 3

For each verbal phrase, define a variable to represent the unknown quantity. Then write the phrase as an algebraic expression. (Examples 2–4) 4–13. Sample answers shown.

4. three more pancakes than Hector ate
Let p represent the number of pancakes Hector ate; $p + 3$

5. twelve fewer questions than were on the first test
Let q represent the number of questions on the first test; $q - 12$

6. two and one-half times the number of minutes spent exercising
Let m represent the number of minutes spent exercising; $2.5m$

7. one-third the number of yards
Let y represent the number of yards; $\frac{1}{3}y$

8. four less than seven times Lynn's age
Let a represent Lynn's age; $7a - 4$

9. \$2.50 more than one-fourth the cost of a pizza
Let c represent the cost of a pizza; $\frac{1}{4}c + 2.5$

10. A plumber charges \$50 to visit a house plus \$40 for every hour of work. Define a variable to represent the unknown quantity. Then write an expression to represent the total cost of hiring a plumber. (Example 4)
Let h represent the number of hours; $50 + 40h$

11. A gymnastics studio charges an annual fee of \$35 plus \$20 per class. Define a variable to represent the unknown quantity. Then write an expression to represent the total cost of taking classes. (Example 4)
Let c represent the number of classes; $35 + 20c$

12. A rectangle has a length that is half its width. Define a variable to represent the unknown quantity. Then write an expression to represent the perimeter of the rectangle. (Example 5)
Let w represent the width of the rectangle; $w + w + \frac{1}{2}w + \frac{1}{2}w$

13. In a triangle there are two sides that have the same length and the third side is 1.5 times longer than the length of the other two. Define a variable to represent the unknown quantity. Then write an algebraic expression to represent the perimeter of the triangle. (Example 5)
Let ℓ represent the length of one of the equal sides; $\ell + \ell + 1.5\ell$

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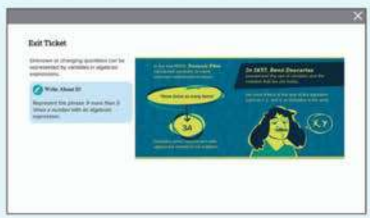
Lesson 5-3 • Write Algebraic Expressions 285

Interactive Presentation



Exit Ticket

T est Practice

14. Open Response Nate scored 5 more than twice the number of points as Jake scored. Write an expression that represents the relationship of the number of points Nate scored in terms of the number of points Jake scored, *p*.

$2p + 5$

Higher-Order Thinking Problems

15. MP **Identify Structure** Write an expression that has four terms and at least one constant. Identify the like terms, coefficients, and constants in your expression.

Sample answer: $2x + 8 + x + 6$; like terms: $2x$, x; 8, 6; coefficients: 2, 1; constants: 8, 6

16. If *x* represents the number of questions on a test, analyze the meaning of each expression: $x + 4$, $x - 5$, $2x$, and $x \div 3$.

Sample answer: 4 more than the number of questions, 5 fewer than the number of questions, 2 times the number of questions, and one third the number of questions

17. MP **Persevere with Problems** Norman earns $8 for every dog he washes plus 25% of the cost of the dog wash. Write an expression that represents the total amount of money Norman earns for one dog wash with a cost, *c*.

$8 + 0.25c$

18. Create Write about a real-world situation that can be represented with an algebraic expression. Then represent the situation with the expression.

Sample answer: The English class has half as many students as the math class. Let *s* represent the unknown value.; $s \div 2$

Teaching the Mathematical Practices

7 Look For and Make Use of Structure In Exercise 15, students will write an expression using the guidelines and then identify the like terms, coefficients, and constants.

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 17, students will write an expression that represents the total amount of money Norman earns for one dog wash. Encourage students to identify the important information in the problem and identify what they are being asked to do.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Clearly explain your strategy.

Use with Exercise 17 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would write the expression, without actually writing it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 13, 15–18
- ALEKS Evaluating and Writing Expressions

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–11, 15, 16
- Personal Tutor
- Extra Examples 1–5
- ALEKS Exponents and Order of Operations

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- ArriveMATH Take Another Look
- ALEKS Exponents and Order of Operations

Learn Evaluate Algebraic Expressions

Objective

Students will learn how to evaluate algebraic expressions.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use clear and precise mathematical language, such as *evaluate*, *variables*, and the value of *y*, to explain why the expression cannot be evaluated without further information.

Teaching Notes

SLIDE 1

Students will learn how to evaluate an algebraic expression given the value for the variable. Ask students to explain why two quantities that are equal can be substituted for one another in an expression without changing the value of the expression.

Talk About It!

SLIDE 2

Mathematical Discourse

Can you evaluate the expression $2x + 5y - 1$ if you know that $x = 3$? Explain your reasoning. no; Sample answer: The expression contains two variables, so it cannot be evaluated completely. In order to evaluate the expression, I would need to know the value of *y*.

DIFFERENTIATE

Reteaching Activity AL

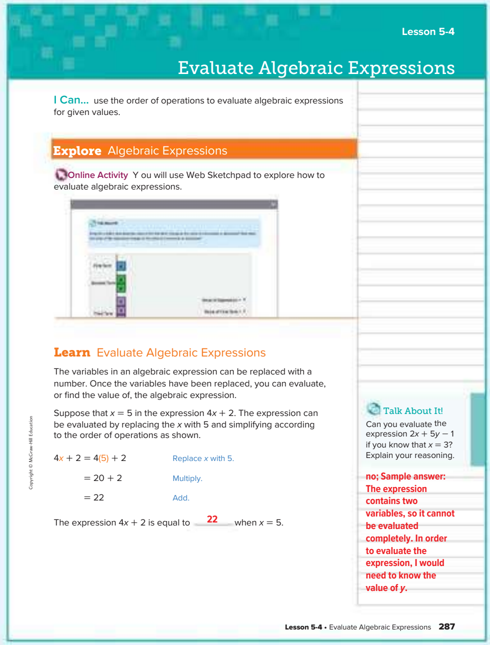
If any of your students have difficulty evaluating algebraic expressions, use the following activity to remind them of the order of operations. Have them work with a partner to simplify each of the following numerical expressions. For each expression, have them explain which operations they should perform first and why. Then have them rewrite each of the original expressions by replacing the number 2 with the variable *x*. For example, the first expression would be written as $3 - x$. Have them explain if replacing the number 2 with a variable changes the order of how they would evaluate the expressions and explain why or why not.

$3 - 1 \cdot 2$ 1

$5 + 4 \div 2$ 7

$10 - 2 + 5 + 4 \cdot 2$ 21

$24 - 4 \cdot 2$ 16


Lesson 5-4

Evaluate Algebraic Expressions

I Can... use the order of operations to evaluate algebraic expressions for given values.

Explore Algebraic Expressions

Online Activity You will use Web Sketchpad to explore how to evaluate algebraic expressions.

Learn Evaluate Algebraic Expressions

The variables in an algebraic expression can be replaced with a number. Once the variables have been replaced, you can evaluate, or find the value of, the algebraic expression.

Suppose that $x = 5$ in the expression $4x + 2$. The expression can be evaluated by replacing the x with 5 and simplifying according to the order of operations as shown.

$4x + 2 = 4(5) + 2$ Replace *x* with 5.

$= 20 + 2$ Multiply.

$= 22$ Add.

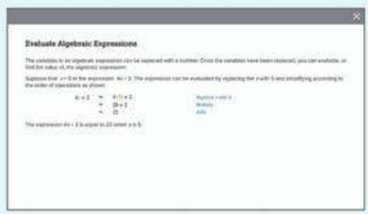
The expression $4x + 2$ is equal to 22 when $x = 5$.

Talk About It! Can you evaluate the expression $2x + 5y - 1$ if you know that $x = 3$? Explain your reasoning.

no; Sample answer: The expression contains two variables, so it cannot be evaluated completely. In order to evaluate the expression, I would need to know the value of *y*.

Lesson 5-4 • Evaluate Algebraic Expressions 287

Interactive Presentation



Learn, Evaluate Algebraic Expressions, Slide 1 of 2

Lesson 5-4

Evaluate Algebraic Expressions

6.EE.A.2, 6.EE.B.6

LESSON GOAL

Students will evaluate algebraic expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Algebraic Expressions

Learn: Evaluate Algebraic Expressions

Example 1: Evaluate One-Step Algebraic Expressions

Example 2: Evaluate One-Step Algebraic Expressions

Example 3: Evaluate Multi-Step Algebraic Expressions

Example 4: Use Algebraic Expressions

Apply: Woodworking

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 30 of the *Language Development Handbook* to help your students build mathematical language related to evaluating algebraic expressions.

ELL You can use the tips and suggestions on page T30 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Expressions and Equations

Major Cluster(s): In this lesson, students address major clusters **6.EE.A** and **6.EE.B** by evaluating algebraic expressions.

Standards for Mathematical Content: **6.EE.A.2, 6.EE.A.2.C, 6.EE.B.6**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous

Students wrote algebraic expressions.
6.EE.A.2.A, 6.EE.A.2.B, 6.EE.B.6

Now

Students evaluate algebraic expressions.
6.EE.A.2, 6.EE.A.2.C, 6.EE.B.6

Next

Students will solve problems by finding the greatest common factor and least common multiple of two whole numbers.
6.NS.B.4

Rigor

The Three Pillars of Rigor

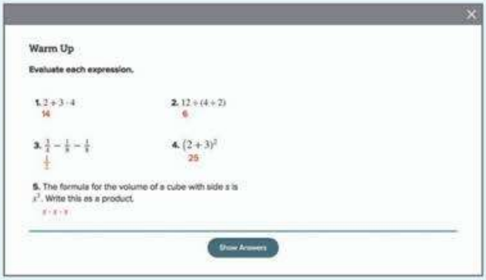
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of evaluating algebraic expressions. Using given rational values, they build *fluency* with evaluating one-step and multi-step algebraic expressions. They also *apply* their understanding of evaluating algebraic expressions to solve real-world problems.

Mathematical Background

To *evaluate* an algebraic expression, substitute specific values for the variables, then evaluate the resulting numerical expression. The specific values for all variables present are needed in order to evaluate an expression.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- performing operations with positive rational numbers (Exercises 1–3)
- understanding exponents (Exercises 4–5)

Answers

1. 14
2. 6
3. $\frac{1}{2}$
4. 25
5. $s \cdot s \cdot s$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about making posters for a school dance, and the use of an algebraic expression to determine supplies.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- How might you apply the meaning of the term *evaluate* and your prior experience with evaluating numerical expressions to evaluating algebraic expressions? Sample answer: I know that to *evaluate* a numerical expression I find the value of the expression. To *evaluate* an algebraic expression, I will need to use the value(s) of the variable(s) in order to find the value of the expression.

Explore Algebraic Expressions

Objective

Students will use Web Sketchpad to explore algebraic expressions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will examine what happens to the value of an algebraic expression as the values of each of the variables change. Throughout this activity, students will use Web Sketchpad to explore the changing values of variables by using a slider. Students will use their observations to make conjectures about how the values of the variables impact the value of the algebraic expression.

Inquiry Question

How can you determine the value of an algebraic expression for different given values? Sample answer: I can replace each variable with a given value and then evaluate the resulting numerical expression.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

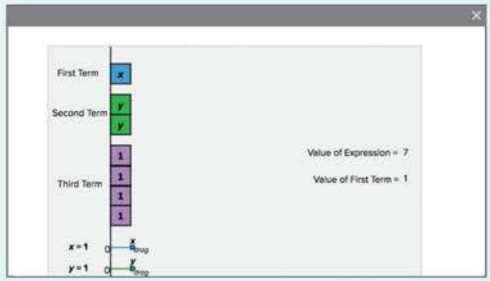
Mathematical Discourse

Drag the x slider. How does the value of the first term change as the value of x increases or decreases? How does the value of the expression change as the value of x increases or decreases? Sample answer: The value of the first term increases by one as x increases by one. The value of the expression increases by one as x increases by one.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 9



Explore, Slide 3 of 9

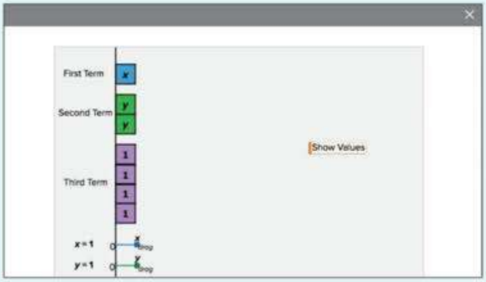
SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore algebraic expressions.

TYPE

On Slide 2, students enter the missing values in the expression.

Interactive Presentation



Explore, Slide 6 of 9

TYPE

On Slide 8, students enter the missing values in the expression.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Write Algebraic Expressions *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to represent the parts of algebraic expressions. Encourage students to think about the meaning of the different colors of the tiles and how the manipulation of them can help when writing an algebraic expression.

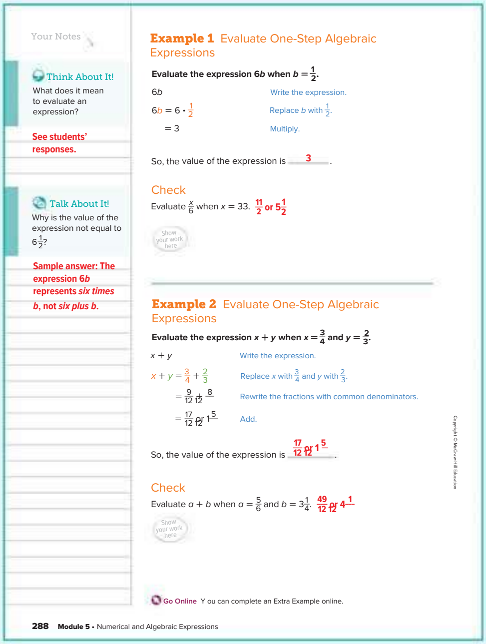
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Predict the value of the expression when $x = 3$ and $y = 4$. Explain your reasoning. Drag the sliders and press *Show Values* to check your answer. See students' responses.



Your Notes

Think About It!
What does it mean to evaluate an expression?

See students' responses.

Talk About It!
Why is the value of the expression not equal to $6\frac{1}{2}$?

Sample answer: The expression 6*b* represents *six times b*, not *six plus b*.

Example 1 Evaluate One-Step Algebraic Expressions

Evaluate the expression 6*b* when $b = \frac{1}{2}$.

$6b$	Write the expression.
$6b = 6 \cdot \frac{1}{2}$	Replace *b* with $\frac{1}{2}$.
$= 3$	Multiply.

So, the value of the expression is **3**.

Check

Evaluate $\frac{x}{6}$ when $x = 33$. $\frac{11}{2}$ **or** $5\frac{1}{2}$

Show your work here

Example 2 Evaluate One-Step Algebraic Expressions

Evaluate the expression $x + y$ when $x = \frac{3}{4}$ and $y = \frac{2}{3}$.

$x + y$	Write the expression.
$x + y = \frac{3}{4} + \frac{2}{3}$	Replace *x* with $\frac{3}{4}$ and *y* with $\frac{2}{3}$.
$= \frac{9}{12} + \frac{8}{12}$	Rewrite the fractions with common denominators.
$= \frac{17}{12}$ or $1\frac{5}{12}$	Add.

So, the value of the expression is $\frac{17}{12}$ **or** $1\frac{5}{12}$.

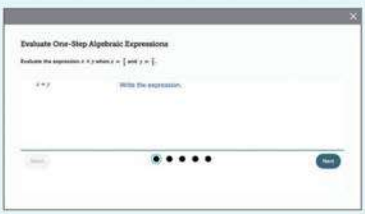
Check

Evaluate $a + b$ when $a = \frac{5}{6}$ and $b = 3\frac{1}{4}$. $\frac{49}{12}$ **or** $4\frac{1}{12}$

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 2, Evaluate One-Step Algebraic Expressions, Slide 1 of 2

CLICK

On Slide 2 of Example 1, students move through the steps to evaluate the expression.

TYPE

On Slide 1 of Example 2, students determine the value of the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Evaluate One-Step Algebraic Expressions

Objective

Students will evaluate one-step algebraic expressions for given rational-number values.

Questions for Mathematical Discourse

SLIDE 2

AL What is *b* equal to? $\frac{1}{2}$

AL Why is this a multiplication expression when there is no multiplication sign? Sample answer: When coefficients are used, multiplication is implied and no multiplication sign is needed.

OL Why is the value of 6*b* less than the value of 6? Sample answer: 6 is multiplied by a fraction between 0 and 1, which means the product is less than the original number.

BL A classmate says that the answer is 3 because the expression means *one half of 6*. Is this correct? yes; Sample answer: Multiplying a number by $\frac{1}{2}$ means the same as one half of the number.

Example 2 Evaluate One-Step Algebraic Expressions

Objective

Students will evaluate one-step algebraic expressions for given rational-number values.

Questions for Mathematical Discourse

SLIDE 1

AL What is the value of *x*? the value of *y*? $x = \frac{3}{4}$ and $y = \frac{2}{3}$

OL Estimate the value of the expression. Sample answer: $\frac{3}{4}$ is close to 1, and $\frac{2}{3}$ is also close to 1. So, the value of the expression will be close to 2, but less than 2.

BL A student mistakenly switched the values for *x* and *y* when substituting, but the student obtained the correct answer for the value of the expression. Why? Sample answer: Because the problem only involves the addition of two numbers, and because addition is commutative, switching the values of *x* and *y* won't change the final value of the expression.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Evaluate Multi-Step Algebraic Expressions

Objective

Students will evaluate multi-step algebraic expressions for given rational-number values.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the variable quantities in the expression, and be able to efficiently and accurately find the value of the expression.

6 Attend to Precision Students should be able to flexibly use the order of operations, by first evaluating the expression inside the parentheses.

Questions for Mathematical Discourse

SLIDE 1

AL How many variables are in this expression? Identify them. There are three variables: *x*, *y*, and *z*.

OL After substituting the values, what should you do first? Explain. Sample answer: Following the order of operations, first evaluate the expressions inside the parentheses.

BL Are parentheses necessary in this expression? Explain. yes; Sample answer: Removing the parentheses would cause the division operation to be 2 ÷ 9, which would cause the value of the expression to change.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of evaluating multi-step algebraic expressions, have them create their own algebraic expressions that satisfy the given conditions. They should be sure to include the values of the variables that need to be substituted into the expressions, in order to evaluate them.

- The expression should have at least four terms.
- The expression should have at least two different variables.
- There should be at least three operations, one of which is a power.

Then have them trade expressions with another pair of students. Each pair should evaluate the expression using the given values of the variables. Have pairs check each other's work.

Example 3 Evaluate Multi-Step Algebraic Expressions

Evaluate $(5x - 4y) \div z^2$ when $x = 4$, $y = \frac{1}{2}$, and $z = 3$.

$(5x - 4y) \div z^2$	Write the expression.
$(5x - 4y) \div z^2 = \left(5 \cdot 4 - 4 \cdot \frac{1}{2}\right) \div 3^2$	Replace *x* with 4, *y* with $\frac{1}{2}$, and *z* with 3.
$= (20 - 2) \div 9$	Multiply.
$= 18 \div 9$	Subtract.
$= 2$	Divide.

So, the value of the expression is 2.

Check

Evaluate $\frac{x}{4} + 2(y^2 - 3z)$ when $x = 12$, $y = 7$, and $z = 8$. 53

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare and contrast evaluating one-step and multi-step algebraic expressions. Do the differences affect your approach to evaluating the expressions? If yes, what do you do differently? If no, then explain why your approach remains the same.

Record your observations here

See students' observations.

Lesson 5-4 • Evaluate Algebraic Expressions 289

Interactive Presentation

Example 3, Evaluate Multi-Step Algebraic Expressions, Slide 1 of 2

CLICK On Slide 1, students move through the steps to evaluate the expression.

TYPE On Slide 1, students determine the value of the expression.

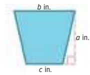
CHECK Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
How would you begin solving the problem?
See students' responses.

Talk About It!
Explain how the Commutative Property allows you to multiply $\frac{1}{2} \cdot 19 \cdot 9.8$.
Sample answer: This expression contains only multiplication, and the Commutative Property allows me to multiply $\frac{1}{2}$, 19, and 9.8 in any order without changing the result.

Example 4 Use Algebraic Expressions

The expression $\frac{1}{2}a(b + c)$ can be used to find the area of the trapezoid.



What is the area of the trapezoid when $a = 9.8$, $b = 12$, and $c = 7$?

$\frac{1}{2}a(b + c)$ — Write the expression.

$\frac{1}{2}a(b + c) = \frac{1}{2} \cdot 9.8(12 + 7)$ — Replace *a* with 9.8, *b* with 12, and *c* with 7.

$= \frac{1}{2} \cdot 9.8(19)$ — Simplify inside the parentheses.

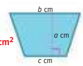
$= 4.9(19)$ — Multiply.

$= 93.1$ — Multiply.

So, the area of the trapezoid is 93.1 square inches.

Check

The expression $\frac{1}{2}a(b + c)$ can be used to find the area of the trapezoid. What is the area of the trapezoid when $a = 3.7$, $b = 6.4$, and $c = 3.6$? 18.5 cm^2



Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you encounter difficulty when using algebraic expressions in this Example and Check? What are some helpful tips you could give a classmate who encountered difficulty when using algebraic expressions?

Record your observations here

See students' observations.

Interactive Presentation

Example 4, Use Algebraic Expressions, Slide 2 of 4

CLICK
On Slide 2, students move through the steps to evaluate the expression.

TYPE
On Slide 2, students determine the area of the trapezoid.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Use Algebraic Expressions

Objective

Students will evaluate multi-step algebraic expressions for given rational-number values to find area.

Teaching the Mathematical Practices

6 Attend to Precision Students should be able to find the value of the expression precisely and accurately, adhering to the order of operations.

As students discuss the *Talk About It!* question on Slide 3, encourage them to clearly and precisely explain how the Commutative Property allows them to multiply in any order.

Questions for Mathematical Discourse

SLIDE 2

AL Is $\frac{1}{2}$ a constant or a coefficient? It is a coefficient since it immediately precedes a variable.

OL Why are the parentheses necessary? Sample answer: Without the parentheses, the addition will not be performed first.

BL How could you use mental math to tell that the area of the trapezoid is less than 100 square inches? Sample answer: Since $9.8 < 10$ and $19 < 20$, then $\frac{1}{2} \cdot 19 \cdot 9.8 < \frac{1}{2} \cdot 20 \cdot 10$, or 100.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Woodworking

Objective

Students will come up with their own strategy to solve an application problem involving perimeter of picture frames.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is perimeter?
- How do you find the perimeter of a rectangle?
- How would you determine the amount of wood needed for one of each type of picture frame?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Woodworking

The table shows the dimensions of three rectangular picture frame sizes that Martina is making. How much wood is needed to make two small frames and three large frames? The perimeter of a rectangle is $2\ell + 2w$, where ℓ is the length and w is the width.

Picture Frame Size	Length (in.)	Width (in.)
Small	3	5
Medium	5	7
Large	8	10

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

140 in.; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

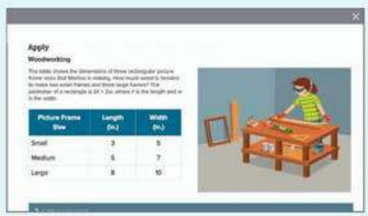
See students' arguments.

Talk About It!

Suppose the dimensions of the frames were doubled. How would this affect the perimeter of the frames?

Sample answer: The perimeter of the frames would also double.

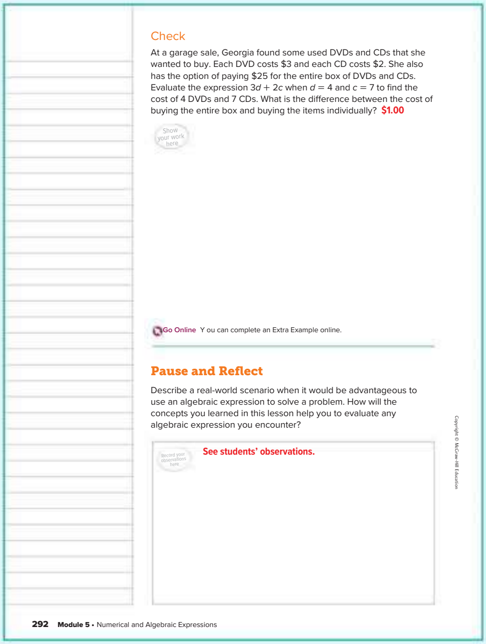
Interactive Presentation



Apply, Woodworking

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

At a garage sale, Georgia found some used DVDs and CDs that she wanted to buy. Each DVD costs $3 and each CD costs $2. She also has the option of paying $25 for the entire box of DVDs and CDs. Evaluate the expression $3d + 2c$ when $d = 4$ and $c = 7$ to find the cost of 4 DVDs and 7 CDs. What is the difference between the cost of buying the entire box and buying the items individually? **$1.00**

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe a real-world scenario when it would be advantageous to use an algebraic expression to solve a problem. How will the concepts you learned in this lesson help you to evaluate any algebraic expression you encounter?

Record your observations here

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can we communicate algebraic relationships with mathematical symbols? In this lesson, students learned how to evaluate algebraic expressions for specific values of the variables. Encourage them to discuss with a partner some algebraic expressions they have used and evaluated in different situations. For example, how the expression $4s$ can represent the perimeter of a square with side length s and how the expression can be evaluated to find the perimeter of any square.

Exit Ticket

Refer to the Exit Ticket slide. Write an expression for the number of markers, and evaluate the expression if the container originally has 100 markers. $\frac{1}{2}m - 10$; If $m = 100$, the number of markers left is $\frac{1}{2}(100) - 10 = 50 - 10 = 40$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 13, 15–19
- ALEKS Evaluating and Writing Expressions

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–11, 13, 15, 17, 19
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Exponents and Order of Operations

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Exponents and Order of Operations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

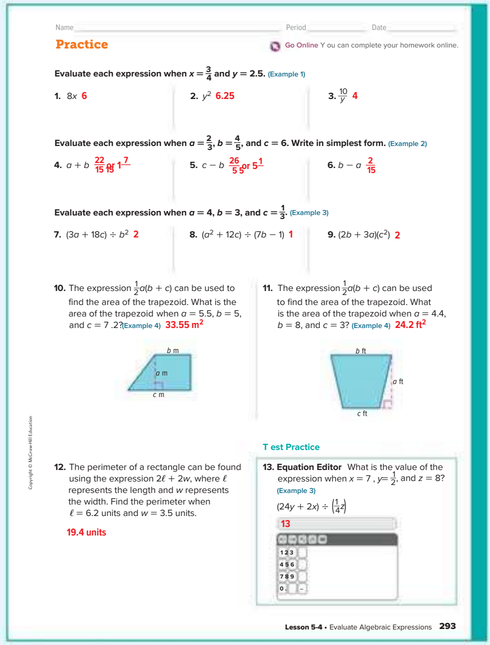
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	evaluate one-step algebraic expressions for given rational-number values	1–6
1	evaluate multi-step algebraic expressions for given rational-number values	7–9
1	evaluate multi-step algebraic expressions for given rational-number values to find area	10, 11
2	extend concepts learned in class to apply them in new contexts	12, 13
3	solve application problems that involve evaluating algebraic expressions	14, 15
3	higher-order and critical thinking skills	16–19

Common Misconception

Some students may incorrectly evaluate algebraic expressions by either incorrectly substituting values, or by not following the order of operations. Encourage students to use precision when substituting the values of the variables in the expressions. Students should adhere to the order of operations to evaluate the expressions.


Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

Evaluate each expression when $x = \frac{3}{4}$ and $y = 2.5$. (Example 1)

1. $8x$ 6
2. y^2 6.25
3. $\frac{10}{y}$ 4

Evaluate each expression when $a = \frac{2}{3}$, $b = \frac{4}{5}$, and $c = 6$. Write in simplest form. (Example 2)

4. $a + b$ $\frac{22}{15}$ or $1\frac{7}{15}$
5. $c - b$ $\frac{26}{5}$ or $5\frac{1}{5}$
6. $b - a$ $\frac{2}{15}$

Evaluate each expression when $a = 4$, $b = 3$, and $c = \frac{1}{3}$. (Example 3)

7. $(3a + 18c) \div b^2$ 2
8. $(a^2 + 12c) \div (7b - 1)$ 1
9. $(2b + 3a)(c^2)$ 2

10. The expression $\frac{1}{2}a(b + c)$ can be used to find the area of the trapezoid. What is the area of the trapezoid when $a = 5.5$, $b = 5$, and $c = 7.2$? (Example 4) 33.55 m^2

11. The expression $\frac{1}{2}a(b + c)$ can be used to find the area of the trapezoid. What is the area of the trapezoid when $a = 4.4$, $b = 8$, and $c = 3$? (Example 4) 24.2 ft^2

12. The perimeter of a rectangle can be found using the expression $2\ell + 2w$, where ℓ represents the length and w represents the width. Find the perimeter when $\ell = 6.2$ units and $w = 3.5$ units.

19.4 units

T est Practice

13. **Equation Editor** What is the value of the expression when $x = 7$, $y = \frac{1}{2}$, and $z = 8$? (Example 3)

$(24y + 2x) \div \left(\frac{1}{4}z\right)$

13

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Apply *indicates multi-step problem

*14. Mr . Y oung is replacing the fencing around his rabbit pens and garden. The table shows the dimensions of the different areas. How many feet of fencing will he need to replace two rabbit pens and his garden? The perimeter of a rectangle is $2\ell + 2w$, where ℓ is the length and w is the width.

Item	Length (ft)	Width (ft)
Rabbit Pen	3.5	4.5
Garden	12	10

76 ft

*15. Angel is comparing the price to print shirts for summer camp at two companies. Company A charges an initial fee of $50 and $12 per shirt. Company B charges an initial fee of $10 and $15 per shirt. Evaluate the expressions $50 + 12x$ and $10 + 15x$ for $x = 40$ to find the total cost to print 40 shirts at each company. What is the difference in cost between the companies?

$80

Higher-Order Thinking Problems

16. **Which One Doesn't Belong?** Circle the expression that does not equal 13 when $x = 3$.

$5x - 2$ $\quad$ $5x^2 - 27 + 5$ (circled)

$(x^3 - 1) \div 2$ $\quad$ $x + 13 - x$

17. **Find the Error** A student was evaluating $4b + c$ for $b = 2$ and $c = 3$. Find the student's mistake and correct it.

$$4b + c = 4(3) + 2$$
$$= 12 + 2$$
$$= 14$$

Sample answer: The student replaced the variables with the incorrect values. The correct value should be $4(2) + 3$ or 11.

18. **Be Precise** Compare and contrast algebraic expressions and numerical expressions.

Sample answer: To evaluate both algebraic expressions and numerical expressions, use the order of operations. An algebraic expression contains numbers and variables. Numerical expressions contain only numbers.

19. Give an example of an algebraic expression and a numerical expression that have the same value when evaluated.

Sample answer: If $a = 2$, then $a + 10 = 12$; $(15 + 5) - 8 = 12$

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students explain why another student's solution is incorrect and then correct the solution.

6 Attend to Precision In Exercise 18, students compare and contrast algebraic expressions and numerical expressions. Encourage students to use proper mathematical terminology in their explanations.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.

Use with Exercises 14–15 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Interview a student.

Use with Exercise 17 Have pairs of students interview each other as they complete this problem. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 17 might be, "How do you know that the student made a mistake while evaluating the expression?"

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Greatest Common Factor

Objective

Students will learn how to find the greatest common factor of two whole numbers.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to think about the process they use to find factors of very large numbers in order to make a plausible argument and justify their reasoning.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how to find the GCF by listing the factors.

Talk About It!

SLIDE 3

Mathematical Discourse

When is making a list of the factors difficult to do? Sample answer: It is difficult to do when the numbers are very large and have many factors.

DIFFERENTIATE

Reteaching Activity AL

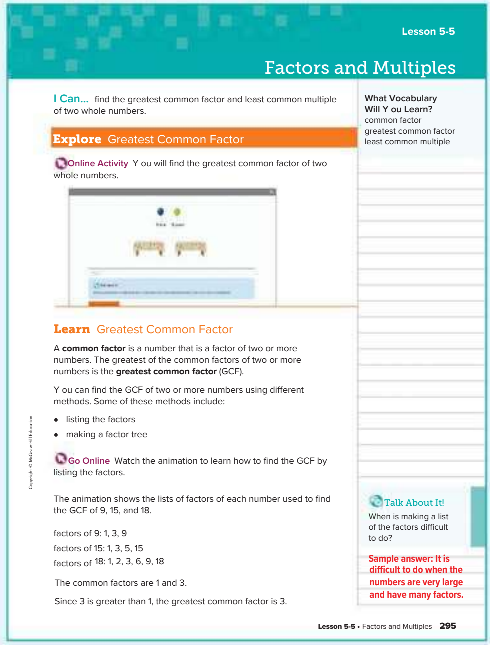
If any of your students are having difficulty making a list of factors, have them write down the definition of a factor, and use it to fill in the blanks for the following questions.

6 is a factor of 24 because 6 × 4 = 24

2 is a factor of 40 because 2 × 20 = 40

13 is a factor of 26 because 13 × 2 = 26

5 is a factor of 55 because 5 × 11 = 55



Lesson 5-5

Factors and Multiples

I Can... find the greatest common factor and least common multiple of two whole numbers.

What Vocabulary Will You Learn?
common factor
greatest common factor
least common multiple

Explore Greatest Common Factor

Online Activity You will find the greatest common factor of two whole numbers.

Learn Greatest Common Factor

A **common factor** is a number that is a factor of two or more numbers. The greatest of the common factors of two or more numbers is the **greatest common factor** (GCF).

You can find the GCF of two or more numbers using different methods. Some of these methods include:

- listing the factors
- making a factor tree

Go Online Watch the animation to learn how to find the GCF by listing the factors.

The animation shows the lists of factors of each number used to find the GCF of 9, 15, and 18.

factors of 9: 1, 3, 9
factors of 15: 1, 3, 5, 15
factors of 18: 1, 2, 3, 6, 9, 18

The common factors are 1 and 3.

Since 3 is greater than 1, the greatest common factor is 3.

Talk About It! When is making a list of the factors difficult to do?

Sample answer: It is difficult to do when the numbers are very large and have many factors.

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Interactive Presentation

Learn, Greatest Common Factor, Slide 2 of 3

WATCH

On Slide 2, students watch an animation that explains what a greatest common factor is, and how to find the GCF of two numbers.

Lesson 5-5

Factors and Multiples

6.NS.B.4

LESSON GOAL

Students will solve problems by finding the greatest common factor and least common multiple of two whole numbers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Greatest Common Factor

Learn: Greatest Common Factor
Example 1: Find the GCF by Using a List of Factors
Example 2: Find the GCF by Using a Factor Tree

Explore: Least Common Multiple

Learn: Least Common Multiple
Example 3: Find the LCM by Using a List of Multiples
Example 4: Find the LCM by Using a Number Line
Apply: School Supplies

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

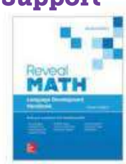
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 31 of the *Language Development Handbook* to help your students build mathematical language related to factors and multiples.



ELL You can use the tips and suggestions on page T31 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System
Additional Cluster(s): In this lesson, students address additional cluster **6.NS.B** by solving problems involving greatest common factor and least common multiple.
Standards for Mathematical Content: **6.NS.B.4**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students evaluated algebraic expressions.
6.EE.A.2, 6.EE.B.6

Now
Students solve problems by finding the greatest common factor and least common multiple of two whole numbers.
6.NS.B.4

Next
Students will use the Distributive Property.
6.EE.A.2.B, 6.EE.A.3, 6.NS.B.4

Rigor

The Three Pillars of Rigor

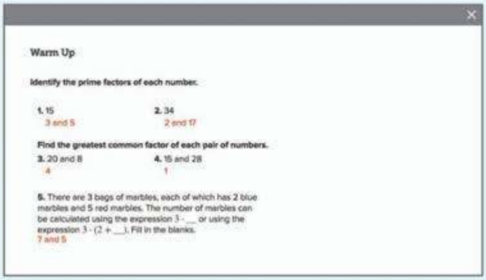
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of factors and multiples (gained in Grade 4) and their *understanding* of prime factorization to build *fluency* with finding the greatest common factor and least common multiple of two whole numbers. They *apply* their understanding of greatest common factor and least common multiple to solve real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- understanding prime numbers by finding factors (Exercises 1–5)

Answers

1. 1, 2, 3, 4, 6, 12

2. 1, 2, 11, 22

3. 1, 2, 4, 8, 16, 32

4. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

5. 7 groups; Sample answer: They can be divided into 2 groups of 18 students, 3 groups of 12 students, 4 groups of 9 students, 6 groups of 6 students, 9 groups of 4 students, 12 groups of 3 students or 18 groups of 2 students.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about designing floral centerpieces for dinner tables.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What part of speech is the term *common*? How can this term help you understand what a *common factor* might be? adjective; Sample answer: Something that is common is shared; A common factor might be a number that is a shared factor of two or more numbers.
- Use your knowledge of the terms greatest, common, and factor to help you understand what the *greatest common factor* might be. Sample answer: The greatest common factor might be the largest (greatest) factor that is shared between two or more numbers.
- What parts of speech are the terms least and common? Use your understanding of multiple to describe what you think the least common multiple might be. Sample answer: Least and common are adjectives that describe the term multiple. A multiple of a number is a product of that number and another number. The least common multiple might be the least (lowest) number that is a multiple of two or more numbers (making it common).

Explore Greatest Common Factor

Objective

Students will explore how to find the greatest common factor of two whole numbers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a situation where Lucy has 8 blue balloons and 12 green balloons and needs to make identical table arrangements for a birthday party. Throughout this activity, students will explore the possibility of making various numbers of arrangements (2, 3, 4, etc.). Students will use their observations to connect the greatest number of possible identical balloon arrangements with the greatest common factor.

Inquiry Question

How can finding the greatest common factor help solve a real-world problem? Sample answer: I can use the greatest common factor to help make identical groups of different types of objects.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

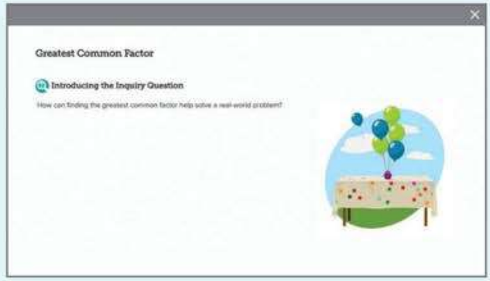
SLIDE 2

Mathematical Discourse

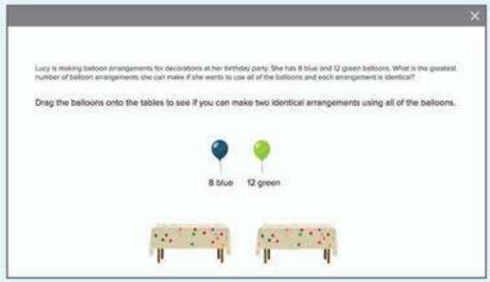
Share your balloon arrangement with a classmate. How many balloons of each color are in each arrangement? Sample answer: 4 blue, 6 green

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 9



Explore, Slide 2 of 9

DRAG & DROP

On Slide 2, students drag the blue balloons and the green balloons to make identical arrangements on two tables.

Interactive Presentation

Explore, Slide 5 of 9

DRAG & DROP

On Slide 5, students drag the balloons to test their prediction for four balloon arrangements.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Greatest Common Factor *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to examine the relationship between divisibility, factors, common factors, and the greatest common factor.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Did your prediction hold true? Explain why the balloons can be divided into 4 identical arrangements. Sample answer: Both 8 and 12 are divisible by 4.

Your Notes

Example 1 Find the GCF by Using a List of Factors

Use a list of factors to find the greatest common factor of 12 and 28.

Step 1 List the factors of each number.

factors of 12: 1, 2, 3, 4, 6, 12

factors of 28: 1, 2, 4, 7, 14, 28

Step 2 Identify the common factors.

common factors: 1, 2, 4

Step 3 Identify the greatest common factor.

greatest common factor: 4

So, the greatest common factor of 12 and 28 is 4.

Talk About It! Can the greatest common factor of two numbers ever be 1? If so, give an example.

yes; Sample answer: 22 and 25 have a GCF of 1.

Check

Use a list of factors to find the GCF of 9 and 20. 1

Show your work here

Go Online You can complete an Extra Example online.

Example 2 Find the GCF by Using a Factor Tree

Use factor trees to find the greatest common factor of 52 and 78.

For greater numbers, listing all of the factors can be inefficient. A factor tree is another method you can use to find the GCF.

Complete each factor tree.

52: 2 × 26; 2 × 2 × 13

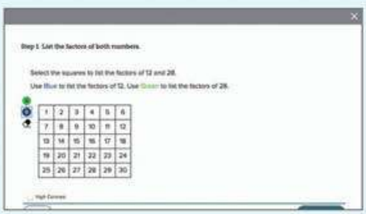
78: 2 × 39; 2 × 3 × 13

Think About It! How will you set up a factor tree for each number?

See students' responses.

(continued on next page)

Interactive Presentation



Example 1, Find the GCF by Using a List of Factors, Slide 2 of 6

CLICK

On Slide 2 of Example 1, students select values that are the factors of 12 and 28.

DRAG & DROP

On Slide 3 of Example 1, students drag the values to indicate the common factors of 12 and 28 in order from least to greatest.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find the GCF by Using a List of Factors

Objective

Students will find the greatest common factor of two whole numbers by listing the factors.

Questions for Mathematical Discourse

SLIDE 2

AL What are the factors of 12? 28? The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 28 are 1, 2, 4, 7, 14, and 28.

OL After selecting the factors of 12, what happens when we select the factors of 28? Sample answer: Some of the factors are factors of both numbers, such as 1, 2, and 4.

BL *Natural* numbers are the set of whole numbers excluding zero. Which natural number will always be a common factor of any two natural numbers? Explain. The number 1 is always a factor of any natural number, so it will always be a common factor of any two natural numbers.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find the GCF by Using a Factor Tree

Objective

Students will find the greatest common factor of two whole numbers by using a factor tree.

Questions for Mathematical Discourse

SLIDE 2

AL What is a prime number? How do you know if a number is prime? Sample answer: A prime number has exactly two different factors: the number 1 and itself. I know a number is prime if I test all possible factors and find that the only factor pair is 1 and the number itself.

OL Where do you look to find common prime factors in a factor tree? Sample answer: Common prime factors are numbers that are in the bottom rows of both numbers.

BL Suppose the prime factorization of Number A is 5 × 5 × 5 and the prime factorization of Number B is 5 × 5 × 5 × 5. How many 5s would be included in the list of common prime factors? Sample answer: Three 5s, because the two numbers have three 5s in common.

(continued on next page)

Example 2 Find the GCF by Using a Factor Tree *(continued)*

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Look at the bottom row of the factor trees. The common prime factors are **2** and **13**.

Because 2 and 13 are factors of both 52 and 78, the product of 2 and 13 is also a factor of both numbers. Multiply the common prime factors to find the GCF.

So, the GCF of 52 and 78 is 2×13, or 26.

Talk About It!
Does changing the order of the factors result in a different GCF? Explain.

no; Sample answer: Because of the Commutative Property, the order of the factors can be different.

Check

Use factor trees to find the GCF of 45 and 75. **15**

Show your work here

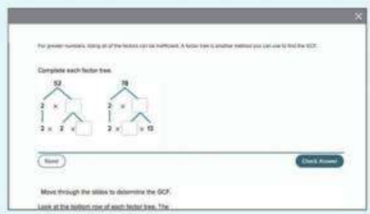
Go Online You can complete an Extra Example online.

Explore Least Common Multiple

Online Activity You will use Web Sketchpad to find the least common multiple of two whole numbers.

Lesson 5-5 • Factors and Multiples 297

Interactive Presentation



Example 2, Find the GCF by Using a Factor Tree, Slide 2 of 4

TYPE

On Slide 2 of Example 2, students enter the missing values to complete the factor tree.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Least Common Multiple

The least nonzero number that is a multiple of two or more whole numbers is the **least common multiple** (LCM) of the numbers.

Y ou can find the least common multiple of a set of whole numbers by:

- listing the multiples
- using a number line

Go Online Watch the animation to learn how to find the LCM of 4 and 6 by listing the nonzero multiples.

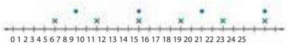
The animation shows the lists of the first six nonzero multiples of each number.

multiples of 4: 4, 8, 12, 16, 20, 24, ...
multiples of 6: 6, 12, 18, 24, 30, 36, ...

The common multiples in the list are 12 and 24.

Since 12 is less than 24, the least common multiple is 12.

Y ou can also find the least common multiple of a set of whole numbers by using a number line.



The least number with both an X and a dot is 12.

So, the least common multiple is 12.

Example 3 Find the LCM by Using a List of Multiples

Ernesto is at the community center every 8 weeks for his painting class. Kamala is at the community center every 6 weeks for her pottery class. They were both at the center for their classes this week.

How many weeks will it be until they both have their classes in the same week again?

Step 1 List the multiples of each number.

multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64...

multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48...

(continued on next page)

Think About It!
When will Ernesto be back at the community center? Will Kamala be there? How do you know?

See students' responses.

Interactive Presentation

Learn, Least Common Multiple, Slide 2 of 3

WATCH

On Slide 2 of the Learn, students watch an animation that explains how to find the least common multiple of two whole numbers.

CLICK

On Slide 3, students select each button to view the multiples of 4 and 6 on a number line.

Learn Least Common Multiple

Objective

Students will learn how to find the least common multiple of two whole numbers.

Teaching Notes

SLIDE 1

Students will learn the definition of *least common multiple*, and several methods for finding the least common multiple. Note that only nonzero multiples will be considered when finding the LCM.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation explains how to find the least common multiple of two whole numbers.

Example 3 Find the LCM by Using a List of Multiples

Objective

Students will find the least common multiple of two whole numbers by listing the multiples.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to make a plausible argument, including an example, for why the LCM of two numbers can be one of the numbers.

5 Use Appropriate Tools Strategically On Slide 2, students will use the shading tool to list the multiples of 6 and 8 in order to find the number of weeks until Ernesto and Kamala have their classes in the same week.

Questions for Mathematical Discourse

SLIDE 2

AL From 1 to 30, what are the multiples of 6? 8? The multiples of 6 are 6, 12, 18, 24, and 30. The multiples of 8 are 8, 16, and 24.

OL Are there more multiples of 6 and 8 than are shown on the shading tool? Explain. yes; Sample answer: There are an infinite number of multiples. Some of the other multiples of 6 are 36 and 48. Some of the other multiples of 8 are 32 and 40.

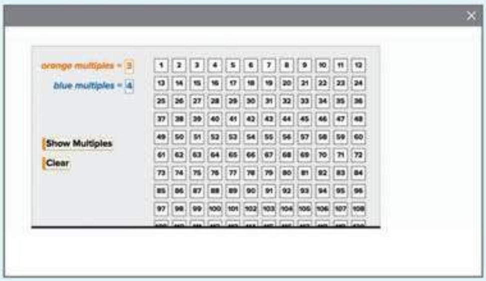
OL How are the common multiples of 6 and 8 shown on the shading tool? They are shaded twice.

BL Does every pair of positive whole numbers have a common multiple? Explain. yes; Sample answer: The product of the two numbers is always a common multiple because it is a multiple of each of the numbers individually.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 10



Explore, Slide 4 of 10

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to find the least common multiple of two numbers.

Explore Least Common Multiple

Objective

Students will use Web Sketchpad to explore how to find the least common multiple of two whole numbers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with several pairs of numbers and their multiples. Throughout this activity, students will use interactive grids of multiples to recognize the patterns and to find the least common multiple of two numbers.

Inquiry Question

How can you find the least number that is a multiple of two whole numbers? Sample answer: I could make a list of the multiples, find the multiples that they share, and then find the least number that is a multiple of both.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What patterns do you notice? Sample answer: The multiples they have in common are also multiples of 12. In this chart, they are the numbers in the last column.

(continued on next page)

Explore Least Common Multiple *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically. Students will use Web Sketchpad to carefully identify any patterns in finding the common multiples. Encourage students to explore the sketches and deepen their understanding of the least common multiple.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 7 are shown.

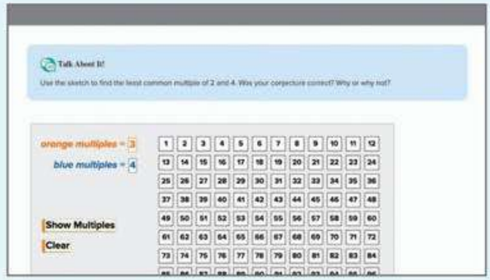
Talk About It!

SLIDE 7

Mathematical Discourse

Use the sketch to find the least common multiple of 2 and 4. Was your conjecture correct? Why or why not? Sample answer: Y es, my conjecture was that it is not always true. The least common multiple of 2 and 4 is 4, but the product of 2 and 4 is 8.

Interactive Presentation



Explore, Slide 7 of 10

TYPE

On Slide 10, students respond to the Inquiry Question and view a sample answer.

Example 3 Find the LCM by Using a List of Multiples *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL From 1 to 30, what multiple(s) do 6 and 8 have in common? 24

AL Are there other multiples, beyond 30, that 6 and 8 have in common? If so, identify one. yes; Sample answer: 6 and 8 both have the multiple 48 in common.

OL Why does the least common multiple help solve the problem? Sample answer: Ernesto and Kamala will both be at the center in 24 weeks, since 24 is the LCM of their respective cadences.

BL Name some other weeks that Ernesto and Kamala will both be at the center for their classes. Sample answers: 48, 72, 96, etc. The least common multiple is 24 weeks, so they will both be at the center every 24 weeks.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

If students have trouble identifying multiples, have them work in pairs to generate two different numbers. The students should write two sentences indicating whether or not one number is a multiple of the other. For example, if one student chooses 4 and the other student chooses 12, the students would work together to write, "12 is a multiple of 4," and "4 is not a multiple of 12." Then have each pair fill in the blank for the following sentences.

18 ______ a multiple of 6. is

25 ______ a multiple of 10. is not

4 ______ a multiple of 40. is not

27 ______ a multiple of 9. is

Step 2 Identify the least common multiple.

Circle the multiples that 8 and 6 have in common.

multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64...

multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48...

least common multiple: 24

So, Ernesto and Kamala have their classes in the same week again in 24 weeks.

Talk About It!

Can the least common multiple of two numbers ever be one of the given numbers? If so, give an example.

yes; Sample answer: The LCM of 2 and 4 is 4.

Check

Every 10th week Tamika visits the zoo. Every 12th week she visits the local pet rescue. If she visited both this week, how many weeks will it be until she visits both in the same week again? 60 weeks

Show your work here

Pause and Reflect

Describe the differences between finding the greatest common factor and finding the least common multiple. How will knowing these differences be helpful when checking the accuracy of your answers?

Record your observations here

See students' observations.

Lesson 5-5 • Factors and Multiples 299

Interactive Presentation

Example 3, Find the LCM by Using a List of Multiples, Slide 3 of 5

DRAG & DROP

On Slide 3, students drag the circle to select the multiples that 6 and 8 have in common.

CHECK

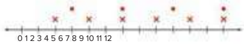
Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find the LCM by Using a Number Line

Use a number line to find the least common multiple of 2 and 3.

Place an X above each multiple of 2.

Place a dot above each multiple of 3.

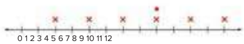


What is the least number with both an X and a dot? 6

So, the least common multiple of 2 and 3 is 6.

Check

Use a number line to find the LCM of 2 and 8. 8



Show your work here

Go Online You can complete an Extra Example online.

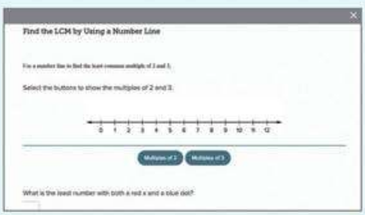
Pause and Reflect

Compare and contrast using a number line and using a list of multiples to find the least common multiple. When might using a number line be more advantageous? When might using a list be more advantageous?

Record your observations here

See students' observations.

Interactive Presentation



Example 4, Find the LCM by Using a Number Line, Slide 1 of 2

CLICK

On Slide 1, students select to view the multiples of 2 and 3.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find the LCM by Using a Number Line

Objective

Students will find the least common multiple of two whole numbers by using a number line.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use the number line to help them represent the multiples of 2 and 3, and to help them find the least common multiple by noting which multiples are common to both 2 and 3.

Questions for Mathematical Discourse

SLIDE 1

AL What are the multiples of 2? The multiples of 3? Sample answer: The multiples of 2 are 2, 4, 6, 8, 10, 12, ... The multiples of 3 are 3, 6, 9, 12, ...

OL How do the buttons reveal the common multiples? the least common multiple? The common multiples have both an x and a dot above them. The least common multiple is the least number on the number line with both an x and a dot above it.

BL What is the least common multiple of 2, 3, and 4? Explain. 12; Sample answer: By listing the multiples of 2, 3, and 4, the least of the common multiples is 12.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply School Supplies

Objective

Students will come up with their own strategy to solve an application problem involving items in the school store.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What types of supplies might be sold in a school store?
- How do you find the GCF of 48 and 36?
- What type of model could you use to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply School Supplies

The table shows the supplies a school supply store has left at the end of the week. The store manager wants to put pencils and notepads together in bags to sell as a combo pack, and he wants to make the greatest number of bags possible. If all of the pencils and notepads are distributed evenly among all of the bags, and the store charges $4 per bag, how much money will the store bring in if they sell all of the bags?

Item	Number
Pencils	48
Pens	32
Erasers	60
Notepads	36

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

$48.00; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online watch the animation.

Talk About It!

Suppose the manager wanted to distribute the erasers evenly to the combo packs in addition to the pencils and notepads. Would adding the erasers alter the number of combo packs that can be made? Explain your reasoning.

no; Sample answer: The greatest common factor of 36, 48, and 60 is still 12, so the number of combo packs that can be made will not change.

Lesson 5-5 • Factors and Multiples 301

Interactive Presentation

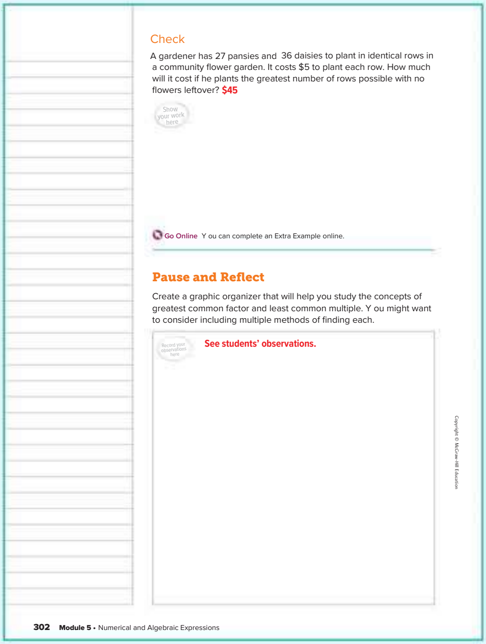
Apply, School Supplies

WATCH

Students watch an animation that demonstrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

A gardener has 27 pansies and 36 daisies to plant in identical rows in a community flower garden. It costs $5 to plant each row. How much will it cost if he plants the greatest number of rows possible with no flowers leftover? $45

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that will help you study the concepts of greatest common factor and least common multiple. You might want to consider including multiple methods of finding each.

Record your observations here

See students' observations.

302 Module 5 • Numerical and Algebraic Expressions

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Samantha and her sister have 250 roses and 175 peonies. If they want to use all of the flowers, how many identical centerpieces can they make, if they want to have as many as possible? Write a mathematical argument that can be used to defend your solution. 25 identical centerpieces; Sample answer: Find the prime factorization of each number. Then find the common prime factors and the GCF. Since the GCF is 25, they can make 25 identical centerpieces.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 11–13, 15–18
- Extension: Extension Resources
- ALEKS Prime Numbers, Factors, and Multiples

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–10, 12, 14–16
- Extension: Extension Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Prime Numbers, Factors, and Multiples

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Prime Numbers, Factors, and Multiples

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

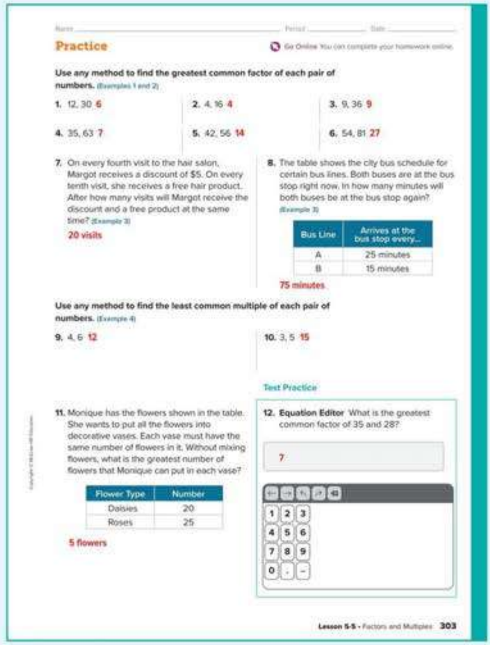
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the GCF	1–6
1	find the LCM	7–10
2	extend concepts learned in class to apply them in new contexts	11, 12
3	solve application problems involving the GCF and LCM of whole numbers	13, 14
3	higher-order and critical thinking skills	15–18

Common Misconception

When finding the least common multiple, some students may incorrectly list the common factor of 1, because they are accustomed to including it when listing factors. Remind students to reason that multiples of a given number will be equal to or greater than the number itself.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Use any method to find the greatest common factor of each pair of numbers. (Examples 1 and 2)

1. 12, 30 **6**
2. 4, 16 **4**
3. 9, 36 **9**
4. 35, 63 **7**
5. 42, 56 **14**
6. 54, 81 **27**

7. On every fourth visit to the hair salon, Margot receives a discount of $5. On every tenth visit, she receives a free hair product. After how many visits will Margot receive the discount and a free product at the same time? (Example 3) **20 visits**

8. The table shows the city bus schedule for certain bus lines. Both buses are at the bus stop right now. In how many minutes will both buses be at the bus stop again? (Example 3)

Bus Line	Arrives at the bus stop every...
A	25 minutes
B	15 minutes

75 minutes

Use any method to find the least common multiple of each pair of numbers. (Example 4)

9. 4, 6 **12**
10. 3, 5 **15**

Test Practice

11. Monique has the flowers shown in the table. She wants to put all the flowers into decorative vases. Each vase must have the same number of flowers in it. Without mixing flowers, what is the greatest number of flowers that Monique can put in each vase?

Flower Type	Number
Daisies	20
Roses	25

5 flowers

12. **Equation Editor** What is the greatest common factor of 35 and 28?

7

Lesson 5-5 • Factors and Multiples 303

Apply *indicates multi-step problem

*13. The table shows the number of each type of cookie a bakery has left at the end of the day. The baker wants to make the greatest number of cookie boxes to sell, using chocolate chip and sugar cookies together. If all of the chocolate chip and sugar cookies are distributed evenly among the boxes and the baker charges $5 per box, how much money will the bakery bring in if they sell all of the boxes? **$65**

Type of Cookie	Number
Chocolate Chip	26
Oatmeal Raisin	34
Peanut Butter	18
Sugar	39

*14. A teacher needs to purchase notebooks and pencils for her students. Notebooks come in packages of 6 and pencils in packages of 10. The table shows the cost of the items. What is the least amount of money the teacher can spend and have the same number of notebooks and pencils? **$31.00**

Item	Cost ($)
Folder Packages	3.50
Notebook Packages	5.00
Pencil Packages	2.00

Higher-Order Thinking Problems

15. MP **Identify Structure** Explain how the Commutative Property is applied when finding the GCF using factor trees.

Sample answer: The bottom row of the factor trees may not show the factors listed in order from least to greatest. I can use the Commutative Property to write the factors in order from least to greatest.

16. MP **Make a Conjecture** A student is finding the GCF of 6 and 12. Without computing, will the GCF be odd or even? Explain.

even; Sample answer: Even numbers have a factor of 2. The GCF will therefore have 2 as a factor. So, the GCF must be even.

17. MP **Use a Counterexample** Determine if the statement is *true* or *false*. If true, support with an example. If false, give a counterexample.

If one number is a multiple of another number, the LCM is the lesser of the two numbers.

false; Sample answer: 25 and 50; 50 is a multiple of 25, 50 is the LCM, and 50 is the greater number. The LCM is the greater of the two numbers.

18. MP **Make a Conjecture** Can two different pairs of numbers have the same LCM? Explain.

yes; Sample answer: The numbers 3 and 8 have a LCM of 24 and the numbers 12 and 24 also have a LCM of 24.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 15, students will analyze the structure of a factor tree and explain how the Commutative Property is important for finding the greatest common factor of two numbers. Because the greatest common factor is the product of the common prime factors of the two numbers, it is important to remember that these factors can be reordered and multiplied to obtain the greatest common factor.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students will determine if a statement about least common multiples is true or false. In the event that the statement is false, students will provide a counterexample.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.

Use with Exercise 13 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Clearly and precisely explain.

Use with Exercise 17–18 Have students work in groups of 3–4 to solve the problem in Exercise 17. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 18.

Learn The Distributive Property

Objective

Students will understand how the Distributive Property can be applied to multiply a sum by a number.

Teaching the Mathematical Practices

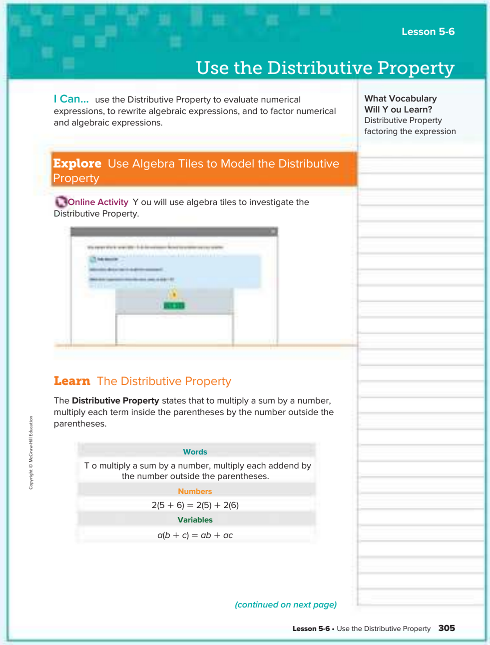
6 Attend to Precision As students discuss the *Talk About It!* questions on Slide 3, encourage them to use precise and clear mathematical language, such as order of operations, in their explanations.

Teaching Notes

SLIDE 1

You may wish to have students select the *Numbers* flashcard prior to the other flashcards in order to understand, using a concrete example, what it means to multiply a sum by a number. You may wish to have them first evaluate the sum 5 + 6, and then multiply by 2, in order to illustrate that the result is equivalent to first multiplying 2 by each of the addends 5 and 6. You may also wish to point out that, when they apply the Distributive Property, they are writing the product 2(5 + 6) as the sum 2(5) + 2(6).

(continued on next page)


Lesson 5-6

Use the Distributive Property

I Can... use the Distributive Property to evaluate numerical expressions, to rewrite algebraic expressions, and to factor numerical and algebraic expressions.

What Vocabulary Will You Learn?
Distributive Property
factoring the expression

Explore Use Algebra Tiles to Model the Distributive Property

Online Activity You will use algebra tiles to investigate the Distributive Property.

Learn The Distributive Property

The **Distributive Property** states that to multiply a sum by a number, multiply each term inside the parentheses by the number outside the parentheses.

Words
To multiply a sum by a number, multiply each addend by the number outside the parentheses.
Numbers
2(5 + 6) = 2(5) + 2(6)
Variables
$a(b + c) = ab + ac$

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(continued on next page)

Lesson 5-6 • Use the Distributive Property 305

Interactive Presentation

Learn, The Distributive Property, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to learn about the Distributive Property.

DIFFERENTIATE

Enrichment Activity BL

For students that may need more of a challenge, have them work with a partner to expand each of the following expressions using the Distributive Property. Then have them generate their own expressions and trade them with another pair of students. Each pair should expand the expressions and compare their results. Have pairs discuss and resolve any differences.

$2x(5y + 8)$ $10xy + 16x$

$3m(7n + 4p)$ $21mn + 12mp$

$4a(6bc + 11d)$ $24abc + 44ad$

Lesson 5-6

Use the Distributive Property

6.NS.B.4, 6.EE.A.3

LESSON GOAL

Students will use the Distributive Property to expand and factor expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Algebra Tiles to Model the Distributive Property

Learn: The Distributive Property

Example 1: Use the Distributive Property

Example 2: Use the Distributive Property

Learn: Greatest Common Factor and the Distributive Property

Example 3: Use GCF to Factor Numerical Expressions

Example 4: Use GCF to Factor Algebraic Expressions

Apply: Money

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: The FOIL Method		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 32 of the *Language Development Handbook* to help your students build mathematical language related to the Distributive Property.

ELL You can use the tips and suggestions on page T32 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Expressions and Equations

Major Cluster(s): In this lesson, students address major cluster **6.EE.A** and the supporting cluster **6.NS.B** by using the Distributive Property.

Standards for Mathematical Content: **6.NS.B.4, 6.EE.A.3,** *Also addresses 6.EE.A.2.B*

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous

Students found the greatest common factor and least common multiple of two whole numbers.
6.NS.B.4

Now

Students use the Distributive Property.
6.NS.B.4, 6.EE.A.2.B, 6.EE.A.3

Next

Students will identify and generate equivalent algebraic expressions.
6.EE.A.3, 6.EE.A.4

Rigor

The Three Pillars of Rigor

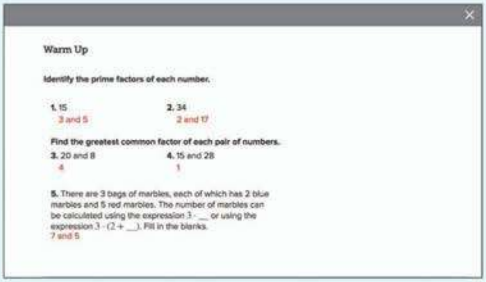
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their *understanding* of numerical and algebraic expressions as they explore the Distributive Property. They use the Distributive Property to build *fluency* with expanding an expression, multiplying a whole number and a rational number, and factoring expressions using greatest common factors. They also *apply* their understanding of the Distributive Property to solve real-world problems.

Mathematical Background

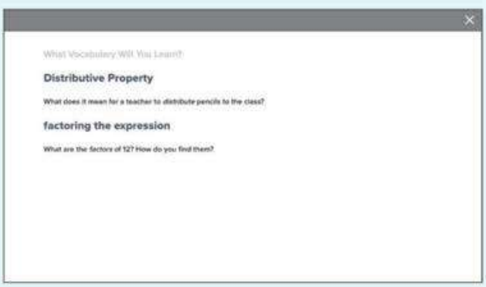
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding prime factors (Exercises 1–2)
- finding the greatest common factor (Exercises 3–4)
- understanding operations with numbers (Exercise 5)

Answers

1. 3 and 5

2. 2 and 17

3. 4

4. 1

5. 7 and 5

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about finding the cost of concert tickets.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does it mean for a teacher to *distribute* pencils to the class? Sample answer: It would mean that the teacher gives each student in the class a pencil or pencils.
- What are the *factors* of 12? How do you find them? The factors of 12 are 1, 2, 3, 4, 6, and 12. Sample answer: I can find them by listing the numbers that divide evenly into 12.

Explore Use Algebra Tiles to Model the Distributive Property

Objective

Students will use algebra tiles to explore the Distributive Property.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use algebra tiles to explore the Distributive Property. Students will begin with familiar examples using only numerical expressions. They will then extend their knowledge to the use of the Distributive Property in the context of algebraic expressions. Throughout the activity students will make and test conjectures about equivalent algebraic expressions.

Inquiry Question

How can you use algebra tiles to model the Distributive Property? Sample answer: Place equal groups of tiles on the mat to represent the multiplication expression. Then group like tiles together to represent an equivalent addition expression.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What steps did you take to model the expression? See students' responses.

What other expressions have the same value as $2(2x + 1)$? Sample answer: Some other expressions that have the same value are $4\left(x + \frac{1}{2}\right)$ and $\frac{1}{2}(8x + 4)$.

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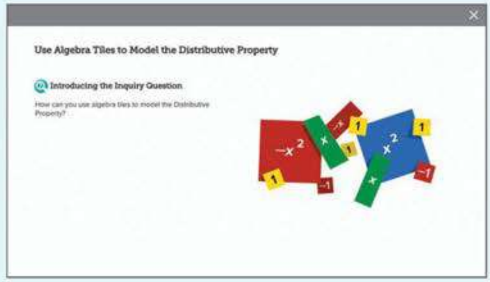
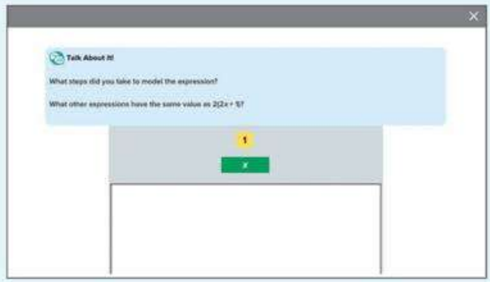
Interactive Presentation



Explore, Slide 1 of 8



Explore, Slide 4 of 8

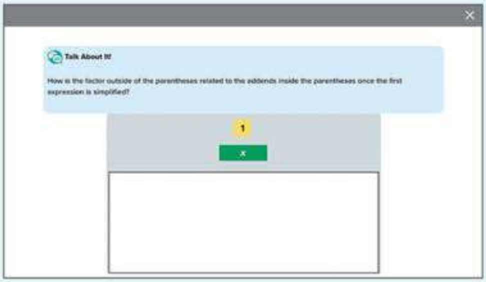
DRAG & DROP

Throughout the Explore, students drag algebra tiles to explore and model the Distributive Property.

WATCH

On Slide 3, students watch a video to learn about using algebra tiles to model the Distributive Property.

Interactive Presentation



Explore, Slide 7 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Use Algebra Tiles to Model the Distributive Property *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Explain to students the benefit of using algebra tiles as they can manipulate the tiles to represent and simplify the expressions, visualize the results, and make conjectures about how to use algebra tiles to model using the Distributive Property.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

How is the factor outside of the parentheses related to the addends inside the parentheses once the Distributive Property is applied? Sample answer: The factor outside of the parentheses is multiplied by the addends inside of the parentheses.

Your Notes

Go Online Watch the animation to see how to use the Distributive Property to expand an expression.

The animation shows how to expand the expression $a(b + c)$.

$a(b + c) = a \cdot b + a \cdot c$ — Multiply each term inside the parentheses by a. Then simplify.

The expression can be written as $ab + ac$.

Consider the expression $2(3 + 5)$.

$2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$ — Multiply each term inside the parentheses by 2.

$6 + 10$ — Multiply 2 by 3 and 2 by 5.

The expression can be written as $6 + 10$ or 16.

Consider the expression $3(x + 4)$.

$3(x + 4) = 3 \cdot x + 3 \cdot 4$ — Multiply each term inside the parentheses by 3.

$3x + 12$ — Simplify.

The expression can be written as $3x + 12$.

Talk About It! When there are only numbers in an expression, such as $3(4 + 9)$, is the Distributive Property the only way to evaluate the expression?

no; Sample answer: I can use the order of operations to add 4 and 9 to obtain 13. Then multiply 13 by 3 to obtain 39.

Talk About It! Does the Distributive Property apply to subtraction? For example, does $3(8-2) = 3(8) - 3(2)$? Does it apply to all numbers? Explain.

yes; Sample answer: $3(8-2) = 18$ and $3(8) - 3(2) = 18$. If I try other numbers the expressions are still equal.

Pause and Reflect

What questions do you have about the Distributive Property as a result of this Learn? Can you begin to think of an instance where the Distributive Property could be beneficial?

Record your observations here

See students' observations.

Interactive Presentation

Learn, The Distributive Property, Slide 2 of 3

WATCH

On Slide 2, students watch the animation to learn about how the Distributive Property can be used to expand an expression.

Learn The Distributive Property *(continued)*

Go Online to have your students watch the animation on Slide 2. The animation illustrates the Distributive Property.

Teaching Notes

SLIDE 2

You may wish to pause the animation after the Distributive Property was used to expand the expression $a(b + c)$. Some students may incorrectly apply the Distributive Property and think that the expanded expression is equivalent to $ab + c$. Remind them that the term outside the parentheses is multiplied by each term inside the parentheses, so a must also be multiplied by c. For each of the next two expressions in the animation, $2(3 + 5)$ and $3(x + 4)$, you may wish to pause the animation and have students expand the expression. Then replay the animation and have students check their work.

Talk About It!

SLIDE 3

Mathematical Discourse

When there are only numbers in an expression, such as $3(4 + 9)$, is the Distributive Property the only way to evaluate the expression? no; Sample answer: I can use the order of operations to add 4 and 9 to obtain 13. Then multiply 13 by 3 to obtain 39.

Does the Distributive Property apply to subtraction? For example, does $3(8 - 2) = 3(8) - 3(2)$? Does it apply to all numbers? Explain. yes; Sample answer: $3(8 - 2) = 18$ and $3(8) - 3(2) = 18$. If I try other numbers, the expressions are still equal.

Example 1 Use the Distributive Property

Objective

Students will use the Distributive Property to expand algebraic expressions.

Questions for Mathematical Discourse

SLIDE 2

AL Which quantity is being distributed? To which quantities is it being distributed? The quantity 2 is being distributed to x and 3.

AL What operation represents the act of the distribution? multiplication

OL Explain why the expanded expression $2x + 6$ cannot be further evaluated or simplified. Sample answer: The expression contains a variable and is currently in simplest form, since we do not know the value of the variable.

BL A classmate obtained the answer $2x + 5$. What was the most likely mistake? Sample answer: The classmate may have added 2 and 3 instead of multiplying 2 by 3.

Example 2 Use the Distributive Property

Objective

Students will use the Distributive Property to multiply a whole number and a rational number.

Questions for Mathematical Discourse

SLIDE 2

AL How can you write $3\frac{1}{2}$ as a sum of two numbers? Sample answer: $3\frac{1}{2} = 3 + \frac{1}{2}$

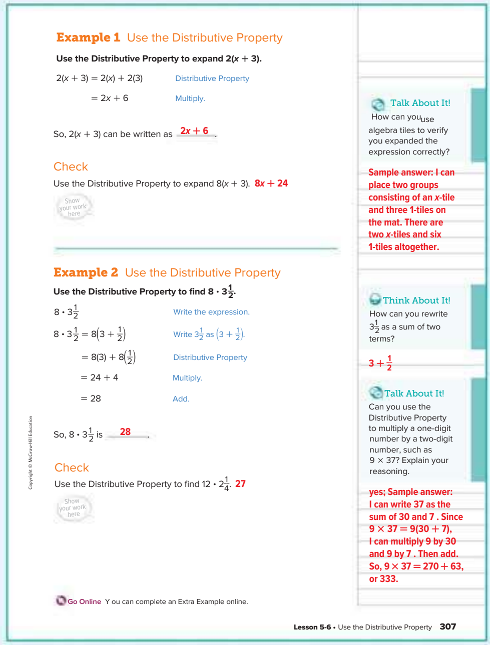
OL Why does writing $3\frac{1}{2}$ as a sum of two terms and using the Distributive Property help you find the product mentally? Sample answer: I can mentally find the product of 8 and 3, and the product of 8 and $\frac{1}{2}$. Then I can add them mentally.

OL How can you estimate the product? Sample answer: $3\frac{1}{2}$ is between 3 and 4. So the product of 8 and $3\frac{1}{2}$ will be between 8(3), or 24, and 8(4), or 32.

BL Can you use a similar process to find $8\frac{1}{2} \cdot 3$? Explain. yes; Sample answer: Since multiplication is commutative, I can write $8\frac{1}{2} \cdot 3$ as $3 \cdot 8\frac{1}{2}$. Then I can write $8\frac{1}{2}$ as $8 + \frac{1}{2}$ and use the Distributive Property.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Use the Distributive Property

Use the Distributive Property to expand $2(x + 3)$.

$2(x + 3) = 2(x) + 2(3)$ — Distributive Property

$= 2x + 6$ — Multiply.

So, $2(x + 3)$ can be written as $2x + 6$.

Check

Use the Distributive Property to expand $8(x + 3)$. $8x + 24$

Show your work here

Talk About It!

How can you use algebra tiles to verify you expanded the expression correctly?

Sample answer: I can place two groups consisting of an x-tile and three 1-tiles on the mat. There are two x-tiles and six 1-tiles altogether.

Example 2 Use the Distributive Property

Use the Distributive Property to find $8 \cdot 3\frac{1}{2}$.

$8 \cdot 3\frac{1}{2}$ — Write the expression.

$8 \cdot 3\frac{1}{2} = 8\left(3 + \frac{1}{2}\right)$ — Write $3\frac{1}{2}$ as $\left(3 + \frac{1}{2}\right)$.

$= 8(3) + 8\left(\frac{1}{2}\right)$ — Distributive Property

$= 24 + 4$ — Multiply.

$= 28$ — Add.

So, $8 \cdot 3\frac{1}{2}$ is 28.

Think About It!

How can you rewrite $3\frac{1}{2}$ as a sum of two terms?

$3 + \frac{1}{2}$

Talk About It!

Can you use the Distributive Property to multiply a one-digit number by a two-digit number, such as 9×37? Explain your reasoning.

yes; Sample answer: I can write 37 as the sum of 30 and 7. Since $9 \times 37 = 9(30 + 7)$, I can multiply 9 by 30 and 9 by 7. Then add. So, $9 \times 37 = 270 + 63$, or 333.

Check

Use the Distributive Property to find $12 \cdot 2\frac{1}{4}$. 27

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation

Example 2, Use the Distributive Property, Slide 2 of 4

CLICK

On Slide 2 of Example 2, students move through the steps to simplify the expression.

TYPE

On Slide 2 of Example 2, students determine the value of the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Learn Greatest Common Factor and the Distributive Property

You can use the greatest common factor (GCF) to rewrite the sum of two whole numbers with a common factor as a product. The Distributive Property allows you to write the sum as the product of the greatest common factor and the sum of the remaining factors.

When numerical or algebraic expressions are written as a product of their factors, the process is called **factoring the expression**. To factor an expression, follow these steps.

1. Find the GCF of the terms.

2. Write the terms as a product of factors.

3. Rewrite the expression as the product of two terms.

Go Online Watch the animation to see how to use the GCF and the Distributive Property to factor an expression.

The animation explains how to factor the expression 8 + 56.

$8 = 2 \cdot 2 \cdot 2$
$56 = 2 \cdot 2 \cdot 2 \cdot 7$ — Find the GCF of the terms.

The GCF is 8.

$8 + 56 = 8(1) + 8(7)$ — Use the GCF to write each term as a product of factors.

$= 8(1 + 7)$ — Rewrite the expression as a product of two terms.

You can also use the GCF to factor expressions containing variables, such as $45x + 6$.

$45x = 3 \cdot 3 \cdot 5 \cdot x$
$6 = 3 \cdot 2$ — Find the GCF of the terms.

The GCF is 3.

$45x + 6 = 3(15x) + 3(2)$ — Use the GCF to write each term as a product of factors.

$= 3(15x + 2)$ — Rewrite the expression as a product of two terms.

Talk About It!
How can you determine what remains in the parentheses after the GCF has been factored out of the expression?

Sample answer: The remaining terms in the parentheses are the quotients of the original terms each divided by the GCF.

Interactive Presentation

Learn, Greatest Common Factor and the Distributive Property, Slide 2 of 3

WATCH

On Slide 2, students watch an animation to learn about how to use the GCF and the Distributive Property to factor an expression.

Learn Greatest Common Factor and the Distributive Property

Objective

Students will learn how to factor expressions using the greatest common factor.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to use reasoning about how multiplication and division are inverse operations.

Teaching Notes

SLIDE 1

Students will learn how to use the greatest common factor to rewrite the sum of two whole numbers as the product of the greatest common factor and the sum of the remaining factors. This process is called *factoring the expression*. You may wish to point out that previously, students used the Distributive Property to write a product as a sum, but in this case, they are writing a sum of two terms that have a common factor as a product.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates using the Distributive Property to factor an expression.

Talk About It!

SLIDE 3

Mathematical Discourse

How can you determine what remains in the parentheses after the GCF has been factored out of the expression? Sample answer: The remaining terms in the parentheses are the quotients of the original terms each divided by the GCF.

DIFFERENTIATE

Reteaching Activity AL

If any of your students have difficulty finding the GCF, use the following activity to support their learning. Have students determine the factors of the following numbers. Then have them work with a partner to select 2 or 3 of the numbers and determine their common factors, and their greatest common factor. For example, they may select the numbers 10 and 15. The common factors are 1 and 5. So, the greatest common factor is 5.

12 1, 2, 3, 4, 6, 12

10 1, 2, 5, 10

15 1, 3, 5, 15

8 1, 2, 4, 8

Example 3 Use GCF to Factor Numerical Expressions

Objective

Students will factor numerical expressions using the greatest common factor.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language, such as *commutative*, when explaining why the two expressions are equivalent.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the expression, paying careful attention to rewriting each term as a product of the GCF and its remaining factor.

Questions for Mathematical Discourse

SLIDE 2

AL How can you find the GCF of two numbers? Sample answer: I can write the factors of each number, circle the common factors, and then select the greatest of the circled numbers.

OL Why is 9 the number that is multiplied by both 5 and 8? 9 is the GCF of 45 and 72

OL How can you determine that 5 and 8 are left inside the parentheses? After factoring out 9 from 45 and 72, 5 and 8 are left because 9(5) = 45, and 9(8) = 72.

BL The two numbers inside the parentheses, 5 and 8, do not have any factors in common except for 1. Why is this true? Sample answer: This is true because we factored out the GCF. If they had a factor in common (other than 1), then we would not have been using the GCF.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Use GCF to Factor Numerical Expressions

Use the GCF to factor 45 + 72.

45 + 72	Write the expression.
45 + 72 = 9(5) + 9(8)	Rewrite each term as a product of the GCF, 9, and its remaining factor.
= 9(5 + 8)	Use the Distributive Property to write as the product of two terms.

So, 45 + 72 in factored form is 9(5 + 8).

Think About It! How can you find the GCF of 45 and 72?

See students' responses.

Talk About It! Are the expressions 9(5 + 8) and (5 + 8)9 equal to the same value? Explain your reasoning.

yes; Sample answer: Each of the expressions is equal to 117. The expressions are equivalent because multiplication is commutative.

Check

Use the GCF to factor 80 + 56. 8(10 + 7)

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How did your prior knowledge of greatest common factor help you to understand the concepts in this Learn and Example?

Record your observations here

See students' observations.

Interactive Presentation

Example 3, Use GCF to Factor Numerical Expressions, Slide 2 of 4

CLICK On Slide 2, students move through the steps to factor the numerical expression using the GCF.

TYPE On Slide 2, students determine the GCF.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
What is the GCF of the two terms?

3

Talk About It!
Is it possible to factor an expression using a factor other than the GCF? Explain.

yes; Sample answer: As long as the value that is factored out of the expression is a common factor, it can be used to factor an expression, but it won't be factored completely.

Example 4 Use GCF to Factor Algebraic Expressions

Use the GCF to factor $6x + 15$.

$6x + 15$	Write the expression.
$6x + 15 = 3(2x) + 3(5)$	Rewrite each term as a product of the GCF, 3, and its remaining factor.
$= 3(2x + 5)$	Use the Distributive Property to write as the product of two terms.

So, $6x + 15$ in factored form is $3(2x + 5)$.

Check

Factor $36x + 30$. Use the GCF. $6(6x + 5)$

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you make any errors while factoring the algebraic expression in the Check exercise? If so, was it in finding the GCF or rewriting each term? If not, how could you check the accuracy of your answer?

Record your observations here

See students' observations.

310 **Module 5** • Numerical and Algebraic Expressions

Interactive Presentation

Example 4, Use GCF to Factor Algebraic Expressions, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to factor the algebraic expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Use GCF to Factor Algebraic Expressions

Objective

Students will factor algebraic expressions using the greatest common factor.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of what it means to factor an expression and explain why it is possible to use factors other than the GCF to factor an expression, even though the expression won't be factored completely.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the expression, noting that a variable has been introduced. Students should reason that $6x$ represents *6 times x* in order to factor the expression accurately.

Questions for Mathematical Discourse

SLIDE 2

AL What is the GCF of 6 and 15? 3

OL What is the remaining factor when 3 is factored out of 15? 5

OL What is the remaining factor when 3 is factored out of $6x$? Explain. $2x$; Sample answer: Since $6x$ represents *6 times x*, after factoring out the 3, the remaining expression is $2x$.

BL What would be the GCF of $6x$ and $15x$? Why? $3x$; Sample answer: Both terms contain a factor of 3 and a factor of x.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Money

Objective

Students will come up with their own strategy to solve an application problem involving calculating change.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How much does one bottle of juice cost?
- How would you estimate the cost of 5 bottles?
- Will the change be greater than or less than $10?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Money

Wen is buying bottles of apple juice and wants to mentally calculate how much they will cost. He buys 5 bottles of juice at $2.15 each. Use mental math and the Distributive Property to determine how much change he will receive from $20.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$9.25; See students' work.

Talk About It! Is there another method you could use to solve this problem?

See students' responses.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Interactive Presentation

Apply, Money

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Martin exercised four days for 65 minutes each day. His goal is to exercise for a total of 300 minutes in 5 days. How many minutes does he need to exercise on the fifth day to meet his goal?

Show your work here

40 minutes

Go Online You can complete an Extra Example online.

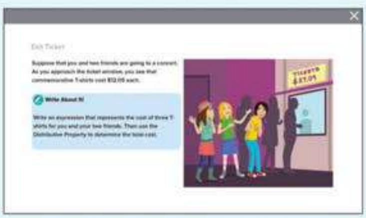
Pause and Reflect

Write a real-world problem that involves writing an expression using the Distributive Property. Explain how you came up with that problem. Exchange problems with a classmate and solve each other's problem.

Record your observations here

See students' observations.

Interactive Presentation



Exit Ticket

Essential Question Follow-Up

How can we communicate algebraic relationships with mathematical symbols? In this lesson, students learned how to rewrite expressions using the Distributive Property. Encourage them to discuss with a partner how using the Distributive Property makes it easier to simplify numerical and algebraic expressions. For example, how rewriting the expression for the perimeter of a rectangle, $2w + 2\ell$, as $2(w + \ell)$ allows you to multiply the sum of one width and one length by two.

Exit Ticket

Refer to the Exit Ticket slide. Write an expression that represents the cost of three T-shirts for you and your two friends. Then use the Distributive Property to determine the total cost. $3(12 + 0.05) = 3(12) + 3(0.05)$, or $36 + 0.15$; The total cost is $36 + $0.15, or $36.15.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 13, 15, 17–20
- Extension: The FOIL Method
- ALEKS The Distributive Property

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–12, 15, 18, 20
- Extension: The FOIL Method
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Evaluating and Writing Expressions

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Evaluating and Writing Expressions

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use the Distributive Property to expand expressions	1–3
1	use the Distributive Property to multiply a whole number and a rational number	4–6
1	factor numerical expressions using the greatest common factor	7–9
1	factor algebraic expressions using the greatest common factor	10–12
2	extend concepts learned in class to apply them in new contexts	13, 14
3	solve application problems that involve the Distributive Property	15, 16
3	higher-order and critical thinking skills	17–20

Common Misconception

Some students may factor algebraic expressions incorrectly when using the greatest common factor. In Exercise 11, students may use the correct GCF, but incorrectly factor part of the expression by neglecting the variable, resulting in 6(4 + 1). Encourage students to remember that $6x$ is really 6 • x, so when factoring out 6, only x remains.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Use the Distributive Property to expand each algebraic expression. (Example 1)

1. $3(x + 8)$ **$3x + 24$**

2. $5(6 + x)$ **$30 + 5x$**

3. $9(3 + x)$ **$27 + 9x$**

Use the Distributive Property to simplify each expression. (Example 2)

4. $12 \cdot 3\frac{3}{4}$ **45**

5. $15 \cdot 2\frac{2}{3}$ **40**

6. $8 \cdot 4\frac{1}{2}$ **36**

Use the GCF to factor each numerical expression. (Example 3)

7. $16 + 48$ **$16(1 + 3)$**

8. $35 + 63$ **$7(5 + 9)$**

9. $26 + 39$ **$13(2 + 3)$**

Use the GCF to factor each algebraic expression. (Example 4)

10. $8x + 16$ **$8(x + 2)$**

11. $24 + 6x$ **$6(4 + x)$**

12. $42 + 7x$ **$7(6 + x)$**

13. Five friends each bought a shirt and a pair of shoes. The table shows the cost of the items. The expression $5(x + 24)$ shows the total amount of money they spent. Expand the expression using the Distributive Property. **$5x + 120$**

Item	Cost ($)
Shirt	x
Shoes	24.00

Test Practice

14. Multiple Choice Which expression has the same value as 9 + 24?

Ⓐ 3(3 + 24)
Ⓑ 3(3 + 8)
Ⓒ 3(9 + 8)
Ⓓ 9(1 + 24)

Apply *indicates multi-step problem

*15. The table shows the cost of snacks at a basketball game. Mrs. Cooper buys 6 nachos for her daughter and 5 friends. Use mental math and the Distributive Property to determine how much change she will receive from $30.

Item	Cost
Nachos	$4.10
Popcorn	$2.85

$5.40

*16. Jeffery is making 4 batches of chocolate chip cookies. Each batch of cookies needs $2\frac{3}{4}$ cups of chocolate chips. If he has 96 ounces of chocolate chips, how many ounces will be left over? Use mental math and the Distributive Property.

8 ounces

Higher-Order Thinking Problems

17. **MP Identify Structure** Write two equivalent numerical expressions involving fractions that illustrate the Distributive Property.

Sample answer: $8\left(4\frac{3}{4}\right) = 8\left(4 + \frac{3}{4}\right)$

18. **MP Justify Conclusions** A student rewrote the expression $4(5 + x)$ as $20 + x$. Did the student rewrite the expression correctly? Justify your reasoning.

no; Sample answer: The student did not multiply both terms inside the parentheses by 4. The correct expression should be $20 + 4x$.

19. **MP Construct an Argument** Is the expression $2(6x)$ equivalent to $(2 \cdot 6)(2 \cdot x)$? Explain why or why not.

no; Sample answer: The Distributive Property combines addition and multiplication. The expression $2(6x)$ is one term with three factors and does not contain addition. $2(6x)$ is equal to $12x$.

20. Are the expressions $4(x + 5) + 1$ and $(2x + 16) + 2x + 5$ equivalent? Explain.

yes; Sample answer: $4(x + 5) + 1$ simplifies to $4x + 21$ and $(2x + 16) + 2x + 5$ simplifies to $4x + 21$. Since both expressions simplify to $4x + 21$ they are equivalent.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 17, students will write two equivalent numerical expressions involving fractions that illustrate the Distributive Property. Encourage students to check that their answer includes fractions and illustrates the Distributive Property correctly.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students will determine if a student used the Distributive Property correctly. Encourage students to support their answer with a well-constructed explanation.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 19, students will determine if two expressions are equivalent. Encourage students to form an explanation that supports their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.
Use with Exercises 15–16 Have students work in groups of 3–4. After completing Exercise 15, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 16.

Create your own higher-order thinking problem.
Use with Exercises 17–20 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Learn Use Properties to Identify Equivalent Expressions

Objective

Students will learn how to use mathematical properties to identify equivalent expressions.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to defend their answer using their understanding of equivalent expressions. Students may have different responses; encourage them to listen to each other and determine whether or not they make sense and/or could be plausible arguments.

6 Attend to Precision In the drag-and-drop activity on Slide 2, encourage students to think about the precise meaning of each property and how the equations illustrate the properties.

7 Look for and Make Use of Structure Students should analyze the structure of each equation in order to drag it into the appropriate bin and complete the activity.

Teaching Notes

SLIDE 1

Students will learn the definition of equivalent expressions and how to use the *Commutative*, *Associative*, *Distributive*, and *Identity Properties* to create pairs of equivalent expressions in the form of equations. You may wish to have students restate the properties in their own words to deepen their understanding.

Go Online to have your students watch the animation on Slide 3. The animation illustrates using properties of operations to identify equivalent expressions.

(continued on next page)

DIFFERENTIATE

Language Development Activity ELL

If any of your students have difficulty remembering the different properties, have students create a table with each property as an entry. Then have students write an example that illustrates each property. Allow students to use this table when working through the problems throughout this lesson.

Lesson 5-7

Equivalent Algebraic Expressions

I Can... use the properties of operations to write expressions in simplest form and check to see if two expressions are equivalent.

What Vocabulary Will Y ou Learn?
Associative Property
Commutative Property
Distributive Property
equivalent expressions
Identity Property
simplest form

Explore Properties and Equivalent Expressions

Online Activity Y ou will use algebra tiles and mathematical properties to identify equivalent expressions.

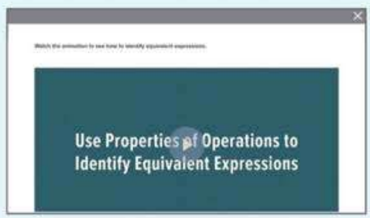
Learn Use Properties to Identify Equivalent Expressions

Equivalent expressions are expressions that have the same value. Algebraic expressions are equivalent when they have the same value, no matter what value is substituted for the variable(s). Y ou can write an equivalent expression by applying the properties of operations to an expression.

Commutative Property	Associative Property
Words	**Words**
The order in which numbers are added or multiplied does not change the sum or product.	The order in which numbers are grouped when added or multiplied does not change the sum or product.
Numbers	**Numbers**
$7 + 9 = 9 + 7$ $8 \cdot 4 = 4 \cdot 8$	$3 + (4 + 7) = (3 + 4) + 7$ $7 \cdot (6 \cdot 2) = (7 \cdot 6) \cdot 2$
Variables	**Variables**
$a + b = b + a$ $a \cdot b = b \cdot a$	$a + (b + c) = (a + b) + c$ $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(continued on next page)

Interactive Presentation



Learn, Use Properties to Identify Equivalent Expressions, Slide 3 of 4

WATCH

On Slide 3, students watch an animation to learn about using properties to identify equivalent expressions.

Lesson 5-7

Equivalent Algebraic Expressions

6.EE.A.3, 6.EE.A.4

LESSON GOAL

Students will identify and generate equivalent algebraic expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Properties and Equivalent Expressions

Learn: Use Properties to Identify Equivalent Expressions
Example 1: Identify Equivalent Expressions
Learn: Use Substitution to Identify Equivalent Expressions
Examples 2–3: Determine Equivalency Using Substitution
Learn: Combine Like Terms
Examples 4–5: Combine Like Terms
Learn: Apply Properties to Write Equivalent Expressions
Example 6: Write Equivalent Expressions
Apply: Shipping

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

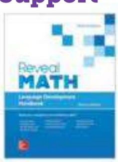
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Multiple Sets of Grouping Symbols		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 33 of the *Language Development Handbook* to help your students build mathematical language related to equivalent algebraic expressions.



ELL You can use the tips and suggestions on page T33 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.A** by identifying and generating equivalent algebraic expressions.
Standards for Mathematical Content: **6.EE.A.3, 6.EE.A.4,** *Also addresses 6.EE.A.2*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students used the Distributive Property.
6.EE.A.2.B, 6.EE.A.3, 6.NS.B.4

Now
Students identify and generate equivalent algebraic expressions.
6.EE.A.3, 6.EE.A.4

Next
Students will use substitution to solve one-step equations.
6.EE.B.5

Rigor

The Three Pillars of Rigor

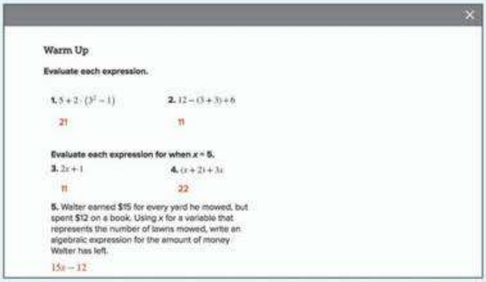
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students identify and generate equivalent algebraic expressions to continue to expand their *understanding* of expressions. They learn how to use the Commutative, Associative, Distributive, and Identity Properties to write equivalent expressions, and build *fluency* by using substitution and combining like terms to simplify expressions. They also *apply* their understanding of equivalent algebraic expressions to solve real-world problems.

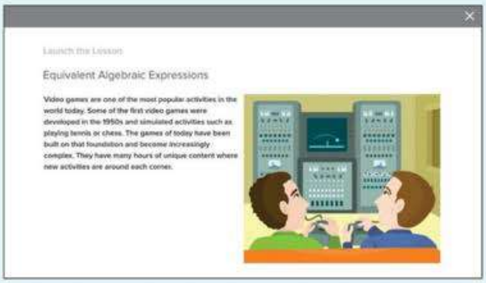
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

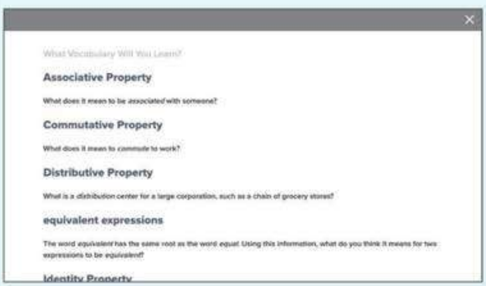
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- evaluate numerical and algebraic expressions using order of operations (Exercises 1–4)
- using variables (Exercise 5)

Answers

1. 21 **4.** 22

2. 11 **5.** $15x - 12$

3. 11

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the cost of video games, using expressions.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- What does it mean to be *associated* with someone? Sample answer: to be connected to, or grouped with, someone
- What does it mean to *commute* to work? Sample answer: to travel back and forth
- What is a *distribution* center for a large corporation, such as a chain of grocery stores? Sample answer: It is a central location that organizes and stores a large quantity of products, and then distributes them to the individual stores.
- The word *equivalent* has the same root as the word *equal*. Using this information, what do you think it means for two expressions to be *equivalent*? Sample answer: It means that the two expressions have the same value.

Explore Properties and Equivalent Expressions

Objective

Students will explore using mathematical properties to identify equivalent expressions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with several algebraic expressions using the same variables. They will be asked to identify groups of expressions that are equivalent. Throughout this activity, students will use appropriate tools, such as algebra tiles, to assist them in making conjectures.

Inquiry Question

How can you use mathematical rules and properties to identify equivalent expressions? Sample answer: Applying properties can help me reorder or regroup like terms and simplify the expression so I can determine which expressions are equivalent.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

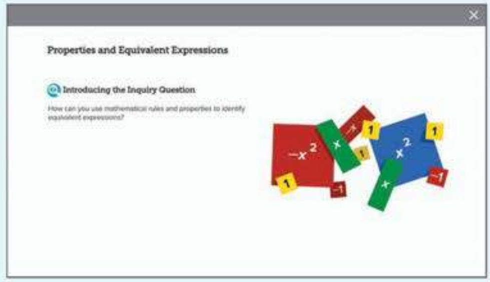
SLIDE 3

Mathematical Discourse

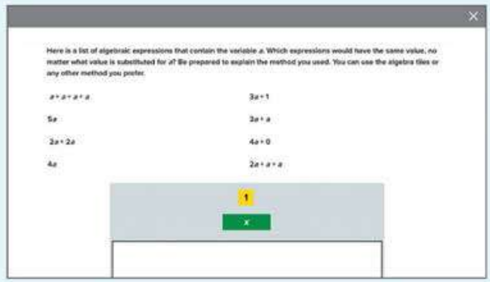
Compare your list with your partner's list. Explain what you did to determine which expressions had the same value, no matter what number *a* represents. Can you use the rules and properties of mathematics to justify your reasoning? See students' responses.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8

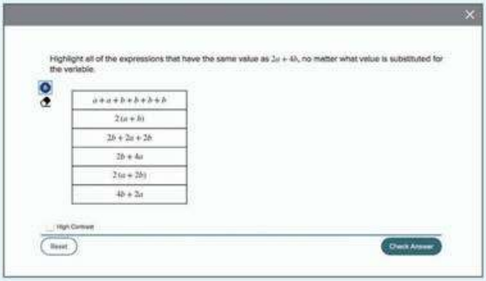


Explore, Slide 2 of 8

DRAG & DROP

On Slide 2, students drag algebra tiles to model mathematical properties and equivalent expressions.

Interactive Presentation



Explore, Slide 6 of 8

CLICK

On Slide 6, students highlight the expressions that have the same value as $2a + 4b$.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Properties and Equivalent Expressions *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use the algebra tiles to model each expression. Students should think about the meaning of the different colors and sizes of the algebra tiles and how the manipulation of them can help when determining the equivalent expressions.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Compare your list with your partner's list. Explain what you did to determine which expressions had the same value, no matter what numbers a and b represent. Can you use the rules and properties of mathematics to justify your reasoning? See students' responses.

Your Notes

Distributive Property	Identity Property
Words	**Words**
To multiply a sum by a number, multiply each addend by the number outside the parentheses.	The sum of an addend and 0 is the addend. The product of a factor and 1 is the factor.
Numbers	**Numbers**
$2(7 + 9) = 2(7) + 2(9)$ $4(5 - 2) = 4(5) - 4(2)$	$13 + 0 = 13$ $7 \cdot 1 = 7$
Variables	**Variables**
$a(b + c) = a(b) + a(c)$ $a(b - c) = a(b) - a(c)$	$a + 0 = a$ $a \cdot 1 = a$

Go Online Watch the animation to see how to use properties to identify equivalent expressions.

The animation explains how to use properties to determine whether or not $4(2x + 3) + 5$ and $(4x + 3) + 5$ are equivalent expressions.

Simplify the expressions and draw a conclusion.

$4(2x + 3) + 5 = 8x + 12 + 5$ Distributive Property

$= 8x + (12 + 5)$ Associative Property

$= 8x + 17$

$(4x + 3) + 5 = 4x + (3 + 5)$ Associative Property

$= 4x + 8$

Because the simplified expressions have different terms, they will *never* have the same value. So, the expressions are *not* equivalent.

Talk About It! Are the expressions that are written before and after a property is applied equivalent? Explain your reasoning.

Yes; Sample answer: The properties applied to expressions can be used to help simplify expressions, and they do not change the value of the expression.

The animation also explains how to use properties to determine whether or not $8 + 3n + 2$ and $3(n + 1) + 7$ are equivalent expressions.

Simplify the expressions and draw a conclusion.

$8 + 3n + 2 = 3n + 8 + 2$ Commutative Property

$= 3n + (8 + 2)$ Associative Property

$= 3n + 10$

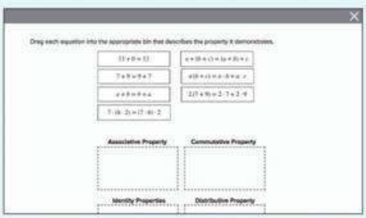
$3(n + 1) + 7 = 3n + 3 + 7$ Distributive Property

$= 3n + (3 + 7)$ Associative Property

$= 3n + 10$

Because the simplified expressions have the same terms, they will *always* have the same value. So, the expressions are equivalent.

Interactive Presentation



Learn, Use Properties to Identify Equivalent Expressions, Slide 2 of 4

DRAG AND DROP

On Slide 2, students drag each equation into the appropriate bin that describes the property it demonstrates.

Learn Use Properties to Identify Equivalent Expressions *(continued)*

Teaching Notes

SLIDE 2

You may wish to have student volunteers come up to the board to drag each equation to the appropriate bin. Ask students to describe what they are looking for in the equation, in order to determine which property is being applied.

Talk About It!

SLIDE 4

Mathematical Discourse

Are the expressions that are written before and after a property is applied equivalent? Explain your reasoning. yes; Sample answer: The properties applied to expressions can be used to help simplify expressions, and they do not change the value of the expression.

Example 1 Identify Equivalent Expressions

Objective

Students will use mathematical properties to identify equivalent expressions.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to simplify each expression accurately and efficiently, making sense of each step and each property applied. They should be able to explain, using the properties of operations, why the expressions are equivalent.

As students discuss the *Talk About It!* question on Slide 4, encourage them to use clear and precise mathematical language to support their reasoning as to why the expressions they generated are not equivalent to $3x + 23$.

Questions for Mathematical Discourse

SLIDE 2

AL What property can be applied to $3(x + 7)$? the Distributive Property

OL In simplifying $3x + 21 + 2$ as $3x + 23$, what property allows you to add 21 and 2? Sample answer: The Associative Property allows us to regroup addition.

BL Apply the Commutative Property of Multiplication to $3(x + 7)$ and then simplify the expression. Did you get a different result? No; the expressions are equivalent; $(x + 7)\,3 + 2 = x(3) + 7(3) + 2 = 3x + 21 + 2$, or $3x + 23$

SLIDE 3

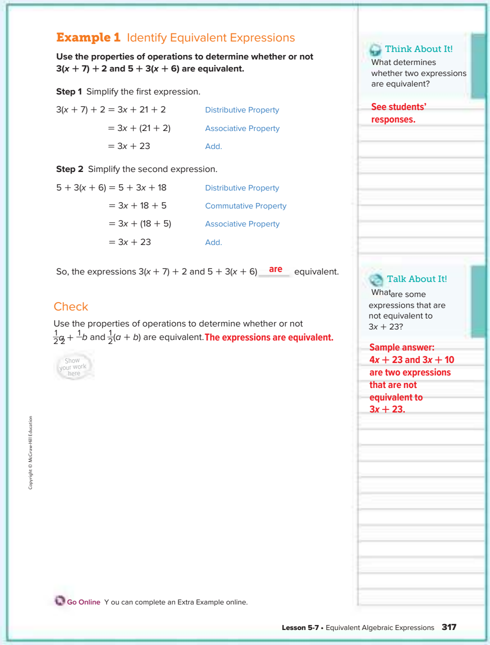
AL When are two expressions equivalent? Sample answer: Two expressions are equivalent when they both simplify to the same expression, or have the same value.

OL Why is the Commutative Property and Associative Property helpful? Sample answer: The like terms are grouped together in order to add them first.

BL Generate a different expression that is equivalent to $3x + 23$. Sample answer: $2(x + 8) + x + 7$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Identify Equivalent Expressions

Use the properties of operations to determine whether or not $3(x + 7) + 2$ and $5 + 3(x + 6)$ are equivalent.

Step 1 Simplify the first expression.

$3(x + 7) + 2 = 3x + 21 + 2$	Distributive Property
$= 3x + (21 + 2)$	Associative Property
$= 3x + 23$	Add.

Step 2 Simplify the second expression.

$5 + 3(x + 6) = 5 + 3x + 18$	Distributive Property
$= 3x + 18 + 5$	Commutative Property
$= 3x + (18 + 5)$	Associative Property
$= 3x + 23$	Add.

So, the expressions $3(x + 7) + 2$ and $5 + 3(x + 6)$ ___are___ equivalent.

Think About It! What determines whether two expressions are equivalent? See students' responses.

Talk About It! What are some expressions that are not equivalent to $3x + 23$? Sample answer: $4x + 23$ and $3x + 10$ are two expressions that are not equivalent to $3x + 23$.

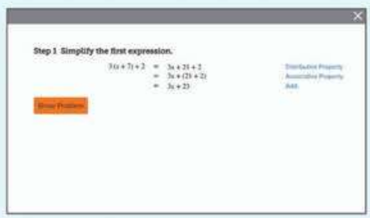
Check

Use the properties of operations to determine whether or not $\frac{1}{2}a + \frac{1}{2}b$ and $\frac{1}{2}(a + b)$ are equivalent. The expressions are equivalent.

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Identify Equivalent Expressions, Slide 2 of 5

CLICK

On Slide 3, students select the correct phrase to identify an equivalent expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Use Substitution to Identify Equivalent Expressions

Go Online Watch the animation to learn about using substitution to identify when expressions are equivalent.

The animation explains how to determine whether $y + y + y$ and $3y$ are equivalent expressions using substitution.

Step 1 Evaluate the expressions for the same value of the variable.

Evaluate each expression when $y = 0$.

$y + y + y = 0 + 0 + 0 = 0$ $\quad$ $3y = 3(0) = 0$

Repeat Step 1 using a different value for the variable. Evaluate each expression when $y = 5$.

$y + y + y = 5 + 5 + 5 = 15$ $\quad$ $3y = 3(5) = 15$

Step 2 Draw a conclusion.

Based on substitution, the expressions appear to be equivalent. If you continue substituting additional values of x, you will see that the expressions will always be equivalent, regardless of the value of the variable being substituted.

The animation also explains how to determine whether $5(x + 4)$ and $5x + 4$ are equivalent expressions using substitution.

Step 1 Evaluate the expressions for the same value of the variable.

Evaluate each expression when $x = 0$.

$5(x + 4) = 5(0 + 4) = 5(4) = 20$ $\quad$ $5x + 4 = 5(0) + 4 = 0 + 4 = 4$

Step 2 Draw a conclusion.

Based on substitution, the expressions are not equivalent.

Talk About It! Why is it important to substitute more than one value into an expression to help determine equivalency?

Sample answer: It is possible that two expressions could show the same value for a particular value, but not for every value. For example, $2x$ and $3x$ have the same value when $x = 0$, but not when $x = 1$.

Interactive Presentation

Learn, Use Substitution to Identify Equivalent Expressions, Slide 1 of 2

WATCH

On Slide 1, students watch the animation to learn how to use substitution to identify when expressions are equivalent.

Learn Use Substitution to Identify Equivalent Expressions

Objective

Students will understand that two expressions are equivalent if they have the same value regardless of which value is substituted into them.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use clear and precise mathematical language in their explanation to explain why it is important to substitute more than one value when determining equivalency.

Go Online to have your students watch the animation onSlide 1. The animation illustrates using substitution to identify whether or not expressions are equivalent.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the expressions $y + y + y$ and $3y$ are shown. Ask students to work with a partner to determine if they are equivalent expressions. They must be able to justify their reasoning mathematically. Have pairs of students share their responses with the class. Then have students continue watching the animation to see how substitution can be used to verify the expressions are equivalent. Repeat a similar process for the second pair of expressions, $5(x + 4)$ and $5x + 4$. Students should note these expressions are not equivalent.

Talk About It!

SLIDE 2

Mathematical Discourse

Why is it important to substitute more than one value into an expression to help determine equivalency? Sample answer: It is possible that two expressions could show the same value for a particular value, but not for every value. For example, $2x$ and $3x$ have the same value when $x = 0$, but not when $x = 1$.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Determine Equivalency Using Substitution

Objective

Students will use substitution to identify equivalent expressions.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to carefully and accurately substitute varying values of x into each expression in order to determine if they are equivalent. Students should be able to explain clearly why checking more than one value is necessary.

Questions for Mathematical Discourse

SLIDE 1

AL What happens when 0 is substituted into each expression? 1? 2? The values of the expressions are equal for these values of x.

OL Why is it important to check more than one value of x? Sample answer: Two expressions might agree for one value of x but not for another.

BL What is another method you can use to determine if the expressions are equivalent? Sample answer: The first expression is equivalent to $4x$, which is the same as the second expression.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Determine Equivalency Using Substitution

Use substitution to determine whether or not $2x + x + x$ and $4x$ are equivalent.

Let $x = 0$, 1, and 2. Substitute those values into both expressions. Then compare to determine whether or not they are equivalent.

$2x + x + x$	Write the expression.	$4x$
$2(0) + 0 + 0 = 0$	$x = 0$	$4(0) = 0$
$2(1) + 1 + 1 = 4$	$x = 1$	$4(1) = 4$
$2(2) + 2 + 2 = 8$	$x = 2$	$4(2) = 8$

When x is replaced with different values, the results are the same for both expressions.

So, the expressions are equivalent because they do have the same value when values are substituted in for the variable.

Talk About It!
Try substituting 3 more values for the variable. What do you notice?

Sample answer: When I substitute other values for the variable, the expressions are still equivalent.

Check

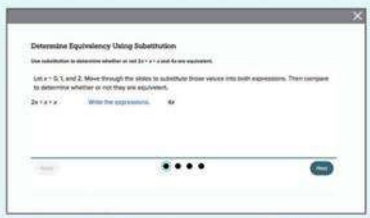
Use substitution to determine whether or not $3x + x + 4$ and $x + 2(x + 1) + x$ are equivalent. **The expressions are not equivalent.**

Show your work here

Go Online You can complete an Extra Example online.

Lesson 5-7 • Equivalent Algebraic Expressions 319

Interactive Presentation



Example 2, Determine Equivalency Using Substitution, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to determine if the expressions are equivalent.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Determine Equivalency Using Substitution

Use substitution to determine whether or not $\frac{1}{2}x + x^2 + \frac{1}{2}$ and $\frac{1}{2}x + 3x^2 + \frac{1}{2} - x^2$ are equivalent.

$\frac{1}{2}x + x^2 + \frac{1}{2}$	Write the expressions.	$\frac{1}{2}x + 3x^2 + \frac{1}{2} - x^2$
$\frac{1}{2}(0) + (0)^2 + \frac{1}{2} = \frac{1}{2}$	$x = 0$	$\frac{1}{2}(0) + 3(0)^2 + \frac{1}{2} - (0)^2 = \frac{1}{2}$
$\frac{1}{2}(1) + (1)^2 + \frac{1}{2} = 2$	$x = 1$	$\frac{1}{2}(1) + 3(1)^2 + \frac{1}{2} - (1)^2 = 3$
$\frac{1}{2}(2) + (2)^2 + \frac{1}{2} = 5\frac{1}{2}$	$x = 2$	$\frac{1}{2}(2) + 3(2)^2 + \frac{1}{2} - (2)^2 = 9\frac{1}{2}$

So, the expressions are **not equivalent** because they **do not** have the same value when $x = 1$ and $x = 2$.

Check

Use substitution to determine whether or not $y + 2y + 3$ and $1 + y + 2(y + 1)$ are equivalent. **The expressions are equivalent.**

Show your work here

Go Online You can complete an Extra Example online.

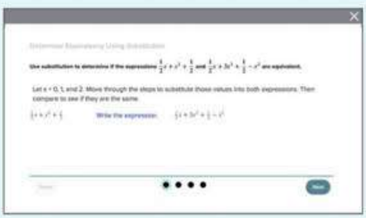
Pause and Reflect

Did you encounter any difficulty when using substitution to determine equivalency? What are some important things to remember as you progress through this lesson?

Record your observations here

See students' observations.

Interactive Presentation



Example 3, Determine Equivalency Using Substitution, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to determine if the expressions are equivalent.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Determine Equivalency Using Substitution

Objective

Students will use substitution to identify equivalent expressions.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to carefully and accurately substitute varying values of x into each expression in order to determine if they are equivalent. Students should be able to explain clearly why the expressions are not equivalent.

Questions for Mathematical Discourse

SLIDE 1

AL What happens when 0 is substituted into each expression? 1? 2? When 0 is substituted, the expressions have the same value, $\frac{1}{2}$. When 1 is substituted, the expressions do not have the same value, since $2 \neq 3$. When 2 is substituted, the expressions do not have the same value, since $5\frac{1}{2} \neq 9\frac{1}{2}$.

OL After you substitute 1 into each expression, do you need to substitute 2? Explain. no; Sample answer: After knowing that the expressions are not equivalent when substituting 1, I can stop.

BL How can you study the structure of each expression in order to determine that the expressions are not equivalent without using substitution? Sample answer: Both expressions have the same constant and the same coefficient on x. But the first expression has a coefficient of 1 on x^2, while the second expression has a coefficient of $3 - 1$, or 2 on x^2. Since these are different, the expressions are not equivalent.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Combine Like Terms

Objective

Students will learn how to combine like terms to simplify expressions.

Teaching Notes

SLIDE 1

Encourage students to understand how bar diagrams can be used to combine variable expressions in which the variable terms are the same. The expression $3x + 5x$ can be thought of as *three groups of x plus five groups of x*. Ask students if they can use a similar method to simplify the expression $3x + 5y$. Students should note that because the variable terms are not the same, the expressions cannot be simplified; *groups of x* and *groups of y* might not refer to groups of the same quantity, unless x and y are equal.

SLIDE 2

You may wish to pause the video after the expression $2x + 4 + 3x$ is shown. Have students work with a partner to use algebra tiles to model the expression. Have them discuss what strategies they can use to simplify the expression. Then have them simplify the expression. Then have them continue watching the video to compare their simplified expression with the one shown. Repeat, using a similar process, for the remaining expressions in the video, $x + 5x + x$ and $1 + x + 4 + 3x$.

Go Online to have your students watch the video on Slide 2. The video illustrates how to use algebra tiles to combine like terms in an algebraic expression.

(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling with combining like terms, have them work with a partner to complete the following activity for the expression $4a + 2b + 6a$. Let a be represented by a real-world object, such as a paper clip. Let b be represented by another real-world object, such as a pencil. Ask pairs to discuss the following questions.

How many paper clips will you use to represent $4a$? 4 paper clips

Is there another term in the expression that you will need paper clips to represent it? Explain. yes; $6a$ will be represented by 6 paper clips.

How can you represent $2b$? Why will you not use paper clips? Use 2 pencils; b is represented by a pencil, not a paper clip.

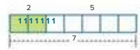
How many total paper clips do you have? What expression can represent it? 10 paper clips; $10a$

What is the simplified expression for $4a + 2b + 6a$? $10a + 2b$

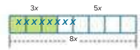
Learn Combine Like Terms

An expression is in **simplest form** if it has no like terms and no parentheses. You can use the structure of an algebraic expression to combine like terms and write it in simplest form.

When you have an expression with only constants, such as 5 + 2, you can combine these terms for a result of 7.



Sometimes you have an expression with like terms, such as $3x + 5x$, which means 3 groups of x plus 5 groups of x. You can combine these terms for a result of $8x$.

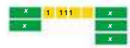


Algebra tiles can also be used to model and simplify an expression that contains like terms.

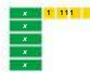
Go Online Watch the animation to learn about using algebra tiles to combine like terms in an algebraic expression.

The animation demonstrates how to simplify the expression $2x + 4 + 3x$.

To model the expression, place two x-tiles, four 1-tiles, and three more x-tiles on the integer mat.



Combine like tiles.



There are five x-tiles and four 1-tiles.

The simplified expression is $5x + 4$.

(continued on next page)

Lesson 5-7 • Equivalent Algebraic Expressions 321

Interactive Presentation

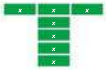
Learn, Combine Like Terms, Slide 2 of 4

WATCH

On Slide 2, students watch a video to learn about how to use algebra tiles to combine like terms in an algebraic expression.

The animation also demonstrates how to simplify the expression $x + 5x + x$.

To model the expression, place one x-tile, then five x-tiles, and then one more x-tile on the integer mat.



Combine like tiles.

There are seven x-tiles.

The simplified expression is $7x$.

You can also use the Distributive Property to combine like terms. This method allows you to simplify by adding or subtracting the coefficients of the terms.

$3x + 5x = x(3 + 5)$	Factor out the common factor in the terms, x.
$= x(8)$	Add inside parentheses.
$= 8x$	Multiply.

Pause and Reflect

How did your prior knowledge of like terms help you to understand the concepts in this Learn?

Record your observations here. See students' observations.

Talk About It!

How can you use the Distributive Property to combine like terms for the expression $2x + 3x + 3x^2 + 6x^2$?

Sample answer: The first two terms share a factor of x. Factoring out the x, $2x + 3x$ can be written as $x(2 + 3)$, or $5x$. The second two terms share a factor of x^2. Factoring out the x^2, $3x^2 + 6x$ can be written as $x^2(3 + 6)$, or $9x^2$. The expression can be written in simplest form as $9x^2 + 5x$.

Learn Combine Like Terms *(continued)*

Teaching Notes

SLIDE 3

Some students may look at the expression $3x + 5x$ and quickly determine that the sum is $8x$. Encourage them to pause and consider how they know these expressions are equivalent. Ask them which mathematical property allows them to combine the like terms. Encourage them to understand that they can rewrite the expression as $x(3 + 5)$ by using the Distributive Property.

Go Online to find Teaching the Mathematical Practices.

Talk About It!

SLIDE 4

Mathematical Discourse

How can you use the Distributive Property to combine like terms for the expression $2x + 3x + 3x^2 + 6x^2$? Sample answer: The first two terms share a factor of x. Factoring out the x, $2x + 3x$ can be written as $x(2 + 3)$, or $5x$. The second two terms share a factor of x^2. Factoring out the x^2, $3x^2 + 6x^2$ can be written as $x^2(3 + 6)$, or $9x^2$. The expression can be written in simplest form as $9x^2 + 5x$.

Example 4 Combine Like Terms

Objective

Students will combine like terms to simplify algebraic expressions.

Questions for Mathematical Discourse

SLIDE 2

AL What property allows you to factor out x in the second step? the Distributive Property

OL How do you know the final expression is equivalent to the original? Sample answer: When the properties of operations are used from one step to the next, each step contains equivalent expressions.

BL A classmate rewrote the expression as $7x + 2x$. What was the likely mistake? The classmate probably thought that the first two terms were like terms and the last two terms were like terms.

Example 5 Combine Like Terms

Objective

Students will combine like terms to simplify algebraic expressions.

Questions for Mathematical Discourse

SLIDE 2

AL What is a constant? a term that does not have a variable

OL How do you know that x^2 and $5x^2$ are like terms? They have the same variable raised to the same power.

OL When dragging the terms to the bins, why is the preceding plus sign also dragged with the term? The terms are all positive.

BL If the expression was $5x^2 + 2x + 2 + x^2 - 6$, what term would be dragged to the *constant* bin? Explain. -6; Sample answer: 6 is being subtracted. In other words, -6 is being added.

SLIDE 3

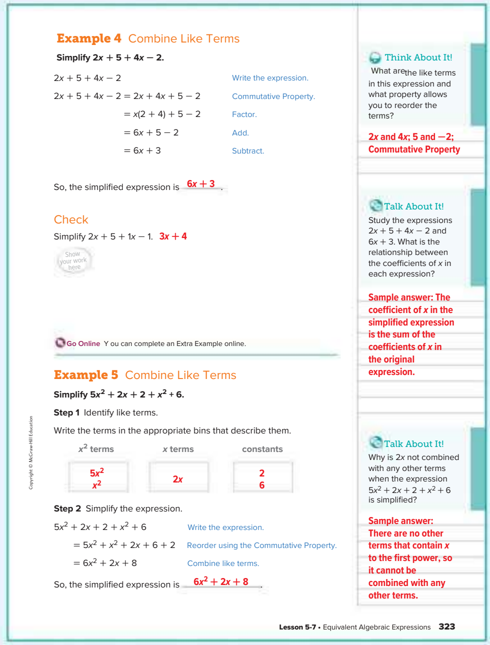
AL What term is created by combining x^2 and $5x^2$? $6x^2$

OL How can you tell how many terms will be in the simplified expression? Sample answer: Since there are two different pairs of like terms, x^2 and constants, and one additional term, x, there will be three terms at most, after combining like terms.

BL Using the Commutative Property, how many different ways can you write the final expression and still keep it in simplest form? Sample answer: six different ways: $6x^2 + 2x + 8$, $6x^2 + 8 + 2x$, $2x + 8 + 6x^2$, $2x + 6x^2 + 8$, $8 + 2x + 6x^2$, and $8 + 6x^2 + 2x$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Combine Like Terms

Simplify $2x + 5 + 4x - 2$.

$2x + 5 + 4x - 2$ — Write the expression.

$2x + 5 + 4x - 2 = 2x + 4x + 5 - 2$ — Commutative Property.

$= x(2 + 4) + 5 - 2$ — Factor.

$= 6x + 5 - 2$ — Add.

$= 6x + 3$ — Subtract.

So, the simplified expression is $6x + 3$.

Check

Simplify $2x + 5 + 1x - 1$. $3x + 4$

Show your work here

Go Online You can complete an Extra Example online.

Think About It!
What are the like terms in this expression and what property allows you to reorder the terms?

$2x$ and $4x$; 5 and -2; Commutative Property

Talk About It!
Study the expressions $2x + 5 + 4x - 2$ and $6x + 3$. What is the relationship between the coefficients of x in each expression?

Sample answer: The coefficient of x in the simplified expression is the sum of the coefficients of x in the original expression.

Example 5 Combine Like Terms

Simplify $5x^2 + 2x + 2 + x^2 + 6$.

Step 1 Identify like terms.

Write the terms in the appropriate bins that describe them.

x^2 terms	x terms	constants
$5x^2$, x^2	$2x$	2, 6

Step 2 Simplify the expression.

$5x^2 + 2x + 2 + x^2 + 6$ — Write the expression.

$= 5x^2 + x^2 + 2x + 6 + 2$ — Reorder using the Commutative Property.

$= 6x^2 + 2x + 8$ — Combine like terms.

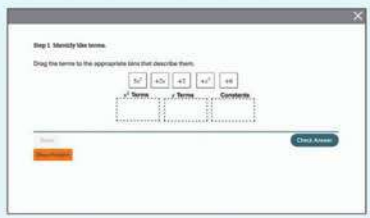
So, the simplified expression is $6x^2 + 2x + 8$.

Talk About It!
Why is $2x$ not combined with any other terms when the expression $5x^2 + 2x + 2 + x^2 + 6$ is simplified?

Sample answer: There are no other terms that contain x to the first power, so it cannot be combined with any other terms.

Lesson 5-7 • Equivalent Algebraic Expressions 323

Interactive Presentation



Example 5, Combine Like Terms, Slide 2 of 5

DRAG & DROP

On Slide 2 of Example 5, students drag each term to the appropriate bin.

TYPE

On Slide 3 of Example 5, students simplify the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Simplify $2x^3 + x^3 + 0.5 + x^2 + 1.5$. $3x^3 + x^2 + 2$

Show your work here

Learn Apply Properties to Write Equivalent Expressions

When you simplify an expression, you can apply properties and combine like terms to write equivalent expressions.

$3(4x + 1) + 2x^2 + x = 12x + 3 + 2x^2 + x$ Distributive Property

$= 2x^2 + 12x + x + 3$ Commutative Property.

$= 2x^2 + 13x + 3$ Combine like terms.

So, $3(4x + 1) + 2x^2 + x$ is equivalent to $2x^2 + 13x + 3$.

Talk About It!
What property allows you to combine like terms?

The Distributive Property allows a common variable factor to be factored out of like terms so that their coefficients can be added.

Example 6 Write Equivalent Expressions

Simplify $\frac{1}{2}\left(2x^2 + \frac{1}{2}\right) + \frac{2}{5}x^2 + 7$.

$\frac{1}{2}\left(2x^2 + \frac{1}{2}\right) + \frac{2}{5}x^2 + 7 = x^2 + \frac{1}{4} + \frac{2}{5}x^2 + 7$ Distributive Property

$= x^2 + \frac{2}{5}x^2 + \frac{1}{4} + 7$ Commutative Property

$= 1\frac{2}{5}x^2 + 7\frac{1}{4}$ Combine like terms.

So, $\frac{1}{2}\left(2x^2 + \frac{1}{2}\right) + \frac{2}{5}x^2 + 7$ is equivalent to $1\frac{2}{5}x^2 + 7\frac{1}{4}$.

Think About It!
What property should be used first to simplify the expression?

the Distributive Property

Talk About It!
What are some other expressions that are equivalent to $\frac{1}{2}\left(2x^2 + \frac{1}{2}\right) + \frac{2}{5}x^2 + 7$?

See students' responses.

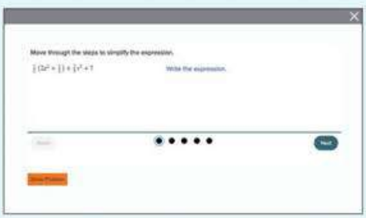
Check

Simplify $\frac{1}{4}(4x + 12) + \frac{1}{2}x + 1 + \frac{3}{2}x$. $3x + 4$

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 6, Write Equivalent Expressions, Slide 2 of 4

CLICK

On Slide 2 of Example 6, students move through the steps to simplify the expression.

TYPE

On Slide 2 of Example 6, students simplify the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Apply Properties to Write Equivalent Expressions

Objective

Students will learn how to write equivalent algebraic expressions using mathematical properties.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use the proper terminology when referring to the property and the parts of the expression. Students should use clear and precise mathematical language to support their claim.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

What property allows you to combine like terms? Sample answer: The Distributive Property allows a common variable factor to be factored out of like terms so that their coefficients can be added.

Example 6 Write Equivalent Expressions

Objective

Students will write equivalent algebraic expressions.

Questions for Mathematical Discourse

SLIDE 2

AL The parentheses in the original expression suggest the use of which property? the Distributive Property

OL How can you apply the Commutative Property? Sample answer: To combine the like terms, I can move the x^2 terms next to each other and the constants next to each other, so that I can combine them more easily.

BL Which expression can you add to the original expression to get 0? Sample answer: $-1\frac{2}{5}x^2 - 7\frac{1}{4}$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Shipping

Objective

Students will come up with their own strategy to solve an application problem involving shipping comic books.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Why does the total cost include shipping and not just the cost of the comic books?
- How would you find the cost of the books, without shipping?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Shipping

Dawit wants to buy some vintage comic books at a local shop and have them shipped to his cousin. The price of a comic book is based on its condition. The table shows the total cost of x number of comic books for each condition. He buys two that are in excellent condition, two that are in good condition, and two that are in fair condition. The shipping cost for the comic books is $5.00. What expression represents the total cost of buying and shipping the comic books?

Condition	Book Costs
Poor	x
Fair	$4.5x$
Good	$9.75x$
Excellent	$18x$
Like New	$25.5x$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$64.5x + 5$; See students' work.

4 How can you show your solution is reasonable?

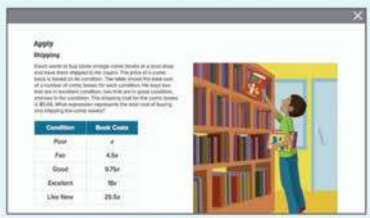
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How would the expression change if you needed to include a tax rate of 8% on the comic books?

Sample answer: The expression would change to $69.66x + 5$, because 8% of $64.5 = 5.16$, so $64.5x + 5.16x = 69.66x$.

Interactive Presentation



Apply, Shipping

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Yasmin bought a case of 144 beach hats for $7.39 per hat and a case of 125 pairs of flip-flops for $2.09 per pair. She sold x number of hats for $15.75 each and y number of pairs of flip-flops for $4.95 each. Write an expression that represents Yasmin's profit.

Show your work here

$15.75x + 4.95y - 1{,}325.41$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how the different properties are used to simplify expressions and determine whether or not two expressions are equivalent. You may wish to have students share their Foldables with a partner.

Essential Question Follow-Up

How can we communicate algebraic relationships with mathematical symbols? In this lesson, students learned how to identify equivalent expressions using the properties of operations and/or substitution. Encourage them to discuss with a partner the benefits of combining like terms when working with algebraic expressions.

Exit Ticket

Refer to the Exit Ticket slide. Assuming the video games cost the same amount of money, write two expressions, one that represents the total value of the gift Andrew received from his mother, and one that represents the total value of the gift he received from his aunt. Are these two expressions equivalent? Write a mathematical argument that can be used to defend your solution. mother: $x + 10$; aunt: $2x + 5$; not equivalent; Sample answer: If the cost of the video games is $10 each, then $x = 10$. If $x = 10$, then the expression $x + 10$ is $10 + 10$, or 20. The expression $2x + 5$ is $2(10) + 5$, or 25.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 11, 13–16
- Extension: Multiple Sets of Grouping Symbols
- ALEKS Simplifying Algebraic Expressions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 14, 16
- Extension: Multiple Sets of Grouping Symbols
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–6
- ALEKS Evaluating and Writing Expressions

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Evaluating and Writing Expressions

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use mathematical properties to identify equivalent expressions	1, 2
1	use substitution to identify equivalent expressions	3, 4
1	combine like terms to simplify expressions	5–8
1	generate equivalent expressions	9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems that involve equivalent algebraic expressions	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may incorrectly use the Distributive Property when determining if two expressions are equivalent. Remind students to multiply the number outside of the parentheses by both terms inside the parentheses. Encourage students to draw arrows from the outside number to the inside numbers so they remember to do so.

Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Use properties of operations to determine whether or not the expressions are equivalent. (Example 1)

1. $(x + 10) + x + 9$ and $2(x + 7) + 5$ **equivalent**

2. $0.5x + 1$ and $1(0.5x)$ **not equivalent**

Use substitution to determine whether or not the expressions are equivalent. (Examples 2 and 3)

3. $3x + 2x + x$ and $7x$ **not equivalent**

4. $x^2 + 1$ and $\frac{2}{3}x^2 + \frac{1}{3}x^2 + 1 + x$ **not equivalent**

Simplify each expression. (Examples 4 and 5)

5. $3x + 4 + 5x - 1$ **$8x + 3$**

6. $10 + 7x - 5 + 4x$ **$11x + 5$**

7. $4x^2 + 6x + 8 + x + 2$ **$4x^2 + 7x + 10$**

8. $\frac{1}{2}x^2 + x + \frac{1}{2} + 2x + \frac{1}{2}x^2$ **$x^2 + 3x + \frac{1}{2}$**

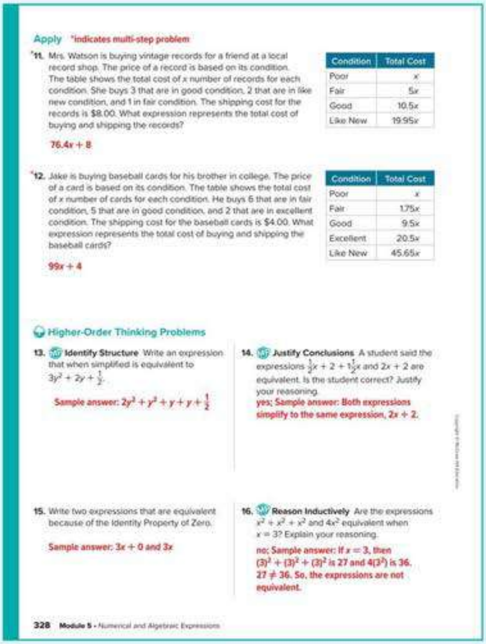
9. Simplify $\frac{3}{4} + \frac{2}{3}(9x + 6) + 4x + 3\frac{1}{4}$. (Example 6)

$10x + 8$

Test Practice

10. Multiselect Which of the following are equivalent to $\frac{3}{4}(8x^2 + 1) + 3x^2 + \frac{1}{4}$? Select all that apply.

- [x] $6x^2 + \frac{3}{4} + 3x^2 + \frac{1}{4}$
- [ ] $6x^2 + 1 + 3x^2 + \frac{1}{4}$
- [ ] $9x^2 + 1\frac{1}{4}$
- [x] $9x^2 + \frac{3}{4} + \frac{1}{4}$
- [ ] $9x^2 + 2$
- [x] $9x^2 + 1$



Apply *indicates multi-step problem

*11. Mrs. Watson is buying vintage records for a friend at a local record shop. The price of a record is based on its condition. The table shows the total cost of x number of records for each condition. She buys 3 that are in good condition, 2 that are in like new condition, and 1 in fair condition. The shipping cost for the records is $8.00. What expression represents the total cost of buying and shipping the records?

Condition	Total Cost
Poor	x
Fair	$5x$
Good	$10.5x$
Like New	$19.95x$

$76.4x + 8$

*12. Jake is buying baseball cards for his brother in college. The price of a card is based on its condition. The table shows the total cost of x number of cards for each condition. He buys 6 that are in fair condition, 5 that are in good condition, and 2 that are in excellent condition. The shipping cost for the baseball cards is $4.00. What expression represents the total cost of buying and shipping the baseball cards?

Condition	Total Cost
Poor	x
Fair	$1.75x$
Good	$9.5x$
Excellent	$20.5x$
Like New	$45.65x$

$99x + 4$

Higher-Order Thinking Problems

13. **Identify Structure** Write an expression that when simplified is equivalent to $3y^2 + 2y + \frac{1}{2}$.

Sample answer: $2y^2 + y^2 + y + y + \frac{1}{2}$

14. **Justify Conclusions** A student said the expressions $\frac{1}{2}x + 2 + 1\frac{1}{2}x$ and $2x + 2$ are equivalent. Is the student correct? Justify your reasoning.

yes; Sample answer: Both expressions simplify to the same expression, $2x + 2$.

15. Write two expressions that are equivalent because of the Identity Property of Zero.

Sample answer: $3x + 0$ and $3x$

16. **Reason Inductively** Are the expressions $x^2 + x^2 + x^2$ and $4x^2$ equivalent when $x = 3$? Explain your reasoning.

no; Sample answer: If $x = 3$, then $(3)^2 + (3)^2 + (3)^2$ is 27 and $4(3^2)$ is 36. $27 \neq 36$. So, the expressions are not equivalent.

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Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 13, students will use the structure of the given expression to write an equivalent expression that can be simplified to the given expression. Encourage students to use the properties of like terms to write the expression.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will determine if the student was correct about the two expressions being equivalent. Encourage students to support their answer with an explanation that supports their reasoning.

2 Reason Abstractly and Quantitatively In Exercise 16, students will reason as to whether or not the two expressions are equivalent and explain their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercises 11–12 Have students work in groups of 3–4 to solve the problem in Exercise 11. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 12.

Listen and ask clarifying questions.

Use with Exercises 14 and 16 Have students work in pairs. Have students individually read Exercise 14 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 16.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of how the properties of addition and multiplication apply to expressions. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

Assessment Resources

Put It All Together: Lessons 5-1, 5-2, 5-3, and 5-4
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Expressions and Equations**.

• Exponents
• Algebraic Expressions


Module 5 • Numerical and Algebraic Expressions

Review

Foldables Use your Foldable to help review the module.

Tab 1 Properties of Addition

Example	Example	Example
Write About It	Write About It	Write About It

Tab 2 Properties of Multiplication

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.	Write about a question you still have.
See students' responses.	See students' responses.

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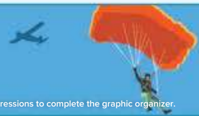
Module 5 • Numerical and Algebraic Expressions 329

Reflect on the Module

Use what you learned about numerical and algebraic expressions to complete the graphic organizer.



Essential Question

How can we communicate algebraic relationships with mathematical symbols?

Expression	Variable	Write a real-world example to represent the given expression. What does the variable represent?
$7x$	x	Each ticket to the school play costs $7. The variable x represents the number of tickets purchased.
$9 + y$	y	Amanda is 9 years older than her brother . The variable y represents the age of her brother in years.
$23 - p$	p	Twenty-three people went on a field trip to the museum. The variable p represents the number of parents who chaperoned. How many students went on the field trip?
$\frac{d}{4}$	d	Mr . Jackson divided the number of dollars in the family fund among his four children. The variable d represents the number of dollars in the family fund.
$\frac{3}{5}c$	c	Three-fifths of the candy in the jar has been eaten. The variable c represents the amount of candy the jar will hold.

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can we communicate algebraic relationships with mathematical symbols? See students' graphic organizers.

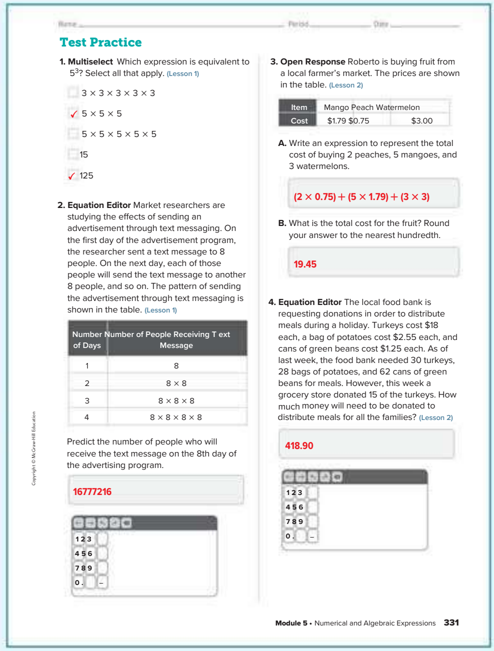
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–11 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	7, 9
Multiselect	Multiple answers may be correct. Students must select all correct answers.	1
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	2, 4
Table Item	Students complete a table by correctly classifying the information.	11
Open Response	Students construct their own response in the area provided.	3, 5, 6, 8–10

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.NS.B.4	5-5	9, 10
6.EE.A.1	5-1, 5-2	1–3
6.EE.A.2	5-3, 5-4	5–8
6.EE.A.2.A	5-3	6
6.EE.A.2.B	5-3	5
6.EE.A.2.C	5-4	7, 8
6.EE.A.3	5-6, 5-7	9–11
6.EE.A.4	5-7	11
6.EE.B.6	5-3, 5-4	4, 6–8



Name ______ Period ______ Date ______

Test Practice

1. **Multiselect** Which expression is equivalent to 5^3? Select all that apply. (Lesson 1)
   - ☐ 3 × 3 × 3 × 3 × 3
   - ✓ 5 × 5 × 5
   - ☐ 5 × 5 × 5 × 5 × 5
   - ☐ 15
   - ✓ 125

2. **Equation Editor** Market researchers are studying the effects of sending an advertisement through text messaging. On the first day of the advertisement program, the researcher sent a text message to 8 people. On the next day, each of those people will send the text message to another 8 people, and so on. The pattern of sending the advertisement through text messaging is shown in the table. (Lesson 1)

Number of Days	Number of People Receiving Text Message
1	8
2	8 × 8
3	8 × 8 × 8
4	8 × 8 × 8 × 8

Predict the number of people who will receive the text message on the 8th day of the advertising program.

16777216

3. **Open Response** Roberto is buying fruit from a local farmer's market. The prices are shown in the table. (Lesson 2)

Item	Mango	Peach	Watermelon
Cost	$1.79	$0.75	$3.00

A. Write an expression to represent the total cost of buying 2 peaches, 5 mangoes, and 3 watermelons.

(2 × 0.75) + (5 × 1.79) + (3 × 3)

B. What is the total cost for the fruit? Round your answer to the nearest hundredth.

19.45

4. **Equation Editor** The local food bank is requesting donations in order to distribute meals during a holiday. Turkeys cost $18 each, a bag of potatoes cost $2.55 each, and cans of green beans cost $1.25 each. As of last week, the food bank needed 30 turkeys, 28 bags of potatoes, and 62 cans of green beans for meals. However, this week a grocery store donated 15 of the turkeys. How much money will need to be donated to distribute meals for all the families? (Lesson 2)

418.90

5. Open Response Identify the terms, like terms, coefficients, and constants in the expression $8p + 6q + 5 + 9q + 12p$. (Lesson 3)

terms: $8p$, $6q$, 5, $9q$, $12p$; like terms: $8p$ and $12p$, $6q$ and $9q$; coefficients: 8, 6, 9, 12; constant: 5

6. Open Response Write *fifteen dollars more than the original cost* as an algebraic expression. Let c represent the original cost. Do not include dollar signs in your expression. (Lesson 3)

$15 + c$

7. Multiple Choice Evaluate $(6x + 3y) - z^2 \div (2x)$ when $x = 3$, $y = 4$, and $z = 6$. (Lesson 4)

- (A) -1
- (B) 11
- (C) 24 ●
- (D) 96

8. Open Response Savannah is choosing between two cell phone plans. Plan A charges $60 a month plus a one-time activation fee of $75. Plan B charges $63 a month plus a one-time activation fee of $15. Evaluate the expressions $60m + 75$ and $63m + 15$ when $m = 18$ to find the total cost for each cell phone plan for 18 months. What is the difference in cost between the two cell phone plans? (Lesson 4)

6

9. Multiple Choice Consider the expression $32x + 56$. (Lesson 6)

A. What is the GCF of 32x and 56?

- (A) 2
- (B) 4
- (C) 8 ●
- (D) 14

B. Use the GCF to factor $32x + 56$.

$8(4x + 7)$

10. Open Response Avery is buying cupcakes for 12 friends at the school bake sale. Each cupcake costs $1.25(Lesson 6)

A. Write an expression using the Distributive Property to find the total cost of 12 cupcakes.

$12(1 + 0.25)$

B. If Avery has $24, how much money will he have left?

$9

11. Table Item Indicate whether or not the two expressions are equivalent using substitution. (Lesson 7)

	Equivalent	Not Equivalent
$5x + 2$ and $4x + 1 + x + 2$		X
$(8y + 4x + 4y + 5)$ and $4(3y + x) + 5$	X	
$y^2 + 4y + 5 - 3y$ and $y^2 + y + 5$	X	

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IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How are the solutions of equations and inequalities different?
See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition.* They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

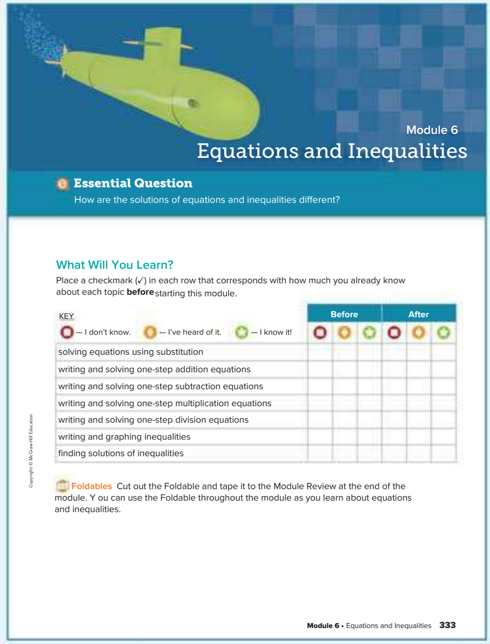
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of scuba diving and hot air balloons to introduce the idea of equations and inequalities. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.


Module 6
Equations and Inequalities

Essential Question
How are the solutions of equations and inequalities different?

What Will You Learn?
Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
solving equations using substitution						
writing and solving one-step addition equations						
writing and solving one-step subtraction equations						
writing and solving one-step multiplication equations						
writing and solving one-step division equations						
writing and graphing inequalities						
finding solutions of inequalities						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. Y ou can use the Foldable throughout the module as you learn about equations and inequalities.

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Module 6 • Equations and Inequalities 333

Interactive Presentation

Module 6

Equations and Inequalities

Module Goal

Write and solve one-step equations and inequalities.

Focus

Domain: Expressions and Equations

Major Cluster(s):

6.EE.B Reason about and solve one-variable equations and inequalities.

Standards for Mathematical Content:

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Also addresses 6.NS.C.6.C, 6.EE.B.5, and 6.EE.B.8.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- fluently apply the Order of Operations to evaluate numerical expressions

Use the Module Pretest to diagnose students' readiness for this module. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students wrote and evaluated numerical and algebraic expressions.
6.NS.B.4, 6.EE.A.1, 6.EE.A.2.A, 6.EE.A.2.B, 6.EE.A.2.C, 6.EE.A.3, 6.EE.A.4, 6.EE.B.6

Now
Students write and solve one-step equations and inequalities.
6.EE.B.5, 6.EE.B.6, 6.EE.B.7, 6.EE.B.8

Next
Students will express relationships between two variables using tables, equations, and graphs.
6.EE.C.9

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of expressions, inequality symbols, and inverse operations to develop *understanding* of equations and inequalities. They use their understanding of models, properties of equality, and substitution to build *fluency* with writing and solving one-step addition, subtraction, multiplication, and division equations. Fluency is also built through writing, solving, and graphing inequalities. They *apply* their understanding of equations and inequalities to solve multi-step, real-world problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

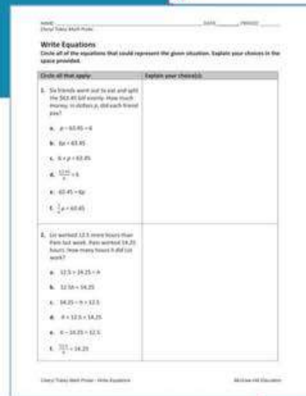
Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
6-1	Use Substitution to Solve One-Step Equations	6.EE.B.5, *Also addresses 6.EE.B.6*	1	0.5
6-2	One-Step Addition Equations	6.EE.B.6, 6.EE.B.7	3	1.5
6-3	One-Step Subtraction Equations	6.EE.B.6, 6.EE.B.7	2	1
6-4	One-Step Multiplication Equations	6.EE.B.6, 6.EE.B.7	2	1
6-5	One-Step Division Equations	6.EE.B.6, 6.EE.B.7	2	1
Put It All Together 1: Lessons 6-1, 6-2, 6-3, 6-4, and 6-5			0.5	0.25
6-6	Inequalities	6.EE.B.5, 6.EE.B.8, *Also addresses 6.NS.C.6.C, 6.EE.B.6*	3	1.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	16.5	8.25

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe
Write Equations

Module Resource


Write Equations
Circle all of the equations that could represent the given situation. Explain your choices in the space provided.

Correct Answers: 1. b, d, e**; 2.** a, e

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students select all of the equations that can represent the given situation, and explain their choices.

Targeted Concept Understand the mathematical meaning of words used to describe relationships between quantities and know that different mathematical equations can be used to represent the same mathematical relationships.

Targeted Misconceptions

- Students may incorrectly determine the operation needed to solve the equation.
- Students may believe there is only one correct equation for solving a problem.

Assign the probe after Lesson 5.

Collect and Assess Student Work

If the student selects...	Then the student likely...
1. a and/or c **2.** b and/or f	incorrectly determines the operation needed to solve the equation. **Example:** If the student chooses a or c for Exercise 1, the student views the problem as an additive relationship, instead of a multiplicative one.
1. b, f, or e **2.** a or e	believes that there is only one correct equation for each problem. **Example:** The student chooses only one of the correct equations for each of the two problems (usually the first correct equation).

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Equations and Inequalities
- Lesson 2, Examples 1–3
- Lesson 3, Examples 1–3
- Lesson 4, Examples 1–3
- Lesson 5, Examples 1–3

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ Addition Property of Equality
- ☐ Division Property of Equality
- ☐ equals sign
- ☐ equation
- ☐ guess, check, and revise strategy
- ☐ inequality
- ☐ inverse operations
- ☐ Multiplication Property of Equality
- ☐ solution
- ☐ solve
- ☐ Subtraction Property of Equality

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1
Subtract decimals.

Find 2.46 − 1.37 .

$$\begin{array}{r} 2.46 \\ -1.37 \\ \hline 1.09 \end{array}$$

Line up the decimal points.
Subtract.

Example 2
Add fractions.

Find $\frac{2}{3} + \frac{1}{5}$.

$\frac{2}{3} + \frac{1}{5}$ Write the problem.

$= \frac{10}{15} + \frac{3}{15}$ Rename using the LCD, 15.

$= \frac{13}{15}$ Add the numerators.

Quick Check

1. Find 14.39 – 7.45. 6.94

2. Find $\frac{3}{4} + \frac{7}{10}$. $1\frac{9}{20}$

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. 1 2

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define The **Addition Property of Equality** states that if you add the same number to each side of an equation, the two sides remain equal.

Example
If you add 3 to each side of the equation $x - 3 = 12.2$, the equation becomes $x = 15.2$, and is an equivalent equation to $x - 3 = 12.2$.

Ask What happens to the equation $m - 9 = 14$ if you add 9 to each side? Sample answer: It becomes $m = 23$ and this equation is equivalent to $m - 9 = 14$.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- writing and simplifying expressions
- performing operations with rational numbers
- using the *guess, check, and revise* strategy
- using bar diagrams
- using algebra tiles
- understanding number lines
- ordering rational numbers

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Equations and Inequalities** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Reward Effort, Not Talent

When adults praise students for their hard work toward a solution, rather than praising them for being smart or talented, it supports students' development of a growth mindset.

How Can I Apply It?

Have students complete the **Performance Task** for the module. Allow students a forum to discuss their process or strategy that they used and give them positive feedback on their diligence in completing the task.

Learn Equations

Objective

Students will learn how to differentiate an equation from an expression.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to think about the precise meanings of an equation and an expression. Students should analyze the structure of an equation and compare it to an expression and note that an expression does not contain an equals sign.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Describe the similarities and differences between equations and expressions. Sample answer: They both can contain numbers, operations, and variables. An equation always contains an equals sign, but an expression does not.

Learn Solve Equations Using Substitution

Objective

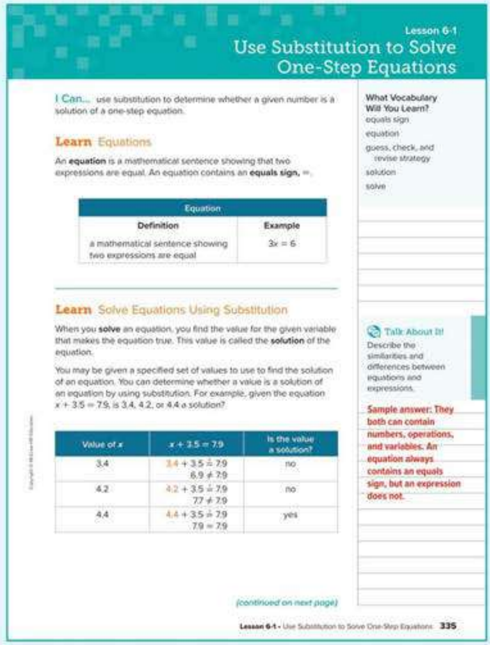
Students will learn how to solve equations using substitution.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the equation $4.5x = 135$ in order to determine that there is only one value of x, that when multiplied by 4.5, yields a product of 135.

Go Online to find additional teaching notes.

(continued on next page)



Lesson 6-1

Use Substitution to Solve One-Step Equations

I Can... use substitution to determine whether a given number is a solution of a one-step equation.

Learn Equations

An **equation** is a mathematical sentence showing that two expressions are equal. An equation contains an **equals sign,** =.

Equation	
Definition	**Example**
a mathematical sentence showing two expressions are equal	$3x = 6$

Learn Solve Equations Using Substitution

When you **solve** an equation, you find the value for the given variable that makes the equation true. This value is called the **solution** of the equation.

You may be given a specified set of values to use to find the solution of an equation. You can determine whether a value is a solution of an equation by using substitution. For example, given the equation $x + 3.5 = 7.9$, is 3.4, 4.2, or 4.4 a solution?

Value of x	$x + 3.5 = 7.9$	Is the value a solution?
3.4	$3.4 + 3.5 \stackrel{?}{=} 7.9$ $6.9 \neq 7.9$	no
4.2	$4.2 + 3.5 \stackrel{?}{=} 7.9$ $7.7 \neq 7.9$	no
4.4	$4.4 + 3.5 \stackrel{?}{=} 7.9$ $7.9 = 7.9$	yes

(continued on next page)

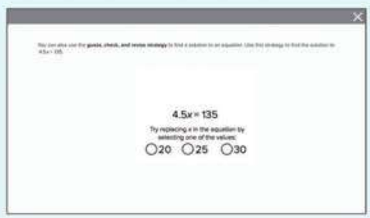
What Vocabulary Will You Learn?
equals sign
equation
guess, check, and revise strategy
solution
solve

Talk About It!
Describe the similarities and differences between equations and expressions.

Sample answer: They both can contain numbers, operations, and variables. An equation always contains an equals sign, but an expression does not.

Lesson 6-1 • Use Substitution to Solve One-Step Equations 335

Interactive Presentation



Learn, Solve Equations Using Substitution, Slide 2 of 3

CLICK

On Slide 1 of Learn, Solve Equations Using Substitution, students move through the steps to solve the equation.

CLICK

On Slide 2 of Learn, Solve Equations Using Substitution, students select possible values of the variable to see if they are solutions of the equation.

Lesson 6-1

6.EE.B.5

Use Substitution to Solve One-Step Equations

LESSON GOAL

Students will use substitution to solve one-step equations.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Equations

Learn: Solve Equations Using Substitution

Example 1: Solve Equations Using Substitution

Example 2: Solve Equations Using Substitution

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Use Substitution to Solve Two-Step Equations		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 34 of the *Language Development Handbook* to help your students build mathematical language related to solving equations by substitution.

ELL You can use the tips and suggestions on page T34 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations

Major Cluster(s): In this lesson, students address major cluster **6.EE.B** by using substitution to solve one-step equations.

Standards for Mathematical Content: **6.EE.B.5,** *Also addresses 6.EE.B.6*

Standards for Mathematical Practice: **MP1, MP2, MP3, MP6, MP7**

Coherence

Vertical Alignment

Previous

Students identified and generated equivalent algebraic expressions. **6.EE.A.3**

Now

Students use substitution to determine whether a given number in a specified set makes an equation true. **6.EE.B.5**

Next

Students will use the Subtraction Property of Equality to write and solve one-step addition equations. **6.EE.B.6, 6.EE.B.7**

Rigor

The Three Pillars of Rigor

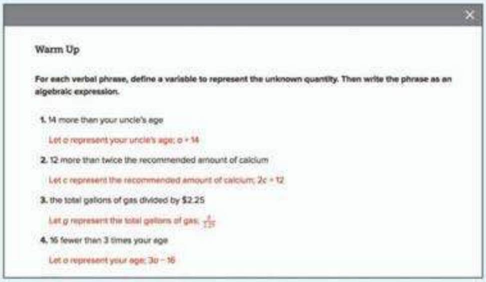
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of equivalent expressions to begin to develop *understanding* of one-step equations. They come to understand that solving an algebraic equation means finding a value for the variable that results in a true sentence, and they build *fluency* with using the substitution method to solve one-step equations. They also *apply* this understanding to solve real-world problems.

Mathematical Background

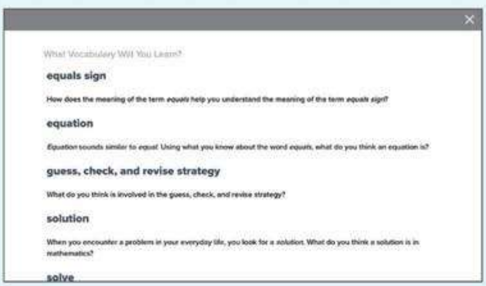
An *equation* is a mathematical sentence showing the equality of two expressions by using an equals sign, =. When the variable is replaced with a value that results in a true sentence, the substituted value is called a *solution* of the equation. Equations can be solved using the *guess, check, and revise strategy*. To use this strategy, first make an initial guess, then substitute this value into the equation. If the guess does not make the equation true, increase or decrease the guess based on the value of the expressions on each side of the equation. Repeat this process until the solution is obtained.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing expressions (Exercises 1–4)
- writing and evaluating expressions (Exercise 5)

Answers

1. Let a represent your uncle's age; $a + 14$
2. Let c represent the recommended amount of calcium; $2c + 12$
3. Let g represent the total gallons of gas; $\frac{g}{2.25}$
4. Let a represent your age; $3a - 16$
5. $1.5x + 25.99$; $213.49

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an equation to determine the cost of books for a book drive.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- How does the meaning of the term *equals* help you understand the meaning of the term *equals sign*? Sample answer: *Equals* means that two things are the same or alike in quantity. An *equals sign* might be a mathematical symbol that sets things equal to one another.
- *Equation* sounds similar to *equal*. Using what you know about the term *equals*, what do you think an *equation* is? Sample answer: An equation might be a way of setting things equal to one another.
- What do you think is involved in the *guess, check, and revise strategy*? Sample answer: The *guess*, *check*, *and revise strategy* might involve guessing numbers and checking them in an equation to find a solution.
- When you encounter a problem in your everyday life, you look for a *solution*. What do you think is a solution in mathematics? Sample answer: A solution in mathematics might be the answer to an equation.
- In your everyday life, you use a solution to solve your problems. Can you infer what *solve* means in mathematics? Sample answer: To solve in mathematics might be to find a solution to a problem or an equation.

Your Notes

Y ou can also use the **guess, check, and revise strategy** to find the solution of an equation. T o find the solution of the equation $4.5x = 135$, begin by choosing a reasonable value for x. For example, try $x = 20$.

Value of x	$4.5x = 135$	Is the value a solution?
20	$4.5(20) \stackrel{?}{=} 135$ $90 \neq 135$	No, because $90 < 135$, the value of x is too small. Try revising the number guessed.
25	$4.5(25) \stackrel{?}{=} 135$ $112.5 \neq 135$	No, because $112.5 < 135$, the value of x is too small. Try revising the number guessed.
30	$4.5(30) \stackrel{?}{=} 135$ $135 = 135$	Y es, because 135 = 135, 30 is the correct solution.

Talk About It!
Is there another value that is a solution of $4.5x = 135$? Explain your reasoning.

no, Sample answer: Since $4.5(30) = 135$, if you substitute a different number for x, then the product of the factors will no longer be 135.

Example 1 Solve Equations Using Substitution

Is 3, 4, or 5 the solution of the equation $p + 9.7 = 13.7$?

Complete the table to find the solution of the equation.

Value of p	$p + 9.7 = 13.7$	Is the value a solution?
3	$3 + 9.7 \stackrel{?}{=} 13.7$ $12.7 \neq 13.7$	no
4	$4 + 9.7 \stackrel{?}{=} 13.7$ $13.7 = 13.7$	yes
5	$5 + 9.7 \stackrel{?}{=} 13.7$ $14.7 \neq 13.7$	no

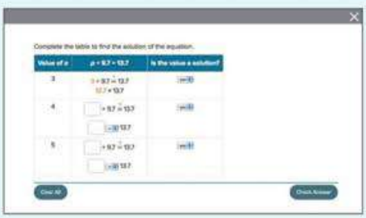
So, the solution is 4.

Check
Is 1, 2, or 3 the solution of the equation $m + \frac{4}{5} = 2\frac{4}{5}$? 2

Show your work here

Go Online Y ou can complete an Extra Example online.

Interactive Presentation



Example 1, Solve Equations Using Substitution, Slide 1 of 2

TYPE
On Slide 1, students enter each value into each equation to determine if it is a solution.

CLICK
On Slide 1, students select yes or no to determine if each value is a solution.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Learn Solve Equations Using Substitution *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

Is there another value that is a solution of $4.5x = 135$? Explain your reasoning. no; Sample answer: Since $4.5(30) = 135$, if you substitute a different number for x, then the product of the factors will no longer be 135.

Example 1 Solve Equations Using Substitution

Objective

Students will use the substitution method to solve one-step equations.

Questions for Mathematical Discourse

SLIDE 1

AL What is the unknown in the given equation? p

OL Why is 3 not a solution? When 3 is substituted for p, the statement $12.7 = 13.7$ is not true. So, 3 is not a solution.

OL Once you know that 3 is not a solution, how do you know to check numbers greater than 3, as opposed to less than 3? Sample answer: Since $3 + 9.7 = 12.7$, and $12.7 < 13.7$, I need to try values that are greater than 3.

BL Once you know that 4 is a solution, do you need to check whether 5 is a solution? Explain. no; Sample answer: There is only one number than when added to 9.7 yields a sum of 13.7.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

Have students work with a partner to use substitution and the *guess, check, and revise* strategy to solve each equation. Then have them analyze the structure of each equation to make a conjecture as to how they might solve the equation without having to use the *guess, check, and revise* strategy.

$22 = x + 2$ $x = 20$; Sample answer: Subtract 2 from each side.
$y - 5.1 = 3.7$ $y = 8.8$; Sample answer: Add 5.1 to each side.
$m + 3\frac{1}{2} = 9\frac{3}{4}$ $m = 6\frac{1}{4}$; Sample answer: Subtract $3\frac{1}{2}$ from each side.
$13.75 = b - 0.8$ $b = 14.55$; Sample answer: Add 0.8 to each side.

Example 2 Solve Equations Using Substitution

Objective

Students will use the substitution method to solve one-step equations.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should pause after checking each value to determine whether or not the value is a solution of the equation. If not, students should move to the next value, continuing to use the *guess, check and revise* strategy.

As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the real-world problem and that two planks together have a combined width of 9 inches. Students can solve the problem by determining the combined width, and then reason about how many total planks are needed.

6 Attend to Precision Encourage students to calculate accurately and efficiently, paying careful attention to the values on each side of the equals sign as to whether or not they are equivalent.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown value for which we are solving the equation? *p*, the number of planks Nevaeh will need

OL If you substitute a number into this equation and find that the value of the left side of the equation is less than the right side, what does this tell you about your guess? Sample answer: This tells me that I need to increase the value of my guess.

OL Use estimation to determine that the number of planks must be greater than 7. Sample answer: $4\frac{1}{2}$ is halfway between 4 and 5. Since 4(7) = 28, and 5(7) = 35, and both products are still less than 36, then I know the number of planks must be greater than 7.

BL How many planks will be needed if each plank is 6 inches wide? Explain. 6; Sample answer: The solution to $6p = 36$ is $p = 6$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Solve Equations Using Substitution

Navaeh is building a door that is 36 inches wide using wooden planks that are $4\frac{1}{2}$ inches wide.

Use the *guess, check, and revise* strategy to solve the equation $4\frac{1}{2}p = 36$ to find *p*, the number of planks Nevaeh will need to make her door.

Begin by substituting 6 into the equation.

$4\frac{1}{2}p = 36$

$4\frac{1}{2}(6) \stackrel{?}{=} 36$

$27 \neq 36$

Since 27 < 36, try a greater number of planks.

Substitute 7 into the equation.

$4\frac{1}{2}p = 36$

$4\frac{1}{2}(7) \stackrel{?}{=} 36$

$31\frac{1}{2} \neq 36$

Since $31\frac{1}{2} < 36$, try a greater number of planks.

Substitute 8 into the equation.

$4\frac{1}{2}p = 36$

$4\frac{1}{2}(8) \stackrel{?}{=} 36$

$36 = 36$

The sentence is true, so 8 is the solution of the equation $4\frac{1}{2}p = 36$.

So, Nevaeh needs to use 8 planks to build the door.

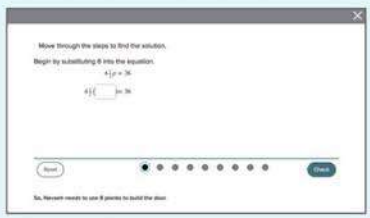
Think About It! How will you make your first *guess*?

See students' responses.

Talk About It! How can you use mental math to solve the equation?

Sample answer: Two planks together have a combined width of $4\frac{1}{2} + 4\frac{1}{2}$, or 9 inches. Since 9(4) = 36, Nevaeh needs 4 sections of two planks. This means, she needs 8 planks.

Interactive Presentation



Example 2, Solve Equations Using Substitution, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the solution.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

This year, students ate 100 pounds of broccoli in the Walnut Springs Middle School cafeteria. This is $6\frac{1}{4}$ times as much as they ate in the previous year. Use the *guess, check, and revise* strategy to solve the equation $6\frac{1}{4}b = 100$ to find b, the number of pounds of broccoli the students ate the previous year. **16 pounds**

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Write a real-world problem that uses the guess, check, and revise strategy to solve an equation. Explain how you came up with that problem. Exchange problems with a classmate and solve each other's problem.

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. The equation $23b = 61.87$ can be used to represent Rylie's purchase. Solve the equation to find b the cost of each book. $2.69

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11, 13–16
- Extension: Use Substitution to Solve Two-Step Equations
- ALEKS One-Step Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–10, 14, 15
- Extension: Use Substitution to Solve Two-Step Equations
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Introduction to One-Step Equations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Introduction to One-Step Equations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use the substitution method to solve one-step equations	1–10
2	extend concepts learned in class to apply them in new contexts	11, 12
3	higher-order and critical thinking skills	13–16

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Identify the solution of each equation from the specified set. (Example 1)

1. $x + 5.6 = 11.6$; 5, 6, 7 **6**

2. $4.2 + z = 11.2$; 6, 7, 8 **7**

3. $b - 9.7 = 13.3$; 23, 24, 25 **23**

4. $d - 8.4 = 8.6$; 15, 16, 17 **17**

5. $4.5x = 18$; 3, 4, 5 **4**

6. $2.25c = 27$; 12, 13, 14 **12**

7. $d \div 5.5 = 4$; 22, 23, 24 **22**

8. $36.3 \div y = 12.1$; 2, 3, 4 **3**

9. Brinley is making headbands for her friends. Each headband needs $16\frac{1}{2}$ inches of elastic and she has 132 inches of elastic. Use the *guess, check, and revise* strategy to solve the equation $16\frac{1}{2}h = 132$ to find h, the number of headbands Brinley can make. (Example 2)

   8 headbands

10. Maddox has $12.25 to spend on sports drinks. Each drink costs $1.75. Use the *guess, check, and revise* strategy to solve the equation $1.75d = \$12.25$ to find d, the number of drinks Maddox can buy (Example 2)

    7 sports drinks

11. Manuel has two different recipes for chocolate chip muffins. The table shows the amount of chocolate chips needed per batch for each recipe. He has $8\frac{3}{4}$ cups of chocolate chips. Use the *guess, check, and revise* strategy to solve the equation $1\frac{1}{4}b = 8\frac{3}{4}$ to find b, the number of batches of muffins he can make if he uses Recipe 2.

    7 batches

Recipe	Chocolate Chips (cups)
1	$\frac{3}{4}$
2	$1\frac{1}{4}$

T est Practice

12. Multiple Choice Consider the following equation.

$x + 9 = 17$

Which of the values can be substituted for x to make the equation true?

(A) 7
(B) 8
(C) 9
(D) 26

Higher-Order Thinking Problems

13. Create Write a real-world problem that can be solved using the equation $7.5 + x = 16$.

Sample answer: Jack had $7.50. His mother gave him his allowance at the end of the week. Now Jack has $16. Solve the equation $7.5 + x = 16$ to find how much money his mother gave him.

14. MP Justify Conclusions A student said that for $x + 7 = 11$, the value of x can be any value. Is the student correct? Write an argument that can be used to defend your response.

no; Sample answer: There is only one value for x that makes this equation true. The only value for x is 4.

15. MP Be Precise Compare and contrast the expression $x + 1$ and the equation $x + 1 = 2$.

Sample answer: $x + 1$ is an algebraic expression and is not equal to a specific value. So, there are no restrictions placed on the value of x. $x + 1 = 2$ is an algebraic equation. Each side of an algebraic equation must be equal, so x can only be equal to one value. In this case, $x = 1$.

16. Give an example of an addition equation and a subtraction equation that each have a solution of 10.

Sample answer: $x + 10 = 20$; $20 - x = 10$

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will determine if the statement made by the student is correct and justify their reasoning. Encourage students to determine the error in the statement and support their findings with a valid explanation.

6 Attend to Precision In Exercise 15, students will compare and contrast the given expression and equation. Encourage students to use precision when comparing and contrasting the equation and expression.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 16, students will give an example of an addition equation and a subtraction equation that each have a solution of 10.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Listen and ask clarifying questions.

Use with Exercises 14–15 Have students work in pairs. Have students individually read Exercise 14 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 15.

Learn Write Addition Equations

Objective

Students will learn how to model a real-world problem with a one-step addition equation.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language, such as *variable* and *equation*, to explain why defining a variable is an important step in modeling a real-world problem with an equation.

Teaching Notes

SLIDE 1

Be sure students understand the importance of defining the variable when writing an equation to model a real-world problem. You may wish to have students create their own word problem that involves addition, such as *Today, the temperature was eight degrees warmer than yesterday. Today's temperature was 66 degrees Fahrenheit. What was yesterday's temperature?* Have students choose a variable, such as x or t, and clearly explain what that variable represents (yesterday's temperature). Then have them write an equation that models the problem, such as $x + 8 = 66$, $t + 8 = 66$, $66 = 8 + x$, or $66 = 8 + t$. Be sure they understand there can be more than one way to write the equation.

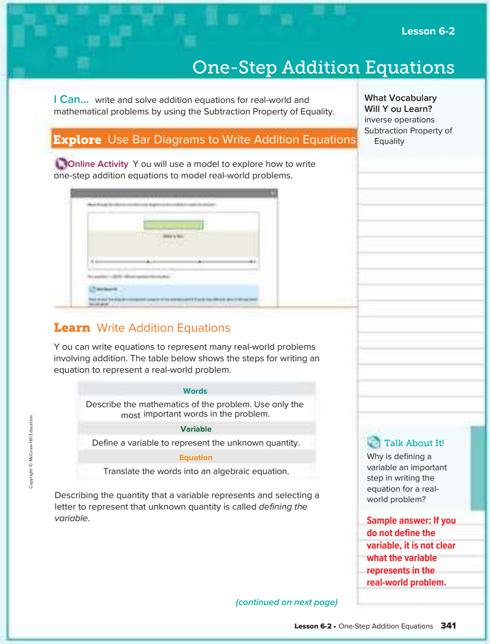
Talk About It!

SLIDE 3

Mathematical Discourse

Why is defining a variable an important step in writing the equation for a real-world problem? Sample answer: If you do not define the variable, it is not clear what the variable represents in the real-world problem.

(continued on next page)


Lesson 6-2

One-Step Addition Equations

I Can... write and solve addition equations for real-world and mathematical problems by using the Subtraction Property of Equality.

What Vocabulary Will You Learn?
inverse operations
Subtraction Property of Equality

Explore Use Bar Diagrams to Write Addition Equations

Online Activity You will use a model to explore how to write one-step addition equations to model real-world problems.

Learn Write Addition Equations

You can write equations to represent many real-world problems involving addition. The table below shows the steps for writing an equation to represent a real-world problem.

Words
Describe the mathematics of the problem. Use only the most important words in the problem.
Variable
Define a variable to represent the unknown quantity.
Equation
Translate the words into an algebraic equation.

Describing the quantity that a variable represents and selecting a letter to represent that unknown quantity is called *defining the variable*.

Talk About It!
Why is defining a variable an important step in writing the equation for a real-world problem?

Sample answer: If you do not define the variable, it is not clear what the variable represents in the real-world problem.

(continued on next page)

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Lesson 6-2 • One-Step Addition Equations 341

Interactive Presentation

Learn, Write Addition Equations, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view the steps for writing an equation to model a real-world problem.

DIFFERENTIATE

Language Development Activity ELL

Some students may struggle with identifying words that signify addition. Have students work with a partner to brainstorm words that signify addition. Have them create a poster to display in the classroom.

Sample answer: add, join, both, combined, how many, increase, plus, sum, total

Lesson 6-2

One-Step Addition Equations

6.EE.B.6, 6.EE.B.7

LESSON GOAL

Students will use the Subtraction Property of Equality to write and solve one-step addition equations.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Bar Diagrams to Write Addition Equations

Learn: Write Addition Equations
Example 1: Write Addition Equations

Explore: One-Step Addition Equations

Learn: Solve Addition Equations
Examples 2–3: Solve Addition Equations
Apply: Money

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

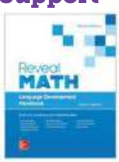
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 35 of the *Language Development Handbook* to help your students build mathematical language related to solving one-step addition equations.



ELL You can use the tips and suggestions on page T35 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min — 1.5 days
45 min — 3 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.B** by using the Subtraction Property of Equality to write and solve one-step addition equations.
Standards for Mathematical Content: **6.EE.B.6, 6.EE.B.7**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students used substitution to solve one-step equations.
6.EE.B.5

Now
Students use the Subtraction Property of Equality to write and solve one-step addition equations.
6.EE.B.6, 6.EE.B.7

Next
Students will use the Addition Property of Equality to write and solve one-step subtraction equations.
6.EE.B.6, 6.EE.B.7

Rigor

The Three Pillars of Rigor

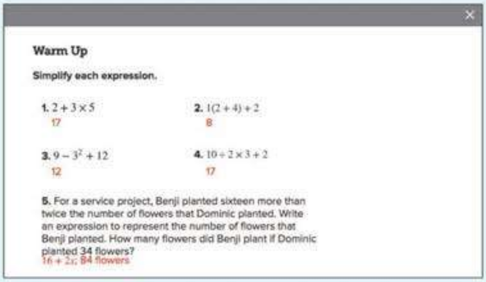
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of one-step addition equations. They learn how to use a model and the Subtraction Property of Equality to build *fluency* with solving one-step addition equations involving whole numbers and fractions. They *apply* their understanding of writing and solving one-step addition equations to solve multi-step, real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- performing operations with whole numbers (Exercises 1–4)
- writing and evaluating expressions (Exercise 5)

Answers

1. 17
2. 8
3. 12
4. 17
5. $16 + 2x$; 84 flowers

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about finding the lifespans of plants and trees, using the information given.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The inverse of walking forward is walking backward. How does what you know about mathematical operations help you understand the meaning of the term *inverse operations*? Sample answer: Inverse operations must be operations that are opposite of one another, like addition and subtraction.
- Describe how you have used the word *equality* in everyday life. How can this help you determine the meaning of the *Subtraction Property of Equality*? Sample answer: Equality means that two things are equal. The Subtraction Property of Equality might involve subtracting a number from each side of an equation to keep both sides equal.

Explore Use Bar Diagrams to Write Addition Equations

Objective

Students will explore how to use a model to write addition equations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the Talk About It! questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a word problem involving an unknown value. Throughout this activity, students will use various strategies, including drawing bar diagrams, to write addition equations for real-world problems.

Inquiry Question

How can you use a model to write addition equations? Sample answer: I can write an addition equation using a bar diagram with a section representing what I know and a section representing what I don't know. The entire bar represents the total.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

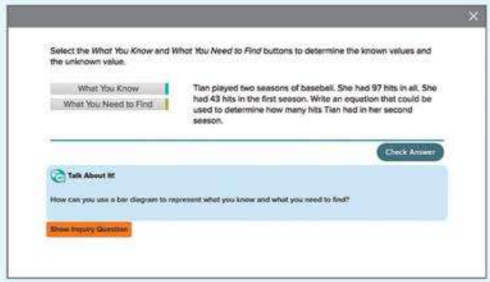
Mathematical Discourse

How can you use a bar diagram to represent what you know and what you need to find? Sample answer: Draw a bar diagram to represent the total, 97, and then divide the bar into two seasons.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 3 of 8

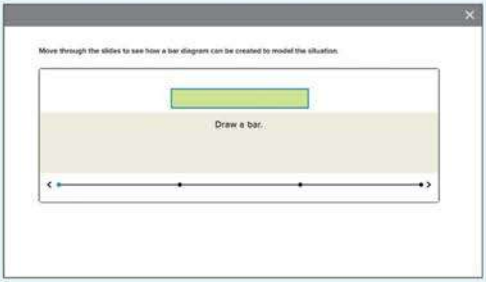
CLICK

On Slide 3, students highlight what they know and what they need to find.

CLICK

On Slide 4, students move through the slides to see how a bar diagram can be created to model the real-world scenario.

Interactive Presentation



Explore, Slide 7 of 8

CLICK

On Slide 7, students move through the slides to see how a bar diagram can be created to model the situation.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Use Bar Diagrams to Write Addition Equations *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should be able to identify the important quantities in the real-world situation, decontextualize them, and use the bar diagrams to represent them symbolically.

5 Use Appropriate Tools Strategically Encourage students to examine the correspondences between the bar diagrams and equations, and how they could eventually transition from the problem statement to writing the equation without the bar diagram.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

What is the known and what is the unknown in the situation? How did you set up the bar diagram? Sample answer: The known is the total amount of money Terrell started with, and the amount he spent on the music service. The unknown is the amount he spent on the digital music player. Draw a bar diagram and label the total $135. Divide the bar into two sections, label one $25.95 and the other with the variable.

Your Notes

Go Online Watch the animation to learn how to write an addition equation to represent the following real-world problem.

In a recent Summer Olympics, the United States won 23 more medals in swimming than Australia. The United States won a total of 33 swimming medals. Write an addition equation that can be used to determine the number of medals won by Australia.

Words
Describe the mathematics of the problem.
medals for Australia plus 23 equals 33 medals for the U.S.
Variable
Define the variable.
Let m represent the number of medals for Australia.
Equation
Write an equation.
$m + 23 = 33$

Example 1 Write Addition Equations

Together, Ruben and Tariq downloaded 245.5 megabytes (MB) of music. Ruben downloaded 132 MB of that total.

Write an addition equation that can be used to find how many megabytes of music Tariq downloaded.

Words
Describe the mathematics of the problem.
Ruben and Tariq downloaded a total of 245.5 MB of music.
Of this total, Ruben downloaded 132 MB
Variable
Define the variable.
Let m represent the MB that Tariq downloaded.
Equation
Write an equation.
$132 + m = 245.5$

So, the equation $132 + m = 245.5$ can be used to find the MB that Tariq downloaded.

Think About It! What is the unknown in this problem?

the megabytes of music downloaded by Tariq

Talk About It! What are some other ways to write the equation based on the real-world problem?

Sample answers: $m + 132 = 245.5$; $245.5 - m = 132$; $245.5 - 132 = m$

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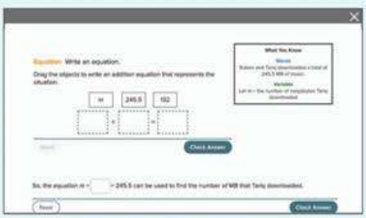
Interactive Presentation



Example, Write Addition Equations, Slide 4 of 6

WATCH

On Slide 2 of the Learn, students watch an animation to model a real-world problem with an addition equation.

DRAG & DROP

On Slide 4 of Example 1, students drag the objects to write an addition equation that models the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Write Addition Equations *(continued)*

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how to write an addition equation to model a real-world problem.

Example 1 Write Addition Equations

Objective

Students will model a real-world problem with a one-step addition equation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the real-world problem by representing it symbolically with a correct addition equation, paying careful attention to each quantity and how it can be represented in the equation.

As students discuss the *Talk About It!* question on Slide 5, encourage them to determine other equations that can model the problem. They should be able to explain why the equations are equivalent.

6 Attend to Precision Students should use precision in defining the variable prior to writing the equation.

Questions for Mathematical Discourse

SLIDE 2

AL What information are you given? Ruben and Tariq downloaded 245.5 megabytes of music, and Ruben downloaded 132 MB of that total.

OL How many megabytes of music did Ruben and Tariq download altogether? 245.5 megabytes

OL Why does it make sense that Ruben downloaded a value that is less than 245.5 MB? Sample answer: Ruben downloaded a subset of the total amount, 245.5 MB. This means the amount he downloaded must be less than 245.5 MB.

BL How many more MB did Ruben download than Tariq? Explain. 18.5 MB; Sample answer: Ruben downloaded 132 MB. Tariq downloaded 245.5 − 132, or 113.5 MB. So, Ruben downloaded 132 − 113.5, or 18.5 more MB than Tariq.

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Write Addition Equations *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why is it important to define the variable? Sample answer: We need to specify what we mean by using m in the equation. Otherwise, it is not clear.

OL How do you know that m needs to represent the number of MB that Tariq downloaded? Sample answer: I know how many megabytes Ruben downloaded, but I do not know how much music Tariq downloaded. That is the unknown.

OL Can you use any other letter for the unknown, other than m? Explain. yes; Sample answer: It does not matter what letter is used, as long as the variable is defined correctly.

BL Maria downloaded 69.7 MB of music. How would you write an equation to represent the total amount that Ruben, Tariq, and Maria downloaded? $132 + m + 69.7 = 315.2$

SLIDE 4

AL Explain why it makes sense that this is an addition equation. Sample answer: We are given the total amount that Tariq and Ruben downloaded, which represents an addition problem.

OL How can you determine which quantities belong to the left of the equals sign? Sample answer: The left side of the equation uses addition. The two quantities that are added together are m and 132, the number of megabytes downloaded by each person belong to the left side of the equation.

BL Are the equations $m + 132 = 245.5$ and $132 + m = 245.5$ equivalent? Explain. yes; Sample answer: Since addition is commutative, the terms on the left side of the equals sign can be added in any order.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

Together, Zacharias and Paz have $756.80. If Zacharias has $489.50, how much does Paz have? Write an addition equation that can be used to find the amount of money that belongs to Paz.

Show your work here

Sample answer: $489.50 + p = 756.80$

Go Online You can complete an Extra Example online.

Explore One-Step Addition Equations

Online Activity You will use a balance to explore how to solve one-step addition equations.

Pause and Reflect

In the Explore, you used a balance to solve equations, such as $x + 3 = 5$ and $x + 3 = 7$. Then you made a conjecture as to how to solve an addition equation without using a balance. When might a balance not be the most advantageous method to use?

Record your observations here

See students' responses.

Learn Solve Addition Equations

You can use substitution, models, or properties of mathematics to solve addition equations.

Go Online Watch the video to learn how to solve one-step addition equations using algebra tiles.

The video demonstrates how to find the value of x for the equation $x + 4 = 7$.

To model the equation, place one x-tile and four 1-tiles on the left side of the mat. Place seven 1-tiles on the right side of the mat.

To isolate the variable, or the x-tile, remove the same number of 1-tiles from each side of the mat until the x-tile is by itself.

There are three 1-tiles remaining on the right side, so $x = 3$.

Another way to solve an addition equation is to use **inverse operations**, which are operations that undo each other. When you solve an addition equation by subtracting the same number from each side of the equation, you are using the **Subtraction Property of Equality**.

Words	Examples
If you subtract the same number from each side of an equation, the two sides remain equal.	If $5 = 5$, then $5 - 2 = 5 - 2$. If $x + 2 = 3$, then $x + 2 - 2 = 3 - 2$.

To solve the addition equation $x + 4 = 7$ by using inverse operations, undo the addition of 4 by subtracting 4 from each side of the equation.

$x + 4 = 7$	Write the equation.
$-4 \quad -4$	Subtraction Property of Equality
$x = 3$	The solution is $x = 3$.

Talk About It! How does using algebra tiles to solve an addition equation model the Subtraction Property of Equality?

Sample answer: To keep each side of the mat equal, I need to remove equal numbers of tiles from each side. This models the Subtraction Property of Equality.

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Interactive Presentation

Learn, Solve Addition Equations, Slide 2 of 3

WATCH

On Slide 1, students watch a video to learn about how to use algebra tiles to solve one-step addition equations.

FLASHCARDS

On Slide 2, students use Flashcards to learn more about the Subtraction Property of Equality.

Learn Solve Addition Equations

Objective

Students will learn how to solve one-step addition equations using a model and the Subtraction Property of Equality.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to explain how the structure of using algebra tiles to remove equal numbers of tiles from each side of the workmat mirrors the Subtraction Property of Equality.

Go Online to have your students watch the video on Slide 1. The video illustrates how to solve one-step addition equations using algebra tiles.

Teaching Notes

SLIDE 1

You may wish to pause the video after the equation $x + 4 = 7$ is shown, and ask students to work with a partner to use algebra tiles to model and solve the equation. Have them share their process and solution with another pair of students, or the entire class. Then have them continue watching the video to compare their process and solution with the one shown. Repeat using a similar process for the second equation in the video, $3 + x = 8$.

SLIDE 2

Remind students that addition and subtraction are inverse operations. To solve an addition equation for a variable, such as $x + 2 = 3$, students can undo the addition of 2 by subtracting 2. Point out that the same number must be subtracted from each side of the equation, in order for the equation to remain equal. Have students select the *Words* and *Examples* flashcards to view the *Subtraction Property of Equality* expressed in these multiple representations.

Talk About It!

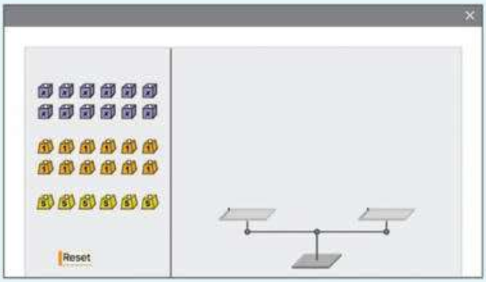
SLIDE 3

Mathematical Discourse

How does using algebra tiles to solve an addition equation model the Subtraction Property of Equality? Sample answer: To keep each side of the mat equal, I need to remove equal numbers of tiles from each side. This models the Subtraction Property of Equality.

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 2 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore solving one-step addition equations using a balance.

TYPE

On Slide 2, students enter the weight of the *x*-weight.

Explore One-Step Addition Equations

Objective

Students will explore solving one-step addition equations using a balance.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the Talk About It! questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a balance and explore the idea of keeping the scale balanced. Students will relate the idea of the balance to an equation and learn that an equation is similar to a balance, in that both sides need to be equal at all times.

Inquiry Question

How is solving an addition equation like using a balance? Sample answer: To keep a scale in balance, you need to subtract the same weight from each side. To keep an equation in balance, you need to subtract the same number from each side. Otherwise the two sides of the equation will not be equal.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

How could you use the balance to find the weight of the *x*-weight? Sample answer: By adding 1-weights to the opposite side of the balance until the sides are equal.

(continued on next page)

Explore One-Step Addition Equations *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how an equation is similar to a balance.

8 Look for and Express Regularity in Repeated Reasoning Encourage students to use repetitive reasoning when finding the *x*-weight on the balance in order to find an equation that represents the balance.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

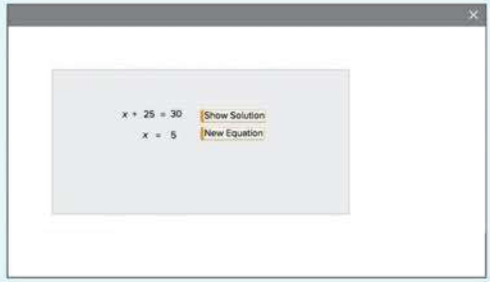
Talk About It!

SLIDE 6

Mathematical Discourse

What steps did you take to solve the equation without the model? Some methods could include subtracting the same number from each side of the equation to isolate the variable.

Interactive Presentation



Explore, Slide 6 of 8

WEB SKETCHPAD

On Slide 6, students use Web Sketchpad to practice solving one-step equations.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Example 2 Solve Addition Equations

Objective

Students will solve one-step addition equations involving whole numbers, using a model and the Subtraction Property of Equality.

Questions for Mathematical Discourse

SLIDE 2

AL How can you represent 8 on the left side of the mat? Place eight 1-tiles on the left side of the mat.

AL How can you represent $x + 3$ on the right side of the mat? Place one x-tile and three 1-tiles on the right side of the mat.

OL In order to isolate x on the right side of the mat, what do you need to do? Sample answer: Remove three 1-tiles from each side of the mat, in order to isolate x.

OL Why is it not enough to only remove three 1-tiles from the right side of the mat? Sample answer: By doing so, the equation would not remain balanced. If I subtract three 1-tiles from the right side, I need to subtract three 1-tiles from the left side.

BL If the equation is written as $8 = 3 + x$, would the process and/or solution change? Explain. no; Sample answer: $3 + x$ is equivalent to $x + 3$ because addition is commutative. I would still need to subtract three 1-tiles from each side of the mat.

SLIDE 3

AL Why is it important to subtract 3 from each side of the equation? Sample answer: In order to keep each side of the equation balanced, 3 needs to be subtracted from each side of the equation.

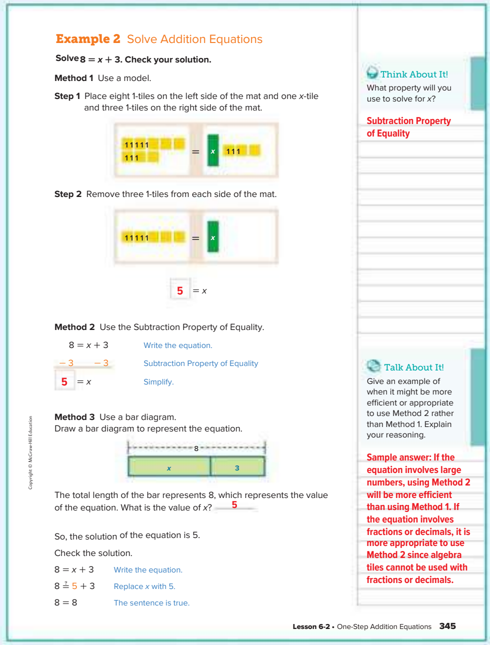
OL Explain how solving the equation algebraically mirrors solving the equation using algebra tiles. Sample answer: Subtracting 3 from each side of the equation is the same as removing three 1-tiles from each side of the mat.

OL How can you check your solution? Sample answer: To check my solution, I can substitute 5 for x in the equation to verify that the statement $8 = 5 + 3$ is true, which it is.

BL If x is equal to twice the value of y, and $8 = x + 3$, what is the value of y? Explain. $y = 2.5$; Sample answer: Since $x = 5$, then $y = 2.5$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Solve Addition Equations

Solve $8 = x + 3$. Check your solution.

Method 1 Use a model.

Step 1 Place eight 1-tiles on the left side of the mat and one x-tile and three 1-tiles on the right side of the mat.

Step 2 Remove three 1-tiles from each side of the mat.

$5 = x$

Method 2 Use the Subtraction Property of Equality.

$8 = x + 3$	Write the equation.
$-3 \quad -3$	Subtraction Property of Equality
$5 = x$	Simplify.

Method 3 Use a bar diagram.
Draw a bar diagram to represent the equation.

The total length of the bar represents 8, which represents the value of the equation. What is the value of x? 5

So, the solution of the equation is 5.

Check the solution.

$8 = x + 3$	Write the equation.
$8 \stackrel{?}{=} 5 + 3$	Replace x with 5.
$8 = 8$	The sentence is true.

Think About It! What property will you use to solve for x?

Subtraction Property of Equality

Talk About It! Give an example of when it might be more efficient or appropriate to use Method 2 rather than Method 1. Explain your reasoning.

Sample answer: If the equation involves large numbers, using Method 2 will be more efficient than using Method 1. If the equation involves fractions or decimals, it is more appropriate to use Method 2 since algebra tiles cannot be used with fractions or decimals.

Lesson 6-2 • One-Step Addition Equations 345

Interactive Presentation

Example 2, Solve One-Step Addition Equations, Slide 2 of 5

DRAG & DROP

On Slide 2, students use algebra tiles to model the equation.

CLICK

On Slide 3, students move through the steps to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Solve $570 = 33 + x$ for x. **537**

Show your work here

Example 3 Solve Addition Equations

Solve $3\frac{3}{4} + m = 7\frac{1}{2}$. Check your solution.

$3\frac{3}{4} + m = 7\frac{1}{2}$ — Write the equation.

$3\frac{3}{4} + m = 7\frac{2}{4}$ — Rewrite with like denominators.

$-3\frac{3}{4} \quad -3\frac{3}{4}$ — Subtraction Property of Equality

$m = 3\frac{3}{4}$

So, the solution of the equation is $3\frac{3}{4}$.

Check the solution.

$3\frac{3}{4} + m = 7\frac{1}{2}$ — Write the equation.

$3\frac{3}{4} + 3\frac{3}{4} \stackrel{?}{=} 7\frac{1}{2}$ — Replace m with $3\frac{3}{4}$.

$7\frac{1}{2} = 7\frac{1}{2}$ — The sentence is true.

Think About It! What do you notice about the equation? Does this change your approach to solving it? Why or why not?

Sample answer: The equation involves mixed numbers. No, the process for solving the equation remains the same, except I need to calculate with mixed numbers.

Check

Solve $k + \frac{5}{8} = 2\frac{1}{4}$ for k. Check your solution. $1\frac{5}{8}$

Show your work here

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3, Solve One-Step Addition Equations, Slide 2 of 3

CLICK

On Slide 2, students move through the steps to solve the equation.

TYPE

On Slide 2, students determine the value of m.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Solve Addition Equations

Objective

Students will solve one-step addition equations involving fractions using the Subtraction Property of Equality.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to adhere to the Subtraction Property of Equality to solve the equation algebraically. Students should be able to calculate with mixed numbers and fractions efficiently and accurately.

Questions for Mathematical Discourse

SLIDE 1

AL What property can you use to isolate the variable? Subtraction Property of Equality

OL Why is it important to rewrite the mixed numbers with like denominators? Sample answer: I need to subtract $3\frac{3}{4}$ from each side of the equation. In order to subtract $3\frac{3}{4}$ from $7\frac{1}{2}$, the fractions need to have a common denominator.

OL How can you use mental math to solve the equation? Sample answer: I know that $3\frac{3}{4} + 4 = 7\frac{3}{4}$. Since the sum needs to be $7\frac{1}{2}$, not $7\frac{3}{4}$, the second addend should be $\frac{1}{4}$ less than 4, which is $3\frac{3}{4}$.

BL A classmate stated that $m = 11\frac{1}{4}$. Explain the mistake. Sample answer: Instead of subtracting $3\frac{3}{4}$ from each side of the equation, the classmate subtracted $3\frac{3}{4}$ from the left side of the equation and added $3\frac{3}{4}$ to the right side of the equation.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Money

Objective

Students will come up with their own strategy to solve an application problem involving buying books from an online bookstore.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What information from the table is extra?
- How much did she spend altogether?
- What is the first step to writing the equation?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Money

An online bookstore is having a sale on mysteries. The table shows the cost of each book format. Abigail has $70 to spend. She bought two paperbacks, one hardcover, and one e-book. Write an addition equation that can be used to determine how much more money Abigail still has to spend. Then solve the equation.

Books	Hardcover	Paperback	E-book	Audio book
Cost ($)	19.49	8.25	10.99	25.19

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Sample answer: $2(8.25) + 19.49 + 10.99 + x = 70$; $23.02; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How can you solve the problem another way?

See students' responses.

Interactive Presentation

Apply, Money

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Miguel has $2\frac{1}{2}$ hours to work on his homework. The table shows how much time he spent working on his English homework and his math homework. Write an equation that can be used to find how much time, in minutes, he has left to work on his science project if he wants to take a 15-minute snack break. Then solve the equation.

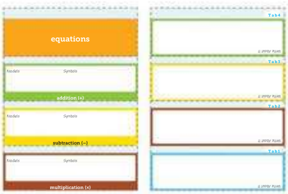
Homework	Time Spent (min)
English	45
Math	28
Science Project	?

Show your work here

Sample answer: $45 + 28 + 15 + x = 150$; 62 minutes

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how to write and solve one-step addition equations. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Suppose the lifespan of a tumbo tree is 1,850 years. Suppose you also know that the combined life span of a tumbo tree and a bristlecone pine is 6,910 years. Write and solve an equation to find the lifespan of a bristlecone pine. Sample answer: Let x represent the lifespan of the Bristlecone pine tree; $1{,}850 + x = 6{,}910$; $x = 5{,}060$

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11, 13–16
- ALEKS One-Step Equations, Applications of Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 13, 14
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Introduction to One-Step Equations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Introduction to One-Step Equations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

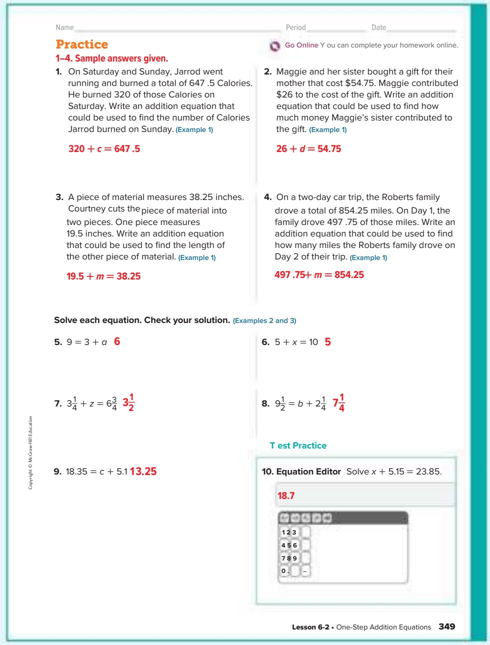
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	model a real-world problem with a one-step addition equation.	1–4
1	solve one-step addition equations involving whole numbers using a model and the Subtraction Property of Equality	5, 6
1	solve one-step addition equations involving fractions using the Subtraction Property Equality	7, 8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving one-step addition equations	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may struggle with subtracting fractions and/or mixed numbers. In Exercises 7 and 8, students may understand how to solve one-step addition equations, but incorrectly find the solution because they made a mistake in their calculations with the mixed numbers. Remind students that, when subtracting with mixed numbers, it is often helpful to write the mixed numbers as improper fractions first. Then they can rewrite the improper fractions with a common denominator. You may wish to have students practice their fluency with operations of fractions.



Name Period Date

Practice Go Online You can complete your homework online.

1–4. Sample answers given.

1. On Saturday and Sunday, Jarrod went running and burned a total of 647.5 Calories. He burned 320 of those Calories on Saturday. Write an addition equation that could be used to find the number of Calories Jarrod burned on Sunday. (Example 1)
$320 + c = 647.5$

2. Maggie and her sister bought a gift for their mother that cost $54.75. Maggie contributed $26 to the cost of the gift. Write an addition equation that could be used to find how much money Maggie's sister contributed to the gift. (Example 1)
$26 + d = 54.75$

3. A piece of material measures 38.25 inches. Courtney cuts the piece of material into two pieces. One piece measures 19.5 inches. Write an addition equation that could be used to find the length of the other piece of material. (Example 1)
$19.5 + m = 38.25$

4. On a two-day car trip, the Roberts family drove a total of 854.25 miles. On Day 1, the family drove 497.75 of those miles. Write an addition equation that could be used to find how many miles the Roberts family drove on Day 2 of their trip. (Example 1)
$497.75 + m = 854.25$

Solve each equation. Check your solution. (Examples 2 and 3)

5. $9 = 3 + a$ **6**

6. $5 + x = 10$ **5**

7. $3\frac{1}{4} + z = 6\frac{3}{4}$ **$3\frac{1}{2}$**

8. $9\frac{1}{2} = b + 2\frac{1}{4}$ **$7\frac{1}{4}$**

9. $18.35 = c + 5.1$ **13.25**

Test Practice

10. **Equation Editor** Solve $x + 5.15 = 23.85$.
18.7

Lesson 6-2 • One-Step Addition Equations 349

Apply *indicates multi-step problem

*11. Jeremiah has $35 to spend on items for his dog at the pet store. The table shows the cost of the items. He bought a collar, two toys, two biscuits, and a ball. Write an addition equation that can be used to determine how much more money Jeremiah still has to spend. Then solve the equation.

Item	Cost ($)
Ball	3.45
Biscuit	1.15
Bone	2.50
Collar	8.99
T oy	5.75

Sample equation: $8.99 + 2(5.75) + 2(1.15) + 3.45 + x = 35$; $8.76

*12. Jasmine has $30 to spend on ice cream for a party. The table shows the cost of each size of ice cream. She bought five quarts and one gallon. Write an addition equation that can be used to determine how much more money Jasmine still has to spend. Then solve the equation.

Ice Cream Size	Cost ($)
Gallon	6.99
Pint	1.59
Quart	3.35

Sample equation: $5(3.35) + 6.99 + x = 30$; $6.26

Higher-Order Thinking Problems

13. MP **Reason Abstractly** Suppose $a + b = 20$ and the value of a is increased by 1. If the sum of a and b remains the same, what must happen to the value of b?

The value of b must be decreased by 1.

14. MP **Find the Error** A student is solving the equation $x + 9 = 14$. Find the student's mistake and correct it.

$$\begin{aligned} x + 9 &= 14 \\ +9 &= +9 \\ \hline x &= 23 \end{aligned}$$

The student added 9 to each side of the equation instead of subtracting 9 from each side. The correct solution should be $x = 5$.

15. MP **Persevere with Problems** In the equation $m + n = 12$, the value for m is a whole number greater than 5 but less than 9. Determine the possible solutions for n.

4, 5, 6

16. **Create** Write and solve a real-world problem that can be solved with a one-step addition equation.

Sample answer: Chad is saving money to buy a scooter that costs $47 . He has already saved $25. Write and solve an equation to find how much more money Chad needs to save; $25 + m = 47$; $22

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Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 13, students will describe what happens to the value of b in the given situation. Encourage students to use abstract reasoning to determine the outcome when the sum stays the same and the value of a increases.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students find the mistake and correct it. Encourage students to find the error and then rework the given problem correctly.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 15, students determine the possible solutions for n, given certain situations. Encourage students to think about the given equation and how the given restrictions on the variable would affect the equation.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.

Use with Exercises 11–12 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Explore the truth of statements created by others.

Use with Exercises 13–16 Have students work in pairs. After completing the exercises, have students write two true statements about one-step addition equations and one false statement. An example of a true statement might be, "You can use the Subtraction Property of Equality to solve one-step addition equations." An example of a false statement might be, "You can use the Addition Property of Equality to solve one-step addition equations." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. For each false statement, have them generate a counterexample. Have them discuss and resolve any differences.

Learn Write Subtraction Equations

Objective

Students will learn how to model a real-world problem with a one-step subtraction equation.

Teaching the Mathematical Practices

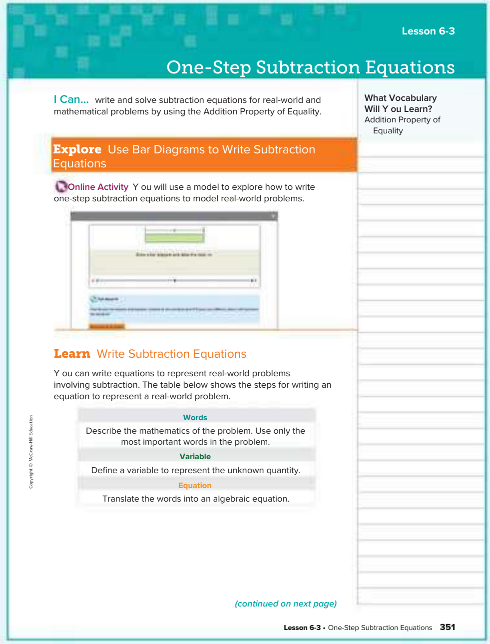
6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language to share which key words, from the animation, they thought were important and why those are important in writing the equation.

Teaching Notes

SLIDE 1

Be sure students understand the importance of defining the variable when writing an equation to model a real-world problem. You may wish to have students create their own word problem that involves subtraction, such as *Jackson spent $3.15 less on lunch than his sister. Jackson spent $5.50 on lunch. How much did his sister spend?* Have students choose a variable, such as x or s, and clearly explain what that variable represents (the dollar amount his sister spent on lunch). Then have them write an equation that models the problem, such as $x - 3.15 = 5.50$ or $s - 3.15 = 5.50$. Be sure they understand there can be more than one way to write the equation. Have them explain, however, why the equation $3.15 - x = 5.50$ does not model the problem.

(continued on next page)



Lesson 6-3

One-Step Subtraction Equations

I Can... write and solve subtraction equations for real-world and mathematical problems by using the Addition Property of Equality.

What Vocabulary Will You Learn?
Addition Property of Equality

Explore Use Bar Diagrams to Write Subtraction Equations

Online Activity You will use a model to explore how to write one-step subtraction equations to model real-world problems.

Learn Write Subtraction Equations

You can write equations to represent real-world problems involving subtraction. The table below shows the steps for writing an equation to represent a real-world problem.

Words
Describe the mathematics of the problem. Use only the most important words in the problem.
Variable
Define a variable to represent the unknown quantity.
Equation
Translate the words into an algebraic equation.

(continued on next page)

Lesson 6-3 • One-Step Subtraction Equations 351

Interactive Presentation

Learn, Write Subtraction Equations, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view the steps for writing an equation to model a real-world problem.

DIFFERENTIATE

Language Development Activity ELL

Some students may struggle with identifying words that signify subtraction. Have students work with a partner to brainstorm words that signify subtraction. Have them create a poster to display in the classroom.

Sample answers: how many more, less than, subtract, take away, remain, minus, difference, left

Lesson 6-3

6.EE.B.6, 6.EE.B.7

One-Step Subtraction Equations

LESSON GOAL

Students will use the Addition Property of Equality to write and solve one-step subtraction equations.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Bar Diagrams to Write Subtraction Equations

Learn: Write Subtraction Equations

Example 1: Write Subtraction Equations

Learn: Solve Subtraction Equations

Example 2: Solve Subtraction Equations

Example 3: Solve Subtraction Equations

Apply: Shopping

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 36 of the *Language Development Handbook* to help your students build mathematical language related to solving one-step subtraction equations.

ELL You can use the tips and suggestions on page T36 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min — 1 day

45 min — 2 days

Focus

Domain: Expressions and Equations

Major Cluster(s): In this lesson, students address major cluster **6.EE.B** by using the Addition Property of Equality to write and solve one-step subtraction equations.

Standards for Mathematical Content: **6.EE.B.6, 6.EE.B.7**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous

Students used the Subtraction Property of Equality to write and solve one-step addition equations.
6.EE.B.6, 6.EE.B.7

Now

Students use the Addition Property of Equality to write and solve one-step subtraction equations.
6.EE.B.6, 6.EE.B.7

Next

Students will use the Division Property of Equality to write and solve one-step multiplication equations.
6.EE.B.6, 6.EE.B.7

Rigor

The Three Pillars of Rigor

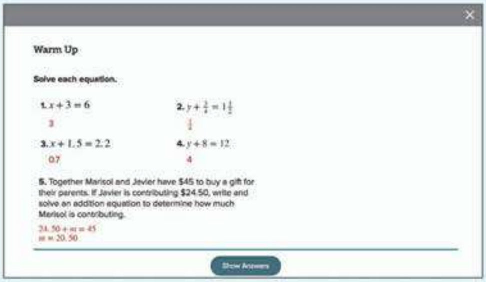
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of one-step subtraction equations. They learn how to use a model and the Addition Property of Equality to build *fluency* with solving one-step subtraction equations involving whole numbers and fractions. They *apply* their understanding of writing and solving one-step subtraction equations to solve multi-step, real-world problems.

Mathematical Background

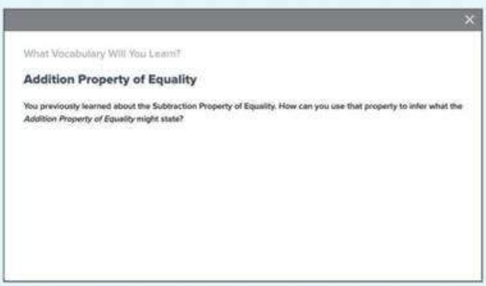
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- solving one-step addition equations (Exercises 1–4)
- writing and solving one-step addition equations (Exercise 5)

Answers

1. 3

2. $\frac{3}{4}$

3. 0.7

4. 4

5. $24.50 + m = 45$; $m = 20.50$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the record for most runs scored in the Cricket World Cup.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- You previously learned about the Subtraction Property of Equality. How can you use that property to infer what the *Addition Property of Equality* might state? Sample answer: The Subtraction Property of Equality states that, as long as the same number is subtracted from each side of an equation, the sides of the equation remain balanced. The Addition Property of Equality might mean that, as long as the same number is added to each side of an equation, the sides of the equation will remain balanced.

Explore Use Bar Diagrams to Write Subtraction Equations

Objective

Students will explore how to use a model to write subtraction equations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will work through two real-world situations, first using a bar diagram and then writing an equation, to illustrate the situation. Students will explore how to use a model to write subtraction equations.

Inquiry Question

How can you use a model to write subtraction equations? Sample answer: I can write a subtraction equation using a bar diagram with two sections representing what I know. The total bar represents the original number, which is the unknown.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

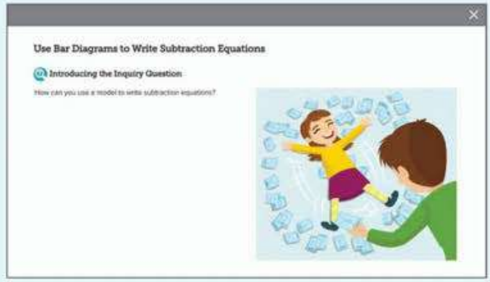
SLIDE 3

Mathematical Discourse

How could you use a bar diagram to represent what you know and what you need to find? Sample answer: I could draw a bar and label the total c. I could separate the bar into two sections labeled 17 and 41.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8

Explore, Slide 3 of 8

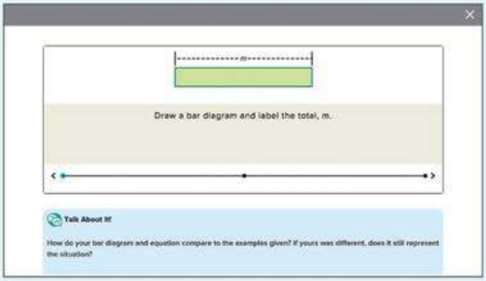
CLICK

On Slide 3, students highlight what they know and what they need to find.

CLICK

On Slide 4, students move through the slides to see how a bar diagram can be created to model the situation.

Interactive Presentation



Explore, Slide 7 of 8

CLICK

On Slide 7, students move through the steps used to draw the bar diagram.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Use Bar Diagrams to Write Subtraction Equations *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should identify the important quantities in the real-world situation, decontextualize them, and use the bar diagram to represent them symbolically.

5 Use Appropriate Tools Strategically Encourage students to examine the correspondences between the bar diagrams and equations, and how they could eventually transition from the problem statement to writing the equation without the bar diagram.

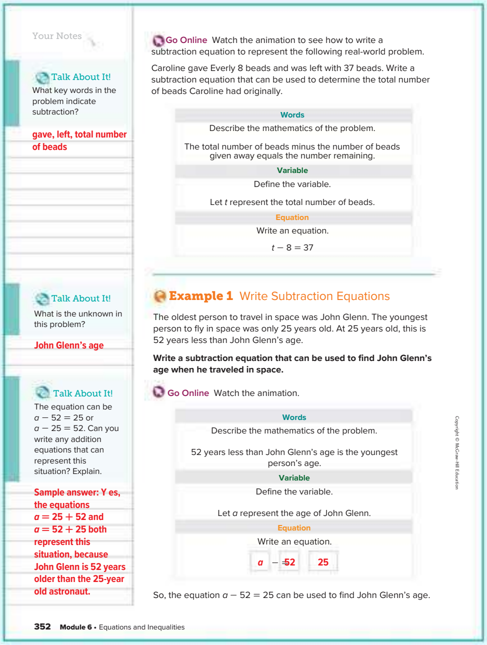
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

Talk About It!

SLIDE 6

Mathematical Discourse

What is known and what is unknown in the situation? How did you set up the bar diagram? Sample answer: The amount Logan had originally is unknown, the snack amount and change is known. The bar diagram could be set up with x being the total and the bar being split into two sections, $5.33 and $12.67.



Your Notes

Talk About It!
What key words in the problem indicate subtraction?

gave, left, total number of beads

Go Online Watch the animation to see how to write a subtraction equation to represent the following real-world problem.

Caroline gave Everly 8 beads and was left with 37 beads. Write a subtraction equation that can be used to determine the total number of beads Caroline had originally.

Words
Describe the mathematics of the problem.
The total number of beads minus the number of beads given away equals the number remaining.
Variable
Define the variable.
Let t represent the total number of beads.
Equation
Write an equation.
$t - 8 = 37$

Talk About It!
What is the unknown in this problem?

John Glenn's age

Example 1 Write Subtraction Equations

The oldest person to travel in space was John Glenn. The youngest person to fly in space was only 25 years old. At 25 years old, this is 52 years less than John Glenn's age.

Write a subtraction equation that can be used to find John Glenn's age when he traveled in space.

Go Online Watch the animation.

Words
Describe the mathematics of the problem.
52 years less than John Glenn's age is the youngest person's age.
Variable
Define the variable.
Let a represent the age of John Glenn.
Equation
Write an equation.
a − 52 = 25

So, the equation $a - 52 = 25$ can be used to find John Glenn's age.

Talk About It!
The equation can be $a - 52 = 25$ or $a - 25 = 52$. Can you write any addition equations that can represent this situation? Explain.

Sample answer: Yes, the equations $a = 25 + 52$ and $a = 52 + 25$ both represent this situation, because John Glenn is 52 years older than the 25-year old astronaut.

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Learn Write Subtraction Equations *(continued)*

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how to write a subtraction equation to model a real-world problem.

Talk About It!

SLIDE 3

Mathematical Discourse

What key words are in the situation? gave, left, total number of beads

Example 1 Write Subtraction Equations

Objective

Students will model a real-world problem with a one-step subtraction equation.

Questions for Mathematical Discourse

SLIDE 3

AL What is the unknown you are trying to find? John Glenn's age

AL How do you know that the unknown is not the youngest person's age? I know they were 25 years old.

OL How do you know that the equation is not $52 - a = 25$? Sample answer: John Glenn's age, represented by a, is the greater age. So, it cannot be subtracted from 52. This would mean that John Glenn's age was 25 years younger than the age of 52, which is not correct.

BL Why is it correct to write the equation as either $a - 52 = 25$ or $a - 25 = 52$? Sample answer: The equation $a - 52 = 25$ states that John Glenn's age minus the difference between the ages equals 25, which is correct. The equation $a - 25 = 52$ states that John Glenn's age minus the age of the youngest person to travel in space is equal to the difference in their ages, 52. This is also correct.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Interactive Presentation

Example 1, Write Subtraction Equations, Slide 1 of 6

WATCH

On Slide 2 of the Learn, students watch the animation to learn how to model a real-world problem with a one-step subtraction equation.

DRAG & DROP

On Slide 3 of Example 1, students drag the objects to write a subtraction equation to model the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Solve Subtraction Equations

Objective

Students will learn how to solve one-step subtraction equations using a model and the Addition Property of Equality.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to reason about the similarities and differences between the processes used to solve one-step addition equations and subtraction equations.

Go Online to have your students watch the video on Slide 1. The video illustrates how to solve one-step subtraction equations using a bar diagram.

Teaching Notes

SLIDE 1

You may wish to pause the video after the equation $x - 15 = 11$ is shown, and ask students to work with a partner to use bar diagrams to model and solve the equation. Have them share their process and solution with another pair of students, or the entire class. Then have them continue watching the video to compare their process and solution with the one shown. Repeat using a similar process for the second equation in the video, $x - 32 = 14$.

SLIDE 2

Remind students that addition and subtraction are inverse operations. To solve a subtraction equation for a variable, such as $n - 6 = 7$, students can undo the subtraction of 6 by adding 6. Point out that the same number must be added to each side of the equation, in order for the equation to remain equal. Have students select the *Words* and *Examples* flashcards to view the *Addition Property of Equality* expressed in these multiple representations.

Talk About It!

SLIDE 3

Mathematical Discourse

Compare and contrast solving one-step addition equations and solving one-step subtraction equations. Sample answer: They are both similar in that I use inverse operations to undo the addition or subtraction. They are different in that to undo addition, I use subtraction, and to undo subtraction, I use addition.

Check

An e-Book costs \$14.95. This is \$7 .55 less than the cost of the hardback version of the same book. Write a subtraction equation that can be used to find the cost of the hardback book.

Sample answer: $h - 7.55 = 14.95$

Show your work here

Go Online Y ou can complete an Extra Example online.

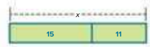
Learn Solve Subtraction Equations

Y ou can use substitution, models, or properties of mathematics to solve subtraction equations.

Go Online Watch the video to learn how to solve one-step subtraction equations using a bar diagram.

The video demonstrates how to find the value of x in the equation $x - 15 = 11$.

Draw a bar to represent the total. The total length of the bar represents the original amount, x. Divide the bar into two sections to show the known values, 15 and 11.



Because x represents the length of the entire bar, add 15 and 11 to find the value of x.

So, $x = 26$.

T o solve a subtraction equation, use the inverse operation, which is addition. When you solve an equation by adding the same number to each side of the equation, you are using the **Addition Property of Equality**.

Words	Examples
If you add the same number to each side of an equation, the two sides remain equal.	If $10 = 10$, then $10 + 3 = 10 + 3$. If $n - 6 = 7$, then $n - 6 + 6 = 7 + 6$.

Talk About It! Compare and contrast solving one-step addition equations and solving one-step subtraction equations.

Sample answer: They are both similar in that I use inverse operations to undo the addition or subtraction. They are different in that to undo addition, I use subtraction, and to undo subtraction, I use addition.

Interactive Presentation

Learn, Solve Subtraction Equations, Slide 2 of 3

WATCH

On Slide 1, students watch a video to learn about how to use a model to solve a one-step subtraction equation.

FLASHCARDS

On Slide 2, students use Flashcards to learn more about the Addition Property of Equality.

Example 2 Solve Subtraction Equations

Solve $32 = x - 7$. Check your solution.

$32 = x - 7$	Write the equation.
$+7 = \quad +7$	Addition Property of Equality
$39 = x$	

So, the solution of the equation is **39**.

Check the solution.

$32 = x - 7$	Write the equation.
$32 \stackrel{?}{=} 39 - 7$	Replace x with 39.
$32 = 32$	The sentence is true.

Check

Solve $2{,}019 = x - 731$ for x. **2,750**

Show your work here

Example 3 Solve Subtraction Equations

Solve $m - 13\frac{2}{3} = 2\frac{1}{6}$.

$m - 13\frac{2}{3} = 2\frac{1}{6}$	Write the equation.
$m - 13\frac{4}{6} = 2\frac{1}{6}$	Rewrite with like denominators.
$+13\frac{4}{6} \quad +13\frac{4}{6}$	Addition Property of Equality
$m = 15\frac{5}{6}$	

So, the solution of the equation is $15\frac{5}{6}$.

Talk About It! How can you check your solution?

Sample answer: I can check my solution by substituting my solution back into the equation. If the sentence is true, then my solution is correct.

Check

Solve $p - \frac{3}{4} = 4\frac{2}{5}$ for p. $\frac{103}{20}$ or $5\frac{3}{20}$

Show your work here

Go Online You can complete an Extra Example online.

Example 2 Solve Subtraction Equations

Objective

Students will solve one-step subtraction equations involving whole numbers using the Addition Property of Equality.

Questions for Mathematical Discourse

SLIDE 1

AL What is the inverse operation of subtraction? addition

OL How is the Addition Property of Equality relevant to solving this equation? Sample answer: The Addition Property of Equality states that the equation is true as long as the same number is added to each side of the equation. This allows me to add 7 to each side, and isolate the variable.

OL How can you check your solution? Sample answer: To check my solution, I can substitute 39 for x in the equation to verify that the statement $32 = 39 - 7$ is true, which it is.

BL If x is equal to twice the value of y, and $32 = x - 7$, what is the value of y? Explain. $y = 19.5$; Sample answer: Since $x = 39$, then $y = 19.5$.

Example 3 Solve Subtraction Equations

Objective

Students will solve one-step subtraction equations involving fractions using the Addition Property of Equality.

Questions for Mathematical Discourse

SLIDE 1

AL What property can you use to isolate the variable? Addition Property of Equality

OL Why is it important to rewrite the mixed numbers with like denominators? Sample answer: I need to add $13\frac{2}{3}$ to each side of the equation. In order to add $13\frac{2}{3}$ to $2\frac{1}{6}$, the fractions need to have a common denominator.

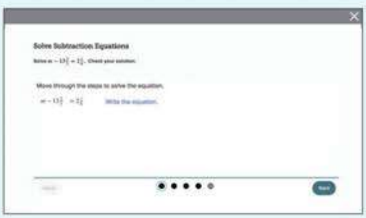
OL How can you use mental math to solve the equation? Sample answer: I know that m has to be $2\frac{1}{6}$ more than $13\frac{2}{3}$. This means that m must be the sum of $2\frac{1}{6}$ and $13\frac{2}{3}$. Add two wholes to $13\frac{2}{3}$ to obtain $15\frac{2}{3}$. Then add $\frac{1}{6}$: $15\frac{2}{3} + \frac{1}{6} = 15\frac{5}{6}$.

BL Explain how someone might get $m = 11\frac{1}{2}$ as the solution. Sample answer: If you subtract $2\frac{1}{6}$ from $13\frac{2}{3}$, you will get $11\frac{1}{2}$, which is incorrect.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Interactive Presentation



Example 3, Solve Subtraction Equations, Slide 1 of 2

CLICK On Slide 1 of Example 2, students move through the steps to solve the equation.

TYPE On Slide 1 of Example 3, students determine the solution of the equation.

CHECK Students complete the Check exercises online to determine if they are ready to move on.

Apply Shopping

Objective

Students will come up with their own strategy to solve an application problem involving shopping.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How much did Tyson spend on each item?
- What operation would you use to determine how much he spent altogether?
- Is the amount that he originally had in his savings account going to be greater than or less than the amount he had after withdrawing money?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Shopping

Tyson had $302.87 in his savings account after he withdrew money to go shopping. He spent the amounts shown, and he had $18.25 remaining. Use an equation to find how much Tyson originally had in his savings account.

Item	Total Spent ($)
Clothes	95.21
Gifts	42.79
Soccer ball	23.75

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

$482.87; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How can you solve the problem another way?

See students' responses.

Lesson 6-3 • One-Step Subtraction Equations 355

Interactive Presentation

Apply, Shopping

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

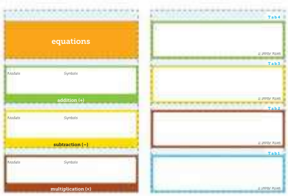
Check

Nina used $12\frac{1}{3}$ yards of ribbon to make hair bows and 5 yards of ribbon to wrap gifts. She has $17\frac{2}{3}$ yards of ribbon left. Use a subtraction equation to find how many yards of ribbon she had to start. **35 yards**

Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how to write and solve one-step subtraction equations. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. The current runner-up for the most runs scored in the Cricket World Cup has 1,743 runs, which is 535 runs fewer than the current record holder. Write and solve an equation that represents the number of runs of the current record holder. Sample answer: Let x represent the number of runs of the record holder; $x - 535 = 1{,}743$; $x = 2{,}278$

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11, 13–16
- ALEKS One-Step Equations, Applications of Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 13, 14
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Introduction to One-Step Equations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Introduction to One-Step Equations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

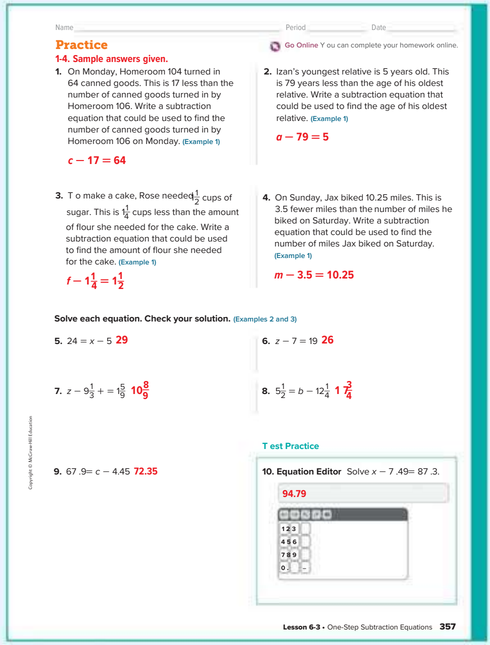
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	model a real-world problem with a one-step subtraction equation	1–4
1	solve one-step subtraction equations involving whole numbers using the Addition Property of Equality	5, 6
1	solve one-step subtraction equations involving fractions using the Addition Property of Equality	7, 8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving one-step subtraction equations	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may struggle with adding or subtracting decimals. In exercises 9 and 10, students may understand how to solve one-step subtraction equations, but incorrectly find the solution because they made a mistake in their calculations with decimals. Remind students that, when adding vertically, they need to line up the decimal places and annex a zero if necessary. You may wish to have students practice their fluency with operations with decimals.



Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

1-4. Sample answers given.

1. On Monday, Homeroom 104 turned in 64 canned goods. This is 17 less than the number of canned goods turned in by Homeroom 106. Write a subtraction equation that could be used to find the number of canned goods turned in by Homeroom 106 on Monday. (Example 1)

   $c - 17 = 64$

2. Izan's youngest relative is 5 years old. This is 79 years less than the age of his oldest relative. Write a subtraction equation that could be used to find the age of his oldest relative. (Example 1)

   $a - 79 = 5$

3. T o make a cake, Rose needed $1\frac{1}{2}$ cups of sugar. This is $1\frac{1}{4}$ cups less than the amount of flour she needed for the cake. Write a subtraction equation that could be used to find the amount of flour she needed for the cake. (Example 1)

   $f - 1\frac{1}{4} = 1\frac{1}{2}$

4. On Sunday, Jax biked 10.25 miles. This is 3.5 fewer miles than the number of miles he biked on Saturday. Write a subtraction equation that could be used to find the number of miles Jax biked on Saturday. (Example 1)

   $m - 3.5 = 10.25$

Solve each equation. Check your solution. (Examples 2 and 3)

5. $24 = x - 5$ **29**
6. $z - 7 = 19$ **26**
7. $z - 9\frac{1}{3} + = 1\frac{5}{9}$ **$10\frac{8}{9}$**
8. $5\frac{1}{2} = b - 12\frac{1}{4}$ **$1 7\frac{3}{4}$**
9. 67 .9= $c - 4.45$ **72.35**

T est Practice

10. **Equation Editor** Solve $x - 7 .49= 87 .3$.

    94.79

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Lesson 6-3 • One-Step Subtraction Equations 357

Apply *indicates multi-step problem

*11. After spending money for a golf outing, Gus had $517 .92 remaining in his checking account. The table shows how much money he spent on different items to participate in the outing. Use an equation to find how much money Gus originally had in his checking account.

Item	Cost ($)
Entry Fee	94.50
Golf Shoes	44.25
Gloves	11.25

$667 .92

*12. Robin made two batches of every item shown in the table. At the end of the day, she had $1\frac{1}{4}$ cups of flour left. Use an equation to find how much flour Robin originally had on Saturday.

Baking Item	Amount of Flour
Bread	$1\frac{3}{4}$ cups
Muffins	2 cups
Pancakes	$1\frac{1}{2}$ cups

$11\frac{3}{4}$ cups of flour

Higher-Order Thinking Problems

13. MP Reason Abstractly During a test flight, Jeri's rocket reached a height of 18 yards above the ground. This was 7 yards less than the height that Devon's rocket reached. Did Devon's rocket reach a height greater than 23 yards? Explain.

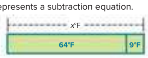
yes; Sample answer: Solve the equation $x - 7 = 18$ to find the height of Devon's rocket. Devon's rocket reached a height of $18 + 7$ or 25 yards. Since $25 > 23$, Devon's rocket reached a height greater than 23 yards.

14. MP Find the Error A student is solving the equation $x - 3.2 = 5.5$. Find the student's mistake and correct it.

$$\begin{array}{rcl} x - 3.2 & = & 5.5 \\ -3.2 & & -3.2 \\ \hline x & = & 2.3 \end{array}$$

Sample answer: The student subtracted 3.2 from each side of the equation instead of adding 3.2. The solution should be $x = 8.7$.

15. Multiple Representations The bar diagram represents a subtraction equation.



a. Words Write a real-world situation for the bar diagram.
Sample answer: Today's high temperature is 64°F. This is 9°F less than yesterday's high temperature. What was yesterday's high temperature?

b. Algebra Write a subtraction equation for the bar diagram. **$x - 9 = 64$**

c. Numbers Solve the equation from part b.
73°F

16. Create Write and solve a real-world problem involving decimals that can be solved with a one-step subtraction equation.

Sample answer: Frank's allowance is $8.50 a week. This is $0.75 less than Bonnie's weekly allowance. Write and solve a subtraction equation to find Bonnie's weekly allowance; $a - 8.5 = 0.75$; $9.25

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 13, students will determine if the rocket reached a height greater than 23 yards. Encourage students to use reasoning and an equation to answer the problem.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will find the error made by the student. Encourage students to find the error and then rework the problem correctly.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 11 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 14 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks you need to subtract 3.2 from each side of the equation. Have each pair or group of students present their explanations to the class.

Learn Write Multiplication Equations

Objective

Students will learn how to model a real-world problem with a one-step multiplication equation.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language to share which key words they thought were important and how they are helpful in writing the equation.

Teaching Notes

SLIDE 1

You may wish to have students create their own word problem that involves multiplication, such as *Felicia sent 3 times as many text messages as Hector. Felicia sent 36 text messages. How many text messages did Hector send?* Have students choose a variable, such as x or t, and clearly explain what that variable represents (the number of text messages Hector sent). Then have them write an equation that models the problem, such as $3x = 36$, $3t = 36$, $36 = 3x$, or $36 = 3t$. Be sure they understand there can be more than one way to write the equation.

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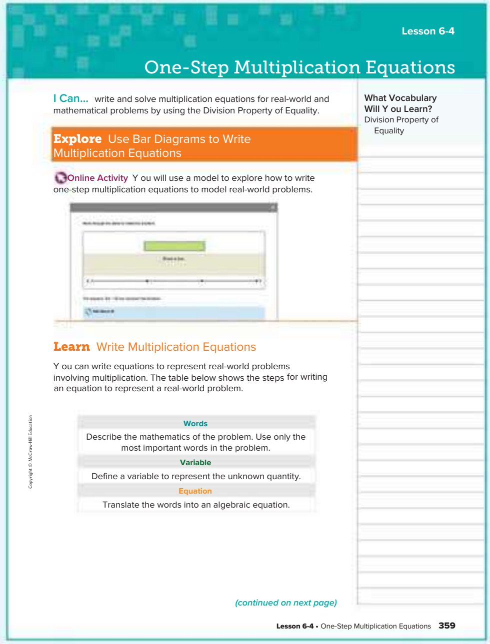

Lesson 6-4

One-Step Multiplication Equations

I Can... write and solve multiplication equations for real-world and mathematical problems by using the Division Property of Equality.

What Vocabulary Will Y ou Learn?
Division Property of Equality

Explore Use Bar Diagrams to Write Multiplication Equations

Online Activity Y ou will use a model to explore how to write one-step multiplication equations to model real-world problems.

Learn Write Multiplication Equations

Y ou can write equations to represent real-world problems involving multiplication. The table below shows the steps for writing an equation to represent a real-world problem.

Words
Describe the mathematics of the problem. Use only the most important words in the problem.
Variable
Define a variable to represent the unknown quantity.
Equation
Translate the words into an algebraic equation.

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(continued on next page)

Lesson 6-4 • One-Step Multiplication Equations 359

Interactive Presentation

Learn, Write Multiplication Equations, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view the steps for writing an equation to represent a real-world problem.

DIFFERENTIATE

Enrichment Activity BL

If any of your students need more of a challenge, have students work with a partner to create three different real-world problems. One problem should be able to be modeled with a one-step addition equation. Another problem should be able to be modeled with a one-step subtraction equation. The third problem should be able to be modeled with a one-step multiplication equation.

Have pairs trade problems with another pair of students. Each pair should generate the appropriate equations that model each problem. Have them check their equations with the original pair of students, and discuss and resolve any differences. Finally, have pairs solve the addition and subtraction equations using inverse operations and properties of equality, and then make a conjecture as to how they might be able to solve the multiplication equations.

Lesson 6-4

One-Step Multiplication Equations

6.EE.B.6, 6.EE.B.7

LESSON GOAL

Students will use the Division Property of Equality to write and solve one-step multiplication equations.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Bar Diagrams to Write Multiplication Equations

Learn: Write Multiplication Equations
Example 1: Write Multiplication Equations
Learn: Solve Multiplication Equations
Example 2: Solve Multiplication Equations
Example 3: Solve Multiplication Equations
Apply: Nutrition

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 37 of the *Language Development Handbook* to help your students build mathematical language related to solving one-step multiplication equations.

ELL You can use the tips and suggestions on page T37 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.B** by using the Division Property of Equality to write and solve one-step multiplication equations.
Standards for Mathematical Content: **6.EE.B.6, 6.EE.B.7**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students used the Addition Property of Equality to write and solve one-step subtraction equations.
6.EE.B.6, 6.EE.B.7

Now
Students use the Division Property of Equality to write and solve one-step multiplication equations.
6.EE.B.6, 6.EE.B.7

Next
Students will use the Multiplication Property of Equality to write and solve one-step division equations.
6.EE.B.6, 6.EE.B.7

Rigor

The Three Pillars of Rigor

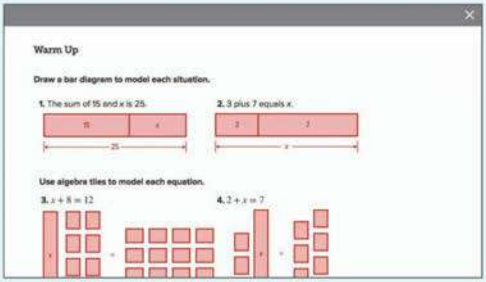
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of one-step multiplication equations. They learn how to use a model and the Division Property of Equality to build *fluency* with solving one-step multiplication equations involving whole numbers and fractions. They *apply* their understanding of writing and solving one-step multiplication equations to solve multi-step, real-world problems.

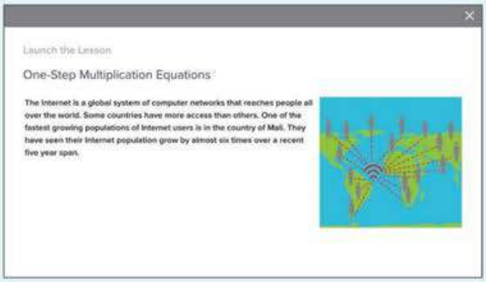
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

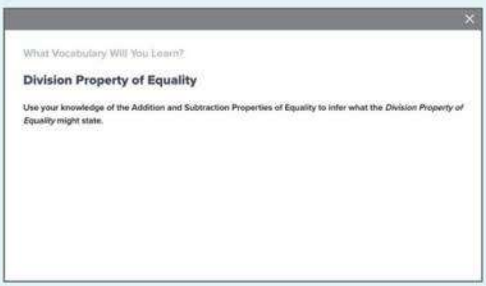
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- using bar diagrams (Exercises 1–2)
- using algebra tiles (Exercises 3–4)
- solving one-step multiplication equations using the *guess*, *check*, and *revise* strategy (Exercise 5)

Answers

1–4. See Warm Up slide online for correct answers.

5. 6 additional toppings

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about Mali's population in relation to its percentage of Internet users.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Use your knowledge of the Addition and Subtraction Properties of Equality to infer what the *Division Property of Equality* might state. Sample answer: The Addition and Subtraction Properties of Equality state that as long as I perform the same operation (addition or subtraction of the same number) to each side of an equation, the equation remains unchanged. The Division Property of Equality might state that if I divide each side of an equation by the same nonzero number, the equation remains unchanged.

Explore Use Bar Diagrams to Write Multiplication Equations

Objective

Students will explore how to use a model to write multiplication equations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a real-world situation that can be represented by a multiplication equation. Throughout this activity, students will explore how to solve the real-world problem by identifying key information and writing an equation modeled by a bar diagram that could be solved to find the missing piece of information.

Inquiry Question

How can you use a model to write multiplication equations? Sample answer: I can write a multiplication equation using a bar diagram. The total is represented by the entire bar. The bar is divided into the same number of sections as the factor you know. Each section represents the value of the variable.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

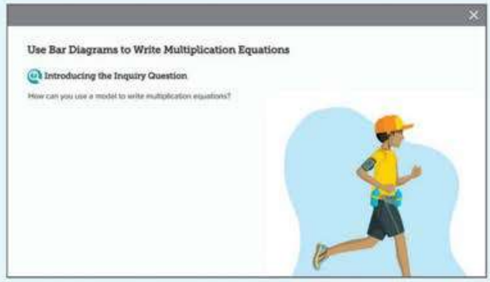
SLIDE 3

Mathematical Discourse

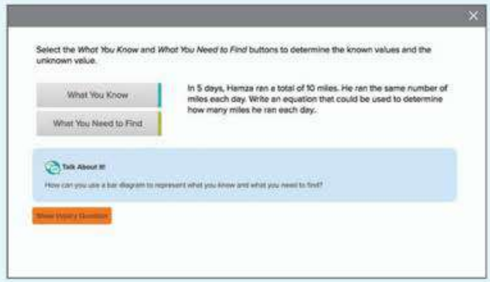
How can you use a bar diagram to represent what you know and what you need to find? Sample answer: Label the total 10 miles, and divide the bar into 5 equal parts. Label each part d, for the number of miles Hamza ran each day.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8



Explore, Slide 3 of 8

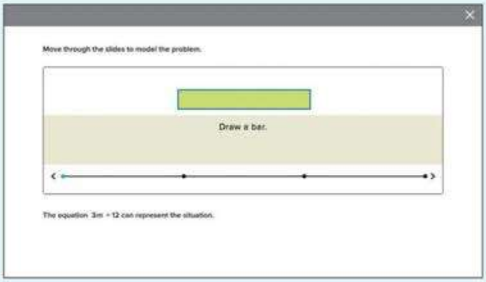
CLICK

On Slide 3, students highlight what they know and what they need to find.

CLICK

On Slide 4, students move through the slides to see how a bar diagram can be created to model the situation.

Interactive Presentation



Explore, Slide 7 of 8

CLICK

On Slide 7, student move through the sides to model the problem with a bar diagram.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Use Bar Diagrams to Write Multiplication Equations *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should identify the important quantities in the real-world situation, decontextualize them, and use the bar diagram to represent them symbolically.

5 Use Appropriate Tools Strategically Encourage students to examine the correspondences between the bar diagrams and equations, and how they could eventually transition from the problem statement to writing the equation without the bar diagram.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

Talk About It!

SLIDE 6

What is known and what is unknown in the situation? How did you set up the bar diagram? Sample answer: Label the total 12 months, and divide the bar into 3 equal parts. Label each part m, for the number of months Allie has owned her cell phone.

Your Notes

Talk About It! What key words in the problem indicate multiplication?

Sample answer: 4 weeks, equal amount, amount saved, total amount

Go Online Watch the animation to see how to write a multiplication equation to represent the following real-world problem.

Kosumi is saving an equal amount each week for 4 weeks to buy a video game for $55. Write a multiplication equation that can be used to determine the amount she is saving each week.

Words
Describe the mathematics of the problem.
The number of weeks times the amount saved each week equals the total amount saved.
Variable
Define the variable.
Let a represent the amount saved each week.
Equation
Write an equation.
$4a = 55$

Example 1 Write Multiplication Equations

Vincent and some friends shared the cost of a season ticket package for the local football team. The package cost $745 and each person contributed $186.25.

Write a multiplication equation that can be used to find how many friends contributed to the ticket purchase.

Words
Describe the mathematics of the problem.
The number of friends times the amount each person paid equals the total cost.
Variable
Define the variable.
Let f represent the number of friends who contributed.
Equation
Write an equation.
$f \cdot 186.25 = 745$

So, the equation $f \cdot 186.25 = 745$ can be used to find the number of friends that contributed. This equation can also be written as $186.25f = 745$.

Think About It! How do you know this equation uses multiplication?

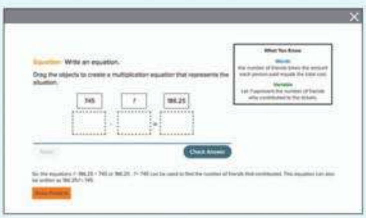
Each person contributed the same amount.

Talk About It! The equation can also be written as $186.25f = 745$. Does removing the multiplication symbol make it easier or more difficult to understand? Explain your reasoning.

See students' responses.

360 Module 6 • Equations and Inequalities

Interactive Presentation



Example 1, Write Multiplication Equations, Slide 4 of 6

WATCH

On Slide 2 of the Learn, students watch an animation to learn how to model a real-world problem with a one-step multiplication equation.

DRAG & DROP

On Slide 4 of Example 1, students drag the objects to write a multiplication equation that models the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Write Multiplication Equations *(continued)*

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how to write a multiplication equation to model a real-world problem.
- Find a sample answer for the *Talk About It!* question on Slide 3.

Example 1 Write Multiplication Equations

Objective

Students will model a real-world problem with a one-step multiplication equation.

Questions for Mathematical Discourse

SLIDE 3

AL Why is it important to define a variable? In order to write an equation, we need to use a variable to represent the unknown. It is important to state what that unknown represents, so that it is clear.

OL Why do you think f was used as the variable? Sample answer: f may have been used because the first letter of friends is f.

OL Does it matter what letter is used for the variable? Sample answer: No, any letter can be used as long as the variable is defined.

BL Why is the unknown neither the total cost, nor the cost per ticket? Both of those values are known.

SLIDE 4

AL How do you know that this should be a multiplication equation? Each friend contributed an equal amount, that when multiplied by the total number of friends, equals the total cost.

OL Why is the equation not $745f = 186.25$? Sample answer: If the equation was $745f = 186.25$, then each friend would have contributed $745 for a total of $186.25. The number of friends would be a fraction or decimal between 0 and 1, and that is impossible.

BL Can you solve the problem without writing an equation? Explain. yes; Sample answer: Divide $745 by $186.25 to find the number of people that contributed, 4.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Solve Multiplication Equations

Objective

Students will learn how to solve one-step multiplication equations using a model and the Division Property of Equality.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language, such as *inverse operations*, in order to explain why the Division Property of Equality is used when solving a multiplication equation.

Go Online to have your students watch the video on Slide 1. The video illustrates how to solve one-step multiplication equations using algebra tiles.

Teaching Notes

SLIDE 1

You may wish to pause the video after the equation $4x = 12$ is shown, and ask students to work with a partner to use algebra tiles to model and solve the equation. Have them share their process and solution with another pair of students, or the entire class. Then have them continue watching the video to compare their process and solution with the one shown. Repeat using a similar process for the second equation in the video, $18 = 3x$.

(continued on next page)

Check

A jewelry store is selling a set of 4 pairs of earrings for $58.85 including tax. Neva and three of her friends want to buy the set so each could have one pair of earrings. Write a multiplication equation that could be used to find how much each person should pay.

Show your work here

Sample answer: $4e = 58.85$

Go Online Y ou can complete an Extra Example online.

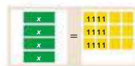
Learn Solve Multiplication Equations

Y ou can use substitution, models, or properties of mathematics to solve multiplication equations.

Go Online Watch the video to learn how to solve one-step multiplication equations using algebra tiles.

The video demonstrates how to find the value of x for $4x = 12$.

T o model the equation, place four x-tiles on the left side of the mat to represent $4x$. Place twelve 1-tiles on the right side of the mat to represent 12.



Arrange the tiles into equal groups on each side of the mat. This will allow you to group the tiles into 4 equal groups to find the value of x.

For each x-tile, there are three 1-tiles, so $x = 3$.

(continued on next page)

Interactive Presentation

Learn, Solve Multiplication Equations, Slide 1 of 3

WATCH

On Slide 1, students watch a video to learn about how to use algebra tiles to solve a one-step multiplication equation.

Talk About It!
Why is the Division Property of Equality used when solving a multiplication equation?

Sample answer: When solving equations, use the inverse operation to undo the operation in the equation. The inverse operation of multiplication is division.

Think About It!
What property will you use to solve for x?

Division Property of Equality

Talk About It!
Describe a multiplication equation for which it would not be possible to use algebra tiles to solve.

Sample answer: If a multiplication equation contained fractions or decimals, instead of whole numbers, it would not be possible to use algebra tiles to solve it.

T o solve a multiplication equation, use the inverse operation, which is division. When you solve a multiplication equation by dividing each side of the equation by the same nonzero number, you are using the **Division Property of Equality**.

Words	Examples
If you divide each side of an equation by the same nonzero number, the two sides remain equal.	If $9 = 9$, then $9 \div 3 = 9 \div 3$. If $4x = 8$, then $4x \div 4 = 8 \div 4$.

Example 2 Solve Multiplication Equations

Solve $2x = 10$. Check your solution.

Method 1 Use a model.

Step 1 Place two x-tiles on the left side of the mat to represent $2x$ and ten 1-tiles on the right side of the mat to represent 10.

Step 2 Group the 1-tiles on the right side into two equal groups because there are two x-tiles on the left side.

Because there are five 1-tiles for every x-tile, the value of x is 5.

$x =$ **5**

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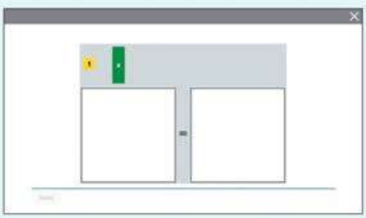
Interactive Presentation



Example 2, Solve Multiplication Equations, Slide 2 of 5

FLASHCARDS

On Slide 2 of the Learn, students use Flashcards to learn more about the Division Property of Equality.

DRAG & DROP

On Slide 2 of Example 2, students use algebra tiles to model the equation.

CLICK

On Slide 2 of Example 2, students move through the steps to model the equation.

Learn Solve Multiplication Equations *(continued)*

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Why is the Division Property of Equality used when solving a multiplication equation? Sample answer: When solving equations, use the inverse operation to undo the operation in the equation. The inverse operation of multiplication is division.

Example 2 Solve Multiplication Equations

Objective

Students will solve one-step multiplication equations using a model and the Division Property of Equality.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others, 5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 4, encourage them to present a plausible argument for a situation in which it might not be possible to use algebra tiles to solve a multiplication equation.

Questions for Mathematical Discourse

SLIDE 2

AL How can you represent $2x$ on the left side of the mat? Place two x-tiles on the left side of the mat.

AL How can you represent 10 on the right side of the mat? Place ten 1-tiles on the right side of the mat.

OL In order to determine the value of x, what do you need to do? Sample answer: Group the tiles on each side of the mat into two equal groups, since there are two x-tiles. Then count the number of 1-tiles that are in one group. This represents the value of x.

OL How can you check the solution? Sample answer: Substitute 5 in for x in the equation $2x = 10$ to verify that it is a true statement.

BL Can you model the same equation by placing ten 1-tiles on the left side of the mat and two x-tiles on the right side? Explain. yes; Sample answer: The equations $2x = 10$ and $10 = 2x$ are equivalent.

(continued on next page)

Example 2 Solve Multiplication Equations *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why is it important to divide each side of the equation by 2? Sample answer: In order to keep the equation balanced, each side of the equation needs to be divided by 2.

OL Explain how solving the equation algebraically mirrors solving the equation using algebra tiles. Sample answer: Dividing each side of the equation by 2 is the same as grouping the ten 1-tiles into two equal groups using algebra tiles.

OL How can you check your solution? Sample answer: To check my solution, I can substitute 5 for x in the equation to verify that the statement $2x = 10$ is true, which it is.

BL If x is equal to four times the value of y, and $2x = 10$, what is the value of y? Explain. $y = 1.25$; Sample answer: Since $x = 5$, then $y = 1.25$ because $4(1.25) = 5$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Method 2 Use the Division Property of Equality.

$2x = 10$	Write the equation.
$\frac{2x}{2} = \frac{10}{2}$	Division Property of Equality
$x = 5$	Simplify.

So, the solution of the equation is 5.

Check the solution.

$2x = 10$	Write the equation.
$2(5) \stackrel{?}{=} 10$	Replace x with 5.
$10 = 10$	The sentence is true.

Check

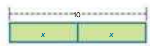
Solve $84 = 7x$. **12**

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

You can also use the bar diagram shown to represent and solve the equation $2x = 10$.



Compare and contrast each of these methods: algebra tiles, properties of equality, and bar diagrams.

Record your observations here

See students' observations.

Interactive Presentation

Example 2, Solve Multiplication Equations, Slide 3 of 5

CLICK

On Slide 3, students move through the steps to solve the equation.

TYPE

On Slide 3, students determine the value of x.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
By what number will you need to divide each side of the equation to undo multiplying by $\frac{2}{3}$?

$\frac{2}{3}$

Talk About It!
Why did you need to multiply each side of the equation by $\frac{3}{2}$, even though you used the Division Property of Equality?

Sample answer: To divide by a fraction, you multiply by its reciprocal. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Example 3 Solve Multiplication Equations

Solve $\frac{2}{3}m = \frac{5}{8}$. Check your solution.

$\frac{2}{3}m = \frac{5}{8}$	Write the equation.
$\frac{\frac{2}{3}m}{\frac{2}{3}} = \frac{\frac{5}{8}}{\frac{2}{3}}$	Division Property of Equality
$\frac{2}{3}\left(\frac{3}{2}\right)m = \frac{5}{8}\left(\frac{3}{2}\right)$	Multiply by the reciprocal.
$m = \frac{15}{16}$	

So, the solution of the equation is $\frac{15}{16}$.

Check the solution.

$\frac{2}{3}m = \frac{5}{8}$	Write the equation.
$\frac{2}{3}\left(\frac{15}{16}\right) \stackrel{?}{=} \frac{5}{8}$	Replace m with $\frac{15}{16}$.
$\frac{30}{48} \stackrel{?}{=} \frac{5}{8}$	Multiply.
$\frac{5}{8} = \frac{5}{8}$	Simplify. The sentence is true.

Check

Solve $\frac{4}{5}k = \frac{1}{3}$ for k. $\frac{5}{12}$

Show your work here

Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Did you struggle with any Examples about solving multiplication equations? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

Record your observations here

See students' observations.

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Interactive Presentation

Example 3, Solve Multiplication Equations, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to solve the equation.

TYPE

On Slide 2, students determine the value of *m*.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Solve Multiplication Equations

Objective

Students will solve one-step multiplication equations involving fractions using the Division Property of Equality.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to adhere to the Division Property of Equality to solve the equation algebraically. Students should be able to calculate with fractions efficiently and accurately.

As students discuss the *Talk About It!* question on Slide 3, encourage them to precisely explain the property that they think will be used to solve division equations, using what they know about the inverse operations and how to solve multiplication equations.

Questions for Mathematical Discourse

SLIDE 2

AL What is the operation on the left side of the equation? How do you undo this operation? multiplication; Use division to undo multiplication.

OL How do you divide fractions? Sample answer: To divide fractions, multiply the first fraction by the reciprocal of the second fraction.

OL Is it necessary to find a common denominator first? Explain. no; Sample answer: Common denominators are only needed when fractions are added or subtracted, not when multiplying or dividing fractions.

BL A classmate solved the equation by first multiplying each side of the equation by 3 to eliminate the fraction, thus obtaining the equation $2m = \frac{15}{8}$. Then they divided each side of the equation by 2 to obtain $m = \frac{15}{16}$. Is this a correct method? Explain. yes; Sample answer: Dividing each side of the equation by $\frac{2}{3}$ is the same as multiplying each side by 3, and then dividing by 2.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Nutrition

Objective

Students will come up with their own strategy to solve an application problem involving grams of sugar per serving.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the serving size for each brand of tea?
- How would you determine the amount of sugar per serving for each brand?
- How many grams of sugar are in each serving of each type of tea?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Nutrition

The nutrition information for two different bottles of iced tea is shown. Alicia wants to compare the grams of sugar in a single serving for each brand. Which brand has more sugar per serving? How much more?

Aunt Maggie's Iced T ea (3 servings)	
Calories	120
Sodium (mg)	75
Sugar (g)	63

Southern Goodness Sweet T ea (4 servings)	
Calories	125
Sodium (mg)	82
Sugar (g)	74

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Aunt Maggie's Iced Tea; 2.5 grams; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Suppose a third brand of tea has 42 grams of sugar in 2 servings. How does this compare to the brand that has more sugar per serving?

Sample answer: This brand has the same amount of sugar per serving as Aunt Maggie's Iced Tea.

Lesson 6-4 • One-Step Multiplication Equations 365

Interactive Presentation

Apply, Nutrition

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

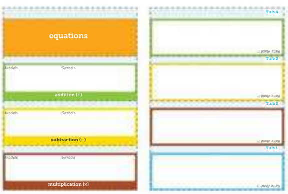
The nutrition information for two different bags of chips is shown. Frederick wants to compare the milligrams of sodium per serving in each bag of chips. Which brand has more milligrams of sodium per serving? How much more? **Heartland; 3 mg**

	Northern Grown (9 servings)	Heartland (7 servings)
Calories	1,440	1,250
Sodium	1,530 mg	1,211 mg
Sugar	9 g	5 g

Show your work here

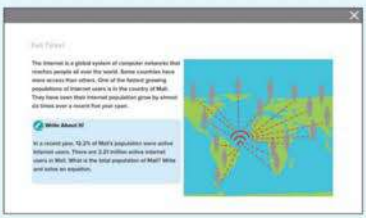
Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



366 Module 6 • Equations and Inequalities

Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how to write and solve one-step multiplication equations. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. In a recent year, 12.2% of Mali's population were active Internet users. There are 2.21 million active Internet users in Mali. What is the total population of Mali? Write and solve an equation.
Let p represent the total population of Mali in millions; $0.122p = 2.21$; $p \approx 18.11$ million people

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11, 13–16
- Extension: Extension Resources
- ALEKS One-Step Equations, Applications of Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 13, 14
- Extension: Extension Resources
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Introduction to One-Step Equations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Introduction to One-Step Equations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	model a real-world problem with a one-step multiplication equation	1–4
1	solve one-step multiplication equations using a model and the Division Property of Equality	5, 6
1	solve one-step multiplication equations involving fractions using the Division Property of Equality	7, 8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving one-step multiplication equations	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Students may struggle with division of fractions. In Exercises 7–8, students may confuse their understanding of dividing fractions with other operations with fractions. They may try to find a common denominator before dividing. You may wish to have students practice their fluency with all of the four operations with fractions. It may be helpful to have them create a chart that illustrates how to add, subtract, multiply, and divide with fractions.

Name ____________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

1–4. Sample variables given.

1. Maribel and some friends went to an adventure park. The total cost of their tickets was $374 and each person paid $46.75. Write a multiplication equation that can be used to find how many people bought tickets to the adventure park. (Example 1)

$46.75p = 374$

2. It takes Samuel $\frac{1}{5}$ hour to walk a mile. Yesterday, Samuel walked for $1\frac{1}{2}$ hours. Write a multiplication equation that can be used to find the number of miles Samuel walked. (Example 1)

$\frac{1}{5}m = 1\frac{1}{2}$

3. The distance around a lake is 2.6 miles. On Saturday, Doug biked a total of 18.2 miles around the lake. Write a multiplication equation that can be used to find how many times Doug biked around the lake. (Example 1)

$2.6t = 18.2$

4. An express delivery company charges $3.25 per pound to mail a package. Georgia paid $9.75 to mail a package. Write a multiplication equation that can be used to find the weight of the package in pounds. (Example 1)

$3.25p = 9.75$

Solve each equation. Check your solution. (Examples 2 and 3)

5. $12 = 6x$ **2**

6. $3z = 15$ **5**

7. $\frac{3}{4}z = \frac{2}{3}$ $\frac{8}{9}$

8. $\frac{1}{2} = \frac{5}{8}w$ $\frac{4}{5}$

9. $60.536 = 9.2j$ **6.58**

Test Practice

10. Equation Editor Solve $3.9x = 16.068$.

4.12

Apply *indicates multi-step problem

*11. Mira is comparing two different types of popcorn. The table shows the nutritional information. She wants to compare the number of Calories per cup for each type of popcorn. Which type has more Calories per cup? How many more?

Light Popcorn $3\frac{1}{2}$ cups	Caramel Popcorn $2\frac{1}{2}$ cups
Calories: 105	Calories: 170
Carbohydrates: 21 g	Carbohydrates: 15 g
Fat: 0 g	Fat: 11 g

Caramel Popcorn; 38 Calories

*12. The table shows the nutritional information for two different brands of apple juice. Marcus wants to compare the number of carbohydrates in a single serving of each brand. Which brand has more carbohydrates per serving? How many more?

Brand A (4 servings)	Brand B (3 servings)
Calories: 480	Calories: 360
Carbohydrates: 120 g	Carbohydrates: 87 g
Sugars: 120 g	Sugar: 78 g

Brand A; 1 g

Higher-Order Thinking Problems

13. MP **Reason Abstractly** Earline needs to save $367 .50 for her summer vacation. She plans on saving $52.50 per week. In 6 weeks, will she have enough money? Explain.

no; Sample answer: Solve the equation $52.5x = 367.50$ to find the number of weeks she needs to save. She needs to save for 7 weeks. Since $7 > 6$, she will not have enough money in 6 weeks.

14. MP **Find the Error** A student is solving the equation $3x = 9$. Find the student's mistake and correct it.

$3x = 9$

$3 \cdot 3x = 9 \cdot 3$

$x = 27$

Sample answer: The student multiplied each side by 3 instead of dividing each side by 3. The correct solution should be $x = 3$.

15. MP **Persevere with Problems** Do the equations $\frac{1}{3} = 3x$ and $\frac{1}{3} \div x = 3$ have the same solution? Explain why or why not.

yes; Sample answer: If you solve each equation you get a value of $x = \frac{1}{9}$. If you replace x with $\frac{1}{9}$ in each equation it makes the equation true. So, $\frac{1}{3} = 3 \times \frac{1}{9}$ or $\frac{1}{3}$ and $\frac{1}{3} \div \frac{1}{9} = 3$.

16. **Create** Write and solve a real-world problem involving decimals that can be solved with a one-step multiplication equation.

Sample answer: A grocery store is selling a can of cat food for $0.60. Piper spent $3.60 on cans of cat food. Write and solve a multiplication equation to find how many cans she bought; $0.6 \cdot c = 3.6$; 6 cans

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Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 13, students will determine if Earline will have enough money. Encourage students to use reasoning to determine if she will have enough money.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will find the error made by another student and correct it. Students should be able to explain how the error was made and how to fix it.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 15, students will determine if the equations have the same solution. Encourage students to solve each equation and then compare the solutions to each other.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercises 11–12 Have students work in groups of 3–4 to solve the problem in Exercise 11. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 12.

Create your own higher-order thinking problem.

Use with Exercises 13–16 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Learn Write Division Equations

Objective

Students will learn how to model a real-world problem with a one-step division equation.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to construct a plausible argument to explain why defining a variable is an important part of the process of writing an equation.

Teaching Notes

SLIDE 1

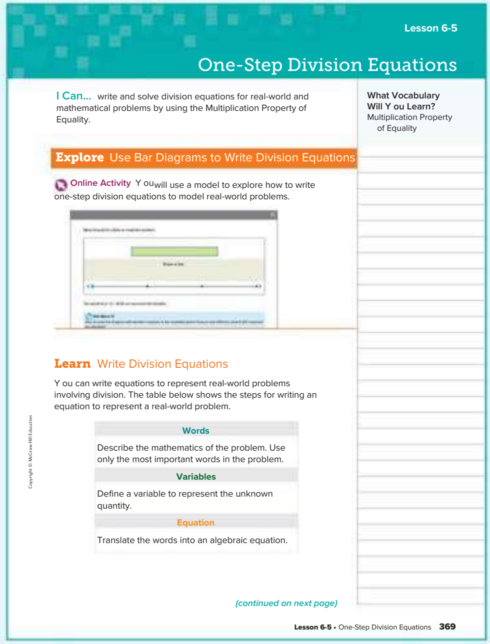
You may wish to have students create their own word problem that involves division, such as *One fourth of the students at Hamilton Middle School play a sport. There are 180 students that play a sport. How many students attend Hamilton Middle School?* Have students choose a variable, such as x or s, and clearly explain what that variable represents (the number of students attending Hamilton Middle School). Then have them write an equation that models the problem, such as $\frac{x}{4} = 180$, $\frac{s}{4} = 180$, $180 = \frac{x}{4}$, or $180 = \frac{s}{4}$. Be sure they understand there can be more than one way to write the equation. Have them explain, however, why the equation $4x = 180$ does not model the problem. Some students may choose to write a multiplication equation, such as $\frac{1}{4}x = 180$. Have students explain why the expressions $\frac{x}{4}$ and $\frac{1}{4}x$ are equivalent.

(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

Have students work with a partner to create two different real-world problems. One problem should be able to be modeled with a one-step multiplication equation. The other problem should be able to be modeled with a one-step division equation. Have pairs trade problems with another pair of students. Each pair should generate the appropriate equations that model each problem.
Have them check their equations with the original pair of students, and discuss and resolve any differences. Finally, have pairs solve the multiplication equation using inverse operations and properties of equality, and then make a conjecture as to how they might be able to solve the division equations.



Lesson 6-5

One-Step Division Equations

I Can... write and solve division equations for real-world and mathematical problems by using the Multiplication Property of Equality.

What Vocabulary Will You Learn?
Multiplication Property of Equality

Explore Use Bar Diagrams to Write Division Equations

Online Activity You will use a model to explore how to write one-step division equations to model real-world problems.

Learn Write Division Equations

You can write equations to represent real-world problems involving division. The table below shows the steps for writing an equation to represent a real-world problem.

Words
Describe the mathematics of the problem. Use only the most important words in the problem.
Variables
Define a variable to represent the unknown quantity.
Equation
Translate the words into an algebraic equation.

(continued on next page)

Lesson 6-5 • One-Step Division Equations 369

Interactive Presentation

Learn, Write Division Equations, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view the steps for writing an equation to represent a real-world problem.

One-Step Division Equations

6.EE.B.6, 6.EE.B.7

LESSON GOAL

Students will use the Multiplication Property of Equality to write and solve one-step division equations.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Bar Diagrams to Write Division Equations

Learn: Write Division Equations
Example 1: Write Division Equations
Learn: Solve Division Equations
Example 2: Solve Division Equations
Example 3: Solve Division Equations
Apply: Catering

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

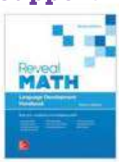
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Solve One-Step Literal Equations		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 38 of the *Language Development Handbook* to help your students build mathematical language related to solving one-step division equations.



ELL You can use the tips and suggestions on page T38 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations

Major Cluster(s): In this lesson, students address major cluster **6.EE.B** by using the Multiplication Property of Equality to write and solve one-step division equations.

Standards for Mathematical Content: **6.EE.B.6, 6.EE.B.7**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5**

Coherence

Vertical Alignment

Previous

Students used the Division Property of Equality to write and solve one-step multiplication equations.
6.EE.B.6, 6.EE.B.7

Now

Students use the Multiplication Property of Equality to write and solve one-step division equations.
6.EE.B.6, 6.EE.B.7

Next

Students will write, solve, and graph inequalities.
6.EE.B.5, 6.EE.B.8

Rigor

The Three Pillars of Rigor

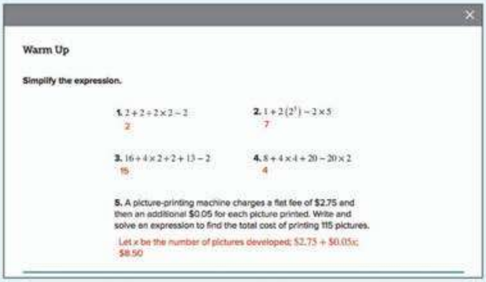
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of one-step division equations. They learn how to use a model and the Multiplication Property of Equality to build *fluency* with solving one-step division equations involving whole numbers and fractions. They *apply* their understanding of writing and solving one-step division equations to solve multi-step, real-world problems.

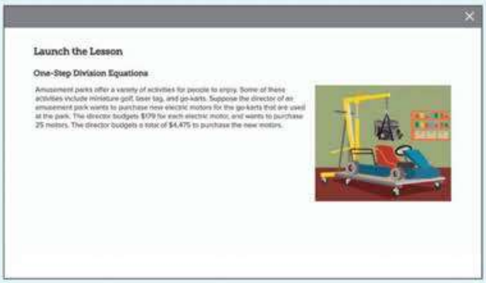
Mathematical Background

To solve a one-step division equation of the form $\frac{x}{a} = b$, where a and b are given values, $a \neq 0$, and x is an unknown, use the *Multiplication Property of Equality* to multiply each side of the equation by a. The solution is $x = ab$.

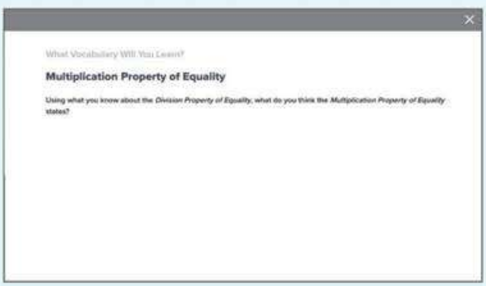
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- performing operations with whole numbers (Exercises 1–4)
- writing and evaluating expressions (Exercise 5)

Answers

1. 2

2. 7

3. 15

4. 4

5. Let x be the number of pictures developed; $\$2.75 + \$0.05x$; $8.50

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the budget of an amusement park, as an equation.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Using what you know about *the Division Property of Equality*, what do you think *the Multiplication Property of Equality* states? Sample answer: I think the Multiplication Property of Equality might state that when each side of an equation is multiplied by the same number, the equation remains equal.

Explore Use Bar Diagrams to Write Division Equations

Objective

Students will explore how to use a model to write division equations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with two word problems that can be represented by a division equation. Throughout this activity, students will identify known and unknown pieces of information and then use a bar diagram to write an equation for each situation.

Inquiry Question

How can you use a model to write division equations? Sample answer: The bar is divided into the same number of sections as the given divisor. The unknown is represented by the entire bar.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

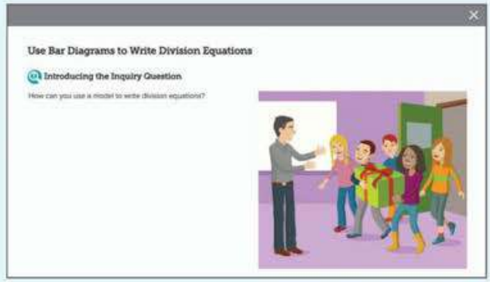
SLIDE 3

Mathematical Discourse

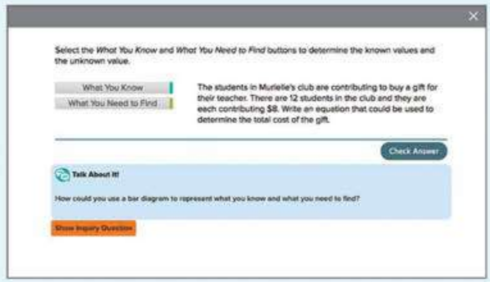
How could you use a bar diagram to represent what you know and what you need to find? Sample answer: Label the total with c, the total amount available for the gift. Divide the bar into 12 equal sections and label each section \$8.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8



Explore, Slide 3 of 8

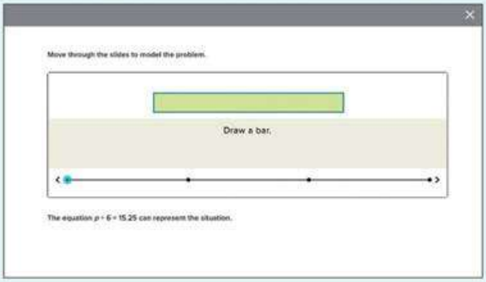
CLICK

On Slide 3, students highlight what they know and what they need to find.

CLICK

On Slide 4, students move through the slides to see how a bar diagram can be created to model the situation.

Interactive Presentation



Explore, Slide 7 of 8

CLICK

On Slide 7, students move through the slides to model the problem with a bar diagram.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Use Bar Diagrams to Write Division Equations *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should identify the important quantities in the real-world problem, decontextualize them, and use the bar diagram to represent them symbolically.

5 Use Appropriate Tools Strategically Encourage students to examine the correspondences between the bar diagrams and equations, and how they could eventually transition from the problem statement to writing the equation without the bar diagram.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What was the equation you wrote? How did the bar diagram help you write the equation? See students' responses.

Can you also write a multiplication equation to represent the situation? If so, explain how it relates to the division equation. Sample answer: The multiplication equation 12 times 8 represents the problem. Because division and multiplication are inverse operations, equations can be written using each operation to represent the same situation.

Your Notes

Go Online Watch the animation to see how to write a division equation to represent the following real-world problem.

Cyrus, Breyon, and Michael are sharing a pack of stickers. Each student gets 9 stickers. Write a division equation that can be used to determine the total number of stickers in the pack.

Words
Describe the mathematics of the problem.
The total number of stickers divided by the number of students equals the number of stickers each student receives.
Variable
Define the variable.
Let s represent the total number of stickers.
Equation
Write an equation.
$s \div 3 = 9$

Talk About It! Why is it important to define a variable before writing an equation?

Sample answer: Before writing an equation, it is important to define what is the unknown quantity so that it is clear what is meant by the variable.

Example 1 Write Division Equations

Benji rode his bike from Pittsburgh to Cleveland over the course of a three-day weekend. His average distance was 48.5 miles each day.

What was the total distance he rode?

Words
Describe the mathematics of the problem.
The total distance divided by 3 equals 48.5 miles.
Variable
Define the variable.
Let b represent the total distance he rode.
Equation
Write an equation.
$b \div 3 = 48.5$

So, the equation $b \div 3 = 48.5$ can be used to find the total distance Benji rode.

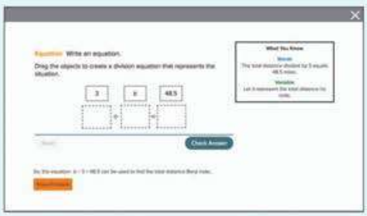
Think About It! How do you know that you will use division when you write the equation?

See students' responses.

Talk About It! Write a multiplication equation that is equivalent to $b \div 3 = 48.5$. Construct a mathematical argument to justify your response.

Sample answer: $\frac{1}{3}b = 48.5$; Dividing b by three is the same as multiplying b by $\frac{1}{3}$.

Interactive Presentation



Example 1, Write Division Equations, Slide 4 of 6

WATCH

On Slide 2 of the Learn, students watch an animation to learn how to model a real-world problem with a one-step division equation.

DRAG & DROP

On Slide 4 of Example 1, students drag the objects to write a division equation to model the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Write Division Equations *(continued)*

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how to write a division equation to model a real-world problem.

Talk About It!

SLIDE 3

Mathematical Discourse

Why is it important to define a variable before writing an equation? Sample answer: Before writing an equation, it is important to define what is the unknown quantity so that it is clear what is meant by the variable.

Example 1 Write Division Equations

Objective

Students will model a real-world problem with a one-step division equation.

Questions for Mathematical Discourse

SLIDE 4

AL How do you know that this should be a division equation? Sample answer: The average distance per day is the quotient of the total distance and the number of days traveled.

OL How can you read the equation in words? Sample answer: The total distance, b, divided by the total number of days, 3, equals the average distance per day, 48.5.

OL Can you write the equation as $3 \div b = 48.5$? Explain. no; Sample answer: Division is not commutative. *The total distance divided by 3* is not equivalent to *3 divided by the total distance.*

BL Can you solve the problem without writing an equation? Explain. yes; Sample answer: Multiply the number of days, 3, by the average distance per day, 48.5, to find the total distance traveled, 145.5.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Solve Division Equations

Objective

Students will learn how to solve one-step division equations using a model and the Multiplication Property of Equality.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively, 5 Use Appropriate Tools Strategically As students discuss the *Talk About It!* question on Slide 3, encourage them to reason about the types of tiles that are available for use when using algebra tiles, and why it might be difficult to use algebra tiles to model a division equation in which the variable is being divided by a number.

Go Online to have your students watch the video on Slide 1. The video illustrates how to solve one-step division equations using a bar diagram.

Teaching Notes

SLIDE 1

You may wish to pause the video after the equation $\frac{x}{4} = 6$ is shown, and ask students to work with a partner to use bar diagrams to model and solve the equation. Have them share their process and solution with another pair of students, or the entire class. Then have them continue watching the video to compare their process and solution with the one shown. Repeat using a similar process for the second equation in the video, $\frac{x}{3} = 7$.

Talk About It!

SLIDE 3

Mathematical Discourse

Why might it be difficult to use algebra tiles to model a division equation, such as $\frac{x}{3} = 4$? Sample answer: There are only whole x-tiles to model quantities such as x, $2x$, and so on. There are no fractional x-tiles to model $\frac{x}{3}$ or $\frac{1}{3}x$.

Check

Sophia has $16.50 to spend on party favors. She wants to spend $2.75 per person. Write a multiplication equation that can be used to find the number of people Sophia can have at the party.

Show your work here

Sample answer: $\frac{16.50}{f} = 2.75$

Go Online Y ou can complete an Extra Example online.

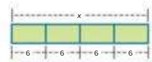
Learn Solve Division Equations

Y ou can use substitution, models, or properties of mathematics to solve division equations.

Go Online Watch the video to learn how to solve one-step division equations using bar diagrams.

The video demonstrates how to find the value of x in the equation $\frac{x}{4} = 6$.

Draw a bar to represent the total. The total length of the bar represents the original amount, x. Divide the bar into four sections to show division by 4. Then work backward to solve the equation.



Because x represents the entire length of the bar, and there are four equal sections of 6, multiply 6 by 4 to find the value of x. So, $x = 24$.

T o solve a division equation, use the inverse operation, multiplication. When you solve an equation by multiplying each side of the equation by the same number, you are using the **Multiplication Property of Equality**.

Words	Examples
If you multiply each side of an equation by the same number, the two sides remain equal.	If $6 = 6$, then $6 \times 5 = 6 \times 5$. If $x \div 3 = 4$, then $x \div 3 \times 3 = 4 \times 3$.

Talk About It!

Why might it be difficult to use algebra tiles to model a division equation, such as $\frac{x}{3} = 4$?

Sample answer: There are only whole x-tiles to model quantities such as x, $2x$, and so on. There are no fractional x-tiles to model $\frac{x}{3}$ or $\frac{1}{3}x$.

Interactive Presentation

Learn, Solve Division Equations, Slide 2 of 3

WATCH

On Slide 1, students watch a video to learn about how to use a bar diagram to solve a one-step division equation.

FLASHCARDS

On Slide 2, students use Flashcards to learn more about the Multiplication Property of Equality.

Think About It!
What operation is represented in the expression $\frac{x}{9}$?

division

Talk About It!
The equation $\frac{x}{9} = 13$ can also be written as $\frac{1}{9}x = 13$. What property of equality can you use to solve this equation? Explain your reasoning.

Division Property of Equality; Sample answer: I can use the Division Property of Equality to divide each side of the equation by $\frac{1}{9}$.

Talk About It!
How can you check your solution?

I can check my solution by substituting my solution back into the equation. If the sentence is true, then my solution is correct.

Example 2 Solve Division Equations

Solve $\frac{x}{9} = 13$. Check your solution.

$\frac{x}{9} = 13$ Write the equation.

$\frac{x}{9}(9) = 13(9)$ Multiplication Property of Equality

$x = 117$ Simplify.

So, the solution of the equation is **117**.

Check the solution.

$\frac{x}{9} = 13$ Write the equation.

$\frac{117}{9} \stackrel{?}{=} 13$ Replace x with 117.

$13 = 13$ The sentence is true.

Check

Solve $9 = \frac{x}{17}$ for x. **153**

Show your work here

Example 3 Solve Division Equations

Solve $\frac{c}{3} = \frac{2}{5}$.

$\frac{c}{3} = \frac{2}{5}$ Write the equation.

$\frac{c}{3}(3) = \frac{2}{5}(3)$ Multiplication Property of Equality

$c = \frac{6}{5}$ or $1\frac{1}{5}$ Simplify.

So, the solution of the equation is **$\frac{6}{5}$**.

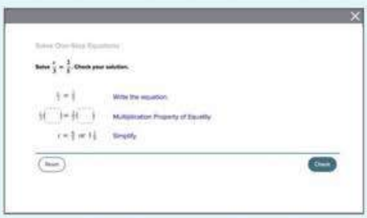
Check

Solve $\frac{k}{4} = 4\frac{2}{3}$ for k. **$18\frac{2}{3}$**

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 3, Solve Division Equations, Slide 1 of 2

TYPE

On Slide 1 of Example 3, students solve the equation.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 2 Solve Division Equations

Objective

Students will solve one-step division equations involving whole numbers using the Multiplication Property of Equality.

Questions for Mathematical Discourse

SLIDE 2

AL How can you read the equation in words? Sample answer: x divided by 9 equals 13.

OL Explain why you multiply each side of the equation by 9. Sample answer: The variable is being divided by 9. To undo that operation, use the inverse operation, which is multiplication.

OL How can you check your solution? Sample answer: Substitute 117 for x in the original equation to verify that the statement is true.

BL Write a real-world scenario that can be represented by this equation. Sample answer: Nine players are splitting the cost of a hotel room. If each player pays \$13, what is the total cost of the room?

Example 3 Solve Division Equations

Objective

Students will solve one-step division equations involving fractions using the Multiplication Property of Equality.

Questions for Mathematical Discourse

SLIDE 1

AL How can you read this equation in words? Sample answer: c divided by three is equal to two fifths.

OL Explain why you multiply each side of the equation by 3. Sample answer: The variable is being divided by 3. To undo that operation, use the inverse operation, which is multiplication.

OL How can you check your solution? Sample answer: Substitute $1\frac{1}{5}$ for c in the original equation to verify that the statement is true.

BL Write a multiplication equation that is equivalent to this equation. Sample answer: $\frac{1}{3}c = \frac{2}{5}$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Catering

Objective

Students will come up with their own strategy to solve an application problem involving serving portions of food during a party.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What are the important numbers and words in the problem?
- How could you define the variables to use in the equations for the ounces of chicken and fish being served?
- What do you notice about the total ounces of chicken and fish being served?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Catering

Dario is catering a party and serves 5.5-ounce servings of chicken to twelve guests, and 5.25-ounce servings of fish to nine guests. Did Dario serve more total ounces of chicken or fish? How much more?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Dario served 18.75 more ounces of chicken than fish; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Summarize the process you took to solve this application problem.

Sample answer: First, I found the total number of ounces for each type of entrée, then, I found the difference between the totals.

Interactive Presentation

Apply, Catering

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Marcel is purchasing boards to build a bookcase. He will use three 4.5-foot boards of pine and four 3.25-foot boards of white oak. Did Marcel use more pine or white oak to build the bookcase? How much more?

Show your work here

Sample answer: Marcel will use 0.5 feet more pine than white oak.

Go Online You can complete an Extra Example online.

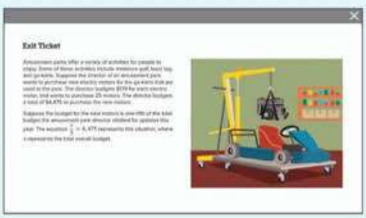
Pause and Reflect

How do you feel when you are asked during class to answer a question or to explain a solution?

Record your observations here

See students' observations.

Interactive Presentation



Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Describe the steps you can take to solve the equation $\frac{x}{5} = 4{,}475$. What is the total budget the amusement park director allotted for updates this year? Sample answer: Multiply each side of the equation by 5. The overall total budget is $22,375.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11, 13–16
- Extension: Solve One-Step Literal Equations
- ALEKS One-Step Equations, Applications of Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 14, 15
- Extension: Solve One-Step Literal Equations
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Introduction to One-Step Equations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Introduction to One-Step Equations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

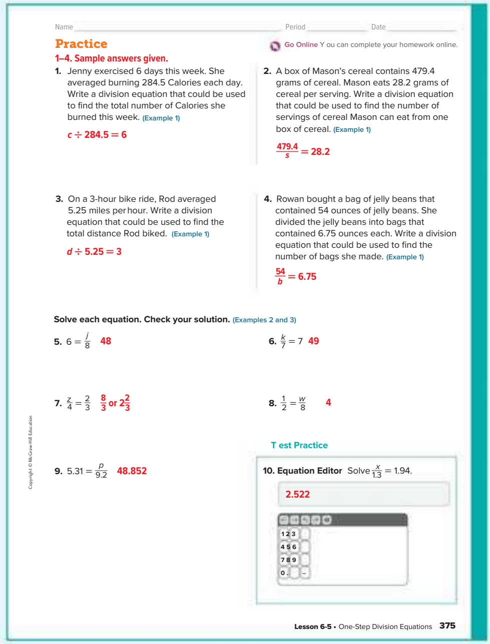
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	model a real-world problem with a one-step division equation	1–4
1	solve one-step division equations involving whole numbers using the Multiplication Property of Equality	5, 6
1	solve one-step division equations involving fractions using the Multiplication Property of Equality	7, 8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving one-step division equations	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

In Exercises 5–10, students may think that they need to multiply by a fraction instead of just the denominator. For example, in Exercise 5, students might multiply each side of the equation by $\frac{1}{8}$ instead of 8. If students are having trouble with the fractions, encourage them to rewrite each expression in long-form instead of fraction form. For Exercise 5, students could rewrite $\frac{j}{8}$ as $j \div 8$. This should help students to understand that they need to multiply by 8 instead of $\frac{1}{8}$.


Name Period Date

Practice

Go Online You can complete your homework online.

1–4. Sample answers given.

1. Jenny exercised 6 days this week. She averaged burning 284.5 Calories each day. Write a division equation that could be used to find the total number of Calories she burned this week. (Example 1)
   $c \div 284.5 = 6$

2. A box of Mason's cereal contains 479.4 grams of cereal. Mason eats 28.2 grams of cereal per serving. Write a division equation that could be used to find the number of servings of cereal Mason can eat from one box of cereal. (Example 1)
   $\frac{479.4}{s} = 28.2$

3. On a 3-hour bike ride, Rod averaged 5.25 miles per hour. Write a division equation that could be used to find the total distance Rod biked. (Example 1)
   $d \div 5.25 = 3$

4. Rowan bought a bag of jelly beans that contained 54 ounces of jelly beans. She divided the jelly beans into bags that contained 6.75 ounces each. Write a division equation that could be used to find the number of bags she made. (Example 1)
   $\frac{54}{b} = 6.75$

Solve each equation. Check your solution. (Examples 2 and 3)

5. $6 = \frac{j}{8}$ **48**

6. $\frac{k}{7} = 7$ **49**

7. $\frac{z}{4} = \frac{2}{3}$ **$\frac{8}{3}$ or $2\frac{2}{3}$**

8. $\frac{1}{2} = \frac{w}{8}$ **4**

9. $5.31 = \frac{p}{9.2}$ **48.852**

Test Practice

10. **Equation Editor** Solve $\frac{x}{1.3} = 1.94$.
    2.522

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Lesson 6-5 • One-Step Division Equations 375

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Apply *indicates multi-step problem

*11. Each month the student council sells snack bags. The table shows the number of ounces in each bag. The first month, the student council sold 50 bags of cheese crackers and 65 bags of pretzels. How many total ounces of each snack did they sell? What is the difference in the total number of ounces?

Snack Type	Amount in Each Bag
Cheese Crackers	2.25 ounces
Pretzels	3.5 ounces

cheese crackers: 112.5 oz; pretzels: 227 .5 oz; 115 oz

*12. Jason bought two different types of boards to make picture frames. He bought a red cedar board and will cut it into eight 10.25-inch pieces. He also bought a tiger maple board that he will cut into sixteen 10.5-inch pieces. Determine the difference between the boards' total lengths.

86 in.

Higher-Order Thinking Problems

13. MP **Reason Abstractly** Shawna noticed that the distance from her house to the ocean, which is 40 miles, was one fifth the distance from her house to the mountains. What is the distance from her house to the mountains? Explain how you solved.

200 miles; Sample answer: Write and solve the division equation $\frac{m}{5} = 40$; 5×40 is 200. So, $m = 200$ miles.

14. MP **Find the Error** A student is solving the equation $\frac{x}{3} = 6$. Find the student's mistake and correct it.

$$\frac{x}{3} = 6$$
$$\frac{x}{3} \div 3 = 6 \div 3$$
$$x = 2$$

Sample answer: The student divided each side by 3 instead of multiplying each side by 3. The correct solution is $x = 18$.

15. MP **Justify Conclusions** A model car is $\frac{1}{24}$ the size of the actual car. If a model car is 7 .75 inches long, how long is the actual car? Justify your answer.

186 in.; Sample answer: The length of the actual car c divided by 24, the scale, equals the length of the model car: $\frac{c}{24} = 7.75$; So $c = 186$ in.

16. **Create** Write and solve a real-world problem that can be solved with a one-step division equation.

Sample answer: At the end of a soccer season, four families decide to buy the coach a gift certificate to a sporting goods store. Each family contributes $25 towards the gift certificate. Write and solve a division equation to find how much the gift certificate is worth.; $\frac{g}{4} = 25$; $100

376 **Module 6** • Equations and Inequalities

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 13, students will determine the distance from Shawna's house to the mountains. Encourage them to use reasoning to determine the distance.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will find the student's mistake and correct it. Students should provide a short explanation of the error.

In Exercise 15, students will determine how long the car actually is. Encourage students to find the length and then construct a explanation that supports their findings.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercises 11–12 Have students work in groups of 3–4. After completing Exercise 11, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 12.

Clearly explain your strategy.

Use with Exercise 15 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would find the length of the actual car, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Learn Inequalities

Objective

Students will understand how to differentiate between an inequality and an equation.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to analyze the structure of an equation in order to identify the differences and similarities between an equation and an inequality.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Compare and contrast an equation and an inequality. Sample answer: They both can contain numbers, operations, and variables. The solution of an equation is exactly one value. The solution of an inequality is a range of values.

DIFFERENTIATE

Enrichment Activity BL

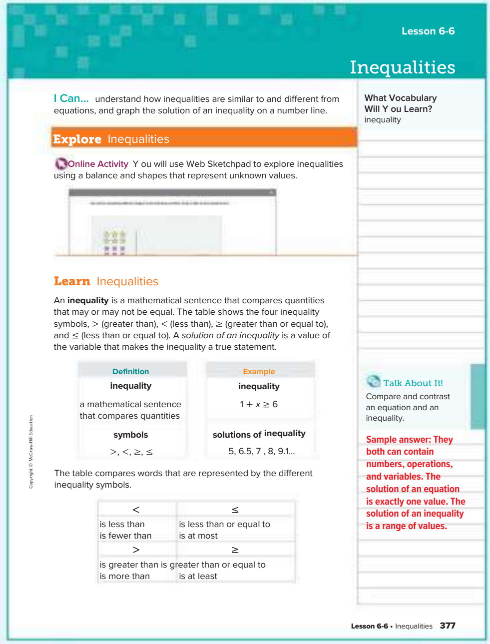
To extend students' understanding of inequalities, have them complete the following activity.

Have students work in pairs to generate four real-world scenarios in which either an equation or an inequality can be used to model each. Have them be sure to define the variable in each of their scenarios. Then have them trade their real-world scenarios with a partner. Each pair should determine whether an equation or an inequality can be used to model each scenario, and then write the corresponding equation or inequality. Have pairs check each other's work, and discuss and resolve any differences. Some sample scenarios are shown.

The minimum cost c of the repairs was \$150.00. inequality; $c \geq 150$

The number of people p who participated in the contest was 400. equation; $p = 400$

A chicken lays less than 3 eggs e per week. inequality; $e < 3$



Lesson 6-6

Inequalities

I Can... understand how inequalities are similar to and different from equations, and graph the solution of an inequality on a number line.

What Vocabulary Will You Learn?
inequality

Explore Inequalities

Online Activity You will use Web Sketchpad to explore inequalities using a balance and shapes that represent unknown values.

Learn Inequalities

An **inequality** is a mathematical sentence that compares quantities that may or may not be equal. The table shows the four inequality symbols, > (greater than), < (less than), ≥ (greater than or equal to), and ≤ (less than or equal to). A *solution of an inequality* is a value of the variable that makes the inequality a true statement.

Definition	Example
inequality	**inequality**
a mathematical sentence that compares quantities	$1 + x \geq 6$
symbols	**solutions of inequality**
>, <, ≥, ≤	5, 6.5, 7, 8, 9.1...

The table compares words that are represented by the different inequality symbols.

<	≤
is less than is fewer than	is less than or equal to is at most
>	≥
is greater than is more than	is greater than or equal to is at least

Talk About It! Compare and contrast an equation and an inequality.

Sample answer: They both can contain numbers, operations, and variables. The solution of an equation is exactly one value. The solution of an inequality is a range of values.

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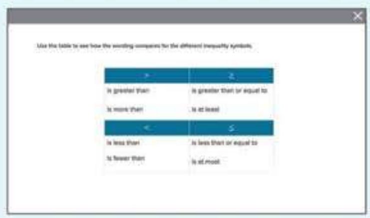
Lesson 6-6 • Inequalities 377

Interactive Presentation



Learn, Inequalities, Slide 2 of 3

Lesson 6-6

Inequalities

6.EE.B.5, 6.EE.B.8

LESSON GOAL

Students will write, solve, and graph inequalities.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Inequalities

Learn: Inequalities
Learn: Write Inequalities
Example 1: Write Inequalities
Learn: Graph Inequalities
Examples 2–3: Graph Inequalities
Learn: Find Solutions of an Inequality
Examples 4–6: Find Solutions of an Inequality
Apply: Earnings

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

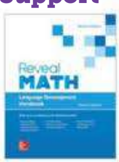
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Graph Compound Inequalities		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 39 of the *Language Development Handbook* to help your students build mathematical language related to understanding inequalities.



ELL You can use the tips and suggestions on page T39 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.B** by writing, solving, and graphing inequalities.
Standards for Mathematical Content: **6.EE.B.5, 6.EE.B.8,** *Also addresses 6.NS.C.6.C, 6.EE.B.6*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students used the Multiplication Property of Equality to write and solve one-step division equations.
6.EE.B.6, 6.EE.B.7

Now
Students write, solve, and graph inequalities.
6.EE.B.5, 6.EE.B.8

Next
Students will identify and use independent and dependent variables in relationships between two variables.
6.EE.C.9

Rigor

The Three Pillars of Rigor

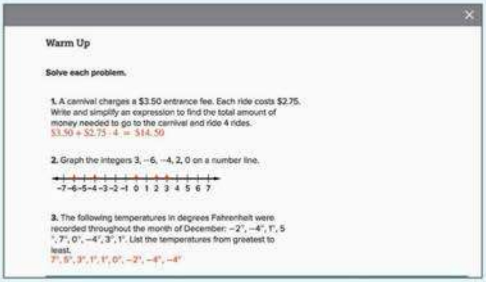
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of the four inequality symbols and the substitution method to develop *understanding* of inequalities. They use this understanding to build *fluency* with writing, solving, and graphing inequalities involving whole numbers, decimals, and fractions. They also *apply* this understanding to solve multi-step, real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing and evaluating expressions, operations with decimals (Exercise 1)
- understanding number lines (Exercise 2)
- ordering rational numbers (Exercise 3)

Answers

1. $3.50 + $2.75 · 4 = $14.50

2. See Warm Up slide online for correct answer.

3. 7°, 5°, 3°, 1°, 1°, 0°, −2°, −4°, −4°

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using inequalities when adhering to fishing regulations.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The term *inequality* has the same prefix as the term *incorrect*. Using what you know about the terms *equality, correct*, and *incorrect*, how might you infer the meaning of the term *inequality*? Sample answer: Since *incorrect* means *not correct*, and *equality* implies that two quantities are equal, an *inequality* might mean that two quantities are *not equal*.

Explore Inequalities

Objective

Students will explore inequalities using a balance and shapes with unknown values.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a balance and various shapes. Throughout this activity, students will use inequalities to express the comparisons they find on the balance. Encourage students to analyze the relationship between the shapes using the movements of the scale when the shapes are added.

Inquiry Question

How can you use a balance to analyze inequalities? Sample answer: I can use a balance to determine which value is greater.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

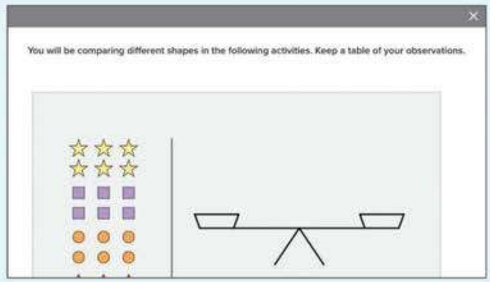
Mathematical Discourse

What happens to the balance? What does this result tell you about the star and the heart? Sample answer: The left side goes up and the right side goes down. The star weighs less than the heart.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

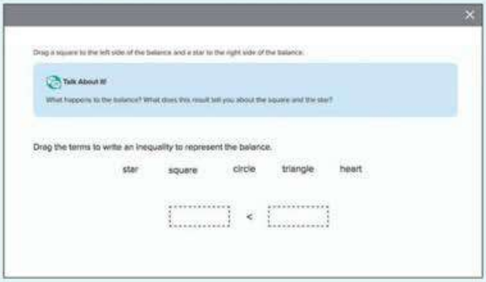


Explore, Slide 2 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore inequalities using a balance and shapes with unknown values.

Interactive Presentation



Explore, Slide 4 of 7

DRAG AND DROP

Throughout the Explore, students use a drag and drop activity to write an inequality to represent the balances.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Inequalities *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to examine the correspondences between the shapes and how they affect the position of the scale.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad in order to write an inequality to represent the balance.

8 Look for and Express Regularity in Repeated Reasoning Students will use repetitive reasoning in order to make observations about the weight of the objects and use precision to order the objects from lightest to heaviest.

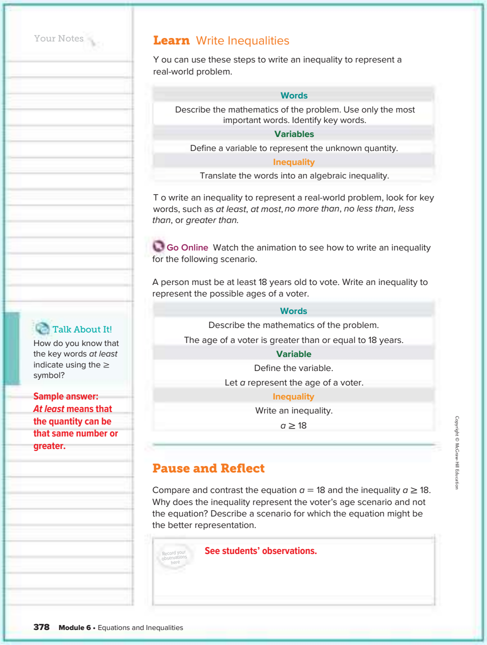
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What happens to the balance? What does this result tell you about the square and the star? Sample answer: The left side goes up and the right side goes down; the square weighs less than the star.



Your Notes

Learn Write Inequalities

You can use these steps to write an inequality to represent a real-world problem.

Words
Describe the mathematics of the problem. Use only the most important words. Identify key words.
Variables
Define a variable to represent the unknown quantity.
Inequality
Translate the words into an algebraic inequality.

To write an inequality to represent a real-world problem, look for key words, such as *at least*, *at most*, *no more than*, *no less than*, *less than*, or *greater than*.

Go Online Watch the animation to see how to write an inequality for the following scenario.

A person must be at least 18 years old to vote. Write an inequality to represent the possible ages of a voter.

Words
Describe the mathematics of the problem.
The age of a voter is greater than or equal to 18 years.
Variable
Define the variable.
Let a represent the age of a voter.
Inequality
Write an inequality.
$a \geq 18$

Talk About It!
How do you know that the key words *at least* indicate using the ≥ symbol?

Sample answer: *At least* means that the quantity can be that same number or greater.

Pause and Reflect

Compare and contrast the equation $a = 18$ and the inequality $a \geq 18$. Why does the inequality represent the voter's age scenario and not the equation? Describe a scenario for which the equation might be the better representation.

Record your observations here

See students' observations.

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Interactive Presentation

Learn, Write Inequalities, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view the steps for writing an inequality to model a real-world problem.

WATCH

On Slide 2, students watch an animation that shows how to model a real-world problem with an inequality.

Learn Write Inequalities

Objective

Students will learn how to model a real-world problem with an inequality.

MP Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language to explain how they know "at least" means using the ≥ symbol.

Go Online to have your students watch the animation on Slide 2. The animation illustrates how to write an inequality to model a real-world problem.

Teaching Notes

SLIDE 1

Point out that the steps for writing an inequality to model a real-world problem are similar to writing an equation to model a real-world problem. You may wish to have students list the key words that would indicate an inequality should be written, as opposed to an equation. Sample responses can include, but are not limited to, *at least*, *at most*, *no more than*, *greater than*, *no less than*, etc.

SLIDE 2

You may wish to pause the animation after the real-world scenario is presented. Ask students to work with a partner to determine the key words from the scenario that indicate an inequality best represents the scenario, as opposed to an equation. Students should note that the key words *at least* indicate an inequality. Have students make a conjecture as to possible inequalities that can be written. Remind them that they must define a variable before writing the inequality. Have them share their inequalities with the class. Then have them continue watching the animation to compare their inequalities with the one shown.

Talk About It!

SLIDE 3

Mathematical Discourse

How do you know that the key words "at least" indicate using the ≥ symbol? Sample answer: "At least" means that the quantity can be that same number or greater.

Example 1 Write Inequalities

Objective

Students will model a real-world problem with an inequality.

Questions for Mathematical Discourse

SLIDE 2

AL In your own words, describe what your age must be in order to have a valid driver's license. Sample answer: My age must be greater than or equal to 16.

OL What key words are used in the statement of the problem? at least

OL Can you be exactly 16 years old? Explain. yes; Sample answer: The words *at least* mean that you can be 16 or older.

BL What are some values of a that would meet the minimum age requirement? Sample answers: $a = 17$, $a = 21$, $a = 35$, $a = 49$

SLIDE 3

AL What is the unknown you need to represent with the variable? the ages at which I can have a driver's license

OL What symbol can be used in the inequality to represent the situation? Explain your reasoning. Sample answer: $\geq$; Because I can be exactly 16 and have a driver's license, the symbol is greater than or equal to.

BL You have to be at least 18 years old to vote in a government election. What variable might you choose to use in this situation? Define that variable. Sample answer: a, representing the ages of people that can vote

SLIDE 4

AL How does the variable a relate to the number 16, within the context of the problem? Sample answer: a, my current age, must be at least 16 in order to have a license

OL Why is the inequality $a \leq 16$ not correct? Sample answer: The inequality $a \leq 16$ means that 16 is the maximum age I can be in order to have a driver's license.

BL Is there another way you can write the inequality? Explain. yes; Sample answer: $16 \leq a$; This inequality means that 16 is the minimum age I can be in order to have a driver's license.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 1 Write Inequalities

In some states, you must be at least 16 years old to have a driver's license.

Write an inequality to represent the age at which you can have a driver's license.

Words
Describe the mathematics of the problem.
In order to have a valid driver's license, your age must be at least 16.
Variable
Define the variable.
Let a represent the age to have a license.
Inequality
Write an inequality.
$a \geq 16$

So, the inequality $a \geq 16$ represents the situation.

Think About It! What are the key words that will help you determine which inequality symbol to use?

See students' responses.

Talk About It! Explain why the inequality is $a \geq 16$ and not $a > 16$.

Sample answer: The inequality $a > 16$ means that I must be older than 16, not exactly 16. But I can be 16 or older to have a driver's license.

Check

A certain hotel only permits dogs that weigh less than 50 pounds to stay with hotel guests. Write an inequality that can be used to represent the weight w of dogs that are permitted to stay at the hotel.

Sample answer: $w < 50$

Go Online You can complete an Extra Example online.

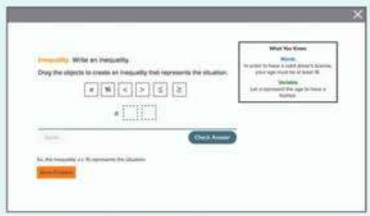
Pause and Reflect

Refer to the Example and Check. Why do both of these situations represent inequalities and not equations? Explain your reasoning.

Record your observations here

See students' observations.

Interactive Presentation



Example 1, Write Inequalities, Slide 4 of 6

CLICK

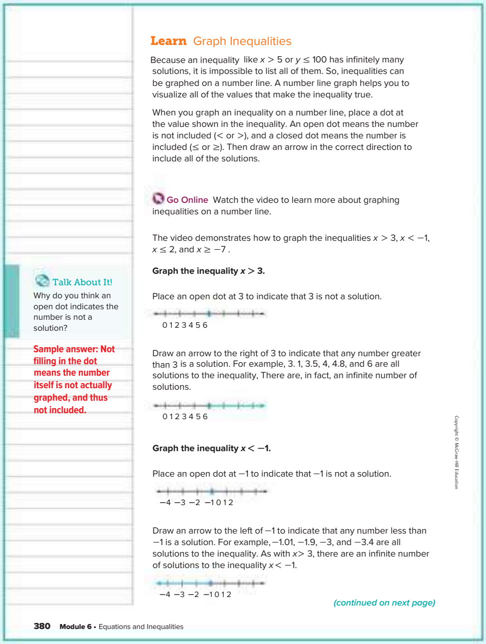
On Slide 2 and Slide 3, students describe the problem and define a variable.

DRAG & DROP

On Slide 4, students drag the objects to write an inequality that models the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Graph Inequalities

Because an inequality like $x > 5$ or $y \leq 100$ has infinitely many solutions, it is impossible to list all of them. So, inequalities can be graphed on a number line. A number line graph helps you to visualize all of the values that make the inequality true.

When you graph an inequality on a number line, place a dot at the value shown in the inequality. An open dot means the number is not included (< or >), and a closed dot means the number is included (≤ or ≥). Then draw an arrow in the correct direction to include all of the solutions.

Go Online Watch the video to learn more about graphing inequalities on a number line.

The video demonstrates how to graph the inequalities $x > 3$, $x < -1$, $x \leq 2$, and $x \geq -7$.

Graph the inequality $x > 3$.

Place an open dot at 3 to indicate that 3 is not a solution.

0 1 2 3 4 5 6

Draw an arrow to the right of 3 to indicate that any number greater than 3 is a solution. For example, 3. 1, 3.5, 4, 4.8, and 6 are all solutions to the inequality, There are, in fact, an infinite number of solutions.

0 1 2 3 4 5 6

Graph the inequality $x < -1$.

Place an open dot at −1 to indicate that −1 is not a solution.

−4 −3 −2 −1 0 1 2

Draw an arrow to the left of −1 to indicate that any number less than −1 is a solution. For example, −1.01, −1.9, −3, and −3.4 are all solutions to the inequality. As with $x > 3$, there are an infinite number of solutions to the inequality $x < -1$.

−4 −3 −2 −1 0 1 2

Talk About It!
Why do you think an open dot indicates the number is not a solution?

Sample answer: Not filling in the dot means the number itself is not actually graphed, and thus not included.

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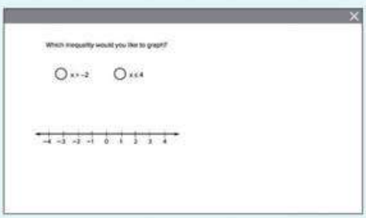
Interactive Presentation



Learn, Graph Inequalities, Slide 2 of 2

WATCH

On Slide 1, students watch a video to learn more about how to graph inequalities on a number line.

CLICK

On Slide 2, students use the interactive tool to understand how to graph inequalities.

Learn Graph Inequalities

Objective

Students will learn how to graph inequalities on a number line.

Teaching Notes

SLIDE 1

To emphasize that an inequality can have infinitely many solutions, you may wish to ask students to name all of the possible solutions of the inequality $x > 5$. Draw a number line on the board and ask students to make a conjecture as to how they can represent all of the possible solutions. Some students may only think of whole-number solutions, such as 6, 7, and 8. Remind them that any rational number greater than 5 is a solution, such as 5.1 or 5.01. Be sure students understand they can draw or shade a line to include all of the possible solutions, and how to indicate whether a number is included (closed dot) or excluded (open dot) from the solution set. Have students watch the video to help solidify their understanding of graphing inequalities on a number line.

Go Online to have your students watch the video on Slide 1. The video illustrates how to graph inequalities on a number line.

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Learn Graph Inequalities *(continued)*

Teaching Notes

SLIDE 2

Have students work with a partner to use the interactive tool on Slide 2. Prior to selecting one of the inequalities, have them describe how they would graph the inequality. After they select each inequality, they will determine whether to use an open or closed dot, and in which direction to draw the line. Have them use the structure of the inequality - in particular, the inequality symbol - to explain why they should use an open or closed dot, and to justify the direction in which they will draw the line.

Example 2 Graph Inequalities

Objective

Students will graph inequalities involving decimals on a number line.

Questions for Mathematical Discourse

SLIDE 2

AL What does < mean? How can you use this information to determine whether or not −5.75 should be included as a solution for *x*? < means strictly less than; Sample answer: Since *x* must be strictly less than −5.75, *x* cannot be equal to −5.75.

OL How does knowing that −5.75 is not included as a solution for *x* help you determine whether the dot is open or closed? Sample answer: Since −5.75 is not a solution for *x*, the dot should be open. An open dot indicates that that value is not part of the graphed solution.

OL Why is the graph a solid line that extends forever to the left? Sample answer: There are infinitely many solutions. All of the possible fractions and decimals between the whole number values that are to the left of −5.75 are solutions of this inequality. To represent them, I draw a solid line that extends forever in that direction.

BL How will the graph change if the inequality was $x \leq -5.75$? How will it remain the same? Sample answer: The dot will be closed because −5.75 would now be a solution. The arrow would still extend forever to the left.

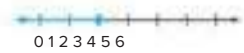
Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Graph the inequality $x \leq 2$.

Place a closed dot at 2 to indicate that 2 is a solution.

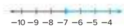
Draw an arrow to the left of 2 to indicate that any number less than 2 is also a solution.



Graph the inequality $x \geq -7$.

Place a closed dot at −7 to indicate that −7 is a solution.

Draw an arrow to the right of −7 to indicate that any number greater than −7 is also a solution.



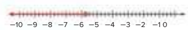
Talk About It! Why do you think a closed dot indicates the number is a solution?

Sample answer: Filling in the dot means the number is graphed and included.

Example 2 Graph Inequalities

Graph the inequality $x < -5.75$.

Place an open dot at −5.75. Draw an arrow to the left of −5.75. The values that lie on the line make the inequality true.



Think About It! What does the symbol < tell you about the graph?

The graph will have an open dot with an arrow that points to the left.

Check

Graph the inequality $x > 1\frac{2}{5}$.

Talk About It! How can you check that your graph is correct?

Sample answer: The numbers that are included in the graph, −6, −7 ,−8... are numbers that are less than −5.75.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 2, Graph Inequalities, Slide 2 of 4

CLICK On Slide 2, students move through the steps to determine how to graph the inequality.

eTOOLS On Slide 2, students use the Number Line eTool to graph the inequality.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Think About It!

What does the symbol ≥ tell you about the graph?

The graph will have a closed dot with an arrow that points to the right.

Talk About It!

How can you check that your graph is correct?

Sample answer: The numbers that are included in the graph, $\frac{3}{5}$, $\frac{4}{5}$, 1, and so on are numbers that are greater than $\frac{2}{5}$.

Example 3 Graph Inequalities

Graph the inequality $x \geq \frac{2}{5}$.

Check

Graph the inequality $x \leq -0.75$.

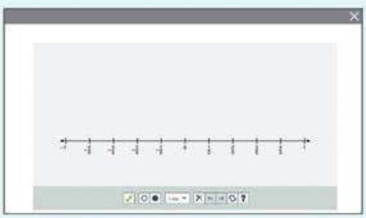
Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Did you make any errors when completing the Check exercise? What can you do to make sure you don't repeat that error in the future?

See students' observations.

Interactive Presentation



Example 3, Graph Inequalities, Slide 2 of 4

eTOOLS

On Slide 2, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Graph Inequalities

Objective

Students will graph inequalities involving fractions on a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the graph by checking possible values of their graphed solution to verify they are solutions of the inequality.

5 Use Appropriate Tools Strategically Students will use the Number Line eTool to graph the inequality.

6 Attend to Precision Have students pay careful attention to whether the dot should be open or closed, and in which direction the arrow should extend.

Questions for Mathematical Discourse

SLIDE 2

AL Between which two whole numbers will you graph $\frac{2}{5}$? between 0 and 1

AL How can you read the inequality? Sample answer: x is greater than or equal to $\frac{2}{5}$.

OL Is $\frac{2}{5}$ a solution of the inequality? How do you know? yes; Sample answer: The inequality symbol means *greater than or equal to*.

OL In which direction does the symbol ≥ indicate that the arrow will be pointing? Why does this make sense? to the right; Sample answer: This makes sense because numbers that are greater than $\frac{2}{5}$ are numbers to the right of $\frac{2}{5}$.

BL The inequality $-\frac{2}{5} < x < \frac{2}{5}$ means that x is between $-\frac{2}{5}$ and $\frac{2}{5}$. What do you think the graph representing this inequality might look like? Sample answer: The graph would have open dots at $-\frac{2}{5}$ and $\frac{2}{5}$ and a solid line between them, since x can be any value that lies between these numbers.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Find Solutions of an Inequality

Objective

Students will learn how to solve one-step inequalities using substitution.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

You found that 8 and 9 are solutions of the inequality $2 + x > 9$. Are there other solutions? Can you list them all? Explain your reasoning. Sample answer: There are other solutions such as 10, 8.1, $9\frac{4}{5}$, etc. It is impossible to list them all, because there are infinitely many solutions.

Example 4 Find Solutions of an Inequality

Objective

Students will solve one-step inequalities using substitution.

Questions for Mathematical Discourse

SLIDE 2

AL How can you check to see if 6 is a solution to the inequality? Sample answer: Replace a with 6, and determine if $6 + 4 \leq 11$.

OL Is the inequality true when $a = 7$? Explain. yes; Sample answer: When $a = 7$, the statement is $11 \leq 11$, which is a true statement.

OL Is 7.1 a solution of the inequality? Explain. no; Sample answer: By replacing a with 7.1, the left side of the inequality is 7.1+ 4, or 11.1, which is not less than or equal to 11.

BL Do you think you can list the minimum number that is not a solution of the inequality? Explain. no; Sample answer: 7.1 is not a solution, but neither are 7.01 or 7.001. Any number that is greater than 7 cannot be a solution, but there are infinitely many numbers between 7 and 7.1, or between 7 and 7.01, or between 7 and 7.001. It is impossible to list the minimum number.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Find Solutions of an Inequality

When you replace a variable with a value that results in a true sentence, you solve the inequality. That value for the variable is a solution of the inequality. Some inequalities have infinitely many solutions. For example, any rational number greater than 4 will make the inequality $x > 4$ true.

Use substitution to determine if the whole numbers 5, 6, 7, 8, and 9 are solutions of the inequality $2 + x > 9$.

Value of x	$2 + x > 9$	Is the inequality true?
5	$2 + 5 > 9$ $7 > 9$	no
6	$2 + 6 > 9$ $8 > 9$	no
7	$2 + 7 > 9$ $9 > 9$	no
8	$2 + 8 > 9$ $10 > 9$	yes
9	$2 + 9 > 9$ $11 > 9$	yes

The whole numbers 8 and 9 are solutions of the inequality.

Example 4 Find Solutions of an Inequality

Which of the following are solutions of the inequality $a + 4 \leq 11$: 6, 7, 8?

Complete the table to determine whether or not each number is a solution of the inequality.

Value of a	$a + 4 \leq 11$	Is the inequality true?
6	$6 + 4 \leq 11$ $10 \leq 11$	yes
7	$7 + 4 \leq 11$ $11 \leq 11$	yes
8	$8 + 4 \leq 11$ $12 \leq 11$	no

So, of the given values, the solutions are 6 and 7.

Sample answer: There are other solutions such as 10, 8. 1, $9\frac{4}{5}$, etc. It is impossible to list them all, because there are infinitely many solutions.

Talk About It!
You found that 8 and 9 are solutions of the inequality $2 + x > 9$. Are there other solutions? Can you list them all? Explain your reasoning.

Sample answer: Because 6 and 7 are included in the graphed inequality, they are solutions. The number 8 is not included, and therefore is not a solution of the inequality.

Talk About It!

The graph shows the solution of $a + 4 \leq 11$. How can you use the graph to see if 6, 7, or 8 are solutions of the inequality?

Lesson 6-6 • Inequalities 383

Interactive Presentation

Example 4, Find Solutions of an Inequality, Slide 2 of 4

CLICK
On Slide 1 of the Learn, students click to see if certain values are solutions of the inequality.

CLICK
On Slide 2 of Example 4, students will select yes or no to determine if each inequality is true.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Math History Minute

The symbols < and > were first introduced in 1631 in a mathematics text. The author of the text was English mathematician **Thomas Harriot (1560 – 1621)** While Harriot initially used triangular symbols to represent inequality, his editor changed them to < and >.

Check

Which of the following are solutions of the inequality $c + 28 > 72$: 44, 45, 46?

Show your work here **45, 46**

Go Online You can complete an Extra Example online.

Example 5 Find Solutions of an Inequality

Which of the following are solutions of the inequality $16b > 5.6$: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$?

Complete the table to determine if the inequality is true for each value of *b*.

Value of *b*	$16b > 5.6$	Is the inequality true?
$\frac{1}{2}$	$16 \cdot \frac{1}{2} > 5.6$ $8 > 5.6$	yes
$\frac{1}{3}$	$16 \cdot \frac{1}{3} > 5.6$ $5\frac{1}{3} > 5.6$	no
$\frac{1}{4}$	$16 \cdot \frac{1}{4} > 5.6$ $4 > 5.6$	no

So, of the given values, the solution is $\frac{1}{2}$.

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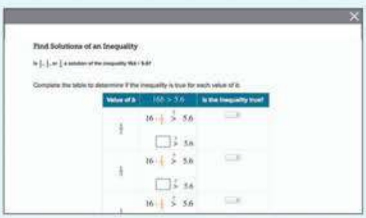
384 **Module 6** • Equations and Inequalities

Interactive Presentation



Example 5, Find Solutions of an Inequality, Slide 1 of 2

TYPE

On Slide 1, students enter the missing values in the table.

CLICK

On Slide 1, students select yes or no to identify whether or not the inequality is true.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Find Solutions of an Inequality

Objective

Students will solve one-step inequalities using substitution.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to calculate accurately and efficiently when substituting the given values into the inequality to check if the inequality is true, paying careful attention to the meaning of the inequality symbol. Students should be able to efficiently and accurately calculate with fractions and decimals.

Questions for Mathematical Discourse

SLIDE 1

AL How can you test to see if the given fractions are solutions of the inequality? Sample answer: I can substitute each fraction into the inequality and check to see if the statement is true or not.

OL Why is $\frac{1}{2}$ the only solution of the given numbers? $\frac{1}{2}$ is the only solution because, when substituted into only that inequality, the statement was true.

OL How can you determine mentally that $\frac{1}{2}$ is a solution, but $\frac{1}{4}$ is not a solution? Sample answer: 16 multiplied by $\frac{1}{2}$ is the same as finding half of 16, which is 8, and I know 8 is greater than 5.6; 16 multiplied by $\frac{1}{4}$ is the same as finding one fourth of 16, which is 4, and I know 4 is not greater than 5.6.

BL Is $\left(3 + \frac{1}{2}\right)$ a solution of the inequality? Explain. yes; Sample answer: $3 + \frac{1}{2} = 3\frac{1}{2}$. When $3\frac{1}{2}$ is substituted into the inequality, the statement is true.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 6 Find Solutions of an Inequality

Objective

Students will solve a real-world inequality using substitution.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the fact that since 7 is a solution of the inequality, any number less than 7 will also be a solution of the inequality.

6 Attend to Precision Encourage students to calculate accurately and efficiently when substituting the given values into the inequality to check if the inequality is true, paying careful attention to the meaning of the inequality symbol.

Questions for Mathematical Discourse

SLIDE 2

AL How can you check to see if 9 is a solution of the inequality? Sample answer: Replace *t* with 9, and determine if $60 \geq 8.40(9)$.

AL What does it mean within the context of the problem that 9 is not a solution? Sample answer: Raven cannot purchase 9 T-shirts, which means that not every teammate could receive a T-shirt.

OL Why does it make sense to only check whole number values? Sample answer: A fraction or a decimal would not make sense within the context of this problem, since Raven can only purchase whole number quantities of T-shirts.

OL How many teammates cannot receive a T-shirt? Explain. 2 teammates; Sample answer: Since 7 is a solution of the inequality, Raven can purchase 7 T-shirts. She cannot purchase 8 or 9 T-shirts, since 8 and 9 are not solutions. This means that 9 — 7, or 2 teammates cannot receive a T-shirt.

BL How much more money would Raven need to have in order to purchase all 9 T-shirts? Explain. $15.60; Sample answer: She has $60. To purchase 9 T-shirts, she needs 9($8.40), or $75.60. So, she needs $75.60 — $60, or $15.60 more.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

Which of the following are solutions of the inequality $11.75b \leq 24.675$: 2.1, 2.3, 2.5?

Show your work here

2.1

Go Online Y ou can complete an Extra Example online.

Example 6 Find Solutions of an Inequality

Raven has $60 to spend on matching T-shirts that cost $8.40 each for her running team. The inequality $60 \geq 8.40t$ represents the number of T-shirts *t* she could buy.

If there are 9 teammates on the team, how many could receive a T-shirt?

T o find a solution of the inequality, substitute varying values for *t*. Try 9 first because that represents the number of teammates. If 9 is a solution of the inequality, then every single one of the 9 members could receive a T-shirt.

Substitute 9	Substitute 8	Substitute 7
$60 \geq 8.40t$	$60 \geq 8.40t$	$60 \geq 8.40t$
$60 \geq 8.40(9)$	$60 \geq 8.40(8)$	$60 \geq 8.40(7)$
$60 \ngeq 75.60$	$60 \ngeq 67.20$	$60 \geq 58.80$

For which values is the inequality true? 7

So, Raven can purchase no more than 7 T-shirts. This means that 7 or fewer teammates could receive a T-shirt.

Think About It! How will the number of teammates help you choose a number to substitute?

See students' responses.

Talk About It! How do you know Raven has enough money to buy 1, 2, 3, 4, 5, or 6 T-shirts?

Sample answer: If Raven has enough money to buy 7 T-shirts, then she also has enough money to buy fewer than 7 T-shirts.

Lesson 6-6 • Inequalities 385

Interactive Presentation

Example 6, Find Solutions of an Inequality, Slide 2 of 4

CLICK

On Slide 2, students identify the solutions of the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

At the end of his vacation, Mr. Otey has $55 left to spend at a souvenir shop. He woud like to buy some picture frames that cost $12.75 each to display some of his vacation photos. The inequality $12.75f < 55$ represents the number of frames f he can choose to buy. What is the greatest number of frames that he can buy?

Show your work here

Mr. Otey can buy no more than 4 picture frames.

Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that shows the different inequality symbols and some key words that are used to indicate which symbol should be used when writing inequalities.

Record your observations here

See students' graphic organizers.

Apply Earnings

Objective

Students will come up with their own strategy to solve an application problem involving earning money to attend a festival.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How will you find the total hours each person worked?
- Since they get paid per hour, what will you do to find the total amount each person earned?
- What do you notice about the total amount earned for each person when compared to the cost to attend the festival?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Earnings

Several friends hope to attend a festival that costs $62.49 each to attend. T o earn money, they mowed lawns for $7 .50 per hour. The table shows the number of hours each person worked each day. Who earned enough money to attend the festival? What inequality can you write to represent this situation?

	Friday (hours)	Saturday (hours)
Emir	$8\frac{1}{2}$	1
Katherine	5	$2\frac{1}{2}$
Dylan	$6\frac{1}{2}$	$2\frac{1}{2}$
Anna	$3\frac{3}{4}$	$3\frac{1}{4}$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Emir and Dylan; 7 .5*h* ≥ 62.49; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

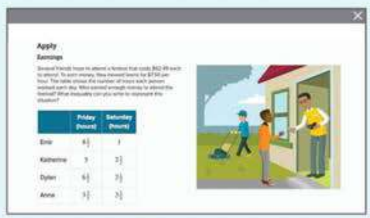
See students' arguments.

Talk About It!
How many more hours would Anna have to work to make enough money to attend?

2 hours

Lesson 6-6 • Inequalities 387

Interactive Presentation



Apply, Earnings

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Several friends each want to buy a ticket to a football game that costs \$75.99. To earn money, they worked extra hours at their job where they each earn \$9.10 per hour. The table shows the number of hours each person worked each day. Who earned enough money to buy a ticket? What inequality can you write to represent this situation?

	Friday (hours)	Saturday (hours)
Aaron	$5\frac{1}{2}$	2
Cliff	$3\frac{1}{2}$	6
Missy	$7\frac{1}{2}$	$2\frac{1}{2}$
Torrance	$2\frac{1}{2}$	1

Show your work here

Cliff and Missy; $91h \geq 75.99$

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you learned today about writing, graphing, and solving inequalities with something similar you learned about writing and solving equations. How are they similar? How are they different?

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How are the solutions of equations and inequalities different?

In Lessons 2–5, students learned how to solve one-step equations. In this lesson, students learned about inequalities. Encourage them to discuss with a partner how the solutions of equations might be different than the solutions of inequalities. For example, have them compare and contrast the statements $x = 5$, $x < 5$, $x > 5$, $x \leq 5$, and $x \geq 5$ and their graphs on a number line.

Exit Ticket

Refer to the Exit Ticket slide. Which fish can Ollie keep? Write a mathematical argument that can be used to defend your solution. Sample answer: Substitute each length, 12, 21, and 28, into the inequality. If the statement is true for each length, then Ollie can keep the fish. Since $12 + 3$, 15, is not greater than or equal to 23, Ollie cannot keep the 12-inch grouper. Ollie can keep the other two groupers because $21 + 3$, 24, is greater than 23, and $28 + 3$, 31, is greater than 23.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 13, 15–18
- Extension: Graph Compound Inequalities
- ALEKS Writing and Graphing Inequalities

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–11, 13, 16, 18
- Extension: Graph Compound Inequalities
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–6
- ALEKS One-Step Equations, Applications of Equations

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS One-Step Equations, Applications of Equations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

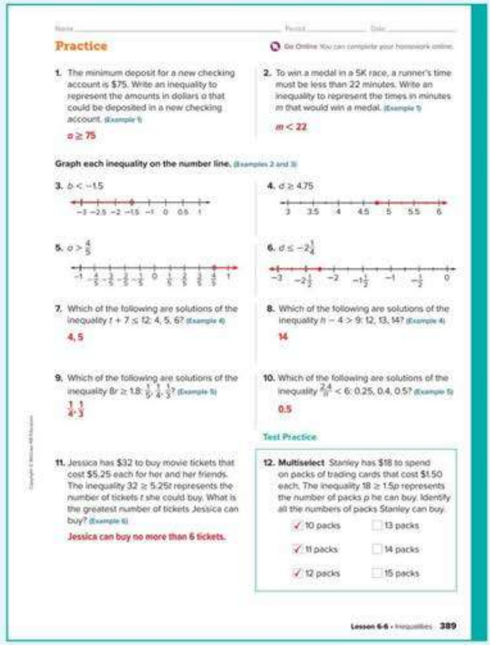
Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	model a real-world problem with an inequality	1, 2
1	graph inequalities involving decimals on a number line	3, 4
1	graph inequalities involving fractions on a number line	5, 6
1	solve one-step inequalities using substitution	7–10
1	solve a real-world inequality problem using substitution	11
2	extend concepts learned in class to apply them in new contexts	12
3	solve application problems involving inequalities	13, 14
3	higher-order and critical thinking skills	15–18

Common Misconceptions

Some students may draw the arrow facing the incorrect direction when graphing the solution of an inequality on a number line. In Exercise3, students may incorrectly draw the arrow facing to the right of −1.5 instead of to the left. They may mistakenly assume that since the less than sign opens to the right, they should draw the arrow to the right. Remind students to adhere to the meaning of each inequality symbol. Encourage them to think about what the inequality *less than* means, and to translate the inequality into words. If *b* is a number less than negative one and five-tenths, then only numbers that are to the left of −1.5 will satisfy the inequality.

Some students may confuse the symbols < and ≤, or ≥ and >. In Exercise 4, students may incorrectly use an open dot to represent the inequality instead of a closed dot. Encourage them to make sense of whether or not a closed dot would include the value 4.75 or not. Have them adhere to the precise definitions of the inequality symbols. You may wish to have them say aloud the meaning of each inequality symbol as they read it.


Name Period Date

Practice Go Online You can complete your homework online.

1. The minimum deposit for a new checking account is \$75. Write an inequality to represent the amounts in dollars a that could be deposited in a new checking account. (Example 1)
$a \ge 75$

2. To win a medal in a 5K race, a runner's time must be less than 22 minutes. Write an inequality to represent the times in minutes m that would win a medal. (Example 1)
$m < 22$

Graph each inequality on the number line. (Examples 2 and 3)

3. $b < -1.5$

4. $d \ge 4.75$

5. $a > \frac{4}{5}$

6. $d \le -2\frac{1}{4}$

7. Which of the following are solutions of the inequality $t + 7 \le 12$: 4, 5, 6? (Example 4)
4, 5

8. Which of the following are solutions of the inequality $h - 4 > 9$: 12, 13, 14? (Example 4)
14

9. Which of the following are solutions of the inequality $8r \ge 1.8$: $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}$? (Example 5)
$\frac{1}{4}, \frac{1}{3}$

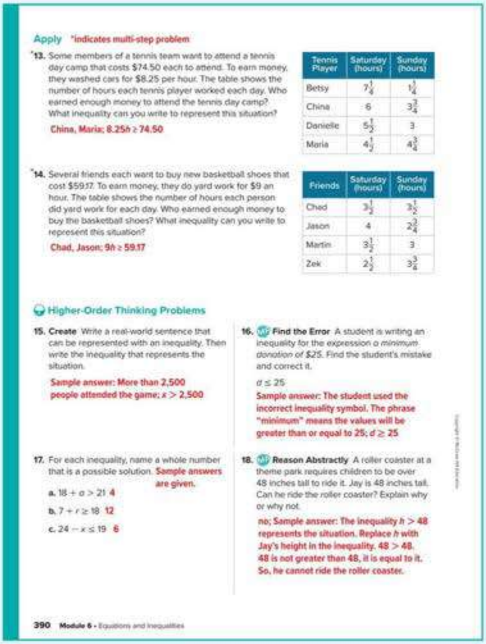
10. Which of the following are solutions of the inequality $\frac{2.4}{n} < 6$: 0.25, 0.4, 0.5? (Example 5)
0.5

11. Jessica has \$32 to buy movie tickets that cost \$5.25 each for her and her friends. The inequality $32 \ge 5.25t$ represents the number of tickets t she could buy. What is the greatest number of tickets Jessica can buy? (Example 6)
Jessica can buy no more than 6 tickets.

Test Practice

12. **Multiselect** Stanley has \$18 to spend on packs of trading cards that cost \$1.50 each. The inequality $18 \ge 1.5p$ represents the number of packs p he can buy. Identify all the numbers of packs Stanley can buy.
✓ 10 packs ☐ 13 packs
✓ 11 packs ☐ 14 packs
✓ 12 packs ☐ 15 packs

Lesson 6-6 • Inequalities 389



Apply *indicates multi-step problem

*13. Some members of a tennis team want to attend a tennis day camp that costs $74.50 each to attend. To earn money, they washed cars for $8.25 per hour. The table shows the number of hours each tennis player worked each day. Who earned enough money to attend the tennis day camp? What inequality can you write to represent this situation?

China, Maria; $8.25h \geq 74.50$

Tennis Player	Saturday (hours)	Sunday (hours)
Betsy	$7\frac{1}{4}$	$1\frac{1}{4}$
China	6	$3\frac{3}{4}$
Danielle	$5\frac{1}{2}$	3
Maria	$4\frac{1}{2}$	$4\frac{3}{4}$

*14. Several friends each want to buy new basketball shoes that cost $59.17. To earn money, they do yard work for $9 an hour. The table shows the number of hours each person did yard work for each day. Who earned enough money to buy the basketball shoes? What inequality can you write to represent this situation?

Chad, Jason; $9h \geq 59.17$

Friends	Saturday (hours)	Sunday (hours)
Chad	$3\frac{1}{2}$	$3\frac{1}{2}$
Jason	4	$2\frac{3}{4}$
Martin	$3\frac{1}{2}$	3
Zek	$2\frac{1}{2}$	$3\frac{3}{4}$

Higher-Order Thinking Problems

15. **Create** Write a real-world sentence that can be represented with an inequality. Then write the inequality that represents the situation.

Sample answer: More than 2,500 people attended the game; $x > 2{,}500$

16. **Find the Error** A student is writing an inequality for the expression *a minimum donation of $25*. Find the student's mistake and correct it.

$d \leq 25$

Sample answer: The student used the incorrect inequality symbol. The phrase "minimum" means the values will be greater than or equal to 25; $d \geq 25$

17. For each inequality, name a whole number that is a possible solution. Sample answers are given.

a. $18 + a > 21$ 4

b. $7 + r \geq 18$ 12

c. $24 - x \leq 19$ 6

18. **Reason Abstractly** A roller coaster at a theme park requires children to be over 48 inches tall to ride it. Jay is 48 inches tall. Can he ride the roller coaster? Explain why or why not.

no; Sample answer: The inequality $h > 48$ represents the situation. Replace h with Jay's height in the inequality. $48 > 48$. 48 is not greater than 48, it is equal to it. So, he cannot ride the roller coaster.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students will find the student's mistake and correct it. Encourage students to find the error and then correct it, supplying a well-constructed explanation.

2 Reason Abstractly and Quantitatively In Exercise 18, students will determine if Jay can ride the roller coaster. Encourage students to use abstract reasoning to answer the question. Students should provide a well-constructed explanation along with their answer.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 13 Have students work together to prepare a brief demonstration that illustrates why this problem might require multiple steps to solve. For example, students must first determine the total hours worked for each person before solving the problem. Have each pair or group of students present their response to the class.

Be sure everyone understands.
Use with Exercises 16 and 18 Have students work in groups of 3–4 to solve the problem in Exercise 16. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 18.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include a review of equations and inequalities. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

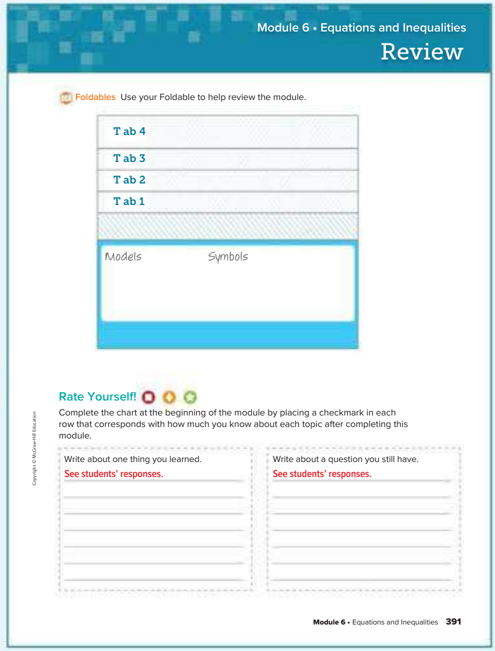
Assessment Resources

Put It All Together: Lessons 6-1, 6-2, 6-3, 6-4, and 6-5
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Expressions and Equations**.

- Equations
- Inequalities


Module 6 • Equations and Inequalities

Review

Foldables Use your Foldable to help review the module.

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.	Write about a question you still have.
See students' responses.	See students' responses.

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Reflect on the Module

Use what you learned about equations and inequalities to complete the graphic organizer.

Essential Question

How are the solutions of equations and inequalities different?

Explain how to solve each equation. Then solve the equation.

$x - 5 = 13$

Sample answer: Use the Addition Property of Equality to add 5 to each side of the equation. So, $x = 18$.

$n + 4 = 9$

Sample answer: Use the Subtraction Property of Equality to subtract 4 from each side of the equation. So, $n = 5$.

$\frac{g}{3} = 4$

Sample answer: Use the Multiplication Property of Equality to multiply each side of the equation by 3. So, $g = 12$.

$6m = 42$

Sample answer: Use the Division Property of Equality to divide each side of the equation by 6. So, $m = 7$.

What are the similarities between solving an equation and solving an inequality?

Sample answer: T o solve an equation or inequality, you can substitute a value for the variable to determine if it results in a true equation or inequality.

What are the differences between the solution of an equation and the solution of an inequality?

Sample answer: An equation has one solution and an inequality has infinitely many solutions.

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How are the solutions of equations and inequalities different? See students' graphic organizers.

Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–13 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 7, 10
Multiselect	Multiple answers may be correct. Students must select all correct answers.	3, 13
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	4, 6
Table Item	Students complete a table by correctly classifying the information.	8, 12
Grid	Students create a graph on an online number line.	11
Open Response	Students construct their own response in the area provided.	2, 5, 9

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.EE.B.5	6-1, 6-6	1, 2, 12, 13
6.EE.B.6	6-2, 6-3, 6-4, 6-5	3, 5, 9, 10
6.EE.B.7	6-2, 6-3, 6-4, 6-5	3–10
6.EE.B.8	6-6	11–13

Name __________ Period ______ Date ______

Test Practice

1. Multiple Choice Which of the following is a solution of the equation $y + \frac{1}{3} = 3\frac{2}{3}$? (Lesson 1)

Ⓐ $2\frac{1}{3}$

Ⓑ $2\frac{2}{3}$

Ⓒ 3

Ⓓ $3\frac{1}{3}$

2. Open Response T onya is using $3\frac{1}{2}$ inch tiles along the 28 inch ledge of her bathroom counter. Use the *guess, check, and revise* strategy to solve the equation $3\frac{1}{2}t = 28$ to find *t*, the number of tiles T onya will need. Show your work. (Lesson 1)

8; Sample work:

$3\frac{1}{2}t = 28$ $\quad$ $3\frac{1}{2}t = 28$

$3\frac{1}{2}(7) \stackrel{?}{=} 28$ $\quad$ $3\frac{1}{2}(8) \stackrel{?}{=} 28$

$24.5 \neq 28$ $\quad$ $28 = 28$

3. Multiselect T ogether, Rhonda and Margo saved \$478.50. If Rhonda saved \$225 of that total, how much did Margo save? Select the addition equation that could be used to find how much money *m* Margo saved. Select all that apply. (Lesson 2)

- ☐ $m + 478.50 = 225$
- ✓ $m + 225 = 478.50$
- ☐ $225 + 478.50 = m$
- ✓ $225 + m = 478.50$
- ☐ $478.50 + m = 225$

4. Equation Editor (Lesson 2)

A. Solve $625 = 219 + x$ for *x*.

$x =$ **406**

B. Check the solution.

Sample work:
$625 = 219 + x$
$625 \stackrel{?}{=} 219 + 406$
$625 = 625$

5. Open Response A one-topping pizza costs \$12.99. This is \$6.50 less than the cost of a specialty pizza. Write a subtraction equation that could be used to find the cost *c* of a specialty pizza. (Lesson 3)

$c - 6.50 = 12.99$

6. Equation Editor Solve $1{,}785 = x - 414$ for *x*. (Lesson 3)

$x =$ **2199**

Module 6 • Equations and Inequalities 393

7. Multiple Choice Solve $\frac{3}{8}d = \frac{5}{24}$ for d. (Lesson 4)

- (A) $d = \frac{5}{64}$
- (B) $d = \frac{5}{9}$
- (C) $d = \frac{1}{6}$
- (D) $d = \frac{3}{4}$

8. Table Item The nutrition information for two different bottles of orange juice is shown. Kylie wants to compare the Calories in a single serving for each brand. (Lesson 4)

	Brand A (3 servings)	Brand B (2 servings)
Calories	150	220
Protein (g)	3	4
Sugar (g)	30	44

A. Find the number of Calories per serving of each brand, then indicate the correct number for each brand in the table.

Calories per Serving	Brand A	Brand B	Neither A nor B
50	X		
85			X
110		X	

B. Which brand has more Calories per serving? How many more?

Brand B; 60 more calories per serving

9. Open Response Solve $\frac{y}{11} = 28$. Show your work. (Lesson 5)

$\frac{y}{11}(11) = 28(11)$
$y = 308$

10. Multiple Choice Mr. Wolfe has a box of 144 pencils. If he wants to give 6 pencils to each of his students, which equation can be used to find the number of students s to whom Mr. Wolfe can give pencils? (Lesson 5)

- (A) $\frac{144}{s} = 6$
- (B) $\frac{s}{144} = 6$
- (C) $144s = 6$
- (D) $144(6) = s$

11. Grid Graph $x < \frac{1}{3}$. (Lesson 6)

12. Table Item Use the table to indicate whether 11, 12, or 13 is a solution of the inequality $b + 8 \geq 20$. (Lesson 6)

Value of *b*	Y es	No
11		X
12	X	
13	X	

13. Multiselect Brandi has $50 to spend on matching bracelets that cost $3.75 each for her volleyball team. The inequality $50 \geq 3.75b$, where b is the number of bracelets, represents the situation. If there are 16 teammates, how many will possibly receive a bracelet? (Lesson 6)

- ☐ 16 teammates
- ☐ 15 teammates
- ☐ 14 teammates
- ☑ 13 teammates
- ☑ 12 teammates

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

What are the ways in which a relationship between two variables can be displayed? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

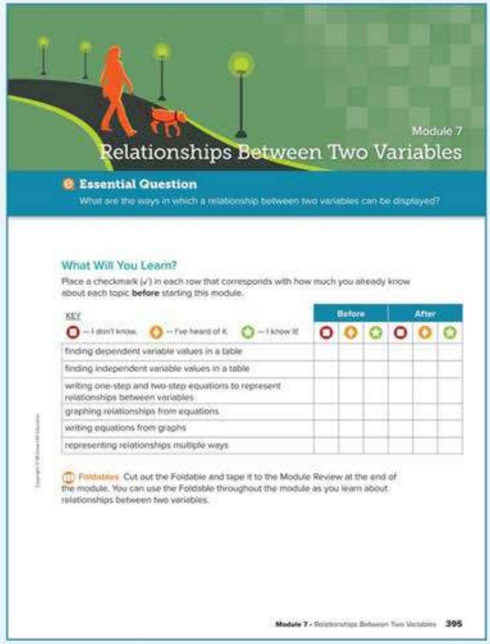
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of converting temperature from degrees Celsius to Fahrenheit and running a dog-sitting business to introduce the idea of relationships between two variables. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 7

Relationships Between Two Variables

Essential Question

What are the ways in which a relationship between two variables can be displayed?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

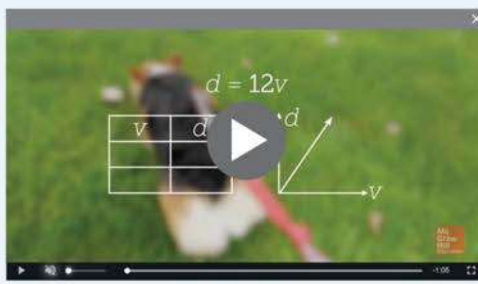
KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know	I've heard of it	I know it!	I don't know	I've heard of it	I know it!
finding dependent variable values in a table						
finding independent variable values in a table						
writing one-step and two-step equations to represent relationships between variables						
graphing relationships from equations						
writing equations from graphs						
representing relationships multiple ways						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about relationships between two variables.

Module 7 • Relationships Between Two Variables 395

Interactive Student Presentation



Module 7

Relationships Between Two Variables

Module Goal

Express relationships between two variables using tables, equations, and graphs.

Focus

Domain: Expressions and Equations
Major Cluster(s): 6.EE.C Represent and analyze quantitative relationships between dependent and independent variables.
Standards for Mathematical Content:
6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- solve one-step equations involving each of the four operations, with positive rational numbers
- graph points in all four quadrants of the coordinate plane

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students represented ratio relationships using tables and graphs.
6.RP.A.3, 6.RP.A.3.A

Now
Students express relationships between two variables using tables, equations, and graphs.
6.EE.C.9

Next
Students will use tables and graphs to determine if a relationship between two quantities is proportional.
7.RP.A.2

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of tables, equations, and the coordinate plane to develop *understanding* of relationships between two variables. They build *fluency* with using a table to find variable values, writing equations, and graphing the relationship. They also *apply* their understanding of relationships between two variables to solve real-world problems.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

EXPLORE → LEARN → EXAMPLE & PRACTICE

Suggested Pacing

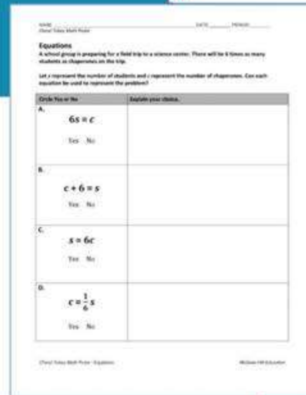
Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
7-1	Relationships Between Two Variables	6.EE.C.9, *Also addresses 6.EE.A.2.C*	3	1.5
7-2	Write Equations to Represent Relationships Represented in Tables	6.EE.C.9, *Also addresses 6.EE.B.6, 6.EE.B.7*	2	1
Put It All Together 1: Lessons 7-1 and 7-2			0.5	0.25
7-3	Graphs of Relationships	6.EE.C.9, *Also addresses 6.RP.A.3.A, 6.NS.C.6.C, 6.EE.B.6, 6.EE.B.7*	1	0.5
7-4	Multiple Representations	6.EE.C.9, *Also addresses 6.RP.A.3.A, 6.NS.C.6.C, 6.EE.B.6, 6.EE.B.7*	1	0.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	10.5	5.25

Module Resource

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe

Equations


Equations

A. $6s = c$ Yes No

B. $c + 6 = s$ Yes No

C. $s = 6c$ Yes No

D. $c = \frac{1}{6}s$ Yes No

Correct Answers: A. No; **B.** No; **C.** Yes; **D.** Yes

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine whether or not the given equation can represent the real-world problem, and explain their choice.

Targeted Concept Equations can be used to represent the changing relationship between a dependent and an independent variable.

Targeted Misconceptions

- Students may rely on "key words" and incorrectly translate the written description as a literal translation.
- Students may believe there is only one correct way to write an equation.

Assign the probe after Lesson 2.

Collect and Assess Student Work

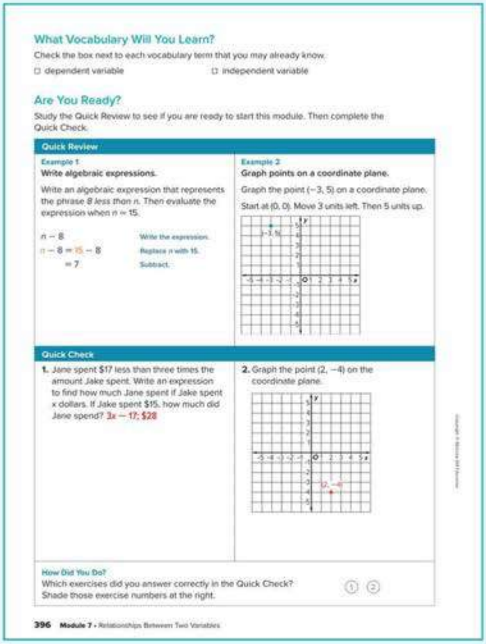
If the student selects...	**Then** the student likely...
A. Yes **C.** No	uses the order of the numbers and the position of key words to incorrectly translate the verbal description.
B. Yes	interprets the phrase "6 times as many" to mean "add 6".
D. No	chooses correctly between equations such as in items A and C, but has difficulty recognizing $c = \frac{1}{6}s$ as a correct equation.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Equations and Inequalities
- Lesson 1, Examples 1–2
- Lesson 2, Examples 1–2
- Lesson 3, Examples 1–2

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ dependent variable ☐ independent variable

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Write algebraic expressions.

Write an algebraic expression that represents the phrase *8 less than n*. Then evaluate the expression when $n = 15$.

$n - 8$ Write the expression.

$n - 8 = 15 - 8$ Replace n with 15.

$= 7$ Subtract.

Example 2

Graph points on a coordinate plane.

Graph the point $(-3, 5)$ on a coordinate plane.

Start at (0, 0). Move 3 units left. Then 5 units up.

Quick Check

1. Jane spent $17 less than three times the amount Jake spent. Write an expression to find how much Jane spent if Jake spent x dollars. If Jake spent $15, how much did Jane spend? $3x - 17$; $28

2. Graph the point $(2, -4)$ on the coordinate plane.

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define The **independent variable** is the variable that does not depend upon the other quantity in a relationship.

Example At top speed, a peregrine falcon can travel about 352 feet in 1 second. The independent variable is the time in seconds.

Ask If Maribella earns $7 per hour babysitting, what is the independent variable in the relationship? the number of hours

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- writing and evaluating algebraic expressions
- solving one-step algebraic equations
- graphing on the coordinate plane

ALEKS®

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Equations and Inequalities** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Mistakes = Learning

When anyone makes a mistake and goes on to learn from it, that person can actually build new connections in his or her brain as he or she determines a new path or process that can be used toward a solution to the problem.

How Can I Apply It?

Have students complete the **Checks** after each Example, either digitally or in their Interactive Student Edition, as a form of student-centered formative assessment. Encourage them to analyze any mistakes they might have made and what they could do to self correct.

ALEKS is a great tool to not only individualize learning for each student, but to also help students understand that making mistakes and trying new problems will help them to learn and grow long term. Have students keep track of their ALEKS Pie Chart to view their progress.

Learn Identify Independent and Dependent Variables

Objective

Students will learn how to identify independent and dependent variables.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use clear and precise mathematical language to explain why the terms *independent* and *dependent* are appropriate names for the variables.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

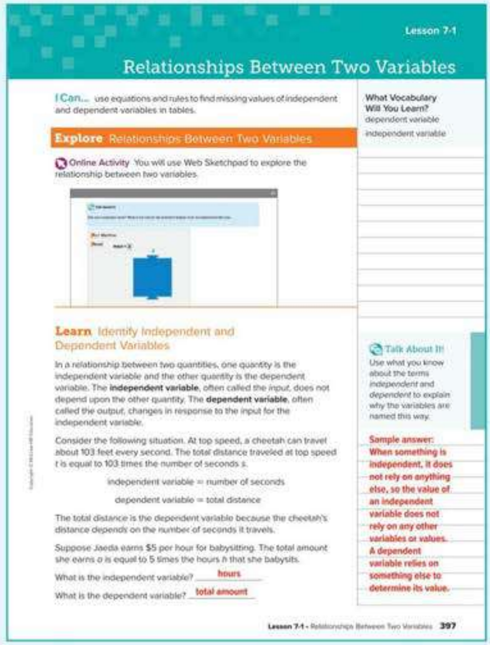
Use what you know about the terms *independent* and *dependent* to explain why the variables are named this way. Sample answer: When something is independent, it does not rely on anything else, so the value of an independent variable does not rely on any other variables or values. A dependent variable relies on something else to determine its value.

DIFFERENTIATE

Language Development Activity ELL

Some students may confuse the independent and dependent variables when analyzing a real-world situation or a rule that describes the relationship between the variables. Encourage them to use the everyday meanings of *independent* and *dependent* to help them determine which variable is the independent variable and which variable is the dependent variable. Remind them that the dependent variable *depends* on the independent variable, and that the independent variable is thus *not dependent* on the dependent variable. Have them work with a partner to respond to each question below about the situations presented in the Learn. Sample responses for the scenario about how fast a cheetah can travel are shown.

1. What are the two quantities, or variables? distance traveled in feet and time in seconds
2. Which variable *depends* on the other variable? Explain. The distance traveled by a cheetah depends on how long the cheetah has been running. So, distance is the dependent variable.
3. Which variable *does not depend* on the other variable? The time a cheetah spends running is not dependent on the distance it travels. So, time is the independent variable.



Lesson 7-1

Relationships Between Two Variables

I Can... use equations and rules to find missing values of independent and dependent variables in tables.

What Vocabulary Will You Learn?
dependent variable
independent variable

Explore Relationships Between Two Variables

Online Activity You will use Web Sketchpad to explore the relationship between two variables.

Learn Identify Independent and Dependent Variables

In a relationship between two quantities, one quantity is the independent variable and the other quantity is the dependent variable. The **independent variable**, often called the *input*, does not depend upon the other quantity. The **dependent variable**, often called the output, changes in response to the input for the independent variable.

Consider the following situation. At top speed, a cheetah can travel about 103 feet every second. The total distance traveled at top speed *t* is equal to 103 times the number of seconds *s*.

independent variable = number of seconds

dependent variable = total distance

The total distance is the dependent variable because the cheetah's distance depends on the number of seconds it travels.

Suppose Jaeda earns $5 per hour for babysitting. The total amount she earns *a* is equal to 5 times the hours *h* that she babysits.

What is the independent variable? hours

What is the dependent variable? total amount

Talk About It! Use what you know about the terms *independent* and *dependent* to explain why the variables are named this way.

Sample answer: When something is independent, it does not rely on anything else, so the value of an independent variable does not rely on any other variables or values. A dependent variable relies on something else to determine its value.

Lesson 7-1 • Relationships Between Two Variables 397

Interactive Presentation

Learn, Identify Independent and Dependent Variables, Slide 1 of 2

CLICK

On Slide 1, students highlight the independent and dependent variables in several real-world situations.

Lesson 7-1

Relationships Between Two Variables

6.EE.C.9

LESSON GOAL

Students will identify and use independent and dependent variables in relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Relationships Between Two Variables

Learn: Identify Independent and Dependent Variables
Learn: Find Dependent Variable Values in a Table
Example 1: Find Dependent Variable Values in a Table
Learn: Find Independent Variable Values in a Table
Example 2: Find Independent Variable Values in a Table
Apply: Measurement

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Domain and Range		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 40 of the *Language Development Handbook* to help your students build mathematical language related to relationships between two variables.

ELL You can use the tips and suggestions on page T40 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Expressions and Equations

Major Cluster(s): In this lesson, students address major cluster **6.EE.C** by identifying and using independent and dependent variables in relationships.

Standards for Mathematical Content: **6.EE.C.9,** *Also addresses 6.EE.A.2.C*

Standards for Mathematical Practice:: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous

Students represented ratio relationships using tables and graphs.
6.RP.A.3, 6.RP.A.3.A

Now

Students identify and use independent and dependent variables in relationships.
6.EE.C.9

Next

Students will write equations to represent relationships.
6.EE.C.9

Rigor

The Three Pillars of Rigor

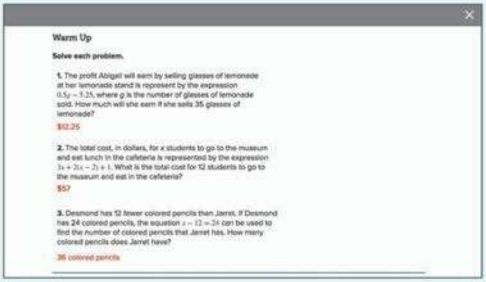
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of simplifying expressions to develop *understanding* of relationships between two variables. They identify independent and dependent variables and use a table to build *fluency* with finding the variable values, given either the independent variable or the dependent variable. They also *apply* this understanding to solve multi-step, real-world problems.

Mathematical Background

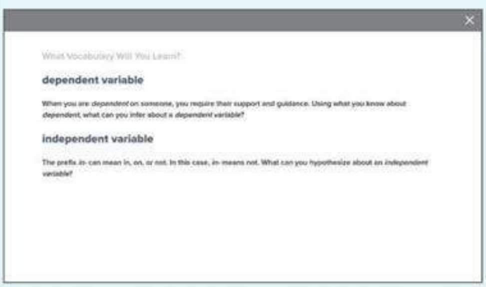
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- evaluating algebraic expressions (Exercises 1–2)
- solving one-step algebraic equations (Exercise 3)

Answers

1. $12.25

2. $57

3. 36 colored pencils

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a fundraiser as an example of an input-output relationship.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- When you are *dependent* on someone, you require their support and guidance. Using what you know about *dependent*, what can you infer about a *dependent variable*? Sample answer: A dependent variable's value may be dependent on another variable's value.
- The prefix *in-* can mean in, on, or not. In this case, *in-* means not. What can you hypothesize about an *independent variable*? Sample answer: An independent variable's value does not depend on another value.

Explore Relationships Between Two Variables

Objective

Students will use Web Sketchpad to explore the relationship between two variables.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the input-output machine. Students need to provide different inputs in order to make conjectures about the outputs. Throughout this activity, students will explore the idea of inputs and outputs and rules that are used to find outputs from inputs.

Inquiry Question

How can you find the rule for a relationship between two variables? Sample answer: Input different numbers into the machine. Then record the different outputs. Look for a pattern that applies to all inputs and outputs.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

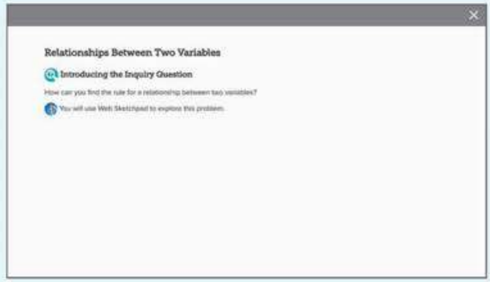
SLIDE 3

Mathematical Discourse

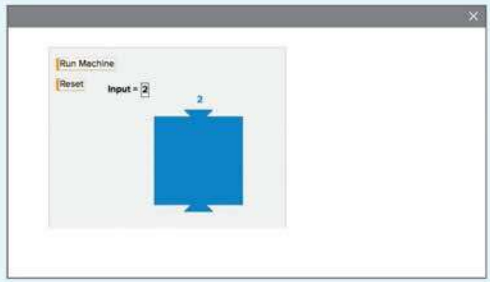
Did your conjecture work? What is the rule for the machine? Explain how you determined the rule. Sample answer: Yes, my conjecture worked. I was able to determine the rule, which is add 2. I had an input of 3 and the output was 5.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 7



Explore, Slide 3 of 7

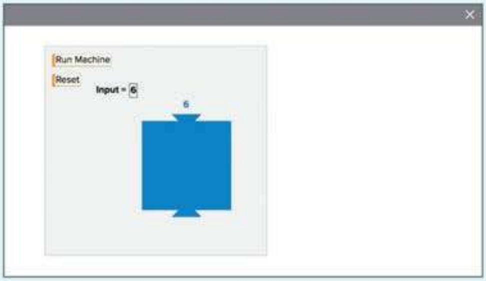
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationship between two variables.

TYPE

On Slide 2, students determine the output.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Relationships Between Two Variables *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to discover a hidden rule that, when given an input value, provides a corresponding output value. Encourage students to input different values and run the machine to deepen their understanding of the rule that gives the output.

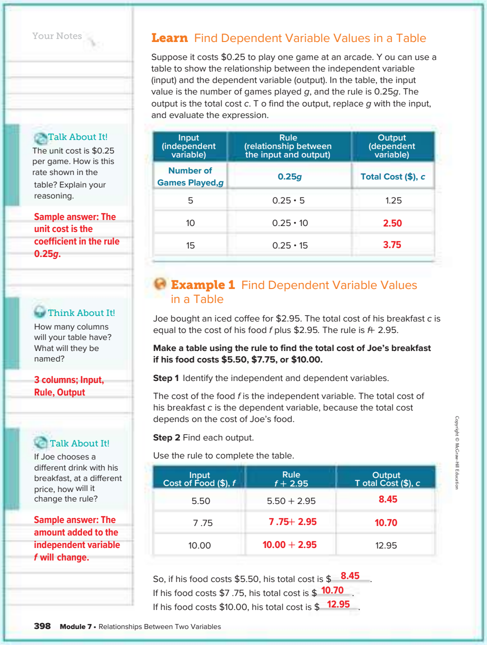
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What is the rule for the new machine? How did you determine the rule? How many numbers did you have to try? subtract 3; Sample answer: I tried an input of 6 and got an output of 3. This was either subtract 3 or divide by 2. I then tried an input of 10 to see if the output was 7 or 5; See students' responses.



Your Notes

Talk About It!
The unit cost is $0.25 per game. How is this rate shown in the table? Explain your reasoning.
Sample answer: The unit cost is the coefficient in the rule 0.25*g*.

Think About It!
How many columns will your table have? What will they be named?
3 columns; Input, Rule, Output

Talk About It!
If Joe chooses a different drink with his breakfast, at a different price, how will it change the rule?
Sample answer: The amount added to the independent variable *f* will change.

Learn Find Dependent Variable Values in a Table

Suppose it costs $0.25 to play one game at an arcade. You can use a table to show the relationship between the independent variable (input) and the dependent variable (output). In the table, the input value is the number of games played *g*, and the rule is 0.25*g*. The output is the total cost *c*. To find the output, replace *g* with the input, and evaluate the expression.

Input (independent variable)	Rule (relationship between the input and output)	Output (dependent variable)
Number of Games Played, *g*	0.25*g*	Total Cost ($), *c*
5	0.25 • 5	1.25
10	0.25 • 10	2.50
15	0.25 • 15	3.75

Example 1 Find Dependent Variable Values in a Table

Joe bought an iced coffee for $2.95. The total cost of his breakfast *c* is equal to the cost of his food *f* plus $2.95. The rule is *f* + 2.95.

Make a table using the rule to find the total cost of Joe's breakfast if his food costs $5.50, $7.75, or $10.00.

Step 1 Identify the independent and dependent variables.

The cost of the food *f* is the independent variable. The total cost of his breakfast *c* is the dependent variable, because the total cost depends on the cost of Joe's food.

Step 2 Find each output.

Use the rule to complete the table.

Input Cost of Food ($), *f*	Rule *f* + 2.95	Output Total Cost ($), *c*
5.50	5.50 + 2.95	8.45
7.75	7.75 + 2.95	10.70
10.00	10.00 + 2.95	12.95

So, if his food costs $5.50, his total cost is $ 8.45.
If his food costs $7.75, his total cost is $ 10.70.
If his food costs $10.00, his total cost is $ 12.95.

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Interactive Presentation

Example 1, Find Dependent Variable Values in a Table, Slide 3 of 5

TYPE

On Slide 3 of Example 1, students complete the table to calculate the dependent variable using the independent variable and the rule.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Find Dependent Variable Values in a Table

Objective

Students will learn how to use a table to find the dependent variable values, given the independent variable values.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

The unit cost is $0.25 per game. How is this rate shown in the table? Explain. Sample answer: The unit cost is the coefficient in the rule 0.25*g*.

Example 1 Find Dependent Variable Values in a Table

Objective

Students will use a table to find the dependent variable values, given the independent variable values.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, they should use clear and precise mathematical language in explaining how different drinks and different prices might affect the rule.

Questions for Mathematical Discourse

SLIDE 3

AL What does *f* represent in the rule? What does 2.95 represent in the rule? the cost of Joe's food; the cost of the iced coffee

OL How did you find the total cost when the cost of the food was $7.75? Sample answer: I substituted 7.75 for *f* in the rule and simplified to find the total cost.

BL How would the rule change if Joe received a $5 discount by using a coupon? What would the total cost be if the cost of the food was $12? You would subtract 5 from *f* + 2.95; $9.95

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Find Independent Variable Values in a Table

Objective

Students will learn how to use a table to find the independent variable values, given the dependent variable values.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
As students discuss the *Talk About It!* questions on Slide 2, encourage them to think about how the *work backward* strategy can be used in this situation and to explain how this process is similar to writing and solving an equation.

Teaching Notes

SLIDE 1

Have students study the table to understand the relationship between the two variables and explain how the rule describes the relationship. Then have them discuss how to determine each input value. Some students may incorrectly think that to find each value of g, they should multiply the corresponding output value by 0.25. Remind them to study the rule carefully. To find each value of g, they must use inverse operations. You may wish to have students work in pairs to complete the table. They should be able to justify how they found each value of g.

Talk About It!

SLIDE 2

Mathematical Discourse
How can you use the *work backward* strategy to find each input value, instead of writing and solving an equation? How are these strategies similar and different? Sample answer: To undo multiplication by 0.25, use the inverse operation to divide each output value by 0.25. Dividing by 0.25 is essentially how you solve the equation.

Check

A grocery store charges $2.50 per gallon of fruit punch. The total cost c of g gallons of fruit punch is equal to 2.5 times g. The rule is $2.5g$. Make a table using the rule to find the total cost of buying 1, 2, or 3 gallons of fruit punch.

Input Number of Gallons, g	Rule $2.5g$	Output Total Cost ($), c
1	2.5(1)	2.50
2	2.5(2)	5.00
3	2.5(3)	7.50

Go Online You can complete an Extra Example online.

Learn Find Independent Variable Values in a Table

Suppose it costs $0.25 to play one game at an arcade. The total cost of playing any number of games can be represented by the rule $0.25g$, where g is the number of games played. You can use a table to find the independent variable (input) if you know the dependent variable (output) and the rule.

Note that in the table below, the output values are $1.75, $3.00, and $4.25. Because the rule is $0.25g$, write and solve an equation to find the input g when the output is $1.75.

$0.25g = 1.75$ The input g multiplied by 0.25 equals the output, $1.75.

$\frac{0.25g}{0.25} = \frac{1.75}{0.25}$ Divide each side by 0.25.

$g = 7$ Simplify. The input value is 7.

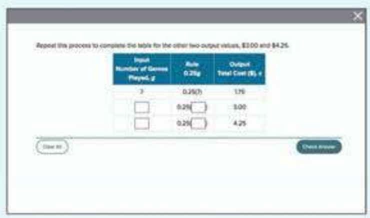
Repeat this process to complete the table for the other two output values, $3.00 and $4.25.

Input Number of Games Played, g	Rule $0.25g$	Output Total Cost ($), c
7	0.25 • 7	1.75
12	0.25 • 12	3.00
17	0.25 • 17	4.25

Talk About It!
How can you use the *work backward* strategy to find each input value, instead of writing and solving an equation? How are these strategies similar and different?

Sample answer: To undo multiplication by 0.25, use the inverse operation to divide each output value by 0.25. Dividing by 0.25 is essentially how you solve the equation

Interactive Presentation



Learn, Find Independent Variable Values in a Table, Slide 1 of 2

TYPE

On Slide 1, students complete the table by computing the independent variable values.

Think About It!
Do you need to find the input values or the output values?

input values

Talk About It!
How is solving this Example related to what you already know about solving equations?

Sample answer: I used inverse operations to find the input value for each output value, which is similar to solving equations.

Example 2 Find Independent Variable Values in a Table

Each small pizza at the local pizza shop costs $6.75. The total cost c of p small pizzas is equal to 6.75 times p.

Make a table to find the number of small pizzas purchased if the total cost is $13.50, $27, or $33.75.

Step 1 Identify the independent and dependent variables.

The number of pizzas p is the input, or independent variable.

The total cost of the pizza c is the output, or dependent variable.

The total cost is 6.75 times p, so the rule is $6.75p$.

Step 2 Find each input.

T o find the number of pizzas for each of the total costs given in the table, use the *work backward* strategy. T o undo multiplication by 6.75, use the inverse operation to divide each output value by 6.75.

Complete the table.

Input Number of Pizzas, p	Rule $6.75p$	Output T otal Cost ($), c
2	6.75(2)	13.50
4	6.75(4)	27 .00
5	6.75(5)	33.75

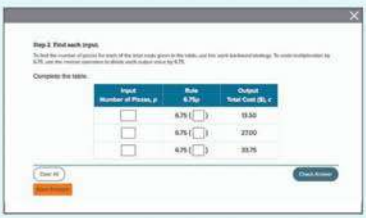
Check

Leslie has 48 stickers to give to her friends. The number of stickers s each friend will receive is equal to 48 divided by f, the number of friends. Complete the table to find the number of friends Leslie gave stickers to if each friend receives 12, 8, or 6 stickers.

Input Number of Friends, f	Rule $48 \div f$	Output Number of Stickers, s
4	$48 \div 4$	12
6	$48 \div 6$	8
8	$48 \div 8$	6

Go Online Y ou can complete an Extra Example online.

Interactive Presentation



Example 2, Find Independent Variable Values in a Table, Slide 3 of 5

CLICK

On Slide 2, students identify the dependent and independent variables.

TYPE

On Slide 3, students complete the table by computing the values of the independent variable.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find Independent Variable Values in a Table

Objective

Students will use a table to find the independent variable values, given the dependent variable values.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should make sense of the real-world situation, the quantities p and c, and the rule that describes the relationship between them, in order to identify the independent and dependent variables.

As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense about what it means to find the input and output values in a table, and how this is related to solving an equation.

Questions for Mathematical Discourse

SLIDE 2

AL Which quantity depends on the other quantity? Is this the independent or the dependent variable? the total cost of the pizza depends on the number of pizzas; dependent variable

OL Why is the independent variable the number of pizzas? The independent variable is the number of pizzas because we can choose this quantity and it does not depend on another value.

BL If the cost for each small pizza were increased, would the total cost still be the dependent variable? Yes, if the cost of each pizza was increased, the total cost would still be the dependent variable.

SLIDE 3

AL In the rule, what operation occurs between 6.75 and p? What is the inverse operation of multiplication? multiplication; division

OL Explain how you would use the *work backward* strategy to find the number of pizzas. Sample answer: I will work backward using the rule and the total cost. The number of pizzas will be equal to the total cost divided by 6.75.

BL Suppose you added a $5 tip to the cost of the pizzas. How will the rule change? Does that change the independent and dependent variables? Explain. Sample answer: You would add $5 to the rule so the new rule would be $6.75p + 5$. This does not change the independent and dependent variables as the total cost still depends on the number of pizzas ordered.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Measurement

Objective

Students will come up with their own strategy to solve an application problem involving comparing measurements.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the best way to compare the three sculptures?
- How do you convert measurements between feet and inches?
- What steps do you need to perform to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Measurement

Sondra is placing sculptures on a 3-foot-tall base to display in a cabinet in the school entryway. The height including the base b is equal to the height h of the sculpture plus 3. If the cabinet is 84 inches tall, which sculpture(s) will fit in the cabinet?

Height of Sculpture (ft), h	Rule $h + 3$	Height with Base (ft), b
$2\frac{1}{4}$	$2\frac{1}{4} + 3$	$5\frac{1}{4}$
$3\frac{3}{4}$	$3\frac{3}{4} + 3$	$6\frac{3}{4}$
$5\frac{1}{8}$	$5\frac{1}{8} + 3$	$8\frac{1}{8}$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

The $2\frac{1}{4}$-foot-tall sculpture and the $3\frac{3}{4}$-foot-tall sculpture will fit; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Why is it helpful to convert the measurements to the same unit?

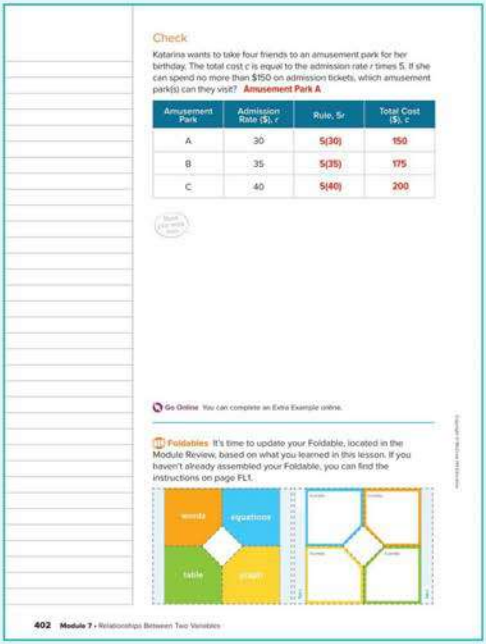
Sample answer: The height of the sculptures and base are given in feet and the height of the cabinet is given in inches. If the measurements are written using the same unit, they can be easily compared.

Interactive Presentation

Apply, Measurement

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Katarina wants to take four friends to an amusement park for her birthday. The total cost c is equal to the admission rate r times 5. If she can spend no more than $150 on admission tickets, which amusement park(s) can they visit? **Amusement Park A**

Amusement Park	Admission Rate ($), r	Rule, $5r$	Total Cost ($), c
A	30	5(30)	150
B	35	5(35)	175
C	40	5(40)	200

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

402 Module 7 • Relationships Between Two Variables

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. You may wish to have students share their Foldables with a partner to compare the information they recorded.

Essential Question Follow-Up

What are the ways in which a relationship between two variables can be displayed? In this lesson, students learned how to identify dependent and independent variables. Encourage them to discuss with a partner the benefits of using a table to display the relationship. Some students may say that they can use the input and output values that are displayed in the table to find the rule that describes the relationship between the two variables.

Exit Ticket

Refer to Exit Ticket slide. How many packages of cookies will they need to sell to cover the registration fee if they charge $1.25 per package? Write a mathematical argument that can be used to defend your solution. 280 packages; Sample answer: You can work backward using the output value $350 and the rule $1.25p$ to find the input value. Since $1.25(280) = 350$, they need to sell 280 packages to cover the $350 registration fee.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1–7 odd, 9–12
- Extension: Domain and Range
- ALEKS Graphs and Functions

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–5, 7, 9, 10
- Extension: Domain and Range
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Ordered Pairs

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ordered Pairs

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

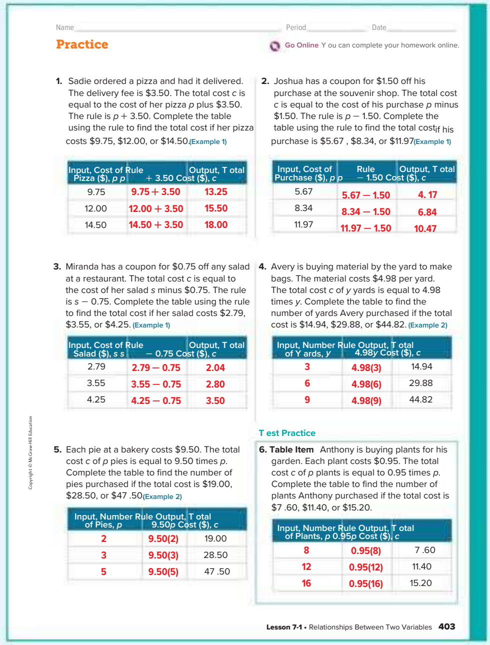
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	use a table to find the dependent variable values, given the independent variable values	1–3
2	use a table to find the independent variable values, given the dependent variable values	4, 5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving the relationship between two variables	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

Students may have difficulty knowing how to use the rule to *work backward* to find the independent variable values given the dependent variable values. In Exercise 4, students may mistakenly attempt to multiply each value of c by 4.98 to determine the value of y. Encourage students to first make sense of the quantities given in the real-world problem and their relationship to one another. Students should make sense of the rule $4.98y$ to determine that each independent variable y is multiplied by 4.98 to find each dependent variable value of c. Since the values of c are known, they must perform the inverse operation and divide each value of c by 4.98 to find each value of y.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Sadie ordered a pizza and had it delivered. The delivery fee is $3.50. The total cost c is equal to the cost of her pizza p plus $3.50. The rule is $p + 3.50$. Complete the table using the rule to find the total cost if her pizza costs $9.75, $12.00, or $14.50. (Example 1)

Input, Cost of Pizza ($), p	Rule $p + 3.50$	Output, Total Cost ($), c
9.75	9.75 + 3.50	13.25
12.00	12.00 + 3.50	15.50
14.50	14.50 + 3.50	18.00

2. Joshua has a coupon for $1.50 off his purchase at the souvenir shop. The total cost c is equal to the cost of his purchase p minus $1.50. The rule is $p - 1.50$. Complete the table using the rule to find the total cost if his purchase is $5.67, $8.34, or $11.97 (Example 1)

Input, Cost of Purchase ($), p	Rule $p - 1.50$	Output, Total Cost ($), c
5.67	5.67 – 1.50	4.17
8.34	8.34 – 1.50	6.84
11.97	11.97 – 1.50	10.47

3. Miranda has a coupon for $0.75 off any salad at a restaurant. The total cost c is equal to the cost of her salad s minus $0.75. The rule is $s - 0.75$. Complete the table using the rule to find the total cost if her salad costs $2.79, $3.55, or $4.25. (Example 1)

Input, Cost of Salad ($), s	Rule $s - 0.75$	Output, Total Cost ($), c
2.79	2.79 – 0.75	2.04
3.55	3.55 – 0.75	2.80
4.25	4.25 – 0.75	3.50

4. Avery is buying material by the yard to make bags. The material costs $4.98 per yard. The total cost c of y yards is equal to 4.98 times y. Complete the table to find the number of yards Avery purchased if the total cost is $14.94, $29.88, or $44.82. (Example 2)

Input, Number of Yards, y	Rule $4.98y$	Output, Total Cost ($), c
3	4.98(3)	14.94
6	4.98(6)	29.88
9	4.98(9)	44.82

5. Each pie at a bakery costs $9.50. The total cost c of p pies is equal to 9.50 times p. Complete the table to find the number of pies purchased if the total cost is $19.00, $28.50, or $47.50 (Example 2)

Input, Number of Pies, p	Rule $9.50p$	Output, Total Cost ($), c
2	9.50(2)	19.00
3	9.50(3)	28.50
5	9.50(5)	47.50

Test Practice

6. Table Item Anthony is buying plants for his garden. Each plant costs $0.95. The total cost c of p plants is equal to 0.95 times p. Complete the table to find the number of plants Anthony purchased if the total cost is $7.60, $11.40, or $15.20.

Input, Number of Plants, p	Rule $0.95p$	Output, Total Cost ($), c
8	0.95(8)	7.60
12	0.95(12)	11.40
16	0.95(16)	15.20

Apply *indicates multi-step problem

*7. Mara lives in a state that has no sales tax on apparel. She has a coupon for $15 off the price of one pair of shoes. The total cost c of a pair of shoes is equal to the original price of the shoes p minus 15. If she only has $60 to spend on a pair of shoes, which pair(s) could she buy?

Original Price ($), p	Rule $p - 15$	Total Cost ($), c
65	65 − 15	50
73	73 − 15	58
79	79 − 15	64

She could buy the pair that originally cost $65 or the pair that originally cost $73.

*8. An empty suitcase weighs 224 ounces. The total weight t of the suitcase is equal to the weight of its contents w plus 224. To not be charged an additional fee for a flight, the total weight must be no more than 50 pounds. Which suitcase(s) would be charged a fee?

Weight of Contents (oz), c	Rule $x + 224$	Weight with Suitcase (oz), t
$575\frac{1}{2}$	$575\frac{1}{2} + 224$	$799\frac{1}{2}$
576	$576 + 224$	800
$576\frac{1}{2}$	$576\frac{1}{2} + 224$	$800\frac{1}{2}$

The suitcase with the contents that weigh $576\frac{1}{2}$ ounces.

Higher-Order Thinking Problems

9. **Identify Structure** Complete the table by finding the input values.

Input, x	Rule, $2x - 2.5$	Output, y
5	2(5) − 2.5	7.5
6.5	2(6.5) − 2.5	10.5
8	2(8) − 2.5	13.5

10. **Reason Inductively** A student said that the independent variable for the following situation is the number of days, d. Is the student correct? Explain.

Jess walks 1.5 miles every day for d days. What is the total number of miles she walks?

yes; Sample answer: The number of days is the independent variable because it does not depend on the other quantity, total mileage. The total number of miles is the dependent variable because it changes with the number of days.

11. **Persevere with Problems** A concession stand sells soft pretzels for $2.75 each and drinks for $1.50 each. The equation $c = 2.75p + 1.50d$ can be used to represent the total cost c of p pretzels and d drinks. What is the total cost of 3 pretzels and 4 drinks? Explain how you solved.

$14.25; Sample answer: In the equation $c = 2.75p + 1.5d$, replace p with 3 and d with 4 and then simplify. $c = 2.75 \times 3 + 1.5 \times 4$ or 14.25.

12. Describe a real-world situation that has an independent variable and a dependent variable. Identify each variable.

Sample answer: A bakery sells muffins for $2.50 each. What is the total cost?; number of muffins is the independent variable; total cost is the dependent variable

Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 9, students will use the structure of an expression and its corresponding equation to find the value of the independent variable given a rule and a value for the dependent variable.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students are asked to evaluate the reasoning of a fellow student in identifying the independent variable in a situation involving the number of miles walked per day, the number of days, and the total number of miles walked.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 11, students will persevere through a problem that has two independent variables in which they are asked to proceed step by step in finding the total cost and explain their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 7–8 Have students work in pairs. Have students individually read Exercise 7 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 8.

Be sure everyone understands.

Use with Exercises 9–10 Have students work in groups of 3–4 to solve the problem in Exercise 9. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 10.

Learn Write One-Step Equations

Objective

Students will learn how to model a relationship shown in a table with a one-step equation.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to recall what they previously learned about rates and to make sense of the words in the scenario that can help them understand where they can see the unit rate in the table.

Teaching Notes

SLIDE 1

Students will learn that an equation can be used to represent the relationship shown in a table. Students should note that equations express a dependent variable in terms of the independent variable. Have students identify which variable in the table represents the dependent variable and which variable represents the independent variable. They should be able to explain that the input value is the independent variable, and the output value is the dependent variable.

Talk About It!

SLIDE 2

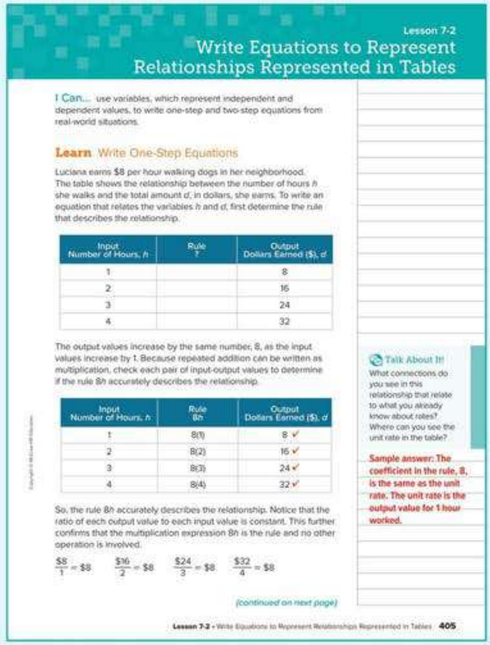
Mathematical Discourse

What connections do you see in this relationship that relate to what you already know about rates? Where can you see the unit rate in the table? Sample answer: The coefficient in the rule, 8, is the same as the unit rate. The unit rate is the output value for 1 hour worked.

Teaching Notes

SLIDE 3

Have students study the relationship between the variables in the table. Ask them to work with a partner to come up with a verbal phrase that describes the relationship, using their own words. For example, some students may say *the value of the output is eight times the input*. Having them understand the relationship and being able to describe it in their own words can help them write the equation that represents the relationship.



Lesson 7-2

Write Equations to Represent Relationships Represented in Tables

I Can... use variables, which represent independent and dependent values, to write one-step and two-step equations from real-world situations.

Learn Write One-Step Equations

Luciana earns $8 per hour walking dogs in her neighborhood. The table shows the relationship between the number of hours h she walks and the total amount d, in dollars, she earns. To write an equation that relates the variables h and d, first determine the rule that describes the relationship.

Input Number of Hours, h	Rule ?	Output Dollars Earned ($), d
1		8
2		16
3		24
4		32

The output values increase by the same number, 8, as the input values increase by 1. Because repeated addition can be written as multiplication, check each pair of input-output values to determine if the rule $8h$ accurately describes the relationship.

Input Number of Hours, h	Rule $8h$	Output Dollars Earned ($), d
1	8(1)	8 ✓
2	8(2)	16 ✓
3	8(3)	24 ✓
4	8(4)	32 ✓

So, the rule $8h$ accurately describes the relationship. Notice that the ratio of each output value to each input value is constant. This further confirms that the multiplication expression $8h$ is the rule and no other operation is involved.

$\frac{\$8}{1} = \$8$ $\frac{\$16}{2} = \$8$ $\frac{\$24}{3} = \$8$ $\frac{\$32}{4} = \$8$

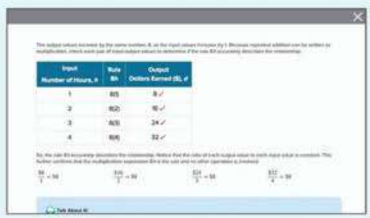
Talk About It! What connections do you see in this relationship that relate to what you already know about rates? Where can you see the unit rate in the table?

Sample answer: The coefficient in the rule, 8, is the same as the unit rate. The unit rate is the output value for 1 hour worked.

(continued on next page)

Lesson 7-2 • Write Equations to Represent Relationships Represented in Tables 405

Interactive Presentation



Learn, Write One-Step Equations, Slide 2 of 3

Lesson 7-2

Write Equations to Represent Relationships Represented in Tables

6.EE.C.9

LESSON GOAL

Students will write equations to represent relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Write One-Step Equations
Example 1: Write One-Step Equations

Explore: Relationships with Rules that Require Two Steps

Learn: Write Two-Step Equations
Example 2: Write Two-Step Equations
Apply: Art

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Write Quadratic Equations for Input-Output Tables		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 41 of the *Language Development Handbook* to help your students build mathematical language related to equations of relationships between two variables.

ELL You can use the tips and suggestions on page T41 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.C** by writing equations to represent relationships.
Standards for Mathematical Content: **6.EE.C.9,** *Also addresses 6.EE.B.6, 6.EE.B.7*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5**

Coherence

Vertical Alignment

Previous
Students identified and used independent and dependent variables in relationships.
6.EE.C.9

Now
Students write equations to represent relationships.
6.EE.C.9

Next
Students will write equations and graph lines to represent relationships.
6.EE.C.9

Rigor

The Three Pillars of Rigor

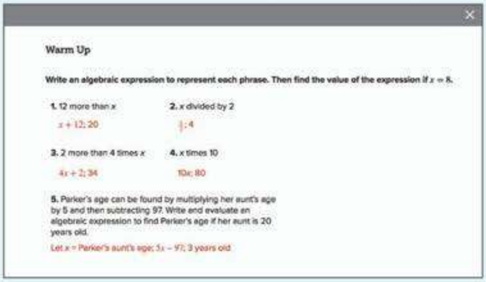
1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of equations to begin to develop *understanding* of writing equations to model relationships represented in tables. They build *fluency* with writing one- and two-step equations to model relationships shown in tables. They *apply* their understanding of writing equations to model relationships represented in tables to solve multi-step, real world problems.

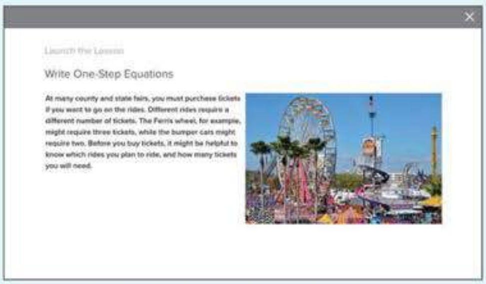
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

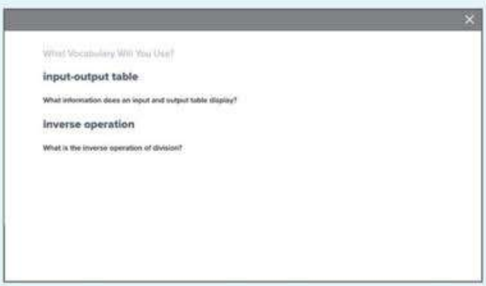
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing and evaluating algebraic expressions (Exercises 1–5)

Answers

1. $x + 12$; 20

2. $\frac{x}{2}$; 4

3. $4x + 2$; 34

4. $10x$; 80

5. Let x = Parker's aunt's age; $5x - 97$; 3 years old

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the cost of ride tickets at a fair.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What information does an input-output table display? Sample answer: An input-output table displays input and output values based on a given rule.
- What is the inverse operation of division? multiplication

Your Notes

Think About It!
How would you begin solving the problem?

See students' responses.

Talk About It!
What connections do you see in this relationship that relate to what you already know about unit price? Where can you see the unit price in the table?

Sample answer: The coefficient in the rule, 9, is the same as the unit price. The unit price is the output value for buying 1 T-shirt.

Use the rule $8h$ to write an equation relating the two variables h and d.

$d = 8h$ ← Each output value is the product of the constant ratio, 8, and the corresponding input value h.

Example 1 Write One-Step Equations

The table shows the total cost c, in dollars, of buying t souvenir T-shirts.

Write an equation to represent the relationship between c and t.

Number of T-shirts, t	Total Cost ($), c
1	9
2	18
3	27
4	36
5	45

Step 1 Identify the variables.

The independent variable is t, the number of shirts.

The dependent variable is c, total cost.

Step 2 Determine the rule.

The output values increase by the same number, 9, as the input values increase by 1. Because repeated addition can be written as multiplication, check each pair of input-output values to determine if the rule $9t$ accurately describes the relationship.

Input Number of T-shirts, t	Rule $9t$	Output Total Cost ($), c
1	9(1)	9 ✓
2	9(2)	18 ✓
3	9(3)	27 ✓
4	9(4)	36 ✓
5	9(5)	45 ✓

So, the rule $9t$ accurately describes the relationship.

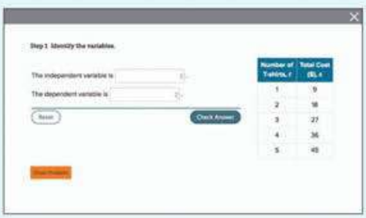
Step 3 Write the equation.

Use the rule $9t$ to write an equation relating the two variables t and c.

$c = 9t$ ← Each output value is the product of the constant ratio, 9, and the corresponding input value t.

So, the equation that represents the total cost c of buying t souvenir T-shirts is $c = 9t$.

Interactive Presentation



Example 1, Write One-Step Equations, Slide 2 of 6

CLICK

On Slide 2, students define the independent and dependent variables.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Write One-Step Equations

Objective

Students will model a relationship shown in a table with a one-step equation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to examine the relationship between the independent and dependent variables represented in the table to decontextulaize the real-world problem by representing it symbolically with the correct one-step equation.

As students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of how the unit price is represented both in the table and in the equation.

Questions for Mathematical Discourse

SLIDE 2

AL What are the two quantities and the variables representing them? The number of T-shirts is represented by t, and the total cost is represented by c.

OL How do you know that the number of T-shirts is the independent variable? Sample answer: The number of T-shirts is the independent variable because it is not affected by the total cost.

BL How could you find the cost of 50 T-shirts? Sample answer: I could use repeated addition, or I could multiply the number of T-shirts by $9, the cost of each shirt.

SLIDE 3

AL What is a coefficient? the numerical factor in a term

AL How do you know the input should be multiplied by 9? Sample answer: If I use repeated addition to find consecutive terms, that is the same as multiplying by 9.

OL Why would you write the equation using multiplication? Sample answer: Writing the equation using multiplication makes it easier to use when working with larger values.

BL Explain why you could not write a multiplication rule for an input of 0, 1, 2, and an output of 2, 4, 8. Sample answer: Since the difference between consecutive terms is not the same, repeated addition does not apply, so multiplication will not work.

(continued on next page)

Example 1 Write One-Step Equations *(continued)*

Questions for Mathematical Discourse

SLIDE 4

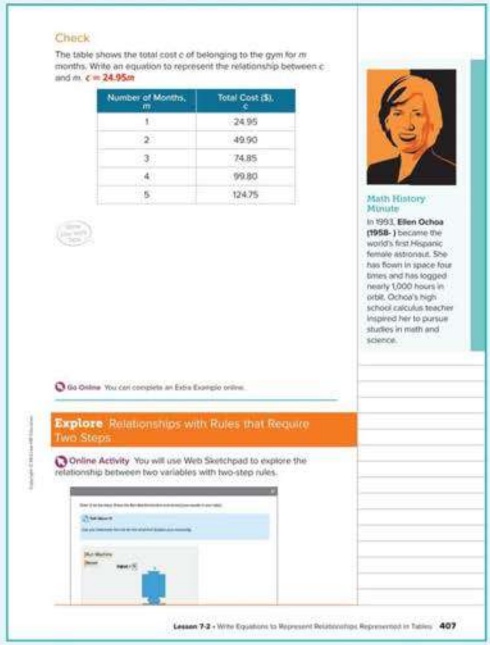
AL What does it mean to substitute a value into the equation? Sample answer: Replace *t* in the equation with one of the input values, and then multiply by 9. Then check to see if it matches the output value.

OL How do you know that the constant is 0? Sample answer: Since the product of the input value and nine is equal to the output value, I know the constant is zero.

BL What is the fewest number of input values you can test for a multiplication rule to make sure your rule is correct and that no constant is added? Explain. 1; Sample answer: Since I know there is repeated addition, if I test one value and find it matches the output, then all of the other values should match as well.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

The table shows the total cost c of belonging to the gym for m months. Write an equation to represent the relationship between c and m. $c = 24.95m$

Number of Months, m	Total Cost ($), c
1	24.95
2	49.90
3	74.85
4	99.80
5	124.75

Math History Minute

In 1993, **Ellen Ochoa (1958-)** became the world's first Hispanic female astronaut. She has flown in space four times and has logged nearly 1,000 hours in orbit. Ochoa's high school calculus teacher inspired her to pursue studies in math and science.

Go Online You can complete an Extra Example online.

Explore Relationships with Rules that Require Two Steps

Online Activity You will use Web Sketchpad to explore the relationship between two variables with two-step rules.

Lesson 7-2 • Write Equations to Represent Relationships Represented in Tables 407

DIFFERENTIATE

Reteaching Activity AL

Some students may struggle with the idea that adding a number to the previous output value is the same as multiplying the input value by that same number. In Example 1, students determine that the total cost increases by $9 with each increase in one T-shirt. To help students understand this concept, it may be beneficial to use counters. Have students make a group of 9 counters to represent the cost of one shirt. Then have them make another group of counters to represent the cost of another shirt. They should continue making groups of counters until there is one group for each of the 5 shirts from the Example. Once they have 5 groups of counters, they should recognize that adding 5 groups of 9 counters is the same as multiplying 5 by 9.

Learn Write Two-Step Equations

Sometimes the relationship between two variables cannot be accurately described by a one-step equation. In those cases, check to see if more than one operation is involved. Consider the following scenario.

An online store sells baseball bats. You will pay for each bat that you order, plus a one-time shipping fee. The table shows the relationship between the number of bats ordered and the total cost. Write a two-step equation to represent the total cost c to ship an order of b baseball bats.

Number of Bats, b	Total Cost ($), c
1	6
2	8
3	10

Step 1 Look for a pattern.

The output values increase by the same number, 2, as the input values increase by 1. The rule includes $2b$.

Step 2 Determine the rule.

Input, Number of Bats, b	Rule $2b$	Output, Total Cost ($), c
1	2(1)	6 ✗
2	2(2)	8 ✗
3	2(3)	10 ✗
4	2(4)	12 ✗
5	2(5)	14 ✗

Check each pair of input-output values to determine if the rule $2b$ accurately describes the relationship. The rule $2b$ does not accurately describe the relationship.

Input, Number of Bats, b	Rule $2b + 4$	Output, Total Cost ($), c
1	2(1) + 4	6 ✓
2	2(2) + 4	8 ✓
3	2(3) + 4	10 ✓
4	2(4) + 4	12 ✓
5	2(5) + 4	14 ✓

To obtain an output value of 6, multiply the input value 1 by 2. Then add 4.

Check each pair of input-output values. The rule $2b + 4$ accurately describes the relationship.

Step 3 Write the equation.

Use the rule $2b + 4$ to write an equation relating the two variables b and c.

$c = 2b + 4$

Talk About It!

If the store did not charge a shipping fee for the bats, how would the equation be different? What is the shipping fee?

Sample answer: The equation would not have a constant. It would be $c = 2b$; $4

Interactive Presentation

Learn, Write Two-Step Equations, Slide 2 of 4

CLICK

On Slide 2, students move through the slides to determine the rule.

Learn Write Two-Step Equations

Objective

Students will learn how to model a relationship shown in a table with a two-step equation.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the equation and how it represents the scenario, in order to understand how the equation would be different without the shipping fee.

Teaching Notes

SLIDE 1

Present the scenario. Before moving on to Step 1, ask students to work with a partner to determine possible strategies they can use to write an equation that represents the relationship between the two variables. Encourage them to first study the table to describe the relationship in their own words. They may use any strategy they wish to write the equation, but they must be able to explain their strategy and defend why it works. Have students share their strategies with the class.

SLIDE 2

After students determine that the rule $2b$ does not accurately describe the relationship, you may wish to have students brainstorm ways in which they can determine the correct rule. Students may try multiplying by a different number for each input. For example, for an input of 1 and an output of 6, they may say to multiply by 6, and for an input of 2 and an output of 8, multiply by 4, and so on. Remind them that they need to determine the rule that describes the relationship for *all* inputs.

SLIDE 3

Remind students that the purpose of determining the rule is to find an output value c for any input value b. So, to write the equation, they should set the rule equal to c.

Talk About It!

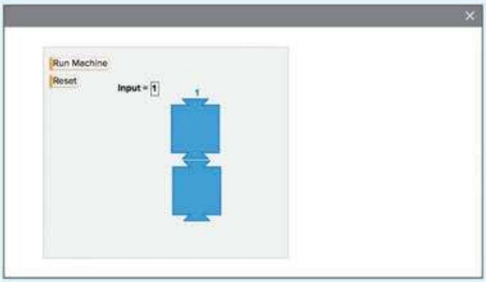
SLIDE 4

Mathematical Discourse

If the store did not charge a shipping fee for the bats, how would the equation be different? What is the cost of the shipping fee? Sample answer: The equation would not have a constant. It would be $c = 2b$; $4

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 3 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationship between two variables when two steps are required.

Explore Relationships with Rules that Require Two Steps

Objective

Students will use Web Sketchpad to explore the relationship between two variables when two steps are required.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with an input-output machine. Throughout the activity, students will use the machine to write a rule that involves two steps to transform the input value into the output value.

Inquiry Question

How can you find a rule involving two steps for a relationship? Sample answer: Look for a pattern between the differences in consecutive outputs to discover the first step. Then determine the additional step needed in order to get the correct output.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

Can you determine the rule for the machine? Explain your reasoning. Sample answer: I still cannot determine the rule. I know the input is not multiplied by 7, nor is 6 added to it. I need to find the outputs for a few more inputs to see if repeated addition occurs.

(continued on next page)

Explore Relationships with Rules that Require Two Steps *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to perform two operations on an input value to obtain a corresponding output value. Encourage students to use the input-output machine to write two-step equations and record their results in the tables.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

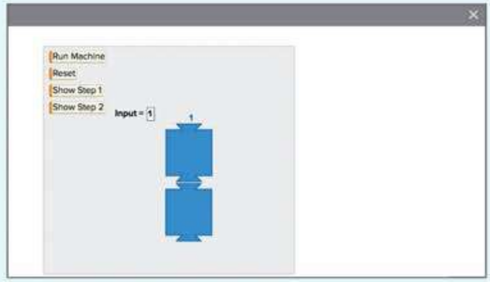
Talk About It!

SLIDE 6

Mathematical Discourse

Make a conjecture about the rule. Explain your reasoning. Use the values in the table to support your conjecture. Then press the *Show Step 2* button to check. Sample answer: With an input of 1, if I multiply by 3, then I need to add 4 to get an output of 7. The rule is multiply by 3, add 4.

Interactive Presentation



Explore, Slide 6 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Example 2 Write Two-Step Equations

Objective

Students will model a relationship shown in a table with a two-step equation.

Questions for Mathematical Discourse

SLIDE 2

AL How will you check for repeated addition in the output? Sample answer: I can subtract the consecutive values for the output and see if the difference is the same.

OL How did you find that the number of necklaces produced increased by 2? Sample answer: I found this by subtracting 5 from 7 necklaces, then 7 from 9.

BL How can you use the table to find the number of necklaces that will have been made after 6 hours? Sample answer: I can extend the pattern in the table to find that 15 necklaces will have been made after 6 hours.

SLIDE 3

AL The coefficient of the input, h, is 2 because there was repeated addition in the output. How do you know you need to find a constant to add to $2h$? Sample answer: If I use the rule $2h$, the input will not match the output. I need to add a number to make them match.

OL The first part of the rule is to multiply by 2. How did you find the second part? Sample answer: To find the second part, I multiplied the input by 2 and then found the difference between the number of necklaces produced in that time and the product.

BL The solution used the first set of information, 1 hour and 5 necklaces, to find the constant. Could you use a different set? Explain. yes; Sample answer: If you used 2 hours, 2 times 2 is 4 so you would still need to add 3 to get 7 necklaces. You could use any set of information to find the constant.

Go Online

- Find additional teaching notes, discussion questions, Teaching the Mathematical Practices, and the *Talk About It!* question to promote matematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Write Two-Step Equations

The table shows the total number of necklaces Ari has made after a certain number of hours.

Write a two-step equation to represent the total number of necklaces n she will have made after h hours.

Time (hours), h	Number of Necklaces, n
1	5
2	7
3	9
4	11

Think About It!
Is there repeated addition in the output?

See students' responses.

Step 1 Look for a pattern.

The output values increase by the same number, 2, as the input values increase by 1. The rule includes $2h$.

Step 2 Determine the rule.

Check each pair of input-output values to determine if the rule $2h$ accurately describes the relationship.

Time (hours), h	Rule $2h$	Number of Necklaces, n
1	2(1)	5 ✗
2	2(2)	7 ✗
3	2(3)	9 ✗
4	2(4)	11 ✗

By itself, the rule $2h$ does not describe the relationship. Check to see if this relationship involves two operations.

Time (hours), h	Rule $2h + 3$	Number of Necklaces, n
1	2(1) + 3	5 ✓
2	2(2) + 3	7 ✓
3	2(3) + 3	9 ✓
4	2(4) + 3	11 ✓

T o obtain an output value of 5, multiply the input value 1 by 2. Then add 3.

Check each pair of input-output values. The rule $2h + 3$ describes the relationship.

Step 3 Write the equation.

Use the rule $2h + 3$ to write an equation relating the two variables h and n.

$n = 2h + 3$

So, the equation used to represent the total number of necklaces n Ari will have made after h hours is $n = 2h + 3$.

Talk About It!
When a relationship can be represented by a two-step equation, is there a constant ratio between the variables? Explain.

no; Sample answer: the constant term in the equation means that there is not a constant ratio between the variables.

Lesson 7-2 • Write Equations to Represent Relationships Represented in Tables 409

Interactive Presentation

Example 2, Write Two-Step Equations, Slide 2 of 6

CLICK

On Slide 3, students determine the rule to describe the relationship.

TYPE

On Slide 4, students write the equation that represents the situation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The table shows the total fees f for d days a library book is overdue. Write a two-step equation to represent the total fee for the number of days the book is overdue. $f = 0.20d + 0.10$

Time (days), d	Total Fee ($), f
1	0.30
2	0.50
3	0.70
4	0.90

Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you struggle more with writing two-step equations as compared to one-step equations? If so, what questions can you ask to better understand the concept? If not, how could you explain the concept to someone who is struggling?

Record your observations here

See students' observations.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of how to model a relationship expressed as a table with a two-step equation, have them work with a partner to complete the following activity.

Present them with the two tables shown. Students should note that the values of the variables B and x do not increase by increments of 1. Students should also note that the values of the variable x do not increase by a consistent number.

Have students use any strategy they wish to write the two-step equation that can model each relationship. Then have them explain their strategy to another pair of students, or to the entire class.

B	p
3	7
6	13
9	19
12	25

$p = 2B + 1$

x	y
2	5
5	14
6	17
10	29

$y = 3x - 1$

Apply Art

Objective

Students will come up with their own strategy to solve an application problem involving painting signs for a school election campaign.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

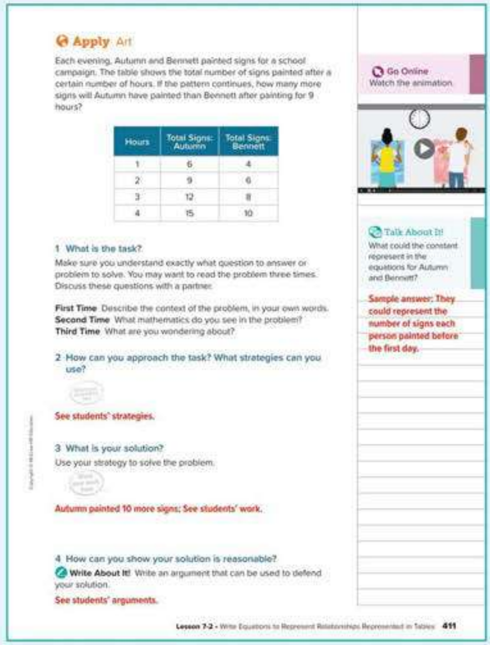
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Is this a one- or two-step rule?
- Who do you think painted more signs?
- What steps do you need to perform to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Art

Each evening, Autumn and Bennett painted signs for a school campaign. The table shows the total number of signs painted after a certain number of hours. If the pattern continues, how many more signs will Autumn have painted than Bennett after painting for 9 hours?

Hours	Total Signs: Autumn	Total Signs: Bennett
1	6	4
2	9	6
3	12	8
4	15	10

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Autumn painted 10 more signs; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online
Watch the animation.

Talk About It!
What could the constant represent in the equations for Autumn and Bennett?

Sample answer: They could represent the number of signs each person painted before the first day.

Lesson 7-2 • Write Equations to Represent Relationships Represented in Tables 411

Interactive Presentation

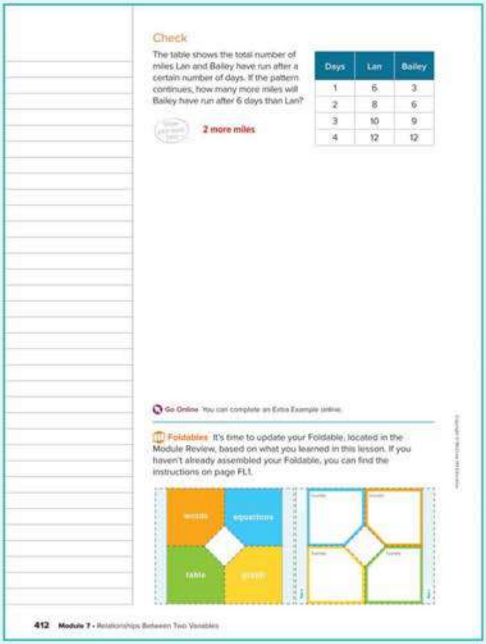
Apply, Art

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows the total number of miles Lan and Bailey have run after a certain number of days. If the pattern continues, how many more miles will Bailey have run after 6 days than Lan?

Days	Lan	Bailey
1	6	3
2	8	6
3	10	9
4	12	12

2 more miles

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

412 Module 7 • Relationships Between Two Variables

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how to write an equation that represents a relationship shown in a table. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What are the ways in which a relationship between two variables can be displayed? In this lesson, students learned how to write one- and two-step equations that describe the relationship between two variables from tables. Encourage them to work with a partner to compare and contrast using tables and equations to display and describe the relationship between two variables. Some students may say that while a table helps them see individual values, the equation describes the relationship between the variables that is true for all of those values.

Exit Ticket

Refer to the Exit Ticket slide. Each ride requires 3 tickets. If a ticket costs \$1.50, write an equation to find the total cost y of x rides. How much will it cost to ride 10 rides? $y = 4.5x$; \$45

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–5 odd, 6–10
- Extension: Write Quadratic Equations for Input-Output Tables
- ALEKS Graphs and Functions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 6, 8, 9
- Extension: Write Quadratic Equations for Input-Output Tables
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Ordered Pairs

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ordered Pairs

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	model a relationship shown in a table with a one-step equation	1, 2
2	model a relationship shown in a table with a two-step equation	3, 4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving writing equations to represent relationships between two variables	6
3	higher-order and critical thinking skills	7–10

Common Misconception

Some students may confuse the independent and dependent variables when writing an equation to model the real-world scenario. In Exercise 1, students may incorrectly write the equation $t = 7c$ instead of $c = 7t$. Encourage them to study the situation and table carefully to determine that the total cost c is dependent on the number of tickets t. Have them make sense of why the equation $t = 7c$ does not represent this situation by substituting the values from the table to determine that the equation is not true. Encourage students to check each equation they write by replacing the values from the table to verify that the equations are true. If the equations they wrote are not true, then they may have confused the variables or made a miscalculation.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The table shows the total cost c of buying t movie tickets. Write an equation to represent the relationship between c and t. (Example 1)

Number of Tickets, t	Total Cost ($), c
1	7
2	14
3	21
4	28

$c = 7t$

2. The table shows the total number of pencils p in b boxes. Write an equation to represent the relationship between p and b. (Example 1)

Number of Boxes, b	Total Number of Pencils, p
1	12
2	24
3	36
4	48

$p = 12b$

3. The table shows the total cost of bowling any number of games and renting bowling shoes. Write a two-step equation to represent the total cost c for bowling g games. (Example 2)

Number of Games, g	Total Cost ($), c
1	6
2	10
3	14
4	18

$c = 4g + 2$

4. The table shows the total cost of renting a canoe based on the number of hours and a one-time rental fee. Write a two-step equation to represent the total cost c of renting a canoe for h hours. (Example 2)

Number of Hours, h	Total Cost ($), c
1	16
2	27
3	38
4	49

$c = 11h + 5$

Test Practice

5. **Open Response** The table shows the total cost of belonging to a fitness center based on the number of months and a one-time registration fee. Write a two-step equation to represent the total cost c for belonging to the fitness center for m months.

Number of Months, m	Total Cost ($), c
1	25
2	40
3	55
4	70

$c = 15m + 10$

Lesson 7-2 • Write Equations to Represent Relationships Represented in Tables 413

Apply *indicates multi-step problem

*6. On weekends, Peter and Aiden washed cars to raise money for a school trip. The table shows the total number of cars washed, after a certain number of hours. If the pattern continues, how many more cars will Aiden have washed than Peter after 8 hours?

Hours	Cars Washed: Peter	Cars Washed: Aiden
1	5	4
2	7	7
3	9	10
4	11	13

6 cars

Higher-Order Thinking Problems

7. **Persevere with Problems** Write an equation to represent the relationship shown in the table.

Input, x	Output, y
3	4
6	5
9	6
12	7
15	8

$y = \frac{1}{3}x + 3$

8. **Reason Abstractly** A dance studio charges $45 per month, plus a $30 registration fee. Willa has $210 for dance lessons. How many months can she take lessons? Explain how you solved.

4 months; Sample answer: $c = 45m + 30$, where c represents the total cost and m represents the number of months

Months, m	1 2 3 4
Cost, c	$75 $120 $165 $210

9. **Find the Error** A student wrote the equation $c = 20h + 12$ to represent the relationship shown in the table. Find the student's error and correct it.

Hours, h	1 2 3 4
Cost, c	$32 $44 $56 $68

Sample answer: The student switched the coefficient and the constant. The coefficient is 12 and the constant is 20. The equation should be $c = 12h + 20$.

10. Write about a real-world situation that can be represented with a two-step equation. Write the equation and explain the meaning of the variables.

Sample answer: To rent a bounce house it costs $20 plus $15 for each hour.; $c = 15h + 20$, where c represents the total cost and h represents the number of hours.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 7, students are asked to write an equation that models the relationship shown in the table without a verbal description of the relationship. Students will have to conjecture, check, and revise their equation until it fits all the values in the table.

2 Reason Abstractly and Quantitatively In Exercise 8, students will write an equation that serves as a model, and they will use this equation to generate data in order to answer the original question.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students will find the error in an equation that is supposed to represent the relationship shown in the table. In order to do this, students will write the correct equation and use it to compare to the incorrect solution.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercise 6 After completing the application problems, have students write their own real-world problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Listen and ask clarifying questions.
Use with Exercises 8–9 Have students work in pairs. Have students individually read Exercise 8 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 9.

Learn Graph a Relationship from an Equation

Objective

Students will learn how to graph a relationship given an equation by creating a table of ordered pairs.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to compare and contrast the difference between graphing ordered pairs from an equation and graphing ordered pairs from a ratio table. Students should analyze the structure of an equation and the structure of a table in order to compare the two methods.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

How is graphing ordered pairs from an equation similar to graphing ordered pairs from a ratio table? How is it different? Explain your reasoning. Sample answer: Graphing ordered pairs from an equation is similar to graphing from a ratio table because there is a relationship between the numbers. The consecutive values always increase by the same number. They are different because the ordered pairs from an equation are not always equivalent ratios.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of graphing a relationship from an equation, have them work in pairs to complete the following.

1. Study the equation presented in the Learn, $y = 2x + 500$. Explain why the ordered pair (0, 500) is a solution of this equation. Sample answer: When $x = 0$, $y = 2(0) + 500$, or 500. So, the equation $y = 2x + 500$ is true for the ordered pair (0, 500).
2. Describe the location of the ordered pair (0, 500) on the graph. Sample answer: It is the point at which the line crosses the y-axis.
3. Why do you think a break was used on the graph between the origin and the point (0, 500)? Sample answer: Without the break, the vertical axis does not increase by even increments. The break is necessary to show the jump from 0 to 500.
4. Study the table. Make a conjecture as to how you can find the next three ordered pairs in the table without referring back to the equation. Sample answer: As the x-values increase by 1, the y-values increase by 2. So, the next three ordered pairs will be (4, 508), (5, 510), and (6, 512).

Lesson 7-3

Graphs of Relationships

I Can... graph a relationship represented by an equation and write an equation represented by a graph by identifying and using the independent and dependent variables.

Learn Graph a Relationship from an Equation

You can use an equation that represents the relationship between an independent variable (input) and a dependent variable (output) to graph the relationship on the coordinate plane. The independent variable is represented by the x-coordinate and the dependent variable is represented by the y-coordinate.

Similar to graphing ratio tables, you can make a table of values to represent the equation, use the values to generate a set of ordered pairs, and graph the relationship. Consider the following equation.

$$y = 2x + 500$$

Make a table of values.

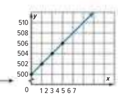
Independent Value, x	Dependent Value, y
0	500
1	502
2	504
3	506

Write the ordered pairs.

x, y
(0, 500)
(1, 502)
(2, 504)
(3, 506)

Graph the ordered pairs. Draw a line to connect the points.

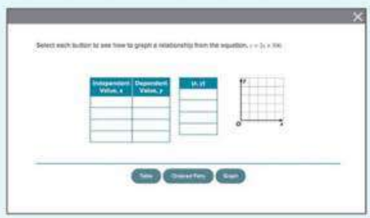
A *break* shows that the there are no values between 0 and 499.



Talk About It!
How is graphing ordered pairs from an equation similar to graphing ordered pairs from a ratio table? How is it different? Explain your reasoning.

Sample answer: Graphing ordered pairs from an equation is similar to graphing from a ratio table because there is a relationship between the numbers. The consecutive values always increase by the same number. They are different because the ordered pairs from an equation are not always equivalent ratios.

Interactive Presentation



Learn, Graph a Relationship from an Equation, Slide 1 of 2

CLICK

On Slide 1, students select the buttons to see how to graph a relationship from an equation.

Lesson 7-3

Graphs of Relationships

6.EE.C.9

LESSON GOAL

Students will write equations and graph lines to represent relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Graph a Relationship from an Equation
Example 1: Graph a Relationship from an Equation
Learn: Write an Equation from a Graph
Example 2: Write an Equation from a Graph

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

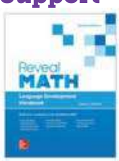
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Linear or Nonlinear Relationships		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 42 of the *Language Development Handbook* to help your students build mathematical language related to graphs of relationships between two variables.



ELL You can use the tips and suggestions on page T42 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **6.EE.C** by writing equations and graphing lines to represent relationships.
Standards for Mathematical Content: **6.EE.C.9,** *Also addresses 6.RP.A.3.A, 6.NS.C.6.C, 6.EE.B.6, 6.EE.B.7*
Standards for Mathematical Practice: **MP2, MP3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students wrote equations to represent relationships.
6.EE.C.9

Now
Students write equations and graph lines to represent relationships.
6.EE.C.9

Next
Students will use tables, equations, and graphs to represent relationships.
6.EE.C.9

Rigor

The Three Pillars of Rigor

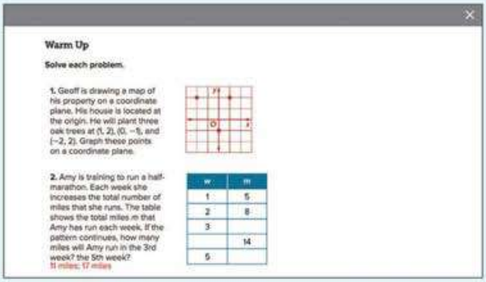
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of tables and the coordinate plane to continue to develop *understanding* of relationships between two variables. They use graphs to build *fluency* with writing equations to model relationships shown on graphs, and with representing a relationship given an equation. They *apply* their understanding of graphs of relationships to solve real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- graphing on the coordinate plane (Exercise 1)
- understanding input-output tables (Exercise 2)

Answers

1. See Warm Up slide online for answer.
2. 11 miles; 17 miles

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about times when displaying information on a graph might be better than a table.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

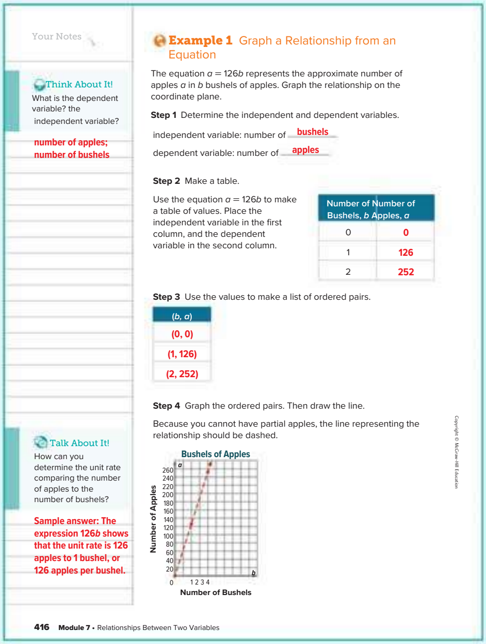
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What are some of the parts of the coordinate plane? Sample answer: origin, *x*-axis, *y*-axis
- What is an example of an ordered pair? Sample answer: (2, 8)



Your Notes

Think About It!
What is the dependent variable? the independent variable?

number of apples; number of bushels

Example 1 Graph a Relationship from an Equation

The equation $a = 126b$ represents the approximate number of apples a in b bushels of apples. Graph the relationship on the coordinate plane.

Step 1 Determine the independent and dependent variables.

independent variable: number of bushels

dependent variable: number of apples

Step 2 Make a table.

Use the equation $a = 126b$ to make a table of values. Place the independent variable in the first column, and the dependent variable in the second column.

Number of Bushels, b	Number of Apples, a
0	0
1	126
2	252

Step 3 Use the values to make a list of ordered pairs.

(b, a)
(0, 0)
(1, 126)
(2, 252)

Step 4 Graph the ordered pairs. Then draw the line.

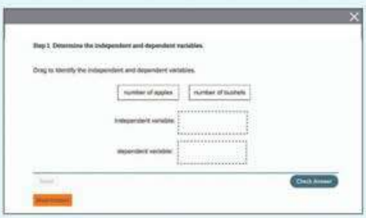
Because you cannot have partial apples, the line representing the relationship should be dashed.

Talk About It!
How can you determine the unit rate comparing the number of apples to the number of bushels?

Sample answer: The expression $126b$ shows that the unit rate is 126 apples to 1 bushel, or 126 apples per bushel.

416 Module 7 • Relationships Between Two Variables

Interactive Presentation



Example 1, Graph a Relationship from an Equation, Slide 2 of 7

DRAG & DROP

On Slide 2, students drag to identify the dependent and independent variables.

TYPE

On Slide 3, students complete the table.

Example 1 Graph a Relationship from an Equation

Objective

Students will graph a relationship given an equation by creating a table of ordered pairs.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 6, encourage them to make sense of how they can use the table to find the unit rate comparing the number of apples to the number of bushels.

5 Use Appropriate Tools Strategically Students will use the Coordinate Graphing eTool to graph the relationship.

6 Attend to Precision Students should use precise mathematical language to correctly identify the independent and dependent variables.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know what the independent variable represents? Sample answer: The number of apples depends on the number of bushels picked, so the number of bushels is the independent variable.

OL How is the independent variable shown in the equation $a = 126b$? the dependent variable? Sample answer: The independent variable is b, the number of bushels multiplied by 126. The dependent variable is the answer to that multiplication problem, the number of apples in b bushels, or a.

BL Describe a situation in which the independent variable is the number of apples. Sample answer: Andrew writes an equation to show how many bottles of apple juice, j, can be made from a apples.

SLIDE 3

AL What do the variables a and b represent? The variable a represents the number of apples and the variable b represents the number of bushels of apples.

OL How can you find the values of the dependent variables? Sample answer: To find the number of apples a, I will substitute each of the b values in the equation and simplify.

BL Why do you think the independent variable is in the first column? Sample answer: Since it is the independent variable, I use that value to substitute in the equation $a = 126b$ to find the value for the dependent variable, so it makes sense that it comes first.

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Graph a Relationship from an Equation *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL Why do we write the variables as ordered pairs? Sample answer: In order to graph the relationship, the values need to be expressed as ordered pairs.

OL Why is the ordered pair (126, 1) incorrect? The ordered pair (126, 1) is incorrect because this ordered pair is in the form (a, b) instead of (b, a).

BL Why is the ordered pair (b, a) and not (a, b)? Sample answer: Ordered pairs are written as (independent variable, dependent variable). The form (a, b) is (dependent variable, independent variable) in this example.

SLIDE 5

AL How will you graph the point (1, 126)? Sample answer: I will find 1 on the horizontal axis, then move up 126 units.

OL What can you tell about the relationship between the apples and the bushels looking at the graph? Sample answer: The graph goes up to the right; as the number of bushels increase, so does the number of apples.

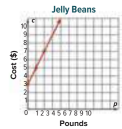
BL What is the unit rate? How can you use the graph to find the unit rate? 126 apples per bushel; Sample answer: Since the points appear to fall in a straight line, I can use the value of a when the independent variable is 1 to find the unit rate.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

The equation $c = 2p + 3$ represents the total cost c, in dollars, of ordering p pounds of personalized jelly beans from an online store with a $3 shipping fee. Graph the relationship on the coordinate plane.



Show your work here

Go Online Y ou can complete an Extra Example online.

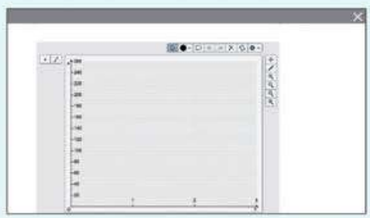
Pause and Reflect

In Example 1, the graph of the line is dashed because you *cannot* have part of an apple in a bushel. In the Check for Example 1, the line is solid because you *can* have part of a pound. Work with a partner to give an example of two real-world relationships that could be represented using a dashed line and two real-world relationships that couldbe represented using a solid line.

Record your observations here

See students' observations.

Interactive Presentation



Example 1, Graph a Relationship from an Equation, Slide 5 of 7

TYPE

On Slide 4, students complete the table to list the ordered pairs.

eTOOLS

On Slide 5, students use the Coordinate Graphing eTool to graph the relationship.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Talk About It! In this situation, why does it not make sense for the graph of the line to cross the y-axis?

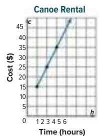
Sample answer: If the line crossed the y-axis, it would mean a rental time of 0 hours for $5. It would not make sense for someone to rent a canoe for 0 hours and pay $5.

Learn Write an Equation from a Graph

An equation can be used to symbolically describe the graph of a relationship. Use the *work backward* strategy to make a table of ordered pairs, then write the equation to represent the relationship.

Go Online Watch the animation to learn how to write an equation from the following graph.

The graph shows the relationship between the cost to rent a canoe and the number of hours the canoe is rented.



Step 1 Determine the independent and dependent variables.

The time h in hours is the independent variable, and the cost c in dollars is the dependent variable.

Step 2 Identify the ordered pairs on the graph.

The graph includes the ordered pairs (1, 15), (2, 25), and (3, 35).

Step 3 Make a table.

Time (h), h	Cost ($), c
1	15
2	25
3	35

Step 4 Write the equation.

The values of the dependent variable c increase by 10 every hour.

After multiplying by 10, you add 5 to obtain the correct value for c.

So, the equation to find the total cost c after h hours is $c = 10h + 5$.

418 Module 7 • Relationships Between Two Variables

Interactive Presentation

Learn, Write an Equation from a Graph, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that explains how to use the *work backward* strategy to make a table of ordered pairs from a graph in order to write the equation.

Learn Write an Equation from a Graph

Objective

Students will learn how to write the equation of a relationship graphed on the coordinate plane by first creating a table of ordered pairs.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the quantities to explain why it does not make sense for the rental time to be 0 hours and still have a payment.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to use an equation to represent a relationship that is displayed in a graph.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the graph is presented, and ask students to work with a partner to determine possible strategies they can use to write an equation that represents the graph. They may use any strategy they wish, but must be able to explain their strategy and defend why it works. Some students may choose to create a table of values, describe the relationship in their own words, and then write an equation. Have students share their strategies with the class. Then have them continue to watch the animation to compare their strategy with the one shown.

Talk About It!

SLIDE 2

Mathematical Discourse

In this situation, why does it not make sense for the graph of the line to cross the y-axis? Sample answer: If the line crossed the y-axis, it would mean a rental time of 0 hours for $5. It would not make sense for someone to rent a canoe for 0 hours and pay $5.

Example 2 Write an Equation from a Graph

Objective

Students will write the equation of a relationship graphed on the coordinate plane, by first creating a table of ordered pairs.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the situation to represent the relationship symbolically with a correct equation. They should carefully study the relationship between the independent and dependent variables.

6 Attend to Precision Students should use precise mathematical language to identify the independent and dependent variables.

As students discuss the *Talk About It!* question on Slide 6, they should be able to explain why it is important to identify the independent and dependent variables before writing the equation illustrated by the graph.

Questions for Mathematical Discourse

SLIDE 2

AL What information does the graph display and how is it displayed? the years of growth are on the x-axis and the heights of Martino's cactus are on the y-axis

OL Describe a different independent variable that would affect the height of the cactus. Sample answer: the amount of sun or water the plant receives

BL In which situation might the height be the independent variable and time be the dependent variable? Sample answer: If Martino constructed a graph to show how many years it takes for cactus to grow to specific heights.

SLIDE 3

AL Using the terms *independent variable* and *dependent variable* what is the general form of an ordered pair? (*independent variable, dependent variable*)

OL How could you check that the ordered pairs are correct? Sample answer: I could plot them on the coordinate plane and make sure they are the points plotted on the original graph.

BL What would the points (1.5, 43) and (2.5, 45) mean in the context of the problem? After 1.5 years, the cactus grew to a height of 43 inches, and after 2.5 years the cactus grew to a height of 45 inches.

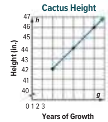
Go Online

- Find additional teaching notes, additional discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Write an Equation from a Graph

Martino constructed the graph that shows the height of his cactus after several years of growth.

Assuming the cactus grows at a constant rate, write an equation from the graph that could be used to find the height h of the cactus after g years.



Step 1 Determine the independent and dependent variables.

The graph shows the relationship between time and the height of the cactus, where time is the independent variable and the height is the dependent variable.

Step 2 Identify the ordered pairs on the graph.

The ordered pairs are (1, 42), (2, 44), and (3, 46).

Step 3 Make a table.

Years of Growth, g	Height (in.), h
1	42
2	44
3	46

Step 4 Write the equation.

The values of the dependent variable h increase by 2 each year.

After multiplying by 2, you add 40 to obtain the correct value for h.

Check each pair of values to determine if the rule $2g + 40$ accurately describes the relationship.

So, the equation to find the height of the cactus h after g years of growth is $h = 2g + 40$.

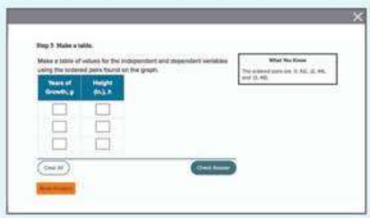
Think About It! What are the ordered pairs you can use to write the equation?

(1, 42), (2, 44), (3, 46)

Talk About It! Why is it important to identify the independent and dependent variables before writing the equation?

Sample answer: When I write an equation I need to first write an expression with the independent variable, and then set the expression equal to the dependent variable.

Interactive Presentation



Example 2, Write an Equation from a Graph, Slide 4 of 7

DRAG & DROP

On Slide 3, students drag the numbers to create the ordered pairs that are in the graph.

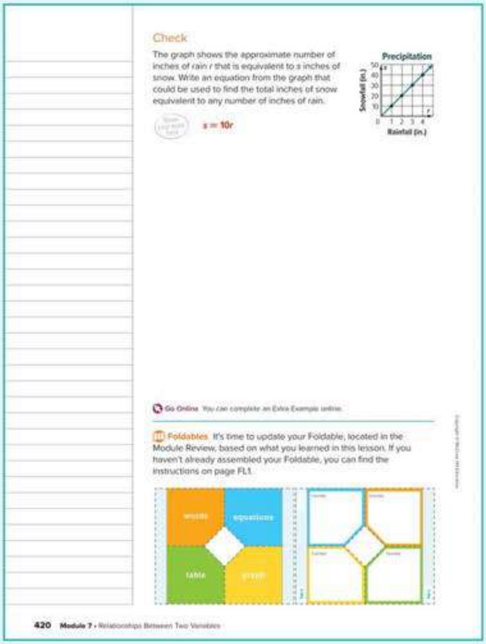
TYPE

On Slide 4, students complete a table to find the values of the dependent and independent variables on the graph.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY **3 APPLICATION**


Check

The graph shows the approximate number of inches of rain r that is equivalent to s inches of snow. Write an equation from the graph that could be used to find the total inches of snow equivalent to any number of inches of rain.

$s = 10r$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

420 Module 7 • Relationships Between Two Variables

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how to graph a relationship given an equation and how to write an equation to represent a relationship shown in a graph. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What are the ways in which a relationship between two variables can be displayed? In this lesson, students learned how to use a graph to describe the relationship between two variables. Encourage them to work with a partner to compare and contrast using tables, graphs, and equations to represent and describe the relationship. Have them state which representation they may prefer over the others. Some students may prefer the graph because it is more visual. Others may prefer the equation because it specifies the operations that relate the variables.

Exit Ticket

Refer to the Exit Ticket slide. Graph the equation for the price of the item after x years on a coordinate plane using at least three points. What is the price after 7 years? $21; See students' graphs.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1, 3, 5–9
- Extension: Linear or Nonlinear Relationships
- ALEKS Graphs and Functions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 5, 7, 8
- Extension: Linear or Nonlinear Relationships
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Ordered Pairs

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ordered Pairs

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	T opic	Exercises
2	graph a relationship given an equation	1, 2
2	write an equation to represent a relationship graphed on the coordinate plane	3, 4
3	solve application problems involving graphs of relationships	5
3	higher-order and critical thinking skills	6–9

Common Misconception

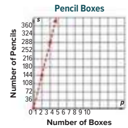
Students may struggle to graph the relationship when the equation contains a constant. For example, in Exercise 2, students may incorrectly write and graph the ordered pairs (1, 2), (2, 4), and (3, 6) instead of the ordered pairs (1, 8), (2, 10), and (3, 12) because they did not include the constant when finding the *y*-coordinate. Remind students that to find the *y*-coordinate, they should substitute a value for *b* and solve the equation, rather than just evaluating the expression $2b$.

Name ______ Period ______ Date ______

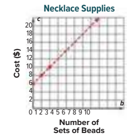
Practice

Go Online Y ou can complete your homework online.

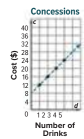
1. The equation $p = 144b$ represents the number of pencils p in b boxes. Graph the relationship on the coordinate plane. (Example 1)



2. The equation $c = 2b + 6$ represents the total cost c of b sets of beads and one necklace string. Graph the relationship on the coordinate plane. (Example 1)

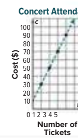


3. The graph shows the total cost c of buying one large bucket of popcorn and d large drinks. Write an equation from the graph that could be used to find the total cost c if you buy one large bucket of popcorn and d large drinks. (Example 2) $c = 4d + 8$



T est Practice

4. **Open Response** The graph shows the total cost c of buying one parking pass and t tickets to a concert. Write an equation from the graph that could be used to find the total cost c if you buy one parking pass and t tickets to a concert. (Example 2)



$c = 20t + 10$

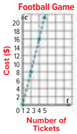
Apply *indicates multi-step problem

*5. Nancy and Elsa like to ride bikes. The equation $m = 12h$ represents the approximate number of miles m Nancy bikes in h hours. The equation $m = 9h$ represents the approximate number of miles m Elsa bikes in h hours. How much longer will it take Elsa to bike 72 miles than Nancy?

2 more hours

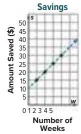
Higher-Order Thinking Problems

6. Write a real-world situation for the graph. Then write the equation that represents the situation.



Sample answer: The total cost c of t tickets to a football game; $c = 8t$

7. **Find the Error** The graph shows the total amount saved s for w weeks. A student said that the equation for the line is $s = 10w + 5$. Find the student's mistake and correct it.



Sample answer: The student switched the coefficient and constant. The correct equation is $s = 5w + 10$.

8. **Reason Inductively** Explain the difference between the graphs $y = 3x$ and $y = 3x + 2$.

Sample answer: The graph of $y = 3x$ is a straight line that passes through the origin. The graph of $y = 3x + 2$ is linear but does not pass through the origin.

9. **Make a Conjecture** What would the graph of $y = \frac{1}{2}x$ look like? Name three ordered pairs that lie on the line.

Sample answer: a straight line through the origin; (0, 0), (2, 1), (4, 2)

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 7, students explain why another student's solution is incorrect and then correct the solution.

In Exercise 8, students are given two similar equations ($y = 3x$ and $y = 3x + 2$) and are asked to reason inductively to explain the difference between their two corresponding graphs.

In Exercise 9, students will make a conjecture about what the graph of $y = \frac{1}{2}x$ looks like and will find three ordered pairs on the graph in order to test their conjecture.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercise 5 Have students work in groups of 3–4 to solve the problem in Exercise 5. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class.

Clearly explain your strategy.

Use with Exercise 8 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would determine the difference between the graphs of $y = 3x$ and $y = 3x + 2$, without actually graphing them. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Learn Multiple Representations of Relationships

Objective

Students will learn that relationships between two variables can be represented in multiple ways (words, tables, equations, and graphs).

MP Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
As students discuss the *Talk About It!* question on Slide 3, they should be able to explain why the table might be a better representation for their situation, even though all the representations are valid and display the same information between the two variables.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

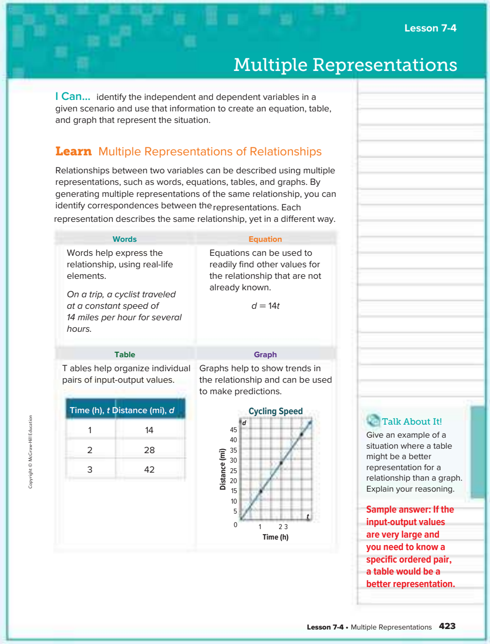
Give an example of a situation where a table might be a better representation for a relationship than a graph. Explain your reasoning
Sample answer: If the input-output values are very large and you need to know a specific ordered pair, a table would be a better representation.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of how relationships can be expressed in multiple ways, have them work in groups of 4 students to complete the following activity.

Assign each member of the group one of the four representations of relationships (words, table, equation, and graph). Have the group work together to create a real-world scenario that is represented in each of these four ways. Have them use one piece of paper for each representation. When each group has completed creating each representation, have them turn in their papers to you. Shuffle the papers so that they are in random order. Distribute one paper to each person at random. Then have the class walk around the room, each student looking for the other three remaining representations that correctly represent the relationship that matches the one they are carrying. To increase the challenge, have each student tape their given representation to their back without looking at it first, so that they do not know which relationship they have. Instead, each student must help others in the classroom find their matches.



Lesson 7-4

Multiple Representations

I Can... identify the independent and dependent variables in a given scenario and use that information to create an equation, table, and graph that represent the situation.

Learn Multiple Representations of Relationships

Relationships between two variables can be described using multiple representations, such as words, equations, tables, and graphs. By generating multiple representations of the same relationship, you can identify correspondences between the representations. Each representation describes the same relationship, yet in a different way.

Words	Equation
Words help express the relationship, using real-life elements. *On a trip, a cyclist traveled at a constant speed of 14 miles per hour for several hours.*	Equations can be used to readily find other values for the relationship that are not already known. $d = 14t$

Table	Graph
Tables help organize individual pairs of input-output values.	Graphs help to show trends in the relationship and can be used to make predictions.

Time (h), t	Distance (mi), d
1	14
2	28
3	42

Talk About It!
Give an example of a situation where a table might be a better representation for a relationship than a graph. Explain your reasoning.

Sample answer: If the input-output values are very large and you need to know a specific ordered pair, a table would be a better representation.

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Lesson 7-4 • Multiple Representations 423

Interactive Presentation

Learn, Multiple Representations of Relationships, Slide 1 of 3

FLASHCARDS

On Slides 1 and 2, students use Flashcards to learn how a relationship can be represented with words, equations, tables, and graphs.

Lesson 7-4

Multiple Representations

6.EE.C.9

LESSON GOAL

Students will use tables, equations, and graphs to represent relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Multiple Representations of Relationships

Example 1: Multiple Representations of Relationships

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 43 of the *Language Development Handbook* to help your students build mathematical language related to multiple representations of relationships between two variables.

ELL You can use the tips and suggestions on page T43 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations

Major Cluster(s): In this lesson, students address major cluster **6.EE.C** by using tables, equations, and graphs to represent relationships.

Standards for Mathematical Content: **6.EE.C.9,** *Also addresses 6.RP.A.3.A, 6.NS.C.6.C, 6.EE.B.6, 6.EE.B.7*

Standards for Mathematical Practice: **MP1, MP2, MP5**

Coherence

Vertical Alignment

Previous

Students wrote equations and graphed lines to represent relationships.
6.EE.C.9

Now

Students use tables, equations, and graphs to represent relationships.
6.EE.C.9

Next

Students will use tables and graphs to determine if a relationship between two quantities is proportional.
7.RP.A.2

Rigor

The Three Pillars of Rigor

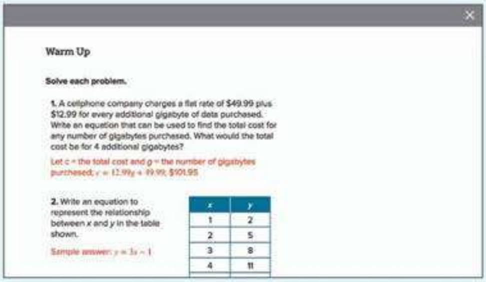
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their *understanding* of relationships between two variables through the use of multiple representations. They build *fluency* by using tables, equations, and graphs to represent relationships between two variables. They also *apply* this understanding by representing real-world relationships between variables.

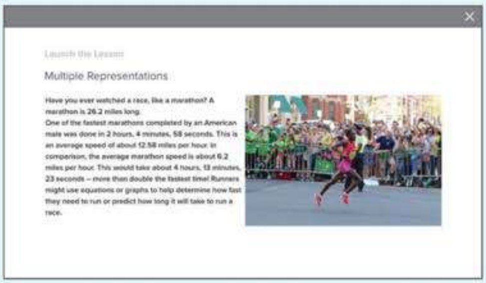
Mathematical Background

A relationship between two quantities can be expressed using words, an equation, a table, or a graph. Each representation has advantages. Words can help express the relationship in a clear way by defining each of the variables and describing how they are related. An equation is useful to finding values of the independent or dependent variable if only one value is known. A table is useful for organizing input/output pairs, and a graph is useful to viewing the trends of the relationship visually.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing algebraic expressions and equations (Exercise 1)
- finding a rule (Exercise 2)
- graphing relationships (Exercise 3)

Answers

1. Let c = the total cost and g = the number of gigabytes purchased; $c = 12.99g + 49.99$; $101.95
2. Sample answer: $y = 3x - 1$
3. See Warm Up slide online for answer.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using different representations to compare specifics about a marathon.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

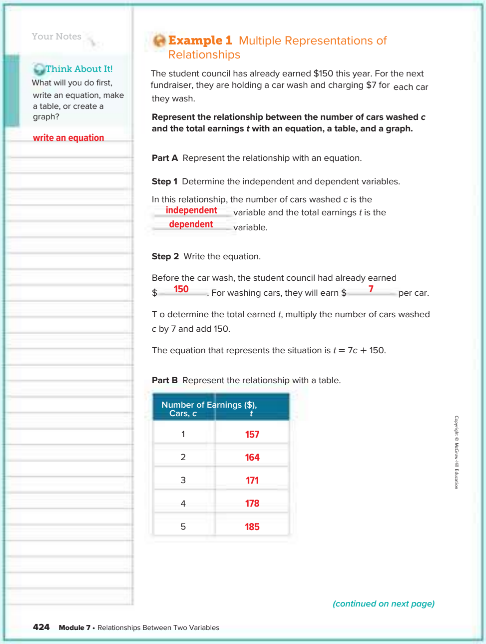
What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- When are equations useful? Sample answer: Equations can be written to represent a situation and find missing input or output values.
- What are the different ways you have learned to represent a relationship between two variables? words, tables, graphs, and equations

Aaron Roeth Photography



Your Notes

Think About It!
What will you do first, write an equation, make a table, or create a graph?

write an equation

Example 1 Multiple Representations of Relationships

The student council has already earned $150 this year. For the next fundraiser, they are holding a car wash and charging $7 for each car they wash.

Represent the relationship between the number of cars washed *c* and the total earnings *t* with an equation, a table, and a graph.

Part A Represent the relationship with an equation.

Step 1 Determine the independent and dependent variables.

In this relationship, the number of cars washed *c* is the independent variable and the total earnings *t* is the dependent variable.

Step 2 Write the equation.

Before the car wash, the student council had already earned $ 150 . For washing cars, they will earn $ 7 per car.

To determine the total earned *t*, multiply the number of cars washed *c* by 7 and add 150.

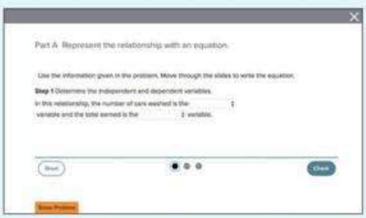
The equation that represents the situation is $t = 7c + 150$.

Part B Represent the relationship with a table.

Number of Cars, *c*	Earnings ($), *t*
1	157
2	164
3	171
4	178
5	185

(continued on next page)

Interactive Presentation



Example 1, Multiple Representations of Relationships, Slide 2 of 7

CLICK

On Slide 2, students move through the steps to write an equation that represents a real-world situation.

TYPE

On Slide 3, students complete the table to represent the situation.

Example 1 Multiple Representations of Relationships

Objective

Students will represent a real-world relationship between variables with an equation, a table, and a graph.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 6, encourage them to make sense of the different types of representations and why some may be more helpful than others, based on what information they might want to highlight in the relationship between the two variables.

Encourage students to represent the real-world situation with multiple representations – an equation, a table, and a graph.

5 Use Appropriate Tools Strategically Students will use the Coordinate Graphing eTool to graph the relationship.

Questions for Mathematical Discourse

SLIDE 2

AL What information do you know? What do the variables *c* and *t* represent? Student council has already earned $150. Students are charging $7 for each car they wash. *c* represents the number of cars they wash. *t* represents their total earnings.

OL Why is the independent variable multiplied by 7? The independent variable needs to be multiplied by $7, the charge for each car washed.

BL If there was an original donation to the student council of $20.00, how would the equation change? $t = 7c + 170$

SLIDE 3

AL How will you find the values in the right column of the table? Sample answer: To find the values in the right column of the table, I will substitute each value of *c* into the equation to find its corresponding *t* value.

OL Why is a table used? Sample answer: A table helps you organize information. It is easy to see the cost for any number of cars washed with a table of values

BL Do you think it would it be easier to create the table after you write the equation or before? Explain. Sample answer: Sometimes I need the table to be able to write the equation so I can see if there is repeated addition in the problem. In this case, the equation was easier to write first since I already knew that "cost per car" would indicate multiplication.

(continued on next page)

Example 1 Multiple Representations of Relationships *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL What is the general form of the ordered pairs for this relationship? Use the variables *c* and *t*. (*c*, *t*)

OL Why do you need to write the ordered pairs in the table? Sample answer: I need to know the ordered pairs in order to graph the relationship on a coordinate plane.

BL What will be the ordered pair when the total earned is $255? (15, 255)

SLIDE 5

AL How will you create the graph? Sample answer: I will use the ordered pairs and graph each one of the points.

OL What do you expect the graph to look like? Sample answer: I expect the points to fall in a straight line.

BL Is (8, 200) a point represented by the equation? Explain. no; Sample answer: (8, 200) is not a point represented by the equation because the total earnings for washing 8 cars will be $206.

Go Online

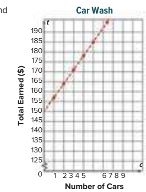
- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Part C Represent the relationship with a graph.

Step 1 Write the ordered pairs.

From the table, the ordered pairs are (1, 157), (2, 164), (3, 171), (4, 178), and (5, 185).

Step 2 Graph the ordered pairs and draw the line.



Talk About It!

Which representation would be good to use if you wanted to see a trend in the amount of money the student council was earning? Explain your reasoning.

Sample answer: a graph; I can graph the values I know, and then look for trends from the graph.

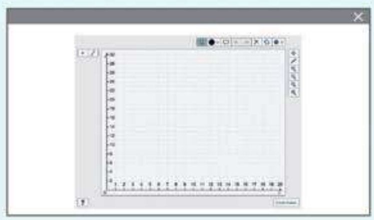
Pause and Reflect

Compare what you learned today with something similar you learned in an earlier module or grade. How are they similar? How are they different?

See students' observations.

Lesson 7-4 • Multiple Representations 425

Interactive Presentation



Example 1, Multiple Representations of Relationships, Slide 5 of 7

eTOOLS

On Slide 5, students use the Coordinate Graphing eTool to graph the relationship.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

An online stores sells trail mix for $2.75 per pound and charges a shipping fee of $4. Represent the relationship between the pounds of trail mix bought *p* and the total cost *c* with an equation, a table, and a graph.

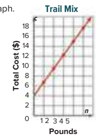
Part A Represent the relationship with an equation.

$c = 2.75p + 4$

Part B Represent the relationship with a table.

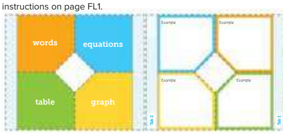
Pounds of Trail Mix, *p*	Total Cost ($), *c*
1	6.75
2	9.50
3	12.25
4	15.00
5	17.75

Part C Represent the relationship with a graph.



Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how to use words, tables, equations, and graphs to represent a relationship. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What are the ways in which a relationship between two variables can be displayed? In this lesson, students summarized all of the representations that can be used to describe the relationship between two variables (words, equations, tables, and graphs). Encourage them to work with a partner to prepare a presentation by generating a real-world relationship that exists between two variables. Have students represent that relationship in each of these four ways, and present their multiple representations to the class. During their presentations, students should clearly explain how each representation shows the same relationship.

Exit Ticket

Refer to the Exit Ticket slide. Decide which representation you would like to use to represent the relationship between the number of laps run by a runner and the total time. Create your representation. Sample answer: I would like to use a graph to represent the relationship. See students' graphs.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1, 3–8
- ALEKS Graphs and Functions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1, 2, 4, 6, 7
- Remediation: Review Resources
- Personal Tutor
- Extra Example 1
- ALEKS Ordered Pairs

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS Ordered Pairs

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	represent a real-world relationship between variables with an equation, a table, and a graph	1, 2
2	extend concepts learned in class to apply them in new contexts	3
3	solve application problems involving multiple representations	4
3	higher-order and critical thinking skills	5–8

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

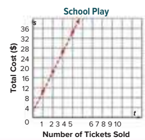
1. A school sells tickets to their school play through an online ticket company. Each ticket costs $8 and the company charges a $2.50 processing fee per order. Represent the relationship between the number of tickets bought *t* and the total cost *c* with an equation, a table, and a graph. (Example 1)

   a. Represent the relationship with an equation. $c = 8t + 2.5$

   b. Represent the relationship with a table.

Number of Tickets, *t*	Total Cost ($), *c*
1	10.50
2	18.50
3	26.50
4	34.50

   c. Represent the relationship with a graph.



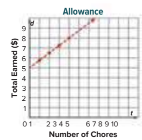
2. Carmelo earns a weekly allowance of $5 plus an additional $0.75 for each chore that he completes. Represent the relationship between the total earned *t* and the number of chores completed *c* with an equation, a table, and a graph. (Example 1)

   a. Represent the relationship with an equation. $t = 0.75c + 5$

   b. Represent the relationship with a table.

Number of Chores, *c*	Total Earned ($), *t*
1	5.75
2	6.50
3	7.25
4	8.00

   c. Represent the relationship with a graph.



Test Practice

3. **Open Response** The table shows the earnings for each pie sold at the sixth grade bake sale. Represent the relationship between the number of pies sold *p* and the total earnings *e* with an equation.

Number of Pies, *p*	Total Earnings ($), *e*
1	6
2	12
3	18

$e = 6p$

Apply *indicates multi-step problem

*4. Zari is comparing the costs of having cupcakes delivered from two different bakeries. Betty's Bakery offers free delivery and sells cupcakes by the dozen. The table shows the total cost c of d dozens from Betty's Bakery. The Sweet Shoppe charges $20 for delivery and $18 per dozen. The equation $c = 18d + 20$ represents the total cost c of d dozens of cupcakes and delivery from the Sweet Shoppe. If Zari has $110 to spend, which bakery should she use to order the greatest number of cupcakes? Explain.

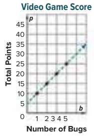
Number of Dozens of Cupcakes, d	T otal Cost ($), c
1	24
2	48
3	72

The Sweet Shoppe; Sample answer: For $110 at The Sweet Shoppe, she can get 5 dozen cupcakes with delivery because $18(5) + $20 is $110. At Betty's Bakery she does not have enough money for 5 dozen because $24(5) is $120 and $120 is greater than $110. So, she could only buy 4 dozen at Betty's Bakery.

Higher-Order Thinking Problems

5. MP **Persevere with Problems**Ryder plays a video game where each player is given points and players earn more points by catching bugs. Write an equation to represent the total number of points p earned for catching b bugs. Use the equation to find Ryder's points after catching 10 bugs.

Video Game Score



$p = 5b + 5$; 55 points

6. **Multiple Representations** Winslow earns $15.50 for each lawn that he mows.

a. Represent the relationship between the number of lawns mowed m and his total earnings e with an equation.

$e = 15.50m$

b. Represent the relationship in a table for 0, 1, 2, and 3 lawns mowed.

Lawns Mowed, m	0	1	2	3
Earnings ($), e	15.50	31.00	46.50	62.00

7. MP **Reason Abstractly** Reese and T amara both babysit. Reese earns $5 per hour and T amara earns $10 per hour. Will the amount earned for each girl ever be the same for the same number of hours after zero hours? Explain.

no; Sample answer: The graphs of the lines will never meet other than zero hours.

8. Write about a real-world situation that could be represented with an equation, a table, and a graph.

Sample answer: Karen earns $9 for every dog she walks.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 5, students will solve an application problem in which they first have to model with an equation a video game situation involving points and then use the equation to find the number of points a player receives for catching 10 bugs.

2 Reason Abstractly and Quantitatively In Exercise 7, students will use abstract reasoning by referring to a graph to see if two babysitters, each charging a different hourly rate, will ever earn the same amount of money.

Common Misconception

In Exercise 5, students may forget to include the y-intercept as part of the equation and instead write $p = 5b$. It is particularly tempting to do so on this problem because both the slope and the y-intercept are 5. Students should be allowed to discover their own error by substituting points from the graph. Each point on the graph should satisfy the equation. If students have written $p = 5b$, the should be able to see that their values of p are off by 5 from what the graph shows.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 4 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Interview a student.

Use with Exercises 6–7 Have pairs of students interview each other as they complete these problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 6 might be, "What are the independent and dependent variables in the situation?"

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include ways to display relationships between two variables, using equations, tables, and graphs. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

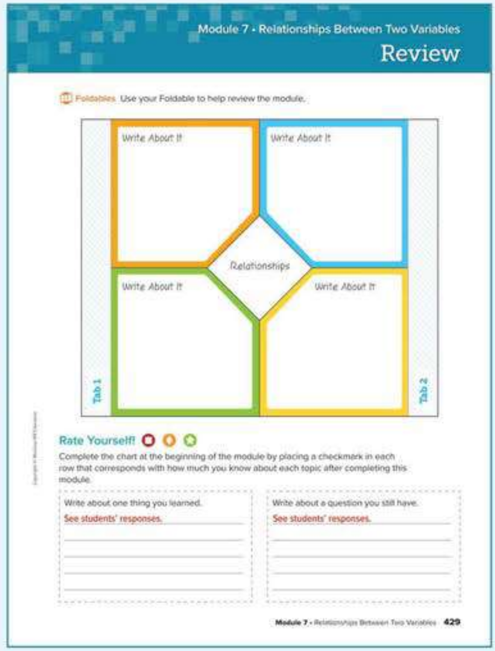
Assessment Resources

Put It All Together: Lessons 7-1 and 7-2
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Expressions and Equations**.

- Algebraic Expressions
- Equations



Module 7 • Relationships Between Two Variables

Review

Foldables Use your Foldable to help review the module.

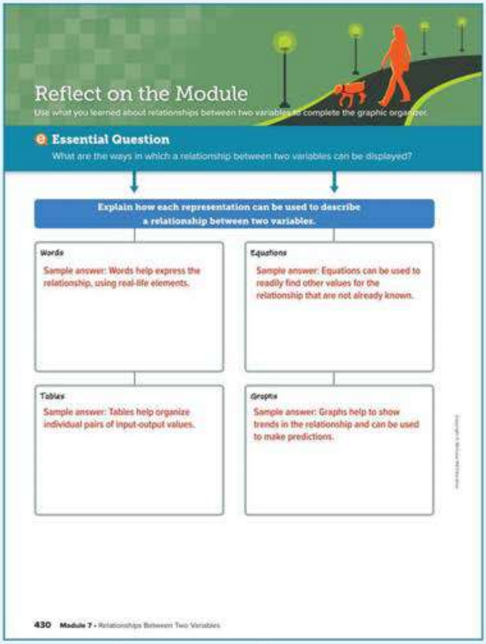
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 7 • Relationships Between Two Variables 429



Reflect on the Module

Use what you learned about relationships between two variables to complete the graphic organizer.

Essential Question

What are the ways in which a relationship between two variables can be displayed?

Explain how each representation can be used to describe a relationship between two variables.

Words Sample answer: Words help express the relationship, using real-life elements.	Equations Sample answer: Equations can be used to readily find other values for the relationship that are not already known.
Tables Sample answer: Tables help organize individual pairs of input-output values.	Graphs Sample answer: Graphs help to show trends in the relationship and can be used to make predictions.

430 **Module 7** • Relationships Between Two Variables

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

What are the ways in which a relationship between two variables can be displayed? See students' graphic organizers.

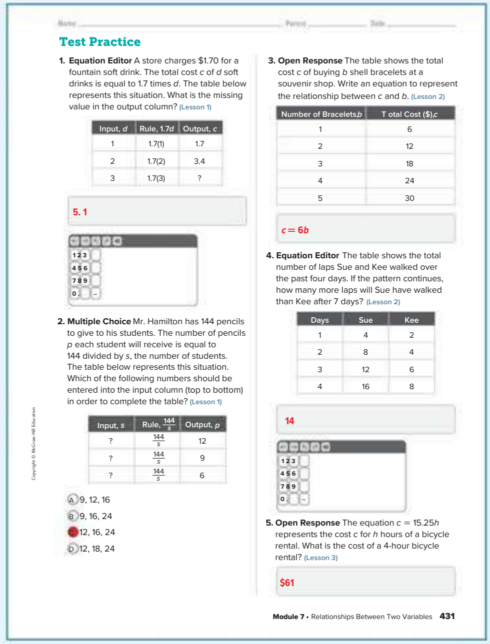
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–9 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	2
Multiselect	Multiple answers may be correct. Students must select all correct answers.	6, 9
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	1, 4
Grid	Students create a graph on an online coordinate plane.	7
Open Response	Students construct their own response in the area provided.	3, 5, 8

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.EE.C.9	7-1, 7-2, 7-3, 7-4	1–9


Name _______ Period _______ Date _______

Test Practice

1. **Equation Editor** A store charges $1.70 for a fountain soft drink. The total cost *c* of *d* soft drinks is equal to 1.7 times *d*. The table below represents this situation. What is the missing value in the output column? (Lesson 1)

Input, *d*	Rule, 1.7*d*	Output, *c*
1	1.7(1)	1.7
2	1.7(2)	3.4
3	1.7(3)	?

5.1

2. **Multiple Choice** Mr. Hamilton has 144 pencils to give to his students. The number of pencils *p* each student will receive is equal to 144 divided by *s*, the number of students. The table below represents this situation. Which of the following numbers should be entered into the input column (top to bottom) in order to complete the table? (Lesson 1)

Input, *s*	Rule, $\frac{144}{s}$	Output, *p*
?	$\frac{144}{s}$	12
?	$\frac{144}{s}$	9
?	$\frac{144}{s}$	6

A. 9, 12, 16
B. 9, 16, 24
C. 12, 16, 24
D. 12, 18, 24

3. **Open Response** The table shows the total cost *c* of buying *b* shell bracelets at a souvenir shop. Write an equation to represent the relationship between *c* and *b*. (Lesson 2)

Number of Bracelets,*b*	T otal Cost ($),*c*
1	6
2	12
3	18
4	24
5	30

$c = 6b$

4. **Equation Editor** The table shows the total number of laps Sue and Kee walked over the past four days. If the pattern continues, how many more laps will Sue have walked than Kee after 7 days? (Lesson 2)

Days	Sue	Kee
1	4	2
2	8	4
3	12	6
4	16	8

14

5. **Open Response** The equation $c = 15.25h$ represents the cost *c* for *h* hours of a bicycle rental. What is the cost of a 4-hour bicycle rental? (Lesson 3)

$61

Module 7 • Relationships Between Two Variables 431

6. Multiselect The table shows the total cost for h hours a plumber charges to make a service call to a customer. Which of the following two-step equations represents the total cost for the number of hours of service the plumber provides? Select all that apply. (Lesson 2)

Number of Hours, h	Total Cost ($), c
1	70
2	110
3	150
4	190

- [ ] $c = 30h + 40$
- [x] $c = 40h + 30$
- [ ] $30h + 40 = c$
- [ ] $20h + 50 = c$
- [x] $40h + 30 = c$

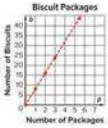
7. Grid The equation $b = 8p$ represents the number of biscuits b in p packages of biscuits. (Lesson 3)

A. Complete the table of values that represents this situation.

Number of Packages, p	Number of Biscuits, b
0	0
1	8
2	16
3	24

B. Graph the equation on the coordinate plane.

Biscuit Packages



8. Open Response The graph shows the amount of money m, in dollars, Stacey earned for h hours of work. Write an equation that could be used to find the amount of money Stacey earns for any number of hours. (Lesson 3)

Wages

$m = 12h$

9. Multiple Choice Heath is selling magazine subscriptions. He earns $10 for every subscription sold. Use s to represent the number sold and t for total earnings. (Lesson 4)

A. Which of the following equations can be used to find Heath's total earnings t given s subscriptions sold?

(A) $t = 10s$ ●

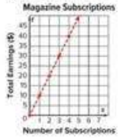
(B) $t = 10 + s$

(C) $s = 10t$

(D) $s = 10 + t$

B. Graph the ordered pairs and draw the line on the coordinate plane.

Magazine Subscriptions



IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How are the areas of triangles and rectangles used to find the areas of other polygons? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

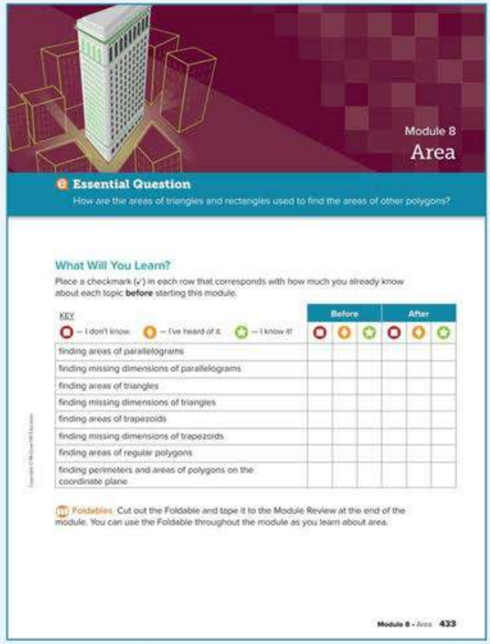
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of buildings and beehives to introduce the idea of area. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 8
Area

Essential Question
How are the areas of triangles and rectangles used to find the areas of other polygons?

What Will You Learn?
Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know	I've heard of it	I know it!	I don't know	I've heard of it	I know it!
finding areas of parallelograms						
finding missing dimensions of parallelograms						
finding areas of triangles						
finding missing dimensions of triangles						
finding areas of trapezoids						
finding missing dimensions of trapezoids						
finding areas of regular polygons						
finding perimeters and areas of polygons on the coordinate plane						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about area.

Module 8 • Area 433

Interactive Student Presentation

Module 8

Area

Module Goal

Find areas of parallelograms, triangles, trapezoids, regular polygons, and polygons on the coordinate plane.

Focus

Domain: Geometry
Major Cluster(s): 6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.
Supporting Cluster(s): 6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.
Standards for Mathematical Content:
6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
Also addresses 6.EE.A.2 and 6.EE.A.2.C.
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- find the area and perimeter of rectangles
- fluently perform all four operations with positive rational numbers
- solve one-step equations

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students classified two-dimensional figures.
5.G.B.3, 5.G.B.4

Now
Students find areas of parallelograms, triangles, trapezoids, regular polygons, and polygons on the coordinate plane.
6.G.A.1, 6.G.A.3, 6.EE.A.2, 6.EE.A.2.C

Next
Students will find volume and surface area of triangular and rectangular prisms and pyramids.
6.G.A.2, 6.G.A.4

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of polygons, basic computation, and the coordinate plane to develop *understanding* of area. They use this understanding to build *fluency* with finding the area of parallelograms, triangles, trapezoids, and regular polygons. They also build *fluency* with finding area by using coordinates of polygons on the coordinate plane. They *apply* their understanding of area to solve multi-step, real-world problems.

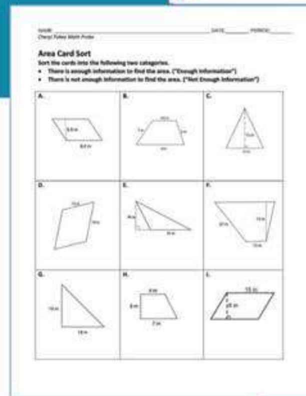
EXPLORE	LEARN	EXAMPLE & PRACTICE
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
8-1	Area of Parallelograms	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C	2	1
8-2	Area of Triangles	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C	3	1.5
8-3	Area of Trapezoids	6.G.A.1, 6.EE.A.2, 6.EE.A.2.C	2	1
8-4	Area of Regular Polygons	6.G.A.1	2	1
Put It All Together 1: Lessons 8-1, 8-2, 8-3, and 8-4			0.5	0.25
8-5	Polygons on the Coordinate Plane	6.G.A.3, *Also addresses 6.G.A.1*	3	1.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	15.5	7.75

Module Resource

Formative Assessment Math Probe
Area Card Sort



Correct Answers:
Enough Information: A, C, E, G, H, I
Not Enough Information: B, D, F

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will sort the cards into two categories determining which figures have enough information to find the area, and which do not.

Targeted Concepts Understand area of triangles and special quadrilaterals, and area of other polygons by composing rectangles and decomposing into triangles and other shapes.

Targeted Misconceptions

- Students may misinterpret the height of a polygon as the length of a diagonal side, not understanding that the height is perpendicular to the base.
- Students may lack understanding of composing and decomposing quadrilaterals.

Assign the probe after Lesson 3.

Collect and Assess Student Work

If the student selects...	Then the student likely...
Enough Information: B, D, F **Not Enough Information:** A, E	used the incorrect measurement as the height of the triangle. **Example:** For items B and D, the student uses the two given measurements to determine the area.
Enough Information: B, F **Not Enough Information:** A, E, G, H, I	does not understand how to compose and decompose the shapes into triangles and rectangles. **Example:** For item A, the student may believe they need to find the area of the triangle.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Perimeters, Areas, and Volumes
- Lesson 1, Examples 1–2
- Lesson 2, Examples 1–3
- Lesson 3, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ base
- ☐ congruent figures
- ☐ height
- ☐ parallelogram
- ☐ regular polygon
- ☐ trapezoid

Are You Ready?

Study the Quick Review to see if you are ready to start this module.

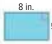
Then complete the Quick Check.

Quick Review

Example 1

Find area of rectangles.

Find the area of the rectangle.



$A = \ell w$ Area of a rectangle

$= 8 \cdot 5$ Replace ℓ with 8 and w with 5.

$= 40$ Multiply.

The area of the rectangle is 40 square inches.

Example 2

Multiply fractions by whole numbers.

Find $\frac{1}{2} \cdot 22$.

$\frac{1}{2} \cdot 22 = \frac{12}{2} \cdot \frac{2}{1}$ Write 22 as $\frac{22}{1}$.

$= \frac{1}{2} \cdot \frac{22}{1}$ (2 and 22 crossed out; 1 and 11 written in red) Divide the numerator and denominator by their GCF, 2.

$= \frac{11}{1}$ or 11 Simplify.

Quick Check

1. A garden is in the shape of a rectangle. The length of the garden is 12 feet and the width is 7 feet. What is the area of the garden?
   84 square feet

2. Find $\frac{1}{2} \cdot 34$.
   17

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define Congruent figures are figures that have the same shape and size.

Example If two right triangles have side lengths of 3 inches, 4 inches, and 5 inches, then the triangles are congruent.

Ask What do you think will be true about the perimeter and area of congruent figures? They will be the same.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- finding the perimeter and area of a rectangle
- solving one-step equations
- classifying quadrilaterals
- performing operations with fractions
- graphing on a coordinate plane

ALEKS®

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Perimeters, Areas, and Volumes** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Promote Process Over Results

The process that a student takes as he or she encounters a new problem is just as important—if not more important—than the results achieved.

How Can I Apply It?

Encourage students to consider the **Think About It!** prompts that precede many of the Examples. These prompts often ask students how they might begin to solve the problem, or have them digest the information they are given in attempts to understand what they might do next. Have students discuss their strategies with a partner and/or engage in a whole-class discussion. Be sure to support the process and reward student effort as they explore and work through problems, instead of merely rewarding the correct answer.

Teaching Notes

Before moving from the *Explore, Area of Parallelograms*, to the *Learn, Area of Parallelograms*, have students discuss the Pause and Reflect question with a partner. Encourage each student to openly talk about what they may have previously learned that might help them prepare for today's lesson. Students should discuss using a formula to find the area of a square or rectangle. They could also discuss multiplying rational numbers. Pairs should work together to determine whether or not this prior knowledge is useful in this context. If they are unable to identify any helpful prior knowledge, have them meet with other pairs of students in the class. Walk around the room, listening to the conversations and encourage students to assist each other before stepping in.

Lesson 8-1

Area of Parallelograms

I Can... understand how a parallelogram can be decomposed into a rectangle to find its area, and use the area formula for a parallelogram to find areas or missing dimensions.

What Vocabulary Will You Learn?
base
height
parallelogram

Explore Area of Parallelograms

Online Activity You will use Web Sketchpad to explore the area of a parallelogram.

Pause and Reflect

Now that you have completed the Explore activity, what are some concepts you learned in a prior grade that might help you find the area of parallelograms in this lesson?

See students' observations.

Lesson 8-1 • Area of Parallelograms 435

DIFFERENTIATE

Enrichment Activity BL

If any of your students need more of a challenge, have them use the formula $A = bh$ to find the area of the parallelograms described below.

Parallelogram A: $b = \frac{1}{2}$ ft, $h = 3$ in. 18 in^2 or $\frac{1}{8}$ ft^2

Parallelogram B: $b = 30\frac{3}{8}$ in., $h = 4\frac{1}{3}$ ft 1,579 $\frac{1}{2}$ in^2 or 10 $\frac{31}{32}$ ft^2

Parallelogram C: $b = 2.8$ m, $h = 350.4$ cm 98,112 cm^2 or 9.8112 m^2

Parallelogram D: $b = 120.9$ mm, $h = 1.5$ m 181,350 mm^2 or 0.18135 m^2

Area of Parallelograms

6.G.A.1, 6.EE.A.2.C

LESSON GOAL

Students will find and use the area of parallelograms.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Area of Parallelograms

Learn: Area of Parallelograms
Example 1: Find Area of Parallelograms
Example 2: Find Missing Dimensions of Parallelograms
Apply: Landscaping

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

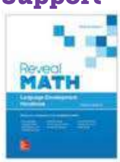
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Rectangle with Maximum Area		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 44 of the *Language Development Handbook* to help your students build mathematical language related to the area of parallelograms.



ELL You can use the tips and suggestions on page T44 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address major cluster **6.EE.A** and the supporting cluster **6.G.A** by finding and using the area of parallelograms.
Standards for Mathematical Content: **6.G.A.1, 6.EE.A.2, 6.EE.A.2.C**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students classified two-dimensional figures.
5.G.B.4

Now
Students find and use the area of parallelograms.
6.G.A.1, 6.EE.A.2, 6.EE.A.2.C

Next
Students will find and use the area of triangles.
6.G.A.1, 6.EE.A.2.C

Rigor

The Three Pillars of Rigor

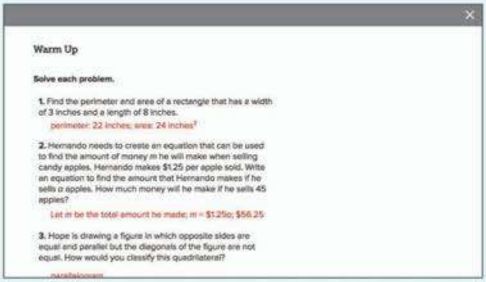
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of polygons and basic computation to develop *understanding* of area of parallelograms. They learn how to find the area of a parallelogram and build *fluency* with finding the area, and finding the missing dimension of a parallelogram when given the area. They *apply* their understanding of area of parallelograms to solve multi-step, real-world problems.

Mathematical Background

A *parallelogram* is a quadrilateral with opposite sides equal in length and parallel. To find the area of a parallelogram, multiply the lengths of the base and the height. The *height* is the perpendicular distance from the base to its opposite side, and the *base* can be any of the sides, often the bottom side.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding the perimeter and area of a rectangle (Exercise 1)
- solving one-step equations (Exercise 2)
- classifying quadrilaterals (Exercise 3)

Answers

1. perimeter: 22 inches; area: 24 inches^2
2. Let m be the total amount he made; $m = \$1.25a$; $56.25
3. parallelogram

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the area of the parallelogram-shaped glass front of the Dockland Office Building in Hamburg, Germany.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- A computer monitor sits on its base, the part that holds the monitor up and supports it. How would this help you infer where the base of a shape is? Sample answer: The base of a shape may be the bottom of the shape.
- When you go to the doctor, the nurse usually measures your *height*. How is your height measured? Sample answer: I stand against a measuring stick that forms a right angle with the platform where I am standing. The marker on the measuring stick that corresponds with the top of my head is my height.
- The word *parallel* is found within the word *parallelogram*. What does parallel mean? Sample answer: Parallel means that two coplanar lines have the same distance between them as far as they are extended. Parallel lines never touch.

Explore Area of Parallelograms

Objective

Students will use Web Sketchpad to explore the area of a parallelogram.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore the similarities between finding the area of a parallelogram and finding the area of a rectangle. Students should manipulate the parallelogram and observe how the area is affected. Encourage students to identify the similar structure between all of the parallelograms.

Inquiry Question

How is finding the area of a parallelogram like finding the area of a rectangle? How is it different? Sample answer: Finding the area of a parallelogram is like finding the area of a rectangle because both involve multiplying the base by the perpendicular distance between the bases. In a parallelogram, you multiply base and height. In a rectangle, you multiply length and width.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

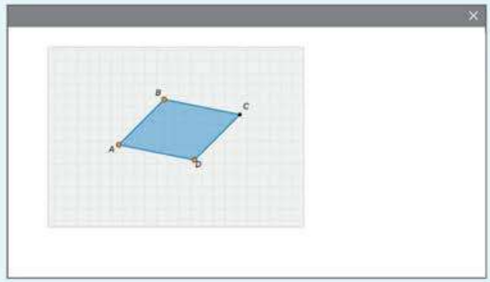
Mathematical Discourse

Share your method of creating a rectangle with your partner. Do you think a rectangle is a parallelogram? Explain your reasoning. Sample answer: I dragged point *B* directly above point *A*, and then point *D* up on the same horizontal line as point *A*. I think a rectangle is a parallelogram because the opposite sides are the same length and parallel.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8

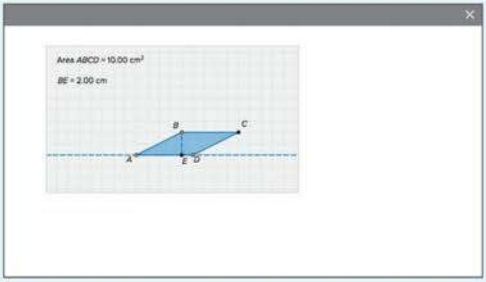


Explore, Slide 2 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the area of a parallelogram.

Interactive Presentation



Explore, Slide 5 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Area of Parallelograms *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how finding the area of a parallelogram is similar to finding the area of a rectangle. Encourage students to deepen their understanding about how the area of a parallelogram is affected when its dimensions are changed.

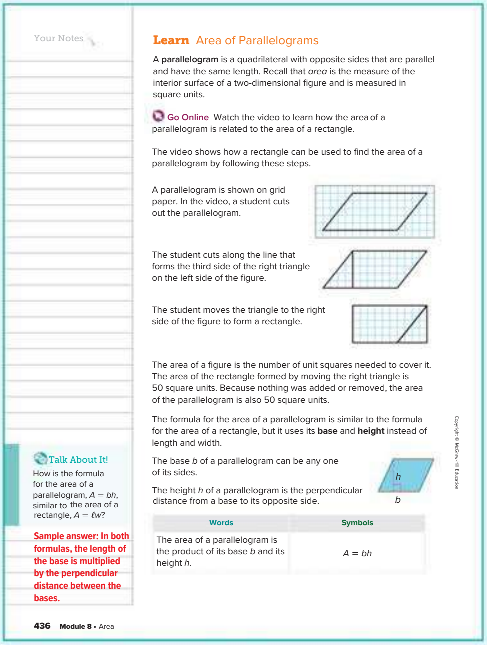
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Describe any changes in the height of the parallelogram. Sample answer: The height was 3 centimeters when I created the rectangle, and stayed 3 centimeters after I dragged point *B* horizontally.



Your Notes

Learn Area of Parallelograms

A **parallelogram** is a quadrilateral with opposite sides that are parallel and have the same length. Recall that *area* is the measure of the interior surface of a two-dimensional figure and is measured in square units.

Go Online Watch the video to learn how the area of a parallelogram is related to the area of a rectangle.

The video shows how a rectangle can be used to find the area of a parallelogram by following these steps.

A parallelogram is shown on grid paper. In the video, a student cuts out the parallelogram.

The student cuts along the line that forms the third side of the right triangle on the left side of the figure.

The student moves the triangle to the right side of the figure to form a rectangle.

The area of a figure is the number of unit squares needed to cover it. The area of the rectangle formed by moving the right triangle is 50 square units. Because nothing was added or removed, the area of the parallelogram is also 50 square units.

The formula for the area of a parallelogram is similar to the formula for the area of a rectangle, but it uses its **base** and **height** instead of length and width.

The base b of a parallelogram can be any one of its sides.

The height h of a parallelogram is the perpendicular distance from a base to its opposite side.

Words	Symbols
The area of a parallelogram is the product of its base b and its height h.	$A = bh$

Talk About It!
How is the formula for the area of a parallelogram, $A = bh$, similar to the area of a rectangle, $A = \ell w$?

Sample answer: In both formulas, the length of the base is multiplied by the perpendicular distance between the bases.

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Learn Area of Parallelograms

Objective

Students will understand how the area of a parallelogram is related to the area of a rectangle.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to analyze the structure of the parallelogram in order to explain how the formula for the area of a parallelogram is similar to the area of a rectangle.

Go Online to have your students watch the video on Slide 1. The video illustrates how to find the area of a parallelogram.

Teaching Notes

SLIDE 1

Students will learn that a *parallelogram* is a quadrilateral with opposite sides that are parallel and have the same length. Play the video to the class. Students will learn how to find the area of a parallelogram. You may wish to have students recreate the activity shown in the video that demonstrates how a parallelogram can be decomposed into a triangle and a trapezoid, and then rearranged to form a rectangle. Have students explain how the area formula of a rectangle can help them come up with the area formula of a parallelogram.

SLIDE 2

Students will learn that the formula to find the area of a parallelogram uses its *base* and *height*. Have students select each button to see how to identify these parts of a parallelogram. A common misconception is that students may think the slanted side is the height of a parallelogram. Remind them that the height of a figure represents its perpendicular distance from the base to the opposite side.

Talk About It!

SLIDE 3

Mathematical Discourse

How is the formula for the area of a parallelogram, $A = bh$, similar to the area of a rectangle, $A = \ell w$? Sample answer: In both formulas, the length of the base is multiplied by the perpendicular distance between the bases.

Interactive Presentation

Learn, Area of Parallelograms, Slide 1 of 3

WATCH

On Slide 1, students watch a video to view how the area of a parallelogram is related to the area of a rectangle.

CLICK

On Slide 2, students select *base* and *height* to view the definitions of the terms in relation to a parallelogram.

FLASHCARDS

On Slide 3, students use Flashcards to view the area formula of a parallelogram expressed in multiple representations.

Example 1 Find Area of Parallelograms

Objective

Students will find the area of parallelograms.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to be precise in their explanations for why the unit for area is square units.

7 Look for and Make Use of Structure Encourage students to study the structure of the flag in order to precisely identify the base and height of the indicated parallelogram.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we need to identify the base and the height of the parallelogram? The base and height are the measurements used in the formula for the area.

AL How will you identify the base? the height? Sample answer: Since the height in this flag is the same as one of the sides of the rectangle, the height is 30 inches. That leaves the portion of the bottom as the base of the parallelogram.

OL How do you know the base of the parallelogram is $6\frac{3}{4}$ inches? Sample answer: The base is $6\frac{3}{4}$ inches because this measurement represents the length of the bottom of the parallelogram. 30 inches is not the base because it represents the vertical height.

BL Suppose the bottom of the flag measured 18 inches. How can you find the area of the flag that is *not* part of the black parallelogram? Sample answer: I can find the area of the entire flag, or 30 • 18, then subtract the area of the parallelogram.

SLIDE 3

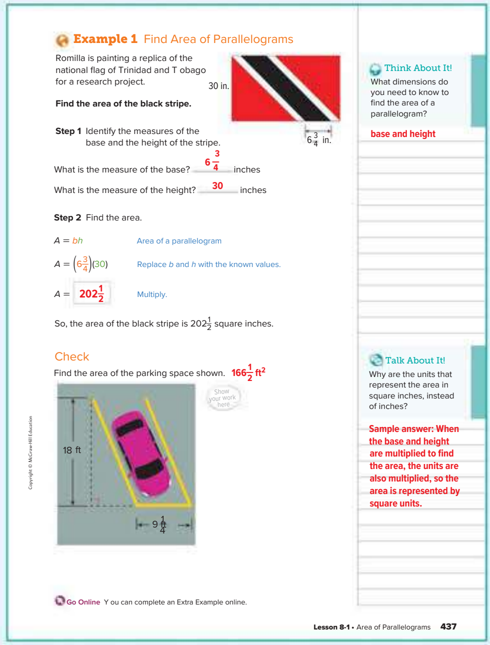
AL What do *b* and *h* represent in the area formula? *b* represents the base and *h* represents the height.

OL How can you mentally multiply $6\frac{3}{4}$ and 30? Sample answer: I can multiply 6 by 30, then find $\frac{3}{4}$ of 30, and add.

BL Suppose each measurement on the flag was doubled. By what number can you multiply $202\frac{1}{2}$ to get the new area? Be prepared to support your answer. Sample answer: The area of the flag would be multiplied by 4. If I double the height and base, I get 60 inches and $13\frac{1}{2}$ inches; $60 \cdot 13\frac{1}{2} = 810$; $4 \cdot 202\frac{1}{2} = 810$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Find Area of Parallelograms

Romilla is painting a replica of the national flag of Trinidad and T obago for a research project.

30 in.

$6\frac{3}{4}$ in.

Find the area of the black stripe.

Step 1 Identify the measures of the base and the height of the stripe.

What is the measure of the base? $6\frac{3}{4}$ inches

What is the measure of the height? 30 inches

Step 2 Find the area.

$A = bh$	Area of a parallelogram
$A = \left(6\frac{3}{4}\right)(30)$	Replace *b* and *h* with the known values.
$A = 202\frac{1}{2}$	Multiply.

So, the area of the black stripe is $202\frac{1}{2}$ square inches.

Check

Find the area of the parking space shown. $166\frac{1}{2}$ ft^2

18 ft

$9\frac{1}{4}$ ft

Show your work here

Think About It! What dimensions do you need to know to find the area of a parallelogram?

base and height

Talk About It! Why are the units that represent the area in square inches, instead of inches?

Sample answer: When the base and height are multiplied to find the area, the units are also multiplied, so the area is represented by square units.

Go Online Y ou can complete an Extra Example online.

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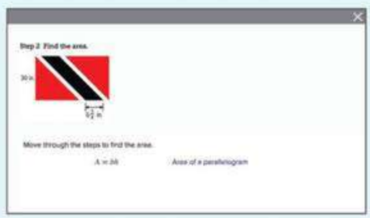
Lesson 8-1 • Area of Parallelograms 437

Interactive Presentation

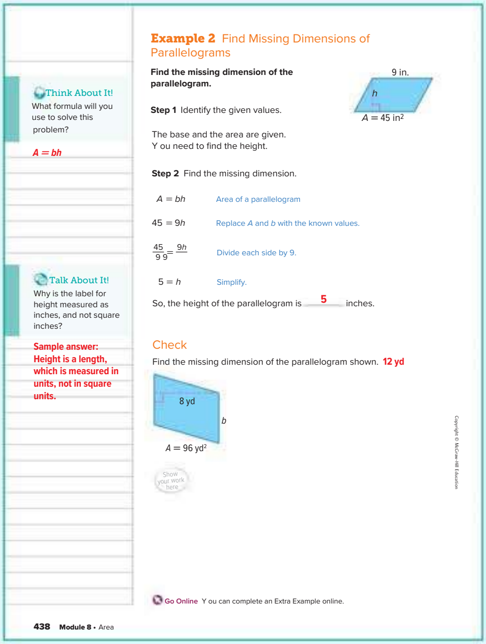


Example 1, Find Area of Parallelograms, Slide 3 of 5

TYPE On Slide 2, students enter the base and height of the black parallelogram.

CLICK On Slide 3, students move through the steps to find the area.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It! What formula will you use to solve this problem?

$A = bh$

Talk About It! Why is the label for height measured as inches, and not square inches?

Sample answer: Height is a length, which is measured in units, not in square units.

Example 2 Find Missing Dimensions of Parallelograms

Find the missing dimension of the parallelogram.

Step 1 Identify the given values.

The base and the area are given. Y ou need to find the height.

Step 2 Find the missing dimension.

$A = bh$	Area of a parallelogram
$45 = 9h$	Replace A and b with the known values.
$\frac{45}{9} = \frac{9h}{9}$	Divide each side by 9.
$5 = h$	Simplify.

So, the height of the parallelogram is **5** inches.

Check

Find the missing dimension of the parallelogram shown. **12 yd**

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Go Online Y ou can complete an Extra Example online.

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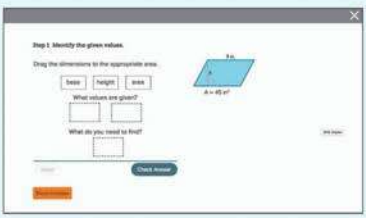
Interactive Presentation



Example 2, Find Missing Dimensions of Parallelograms, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag the dimensions to the appropriate area.

CLICK

On Slide 3, students move through the steps to find the measure of the missing dimension.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find Missing Dimensions of Parallelograms

Objective

Students will find the missing dimension of a parallelogram when given the area.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that the missing dimension is the height? Sample answer: I knew the height was missing because where the label for the measurement should be, the variable h is used.

OL Why is it helpful to identify the information before you begin solving for the missing dimension? Sample answer: Since I am using a formula, I need to identify what to substitute for each variable in the formula. After I substitute, I can solve for the missing variable.

BL In this figure, do you think the height of the parallelogram is the same as the length of the shorter side? Explain. no; Sample answer: Unless the parallelogram is a rectangle, the height is not the same as the length of the shorter side.

SLIDE 3

AL Why is each side of the equation divided by 9? Sample answer: In order to solve for the height, I need to divide each side of the equation by 9 to get the variable by itself.

OL Would you expect the unknown slanted side length in the parallelogram to be greater than, less than, or equal to 5 inches? Explain. Sample answer: Since the height of the parallelogram is inside the figure, I would expect the unknown side length to be greater than the height. Since the height is 5 inches, I think the unknown slanted side length will be greater than 5 inches.

BL Can you find the perimeter of the parallelogram with the information you have? Explain. no; Sample answer: I know the length of two sides, and I know the height of the parallelogram, but I don't know the lengths of the other set of sides. You cannot find the perimeter with the height unless the parallelogram is a rectangle.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Landscaping

Objective

Students will come up with their own strategy to solve an application problem involving landscaping a city park.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

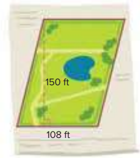
- What mathematical term is related to the word "cover"?
- How do you find the area of a parallelogram?
- What operation will you need to perform so that the pond is not included in the area?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Landscaping

Andy, a city horticulturalist, is developing a new park over an old city lot. The center of the park features a koi pond that will cover 1,245 square feet. The remaining space will need to be covered with grass seed. If a 50-pound bag of grass seed covers up to 7,500 square feet, how many bags of grass seed will Andy need to buy to seed the rest of the park?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

2 bags; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Why is the final answer given as a whole number, when the quotient was a decimal?

Sample answer: You cannot purchase a partial bag of grass seed, so you must round the decimal up to the next whole number.

Interactive Presentation

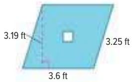
Apply, Landscaping

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

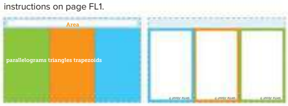
Margie is designing a collage that will be shaped like a parallelogram as shown. The center of the collage will be a square photo that has an area of 0.25 square foot. This will be surrounded by painted, square tiles that each have an area of 0.0625 square foot. How many whole tiles does Margie need to cover the collage? **180 tiles**



Show your work here

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add the formula that is used to find the area of the parallelogram. Then give an example of how to use that formula to find the area of a parallelogram. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are the areas of triangles and rectangles used to find the areas of other polygons? In this lesson, students learned how to use the area formula of a rectangle to discover the area formula of a parallelogram. Encourage them to work with a partner to prepare a brief demonstration (using grid paper or other drawings) that illustrates how the area of a rectangle can be used to find the area of a parallelogram. Have them present their demonstration to the class.

Exit Ticket

Refer to the Exit Ticket slide. The height of the building is about 25 meters and the length of the base is 86 meters. What is the approximate area of the glass front? Write a mathematical argument that can be used to defend your solution. about 2,150 m^2; Sample answer: The glass front is shaped like a parallelogram. To find the area of a parallelogram, multiple the base by the height. 25(86) = 2,150

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 9–12
- Extension: Rectangle with Maximum Area
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 7, 9, 11
- Extension: Rectangle with Maximum Area
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Area of Rectangles

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS Area of Rectangles

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the area of parallelograms	1, 2
1	find the missing dimensions of a parallelogram when given the area	3, 4
2	extend concepts learned in class to apply them in new contexts	5, 6
3	solve application problems involving area of parallelograms	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

When finding the area of a parallelogram, students may incorrectly identify the height of the parallelogram as the length of the slanted side of the parallelogram. If students need to find the area of a parallelogram where both the slanted side length and the height are given, remind students that the height of a parallelogram is the perpendicular distance between the two parallel sides.

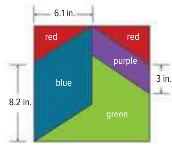
Name ______ Period ______ Date ______

Practice

Go Online Y ou can complete your homework online.

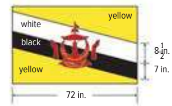
1. The pattern shows the dimensions of a quilting square that Nakida will use to make a quilt. How much blue fabric will she need to make one square? (Example 1)

$50.02\ in^2$



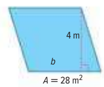
2. A group of students is painting the flag of Brunei for a geography project. Joseph is responsible for painting only the background colors of the flag. How many square inches will he cover with white paint? (Example 1)

$612\ in^2$



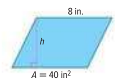
3. Find the missing dimension of the parallelogram. (Example 2)

$b = 7$ m



4. Find the missing dimension of the parallelogram. (Example 2)

$h = 5$ in.



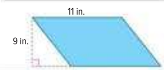
5. Find the area of the yellow striped region of the flag of the Republic of the Congo.

$13.6\ in^2$

T est Practice

6. Open Response What is the area of the parallelogram?

$99\ in^2$



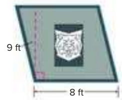
Apply *indicates multi-step problem

*7. Liam is designing a patio and fountain for his backyard. The fountain will cover 50 square feet. The remaining space will be covered with tiles. If one tile covers 2.25 square feet, how many tiles will Liam need?

41 tiles

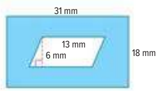
*8. T ara and Veronica are making a parallelogram-shaped banner for a football game. They will paint the entire banner except for a rectangular section where a photo of the school's mascot will be placed. The photo of the mascot has an area of 6 square feet. If a 16-ounce bottle of primer covers 24 square feet, how many bottles of paint will they need?

3 bottles



Higher-Order Thinking Problems

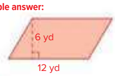
9. **MP Identify Structure** Find the area of the shaded region.



480 mm^2

10. **Create** Draw and label a parallelogram with a base that is 2 times its height and has an area that is less than 100 square yards.

Sample answer:



11. **MP Reason Abstractly** If you were to draw three different parallelograms each with a base of 5 units and a height of 4 units, how would the areas compare? Write an argument that could be used to defend your solution.

Sample answer: Because the area of a parallelogram is found by multiplying the base and height, the area of each of the three parallelograms would be 20 square units, because 5 × 4 = 20.

12. **MP Persevere with Problems** A rectangle and a parallelogram have the same area of 24 square inches. Describe the possible dimensions for each figure.

Sample answer: rectangle: $\ell = 12$ in. and $w = 2$ in.; parallelogram: $h = 4$ in. and $b = 6$ in.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 9, students find the area of the shaded region. Encourage students to use the structure of the blue and white shaded figures to find the area of the shaded region.

2 Reason Abstractly and Quantitatively In Exercise 11, students will determine how the areas compare when drawing three different parallelograms with the same base and height lengths. Encourage students to use reasoning when explaining the comparison.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 12, students describe possible dimensions for each figure. Encourage students to decide the correct pathway that can be implemented to solve the problem.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 7 Have students work together to prepare a brief demonstration that illustrates why this problem might require multiple steps to solve. For example, before they can find the number of tiles needed, they must first find the combined area of the patio and fountain. Then subtract to find the area that will be covered by tiles. Have each pair or group of students present their response to the class.

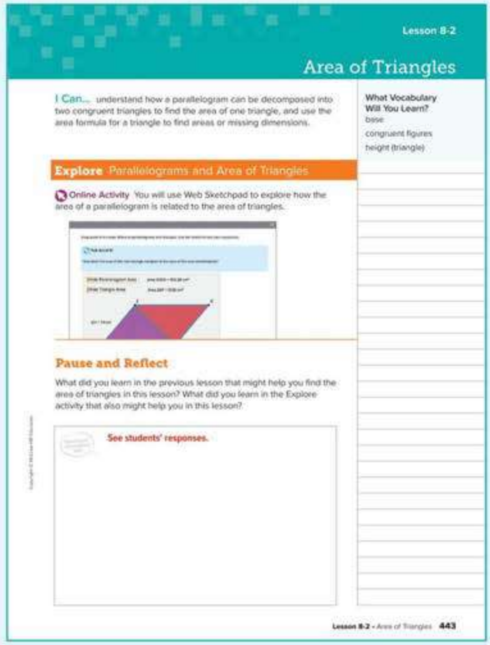
Create your own higher-order thinking problem.

Use with Exercises 9–12 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Teaching Notes

Before moving from the *Explore, Parallelograms and Area of Triangles*, to the *Learn, Area of Triangles*, have students discuss the Pause and Reflect question with a partner. Encourage each student to openly talk about what they may have previously learned that might help them prepare for today's lesson. Students should discuss using a formula to find the area of a parallelogram. They could also discuss multiplying rational numbers. Pairs should work together to determine whether or not this prior knowledge is useful in this context. If they are unable to identify any helpful prior knowledge, have them meet with other pairs of students in the class. Walk around the room, listening to the conversations and encourage students to assist each other before stepping in.



Lesson 8-2

Area of Triangles

I Can... understand how a parallelogram can be decomposed into two congruent triangles to find the area of one triangle, and use the area formula for a triangle to find areas or missing dimensions.

What Vocabulary Will You Learn?
base
congruent figures
height (triangle)

Explore Parallelograms and Area of Triangles

Online Activity You will use Web Sketchpad to explore how the area of a parallelogram is related to the area of triangles.

Pause and Reflect

What did you learn in the previous lesson that might help you find the area of triangles in this lesson? What did you learn in the Explore activity that also might help you in this lesson?

See students' responses.

Lesson 8-2 • Area of Triangles 443

DIFFERENTIATE

Language Development Activity ELL

Some students may struggle with identifying the base and height of different triangles. Have students work in pairs. Together, they should write out the definitions for base and height. If they have difficulty writing the definitions, remind them that any side of a triangle can be the base, but the height must be perpendicular to the base. After writing the definitions, have students draw pictures of several right, acute, and obtuse triangles. They should then draw a height for each triangle and label the base and the height.

Lesson 8-2

Area of Triangles

6.G.A.1, 6.EE.A.2.C

LESSON GOAL

Students will find and use the area of triangles.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Parallelograms and Area of Triangles

Learn: Area of Triangles
Example 1: Find Area of Right Triangles

Explore: Area of Triangles

Example 2: Find Area of Triangles
Example 3: Find Missing Dimensions of Triangles
Apply: Home Improvement

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Area of Kites		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 45 of the *Language Development Handbook* to help your students build mathematical language related to the area of triangles.

ELL You can use the tips and suggestions on page T45 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Geometry

Major Cluster(s): In this lesson, students address major cluster **6.EE.A** and the supporting cluster **6.G.A** by finding and using the area of triangles.

Standards for Mathematical Content: **6.G.A.1, 6.EE.A.2, 6.EE.A.2.C**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found and used the area of parallelograms.
6.G.A.1, 6.EE.A.2.C

Now
Students find and use the area of triangles.
6.G.A.1, 6.EE.A.2, 6.EE.A.2.C

Next
Students will find and use the area of trapezoids by composing and decomposing into other shapes.
6.G.A.1, 6.EE.A.2.C

Rigor

The Three Pillars of Rigor

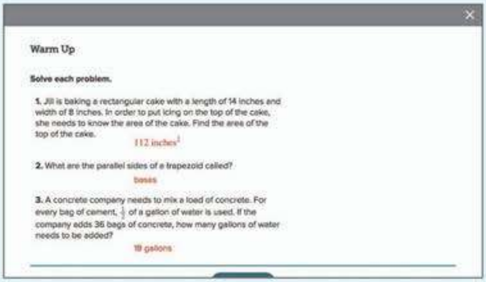
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of area as they explore area of triangles. They build *fluency* with finding the area of a triangle, and finding missing dimensions, given the area. They *apply* their understanding of area of triangles to solve multi-step, real-world problems.

Mathematical Background

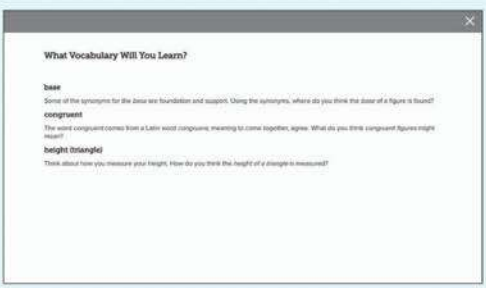
Two figures are *congruent* if they have the same shape and size. A parallelogram can be formed by two congruent triangles, and since the triangles are congruent, they have the same area. This means that the area of each of these triangles is equal to one-half the area of the parallelogram. The *base* can be any side of the triangle and the *height* is the perpendicular distance from the base to the opposite vertex.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding the area of a rectangle (Exercise 1)
- understanding quadrilaterals (Exercise 2)
- performing operations with fractions (Exercise 3)

Answers

1. 112 inches^2
2. bases
3. 18 gallons

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the interlocking triangles of the glass exterior of the Biosphere 2 complex in Tucson, Arizona.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Some of the synonyms for *base* are foundation and support. Using the synonyms, where do you think the *base* of a figure is found? Sample answer: The base of a figure is the side upon which the height rests.
- The word *congruent* comes from a Latin word *congruere*, meaning to come together, to agree. What do you think congruent figures might mean? Sample answer: If two shapes *come together, to agree*, then they may be the same figure, or have the same size and shape.
- Think about how you measure your height. How do you think the *height of a triangle* is measured? Sample answer: I think the height of a triangle is measured by the shortest distance from the base to the highest point.

Explore Parallelograms and Area of Triangles

Objective

Students will use Web Sketchpad to explore how the area of a triangle is related to the area of parallelograms.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a parallelogram using Web Sketchpad. Students will use the parallelogram and what they know about the area of a parallelogram to find the area of a triangle. Students should note how the parallelogram can be decomposed into two congruent triangles.

Inquiry Question

How can you use the area of a parallelogram to find the area of a triangle? Sample answer: To find the area of a triangle I can find the area of a parallelogram with the same base and height measurements as the triangle, and then divide by 2.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

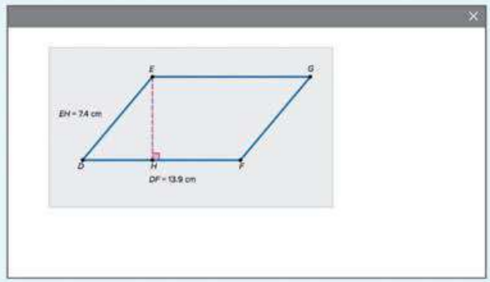
Mathematical Discourse

What line segments represent the bases of the parallelogram and what line segment represents the height? Explain how you know. Sample answer: $\overline{DF}$ or $\overline{EG}$; $\overline{EH}$, it is perpendicular to the bases.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 2 of 8

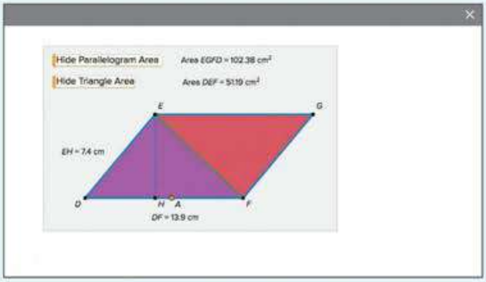
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how the area of a triangle is related to the area of parallelograms.

TYPE

On Slide 2, students enter the area of the parallelogram.

Interactive Presentation



Explore, Slide 7 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Parallelograms and Area of Triangles *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how they can use the area of a parallelogram to find the area of a triangle.

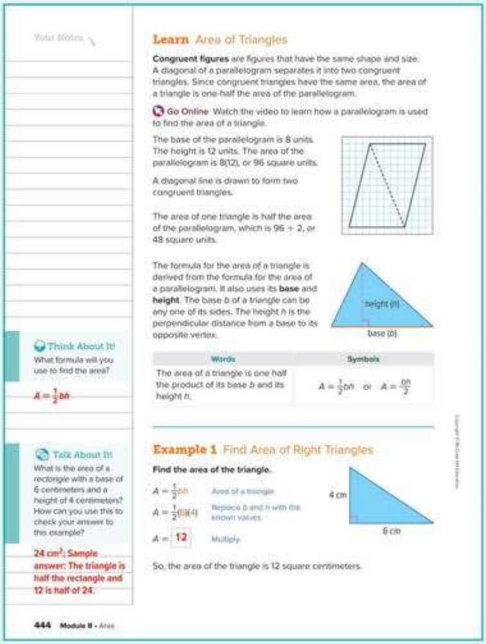
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

How does the area of the new triangle compare to the area of the new parallelogram? Sample answer: The area of the triangle is half the area of the parallelogram.



Your Notes

Learn Area of Triangles

Congruent figures are figures that have the same shape and size. A diagonal of a parallelogram separates it into two congruent triangles. Since congruent triangles have the same area, the area of a triangle is one-half the area of the parallelogram.

Go Online Watch the video to learn how a parallelogram is used to find the area of a triangle.

The base of the parallelogram is 8 units. The height is 12 units. The area of the parallelogram is 8(12), or 96 square units.

A diagonal line is drawn to form two congruent triangles.

The area of one triangle is half the area of the parallelogram, which is 96 ÷ 2, or 48 square units.

The formula for the area of a triangle is derived from the formula for the area of a parallelogram. It also uses its **base** and **height**. The base b of a triangle can be any one of its sides. The height h is the perpendicular distance from a base to its opposite vertex.

height (h)
base (b)

Words	Symbols
The area of a triangle is one half the product of its base b and its height h.	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Think About It! What formula will you use to find the area? $A = \frac{1}{2}bh$

Example 1 Find Area of Right Triangles

Find the area of the triangle.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(6)(4)$ Replace b and h with the known values.

$A = 12$ Multiply.

4 cm
6 cm

So, the area of the triangle is 12 square centimeters.

Talk About It! What is the area of a rectangle with a base of 6 centimeters and a height of 4 centimeters? How can you use this to check your answer to this example? $24\ cm^2$; Sample answer: The triangle is half the rectangle and 12 is half of 24.

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Learn Area of Triangles

Objective

Students will understand how they can use the area formula for a parallelogram to find the area formula for a triangle.

Teaching Notes

SLIDE 2

Students will learn that the formula used to find the area of a triangle uses the triangle's base and height. Students should select each button to see how to identify those parts of a triangle. As with parallelograms, point out to students that the base and height must be perpendicular to each other. You may wish to ask students if they can think of a triangle in which the height is one of the triangle's sides (a right triangle).

Go Online

- Find additional teaching notes.
- Have your students watch the video on Slide 1. The video illustrates how to find the area of a triangle.

Example 1 Find Area of Right Triangles

Objective

Students will find the area of a right triangle.

Questions for Mathematical Discourse

SLIDE 2

AL What formula will you use to find the area? $A = \frac{1}{2}bh$

AL How do you know what values to substitute for b and h? Sample answer: I know the height is perpendicular to the base, so I look for a segment with the right angle symbol. One of the sides of the right angle is the height, the other one is the base.

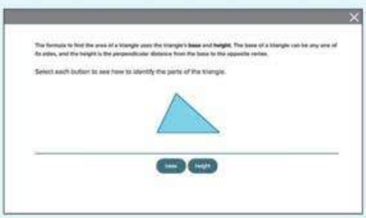
OL In this triangle, does it matter if you substitute 6 in for b or h? Explain. It does not matter; Sample answer: Since the two sides are perpendicular, either one could be the base with the other one being the height.

BL How is the area of the triangle affected if you double the height and the base? Explain your reasoning. The area is multiplied by 4; Sample answer: When the base and height are doubled, the area is $\frac{1}{2} \cdot 12 \cdot 8$ or 48. 48 is 4 times the current area of 12.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Interactive Presentation



Learn, Area of Triangles, Slide 2 of 3

WATCH

On Slide 1 of the Learn, students watch a video to view how a parallelogram can be used to find the area of a triangle.

FLASHCARDS

On Slide 3, students use Flashcards to view the area formula of a triangle expressed in multiple representations.

CLICK

On Slide 2 of Example 1, students move through the steps to find the area of the triangle.

Example 2 Find Area of Triangles

Objective

Students will find the area of a triangle.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to precisely identify the correct base and height and to generate the correct equation that represents the area of the triangle. They should be able to calculate the area accurately and efficiently.

Questions for Mathematical Discourse

SLIDE 2

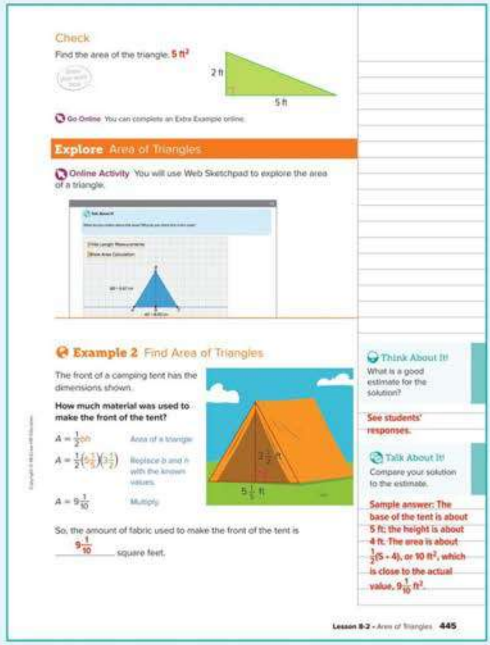
AL How do you know you need to find the area of the front of the tent and not a different measure, like the perimeter? Sample answer: I am asked to find the amount of fabric for the front of the tent, which is the inside of a two-dimensional figure. I am not asked to find the distance around the front of the tent.

OL The lengths are given as mixed numbers. Will you obtain the correct area regardless of whether you use mixed number or decimals in your calculation? Explain. yes; Sample answer: If I am precise in my conversions and calculations, then I will obtain the correct area regardless of whether I use mixed numbers or decimals.

BL If it takes 2 minutes to spray every one-half square foot with a water repellant substance, how long will it take to spray the front of the tent? 36.4 minutes

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

Find the area of the triangle. $5\ ft^2$

Go Online You can complete an Extra Example online.

Explore Area of Triangles

Online Activity You will use Web Sketchpad to explore the area of a triangle.

Example 2 Find Area of Triangles

The front of a camping tent has the dimensions shown.

How much material was used to make the front of the tent?

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}\left(5\frac{1}{5}\right)\left(3\frac{1}{2}\right)$ Replace b and h with the known values.

$A = 9\frac{1}{10}$ Multiply.

So, the amount of fabric used to make the front of the tent is $9\frac{1}{10}$ square feet.

Think About It! What is a good estimate for the solution?

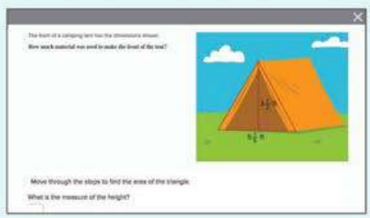
See students' responses.

Talk About It! Compare your solution to the estimate.

Sample answer: The base of the tent is about 5 ft; the height is about 4 ft. The area is about $\frac{1}{2}(5 \cdot 4)$, or $10\ ft^2$, which is close to the actual value, $9\frac{1}{10}\ ft^2$.

Lesson 8-2 • Area of Triangles 445

Interactive Presentation



Example 2, Find Area of Triangles, Slide 2 of 4

TYPE

On Slide 2 of Example 2, students enter the values of *b* and *h*.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Explore Area of Triangles

Objective

Students will use Web Sketchpad to explore the area of a triangle.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to investigate the area of triangles and if it changes depending on the location of the height of the triangle. Students will investigate how the length, base, and position of the triangle affects the area.

Inquiry Question

How does the location of the height of a triangle change the area when the base and height remain the same? Sample answer: The location of the height of the triangle has no effect on the area of the triangle if the base and height are the same.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

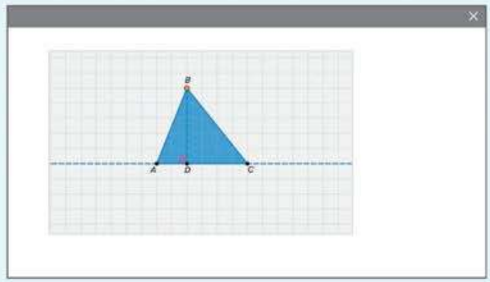
Mathematical Discourse

What do you notice about the location of $\overline{BD}$ as the triangle changes? Sample answer: There are times $\overline{BD}$ is inside the triangle, there are times it is outside the triangle, and there are times it is a side of the triangle.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 2 of 6

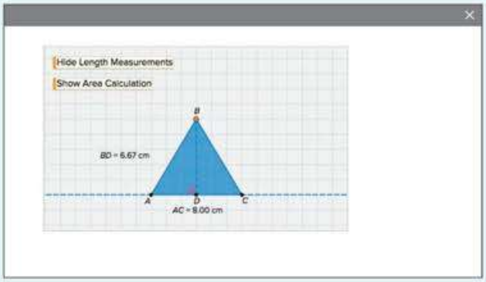
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the area of a triangle.

TYPE

On Slide 3, students make a conjecture about how a triangle's classification relates to the location of its height.

Interactive Presentation



Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Area of Triangles *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and deepen their understanding of the area of the triangle while they change the location of the height of the triangle.

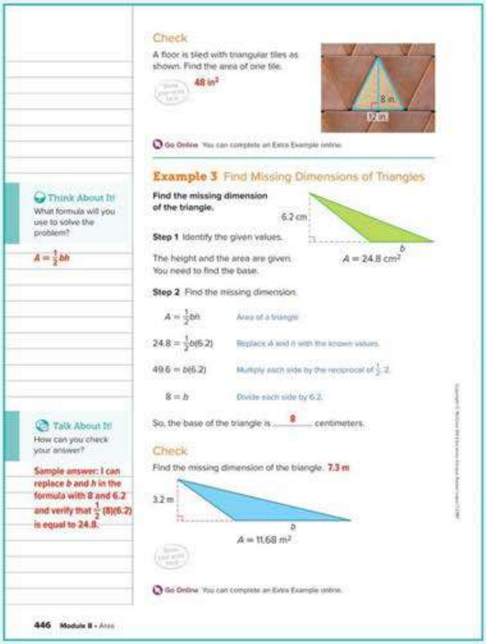
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What do you notice about the area? Why do you think this is the case? Sample answer: The area of the triangle does not change when the height is moved. If the height and base measures are unchanged, then the area will remain the same no matter how the triangle is classified.



Check

A floor is tiled with triangular tiles as shown. Find the area of one tile. 48 in^2

Go Online You can complete an Extra Example online.

Example 3 Find Missing Dimensions of Triangles

Find the missing dimension of the triangle.

Step 1 Identify the given values.

The height and the area are given. You need to find the base.

Step 2 Find the missing dimension.

$A = \frac{1}{2}bh$ Area of a triangle

$24.8 = \frac{1}{2}b(6.2)$ Replace A and h with the known values.

$49.6 = b(6.2)$ Multiply each side by the reciprocal of $\frac{1}{2}$, 2.

$8 = b$ Divide each side by 6.2.

So, the base of the triangle is 8 centimeters.

Check

Find the missing dimension of the triangle. 7.3 m

Go Online You can complete an Extra Example online.

Think About It! What formula will you use to solve the problem? $A = \frac{1}{2}bh$

Talk About It! How can you check your answer? Sample answer: I can replace b and h in the formula with 8 and 6.2 and verify that $\frac{1}{2}(8)(6.2)$ is equal to 24.8.

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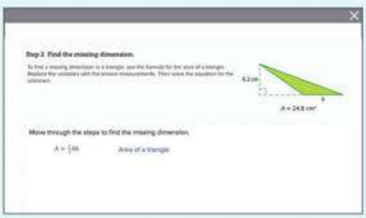
Interactive Presentation



Example 3, Find Missing Dimensions of Triangles, Slide 3 of 5

TYPE

On Slide 3, students enter the values of *A* and *h*.

CLICK

On Slide 3, students move through the steps to find the missing dimension.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Find Missing Dimensions of Triangles

Objective

Students will find the missing dimension of a triangle when given the area.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage them to choose a method they can use to check their answer. They should be able to explain why they chose that method and how it verifies the answer.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the figure in order to identify its known and unknown measures.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the measures that are known and unknown? Sample answer: I am given the area and the height. I need to find the base.

OL Estimate the length of the base. Sample answer: The area is about 24 cm^2, and the height is about 6 cm. So, the base is about 8 cm.

BL If the height doubles, but the base remains the same, what happens to the area of the triangle? Explain. it doubles; Sample answer: In the formula $A = \frac{1}{2}bh$, h is replaced with $2h$, which doubles the total area.

SLIDE 3

AL In the third step, why is each side of the equation multiplied by 2? Sample answer: Multiplying by 2 eliminates the fraction since the denominator is 2.

OL In the third step, instead of multiplying each side by 2, explain how you could simplify the equation in another way. How would that change the rest of the steps? Would the answer change? Sample answer: On the right side of the equation, I can multiply $\frac{1}{2}$ by 6.2 to obtain $3.1b$ remaining on the right side. Then I can divide each side of the equation by 3.1. The answer is the same.

BL Without using the area formula, explain how you can find the area of a parallelogram with the same base and height? Sample answer: Since the area of a triangle is half the area of a corresponding parallelogram, I just need to multiply the area, 24.8, by 2 to find the area of the parallelogram.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Home Improvement

Objective

Students will come up with their own strategy to solve an application problem involving the cost of painting a cabin.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What do you notice about the shape of the cabin?
- How can you find the area of the cabin, without including the windows?
- What operation(s) will help you find the total cost?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Home Improvement

Blossom is painting the outlined section of the cabin shown. A gallon of paint costs $24.95 and covers 250 square feet. If the total area of the windows is 0.75 square feet, how much money will Blossom spend on paint?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$49.90; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

What is the area of a triangle with a base of 35 feet and a height of 25 feet? How can you use this to check your answer to this application problem?

437.5 square feet; Sample answer: If I round the dimensions to find the area of the triangle, I can use it to compare to my answer, to make sure my answer is reasonable.

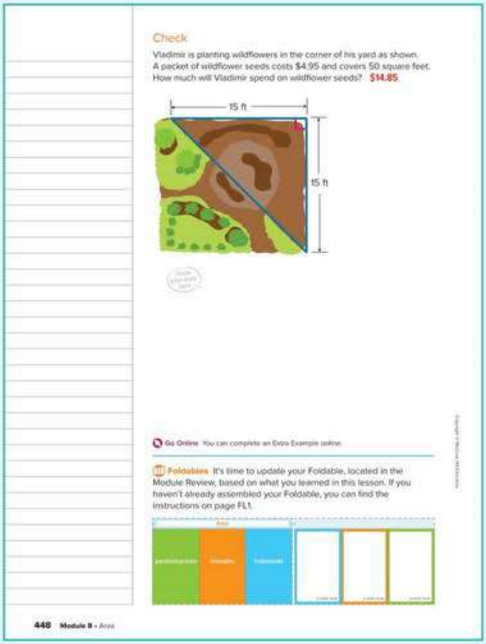
Lesson 8-2 • Area of Triangles 447

Interactive Presentation

Apply, Home Improvement

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Vladimir is planting wildflowers in the corner of his yard as shown. A packet of wildflower seeds costs $4.95 and covers 50 square feet. How much will Vladimir spend on wildflower seeds? $14.85

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

448 Module 8 • Area

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add the formula that is used to find the area of the triangle. Then give an example of how to use that formula to find the area of a triangle. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Each pair of interlocking triangles creates a parallelogram with a base of 4 feet and a height of 2 feet. What is the area of one triangle in the pair of interlocking triangles? Write a mathematical argument that can be used to defend your solution.

4 ft^2; Sample answer: The area of a triangle is one-half the area of a parallelogram formed from two congruent triangles. Since the area of the parallelogram is 4(2) or 8 square feet, the area of one triangle is $\frac{1}{2}(8)$ or 4 square feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7, 9, 11–14
- Extension: Area of Kites
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 9, 11, 13
- Extension: Area of Kites
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Area of Rectangles

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Area of Rectangles

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

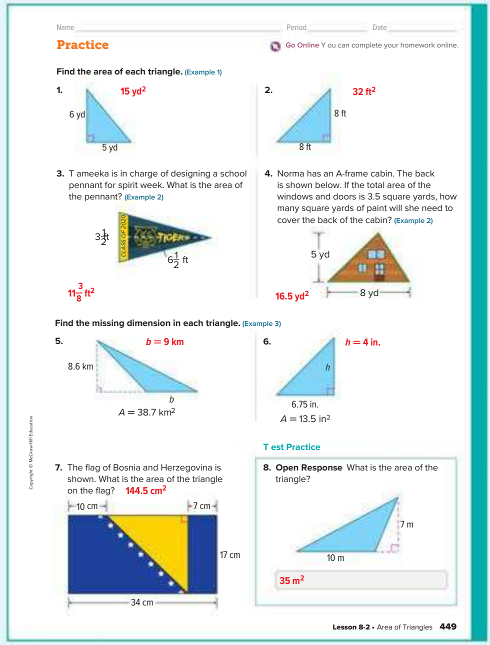
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the area of a right triangle	1, 2
1	find the area of a triangle	3, 4
1	find the missing dimension of a triangle when given the area	5, 6
2	extend concepts learned in class to apply them in new contexts	7, 8
3	solve application problems involving area of triangles	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may use the incorrect formula when finding the area of a triangle. For example, in Exercise 1, students may forget to multiply 30 square yards by $\frac{1}{2}$. Students may benefit from writing the formula for the area of a triangle at the top of their page.



Name ____ Period ____ Date ____

Practice Go Online You can complete your homework online.

Find the area of each triangle. (Example 1)

1. 6 yd, 5 yd — **15 yd²**

2. 8 ft, 8 ft — **32 ft²**

3. Tameeka is in charge of designing a school pennant for spirit week. What is the area of the pennant? (Example 2) $3\frac{1}{2}$ ft, $6\frac{1}{2}$ ft — **$11\frac{3}{8}$ ft²**

4. Norma has an A-frame cabin. The back is shown below. If the total area of the windows and doors is 3.5 square yards, how many square yards of paint will she need to cover the back of the cabin? (Example 2) 5 yd, 8 yd — **16.5 yd²**

Find the missing dimension in each triangle. (Example 3)

5. 8.6 km, b, $A = 38.7$ km² — **$b = 9$ km**

6. h, 6.75 in., $A = 13.5$ in² — **$h = 4$ in.**

7. The flag of Bosnia and Herzegovina is shown. What is the area of the triangle on the flag? **144.5 cm²** 10 cm, 7 cm, 17 cm, 34 cm

Test Practice

8. **Open Response** What is the area of the triangle? 7 m, 10 m — **35 m²**

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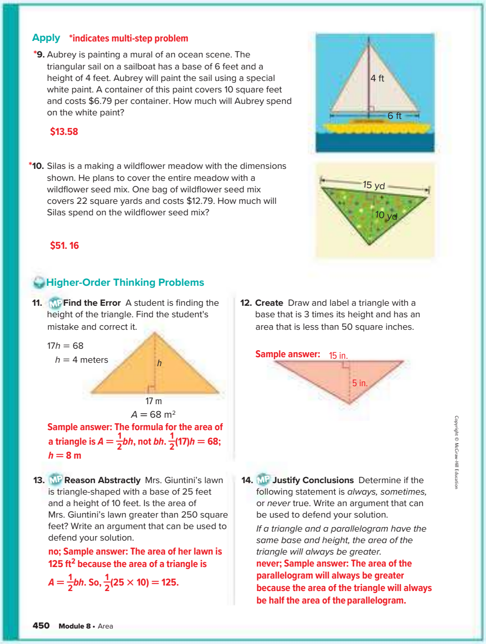
Lesson 8-2 • Area of Triangles **449**



Apply *indicates multi-step problem

*9. Aubrey is painting a mural of an ocean scene. The triangular sail on a sailboat has a base of 6 feet and a height of 4 feet. Aubrey will paint the sail using a special white paint. A container of this paint covers 10 square feet and costs \$6.79 per container. How much will Aubrey spend on the white paint?

4 ft
6 ft

\$13.58

*10. Silas is a making a wildflower meadow with the dimensions shown. He plans to cover the entire meadow with a wildflower seed mix. One bag of wildflower seed mix covers 22 square yards and costs \$12.79. How much will Silas spend on the wildflower seed mix?

15 yd
10 yd

\$51. 16

Higher-Order Thinking Problems

11. **Find the Error** A student is finding the height of the triangle. Find the student's mistake and correct it.

$17h = 68$
$h = 4$ meters

h
17 m
$A = 68 \text{ m}^2$

Sample answer: The formula for the area of a triangle is $A = \frac{1}{2}bh$, not bh. $\frac{1}{2}(17)h = 68$; $h = 8$ m

12. **Create** Draw and label a triangle with a base that is 3 times its height and has an area that is less than 50 square inches.

Sample answer: 15 in.
5 in.

13. **Reason Abstractly** Mrs. Giuntini's lawn is triangle-shaped with a base of 25 feet and a height of 10 feet. Is the area of Mrs. Giuntini's lawn greater than 250 square feet? Write an argument that can be used to defend your solution.

no; Sample answer: The area of her lawn is 125 ft^2 because the area of a triangle is $A = \frac{1}{2}bh$. So, $\frac{1}{2}(25 \times 10) = 125$.

14. **Justify Conclusions** Determine if the following statement is *always*, *sometimes*, or *never* true. Write an argument that can be used to defend your solution.

If a triangle and a parallelogram have the same base and height, the area of the triangle will always be greater.

never; Sample answer: The area of the parallelogram will always be greater because the area of the triangle will always be half the area of the parallelogram.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students find the student's mistake and correct it. Encourage students to find the error and explain how to fix it.

In Exercise 14, students will determine the validity of the statement. Encourage students to determine what makes the statement never true.

2 Reason Abstractly and Quantitatively In Exercise 13, students will determine if the area of the lawn is greater than 250 square feet. Encourage students to use reasoning to explain why the area of the lawn is less than 250 square feet.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 9–10 Have students work in pairs. Have students individually read Exercise 9 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 10.

Make sense of the problem.

Use with Exercise 11 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise used the formula $A = bh$ instead of $A = \frac{1}{2}bh$. Have each pair or group of students present their explanations to the class.

Learn Find Area of Trapezoids by Decomposing

Objective

Students will understand how to decompose a trapezoid and apply the area formulas for a rectangle and a triangle to find the area of the trapezoid.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of how decomposing a figure into familiar figures can help them understand how to find the area of the original figure.

Teaching Notes

SLIDE 1

Students will learn that a *trapezoid* is a quadrilateral with one pair of parallel sides. Students can decompose a trapezoid into triangles and rectangles, find the area of each, and then add to find the total area of the trapezoid. Have students watch the animation to learn how to find the area of a trapezoid by decomposing it into figures with which they are already familiar. You may wish to pause the animation after the dimensions of the trapezoid are given, and ask students to come up with possible strategies for finding the area of the trapezoid. They may use any strategy they wish, but must be able to explain it, and defend why it works. Ask students to share their strategies with the class.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find the area of a trapezoid by decomposing.

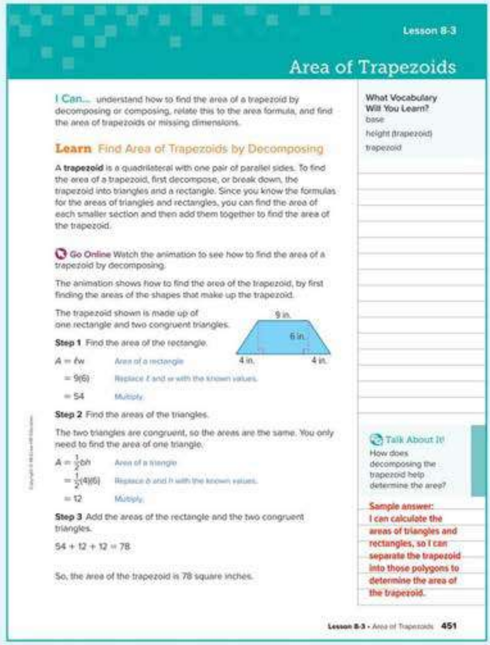
Talk About It!

SLIDE 2

Mathematical Discourse

How does decomposing the trapezoid help determine the area?

Sample answer: I can calculate the area of triangles and rectangles, so I can separate the trapezoid into those polygons to determine the area of the trapezoid.



Lesson 8-3

Area of Trapezoids

I Can... understand how to find the area of a trapezoid by decomposing or composing, relate this to the area formula, and find the area of trapezoids or missing dimensions.

Learn Find Area of Trapezoids by Decomposing

A **trapezoid** is a quadrilateral with one pair of parallel sides. To find the area of a trapezoid, first decompose, or break down, the trapezoid into triangles and a rectangle. Since you know the formulas for the areas of triangles and rectangles, you can find the area of each smaller section and then add them together to find the area of the trapezoid.

Go Online Watch the animation to see how to find the area of a trapezoid by decomposing.

The animation shows how to find the area of the trapezoid, by first finding the areas of the shapes that make up the trapezoid.

The trapezoid shown is made up of one rectangle and two congruent triangles.

Step 1 Find the area of the rectangle.

$A = \ell w$ Area of a rectangle

$= 9(6)$ Replace ℓ and w with the known values.

$= 54$ Multiply.

Step 2 Find the areas of the triangles.

The two triangles are congruent, so the areas are the same. You only need to find the area of one triangle.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(4)(6)$ Replace b and h with the known values.

$= 12$ Multiply.

Step 3 Add the areas of the rectangle and the two congruent triangles.

$54 + 12 + 12 = 78$

So, the area of the trapezoid is 78 square inches.

What Vocabulary Will You Learn?
base
height (trapezoid)
trapezoid

Talk About It!
How does decomposing the trapezoid help determine the area?

Sample answer: I can calculate the areas of triangles and rectangles, so I can separate the trapezoid into those polygons to determine the area of the trapezoid.

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Interactive Presentation

Learn, Find Area of Trapezoids by Decomposing, Slide 1 of 2

WATCH

On Slide 1, students watch the animation to learn how to find the area of a trapezoid by decomposing.

Lesson 8-3

Area of Trapezoids

6.G.A.1, 6.EE.A.2, 6.EE.A.2.C

LESSON GOAL

Students will find and use the area of trapezoids by composing and decomposing into other shapes.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Find Area of Trapezoids by Decomposing
Example 1: Find Area of Trapezoids by Decomposing
Learn: Find Area of Trapezoids by Composing
Learn: Find Area of Trapezoids by Using the Formula
Example 2: Find Area of Trapezoids
Example 3: Find Area of Right Trapezoids by Using the Formula
Example 4: Find Area of Trapezoids
Example 5: Find Missing Dimensions of Trapezoids
Apply: Budgets

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Changes in Dimensions		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 46 of the *Language Development Handbook* to help your students build mathematical language related to the area of trapezoids.

ELL You can use the tips and suggestions on page T46 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address major cluster **6.EE.A** and the supporting cluster **6.G.A** by finding and using the area of trapezoids by composing and decomposing into other shapes.
Standards for Mathematical Content: **6.G.A.1, 6.EE.A.2, 6.EE.A.2.C**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found and used the area of triangles.
6.G.A.1, 6.EE.A.2.C

Now
Students find and use the area of trapezoids by composing and decomposing into other shapes.
6.G.A.1, 6.EE.A.2, 6.EE.A.2.C

Next
Students will find the area of regular polygons by decomposing the figure into other figures.
6.G.A.1

Rigor

The Three Pillars of Rigor

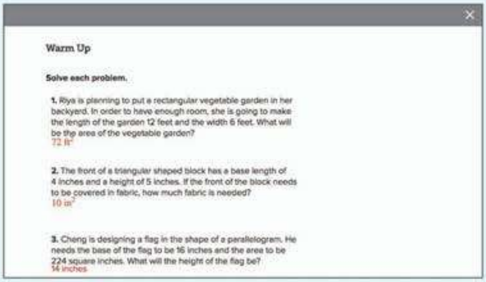
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of area as they explore area of trapezoids. They build *fluency* with finding the area, by composing and decomposing and using the formula, and finding the missing dimension of a trapezoid when given the area. They *apply* their understanding of area of trapezoids to solve multi-step, real-world problems.

Mathematical Background

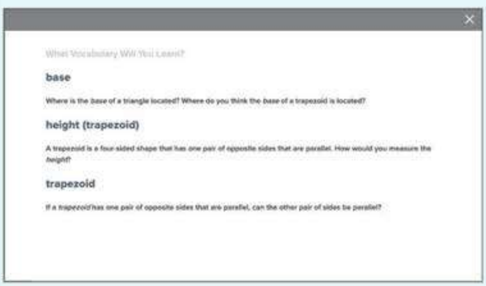
A *trapezoid* is a quadrilateral with one pair of parallel sides. A trapezoid can be decomposed into triangles and rectangles or composed with itself to form a parallelogram. Since the areas of triangles, rectangles, and parallelograms are known, either of these methods can be used to calculate the area of a trapezoid.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding the area of a rectangle (Exercise 1)
- finding the area of a triangle (Exercise 2)
- finding the area of a parallelogram, solving one-step equations (Exercise 3)

Answers

1. 72 ft^2
2. 10 in^2
3. 14 inches

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the trapezoidal walls of the Eye Bank building in Venice, Italy.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Where is the *base* of a triangle located? Where do you think the *base* of a trapezoid is located? Sample answer: The base of a triangle is the side that is perpendicular to the height of the triangle. The base of a trapezoid might be a side that is perpendicular to the height of the trapezoid.
- A trapezoid is a four-sided shape that has one pair of opposite sides that are parallel. How would you measure the *height*? Sample answer: To find the height of a trapezoid, I would measure the shortest distance between the two parallel sides.
- If a *trapezoid* has one pair of opposite sides that are parallel, can the other pair of sides be parallel? Sample answer: No, since a trapezoid has one pair of opposites sides that are parallel, the other pair of opposite sides will not be parallel. If the shape had two pairs of opposite sides that were parallel, it would be a parallelogram.

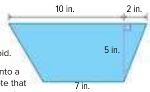
Your Notes

Think About It!
How can you divide the trapezoid into shapes with which you are familiar?

See students' responses.

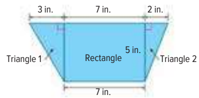
Example 1 Find Area of Trapezoids by Decomposing

Decompose the trapezoid to find its area.



Step 1 Decompose the trapezoid.

The trapezoid is decomposed into a rectangle and two triangles. Note that the two triangles are not congruent.



Step 2 Find the area of each shape.

Triangle 1	Rectangle	Triangle 2
$A = \frac{1}{2}bh$	$A = \ell w$	$A = \frac{1}{2}bh$
$= \frac{1}{2}(3)(5)$	$= 7(5)$	$= \frac{1}{2}(2)(5)$
$= 7.5$	$= 35$	$= 5$

Step 3 Find the total area.

$A = 7.5 + 35 + 5$

So, the area of the trapezoid is 47.5 square inches.

Talk About It!
Is there another way you can decompose the trapezoid? Will this result in the same area measurement? Explain your reasoning.

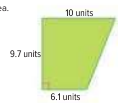
Sample answer: I could draw a line between opposite vertices, resulting in two triangles, each with a height of 5 inches and bases of 7 inches and 12 inches. The area is equal to 17.5 + 42, or 47.5 square inches.

Check

Decompose the trapezoid to find its area.

Show your work here

78.085 units^2



Go Online You can complete an Extra Example online.

Example 1 Find Area of Trapezoids by Decomposing

Objective

Students will decompose a trapezoid and apply the area formulas for a rectangle and a triangle to find the area of the trapezoid.

Questions for Mathematical Discourse

SLIDE 2

AL What does it mean to decompose a figure? Sample answer: When I decompose a figure, I divide it into shapes with which I am familiar.

AL How do you know that the base of the triangle on the left is 3 inches? Sample answer: The length of the top side of the trapezoid is 10 inches plus 2 inches. The 10-inch section is divided into two parts, the base of the triangle and the length of the rectangle. Since the length of the rectangle is 7 inches, the base of the triangle on the left must be 10 — 7, or 3 inches.

OL Why is it helpful to decompose the trapezoid into two right triangles and one rectangle? Sample answer: I was given the distance between the two parallel lines which form right angles with the two sides. Right triangles and rectangles use right angles, so I was given the measures of some of the sides of those figures.

BL Draw and label a different trapezoid into which you can decompose it into one rectangle and one triangle. What kind of trapezoid is it? See students' drawings; a right trapezoid

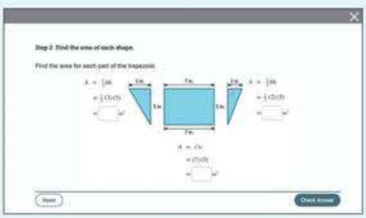
SLIDE 3

AL Why are the formulas you are using different? Sample answer: One of the formulas is for the area of a triangle, the other formula is for the area of a parallelogram.

OL Instead of finding the area of two triangles, can you find the area of one of the triangles and double it? Explain. no; Sample answer: The two triangles have the same height but different bases, so they have different areas.

BL How can you divide the trapezoid into only two triangles? Describe the triangles. Can you use the given information to find the areas of those triangles? Explain. Sample answer: I can draw a diagonal and separate the trapezoid into two triangles. One triangle has a base of 12 inches and a height of 5 inches. The other one has a base of 7 inches and a height of 5 inches. Since I know those measurements, I can find the areas of the two triangles.

Interactive Presentation



Example 1, Find Area of Trapezoids by Decomposing, Slide 3 of 6

CLICK

On Slide 2, students move through the steps to decompose the trapezoid into a rectangle and two triangles.

TYPE

On Slide 3, student determine the area of each part of the trapezoid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Find Area of Trapezoids by Composing

Objective

Students will learn how to compose two congruent trapezoids into a parallelogram to find the area of the trapezoid.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to analyze the structure of a trapezoid in order to explain how they can compose two trapezoids together to form a parallelogram.

Teaching Notes

SLIDE 1

Students will learn that two congruent trapezoids can be composed to create a parallelogram. Students already know how to find the area of a parallelogram. Have them view the video to see how to find the area of a trapezoid by composing. You may wish to have students recreate the activity shown in the video. Have them explain how they can use their understanding of parallelograms to find the area of trapezoids.

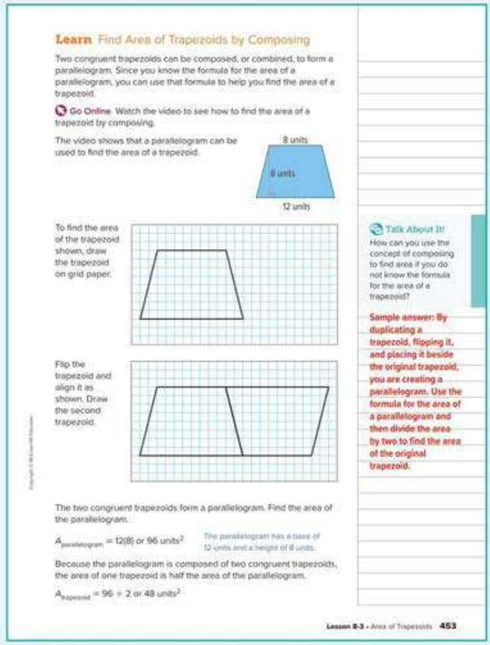
Go Online to have your students watch the video on Slide 1. The video illustrates how to find the area of a trapezoid by composing two congruent trapezoids into a parallelogram.

Talk About It!

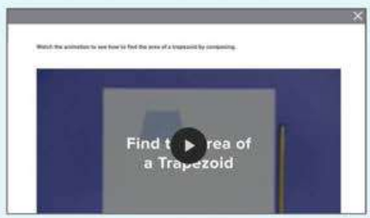
SLIDE 2

Mathematical Discourse

How can you use the concept of composing to find area if you do not know the formula for the area of a trapezoid? Sample answer: By duplicating a trapezoid, flipping it, and placing it beside the original trapezoid, you are creating a parallelogram. Use the formula for the area of a parallelogram and then divide the area by two to find the area of the original trapezoid.



Interactive Presentation



Learn, Find Area of Trapezoids by Composing, Slide 1 of 2

WATCH

On Slide 1, students watch the video to learn how to find the area of a trapezoid by composing two congruent trapezoids into a parallelogram.

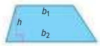
Math History Minute

Júlio César de Mello e Souza (1895–1974) was a Brazilian mathematician, professor , and writer . His writings weave mathematics into entertaining word problems and puzzles. His most famous book, *The Man Who Counted,* tells of the adventures of Beremiz Samir who uses mathematics as a superpower . In Rio de Janeiro, Brazil, his birthday, May 6, is declared as Mathematician's Day.

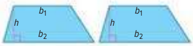
Learn Find Area of Trapezoids by Using the Formula

Go Online Watch the animation to see how the formula for the area of a trapezoid is derived by composing it into a parallelogram.

In the trapezoid shown, base one, b_1, is the shorter base and base two, b_2, is the longer base. The height, h, is the perpendicular distance between the bases.



Step 1 Make a copy of the trapezoid.

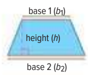


Step 2 Rotate the second trapezoid and align as shown.

The two congruent trapezoids form a parallelogram.

The height of the parallelogram is the same as the height of the trapezoid. The base of the parallelogram is the sum of b_1 and b_2 of the trapezoid. The area of one trapezoid is half the area of the parallelogram.

The formula for the area of a trapezoid is derived from the formula for the area of a parallelogram. It also uses its **base** and **height**. The bases of a trapezoid are the parallel sides, and the height is the perpendicular distance between the bases.



$A_{parallelogram} = bh$ — Write the formula.

$= (b_1 + b_2)h$ — The base of the parallelogram is $b_1 + b_2$.

$A_{trapezoid} = \frac{1}{2}(b_1 + b_2)h$ — The area of one trapezoid is half the area of the parallelogram.

$= \frac{1}{2}h(b_1 + b_2)$ — Commutative Property

Words	Symbols
The area of a trapezoid is one half the product of the height, h, and the sum of its bases, b_1 and b_2.	$A = \frac{1}{2}h(b_1 + b_2)$

Interactive Presentation

Learn, Find Area of Trapezoids by Using the Formula, Slide 2 of 3

CLICK

On Slide 1, students select *base* and *height* to view the definitions of the terms in relation to a trapezoid.

WATCH

On Slide 2, students watch the animation to learn how the formula to find the area of a trapezoid is derived.

FLASHCARDS

On Slide 3, students use the Flashcards to view the area formula of trapezoids expressed in multiple representations.

Learn Find Area of Trapezoids by Using the Formula

Objective

Students will learn the formula used to find the area of a trapezoid.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to derive the formula for the area of a trapezoid.

Teaching Notes

SLIDE 1

Students will learn about the parts of a trapezoid. Be sure they understand that the height is the perpendicular distance between the two parallel bases. In a right trapezoid, the height will be one of the sides of a trapezoid. If the trapezoid is not right, the height will not be one of the sides. You may wish to have students draw several examples of trapezoids to identify the location of the height of each.

SLIDE 2

Play the animation for the class. Students will learn how the formula for the area of the trapezoid is derived. Be sure students understand how the trapezoid and its copy form a parallelogram. Have students explain why the base of the parallelogram can be represented by the sum of the bases of the trapezoid.

SLIDE 3

Have students select the *Words* and *Symbols* flashcards to learn about how the area formula of a trapezoid can be represented in these multiple representations.

DIFFERENTIATE

Language Development Activity ELL

Now that students have learned three different methods for finding the area of a trapezoid (decomposing, composing, and using the formula), have them work with a partner to prepare a brief presentation that summarizes each method, and then compares and contrasts the methods. They should use diagrams and illustrations in their presentation. Have them present to the class. Some students may be uncomfortable speaking in front of others. Encourage them to use clear pronunciation, and speak in a volume that is appropriate for the context.

Example 2 Find Area of Trapezoids

Objective

Students will find the area of a trapezoid by composing and using the formula for the area of a trapezoid.

Questions for Mathematical Discourse

SLIDE 3

AL How can you find the length of the base of the parallelogram? I can add the two base lengths of the trapezoid; $10 + 6 = 16$.

OL Why is the area of the parallelogram not the final answer? The parallelogram is composed of two congruent trapezoids. The final answer will be the area of one trapezoid.

BL Why was the height unchanged when composing the parallelogram? The height was unchanged when composing the parallelogram because the two trapezoids were put side by side. The height was not increased nor decreased; it stayed the same.

SLIDE 4

AL Why is the area of the parallelogram divided by 2? it is composed of two trapezoids

OL Do you prefer finding the area of a trapezoid by composing or decomposing? Explain. Sample answer: composing; when composing I only need to find the area of one figure and then divide by two. When I decomposed, I found the area of 3 figures and then added.

BL How is finding the area of a trapezoid by composing related to finding the area using the formula $A = \frac{1}{2}h(b_1 + b_2)$? Sample answer: When I found the area by composing, I added the two bases, $(b_1 + b_2)$, multiplied the sum by the height, $h(b_1 + b_2)$, and then divided it by two, which is the same as multiplying by one-half. Those are the same steps that are in the formula.

SLIDE 5

AL What do the variables b_1 and b_2 represent? They represent the lengths of the two parallel sides in the trapezoid.

OL When using the formula, how do you know what to do first? The order of operations tells me I need to first find the sum of the two bases.

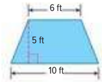
BL In the formula, why do you need to multiply by $\frac{1}{2}$? Sample answer: Since I am adding the two bases, and then multiplying by the height, it is as if I am finding the area of two figures. Multiplying by $\frac{1}{2}$ makes the answer apply to one figure.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

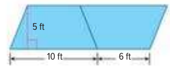
Example 2 Find Area of Trapezoids

Find the area of the trapezoid.



Method 1 Find the area by composing.

Step 1 Compose the trapezoid into a parallelogram.



Step 2 Find the area of the parallelogram.

$A = (16)(5)$ The base is $10 + 6$, or 16 feet. The height is 5 feet.

$= 80$ The area of the parallelogram is 80 square feet.

Step 3 Find the area of the trapezoid.

$80 \div 2 =$ **40** Divide the area of the parallelogram by 2. The area of the trapezoid is 40 square feet.

Method 2 Find the area using the formula.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$A = \frac{1}{2}(5)(10 + 6)$ Replace h, b_1, and b_2 with the known values.

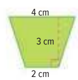
$A = \frac{1}{2}(5)(16)$ Add.

$A = 40$ Multiply.

So, using either method, the area of the trapezoid is **40** square feet.

Check

Find the area of the trapezoid. **9 cm^2**



Show your work here

Go Online Y ou can complete an Extra Example online.

Think About It! How can you find the area of the trapezoid by composing?

See students' responses.

Talk About It! Compare the two methods.

Sample answer: Composing the trapezoid into a parallelogram helps to visualize the relationship between the two polygons. Finding the area using the formula is a more efficient method.

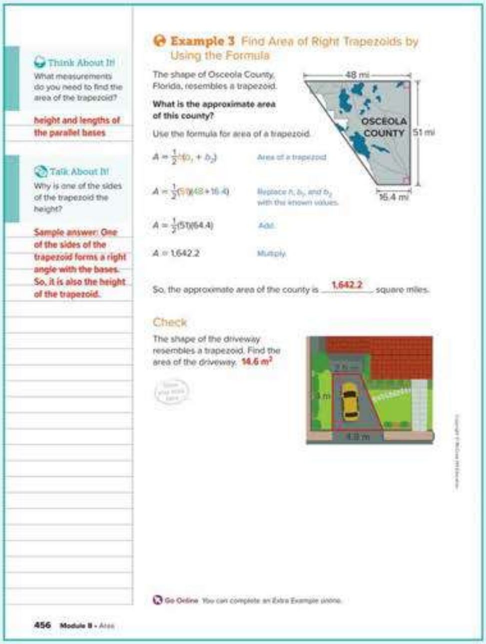
Interactive Presentation

Example 2, Find Area of Trapezoids, Slide 5 of 7

CLICK On Slide 2, students move through the steps to compose the trapezoid into a parallelogram.

TYPE On Slide 5, students determine the area of the trapezoid.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It!

What measurements do you need to find the area of the trapezoid?

height and lengths of the parallel bases

Talk About It!

Why is one of the sides of the trapezoid the height?

Sample answer: One of the sides of the trapezoid forms a right angle with the bases. So, it is also the height of the trapezoid.

Example 3 Find Area of Right Trapezoids by Using the Formula

The shape of Osceola County, Florida, resembles a trapezoid.

What is the approximate area of this county?

Use the formula for area of a trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$A = \frac{1}{2}(51)(48 + 16.4)$ Replace h, b_1, and b_2 with the known values.

$A = \frac{1}{2}(51)(64.4)$ Add.

$A = 1{,}642.2$ Multiply.

So, the approximate area of the county is 1,642.2 square miles.

Check

The shape of the driveway resembles a trapezoid. Find the area of the driveway. 14.6 m^2

Go Online You can complete an Extra Example online.

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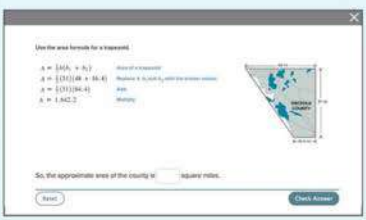
Interactive Presentation



Example 3, Find Area of Right Trapezoids by Using the Formula, Slide 2 of 4

TYPE

On Slide 2, students determine the approximate area of the county.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Find Area of Right Trapezoids by Using the Formula

Objective

Students will use an area formula to solve a real-world problem involving a right trapezoid.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 3, encourage them to pause and consider how the height of different figures may or may not be one of the sides, and to explain why the height is one of the sides in this figure.

6 Attend to Precision Encourage students to generate the correct formula for the area of a trapezoid, and to accurately find the area using the appropriate units.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you think the trapezoid is called a right trapezoid? Sample answer: One of the sides of the trapezoid forms a right angle with both bases, so it is a right trapezoid.

OL In the solution, 48 was substituted for b_1 and 16.4 was substituted for b_2. Can those values be switched? Explain your reasoning. yes; Sample answer: Since I am adding two numbers, the order in which I add them doesn't matter; 48 + 16.4 is the same as 16.4 + 48.

BL There are about 1,051,008 acres in Osceola County. How could you find the unit rate, acres per square mile? About how many acres are in a square mile? Sample answer: I can divide the number of acres, 1,051,008, by the number of square miles, 1,642.2; 640 acres per square mile.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 4 Find Area of Trapezoids

Objective

Students will use an area formula to solve a real-world problem involving a trapezoid.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to use the structure of the shape of the wing to accurately identify the two parallel bases and the height of the trapezoid.

Questions for Mathematical Discourse

SLIDE 1

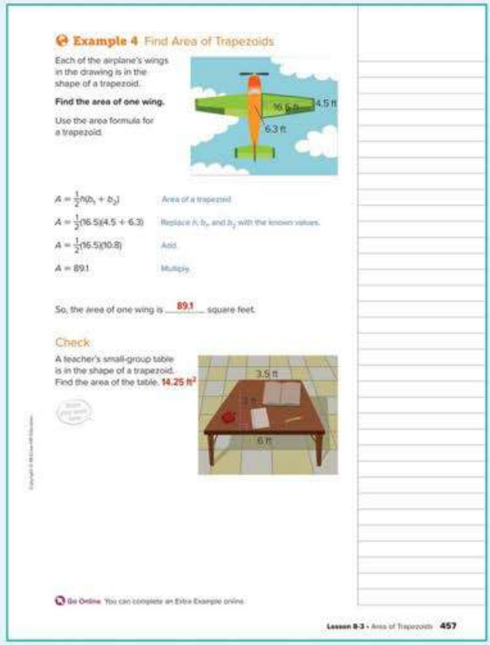
AL What are the measurements you will need to use for the formula? $h = 16.5$, $b_1 = 4.5$ and $b_2 = 6.3$

OL What is the area of both of the plane's front wings? 178.2 ft^2

BL Suppose you wanted to make a model of the plane that is $\frac{1}{50}$ of the actual size of the plane. What is the area of one of the wings of the model? Explain how you found the area. 0.03564 square foot; Sample answer: I converted each measure for the model, and then I found the area using the formula.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Find Area of Trapezoids

Each of the airplane's wings in the drawing is in the shape of a trapezoid.

Find the area of one wing.

Use the area formula for a trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$A = \frac{1}{2}(16.5)(4.5 + 6.3)$ Replace h, b_1, and b_2 with the known values.

$A = \frac{1}{2}(16.5)(10.8)$ Add.

$A = 89.1$ Multiply.

So, the area of one wing is 89.1 square feet.

Check

A teacher's small-group table is in the shape of a trapezoid. Find the area of the table. 14.25 ft^2

Go Online You can complete an Extra Example online.

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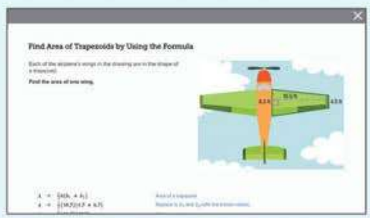
Interactive Presentation



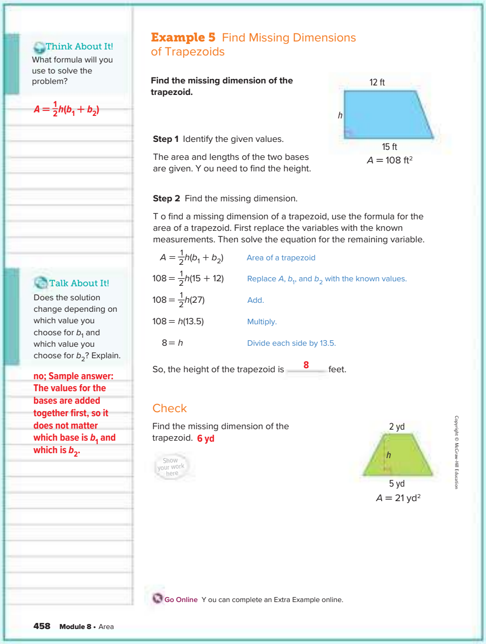
Example 4, Find Area of Trapezoids, Slide 1 of 2

TYPE

On Slide 1, students determine the area of the wing.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It!
What formula will you use to solve the problem?
$A = \frac{1}{2}h(b_1 + b_2)$

Example 5 Find Missing Dimensions of Trapezoids

Find the missing dimension of the trapezoid.

12 ft
h
15 ft
$A = 108$ ft²

Step 1 Identify the given values.

The area and lengths of the two bases are given. Y ou need to find the height.

Step 2 Find the missing dimension.

T o find a missing dimension of a trapezoid, use the formula for the area of a trapezoid. First replace the variables with the known measurements. Then solve the equation for the remaining variable.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$108 = \frac{1}{2}h(15 + 12)$ Replace A, b_1, and b_2 with the known values.

$108 = \frac{1}{2}h(27)$ Add.

$108 = h(13.5)$ Multiply.

$8 = h$ Divide each side by 13.5.

So, the height of the trapezoid is 8 feet.

Talk About It!
Does the solution change depending on which value you choose for b_1 and which value you choose for b_2? Explain.

no; Sample answer: The values for the bases are added together first, so it does not matter which base is b_1 and which is b_2.

Check

Find the missing dimension of the trapezoid. 6 yd

2 yd
h
5 yd
$A = 21$ yd²

Show your work here

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Go Online You can complete an Extra Example online.

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Example 5 Find Missing Dimensions of Trapezoids

Objective

Students will find the missing dimension of a trapezoid given its area.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the bases and how they are used in the formula to explain why the area does not change depending on which values are substituted for b_1 and b_2

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that the missing dimension is the height? Sample answer: I have numerical values for the area and two bases, so the only remaining value I need is the height.

OL Why do you need to identify what you know and what you need to find before using the area formula? Sample answer: I need to be able to identify what variables have values and what variable I need to find.

BL If you were given the area, the height, and the length of one base, how could you find the length of the second base? Sample answer: I would still use the area formula, and work backward. I could multiply the area by 2, then divide the area by the height, and subtract the given base.

SLIDE 3

AL In the fourth step, where did the value 13.5 or $13\frac{1}{2}$ come from? Sample answer: It is the result of multiplying 27 by $\frac{1}{2}$.

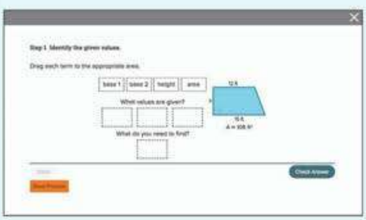
OL In the final step, why did you divide each side by 13.5? Sample answer: To get h by itself on one side of the equation. If one side is divided by 13.5, the other side also needs to be divided by 13.5.

BL You can write the formula $A = \frac{1}{2}h(b_1 + b_2)$ as $A = \frac{h(b_1 + b_2)}{2}$. Explain why the two are equivalent. Sample answer: Multiplying by $\frac{1}{2}$ is the same as dividing by 2. Since the entire expression $h(b_1 + b_2)$ is multiplied by $\frac{1}{2}$, the entire expression needs to be divided by 2.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Interactive Presentation



Example 5, Find Missing Dimensions of Trapezoids, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag each term to identify the given information and what needs to be found.

CLICK

On Slide 3, students move through the steps to find the height of the trapezoid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Apply Budgets

Objective

Students will come up with their own strategy to solve an application problem involving determining if enough money was budgeted for a repaving project.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

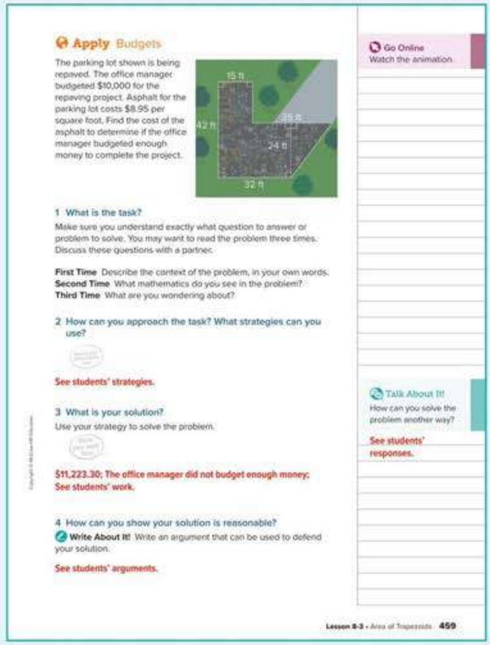
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you decompose the figure into two different polygons?
- How will you use the area formulas to find the area of the original polygon?
- How can you use the cost per square foot to determine if the office manager budgeted enough money?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Budgets

The parking lot shown is being repaved. The office manager budgeted $10,000 for the repaving project. Asphalt for the parking lot costs $8.95 per square foot. Find the cost of the asphalt to determine if the office manager budgeted enough money to complete the project.

15 ft, 42 ft, 35 ft, 24 ft, 32 ft

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

$11,223.30; The office manager did not budget enough money; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

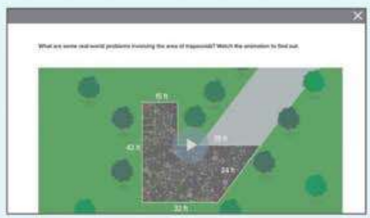
Go Online Watch the animation.

Talk About It! How can you solve the problem another way?

See students' responses.

Lesson 8-3 • Area of Trapezoids 459

Interactive Presentation



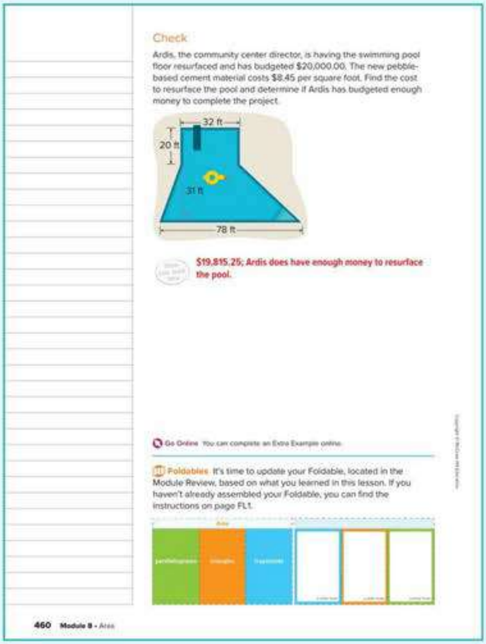
Apply, Budgets

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

Ardis, the community center director, is having the swimming pool floor resurfaced and has budgeted $20,000.00. The new pebble-based cement material costs $8.45 per square foot. Find the cost to resurface the pool and determine if Ardis has budgeted enough money to complete the project.

$19,815.25; Ardis does have enough money to resurface the pool.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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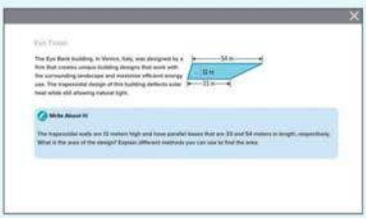
Interactive Presentation


Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add the formula that is used to find the area of a trapezoid. Then give an example of how to use that formula to find the area of a trapezoid.

Essential Question Follow-Up

How are the areas of triangles and rectangles used to find the areas of other polygons? In this lesson, students found the area of a trapezoid by decomposing it into triangles and rectangles, or by composing it into a parallelogram. Encourage them to work with a partner to prepare a brief demonstration that illustrates how the area of these shapes can help them find the area of a trapezoid.

Exit Ticket

Refer to the Exit Ticket slide. What is the area of the design? Explain different methods you can use to find the area. 522 m^2; Sample answer: I can decompose the trapezoid into a parallelogram and a triangle and add the areas of each. I can also compose the trapezoid into a parallelogram, find the area of the parallelogram and then divide that area by 2. Or I can use the area formula for a trapezoid to find the area.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 9–13
- Extension: Changes in Dimensions
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–7, 9, 11, 13
- Extension: Changes in Dimensions
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS Area of Rectangles

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Area of Rectangles

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	decompose a trapezoid and apply the area formulas for a parallelogram and a triangle to find the area of the trapezoid	1, 2
1	find the area of a trapezoid by composing and using the formula for the area of a trapezoid	3, 4
1	use an area formula to solve a real-world problem involving a right trapezoid	5
1	use an area formula to solve a real-world problem involving a trapezoid	6
1	find the missing dimension of a trapezoid given its area	7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving area of trapezoids	9
3	higher-order and critical thinking skills	10–13

Common Misconception

When finding the area of a trapezoid, some students may incorrectly use the formula. As more dimensions and operations are included in mathematical formulas, students have a greater chance for mathematical error. For example, some students may neglect to multiply the sum of the bases by the height or by $\frac{1}{2}$. Students may benefit from writing the formula for the area of a trapezoid at the top of their page and completing a thorough check of each part of the process of solving using the formula.

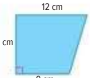
Name ______ Period ______ Date ______

Practice

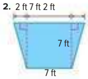
Go Online You can complete your homework online.

Decompose each trapezoid to find its area. (Example 1)

1. **105 cm^2**

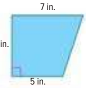


2. **63 ft^2**

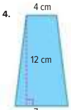


Find the area of each trapezoid. (Example 2)

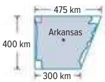
3. **36 in^2**



4. **66 cm^2**

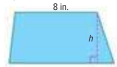


5. The shape of Arkansas resembles a trapezoid. What is the approximate area of Arkansas? (Example 3) **155,000 km^2**



6. The top of the desk shown is in the shape of a trapezoid. What is the area of the top of the desk? (Example 4) **648 in^2**

7. Find the missing dimension of the trapezoid. (Example 5) **4 in.**

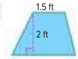


$A = 40$ in^2

Test Practice

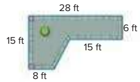
8. Open Response Ciro made a sign in the shape of a trapezoid. What was the area of Ciro's sign?

4.5 ft^2



Apply *indicates multi-step problem

*9. Greta has budgeted $1,500 to have a concrete patio poured in her backyard like the one shown. The cost per square foot of the concrete is $5.50. Find the cost of the patio to determine if Greta has budgeted enough money to complete the project.

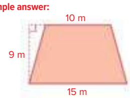


The cost of the patio is $1,443.75. Since this is less than $1,500, Greta has budgeted enough money.

Higher-Order Thinking Problems

10. **Create** Draw and label a trapezoid that has no right angles and an area greater than 75 square meters.

Sample answer:



The area of the trapezoid is 112.5 m^2.

11. Explain the steps needed to rewrite the formula for the area of a trapezoid to find b_2.

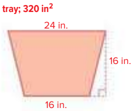
Start with the area formula:

$A = \frac{1}{2}h(b_1 + b_2)$. Multiply each side by 2:

$2A = h(b_1 + b_2)$. Multiply each side by $\frac{1}{h}$: $\frac{2A}{h} = b_1 + b_2$. Subtract b_1 from each side: $\frac{2A}{h} - b_1 = b_2$, or $b_2 = \frac{2A}{h} - b_1$.

12. **Create** Write and solve a real-world problem where you need to find the area of a trapezoid.

Sample answer: A tray in a school cafeteria has the dimensions shown. Find the area of the tray; 320 in^2



13. **Reason Inductively** The area of a trapezoid is 48 square centimeters. The height is 6 centimeters and one base is 3 times the length of the other base. What are the lengths of the bases?

4 cm and 12 cm

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students will determine the lengths of the bases. Encourage students to use reasoning to determine the lengths.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 9 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.

Use with Exercises 11 and 13 Have students work in groups of 3–4 to solve the problem in Exercise 11. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 13.

Learn Area of Regular Polygons

Objective

Students will learn how to find the area of regular polygons by decomposing them into triangles, parallelograms, and trapezoids.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, they should be able to explain whether or not their method is a valid approach.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to decompose a regular polygon to find its area.

Teaching Notes

SLIDE 1

Students will learn that to find the area of a *regular polygon*, they can decompose the figure into triangles, parallelograms, or trapezoids, and then add the individual areas to find the total area. Play the animation for the class. You may wish to pause the animation after the regular hexagon is shown with a side length of 2 centimeters. Ask students to work with a partner to come up with possible strategies for finding the area of the polygon. They may use any strategy they wish, but must be able to explain their strategy and defend why it works. Have students share their strategies with the class. Then have them continue watching the animation to compare their strategy with the one shown. The animation shows the polygon decomposed into two congruent trapezoids. Another possible method is to decompose the polygon into six congruent triangles.

Talk About It!

SLIDE 2

Mathematical Discourse

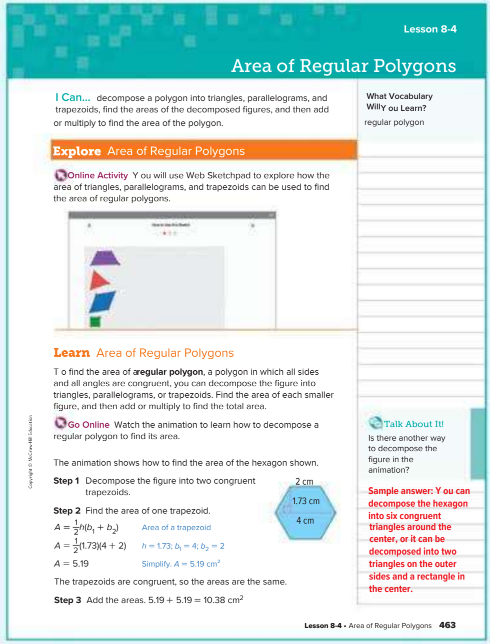
Is there another way to decompose the figure in the animation?

Sample answer: You can decompose the hexagon into six congruent triangles around the center, or it can be decomposed into two triangles on the outer sides and a rectangle in the center.

DIFFERENTIATE

Enrichment Activity BL

For students who need more of a challenge, have pairs of students look around the classroom or around the school for composite figures, figures that are made up of two or more shapes, that include regular polygons. For example, students could use a star from the flag of the United States. Students should find the dimensions of the figure using a ruler and then calculate the area of the composite figure.



Lesson 8-4

Area of Regular Polygons

I Can... decompose a polygon into triangles, parallelograms, and trapezoids, find the areas of the decomposed figures, and then add or multiply to find the area of the polygon.

What Vocabulary Will You Learn?
regular polygon

Explore Area of Regular Polygons

Online Activity You will use Web Sketchpad to explore how the area of triangles, parallelograms, and trapezoids can be used to find the area of regular polygons.

Learn Area of Regular Polygons

To find the area of a **regular polygon**, a polygon in which all sides and all angles are congruent, you can decompose the figure into triangles, parallelograms, or trapezoids. Find the area of each smaller figure, and then add or multiply to find the total area.

Go Online Watch the animation to learn how to decompose a regular polygon to find its area.

The animation shows how to find the area of the hexagon shown.

Step 1 Decompose the figure into two congruent trapezoids.

Step 2 Find the area of one trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$A = \frac{1}{2}(1.73)(4 + 2)$ $h = 1.73$; $b_1 = 4$; $b_2 = 2$

$A = 5.19$ Simplify. $A = 5.19$ cm^2

The trapezoids are congruent, so the areas are the same.

Step 3 Add the areas. $5.19 + 5.19 = 10.38$ cm^2

Talk About It! Is there another way to decompose the figure in the animation?

Sample answer: You can decompose the hexagon into six congruent triangles around the center, or it can be decomposed into two triangles on the outer sides and a rectangle in the center.

Interactive Presentation

Learn, Area of Regular Polygons, Slide 1 of 2

WATCH

On Slide 1, students watch an animation to learn how to decompose a regular polygon to find its area.

Lesson 8-4

Area of Regular Polygons

6.G.A.1

LESSON GOAL

Students will find the area of regular polygons by decomposing the figure into other figures.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Area of Regular Polygons

Learn: Area of Regular Polygons
Example 1: Find Area of Regular Polygons
Apply: Home Improvement

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

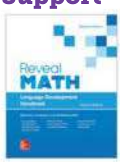
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Area of Circles		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 47 of the *Language Development Handbook* to help your students build mathematical language related to the area of regular polygons.



ELL You can use the tips and suggestions on page T47 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Supporting Cluster(s): In this lesson, students address supporting cluster **6.G.A** by finding the area of regular polygons by decomposing the figure into other figures.
Standards for Mathematical Content: **6.G.A.1**
Standards for Mathematical Practice:e: **MP1, MP2, MP3,MP4, MP5, MP7**

Coherence

Vertical Alignment

Previous
Students found and used the area of trapezoids by composing and decomposing into other shapes.
6.G.A.1, 6.EE.A.2, 6.EE.A.2.C

Now
Students find the area of regular polygons by decomposing the figure into other figures.
6.G.A.1

Next
Students will use the coordinate plane to draw and find attributes of polygons.
6.G.A.3

Rigor

The Three Pillars of Rigor

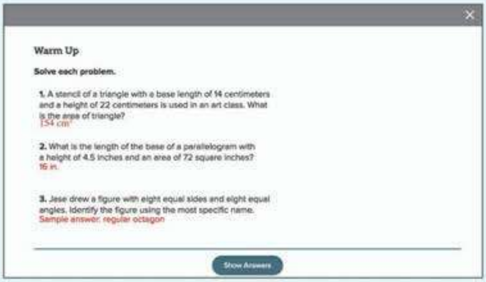
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their *understanding* of area as they explore regular polygons. They learn how to compose and decompose regular polygons into triangles, parallelograms, and trapezoids to build *fluency* with finding the area. They *apply* their understanding of area of regular polygons to solve multi-step, real-world problems.

Mathematical Background

A *regular polygon* is a polygon that has congruent sides and congruent angles. To find the area of a regular polygon, decompose it into triangles, parallelograms, and trapezoids. Add the areas of the smaller shapes to find the area of the regular polygon.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding the area of a triangle (Exercise 1)
- Using the area of a parallelogram to find the base (Exercise 2)
- classifying polygons (Exercise 3)

Answers

1. 154 cm^2

2. 16 in.

3. Sample answer: regular octagon

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the shape of many road signs, including the pentagonal shape of the school zone road sign.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The word polygon comes from the Greek work *polygonon*, where *polys* means many, and *gonon* means angled. What do you think the word *polygon* might mean? Sample answer: A polygon is a figure with many angles like a triangle or a square.

Explore Area of Regular Polygons

Objective

Students will use Web Sketchpad to explore how the area of triangles, parallelograms, and trapezoids can be used to find the area of regular polygons.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore how the areas of triangles, parallelograms, and trapezoids can be used to find the area of polygons. Encourage students to think of how they could use what they know to further their investigation.

Inquiry Question

How can you use the areas of triangles, parallelograms, and trapezoids to find the areas of other polygons? Sample answer: If I can divide a polygon into triangles, parallelograms, or trapezoids, without any gaps or overlap, I can add the areas of those to find the area of the larger polygon.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

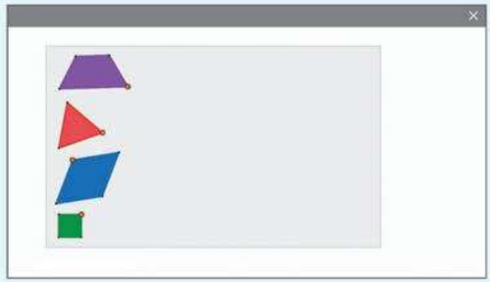
Mathematical Discourse

How did you use triangles to create a six-sided figure? See students' workspace. Sample answer: I lined up a vertex of six triangles in a single location, then moved them around until they formed a six-sided figure.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8

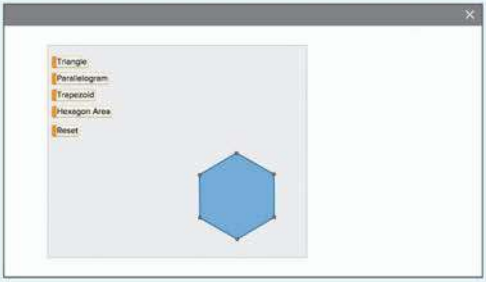


Explore, Slide 2 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how the area of triangles, parallelograms, and trapezoids can be used to find the area of regular polygons.

Interactive Presentation



Explore, Slide 7 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and can view a sample answer.

Explore Area of Regular Polygons *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and examine the correspondences between the areas of the triangles, parallelograms, and trapezoids and polygons.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

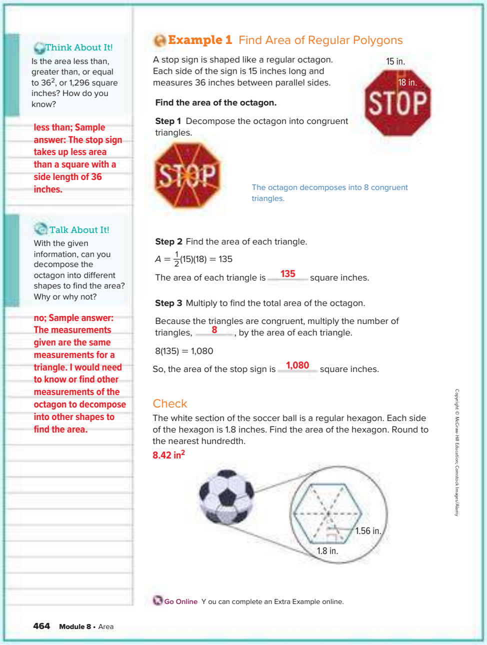
Talk About It!

SLIDE 6

Mathematical Discourse

Can you use any of the other figures to create a hexagon. If so, describe what you did. Sample answer: I used two trapezoids. I rotated one of them so the two longer bases matched up.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Think About It!
Is the area less than, greater than, or equal to 36^2, or 1,296 square inches? How do you know?

less than; Sample answer: The stop sign takes up less area than a square with a side length of 36 inches.

Talk About It!
With the given information, can you decompose the octagon into different shapes to find the area? Why or why not?

no; Sample answer: The measurements given are the same measurements for a triangle. I would need to know or find other measurements of the octagon to decompose into other shapes to find the area.

Example 1 Find Area of Regular Polygons

A stop sign is shaped like a regular octagon. Each side of the sign is 15 inches long and measures 36 inches between parallel sides.

15 in.
18 in.
STOP

Find the area of the octagon.

Step 1 Decompose the octagon into congruent triangles.

The octagon decomposes into 8 congruent triangles.

Step 2 Find the area of each triangle.

$A = \frac{1}{2}(15)(18) = 135$

The area of each triangle is 135 square inches.

Step 3 Multiply to find the total area of the octagon.

Because the triangles are congruent, multiply the number of triangles, 8, by the area of each triangle.

$8(135) = 1{,}080$

So, the area of the stop sign is 1,080 square inches.

Check
The white section of the soccer ball is a regular hexagon. Each side of the hexagon is 1.8 inches. Find the area of the hexagon. Round to the nearest hundredth.
8.42 in^2

1.56 in.
1.8 in.

Go Online You can complete an Extra Example online.

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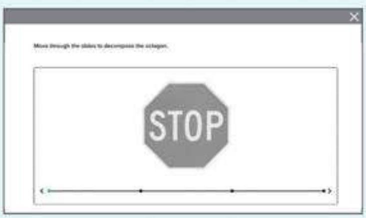
464 Module 8 • Area

Interactive Presentation



Example 1, Find Area of Regular Polygons, Slide 2 of 6

CLICK

On Slide 2, students move through the slides to decompose the octagon into eight congruent triangles.

TYPE

On Slide 4, students determine the area of the octagon.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find Area of Regular Polygons

Objective

Students will find the area of regular polygons by decomposing them into triangles, parallelograms, and trapezoids.

Questions for Mathematical Discourse

SLIDE 2

AL How did you know there were 8 triangles? Sample answer: I drew diagonals from all the vertices through the center of the octagon and then counted the triangles.

OL How would you describe the *center* of the octagon? Sample answer: the point inside the octagon where the diagonals intersect

BL How many triangles would be needed to compose a regular pentagon? a regular hexagon? What pattern do you notice? 5 triangles; 6 triangles; Sample answer: the numbers of triangles needed is equal to the number of sides of the regular polygon.

SLIDE 3

AL How will finding the area of one triangle help you find the area of the octagon? Sample answer: I divided the octagon into 8 triangles with the same shape and size. If I find the area of one of the triangles, I can multiply it by 8 to find the area of the octagon.

OL Explain what 135 square inches means in the context of the problem. Sample answer: The area of one triangle is 135 square inches. The octagon is made up of 8 triangles with that area.

BL Suppose a model of the sign has dimensions that are one-third of the original sign. What fraction of the original triangle area is the area of one triangle in the model? Explain. $\frac{1}{9}$; Sample answer: The base length of the model is 5 inches and the height is 6 inches, so the area of that triangle is 15 square inches; 15 is $\frac{1}{9}$ of 135.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

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Apply Home Improvement

Objective

Students will come up with their own strategy to solve an application problem involving the cost to cover a floor with tiles.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

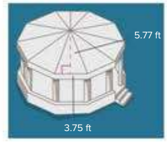
- What do you notice about the measurements given and the smaller shapes within the decagon?
- How will you find the total area?
- How will you need to use the cost per square foot?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Home Improvement

Keilani designed a gazebo and wants to cover the floor with tiles. The gazebo is shaped like a decagon with 3.75 foot sides. If floor tiles cost $2.89 per square foot, what is the least amount she will spend on the tiles?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Answers may vary due to rounding; Sample answer: $312.66; See students' work.

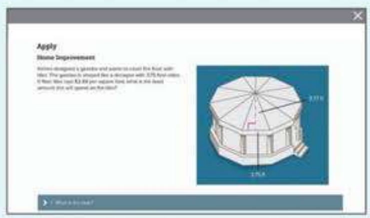
4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.
See students' arguments.

Talk About It!
How can you solve the problem another way?

See students' responses.

Interactive Presentation



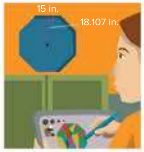
Apply, Home Improvement

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Morgan designed a stained glass window to be added above the door at the community center. The window is shaped like an octagon with 15-inch sides. If stained glass costs $0.70 per square inch, how much will she spend on the window? **$760.49**



Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare finding the area of regular polygons with finding the area of irregular polygons. What are some similarities? What are some differences?

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How are the areas of triangles and rectangles used to find the areas of other polygons? In this lesson, students learned how to find the area of a regular polygon by decomposing it into trapezoids and/or triangles. Encourage them to work with a partner to prepare a brief demonstration (using grid paper or other drawings) that illustrates how the area of trapezoids and triangles can help them find the area of a regular polygon. Have them present their demonstration to the class.

Exit Ticket

Refer to the Exit Ticket slide. A stop sign has eight sides. Suppose each side has a length of 12 inches and the perpendicular distance from the center of the sign to one of the sides is 14.5 inches. What is the area of the sign? Write a mathematical argument that can be used to defend your solution. 696 square inches; Sample answer: I decomposed the octagon into 8 congruent triangles, found the area of each triangle, and then multiplied by 8.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 3, 5–9
- Extension: Area of Circles
- ALEKS® Area of Parallelograms, Triangles, and Trapezoids

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–3, 5, 7, 9
- Extension: Area of Circles
- Remediation: Review Resources
- Personal Tutor
- Extra Example 1
- ALEKS® Polygons and Quadrilaterals

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Polygons and Quadrilaterals

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

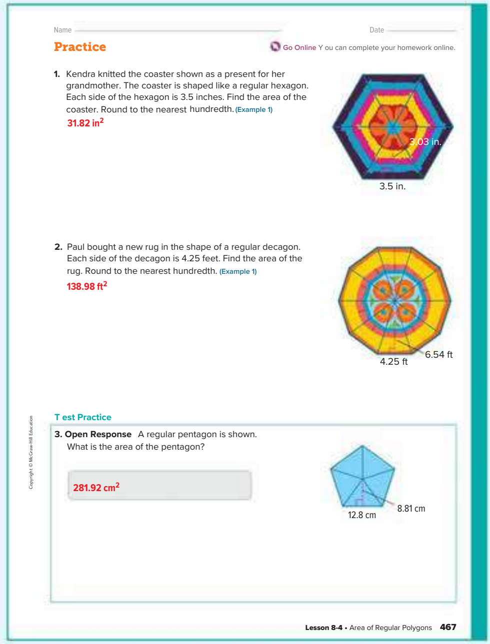
The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the area of regular polygons by decomposing them into triangles, parallelograms, and trapezoids	1, 2
2	extend concepts learned in class to apply them in new contexts	3
3	solve application problems involving area of regular polygons	4, 5
3	higher-order and critical thinking skills	6–9



Name ______ Date ______

Practice

Go Online You can complete your homework online.

1. Kendra knitted the coaster shown as a present for her grandmother. The coaster is shaped like a regular hexagon. Each side of the hexagon is 3.5 inches. Find the area of the coaster. Round to the nearest hundredth. (Example 1)
   31.82 in²

2. Paul bought a new rug in the shape of a regular decagon. Each side of the decagon is 4.25 feet. Find the area of the rug. Round to the nearest hundredth. (Example 1)
   138.98 ft²

Test Practice

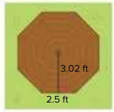
3. **Open Response** A regular pentagon is shown. What is the area of the pentagon?

281.92 cm²

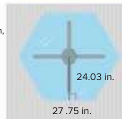
Lesson 8-4 • Area of Regular Polygons 467

Apply *indicates multi-step problem

*4. Julian is going to build a picnic table. The top of the picnic table is shaped like an octagon with sides measuring 2.5 feet. If the wood costs $3.95 per square foot, what is the least he will spend on the top of the picnic table?
$119.29

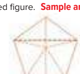


*5. Williana's mother wants to buy a glass tabletop for their dining room table. The tabletop is shaped like a hexagon with sides measuring 27 .75 inches. If the glass costs $0.06 per square inch, how much will she spend on the glass table top?
$120.03



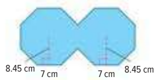
Higher-Order Thinking Problems

6. Draw a regular pentagon and use dashed lines to show the ways it can be decomposed. Describe the shapes in the decomposed figure. Sample answer:



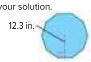
Sample answer: 5 triangles; 1 triangle and 1 trapezoid

7 . **Identify Structure** What is the area of the figure below?
473.2 cm^2



8. **Reason Abstractly** The area of a regular hexagon is about 65 square units. Y ou decompose the figure into 6 triangles. The height of one triangle is about 4.3 units. What is the approximate length of the base of the triangle?
5 units

9. **Reason Inductively** The figure shown is a regular decagon. If the perimeter is 80 inches, what is the area of the decagon? Write an argument that can be used to defend your solution.



492 in^2; the base length of each triangle is $80 \div 10$ or 8 in. So, $10\left(\frac{1}{2} \times 8 \times 12.3\right) = 492$.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 7, students determine the area of the figure. Encourage students to use the structure to find the area of the figure without doing a lot of unnecessary calculations.

2 Reason Abstractly and Quantitatively In Exercise 8, students use reasoning to determine the base of the triangle.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students find the area of the decagon. Encourage students to plan how to solve the problem and then reason inductively to find the area.

Common Misconception

In Exercise 4, some students may multiply incorrectly when finding the total cost of the top of the table. Students may incorrectly place the decimal after multiplying and find the total cost to be $1,192.90 instead of $119.29.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 4–5 After completing the application problems, have students write their own real-world problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Solve the problem another way.
Use with Exercise 7 Have students work in groups of 3–4. After completing Exercise 7, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Learn Draw Polygons on the Coordinate Plane

Objective

Students will learn how to draw polygons in the first quadrant of the coordinate plane given coordinates for the vertices.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make connections between the number of points plotted (vertices) and the number of sides of the polygons.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the polygons that were plotted as they discuss the *Talk About It!* question on Slide 3.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

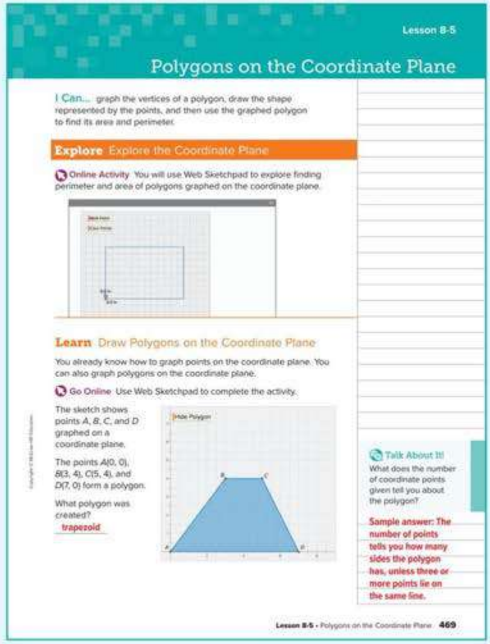
What does the number of coordinate points given tell you about the polygon? Sample answer: The number of points tells you how many sides the polygon has, unless three or more points lie on the same line.

DIFFERENTIATE

Reteaching Activity AL

If students have difficulty with graphing points on the coordinate plane, have pairs of students interview each other about how to graph a point on the coordinate plane. Students could ask questions similar to the following:

- Which coordinate is listed first in an ordered pair, the *x*-coordinate or the *y*-coordinate?
- If the ordered pair is (4, 2), which value is the *x*-coordinate? the *y*-coordinate?
- When graphing, does the *x*-coordinate indicate horizontal or vertical movement on the coordinate plane?
- When graphing, does the *y*-coordinate indicate horizontal or vertical movement on the coordinate plane?


Lesson 8-5

Polygons on the Coordinate Plane

I Can... graph the vertices of a polygon, draw the shape represented by the points, and then use the graphed polygon to find its area and perimeter.

Explore Explore the Coordinate Plane

Online Activity You will use Web Sketchpad to explore finding perimeter and area of polygons graphed on the coordinate plane.

Learn Draw Polygons on the Coordinate Plane

You already know how to graph points on the coordinate plane. You can also graph polygons on the coordinate plane.

Go Online Use Web Sketchpad to complete the activity.

The sketch shows points A, B, C, and D graphed on a coordinate plane.

The points A(0, 0), B(3, 4), C(5, 4), and D(7, 0) form a polygon.

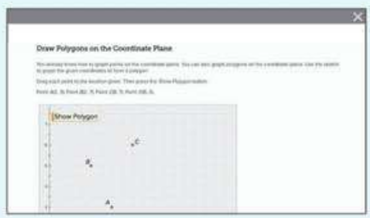
What polygon was created? trapezoid

Talk About It! What does the number of coordinate points given tell you about the polygon?

Sample answer: The number of points tells you how many sides the polygon has, unless three or more points lie on the same line.

Lesson 8-5 • Polygons on the Coordinate Plane 469

Interactive Presentation



Learn, Draw Polygons on the Coordinate Plane, Slide 1 of 3

WEB SKETCHPAD

On Slides 1 and 2, students use Web Sketchpad to graph polygons on the coordinate plane.

CLICK

On Slides 1 and 2, students select the correct term to identify the polygons.

Lesson 8-5

6.G.A.3

Polygons on the Coordinate Plane

LESSON GOAL

Students will use the coordinate plane to draw and find attributes of polygons.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Explore the Coordinate Plane

Learn: Draw Polygons on the Coordinate Plane
Learn: Find Perimeter and Area on the Coordinate Plane
Example 1: Find Perimeter of an Irregular Figure
Example 2: Find Perimeter Using Coordinates
Example 3: Find Area Using Coordinates
Apply: Business Finance

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Pick's Theorem		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 48 of the *Language Development Handbook* to help your students build mathematical language related to polygons on the coordinate plane.

ELL You can use the tips and suggestions on page T48 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Geometry

Supporting Cluster(s): In this lesson, students address supporting cluster **6.G.A** by finding the area of regular polygons by using the coordinate plane to draw and find attributes of polygons.

Standards for Mathematical Content: **6.G.A.3,** *Also addresses 6.G.A.1*

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the area of regular polygons by decomposing the figure into other figures.
6.G.A.1

Now
Students use the coordinate plane to draw and find attributes of polygons.
6.G.A.3

Next
Students will find and use the volume of rectangular prisms.
6.G.A.2

Rigor

The Three Pillars of Rigor

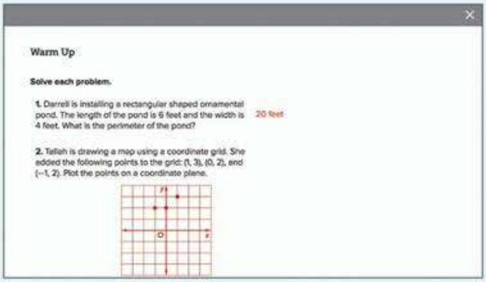
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of area to expand their *understanding* to area of polygons on the coordinate plane. They build *fluency* with finding perimeter and area of polygons and irregular figures on the coordinate plane. They *apply* their understanding to solve multi-step problems.

Mathematical Background

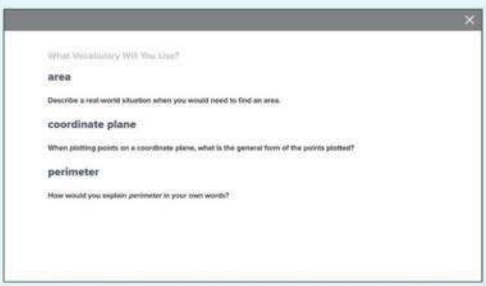
The *perimeter* or *area* of a polygon graphed on the *coordinate plane* can be found by finding the distance between vertices. To find the distance between two points with the same *x*- or *y*-coordinates, subtract the *y*- or *x*-coordinates, respectively. The distances between points correspond to the side lengths of the polygon. Perimeter or area formulas as well as decomposition can be used to find the perimeter or area of the polygon.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding perimeter (Exercise 1)
- graphing on a coordinate plane (Exercises 2–3)

Answers

1–3. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about video game designers using the coordinate plane system.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Describe a real-world situation when you would need to find an area. Sample answer: If I'm covering the top of a box with fabric, I would need to know how much fabric I need.
- When plotting points on a coordinate plane, what is the general form of the points plotted? *(x, y)*
- How would you explain perimeter in your own words? Sample answer: The perimeter is the distance around an object.

Explore Explore the Coordinate Plane

Objective

Students will use Web Sketchpad to explore finding perimeter and area on the coordinate plane.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to form shapes in the coordinate plane. Students will then investigate the perimeter and area of the shapes created applying what they have learned in the previous lessons.

Inquiry Question

How can you use the coordinate plane to find perimeter and area of a polygon? Sample answer: The coordinate plane can be used to find the dimensions of the polygon. To find the perimeter, find the sum of all the sides. To find the area, I can count the squares inside the polygon or use the polygon's dimensions and the area formula.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

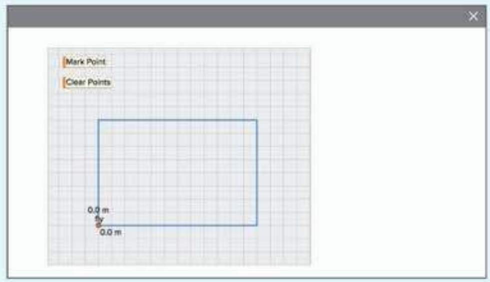
Mathematical Discourse

How did you find the perimeter and area of the rectangle? How did the coordinate plane help you? Sample answer: I can count the units around the figure to find the perimeter. Multiply the length and width to find the area. The coordinate plane is structured using a grid of unit squares that are easily counted.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

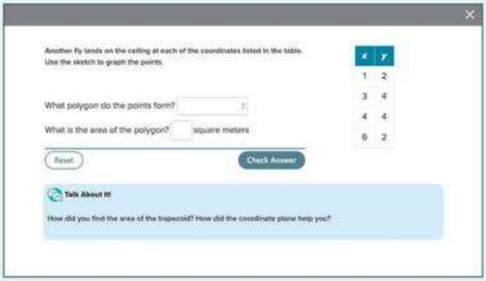


Explore, Slide 3 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore finding perimeter and area on the coordinate plane.

Interactive Presentation



Explore, Slide 4 of 7

TYPE

On Slide 4, students enter the area of the polygon.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Explore the Coordinate Plane *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how they can find the perimeter and area of polygons plotted on the coordinate plane.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

How did you find the area of the trapezoid? How did the coordinate plane help you? Sample answer: I used the coordinates of the vertices to find the lengths of the bases and the height of the trapezoid. Then I used the area formula.

Your Notes

Learn Find Perimeter and Area on the Coordinate Plane

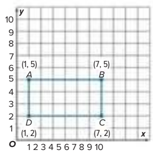
You can use the coordinates of a polygon to find its dimensions by finding the distance between two points.

To find the distance between two points with the same *x*-coordinates, subtract their *y*-coordinates. To find the distance between two points with the same *y*-coordinates, subtract their *x*-coordinates. You can use those dimensions to find the perimeter and area of a polygon.

Go Online Watch the animation to learn about finding perimeter on the coordinate plane.

The animation shows rectangle *ABCD* graphed on the coordinate plane with vertices at *A*(1, 5), *B*(7, 5), *C*(7, 2), and *D*(1, 2).

To find the perimeter, start by using the coordinates to find the distance between two points, which gives the length of the side.



Step 1 Subtract the *y*-coordinates.

Subtract the *y*-coordinates to find the lengths of sides *AD* and *BC*.

side *AD*: 5 − 2, or 3
side *BC*: 5 − 2, or 3

Both sides *AD* and *BC* are 3 units long.

Step 2 Subtract the *x*-coordinates.

Subtract the *x*-coordinates to find the lengths of sides *AB* and *CD*.

side *AB*: 7 − 1, or 6
side *CD*: 7 − 1, or 6

Both sides *AB* and *CD* are 6 units long.

Step 3 Add the lengths of the four sides.

6 + 3 + 6 + 3 = 18 units

So, rectangle *ABCD* has a perimeter of ___18___ units.

Talk About It! How can the coordinates of vertices be used to find the area of a polygon?

Sample answer: I can use the *x*- and *y*-coordinates to find the lengths of the polygon, and then use the polygon's area formula to calculate the area.

Interactive Presentation

Learn, Find Perimeter and Area of the Coordinate Plane, Slide 1 of 2

WATCH

On Slide 1, students watch an animation to learn how to find the perimeter of a polygon on the coordinate plane.

Learn Find Perimeter and Area on the Coordinate Plane

Objective

Students will learn how to use coordinates to find the perimeter of a polygon on the coordinate plane.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, they should make sense of the vertices and dimensions of the polygon in order to use the dimensions in the polygon's area formula.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the polygon and how it is plotted on the coordinate plane, as they discuss the *Talk About It!* question on Slide 2.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find the perimeter of a polygon on the coordinate plane.

Teaching Notes

SLIDE 1

Play the animation for the class. Students will learn that the coordinates of a polygon can be used to find its dimensions. Students will also learn how to find the distance between two points with the same *x*- or *y*-coordinates. Students will view the animation to learn how to apply these techniques in order to find the perimeter and/or area of these polygons.

Talk About It!

SLIDE 2

Mathematical Discourse

How can the coordinates of vertices be used to find the area of a polygon? Sample answer: I can use the *x*- and *y*-coordinates to find the lengths of the sides of the polygon, and then use the polygon's area formula to calculate the area.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Find Perimeter of an Irregular Figure

Objective

Students will find the perimeter of an irregular figure given the figure and coordinates drawn on a coordinate plane.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

2 Reason Abstractly and Quantitatively Encourage students to make sense of the coordinates to find the perimeter of the irregular figure by using the length of each side.

Questions for Mathematical Discourse

SLIDE 2

AL How will you keep track of what you have counted? Sample answer: I will count the units on each side of the figure and write it down.

OL Why is it not efficient to count the units when finding the perimeter? Sample answer: I can forget where I started or what I have already counted.

BL Suppose each interval was 0.25 mile. What would the perimeter of the exhibit be? 10.5 miles

SLIDE 3

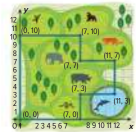
AL Why do you subtract the x-coordinates when finding the length of a horizontal line segment? Sample answer: The y-coordinates of a horizontal line segment are the same, so to find the length of the line segment, you need to subtract the x-coordinates.

OL Could you use Method 2 if the aquarium was located at (10, 2)? Explain your reasoning. no; Sample answer: To use this method, two points need to have the same x-coordinate or the same y-coordinate. If the aquarium was at (10,2) it does not have the same x-coordinate as the rhinoceros, nor does it have the same y-coordinate as the tiger.

BL If each unit requires 2 shrubs, how many shrubs will be needed to go around the entire exhibit? 84 shrubs

Example 1 Find Perimeter of an Irregular Figure

Find the perimeter of the exhibit shown on the coordinate plane.



Method 1 Count the units.

Count the units as you move along the perimeter of the exhibit.

Start at the entrance, or (0, 0). How many units do you need to travel along the y-axis to reach the monkeys? 10 units

How many units do you need to travel along the x-axis from the monkeys to reach the gorillas? 7 units

Continue counting along the perimeter until you return to the entrance.

Add to find the perimeter.

$10 + 7 + 3 + 4 + 4 + 4 + 3 + 7 =$ **42** units

Method 2 Use the coordinates to find the distances.

Find the lengths of the horizontal line segments by subtracting the x-coordinates.

tigers to elephants: $11 - 7 = 4$

aquarium to rhinoceros: $11 - 7 = 4$

reptiles to entrance: $7 - 0 = 7$

gorillas to monkeys: $7 - 0 = 7$

Find the lengths of the vertical line segments by subtracting the y-coordinates.

gorillas to elephants: $10 - 7 = 3$

tigers to aquarium: $7 - 3 = 4$

rhinoceros to reptiles: $3 - 0 = 3$

monkeys to entrance: $10 - 0 = 10$

Find the sum of the sides.

$4 + 4 + 7 + 7 + 3 + 4 + 3 + 10 =$ **42**

So, using either method, the perimeter of the exhibit is 42 units.

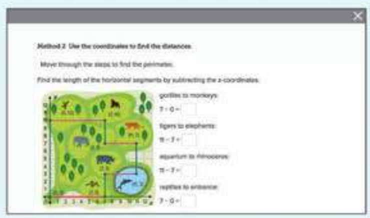
Think About It! How can you find the distance between two points on the coordinate plane?

See students' responses.

Talk About It! Compare the two methods.

Sample answer: Counting the units helps to visualize the perimeter, while finding the differences between the points is helpful when the figure is large or has many sides.

Interactive Presentation



Example 1, Find Perimeter of an Irregular Figure, Slide 3 of 5

TYPE On Slide 3, students enter the missing values to show the distance between two vertices.

CLICK On Slide 3, students move through the steps to find the perimeter.

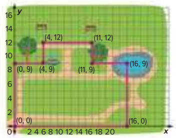
CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

Find the perimeter of the outlined section of the park shown on the coordinate plane.

Show your work here

56 units



Go Online You can complete an Extra Example online.

Example 2 Find Perimeter Using Coordinates

A rectangle has vertices *A*(2, 8), *B*(7, 8), *C*(7, 5), and *D*(2, 5).

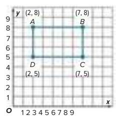
Use the coordinates to find the perimeter of the rectangle.

Think About It!
How can you find the length or horizontal distance? How can you find the width or vertical distance?

See students' responses.

Step 1 Identify the sides of the rectangle.

Graph the vertices on the coordinate plane. Then draw line segments to connect them to form a rectangle.



The horizontal sides are $\overline{AB}$ and $\overline{CD}$. You can also determine this from studying the coordinates.

Points *A* and *B* have the same *y*-coordinate, so they are endpoints of horizontal side $\overline{AB}$.

Points *C* and *D* have the same *y*-coordinate, so they are endpoints of horizontal side $\overline{CD}$.

The vertical sides are $\overline{AD}$ and $\overline{BC}$. You can also determine this from studying the coordinates.

Points *A* and *D* have the same *x*-coordinate, so they are endpoints of vertical side $\overline{AD}$.

Points *B* and *C* have the same *x*-coordinate, so they are endpoints of vertical side $\overline{BC}$.

(continued on next page)

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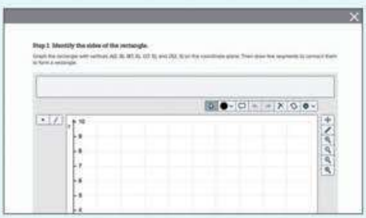
Interactive Presentation



Example 2, Find Perimeter Using Coordinates, Slide 2 of 6.

eTOOLS

On Slide 2, Students use the Coordinate Graphing eTool to graph vertices to form a rectangle.

Example 2 Find Perimeter Using Coordinates

Objective

Students will find the perimeter of a polygon given the coordinates of the vertices.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the coordinates given in order to identify the sides of the polygon.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, they should use clear and precise mathematical language, such as *x*- and *y*-coordinates, in their response.

Questions for Mathematical Discourse

SLIDE 3

AL How do you know that a segment is vertical? Sample answer: If the two endpoints have the same *x*-coordinate, the segment is vertical.

OL Since the polygon is a rectangle, what do you know about $\overline{AB}$ and $\overline{BC}$? Sample answer: They are the length and width of the rectangle, and they are perpendicular.

BL How do you know that $\overline{AD}$ and $\overline{AB}$ are not parallel sides of the polygon? Sample answer: Since they share an endpoint, *A*, they cannot be parallel.

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Find Perimeter Using Coordinates *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL Why do you subtract the *y*-coordinates to find the length of $\overline{AD}$ or $\overline{BC}$? Sample answer: $\overline{AD}$ or $\overline{BC}$ are the vertical sides of the rectangle, so the *x*-coordinates are the same. To find a vertical distance on the coordinate plane, you subtract the *y*-coordinates.

OL Why is it necessary to only calculate the length of one horizontal side and one vertical side? Sample answer: The figure is a rectangle so the two parallel sides are the same length.

BL Could another rectangle have a perimeter of 16 without having the same dimensions? If so, give an example using coordinates. If not, explain why not. yes; Sample answer: The width could be 2 units and the length could be 6 units, so the points could be *A*(0, 0), *B*(0, 2), *C*(6, 0), and *D*(6, 2).

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Step 2 Find the perimeter of the rectangle.

Find the length of each side. Y ou can count the units along each side of the rectangle's graph, or you can use the coordinates of the vertices and subtract to find the length of each side.

Length of $\overline{AB}$: 5 units   Length of $\overline{CD}$: 5 units

Length of $\overline{AD}$: 3 units   Length of $\overline{BC}$: 3 units

So, rectangle *ABCD* has a perimeter of 5 + 5 + 3 + 3, or 16 units.

Talk About It!

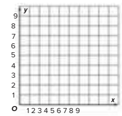
Why do the vertical sides share *x*-coordinates and horizontal sides share *y*-coordinates? Explain your reasoning.

Sample answer: The vertical sides are the same distance from the *y*-axis, so the *x*-coordinates are the same. The horizontal sides are the same distance from the *x*-axis, so the *y*-coordinates are the same.

Check

A rectangle has vertices *A*(1, 4), *B*(1, 9), *C*(8, 9), and *D*(8, 4). Find the perimeter of the rectangle. Use the coordinate plane if needed.

Show your work here



24 units

Go Online Y ou can complete an Extra Example online.

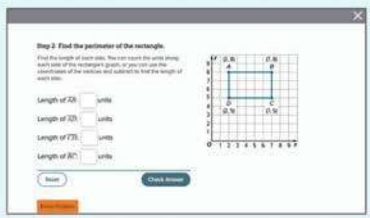
Pause and Reflect

Suppose a classmate was having difficulty finding perimeter on the coordinate plane. How can you explain how to use the different methods to help the classmate understand?

Record your observations here

See students' observations.

Interactive Presentation



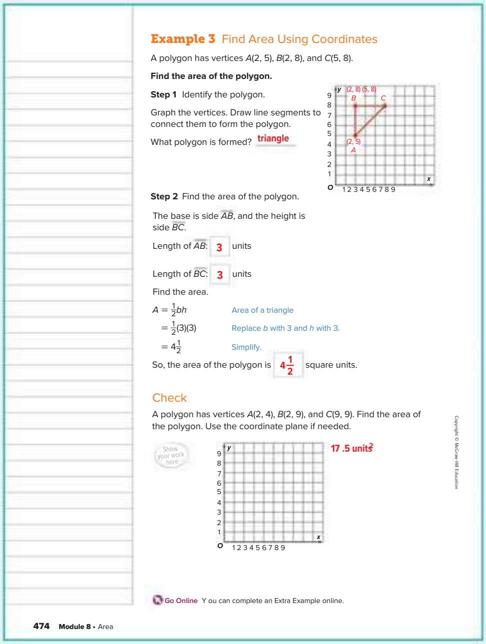
Example 2, Find Perimeter Using Coordinates, Slide 4 of 6

TYPE

On Slide 4, students determine the perimeter of the figure.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Find Area Using Coordinates

A polygon has vertices $A(2, 5)$, $B(2, 8)$, and $C(5, 8)$.

Find the area of the polygon.

Step 1 Identify the polygon.

Graph the vertices. Draw line segments to connect them to form the polygon.

What polygon is formed? triangle

Step 2 Find the area of the polygon.

The base is side $\overline{AB}$, and the height is side $\overline{BC}$.

Length of $\overline{AB}$: 3 units

Length of $\overline{BC}$: 3 units

Find the area.

$A = \frac{1}{2}bh$ — Area of a triangle

$= \frac{1}{2}(3)(3)$ — Replace b with 3 and h with 3.

$= 4\frac{1}{2}$ — Simplify.

So, the area of the polygon is $4\frac{1}{2}$ square units.

Check

A polygon has vertices $A(2, 4)$, $B(2, 9)$, and $C(9, 9)$. Find the area of the polygon. Use the coordinate plane if needed.

Show your work here

17.5 units²

Go Online You can complete an Extra Example online.

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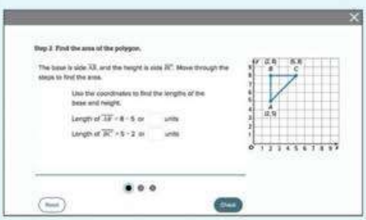
Interactive Presentation



Example 3, Find Area Using Coordinates, Slide 3 of 4

CLICK

On Slide 3, students move through the steps to find the area of the polygon.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Find Area Using Coordinates

Objective

Students will find the area of a polygon given the coordinates of the vertices.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the given coordinates in order to identify the sides of the polygon.

6 Attend to Precision Students should calculate accurately and efficiently, paying careful attention to the order of operations.

Questions for Mathematical Discourse

SLIDE 2

AL Without graphing, how do you know the number of sides of the polygon? Sample answer: I am given three points, or vertices of the polygon, so I know it is a triangle.

AL What formula will you use to find the area? $A = \frac{1}{2}bh$

OL Suppose point A was located at (3, 4). Explain how you could find the area of the triangle. Sample answer: Since $\overline{BC}$ is horizontal, that can be my base. I need to find the height, which will be a vertical line from point A to $\overline{BC}$. I can subtract the y-coordinates, 8 − 4, to get a height of 4 and use the area formula to find the area.

BL Can you classify the triangle by its sides and angles? Explain your reasoning. Sample answer: yes; right isosceles; The endpoints of $\overline{AB}$ have the same x-coordinate so that side is a vertical line. The endpoints of $\overline{BC}$ have the same y-coordinate so that side is a horizontal line. The two sides are perpendicular so the triangle is a right triangle. The two sides are the same length so it is an isosceles triangle.

Go Online

- Find additional teaching notes and discussion questions.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Business Finance

Objective

Students will come up with their own strategy to solve an application problem involving selecting a rental space in a mall.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

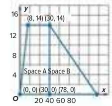
- How can you find the dimensions needed to find each area?
- How will you need to use the cost per square foot of each space?
- What do you notice about the total monthly rental cost for the two spaces?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Business Finance

Miyu, a craft store owner, plans to rent a location in the mall and is considering the two spaces shown. On the map, one unit is equal to one foot. Space A has a monthly rental cost of $13.89 per square foot. Space B has a monthly rental cost of $13.49 per square foot. Miyu wants to pay the lower total monthly rental price. Which location should she choose to rent?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Space B; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

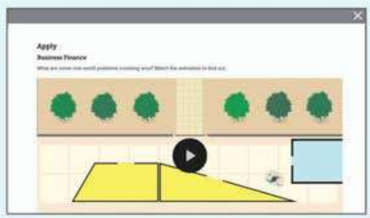
See students' arguments.

Go Online Watch the animation.

Talk About It! In this problem, why is it possible to determine which space she should rent, without figuring out the monthly rental cost?

Sample answer: The area and the cost per square foot of Space B is less than the area and cost per square foot of Space A, so the total monthly rental cost would also be lower.

Interactive Presentation



Apply, Business Finance

WATCH

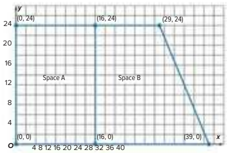
Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Jackie, a sports store owner, plans to rent a location in a strip mall and is considering the two spaces shown. On the map, one unit is equal to one foot. Space A has a monthly rental cost of $14.59 per square foot. Space B has a monthly rental cost of $15.15 per square foot. Jackie wants to pay the lower total monthly rental price. Which location should she choose to rent? **Space A**



Show your work here

Go Online You can complete an Extra Example online.

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Exit Ticket

Refer to the Exit Ticket slide. A video game designer graphed the points (2, 3), (10, 3), (10, 6), and (2, 6) on a coordinate plane. What is the perimeter and the area of the figure formed by the points? Write a mathematical argument that can be used to defend your solution.
perimeter: 22 units; area: 24 units^2; Sample answer: When the points are graphed on a coordinate plane, they form a rectangle. The vertical sides of the rectangle are 6 − 3 or 3 units long and the horizontal sides are 10 − 2 or 8 units long. So, the perimeter of the rectangle is 3 + 3 + 8 + 8 or 22 units and the area is 3(8) or 24 square units.

Interactive Presentation

Exit Ticket

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 7–11
- Extension: Pick's Theorem
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–7, 9, 11
- Extension: Pick's Theorem
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Area of Rectangles

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Area of Rectangles

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the perimeter of an irregular figure given the figure and coordinates drawn on a coordinate plane	1, 2
1	find the perimeter of a polygon given the coordinates of the vertices	3, 4
1	find the area of a polygon given the coordinates of the vertices	5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving polygons on the coordinate plane	7
3	higher-order and critical thinking skills	8–11

Common Misconception

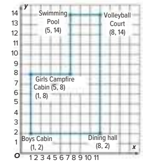
Some students may prefer to count the units on the coordinate plane to find the perimeter. However, if given only the coordinates of a polygon, then they may obtain an incorrect answer due to neglecting to include measurements for all sides, or including a side more than once. Students may benefit by writing a list of sides with the corresponding coordinates, so that they can be certain all sides are included appropriately.

Name ____ Period ____ Date ____

Practice

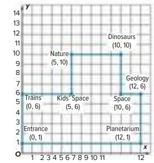
Go Online You can complete your homework online.

1. Find the perimeter of the summer camp shown on the coordinate plane. (Example 1)



38 units

2. Find the perimeter of the science center shown on the coordinate plane. (Example 1)



42 units

3. A rectangle has vertices *W*(2, 7), *X*(2, 0), *Y*(6, 0), and *Z*(6, 7). Use the coordinates to find the perimeter of the rectangle. (Example 2)

22 units

4. A rectangle has vertices *H*(3, 0), *I*(3, 7), *J*(6, 7), and *K*(6, 0). Use the coordinates to find the perimeter of the rectangle. (Example 2)

20 units

5. A polygon has vertices *A*(3, 3), *B*(3, 6), and *C*(9, 3). Find the area of the polygon. (Example 3)

9 square units

Test Practice

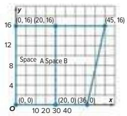
6. **Multiple Choice** A polygon has vertices *J*(2, 3), *K*(4, 3), *L*(4, 7), and *M*(2, 7). What is the area of the polygon? (Example 3)

- (A) 8 square units
- (B) 10 square units
- (C) 12 square units
- (D) 16 square units

Lesson 8-5 • Polygons on the Coordinate Plane 477

Apply *indicates multi-step problem

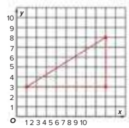
*7. Ethan wants to open a pet store in a town mall and is considering the two spaces shown. On the map, one unit is equal to one foot. Space A has a monthly rental cost of $14.75 per square foot. Space B has a monthly rental cost of $14.50 per square foot. Ethan wants to pay the lower total monthly rental price. Which location should he choose to rent? Write an argument that can be used to justify your solution.



Space A; Sample answer: The monthly rental price of Space A is $4,720. The monthly rental price of Space B is $4,756. $4,720 is less than $4,756.

Higher-Order Thinking Problems

8. Draw and label a triangle on the coordinate plane that has an area of 20 square units.
Sample answer:



9. MP **Reason Inductively** A certain rectangle has a perimeter of 10 units and an area of 6 units. Two of the vertices have coordinates (1, 7) and (1, 4). Find the two missing coordinates.
Sample answer: (3, 4) and (3, 7)

10. MP **Persevere with Problems** Mrs. Palmer is placing a retaining wall around a garden. The coordinates of the vertices of the wall are (1, 1), (1, 5), (6, 5), and (6, 1). If each grid square has a length of 2 feet, what is the perimeter of the area? Write an argument that can be used to justify your solution.
36 ft; Sample answer: The perimeter of the figure is $4 + 5 + 4 + 5$ or 18 units. Since each grid square represents 2 feet, then 18×2 feet is 36 feet.

11. MP **Find the Error** Rectangle *ABCD* has vertices *A*(2, 1), *B*(2, 7), *C*(10, 7), and *D*(10, 1). A classmate states that the perimeter of the rectangle is 16 units. Find the student's mistake and correct it.
Sample answer: The student subtracted $10 - 7$ and $7 - 2$ to find lengths 3 and 5. The student should have subtracted $7 - 1$ and $10 - 2$ to find lengths 6 and 8. The perimeter is 28 units.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 10, students will find the perimeter of the garden area. Encourage students to plan a solution pathway that can be implemented to solve the problem.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students will find the missing coordinates. Encourage students to determine what they need to do in order to find the missing coordinates.

In Exercise 11, students will find another student's mistake and correct it. Encourage students to support their answer with a well-constructed explanation.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.
Use with Exercise 7 Have students work in groups of 3–4 to solve the problem in Exercise 7. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class.

Clearly explain your strategy.
Use with Exercise 9 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would find the missing coordinates, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of finding the area of parallelograms, triangles, and trapezoids. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

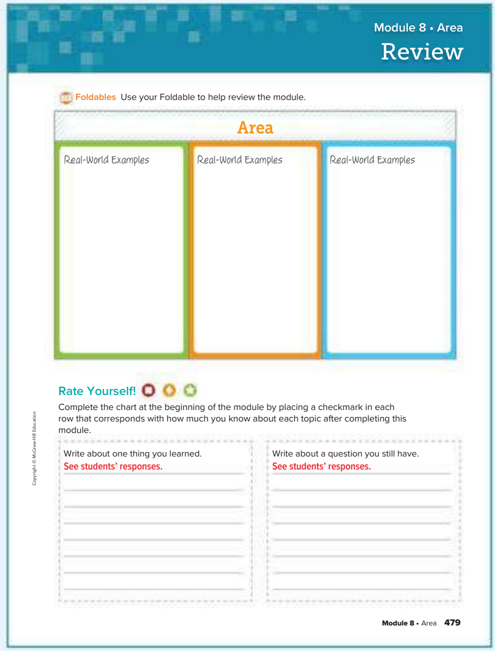
Assessment Resources

Put It All Together: Lessons 8-1, 8-2, 8-3, and 8-4
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Geometry**.

- Area of Parallelograms and Triangles
- Area of Trapezoids and Composite Figures
- Polygons in the Coordinate Plane


Module 8 • Area

Review

Foldables Use your Foldable to help review the module.

Area		
Real-World Examples	Real-World Examples	Real-World Examples

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

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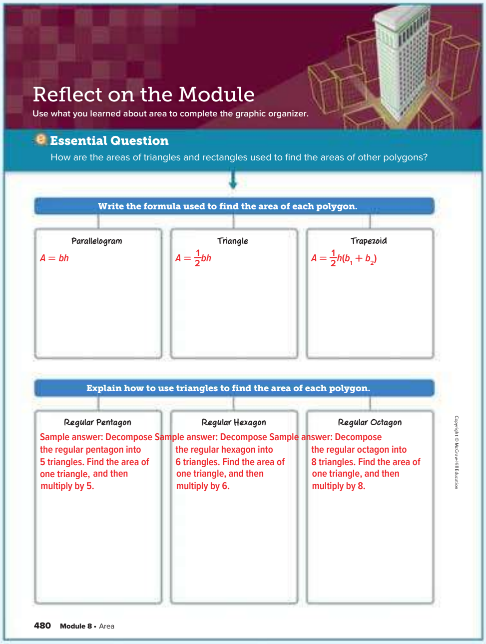


Reflect on the Module

Use what you learned about area to complete the graphic organizer.

Essential Question

How are the areas of triangles and rectangles used to find the areas of other polygons?

Write the formula used to find the area of each polygon.

Parallelogram	Triangle	Trapezoid
$A = bh$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(b_1 + b_2)$

Explain how to use triangles to find the area of each polygon.

Regular Pentagon	Regular Hexagon	Regular Octagon
Sample answer: Decompose the regular pentagon into 5 triangles. Find the area of one triangle, and then multiply by 5.	Sample answer: Decompose the regular hexagon into 6 triangles. Find the area of one triangle, and then multiply by 6.	Sample answer: Decompose the regular octagon into 8 triangles. Find the area of one triangle, and then multiply by 8.

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How are the areas of triangles and rectangles used to find the areas of other polygons? See students' graphic organizers.

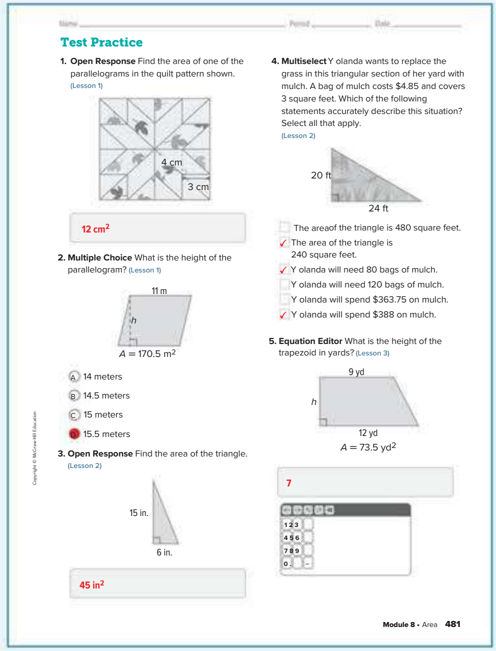
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–9 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	2, 6
Multiselect	Multiple answers may be correct. Students must select all correct answers.	4
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	5, 9
Open Response	Students construct their own response in the area provided.	1, 3, 7, 8

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.EE.A.2	8-1, 8-2, 8-3	1–6
6.EE.A.2.C	8-1, 8-2, 8-3	1, 4, 6
6.G.A.1	8-1, 8-2, 8-3, 8-4	1–8
6.G.A.3	8-5	9


Name ______ Period ______ Date ______

Test Practice

1. **Open Response** Find the area of one of the parallelograms in the quilt pattern shown. (Lesson 1)

12 cm^2

2. **Multiple Choice** What is the height of the parallelogram? (Lesson 1)

A) 14 meters
B) 14.5 meters
C) 15 meters
D) 15.5 meters

3. **Open Response** Find the area of the triangle. (Lesson 2)

45 in^2

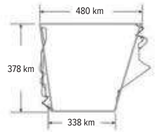
4. **Multiselect** Y olanda wants to replace the grass in this triangular section of her yard with mulch. A bag of mulch costs $4.85 and covers 3 square feet. Which of the following statements accurately describe this situation? Select all that apply. (Lesson 2)

- ☐ The areaof the triangle is 480 square feet.
- ✓ The area of the triangle is 240 square feet.
- ✓ Y olanda will need 80 bags of mulch.
- ☐ Y olanda will need 120 bags of mulch.
- ☐ Y olanda will spend $363.75 on mulch.
- ✓ Y olanda will spend $388 on mulch.

5. **Equation Editor** What is the height of the trapezoid in yards? (Lesson 3)

7

6. Multiple Choice The shape of the park resembles a trapezoid. Which of the following is the approximate area of the park? (Lesson 3)



(A) 109,242 km^2

(B) 154,602 km^2

(C) 231,903 km^2

(D) 309,204 km^2

7 .Open Response A tapestry is shaped like a regular hexagon. (Lesson 4)

A. Explain how you can decompose the hexagon in order to find its area.

Sample answer: I would decompose the hexagon into two trapezoids.

B. Find the area of the tapestry.

261 cm^2

8. Open Response Kim wants to replace the area covered by this rug with hardwood flooring. The rug is shaped like a regular octagon with 3-foot sides. (Lesson 4)

A. What is the area of the floor?

(A) 38.2 ft^2

(B) 40.1 ft^2

(C) 43.2 ft^2

(D) 45.0 ft^2

B. If hardwood flooring costs $9.50 per square foot, how much will she spend to resurface the floor? Explain why you need to round the area up to the nearest whole square foot in order to calculate the cost.

$418; Sample answer: It is necessary to round the area upto 44 ft^2 because hardwood flooring is sold by the whole square foot, and 43 square feet would not be enough to cover the area of the floor .

9. Equation Editor A rectangle has vertices *A*(1,2), *B*(1, 9), *C*(7 , 9), and *D*(7 , 2). Find the perimeter of the rectangle in units. (Lesson 5)

26

482 Module 8 • Area

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can you describe the size of a three-dimensional figure?
See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of cereal boxes to introduce the idea of volume and surface area. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.

Module 9
Volume and Surface Area

Essential Question
How can you describe the size of a three-dimensional figure?

What Will You Learn?
Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
finding volume of rectangular prisms						
finding missing dimensions of rectangular prisms						
making nets to represent rectangular prisms						
finding surface areas of rectangular prisms						
making nets to represent triangular prisms						
finding surface areas of triangular prisms						
making nets to represent pyramids						
finding surface areas of pyramids						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about volume and surface area.

Module 9 • Volume and Surface Area 483

Interactive Student Presentation

Module 9

Volume and Surface Area

Module Goal

Find volume of rectangular prisms and surface area of triangular and rectangular prisms and pyramids.

Focus

Domain: Geometry

Supporting Cluster(s): 6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

Standards for Mathematical Content:

6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Also addresses 6.EE.B.6.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- find the area of triangles and quadrilaterals
- fluently perform all four operations with positive rational numbers
- solve one-step equations

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students found areas of parallelograms, triangles, trapezoids, regular polygons, and polygons on the coordinate plane.
6.G.A.1, 6.G.A.3, 6.EE.A.2.C

Now
Students find volume of rectangular prisms and surface area of triangular and rectangular prisms and pyramids.
6.G.A.2, 6.G.A.4

Next
Students will solve problems involving volume and surface area of prisms and pyramids.
7.G.B.6

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of polygons and area to develop *understanding* of volume and surface area. They use this understanding to build *fluency* with finding the volume of rectangular prisms, and making and using nets to find the surface area of rectangular prisms, triangular prisms, and pyramids. They also *apply* their understanding of volume and surface area to solve multi-step, real-world problems.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

EXPLORE → LEARN → EXAMPLE & PRACTICE

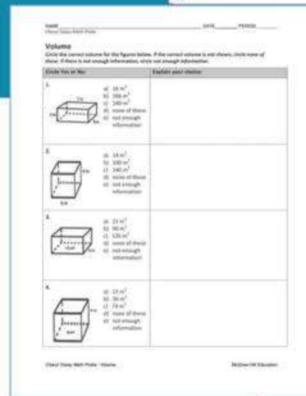
Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
9-1	Volume of Rectangular Prisms	6.G.A.2, *Also addresses 6.EE.B.6*	2	1
9-2	Surface Area of Rectangular Prisms	6.G.A.4	3	1.5
Put It All Together 1: Lessons 9-1 and 9-2			0.5	0.25
9-3	Surface Area of Triangular Prisms	6.G.A.4	3	1.5
9-4	Surface Area of Pyramids	6.G.A.4	2	1
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	13.5	6.75

Module Resource

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe
Volume



Correct Answers: **1.** c; **2.** e; **3.** e; **4.** b

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students determine the correct volume for each figure, and explain their choice.

Targeted Concepts Reason about volume both conceptually and procedurally when given the side lengths, or the area of the base and the height.

Targeted Misconceptions

- Students may only understand volume as the product of three numbers.
- Students may incorrectly find the volume by adding the side lengths, or by confusing volume with surface area.
- Students may not understand volume as multiple layers of the base.

Assign the probe after Lesson 1.

Collect and Assess Student Work

If the student selects...	**Then** the student likely...
1. a **2.** a **3.** a **4.** a	applied a flawed procedure by adding the length, width, and height, instead of multiplying.
1. b **2.** c **3.** c **4.** c	applied a flawed procedure by determining or trying to determine the surface area, instead of volume.
3. b	does not understand how the measures provided relate to finding the volume.
2. b	makes assumptions about measurements based on the drawing of the shape (i.e., assumes the area of the base is found by finding 5×5).

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Perimeters, Areas, and Volumes
- Lesson 1, Examples 1–2

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ cubic units
- ☐ lateral face
- ☐ net
- ☐ prism
- ☐ pyramid
- ☐ rectangular prism
- ☐ slant height
- ☐ surface area
- ☐ three-dimensional figure
- ☐ triangular prism
- ☐ volume

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1	Example 2
Multiply rational numbers. Find 12 × 3.5 × 18.	**Evaluate numerical expressions.** Evaluate (8 × 6) + (3 × 9).
12 × 3.5 × 18 = 42 × 18 Multiply 12 and 3.5. = 756 Multiply by 18.	(8 × 6) + (3 × 9) = 48 + 27 Multiply. = 75 Add.

Quick Check

1. Find 12 × 2.2 × 17.5. 462	**2.** Evaluate (12.5 × 40) + (16.25 × 6). 597.5

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

① ②

484 Module 9 • Volume and Surface Area

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define Volume is the amount of space inside a three-dimensional figure.

Example A rectangular prism has a length of 4.5 feet, a width of 1.5 feet, and a height of 6.5 feet. The volume of the prism is found by multiplying the length, width, and height, which is 43.875 cubic feet.

Ask Why do you think that volume is measured in cubic units? Sample answer: There are three dimensions (length, width, and height). So, the units will be cubed since the dimensions are multiplied.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- writing and solving one-step equations
- performing operations with rational numbers
- finding area of rectangles and triangles

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Perimeters, Areas, and Volumes** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Promote Growth Over Speed

Learning requires time and effort – time to think, reason, make mistakes, and learn from your mistakes and the mistakes of others. Ultimately, it's about the deep connections students make in their thinking and reasoning that matter more than the speed at which a problem is solved.

How Can I Apply It?

Have students complete the **What Will You Learn?** chart in their *Interactive Student Edition* before beginning each module and note the topics they don't know very well. At the end of each module, have them follow the **Rate Yourself!** directions in the module review by returning to this chart to view how their knowledge has increased throughout the module. Encourage them to celebrate the topics with which their knowledge has increased, and take steps to strategize over how they can continue to grow in the topics about which they still might have questions.

Learn Volume

Objective

Students will learn about volume of prisms.

Teaching Notes

SLIDE 1

Be sure students understand the vocabulary presented: *three-dimensional figure*, *prism*, *rectangular prism*, *volume*, and *cubic units*. Students previously learned about volume of rectangular prisms, involving whole-number measurements. Remind them that they can find the volume of a rectangular prism with whole-number measurements by packing the prism with unit cubes. To preview what students are about to learn, draw two rectangular prisms on the board - one with whole-number measurements and one with fractional measurements. Ask students how they can use reasoning about unit cubes to find the volume of the prism with whole-number measurements. Then ask them to make a conjecture about how they might be able to use reasoning to find the volume of the prism with fractional measurements.

DIFFERENTIATE

Reteaching Activity AL

Students may have difficulty counting cubes that they cannot see when finding the volume of a rectangular prism. It may be helpful to give students a rectangular prism that has a grid of cubes drawn on the faces. Students can turn the prism in a variety of ways to count the cubes. As students examine the prisms, ask them the following questions.

How many cubes are in the bottom layer?

How many cubes are in the top layer?

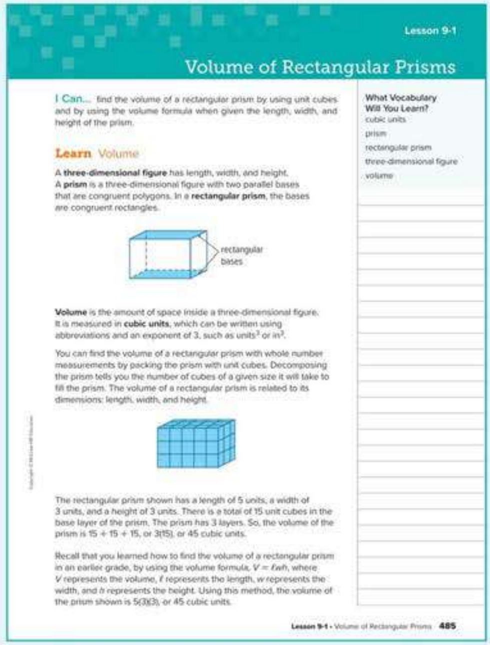
How many cubes are on the front face?

How many cubes are on the back face?

How many cubes are on the side faces?

How many layers are there?

How many cubes are there in all?



Lesson 9-1

Volume of Rectangular Prisms

I Can... find the volume of a rectangular prism by using unit cubes and by using the volume formula when given the length, width, and height of the prism.

What Vocabulary Will You Learn?
cubic units
prism
rectangular prism
three-dimensional figure
volume

Learn Volume

A **three-dimensional figure** has length, width, and height. A **prism** is a three-dimensional figure with two parallel bases that are congruent polygons. In a **rectangular prism**, the bases are congruent rectangles.

rectangular bases

Volume is the amount of space inside a three-dimensional figure. It is measured in **cubic units**, which can be written using abbreviations and an exponent of 3, such as $units^3$ or in^3.

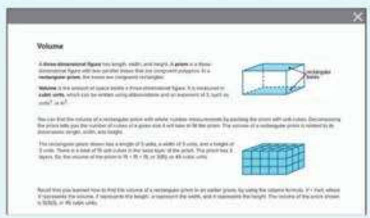
You can find the volume of a rectangular prism with whole number measurements by packing the prism with unit cubes. Decomposing the prism tells you the number of cubes of a given size it will take to fill the prism. The volume of a rectangular prism is related to its dimensions: length, width, and height.

The rectangular prism shown has a length of 5 units, a width of 3 units, and a height of 3 units. There is a total of 15 unit cubes in the base layer of the prism. The prism has 3 layers. So, the volume of the prism is 15 + 15 + 15, or 3(15), or 45 cubic units.

Recall that you learned how to find the volume of a rectangular prism in an earlier grade, by using the volume formula, $V = \ell wh$, where V represents the volume, ℓ represents the length, w represents the width, and h represents the height. Using this method, the volume of the prism shown is 5(3)(3), or 45 cubic units.

Lesson 9-1 • Volume of Rectangular Prisms 485

Interactive Presentation



Learn, Volume

Lesson 9-1

Volume of Rectangular Prisms

6.G.A.2

LESSON GOAL

Students will find and use the volume of rectangular prisms.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Volume
Learn: Volume of a Rectangular Prism
Example 1: Find the Volume of a Rectangular Prism
Learn: Find Missing Dimensions
Example 2: Find Missing Dimensions
Apply: Comparisons

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

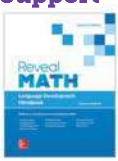
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Volume of a Pyramid		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 49 of the *Language Development Handbook* to help your students build mathematical language related to volume of rectangular prisms.



ELL You can use the tips and suggestions on page T49 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Supporting Cluster(s): In this lesson, students address supporting cluster **6.G.A** by finding and using the volume of rectangular prisms.
Standards for Mathematical Content: **6.G.A.2,** *Also addresses 6.EE.B.6*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6**

Coherence

Vertical Alignment

Previous
Students used the coordinate plane to draw and find attributes of polygons.
6.G.A.3

Now
Students find and use the volume of rectangular prisms.
6.G.A.2

Next
Students will make nets and use them to find the surface area of rectangular prisms.
6.G.A.4

Rigor

The Three Pillars of Rigor

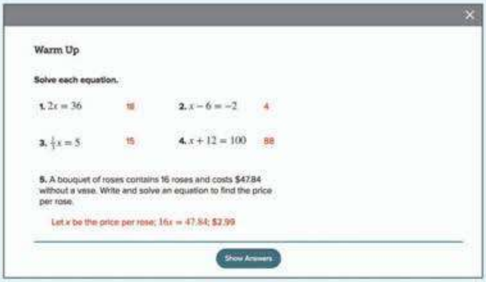
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of attributes of polygons to develop *understanding* of volume of rectangular prisms. They learn how to use cubes and the volume formula to build *fluency* with finding the volume of rectangular prisms with fractional edge lengths, and finding a missing dimension given the volume. They *apply* their understanding of volume to solve multi-step, real-world problems.

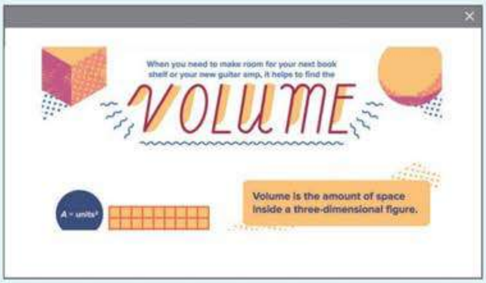
Mathematical Background

A *prism* is a *three-dimensional* figure with two congruent parallel bases. A *rectangular prism* has rectangles on all sides. *Volume* is the measure of the amount of space in a three-dimensional figure. The volume of a rectangular prism is $V = Bh$, where B is the area of the base and h is the height. For a rectangular prism, the volume is $V = \ell wh$, where ℓ is the length of the base, w is the width, and h is the height.

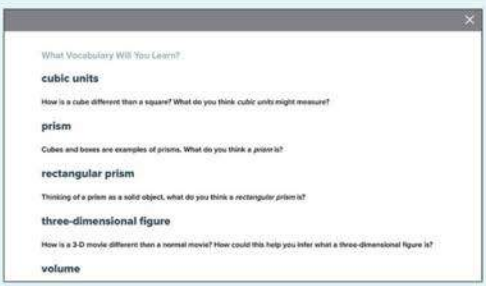
Interactive Presentation



Warm Up



Launch the Lesson



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- solving one-step equations (Exercises 1–4)
- writing and solving one-step equations (Exercise 5)

Answers

1. 18
2. 4
3. 15
4. 88
5. Let x be the price per rose; $16x = 47.84$; $2.99

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about volume, using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- How is a cube different from a square? What do you think *cubic units* might measure? Sample answer: A cube has three dimensions, length, width and height, and they are all the same. A square only has two dimensions, length and width, and they are the same; volume.
- Cubes and boxes are examples of prisms. What do you think a *prism* is? Sample answer: a three-dimensional object made of rectangles or squares
- Thinking of a prism as a three-dimensional object, what do you think a *rectangular prism* is? Sample answer: a three-dimensional figure with 6 rectangular sides
- How is a 3-D movie different than a normal movie? How could this help you infer what a three-dimensional figure is? Sample answer: 3-D movies pop out of the screen. Three-dimensional objects are objects that have height, depth, and width.

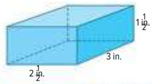
Your Notes

Math History Minute

Benjamin Banneker (1731–1806) was an African-American mathematician, astronomer, inventor, and writer. When he was 22, he used his own drawings and calculations to construct a working clock that was made almost entirely out of wood.

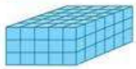
Learn Volume of a Rectangular Prism

You can find the volume of a rectangular prism with fractional measurements using different methods.



Method 1 Use unit cubes.

You can pack a rectangular prism with unit cubes. A cube is a special rectangular prism with all sides congruent. The volume of a cube is found by cubing the side length.



Step 1 Find the number of unit cubes needed to fill the prism. Each unit cube has a side length of $\frac{1}{2}$ inch.

Length The length of the prism is $2\frac{1}{2}$ inches. So, the length is composed of $2\frac{1}{2} \div \frac{1}{2}$ or 5 unit cubes.

Width The width of the prism is 3 inches. So, the width is composed of $3 \div \frac{1}{2}$ or 6 unit cubes.

Height The height of the prism is $1\frac{1}{2}$ inches. So, the height is composed of $1\frac{1}{2} \div \frac{1}{2}$ or 3 unit cubes.

The base layer of the prism contains 5 × 6, or 30 unit cubes. There are three total layers in the prism. So, the rectangular prism contains 30 × 3, or 90 unit cubes.

Step 2 Find the volume of one unit cube.

$V = s^3$	Volume of a cube with side length s.
$= \left(\frac{1}{2}\right)^3$	Replace s with $\frac{1}{2}$.
$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	Definition of exponent
$= \frac{1}{8}$	Multiply. The volume of each cube is $\frac{1}{8}$ in^3.

Step 3 Multiply the volume of each unit cube by the total number of unit cubes, 90.

$V = 90\left(\frac{1}{8}\right)$	There are 90 unit cubes, each with a volume of $\frac{1}{8}$ in^3.
$= 11\frac{1}{4}$	Multiply. The volume of the prism is $11\frac{1}{4}$ in^3.

(continued on next page)

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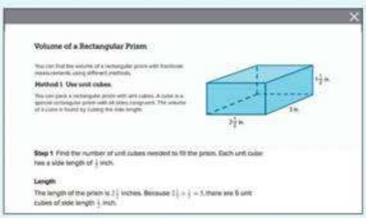
Interactive Presentation



Learn, Volume of a Rectangular Prism, Slide 1 of 3

CLICK

On Slide 1, students move through the steps to use unit cubes to find the volume of the rectangular prism.

Learn Volume of a Rectangular Prism

Objective

Students will understand different methods for finding the volume of a rectangular prism with fractional edge lengths.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the formula for the volume of a rectangular prism and to understand why the formula for the area of the base, ℓw, can be substituted for B.

Teaching Notes

SLIDE 1

Have students imagine the rectangular prism composed of multiple unit cubes, packed together so that there are no gaps and no overlap. Ask students to respond to the following questions.

Why does the unit cube have a side length of $\frac{1}{2}$ inch? The fractional measurements of the prism's dimensions are in multiples of $\frac{1}{2}$ inch.

How can you find the number of unit cubes to fill one layer of the prism? Five unit cubes can fit along the side labeled $2\frac{1}{2}$ inches, and 6 unit cubes can fit along the side labeled 3 inches. So, one layer can hold 5(6), or 30 unit cubes.

How can you find the number of layers? Three unit cubes can fit along the side labeled $1\frac{1}{2}$ inches. So, there are three layers.

SLIDE 2

Ask students to compare and contrast the two formulas shown for finding the volume of a rectangular prism. They should be able to explain how B is equivalent to ℓw. Point out that the capital letter B represents the *area* of the base of the prism. In many area formulas, lowercase b represents the length of the side of the base. Ask students to explain how packing a rectangular prism with unit cubes to find the volume corresponds to using the volume formula. Students should note that the area of the base B represents one layer of unit cubes. Multiplying the number of unit cubes in one layer (B) by the total number of layers (h) gives the volume (V).

Talk About It!

SLIDE 3

Mathematical Discourse

The formula $V = Bh$ can be used to find the volume of any right prism. You know that for a right rectangular prism the area of the base, B, is represented by the expression ℓw. Think of a prism that doesn't have a rectangular base, such as a triangular prism. What expression could you use to represent the area of the base? Sample answer: A prism with a triangular base would use the expression $\frac{1}{2}bh$ to represent B.

Example 1 Find the Volume of a Rectangular Prism

Objective

Students will use unit cubes and the volume formula to find the volume of a rectangular prism with fractional edge lengths.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to reason about how packing a prism with unit cubes can help them find the volume, and how that method corresponds to using the volume formula.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to be precise in their explanation of why the volume of a cube can be found by cubing the side length.

Questions for Mathematical Discourse

SLIDE 2

AL Is each mini sugar cube a unit cube? Explain. yes; Sample answer: Because the measure of each side has a one in the numerator, it represents one unit so it is a unit cube.

AL How will you determine how many cubes will fit along the length of the box? Sample answer: First I need to divide the length of the box, $3\frac{1}{2}$ inches, by the length of the mini sugar cube, $\frac{1}{4}$ inch.

OL The length of one side of the mini sugar cube is $\frac{1}{4}$ inch. How can you find the volume of a unit cube? Because it is a cube, I can find the volume by cubing the length of a side.

OL Why do you multiply the volume of one sugar cube by the total number of sugar cubes to find the volume? Sample answer: The box is completely filled with cubes so the volume of the box is equal to the volume of one cube times the total number of cubes.

BL Is the volume of the box 2,016 cubic inches? Explain. no; Sample answer: 2,016 cubes will fit in the box. Each cube represents $\frac{1}{64}$ cubic inch, not 1 cubic inch.

BL How many cubic inches of empty space would be in the box if there were only 1,500 cubes? 8.0625 cubic inches

(continued on next page)

Method 2 Use the formula.

The formula for the volume of a right prism is $V = Bh$ where B represents the area of the base of the prism and h represents the height of the prism. In a rectangular prism the base is a rectangle, so $B = \ell w$. So, the volume of a right rectangular prism can also be found using the formula $V = \ell wh$.

$V = \ell wh$	Volume formula
$V = 2\frac{1}{2} \cdot 3 \cdot 1\frac{1}{2}$	$\ell = 2\frac{1}{2}, w = 3, h = 1\frac{1}{2}$
$V = 11\frac{1}{4}$	Multiply.

So, using either method, the volume of the rectangular prism is $11\frac{1}{4}$ cubic inches.

Example 1 Find the Volume of a Rectangular Prism

Mini sugar cubes measure $\frac{1}{4}$ inch on each side. The box shown is packed full of sugar cubes.

What is the volume of the box?

Method 1 Use unit cubes.

Step 1 Find the number of mini sugar cubes.

Each sugar cube has a side length of $\frac{1}{4}$ inch.

Length of Prism: $3\frac{1}{2}$ inches

Because $3\frac{1}{2} \div \frac{1}{4} = 14$, there are 14 mini sugar cubes that fit along the length of the prism.

Width of Prism: $1\frac{1}{2}$ inches

Because $1\frac{1}{2} \div \frac{1}{4} = 6$, there are 6 mini sugar cubes that fit along the width of the prism.

Height of Prism: 6 inches

Because $6 \div \frac{1}{4} = 24$, there are 24 mini sugar cubes that fit along the height of the prism.

The base layer of the prism contains 14×6, or 84 mini sugar cubes. There are 24 total layers of unit cubes in the prism. So, the rectangular prism contains 84 $\times$ 24, or 2,016 total mini sugar cubes.

(continued on next page)

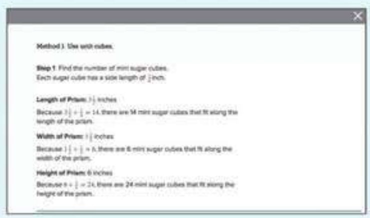
Talk About It! The formula $V = Bh$ can be used to find the volume of any right prism. You know that for a right rectangular prism, the area of the base, B, is represented by the expression ℓw. Think of a prism that doesn't have a rectangular base, such as a triangular prism. What expression can you use to represent the area of the base?

Sample answer: A prism with a triangular base would use the expression $\frac{1}{2}bh$ to represent B.

Think About It! Estimate the volume of the box of mini sugar cubes.

See students' responses.

Interactive Presentation



Example 1, Find the Volume of a Rectangular Prism, Slide 2 of 5

CLICK

On Slide 2, students move through the steps to find the number of cubes needed to fill the box.

TYPE

On Slide 2, students determine the volume by using unit cubes (Method 1).

Step 2 Find the volume of one mini sugar cube.

$V = s^3$ — Volume of a cube with side length s.

$= \left(\frac{1}{4}\right)^3$ — Replace s with $\frac{1}{4}$.

$= \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$ — Definition of exponent

$= \frac{1}{64}$ — Multiply. The volume of each cube is $\frac{1}{64}$ in^3.

Step 3 Multiply the volume of each cube by the total number of unit cubes, 2,016.

$V = 2{,}016\left(\frac{1}{64}\right)$ — There are 2,016 unit cubes, each with a volume of $\frac{1}{64}$ in^3.

$= 31\frac{1}{2}$ — Multiply. The volume of the prism is $31\frac{1}{2}$ in^3.

Method 2 Use the volume formula.

The formula for the area of a right rectangular prism is $V = Bh$ or $V = \ell wh$.

Substitute the dimensions of the box for the variables in the formula and multiply.

$V = \ell wh$ — Write the volume formula.

$V = 3\frac{1}{2} \cdot 1\frac{1}{2} \cdot 6$ — Replace ℓ with $3\frac{1}{2}$, w with $1\frac{1}{2}$, and h with 6.

$V = 31\frac{1}{2}$ in^3 — Multiply.

So, using either method, the total volume of the box is $31\frac{1}{2}$ cubic inches.

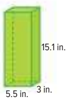
Talk About It!

Use what you know about the formula for the volume of a prism to explain why the volume of a cube can be found by cubing the side length.

Sample answer: The side lengths of a cube are equal, so if I multiply the three dimensions, I am multiplying the same number 3 times. This is the same as cubing the side length.

Check

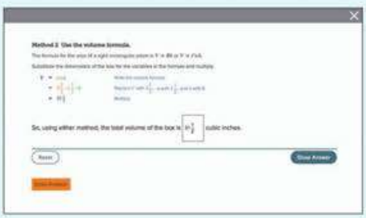
Find the volume of the prism. **249.15 in^3**



Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Find the Volume of a Rectangular Prism, Slide 3 of 5

TYPE

On Slide 3, students use the volume formula to find the volume of the box (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find the Volume of a Rectangular Prism *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL In the problem, why is ℓw used in place of B? Sample answer: B represents the area of the base of the three-dimensional figure. In the figure, the base is a rectangle, so $B = \ell w$.

OL Why are the two formulas, $V = Bh$ and $V = \ell wh$, equivalent? Sample answer: Because the base of the figure is a rectangle, $B = \ell w$, so the two formulas are equal.

BL Suppose the manufacturer wants a larger box. Which side should the company double to get the largest volume? Explain your reasoning. any side; Sample answer: When you double any side, it is the same as multiplying the volume by 2. So when you double the length, the volume is 63 cubic inches. If you double the height, the volume is also 63 cubic inches.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Find Missing Dimensions

Objective

Students will learn how to find a missing dimension in a rectangular prism, given the volume.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the known and unknown values and how using an equation can help them find the unknown value.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find a missing dimension in a rectangular prism.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the prism and its given dimensions are shown. Ask students to work with a partner to come up with a strategy for finding the unknown height of the prism. They may use any strategy they wish, but must be prepared to explain their strategy and defend why it works. Some students may use an equation as the animation suggests. Other students may use reasoning and say that the area of the base is 15 square feet. Since the volume is the product of the area of the base and the height, divide the volume by the area of the base to find the height. Ask students to compare strategies to understand the correspondences between them.

Talk About It!

SLIDE 2

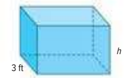
Mathematical Discourse

How can understanding variables and equations help you solve geometry problems? Sample answer: If there is an unknown value in a geometry problem involving area or volume, I can write an equation using variables, and solve for the unknown value.

Learn Find Missing Dimensions

When you know the volume of a rectangular prism and 2 out of 3 dimensions, you can write and solve an equation to find the missing dimension. Using the volume formula, replace the variables with the known values. Then solve the equation to find the unknown value.

Go Online Watch the animation to learn how to find the missing dimension for the rectangular prism shown.



$V = 60\text{ ft}^3$

The rectangular prism shown has a volume of 60 cubic feet. The width of the prism is 3 feet and the length is 5 feet. The height of the prism is unknown.

T o find the unknown height, you can use the formula for volume of a rectangular prism.

$V = \ell wh$	Write the volume formula.
$60 = (5)(3)h$	Replace V with 60, ℓ with 5, and w with 3.
$60 = 15h$	Multiply.
$\frac{60}{15} = \frac{5h}{15}$	Division Property of Equality
$4 = h$	Simplify.

So, the height of the rectangular prism is 4 feet.

Talk About It!
How can understanding variables and equations help you solve geometry problems?

Sample answer: If there is an unknown value in a geometry problem involving area or volume, I can write an equation using variables, replace the variables with the known values, and solve for the unknown value.

Interactive Presentation

Learn, Find Missing Dimensions, Slide 1 of 2

WATCH

On Slide 1, students watch an animation to learn how to use an equation to find a missing dimension in a rectangular prism.

Think About It!
What formula can you use to solve this problem?

$V = \ell wh$

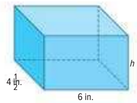
Talk About It!
Why is the unit for the height of the prism inches and not cubic inches or square inches? Explain your reasoning.

Sample answer: Height is a measurement of length, not of volume. Length is given in units, not square or cubic units.

Example 2 Find Missing Dimensions

The rectangular prism shown has a volume of $94\frac{1}{2}$ cubic inches.

What is the height of the prism?



Step 1 Identify the known dimensions.

You know the length, width, and volume. You need to find the height.

Step 2 Find the missing dimension.

$V = \ell wh$	Volume of a rectangular prism
$94\frac{1}{2} = 6 \cdot 4\frac{1}{2} \cdot h$	Substitute the known quantities.
$94\frac{1}{2} = 27h$	Multiply.
$\frac{94\frac{1}{2}}{27} = \frac{27h}{27}$	Divide.
$3\frac{1}{2} = h$	Simplify.

So, the height of the prism is $3\frac{1}{2}$ inches.

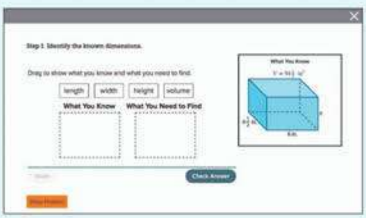
Check

Find the height of a rectangular prism with a volume of 126 cubic inches, a width of $7\frac{7}{8}$ inches, and a length of 2 inches. 8 in.

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 2, Find Missing Dimensions, Slide 2 of 5.

DRAG & DROP

On Slide 2, students drag the terms to identify what they know and what they need to find.

TYPE

On Slide 3, students determine the height of the rectangular prism.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find Missing Dimensions

Objective

Students will find a missing dimension in a rectangular prism, given the volume.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to engage in the drag and drop activity in order to make sense of the dimensions they are given, and which dimension they are asked to find.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know what variable you need to solve for in the formula $V = \ell wh$? Sample answer: I know the values for V, ℓ, and w, so I need to find h.

OL Why is it important to identify the given values? Sample answer: I need to identify what variables in the formula have numeric values so I know what variable to solve for.

BL Suppose you were not given values for the height or the width of the prism. Could you still find the length? Explain your reasoning. no; Sample answer: If I didn't know h and w, I would not have enough information to find the length. When I solve an equation, I can only have one unknown.

SLIDE 3

AL How can you check your value of h? Sample answer: I can substitute all of the values back into the equation to make sure the left side of the equation is equivalent to the right side of the equation.

OL In the third step of the solution, where did the value 27 come from? 27 is the product of 6 and $4\frac{1}{2}$

BL If the length and the width of the prism remained the same but the volume doubled, how would that affect the height? Explain. the height would double; Sample answer: The right side of the equation stays the same if the volume doubles. The left side becomes 189 cubic inches, so when I divide each side by 27, the height is 7, which is two times $3\frac{1}{2}$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Comparisons

Objective

Students will come up with their own strategy to solve an application problem involving comparing the prices of different sizes of theater popcorn.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you find the volume of each container?
- What is the best way to compare the three prices?
- What do you need to do to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Comparisons

A movie theater sells three different-sized boxes of popcorn. If the boxes are rectangular prisms, which size of popcorn is the best buy?

Size	Length (in.)	Width (in.)	Height (in.)	Price ($)
Small	$5\frac{1}{2}$	4	$8\frac{1}{4}$	4.50
Medium	$6\frac{3}{4}$	5	$10\frac{1}{2}$	5.75
Large	$10\frac{1}{4}$	6	$11\frac{1}{2}$	7.00

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

the large box; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

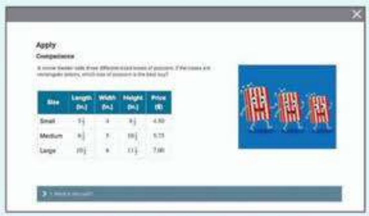
Talk About It!

Suppose the dimensions of each box doubled. Would the answer remain the same? Explain your reasoning.

yes; Sample answer: Changing the dimensions of each box by the same factor results in the volume increasing by the same ratio.

Lesson 9-1 • Volume of Rectangular Prisms 491

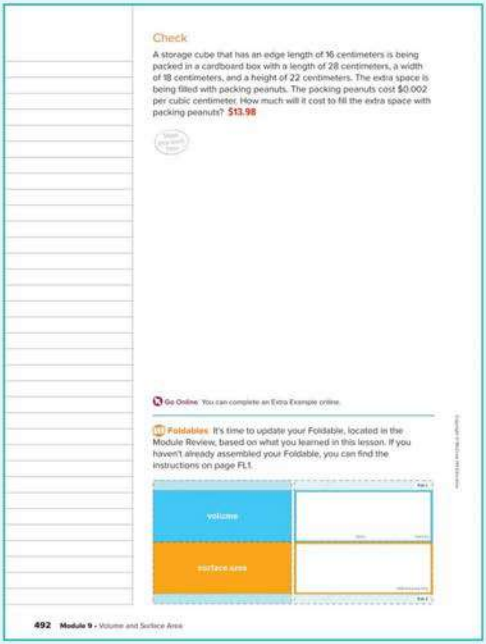
Interactive Presentation



Apply, Comparisons

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

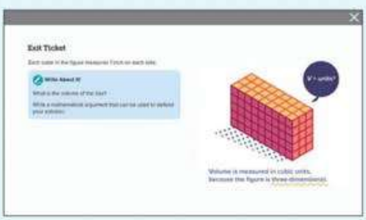
A storage cube that has an edge length of 16 centimeters is being packed in a cardboard box with a length of 28 centimeters, a width of 18 centimeters, and a height of 22 centimeters. The extra space is being filled with packing peanuts. The packing peanuts cost \$0.002 per cubic centimeter. How much will it cost to fill the extra space with packing peanuts? \$13.98

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

492 Module 9 • Volume and Surface Area

Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add the formula that is used to find the volume of a rectangular prism. Then give an example of how to use that formula to find the volume of a rectangular prism. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can you describe the size of a three-dimensional figure?

In this lesson, students learned how to find the volume of rectangular prisms. Encourage them to discuss with a partner what it means to describe the size of a geometric figure, and how volume might be considered one way to do that. For example, they may say that volume describes the amount of space inside a three-dimensional figure. While the term *size* can mean many things, describing the volume of a figure is one way to describe the size of that figure.

Exit Ticket

Refer to the Exit Ticket slide. What is the volume of the box? Write a mathematical argument that can be used to defend your solution. 135 in^3; Sample answer: The figure is 9 cubes long, 3 cubes wide, and 5 cubes tall. So, the volume of the figure is 9(3)(5) or 135 cubic inches.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–7 odd, 8–11
- Extension: Volume of a Pyramid
- ALEKS® Volume of Rectangular Prisms

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 7, 9, 10
- Extension: Volume of a Pyramid
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Area of Parallelograms, Triangles, and Trapezoids

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Area of Parallelograms, Triangles, and Trapezoids

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	T opic	Exercises
1	find the volume of a rectangular prism with fractional edge lengths	1, 2
1	find a missing dimension in a rectangular prism, given the volume	3, 4
2	extend concepts learned in class to apply them in new contexts	5, 6
3	solve application problems involving volume of rectangular prisms	7
3	higher-order and critical thinking skills	8–11

Common Misconception

On exercises where a missing dimension is sought, some students may treat the volume as one of the linear dimensions. For example, in Exercise 3, students may substitute 52 for the width instead of the volume. Have students construct a chart like the one below and fill in the missing values, including a "?" for the value that is asked for in the problem. That value represents the unknown. Completing a chart like this may help students correctly set up the equation to solve for the unknown.

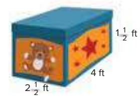
$V =$	ℓ	w	h
52	$6\frac{1}{2}$	?	2

Name ______ Period ______ Date ______

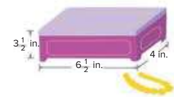
Practice

Go Online Y ou can complete your homework online.

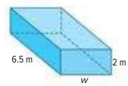
1. Geneva's younger brother has a toy box that is shaped like a rectangular prism with the dimensions shown. What is the volume of the toy box? (Example 1) **$15\ ft^3$**



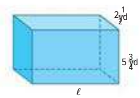
2. Roy made a jewelry box in the shape of a rectangular prism with the dimensions shown. What is the volume of the jewelry box? (Example 1) **$91\ in^3$**



3. The rectangular prism shown has a volume of 52 cubic meters. What is the width of the prism? (Example 2) **4 m**



4. The rectangular prism shown has a volume of 115 cubic yards. What is the length of the prism? (Example 2) **8 yd**

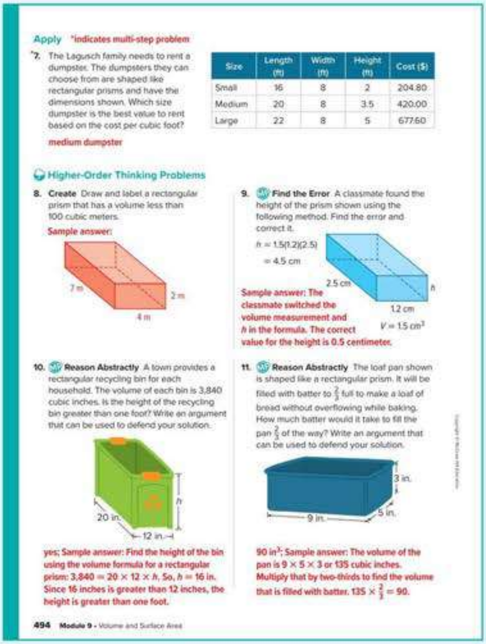


5. Raphael drives a standard-sized dump truck with a rectangular prism shaped bed. The volume of the bed of the truck is 720 cubic feet. If the length of the bed is 15 feet and the width is 8 feet, what is the height of the bed of the dump truck? **6 ft**

T est Practice

6. Open Response A rectangular prism has a length of 8 inches, a width of $7\frac{1}{2}$ inches, and a height of $6\frac{1}{4}$ inches. What is the volume of the prism?

$375\ in^3$



Apply *indicates multi-step problem

*7. The Lagusch family needs to rent a dumpster. The dumpsters they can choose from are shaped like rectangular prisms and have the dimensions shown. Which size dumpster is the best value to rent based on the cost per cubic foot?

Size	Length (ft)	Width (ft)	Height (ft)	Cost ($)
Small	16	8	2	204.80
Medium	20	8	3.5	420.00
Large	22	8	5	677.60

medium dumpster

Higher-Order Thinking Problems

8. Create Draw and label a rectangular prism that has a volume less than 100 cubic meters.

Sample answer:

9. Find the Error A classmate found the height of the prism shown using the following method. Find the error and correct it.

$h = 1.5(1.2)(2.5)$
$= 4.5$ cm

Sample answer: The classmate switched the volume measurement and h in the formula. The correct value for the height is 0.5 centimeter.

10. Reason Abstractly A town provides a rectangular recycling bin for each household. The volume of each bin is 3,840 cubic inches. Is the height of the recycling bin greater than one foot? Write an argument that can be used to defend your solution.

yes; Sample answer: Find the height of the bin using the volume formula for a rectangular prism: $3{,}840 = 20 \times 12 \times h$, So, $h = 16$ in. Since 16 inches is greater than 12 inches, the height is greater than one foot.

11. Reason Abstractly The loaf pan shown is shaped like a rectangular prism. It will be filled with batter to $\frac{2}{3}$ full to make a loaf of bread without overflowing while baking. How much batter would it take to fill the pan $\frac{2}{3}$ of the way? Write an argument that can be used to defend your solution.

90 in^3; Sample answer: The volume of the pan is $9 \times 5 \times 3$ or 135 cubic inches. Multiply that by two-thirds to find the volume that is filled with batter. $135 \times \frac{2}{3} = 90$.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students are presented with a classmates' work for finding the height of a prism, and students must find the error in the work.

2 Reason Abstractly and Quantitatively In Exercise 10, students are given the volume, the length, and the width of a bin, and they must reason quantitatively to determine if the height is greater than or less than 1 foot without actually calculating.

In Exercise 11, students are given the dimensions of a partially filled rectangular prism and are asked to reason through how much more volume it can hold.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 7 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 9 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise substituted the incorrect values to find the missing dimension. Have each pair or group of students present their explanations to the class.

Learn Make a Net to Represent a Rectangular Prism

Objective

Students will learn how to make a net to represent a rectangular prism.

MP Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to analyze the structure of the net in order to explain the similarities between the length, width, and height of the prism and the given dimensions of the net.

Teaching Notes

SLIDE 1

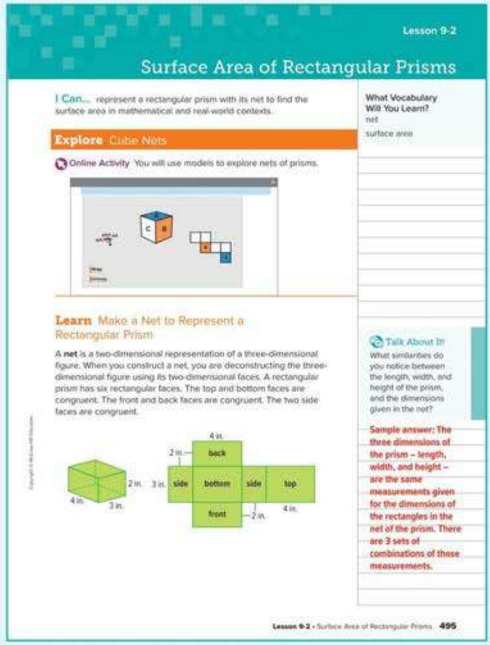
Students will learn that a *net* is a two-dimensional representation of a three-dimensional figure. Have students watch the brief animation that illustrates a rectangular prism unfolding to show its net. You may wish to have students create their own nets by unfolding rectangular prisms by providing students with boxes, such as tissue boxes or cereal boxes. It is important to note that many manufactured boxes have lids and bottom faces that are almost duplicated and glued to each other. Have students cut around the lid so that just an entire face forms that part of the net. Have them label the faces of their net as front, back, top, bottom, side 1, and side 2.

Talk About It!

SLIDE 2

Mathematical Discourse

What similarities do you notice between the length, width, and height of the prism, and the dimensions given in the net? Sample answer: The three dimensions of the prism – length, width, and height – are the same measurements given for the dimensions of the rectangles in the net of the prism. There are 3 sets of combinations of those measurements.


Lesson 9-2

Surface Area of Rectangular Prisms

I Can... represent a rectangular prism with its net to find the surface area in mathematical and real-world contexts.

What Vocabulary Will You Learn?
net
surface area

Explore Cube Nets

Online Activity You will use models to explore nets of prisms.

Learn Make a Net to Represent a Rectangular Prism

A **net** is a two-dimensional representation of a three-dimensional figure. When you construct a net, you are deconstructing the three-dimensional figure using its two-dimensional faces. A rectangular prism has six rectangular faces. The top and bottom faces are congruent. The front and back faces are congruent. The two side faces are congruent.

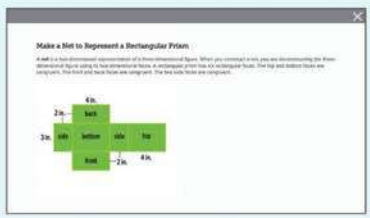
4 in.
2 in.
3 in.
back
side
bottom
side
top
front
2 in.
4 in.

Talk About It!
What similarities do you notice between the length, width, and height of the prism, and the dimensions given in the net?

Sample answer: The three dimensions of the prism – length, width, and height – are the same measurements given for the dimensions of the rectangles in the net of the prism. There are 3 sets of combinations of those measurements.

Lesson 9-2 • Surface Area of Rectangular Prisms 495

Interactive Presentation



Learn, Make a Net to Represent a Rectangular Prism, Slide 1 of 2

WATCH

On Slide 1, students watch a rectangular prism transform into a net of the rectangular prism.

Lesson 9-2

Surface Area of Rectangular Prisms

6.G.A.4

LESSON GOAL

Students will make nets and find surface area of rectangular prims.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Cube Nets

Learn: Make a Net to Represent a Rectangular Prism
Example 1: Make a Net to Represent a Rectangular Prism
Learn: Surface Area of a Rectangular Prism
Example 2: Surface Area of a Rectangular Prism
Apply: Home Improvement

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Changes in Dimension		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 50 of the *Language Development Handbook* to help your students build mathematical language related to surface area of rectangular prisms.

ELL You can use the tips and suggestions on page T50 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Geometry

Supporting Cluster(s): In this lesson, students address supporting cluster **6.G.A** by making nets and finding surface area of rectangular prisms.

Standards for Mathematical Content: **6.G.A.4**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found and used the volume of rectangular prisms.
6.G.A.2

Now
Students make nets and use them to find the surface area of rectangular prisms.
6.G.A.4

Next
Students will make nets and use them to find the surface area of triangular prisms.
6.G.A.4

Rigor

The Three Pillars of Rigor

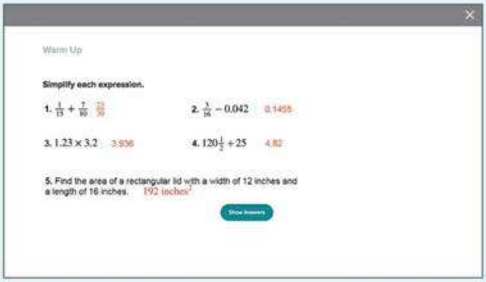
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of rectangular prisms and area to begin to develop *understanding* of surface area of rectangular prisms. They learn to make and use nets to build *fluency* with finding the surface area of rectangular prisms and *apply* their understanding to solve multi-step, real-world problems.

Mathematical Background

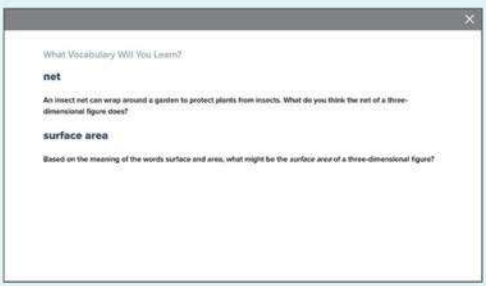
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- performing operations with rational numbers (Exercises 1–4)
- finding area of rectangles (Exercise 5)

Answers

1. $\frac{23}{30}$

2. 0.1455

3. 3.936

4. 4.82

5. 192 in^2

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about covering gift boxes with wrapping paper.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- An insect net can wrap around a garden to protect plants from insects. What do you think the net of a three-dimensional figure does? Sample answer: I think the net would wrap around the figure.
- Based on the meaning of the words surface and area, what might be the *surface area* of a three-dimensional figure? Sample answer: Surface area is the area covering the entire surface of the figure.

Explore Cube Nets

Objective

Students will use Web Sketchpad to explore nets of prisms.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore the idea of nets. Students will use colors to form a net that resembles the cube. Students should end up with the correct net for the cube after using different strategies.

Inquiry Question

How can a net help you visualize a three-dimensional figure? Sample answer: It can help me see each of the faces of the prism and how they connect.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

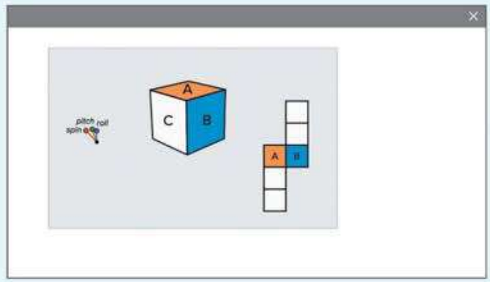
Mathematical Discourse

Label the correct face on the net with the letter C. Explain how you can tell which one is face C. Sample answer: On the net, the face below the A should be labeled C. On the cube, the face labeled C is adjacent to the faces labeled A and B. On the net, the face that should be labeled C is adjacent to A and will be adjacent to B once wrapped around the cube.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8

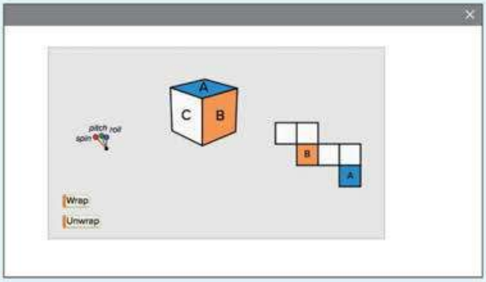


Explore, Slide 3 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore nets of prisms.

Interactive Presentation



Explore, Slide 6 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Cube Nets *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad in order to construct the correct net. Encourage students to think about the strategies they could use to match the colors of the faces.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Did you have to change your strategy from the previous net to place the letters? See students' responses.

What strategies did you use to label the net? Sample answer: This net was more challenging because the A and B faces did not share an edge, but I used the same strategy of identifying which faces will share an edge with the labeled A and B faces on the cube.

Your Notes

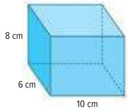
Think About It!
How can the prism be unfolded to make a two-dimensional net?

See students' responses.

Example 1 Make a Net to Represent a Rectangular Prism

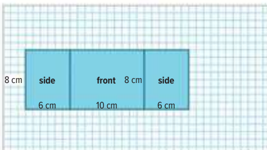
A rectangular prism has a length of 10 centimeters, a width of 6 centimeters, and a height of 8 centimeters.

Draw and label a net to represent the rectangular prism.



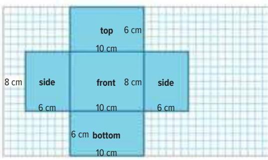
Step 1 Draw and label the front face and side faces.

The dimensions of the front of the prism are 10 centimeters by 8 centimeters. Use grid paper. Let each grid unit represent 1 centimeter. The dimensions of each side are 6 centimeters by 8 centimeters.

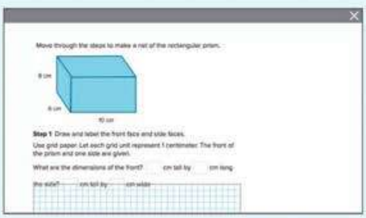


Step 2 Draw and label the top and bottom faces.

The dimensions of the top and bottom are 10 centimeters by 6 centimeters.



Interactive Presentation



Example 1, Make a Net to Represent a Rectangular Prism, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to make a net of the rectangular prism.

TYPE

On Slide 2, students identify the different measurements on the net.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Make a Net to Represent a Rectangular Prism

Objective

Students will make a net to represent a rectangular prism.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the prism in order to construct a net to represent the rectangular prism.

As students discuss the *Talk About It!* question on Slide 3, encourage them to analyze the structure of the prism in order to explain why there are only three measurements for a rectangular prism, with each face using two of the three measurements.

Questions for Mathematical Discourse

SLIDE 2

AL In the first step, how can you tell which measure on the prism is the height of the front? the length? Sample answer: The height of the prism is the same as the height of the front. The length of the front is the same as the length of the prism.

AL In the third step, how do you know that the measurements of the highlighted side are the same as the other side and not the front? Sample answer: Because it is a rectangular prism, the pairs of faces are congruent. The front is congruent to the back.

OL In the first step, why are the heights of the front and the sides the same? Sample answer: Both the sides and the front make up the faces of the prism so they are the same height.

OL In the fifth step, the top of the prism is not labeled. How do you know what the dimensions are? Sample answer: The dimensions are shown on the bottom of the prism. The dimensions of the top are the same as the bottom.

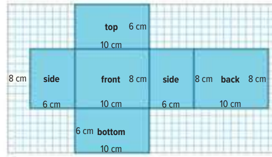
BL Suppose you found the area of all of the rectangles in the net. Is that the same as the volume of the prism? Explain. no; Sample answer: The volume of the prism is the space inside of the prism. The area of the rectangles in the net is the area of the space covering the prism.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Step 3 Draw and label the back face.

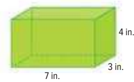
The dimensions of the back are the same as the front, 10 centimeters by 8 centimeters.

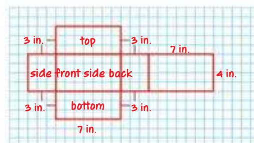


Check

Draw and label a net to represent the rectangular prism. Let each grid unit represent 1 inch.

Show your work here





Talk About It!

Explain why there are only three measurements for a rectangular prism, with each face using two of the three measurements.

Sample answer: The prism has three dimensions – length, width, and height – and each face is two-dimensional. So, the prism has only three measurements, and each face uses two of the three.

Go Online You can complete an Extra Example online.

DIFFERENTIATE

Enrichment Activity BL

There is more than one way to unfold a rectangular prism into a net. That said, not all drawings that consist of the “correct” faces are actual nets of the prism. Give students several nets, some of which are correct and others that are incorrect. Have students identify which ones are incorrect and why. In looking at Step 3 on page 497, ask the students the following questions:

Is it possible to draw a net where the back is not lined up with a side? If so draw it. yes; See students’ drawings.

Is it possible to draw a net that is not a shape that resembles the letter “t”? If so, draw it. yes; See students’ drawings.

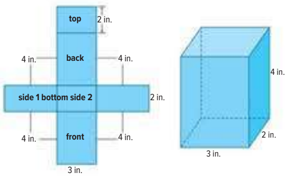
Evie has the correct two sides, top, bottom, front, and back, and has even calculated the surface area correctly. Malik says that Evie’s net is still incorrect. Draw a net that Evie may have drawn that supports Malik’s claim. See students’ drawings.

Learn Surface Area of a Rectangular Prism

The **surface area** of a rectangular prism is the sum of the areas of the faces. Using a net can help you deconstruct the prism into two-dimensional shapes so you can find the area of each face.

Go Online Watch the video to learn how to use a net to find the surface area of the rectangular prism shown.

The video shows the net of a rectangular prism.



The length ℓ of the rectangular prism is 3 inches, the width w is 2 inches, and the height h is 4 inches.

Step 1 Find the area of each face.

Front and Back

The front and back faces are congruent. Find the area of one face. Then multiply by 2 to find the total area of the front and back faces.

$A = \ell h$ The front face has dimensions ℓ and h.
$= 3(4)$ Replace ℓ with 3 and h with 4.
$= 12$ Multiply. The area of the front face is 12 in^2.

The combined area of the front and back faces is 2(12), or 24 square inches.

Top and Bottom

The top and bottom faces are congruent. Find the area of one face. Then multiply by 2 to find the total area of the top and bottom faces.

$A = \ell w$ The top face has dimensions ℓ and w.
$= 3(2)$ Replace ℓ with 3 and w with 2.
$= 6$ Multiply. The area of the top face is 6 in^2.

(continued on next page)

Talk About It!
In the video, the student measured the top and bottom, front and back, and side 1 and side 2. What shortcut can you use when finding the surface area of a rectangular prism?

Sample answer: I can find the area of one of each pair of faces, then multiply each of them by 2.

Interactive Presentation

Learn, Surface Area of a Rectangular Prism, Slide 1 of 2

WATCH

On Slide 1, students watch a video to learn how to use a net to find the surface area of a rectangular prism.

Learn Surface Area of a Rectangular Prism

Objective

Students will learn how to use a net to find the surface area of a rectangular prism.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the area of each face of the prism in order to determine a shortcut that can be used when finding the surface area of a rectangular prism.

Go Online to have your students watch the video on Slide 1. The video illustrates how to use a net to find the surface area of a rectangular prism.

Teaching Notes

SLIDE 1

You may wish to have students recreate the activity shown in the video. Provide them with several rectangular prisms, such as tissue boxes, cereal boxes, or other kinds of boxes you can find at the grocery store. Have them deconstruct the boxes as demonstrated in the video. It is important to note that many manufactured boxes have lids and bottom faces that are almost duplicated and glued to each other. Have students cut around the lid so that just an entire face forms that part of the net. Ask students what they notice about the net. Students should note that there are six rectangular faces, and that opposite faces are congruent.

Talk About It!

SLIDE 2

Mathematical Discourse

In the video, the student measured the top and bottom, front and back, and side 1 and side 2. What shortcut can you use when finding the surface area of a rectangular prism? Sample answer: I can find the area of one of each pair of faces, and then multiply each of them by 2.

Example 2 Surface Area of a Rectangular Prism

Objective

Students will use a net to find the surface area of a rectangular prism.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to use clear and precise mathematical language in their explanations of why the unit of measure is square centimeters, instead of centimeters or cubic centimeters.

7 Look for and Make Use of Structure Encourage students to use the structure of the prism and its net to understand that the corresponding faces (front and back, left and right sides, and top and bottom) are congruent.

Questions for Mathematical Discourse

SLIDE 2

AL What pairs of faces on the net are congruent? front and back, top and bottom, and the two sides

OL Why do the instructions tell you to multiply the area of one face by 2? Sample answer: The areas of opposite faces are equal, so if you find one area, then all you need to do is multiply it by two to find the area of the pair of faces.

BL If this was a cube, how many different areas would you need to find? Explain what you would do. 1; Sample answer: Because a cube has six congruent sides, I would find the area of one side and then multiply it by 6.

SLIDE 3

AL Why do you add the areas of the faces? Sample answer: Surface area is the total area of all of the faces of a prism. You add to find the total.

OL Is surface area the same as volume? Explain your reasoning. Sample answer: no; Sample answer: Surface area is the area that covers a three-dimensional figure. Volume is the space inside the figure.

BL Would a piece of wrapping paper that was 20 centimeters by 25 centimeters be enough? Explain. yes; Sample answer: The area of the wrapping paper is 500 square centimeters which is more than 376 square centimeters.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

The combined area of the top and bottom faces is 2(6), or 12 square inches.

Sides

The two side faces are congruent. Find the area of one. Then multiply by 2 to find the total area of the side faces.

$A = wh$ Each side face has dimensions w and h.
$= 2(4)$ Replace w with 2 and h with 4.
$= 8$ Multiply. The area of each side face is 8 in^2.

The combined area of the two side faces is 2(8), or 16 square inches.

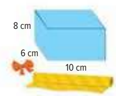
Step 2 Add the areas to find the total surface area.

$24 + 12 + 16 = 52$

So, the total surface area of the rectangular prism is __52__ square inches.

Example 2 Surface Area of a Rectangular Prism

Jon is covering the faces of the gift box shown with wrapping paper.



Use the net to determine the minimum amount of wrapping paper he will need to cover the box.

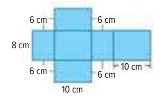
Step 1 Find the area of each face.

Front and Back

The front and back faces are congruent. Find the area of one face. Then multiply by 2 to find the total area of the front and back faces.

$A = \ell h$ The front face has dimensions ℓ and h.
$= 10(8)$ Replace ℓ with 10 and h with 8.
$= 80$ Multiply. The area of the front face is 80 cm^2.

The combined area of the front and back faces is 2(80), or 160 square centimeters.



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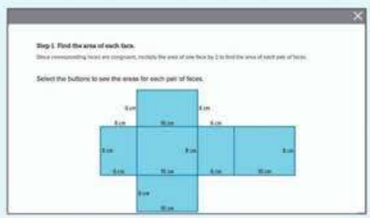
Think About It!
How many different-sized faces are there?
There are 3 different-sized faces.

Interactive Presentation



Example 2, Surface Area of a Rectangular Prism, Slide 2 of 5

CLICK
On Slide 2, students select the buttons to see the area of each pair of faces.

TYPE
On Slide 3, students determine the total surface area of the prism.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Top and Bottom

The top and bottom faces are congruent. Find the area of one face. Then multiply by 2 to find the total area of the top and bottom faces.

$A = \ell w$ The top face has dimensions ℓ and w.
$= 10(6)$ Replace ℓ with 10 and w with 6.
$= 60$ Multiply. The area of the top face is 60 cm^2.

The combined area of the top and bottom faces is 2(60), or 120 square centimeters.

Sides

The two side faces are congruent. Find the area of one face. Then multiply by 2 to find the total area of the side faces.

$A = wh$ Each side face has dimensions w and h.
$= 6(8)$ Replace w with 6 and h with 8.
$= 48$ Multiply. The area of each side face is 48 cm^2.

The combined area of the two side faces is 2(48), or 96 square centimeters.

Step 2 Add the areas to find the total surface area.

$160 + 120 + 96 = 376$

So, Jon will need a minimum of **376** square centimeters of wrapping paper.

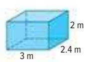
Talk About It!

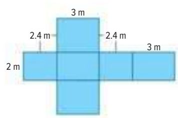
Why is the unit of measure square centimeters rather than centimeters or cubic centimeters?

Sample answer: Surface area measures the two-dimensional area of the faces, so the unit is square centimeters. Centimeters are used to measure length and cubic centimeters are used to measure volume.

Check

A moving crate that is shaped like a rectangular prism with the dimensions shown needs to be painted. Use the net to determine the area that is to be painted. **36 m^2**





Show your work here

Go Online You can complete an Extra Example online.

500 Module 9 • Volume and Surface Area

Apply Home Improvement

Objective

Students will come up with their own strategy to solve an application problem involving painting a room.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

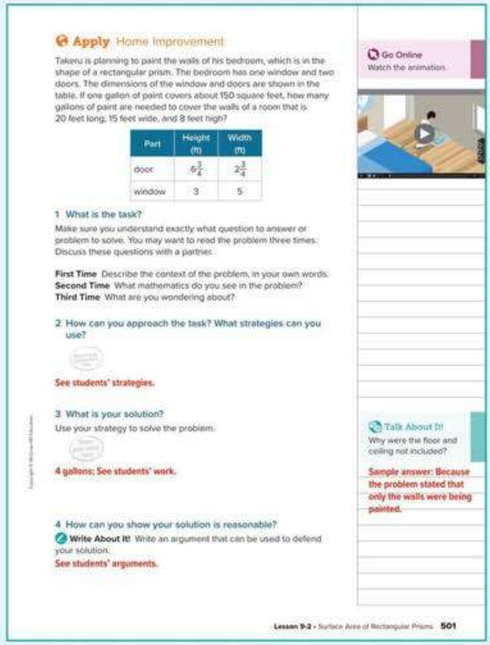
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What effect does the area of the windows and doors have on the problem?
- Why do you need to know how much area each can of paint covers?
- What does it mean if your answer is not a whole number?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Home Improvement

Takeru is planning to paint the walls of his bedroom, which is in the shape of a rectangular prism. The bedroom has one window and two doors. The dimensions of the window and doors are shown in the table. If one gallon of paint covers about 150 square feet, how many gallons of paint are needed to cover the walls of a room that is 20 feet long, 15 feet wide, and 8 feet high?

Part	Height (ft)	Width (ft)
door	$6\frac{3}{4}$	$2\frac{3}{4}$
window	3	5

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

4 gallons; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It! Why were the floor and ceiling not included?

Sample answer: Because the problem stated that only the walls were being painted.

Lesson 9-2 • Surface Area of Rectangular Prisms 501

Interactive Presentation

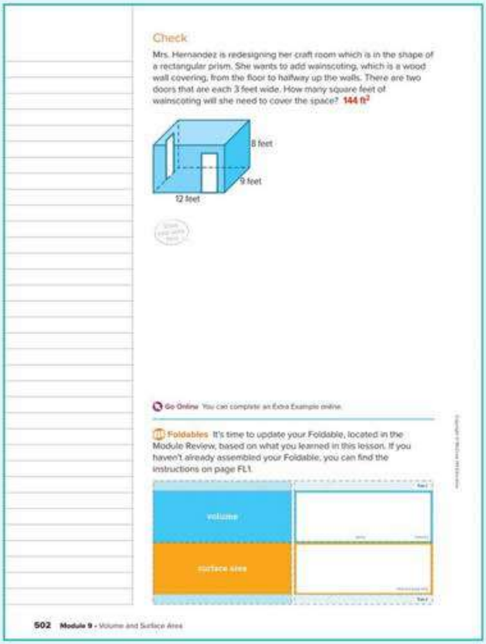
Apply, Home Improvement

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Mrs. Hernandez is redesigning her craft room which is in the shape of a rectangular prism. She wants to add wainscoting, which is a wood wall covering, from the floor to halfway up the walls. There are two doors that are each 3 feet wide. How many square feet of wainscoting will she need to cover the space? 144 ft²

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

502 Module 9 • Volume and Surface Area

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add a description of how to find the surface area of a rectangular prism. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can you describe the size of a three-dimensional figure?

In this lesson, students learned how to use a net to find the surface area of a rectangular prism. Have students work with a partner to compare and contrast volume and surface area of prisms. Some students may say that both can be used to describe the size of figures. While volume is a measure of the space inside a figure, surface area is a measure of space occupied by each two-dimensional surface of the figure. Volume is measured in cubic units, while surface area is measured in square units.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you have a box in the shape of a rectangular prism with a length of 12 inches, a width of 8 inches, and a height of 10 inches. Will a 600-square inch roll of wrapping paper be enough to cover the box? Write a mathematical argument that can be used to defend your solution. yes; Sample answer: The box has a surface area of 592 square inches. Since $592 < 600$, there will be enough paper to cover the box.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1, 3–8
- Extension: Changes in Dimension
- ALEKS Surface Area

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1, 2, 4, 5, 7
- Extension: Changes in Dimension
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

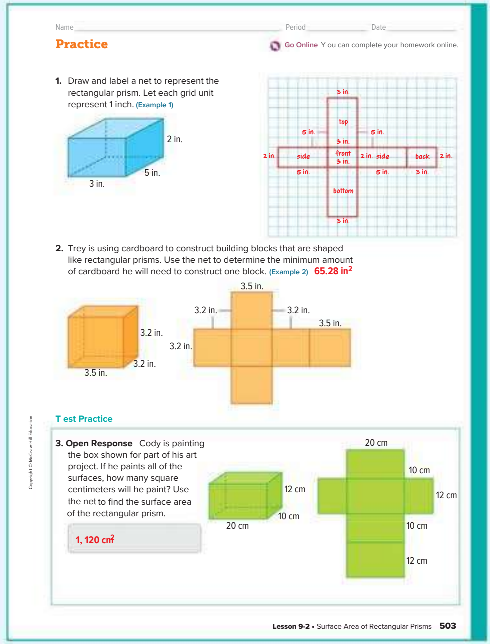
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	make a net to represent a rectangular prism	1
2	use a net to find the surface area of a rectangular prism	2
2	extend concepts learned in class to apply them in new contexts	3
3	solve application problems involving surface area of rectangular prisms	4
3	higher-order and critical thinking skills	5–8

Common Misconception

When drawing and labeling a net, particularly when the faces of the prism are nearly square, students may mix up which label goes on which edge. Give students a net that they can cut out and fold into a prism. When the prism is formed, have them label each edge. That way, when they unfold the prism, the correct labels will be on the correct edges. This will help students visualize the net with the appropriate side measures.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Draw and label a net to represent the rectangular prism. Let each grid unit represent 1 inch. (Example 1)

2. Trey is using cardboard to construct building blocks that are shaped like rectangular prisms. Use the net to determine the minimum amount of cardboard he will need to construct one block. (Example 2) 65.28 in²

Test Practice

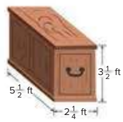
3. **Open Response** Cody is painting the box shown for part of his art project. If he paints all of the surfaces, how many square centimeters will he paint? Use the net to find the surface area of the rectangular prism.

1, 120 cm²

Lesson 9-2 • Surface Area of Rectangular Prisms 503

Apply *indicates multi-step problem

*4. Jing is putting a special restorative stain on the entire surface of her rectangular prism shaped hope chest, except for her name plate that measures $\frac{1}{2}$ foot by $\frac{3}{4}$ foot. If one can of stain covers about 35 square feet, how many cans of stain will she need to buy? **3 cans**



Higher-Order Thinking Problems

5. **Make a Conjecture** Write a formula that could be used to find the surface area of a rectangular prism. Define each variable you choose to use in your formula.

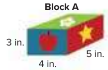
Sample answer: $S.A. = 2\ell w + 2\ell h + 2wh$, where ℓ = length, w = width and h = height.

6. **Create** Draw and label a rectangular prism that has a surface area that is greater than its volume.

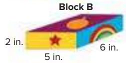
Sample answer:

7. **Reason Abstractly** Find the surface area and volume of each rectangular prism shaped block. Which block has the greater surface area? Does the same block have a greater volume? Write an argument that can be used to defend your solution.

Block A

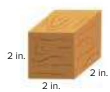


Block B



Block A: 94 in^2; 60 in^3; Block B: 104 in^2; 60 in^3; Block B has a greater surface area. No, the volumes of Blocks A and B are the same.

8. Meredith is painting rectangular prisms like the one shown. If she covers all the surfaces, how many square inches need to be painted? Describe two different ways to solve the problem.



24 in^2; Sample answer: Since a cube has 6 congruent faces, you can multiply the area of one face by 6, 6(2)(2) or you can find the area of the top and bottom 2(2)(2), the sides 2(2)(2), and the front and back 2(2)(2), then add them together 8 + 8 + 8 = 24.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 5, students make a conjecture about the formula for the surface area of a rectangular prism.

2 Reason Abstractly and Quantitatively In Exercise 7, students compare the volumes and surface areas of two different rectangular prisms.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercise 4 Have students work in groups of 3–4. After completing Exercise 4, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Be sure everyone understands.

Use with Exercises 7–8 Have students work in groups of 3–4 to solve the problem in Exercise 7. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 8.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Make a Net to Represent a Triangular Prism

Objective

Students will learn how to make a net to represent a triangular prism.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to analyze the structure of a net of a rectangular prism and a net of a triangular prism in order to find similarities and differences between the nets.

Teaching Notes

SLIDE 1

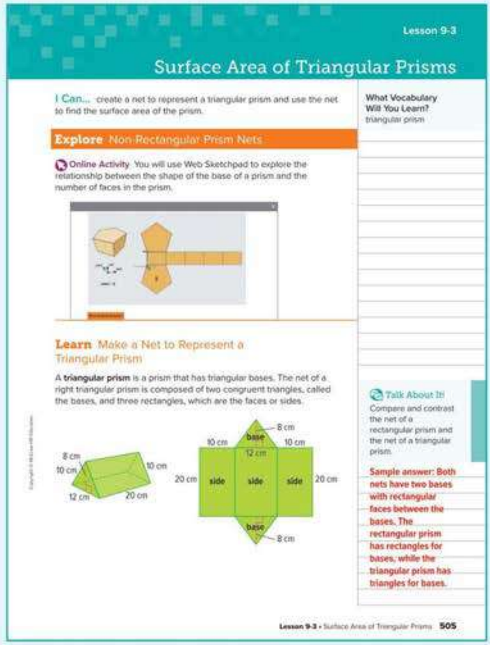
Have students watch the brief animation that illustrates a triangular prism being unfolded to show its net. Be sure students understand why a triangular prism gets its name. Students should be able to explain that prisms are named by the shape of their base. Thus, rectangular prisms have rectangular bases, and triangular prisms have triangular bases. Point out that the remaining faces are rectangles for both rectangular and triangular prisms. Ask students what is true about the two triangular bases. They should note that the bases of any prism are both parallel and congruent.

Talk About It!

SLIDE 2

Mathematical Discourse

Compare and contrast the net of a rectangular prism and the net of a triangular prism. Sample answer: Both nets have two bases with rectangular faces between the bases. The rectangular prism has rectangles for bases, while the triangular prism has triangles for bases.


Lesson 9-3

Surface Area of Triangular Prisms

I Can... create a net to represent a triangular prism and use the net to find the surface area of the prism.

What Vocabulary Will You Learn?
triangular prism

Explore Non-Rectangular Prism Nets

Online Activity You will use Web Sketchpad to explore the relationship between the shape of the base of a prism and the number of faces in the prism.

Learn Make a Net to Represent a Triangular Prism

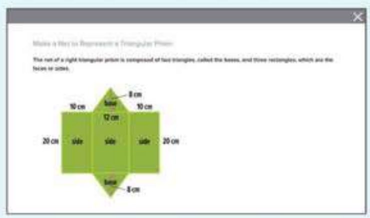
A **triangular prism** is a prism that has triangular bases. The net of a right triangular prism is composed of two congruent triangles, called the bases, and three rectangles, which are the faces or sides.

Talk About It!
Compare and contrast the net of a rectangular prism and the net of a triangular prism.

Sample answer: Both nets have two bases with rectangular faces between the bases. The rectangular prism has rectangles for bases, while the triangular prism has triangles for bases.

Lesson 9-3 • Surface Area of Triangular Prisms 505

Interactive Presentation



Learn, Make a Net to Represent a Triangular Prism, Slide 1 of 2

WATCH

On Slide 1, students watch a triangular prism transform into a net of the triangular prism.

Lesson 9-3

Surface Area of Triangular Prisms

6.G.A.4

LESSON GOAL

Students will make nets and find surface area of triangular prisms.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Non-Rectangular Prism Nets

Learn: Make a Net to Represent a Triangular Prism
Example 1: Make a Net to Represent a Triangular Prism
Learn: Surface Area of a Triangular Prism
Example 2: Surface Area of a Triangular Prism
Example 3: Find Surface Area of a Triangular Prism
Apply: Food

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

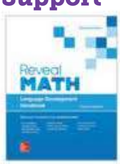
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Find Surface Area of Triangular Prisms Using a Formula		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 51 of the *Language Development Handbook* to help your students build mathematical language related to surface area of triangular prisms.



ELL You can use the tips and suggestions on page T51 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Geometry

Supporting Cluster(s): In this lesson, students address supporting cluster **6.G.A** by making nets and finding the surface area of triangular prisms.

Standards for Mathematical Content: **6.G.A.4**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students made nets and used them to find the surface area of rectangular prisms.
6.G.A.4

Now
Students make nets and use them to find the surface area of triangular prisms.
6.G.A.4

Next
Students will make nets and use them to find the surface area of pyramids.
6.G.A.4

Rigor

The Three Pillars of Rigor

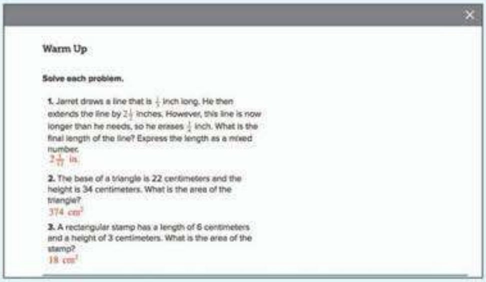
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of surface area as they explore surface area of triangular prisms. They learn to make and use nets to build *fluency* with finding the surface area of triangular prisms. They also *apply* their understanding of surface area of triangular prisms to solve multi-step, real-world problems.

Mathematical Background

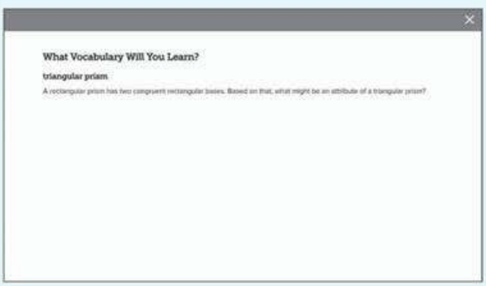
A *triangular prism* is composed of two congruent triangular bases and three rectangular sides. Similar to the process of finding the surface area of a rectangular prism, the surface area of a triangular prism can be found by creating a net. The net of a triangular prism has five components: two triangular bases and three rectangular faces. The surface area is the sum of the areas of these five faces.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- perform operations with rational numbers (Exercise 1)
- finding the area of triangles (Exercise 2)
- finding the area of rectangles (Exercise 3)

Answers

1. $2\frac{1}{12}$ in.
2. 374 cm^2
3. 18 cm^2

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about wheel chair ramps and the amount of material needed to build one.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- A rectangular prism has two congruent rectangular bases. Based on that, what might be an attribute of a triangular prism? Sample answer: a prism with two congruent triangular bases

Explore Non-Rectangular Prism Nets

Objective

Students will use Web Sketchpad to explore nets of prisms with non-rectangular bases.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore how the number of edges on the base of the prism compares to the number of rectangular faces on the net of the prism. Students will also observe how the number of rectangular faces changes with the shape of the bases. Students will then hypothesize how the shape of the base of a prism affects the number of rectangular faces.

Inquiry Question

How does the shape of the base of a prism affect the number of rectangular faces? Sample answer: The number of edges on the base determines the number of rectangular faces needed to make the prism. If the base is a regular polygon, the faces will all be congruent.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

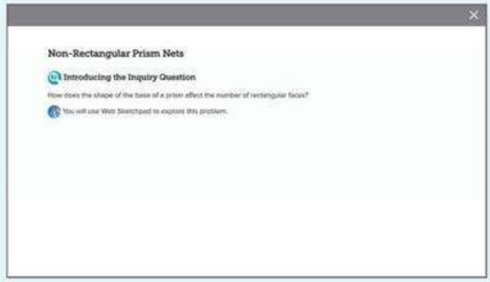
SLIDE 2

Mathematical Discourse

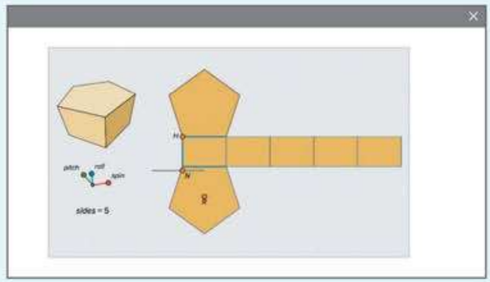
How many edges does the base of the prism have? How does this compare to the number of rectangular faces shown on the net of the prism? Explain why you think this is the case. 5; Sample answer: There are also 5 rectangular faces. There needs to be the same number of rectangular faces as the number of edges on the base in order to connect the two bases together with no curved surfaces or openings in the prism.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 7



Explore, Slide 2 of 7

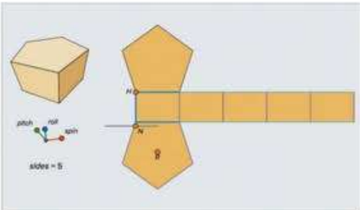
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore nets of prisms with non-rectangular bases.

TYPE

On Slide 4, students make a conjecture about the edge lengths and rectangular faces in a prism.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Non-Rectangular Prism Nets *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and deepen their understanding about the correspondences between the shape of the base of a prism and the number of rectangular faces the prism has.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

An equilateral triangle has three congruent edge lengths. Does your conjecture about the octagonal prism faces hold true for this prism? Explain your reasoning. Sample answer: Yes, if the bases are regular polygons, the rectangular faces of the prism will always be congruent.

Can you draw the net of the triangular prism another way? If so, draw it on a piece of paper and share your work. Explain why your drawing of the net of the prism still represents the prism. See students' drawings and responses.

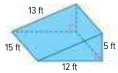
Your Notes

Think About It!
How can the prism be unfolded to make a two-dimensional net?

See students' responses.

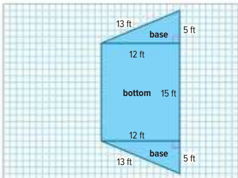
Example 1 Make a Net to Represent a Triangular Prism

Draw and label a net to represent the triangular prism.



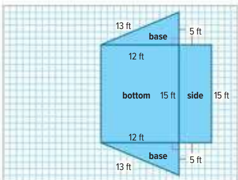
Step 1 Draw and label the bottom rectangular face and the two triangular bases.

Let each grid unit represent 1 foot. The bottom face is a rectangle with side lengths of 12 feet and 15 feet. The bases are triangles that have a base length of 12 feet and a height of 5 feet.



Step 2 Draw and label the side rectangular face.

The side face is a rectangle with side lengths of 15 feet and 5 feet.



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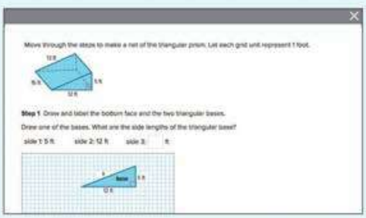
Interactive Presentation



Example 1, Make a Net to Represent a Triangular Prism, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to make a net of the triangular prism.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Make a Net to Represent a Triangular Prism

Objective

Students will make a net to represent a triangular prism.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the base of the triangular prism in order to construct an argument for why there are no pairs of congruent faces in the given prism.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the prism in order to construct and label the net precisely, making sure that the net can be folded to compose the triangular prism.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know which two faces are the bases of the prism? Sample answer: The two bases are the two congruent faces that are parallel.

AL Are any of the three faces of the prism congruent? Explain. no; Sample answer: One face has dimensions of 15 feet by 12 feet, one face has dimensions of 15 feet by 5 feet, and one face has dimensions of 15 feet by 13 feet.

OL Without drawing a net, or seeing the prism, how can you tell that a triangular prism has five faces? Sample answer: There are two triangular bases, and a face that connects the bases along each side of the triangle; $3 + 2 = 5$.

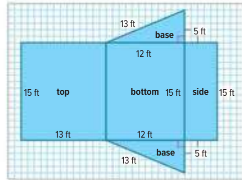
BL Describe a triangular prism where all three of the faces that connect the bases are congruent. Sample answer: The triangular bases of that prism would be congruent equilateral triangles.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Step 3 Draw and label the remaining rectangular face.

The top face is a rectangle with side lengths of 15 feet and 13 feet.



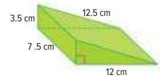
Talk About It!

A rectangular prism has pairs of faces that have the same dimensions. This triangular prism has three rectangular faces that have different dimensions. Explain why there are no pairs of faces with the same dimensions in this prism.

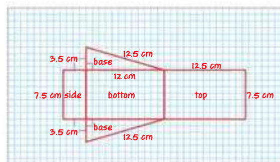
Sample answer: The base of the prism is a scalene triangle, so all three sides are different lengths. This means the rectangular faces have different widths, even though they are all the same height.

Check

Draw and label a net to represent the triangular prism. Let each grid unit represent 1 centimeter.



Show your work here



Go Online You can complete an Extra Example online.

DIFFERENTIATE

Enrichment Activity BL

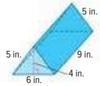
One way to extend the process of drawing nets is to have students think creatively about the possible nets that can be drawn. The prism on page 526 represents an opportunity to draw a unique net. Have students complete the following exercise.

Gregory has drawn a net for the prism in Example 1 that has exactly four rectangles. Draw a net that Gregory could have drawn. Is this net an accurate representation of the triangular prism? Why or why not? Students' drawings should show a total of four rectangles, three that are the faces of the prism, and one that is composed of the two triangular bases; Sample answer: The net is not an accurate representation of the prism because this net cannot be used to make the prism.

Learn Surface Area of a Triangular Prism

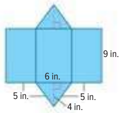
Y ou can use the net of a prism to find the surface area of the prism.

Go Online Watch the animation to learn how to use a net to find the surface area of the prism shown.



The prism has two triangular bases and three rectangular faces.

Step 1 Find the area of the triangular bases.



The triangles are congruent, so the area of each triangular base is the same. Find the area of one base. Then multiply by 2 to find the total area of both bases.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(6)(4)$ $b = 6$ and $h = 4$

$A = 12$ Multiply.

The combined area of the triangular bases is 2(12), or 24 square inches.

Talk About It! Because the bases are isosceles triangles, two of the three rectangular faces are congruent. Is there a way that all three rectangular faces could be congruent? Explain.

yes; Sample answer: If the bases are equilateral triangles, all three rectangular faces will have the same dimensions, so they would be congruent.

Step 2 Find the area of each rectangular face.

Because the triangular bases of the prism are isosceles, two of the rectangular faces of the prism are congruent.

2 Congruent Rectangular Faces
Each face has dimensions of 9 inches and 5 inches. Find the area of one face.

$A = \ell w$ Area of a rectangle
$= 9(5)$ $\ell = 9$ and $w = 5$
$= 45$ Multiply.

The combined area of the two congruent rectangular faces is 2(45), or 90 square inches.

3rd Rectangular Face
The dimensions of the third rectangular face are 9 inches and 6 inches.

$A = \ell w$ Area of a rectangle
$= 9(6)$ $\ell = 9$ and $w = 6$
$= 54$ Multiply.

The third rectangular face has an area of 54 square inches.

Step 3 Add the areas to find the total surface area.
24 + 90 + 54 = 168

So, the surface area of the triangular prism is 168 square inches.

Interactive Presentationz

Learn, Surface Area of a Triangular Prism, Slide 1 of 2

WATCH

On Slide 1, students watch an animation to learn how to use a net to find the surface area of a triangular prism.

Learn Surface Area of a Triangular Prism

Objective

Students will learn how to use a net to find the surface area of a triangular prism.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to study the structure of the base of the prism in order to determine the circumstances in which all three rectangular faces could be congruent.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to use a net to find the surface area of a triangular prism.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the dimensions of the triangular prism are shown. Ask students to work with a partner to draw a net that represents the prism and use their net to find the prism's surface area. Have pairs share their nets with another pair of students or the class. Then have them continue watching the animation to compare their net and surface area with the one shown. Be sure students understand that the three rectangular faces are not all congruent, for this particular prism. Ask them to explain why.

Talk About It!

SLIDE 2

Mathematical Discourse

Because the bases are isosceles triangles, two of the three rectangular faces are congruent. Is there a way that all three rectangular faces could be congruent? Explain. yes; Sample answer: if the bases are equilateral triangles, all three rectangular faces will have the same dimensions, so they would be congruent.

Example 2 Surface Area of a Triangular Prism

Objective

Students will use a net to find the surface area of a triangular prism with bases that are scalene triangles.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to use clear and precise mathematical language to explain why the bases of the triangular prism have the same area.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the net of the prism in order to determine that the three rectangular faces all have different areas, because the triangular bases are scalene.

Questions for Mathematical Discourse

SLIDE 2

AL Are any parts of the net congruent? Explain. yes; Sample answer; The bases are congruent because they are triangles with the same side measures.

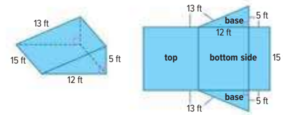
OL After the areas of each figure in the net are found, what is the next step? Sample answer: I will need to find the sum of all of the areas to find the total surface area of the prism.

BL How can the net be drawn a different way? Sketch the net and explain why it works. See students' responses.

(continued on next page)

Example 2 Surface Area of a Triangular Prism

Use the net to find the surface area of the triangular prism.



Step 1 Find the area of the triangular bases.

The triangles are congruent, so the area of each triangular base is the same. Find the area of one base. Then multiply by 2 to find the total area of both bases.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(12)(5)$ $b = 12$ and $h = 5$

$A = 30$ Multiply.

The combined area of the triangular bases is 2(30), or 60 square feet.

Step 2 Find the area of each rectangular face.

Because the triangular bases of the prism are scalene, all three rectangular faces have different dimensions.

Bottom

The length ℓ of the bottom face is 12 feet and the width w is 15 feet.

$A = \ell w$ Area of a rectangle

$= 12(15)$ $\ell = 12$ and $w = 15$

$= 180$ Multiply.

The area of the bottom face is 180 square feet.

Top

The length ℓ of the top face is 13 feet and the width w is 15 feet.

$A = \ell w$ Area of a rectangle

$= 13(15)$ $\ell = 13$ and $w = 15$

$= 195$ Multiply.

The area of the top face is 195 square feet.

Side

The length ℓ of the side face is 15 feet and the width w is 5 feet.

$A = \ell w$ Area of a rectangle

$= 15(5)$ $\ell = 15$ and $w = 5$

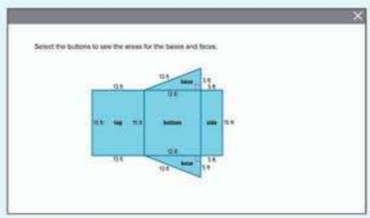
$= 75$ Multiply.

The area of the side face is 75 square feet. *(continued on next page)*

Think About It! What shapes are the faces and bases? What formulas can you use to find the area of each face and base?

rectangles and triangles; $A = \ell w$; $A = \frac{1}{2}bh$

Interactive Presentation



Example 2, Surface Area of a Triangular Prism, Slide 2 of 5

CLICK

On Slide 2, students select to view the areas of the bases and each side of the triangular prism.

Talk About It! Explain why the bases of the triangular prism have the same area.

Sample answer: The definition of a prism is a three-dimensional figure with rectangular faces and two parallel congruent bases. In order for the faces to be rectangular, the bases must be the same size and shape.

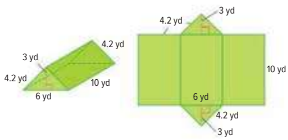
Step 3 Add the areas to find the total surface area.

$60 + 180 + 195 + 75 = 510$

So, the total surface area of the triangular prism is 510 square feet.

Check

Use the net to find the surface area of the triangular prism. 162 yd^2

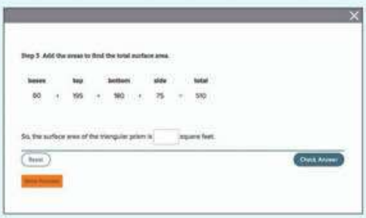


Show your work here

Go Online You can complete an Extra Example online.

510 **Module 9** • Volume and Surface Area

Interactive Presentation



Example 2, Surface Area of a Triangular Prism, Slide 3 of 5

TYPE

On Slide 3, students determine the sum of the areas of the faces.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Surface Area of a Triangular Prism *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why is there only one value shown for the bases when there are two bases? Sample answer: The two bases are congruent, so they have the same area. The area of one of the bases is 30 square feet, so when I calculated that, I multiplied it by 2 to get the area of both bases.

OL The triangular prism has 5 faces. Why is the sum of all the surface areas the sum of four numbers? Sample answer: The two bases have the same area, so the area of the two bases is given as one number. The surface area is the sum of the area of the bases and the areas of the three other faces.

BL Suppose this is a ramp and you want to paint the rectangular sides of the ramp. If a container of paint can cover 170 square feet, how many containers will you need to paint the entire prism? 3

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Find Surface Area of a Triangular Prism

Objective

Students will use a net to find the surface area of a triangular prism with bases that are equilateral triangles.

Questions for Mathematical Discourse

SLIDE 2

AL Why is the area of one of the rectangles multiplied by 3? Since the triangular base has 3 congruent sides, there will be three congruent faces.

OL A classmate says the surface area is 6 square centimeters. What is the likely mistake? Sample answer: 6 square centimeters is the total area of the three faces. To find the total surface area you need to include the area of the bases.

BL In the net, the three rectangular sides form a larger rectangle. How can you use that information to help find the surface area of the figure? Will that only work if the triangular bases are equilateral triangles? Explain. Sample answer: Instead of finding the area of three separate rectangles, I can find the perimeter of one of the bases and then multiply by the height of the faces. In this case it would be $1 + 1 + 1 = 3$, $3 \cdot 2 = 6$. It will work for any prism because when unfolded, the faces form a rectangle.

SLIDE 3

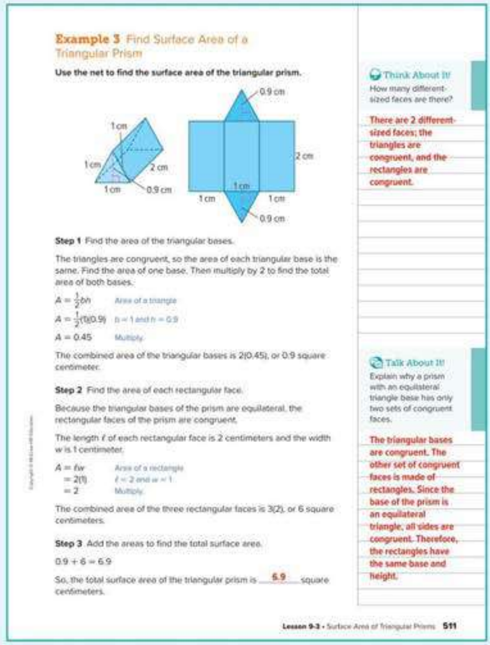
AL How do you know the units for the final answer should be in square centimeters? Sample answer: The measurements on the prism are in centimeters. This problem requires finding the area so the units should be square centimeters.

OL How could you check your answer? Sample answer: To check my answer, I would go back to the original net and make sure I used the correct pieces of information for each calculation. I would then check my final addition.

BL Would the surface area double if the height of the prism changed from 2 centimeters to 4 centimeters? Explain. no; Sample answer: Doubling just that measurement only affects the rectangular bases, the triangular bases are not affected by that change, so the total surface area is not doubled.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Find Surface Area of a Triangular Prism

Use the net to find the surface area of the triangular prism.

Step 1 Find the area of the triangular bases.

The triangles are congruent, so the area of each triangular base is the same. Find the area of one base. Then multiply by 2 to find the total area of both bases.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(1)(0.9)$ $b = 1$ and $h = 0.9$

$A = 0.45$ Multiply.

The combined area of the triangular bases is 2(0.45), or 0.9 square centimeter.

Step 2 Find the area of each rectangular face.

Because the triangular bases of the prism are equilateral, the rectangular faces of the prism are congruent.

The length ℓ of each rectangular face is 2 centimeters and the width w is 1 centimeter.

$A = \ell w$ Area of a rectangle

$= 2(1)$ $\ell = 2$ and $w = 1$

$= 2$ Multiply.

The combined area of the three rectangular faces is 3(2), or 6 square centimeters.

Step 3 Add the areas to find the total surface area.

$0.9 + 6 = 6.9$

So, the total surface area of the triangular prism is 6.9 square centimeters.

Think About It! How many different-sized faces are there?

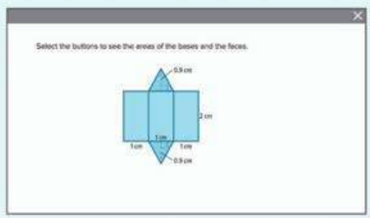
There are 2 different-sized faces; the triangles are congruent, and the rectangles are congruent.

Talk About It! Explain why a prism with an equilateral triangle base has only two sets of congruent faces.

The triangular bases are congruent. The other set of congruent faces is made of rectangles. Since the base of the prism is an equilateral triangle, all sides are congruent. Therefore, the rectangles have the same base and height.

Lesson 9-3 • Surface Area of Triangular Prisms 511

Interactive Presentation



Example 3, Find Surface Area of a Triangular Prism, Slide 2 of 5

CLICK

On Slide 2, students select to view the area of the bases and faces.

TYPE

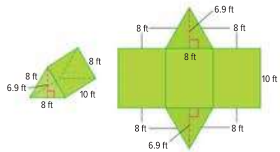
On Slide 3, students determine the surface area of the prism.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Use the net to find the surface area of the triangular prism. **295.2 ft^2**



Show your work here

Go Online You can complete an Extra Example online.

DIFFERENTIATE

Language Development Activity ELL

To help build students' vocabulary, have them create a graphic organizer or table that compares and contrasts nets of rectangular prisms with nets of triangular prisms. Encourage them to use the terms *rectangle*, *triangle*, *face*, *congruent*, etc. in their graphic organizer. Have students share their graphic organizers with another student or the entire class.

A sample table is shown.

Nets of Rectangular Prisms	Nets of Triangular Prisms
Shape of Faces All faces are *rectangles*.	**Shape of Faces** Two faces are *triangles*. The remaining three faces are *rectangles*.
Number of Faces There are six *faces*: front, back, top, bottom, side, side	**Number of Faces** There are five *faces*: two *triangular* bases, three lateral faces
Congruent Faces The front and back faces are *congruent*. The top and bottom faces are *congruent*. The two side faces are *congruent*.	**Congruent Faces** The two triangular faces are *congruent*. The other faces may or may not be congruent depending on what type of triangle forms the bases.

Apply Food

Objective

Students will come up with their own strategy to solve an application problem involving finding the greater unit price.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

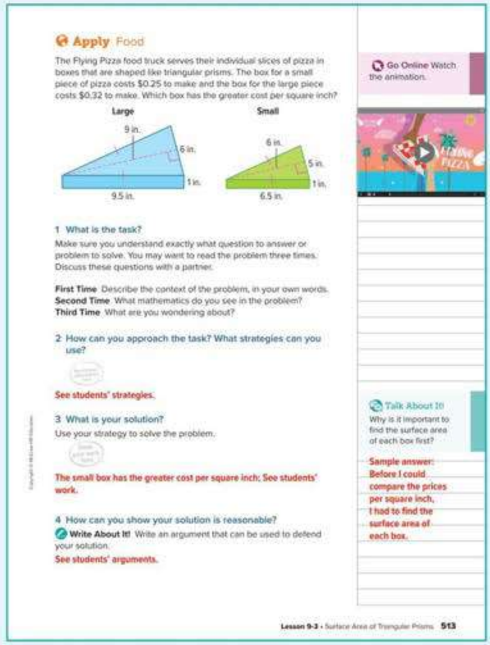
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the difference between the height of the triangular base and the height of the prism?
- Why do you find the surface area and not the volume?
- How do you find the cost per square inch?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Food

The Flying Pizza food truck serves their individual slices of pizza in boxes that are shaped like triangular prisms. The box for a small piece of pizza costs $0.25 to make and the box for the large piece costs $0.32 to make. Which box has the greater cost per square inch?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?
See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.
The small box has the greater cost per square inch; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.
See students' arguments.

Talk About It!
Why is it important to find the surface area of each box first?
Sample answer: Before I could compare the prices per square inch, I had to find the surface area of each box.

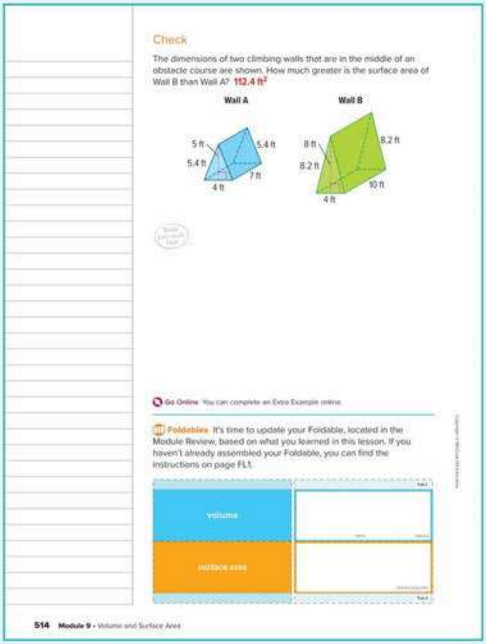
Lesson 9-3 • Surface Area of Triangular Prisms 513

Interactive Presentation

Apply, Food

WATCH
Students watch an animation that illustrates the problem they are about to solve.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.



Check

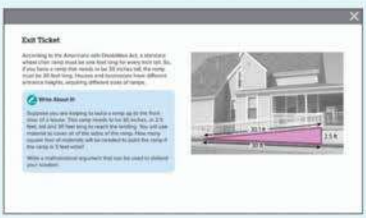
The dimensions of two climbing walls that are in the middle of an obstacle course are shown. How much greater is the surface area of Wall B than Wall A? 112.4 ft²

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

514 Module 9 • Volume and Surface Area

Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add a description of how to find the surface area of a triangular prism. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can you describe the size of a three-dimensional figure?

In this lesson, students learned how to use nets to find the surface area of triangular prisms. Encourage them to work with a partner to compare and contrast the surface area of rectangular prisms and triangular prisms. Some students may say that both have rectangular faces that connect the two bases. While rectangular prisms have six rectangular faces, triangular prisms have two triangular faces and three rectangular faces.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you are helping to build a ramp up to the front door of a house. This ramp needs to be 30 inches, or 2.5 feet, tall and 30 feet long to reach the landing. You will use material to cover all of the sides of the ramp. How many square feet of materials will be needed to build the ramp if the ramp is 3 feet wide? 262.8 ft^2

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 3–8
- Extension: Find Surface Area of Triangular Prisms Using a Formula
- ALEKS® Surface Area

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1, 2, 4, 5, 7
- Extension: Find Surface Area of Triangular Prisms Using a Formula
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Area of Parallelograms, Triangles, and Trapezoids

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Area of Parallelograms, Triangles, and Trapezoids

Susan Law Cain/Shutterstock

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

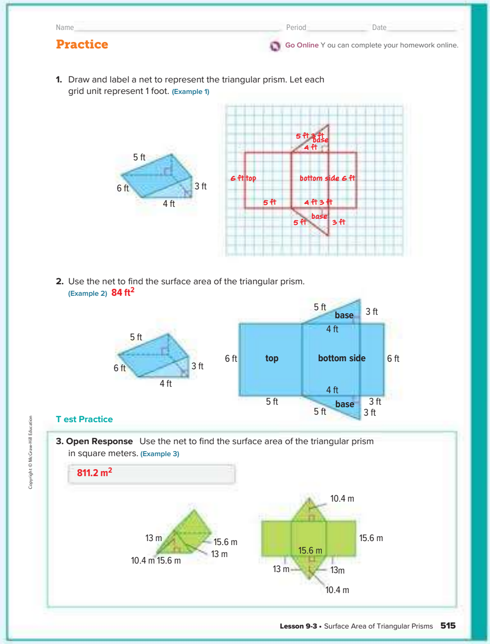
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	make a net to represent a triangular prism	1
2	use a net to find the surface area of a triangular prism	2, 3
3	solve application problems involving surface area of triangular prisms	4, 5
3	higher-order and critical thinking skills	6–8

Common Misconception

Because a rectangular prism has faces that are all rectangles, some students may think that a triangular prism consists of faces that are all triangles, and they may draw their nets accordingly. To dispel this misconception, gives students a triangular prism made out of cardstock, and encourage them to cut it along the edges to make a net. They will see that only two of the faces are triangles. The other faces are rectangles. The number of rectangular faces that are congruent depend on whether the triangular base is scalene, isosceles, or equilateral.


Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

1. Draw and label a net to represent the triangular prism. Let each grid unit represent 1 foot. (Example 1)

2. Use the net to find the surface area of the triangular prism. (Example 2) 84 ft²

Test Practice

3. Open Response Use the net to find the surface area of the triangular prism in square meters. (Example 3)

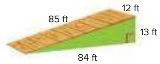
811.2 m²

Lesson 9-3 • Surface Area of Triangular Prisms 515

Apply *indicates multi-step problem

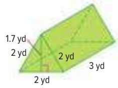
*4. Mr. Saldivar is building a ramp in the shape of a triangular prism with the dimensions shown. Sheets of plywood are 8 feet long and 4 feet wide. What is the minimum number of sheets of plywood he needs to buy in order to have enough to build the ramp?

103 sheets of plywood



*5. A tent is in the shape of the triangular prism with the dimensions shown. If the canvas to make the tent costs $4.99 per square yard, how much will it cost for the fabric to make the tent?

$106.79

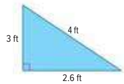


Higher-Order Thinking Problems

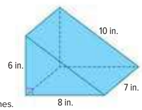
6. **Reason Abstractly** Why is the surface area of a triangular prism measured in square units rather than in cubic units? Explain.

Sample answer: Surface area measures the area of the faces. Area is a two-dimensional measurement, so it is measured in square units.

7. Find the surface area of a triangular prism that has the base triangle shown and a prism height of 7 feet. 75 ft²



8. **Find the Error** A classmate found the surface area of the triangular prism shown. Find the error and correct it.



Area of Bases
$A = 2\left(\frac{1}{2}\right)(6)(8)$
$A = 48$

Area of Faces
$A = 3(7)(10)$
$A = 210$

The surface area of the prism is 48 + 210 or 258 square inches.

Sample answer: The classmate multiplied the area of one face by 3. Since the base is not an equilateral triangle, the bases all have different dimensions. The surface area of the triangular prism is 48 + 7(10) + 7(8) + 7(6) or 216 square inches.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 6, students will explain why surface area is measured in square units instead of cubic units.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 7, students will find the surface area of a triangular prism without being given the original diagram or the net. Instead, they are given a diagram of the base and the height of the prism.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 8, students find and correct a student's mistake.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.

Use with Exercise 4 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Listen and ask clarifying questions.

Use with Exercises 5 and 8 Have students work in pairs. Have students individually read Exercise 5 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 8.

Learn Make a Net to Represent a Pyramid

Objective

Students will learn how to make a net to represent a pyramid.

MP Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to analyze the structure of a prism and a pyramid in order to identify the similarities and differences between prisms and pyramids.

Teaching Notes

SLIDE 1

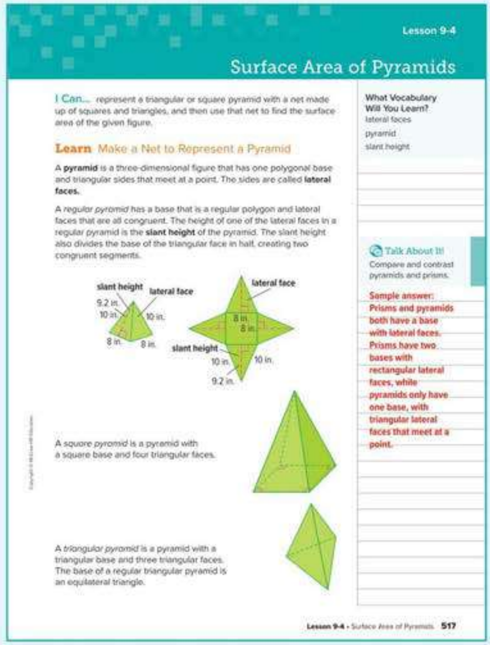
Students will learn that a *pyramid* is a three-dimensional figure that has one polygon for a base and triangles for sides that meet at a point. Students should note that the sides are called *lateral faces* (*lateral* means *side*) and, in a regular pyramid, the height of one of the lateral faces is the *slant height* of the pyramid. Have students compare and contrast the slant height of a pyramid with the height of a pyramid. Students should note the height of a pyramid is perpendicular to the base, while the slant height is at an angle.

Talk About It!

SLIDE 2

Mathematical Discourse

Compare and contrast pyramids and prisms. Sample answer: Prisms and pyramids both have a base with lateral faces. Prisms have two bases with rectangular lateral faces, while pyramids only have one base, with triangular lateral faces that meet at a point.


Lesson 9-4

Surface Area of Pyramids

I Can... represent a triangular or square pyramid with a net made up of squares and triangles, and then use that net to find the surface area of the given figure.

What Vocabulary Will You Learn?
lateral faces
pyramid
slant height

Learn Make a Net to Represent a Pyramid

A **pyramid** is a three-dimensional figure that has one polygonal base and triangular sides that meet at a point. The sides are called **lateral faces.**

A *regular pyramid* has a base that is a regular polygon and lateral faces that are all congruent. The height of one of the lateral faces in a regular pyramid is the **slant height** of the pyramid. The slant height also divides the base of the triangular face in half, creating two congruent segments.

A *square pyramid* is a pyramid with a square base and four triangular faces.

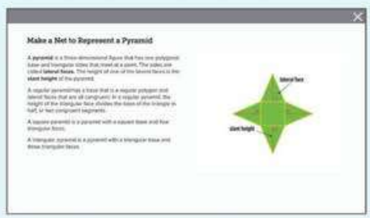
A *triangular pyramid* is a pyramid with a triangular base and three triangular faces. The base of a regular triangular pyramid is an equilateral triangle.

Talk About It!
Compare and contrast pyramids and prisms.

Sample answer: Prisms and pyramids both have a base with lateral faces. Prisms have two bases with rectangular lateral faces, while pyramids only have one base, with triangular lateral faces that meet at a point.

Lesson 9-4 • Surface Area of Pyramids 517

Interactive Presentation



Learn, Make a Net to Represent a Pyramid, Slide 1 of 2

WATCH

On Slide 1, students watch a square pyramid transform into a net of the square pyramid.

Lesson 9-4

Surface Area of Pyramids

6.G.A.4

LESSON GOAL

Students will make nets and find surface area of pyramids.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Make a Net to Represent a Pyramid
Example 1: Make a Net to Represent a Square Pyramid
Example 2: Make a Net to Represent a Triangular Pyramid
Learn: Surface Area of a Pyramid
Example 3: Find Surface Area of a Square Pyramid
Example 4: Find Surface Area of a Triangular Pyramid
Apply: Set Design

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

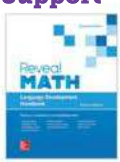
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Surface Area of Cones		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 52 of the *Language Development Handbook* to help your students build mathematical language related to surface area of pyramids.



ELL You can use the tips and suggestions on page T52 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Supporting Cluster(s): In this lesson, students address supporting cluster **6.G.A** by making nets and finding surface area of pyramids.
Standards for Mathematical Content: **6.G.A.4**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students made nets and found surface area of triangular prisms.
6.G.A.4

Now
Students make nets and use them to find the surface area of pyramids.
6.G.A.4

Next
Students will solve problems involving the surface area of prisms and pyramids.
7.G.B.5

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

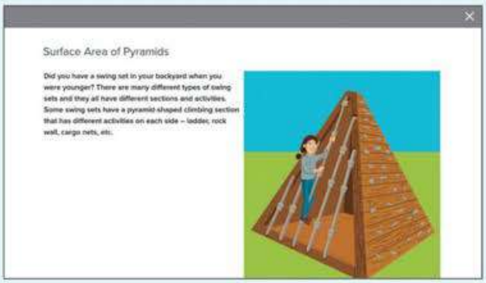
Conceptual Bridge In this lesson, students expand on their *understanding* of surface area to include pyramids. They learn to make and use nets to build *fluency* with finding the surface area of square and triangular pyramids. They also *apply* their understanding of surface area of pyramids to solve multi-step, real-world problems.

Mathematical Background

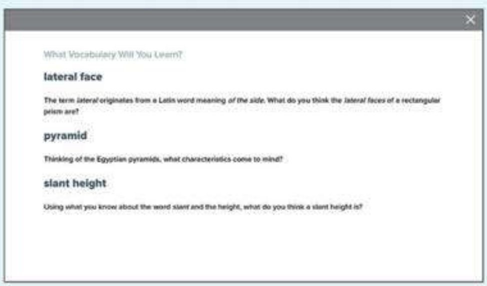
A *pyramid* is a three-dimensional figure with one polygonal base and triangular sides that meet at one point. Each of these triangular sides is referred to as a *lateral face*. In a regular pyramid, the lateral faces are congruent, and the height of each lateral face is its *slant height*. To find the surface area of a square pyramid, a pyramid with a square base, make a net and add the area of the square base to the areas of the triangular lateral faces. To find the surface area of a triangular pyramid, a pyramid with a triangular base, make a net and add the area of the triangular base to the areas of the triangular lateral faces.

Interactive Presentation

Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding area of rectangles (Exercise 1)
- finding area of triangles (Exercise 2)
- operations with rational numbers (Exercise 3)

Answers

1. 153 in^2
2. 6 yd^2
3. 0.91

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a pyramid-shaped climbing section of a swing set.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

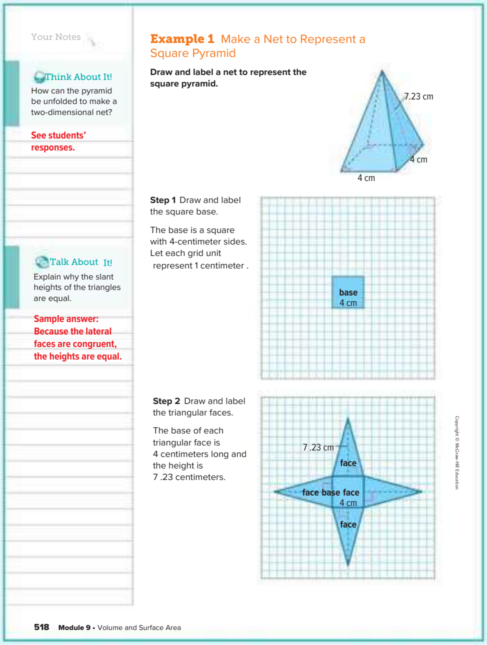
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *lateral* originates from a Latin word meaning *of the side*. What do you think the *lateral faces* of a rectangular prism are? Sample answer: The sides that are not a base.
- Thinking of the Egyptian pyramids, what characteristics come to mind? Sample answer: All of the sides meet together at one point above the base.
- Using what you know about the word *slant* and the height, what do you think a slant height is? Sample answer: Slant height is the height of a lateral face.



Your Notes

Think About It!
How can the pyramid be unfolded to make a two-dimensional net?

See students' responses.

Talk About It!
Explain why the slant heights of the triangles are equal.

Sample answer: Because the lateral faces are congruent, the heights are equal.

Example 1 Make a Net to Represent a Square Pyramid

Draw and label a net to represent the square pyramid.

Step 1 Draw and label the square base.

The base is a square with 4-centimeter sides. Let each grid unit represent 1 centimeter.

Step 2 Draw and label the triangular faces.

The base of each triangular face is 4 centimeters long and the height is 7.23 centimeters.

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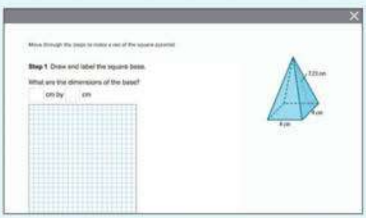
Interactive Presentation



Example 1, Make a Net to Represent a Square Pyramid, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to make a net of the square pyramid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Make a Net to Represent a Square Pyramid

Objective

Students will make a net to represent a square pyramid.

MP Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the pyramid in order to construct the net, making sure that their net can be folded to make the pyramid.

As students discuss the *Talk About It!* question on Slide 3, encourage them to study the structure of the square pyramid to note that the lateral faces are congruent, so the slant heights are equal.

Questions for Mathematical Discourse

SLIDE 2

AL Since the base of the pyramid is a square, how many lateral faces will the net have? Explain. The net will have 4 lateral faces because each side of the square will have an attached face.

OL How do you think the surface area of the pyramid can be found? Sample answer: In order to find the surface area, I will find the area of each part of the net and then find the sum of the areas.

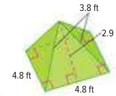
BL The height of a pyramid measures the perpendicular distance from the top point, or vertex of the pyramid to the base. Do you think this is the same length as the slant height? Explain your reasoning. no; Sample answer: You can form a right triangle with the height, the slant height, and a line segment connecting the bottom of the height to the bottom of the slant height. The slant height is opposite the right angle which makes it the hypotenuse of the right triangle. This is the longest side of the triangle, so the two lines cannot be the same length.

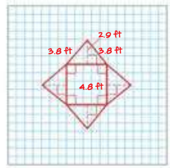
Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

Draw and label a net to represent the pyramid shown. Let each grid unit represent 1 foot.





Go Online Y ou can complete an Extra Example online.

Pause and Reflect

Draw a square pyramid in the space below, one that is different from the ones in Example 1 and Check. Trade your drawing with a partner. Draw and label a net that can be used to represent your partner's pyramid.

See students' observations.

DIFFERENTIATE

Reteaching Activity AL

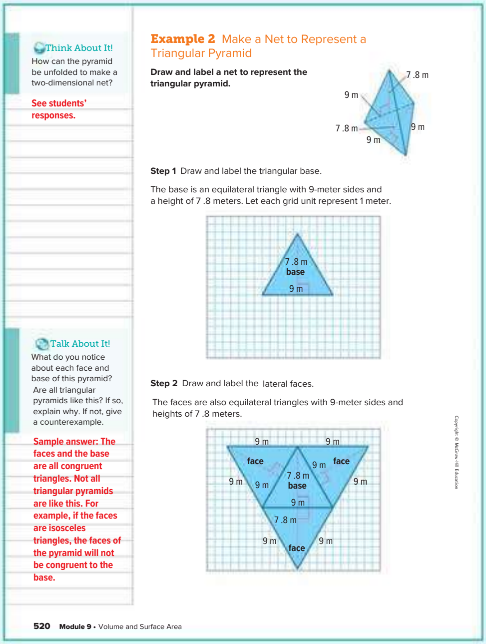
Students often have difficulty visualizing how to draw a net to represent a three-dimensional figure. Using three-dimensional models that can be unfolded to form a net is a great method to help students see how the figure relates to its corresponding net. Give students graph paper and the following instructions:

Draw a net on the graph paper that represents a square pyramid.

Cut out the net and fold it into the pyramid, taping the sides together.

Exchange your pyramid with a partner, and attempt to draw a net of your partner's pyramid without disassembling the pyramid.

Cut and unfold your partner's pyramid to see if your net is correct.


Think About It!
How can the pyramid be unfolded to make a two-dimensional net?
See students' responses.

Talk About It!
What do you notice about each face and base of this pyramid? Are all triangular pyramids like this? If so, explain why. If not, give a counterexample.
Sample answer: The faces and the base are all congruent triangles. Not all triangular pyramids are like this. For example, if the faces are isosceles triangles, the faces of the pyramid will not be congruent to the base.

Example 2 Make a Net to Represent a Triangular Pyramid

Draw and label a net to represent the triangular pyramid.

Step 1 Draw and label the triangular base.

The base is an equilateral triangle with 9-meter sides and a height of 7.8 meters. Let each grid unit represent 1 meter.

Step 2 Draw and label the lateral faces.

The faces are also equilateral triangles with 9-meter sides and heights of 7.8 meters.

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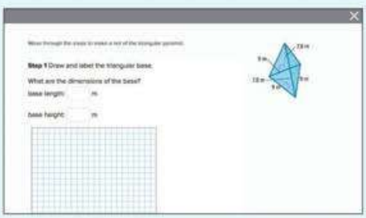
Interactive Presentation



Example 2, Make a Net to Represent a Triangular Pyramid, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to make a net of the pyramid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Make a Net to Represent a Triangular Pyramid

Objective

Students will make a net to represent a triangular pyramid.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to use a counterexample to explain why not all triangular pyramids have congruent faces and bases.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the pyramid in order to construct the net, making sure that their net can be folded to make the pyramid.

Questions for Mathematical Discourse

SLIDE 2

AL How does the completed net help you understand the properties of this pyramid? Sample answer: After I have drawn the net, I can see that there are four triangles that make up the pyramid and that they are all congruent.

OL Compare and contrast the nets for a triangular pyramid and a square pyramid. Sample answer: The nets for both pyramids have triangles for the lateral faces. The net for a triangular pyramid has a triangle for the base so it contains only triangles whereas the net for a square pyramid contains triangles and a square.

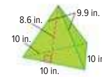
BL This pyramid is made up of four congruent triangles. Do you think the faces of all triangular pyramids are made up of four congruent triangles? Explain. Sample answer: No; you could have a triangular base that has different side lengths. The lateral faces would all be triangles with different bases.

Go Online

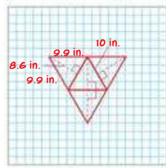
- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

Draw and label a net to represent the pyramid shown.



Show your work here



Go Online You can complete an Extra Example online.

Pause and Reflect

Draw a triangular pyramid in the space below, one that is different from the ones in Example 2 and Check. Trade your drawing with a partner. Draw and label a net that can be used to represent your partner's pyramid.

Record your observations here

See students' observations.

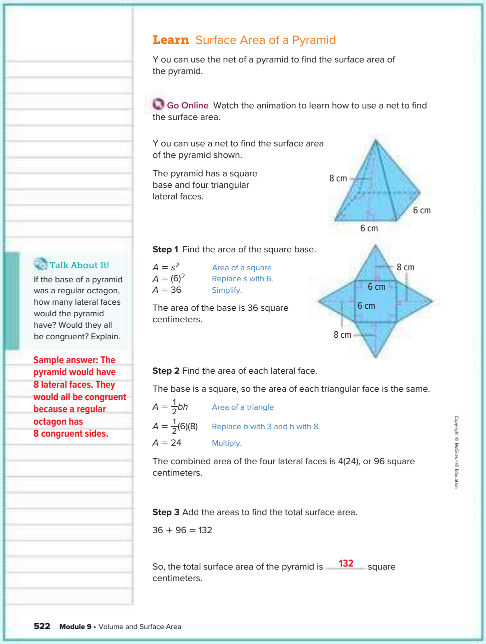
DIFFERENTIATE

Language Development Activity ELL

To help build students' vocabulary, have them create a graphic organizer or table that compares and contrasts nets of rectangular pyramids with nets of triangular pyramids. Encourage them to use the terms *rectangle*, *triangle*, *face*, *congruent*, etc. in their graphic organizer. Have students share their graphic organizers with another student or the entire class.

A sample table is shown.

Nets of Rectangular Pyramids	Nets of Triangular Pyramids
Shape of Faces One face is a *rectangle*. The remaining four faces are *triangles*.	**Shape of Faces** All faces are *triangles*.
Number of Faces There are five *faces*: the rectangular base and four triangular faces.	**Number of Faces** There are four *faces*: one triangular base and three triangular faces.
Congruent Faces There are two pairs of *congruent* triangular faces, formed from opposite sides of the rectangular base. All four triangular faces are *congruent* if the base face is a square.	**Congruent Faces** Two of the three triangular side faces are *congruent* if the base face is an *isosceles* triangle. All three triangular side faces are *congruent* if the base face is an *equilateral* triangle.



Learn Surface Area of a Pyramid

You can use the net of a pyramid to find the surface area of the pyramid.

Go Online Watch the animation to learn how to use a net to find the surface area.

You can use a net to find the surface area of the pyramid shown.

The pyramid has a square base and four triangular lateral faces.

Step 1 Find the area of the square base.

$A = s^2$ Area of a square
$A = (6)^2$ Replace s with 6.
$A = 36$ Simplify.

The area of the base is 36 square centimeters.

Step 2 Find the area of each lateral face.

The base is a square, so the area of each triangular face is the same.

$A = \frac{1}{2}bh$ Area of a triangle
$A = \frac{1}{2}(6)(8)$ Replace b with 3 and h with 8.
$A = 24$ Multiply.

The combined area of the four lateral faces is 4(24), or 96 square centimeters.

Step 3 Add the areas to find the total surface area.

36 + 96 = 132

So, the total surface area of the pyramid is 132 square centimeters.

Talk About It!
If the base of a pyramid was a regular octagon, how many lateral faces would the pyramid have? Would they all be congruent? Explain.

Sample answer: The pyramid would have 8 lateral faces. They would all be congruent because a regular octagon has 8 congruent sides.

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Learn Surface Area of a Pyramid

Objective

Students will learn how to use a net to find the surface area of a pyramid.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* questions on Slide 2, encourage them to make sense of the base of the pyramid to construct an argument about the number of lateral faces and about whether the lateral sides would be congruent using correct mathematical terminology.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the net of the square pyramid is drawn and labeled with its dimensions. Have students work with a partner to determine the surface area, by using the net. Have pairs of students share the process they used and the surface area they found with another pair of students, or with the entire class. Be sure students understand that since the base is a square, the four lateral faces are triangles that have the same base length. Some students may find the area of each lateral face and add them. Others may find the area of one lateral face, and multiply that area by 4. Encourage students to understand that these methods are both valid.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to use a net to find the surface area of a pyramid.

Talk About It!

SLIDE 2

Mathematical Discourse

If the base of a pyramid was a regular octagon, how many lateral faces would the pyramid have? Would they all be congruent? Explain. Sample answer: The pyramid would have 8 lateral faces. They would all be congruent because a regular octagon has 8 congruent sides.

Interactive Presentation

Learn, Surface Area of a Pyramid, Slide 1 of 2

WATCH

On Slide 1, students watch an animation to learn how to use a net to find the surface area of a pyramid.

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** 3 APPLICATION

Example 3 Find Surface Area of a Square Pyramid

Objective

Students will use a net to find the surface area of a square pyramid.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the net of the pyramid in order to find the surface area.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that all of the lateral faces of the pyramid have the same area? Sample answer: The base of the pyramid is a square so the base length of each lateral face is the same. Since the slant height is the same for all four faces, the area of the triangles that makes up the lateral faces is the same.

OL How can you find the total area of all lateral sides? Find the area of one triangle and multiply it by 4.

BL If the base was a rectangle, would the lateral faces have the same area? Explain. no; Sample answer: There would be two pairs of congruent faces, because a rectangle has two pairs of congruent sides.

SLIDE 3

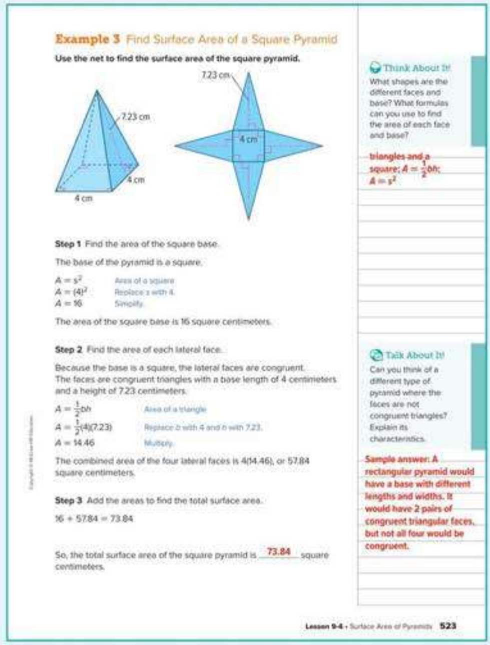
AL How will you find the surface area? I will find the sum of the area of the base and the area of the lateral faces.

OL Suppose the sides of the square base were doubled. How would that affect the total surface area? Sample answer: The area of the base would become 8 • 8 or 64 cm^2 instead of 16 cm^2, and the area of the lateral faces would become $4 \cdot \frac{1}{2} \cdot 8 \cdot 7.23$ or 115.68 cm^2. So, the total surface area would increase from 73.84 cm^2 to 64 + 115.68 or 179.68 cm^2.

BL Suppose the sides of the square base were doubled. Do you think you could still have a pyramid without changing the slant height? How would the pyramid look compared to the original pyramid? Explain. yes; Sample answer: if the slant height is greater than one-half the length of the new base, the lateral faces would still meet at the vertex of the pyramid. The pyramid would be shorter than the original pyramid, with a longer base for the lateral sides.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Find Surface Area of a Square Pyramid

Use the net to find the surface area of the square pyramid.

Step 1 Find the area of the square base.

The base of the pyramid is a square.

$A = s^2$ Area of a square
$A = (4)^2$ Replace s with 4.
$A = 16$ Simplify.

The area of the square base is 16 square centimeters.

Step 2 Find the area of each lateral face.

Because the base is a square, the lateral faces are congruent. The faces are congruent triangles with a base length of 4 centimeters and a height of 7.23 centimeters.

$A = \frac{1}{2}bh$ Area of a triangle
$A = \frac{1}{2}(4)(7.23)$ Replace b with 4 and h with 7.23.
$A = 14.46$ Multiply.

The combined area of the four lateral faces is 4(14.46), or 57.84 square centimeters.

Step 3 Add the areas to find the total surface area.

16 + 57.84 = 73.84

So, the total surface area of the square pyramid is 73.84 square centimeters.

Think About It! What shapes are the different faces and base? What formulas can you use to find the area of each face and base?

triangles and a square; $A = \frac{1}{2}bh$; $A = s^2$

Talk About It! Can you think of a different type of pyramid where the faces are not congruent triangles? Explain its characteristics.

Sample answer: A rectangular pyramid would have a base with different lengths and widths. It would have 2 pairs of congruent triangular faces, but not all four would be congruent.

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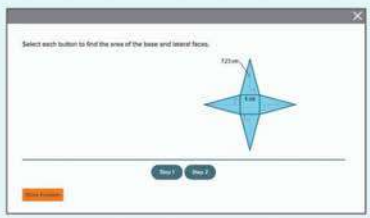
Interactive Presentation



Example 3, Find Surface Area of a Square Pyramid, Slide 2 of 5

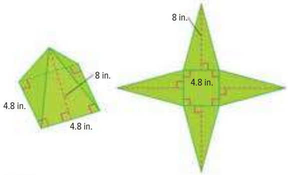
CLICK On Slide 2, students select to find the area of the base and the lateral faces.

TYPE On Slide 3, students determine the surface area of the pyramid.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

Use the net to find the surface area of the square pyramid. **99.84 in^2**



Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

How might you explain how to find the surface area of a square pyramid to a classmate who is encountering difficulty? What vocabulary and steps might be important to include in your explanation?

Record your observations here

See students' observations.

Example 4 Find Surface Area of a Triangular Pyramid

Objective

Students will use a net to find the surface area of a triangular pyramid.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them As students discuss the *Talk About It!* question on Slide 4, encourage them to consider an alternative approach to solving the problem and explain whether the alternative approach will work for all triangular pyramids.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the net of the pyramid in order to find the surface area.

Questions for Mathematical Discourse

SLIDE 2

AL How many equilateral triangles do you see in the net? Describe the triangles. 5; Sample answer: I see the base, the three lateral faces, and then the entire net is an equilateral triangle.

OL What do you notice about the base and the lateral faces? Sample answer: They all have the same dimensions and areas.

BL The outer perimeter of the net is an equilateral triangle. What are the dimensions of this triangle? How could you use this to find the surface area? The length of each side is 18 meters and the height is 2 • 7.8 or 15.6 meters. Sample answer: The area of this triangle is equal to the surface area of the pyramid.

SLIDE 3

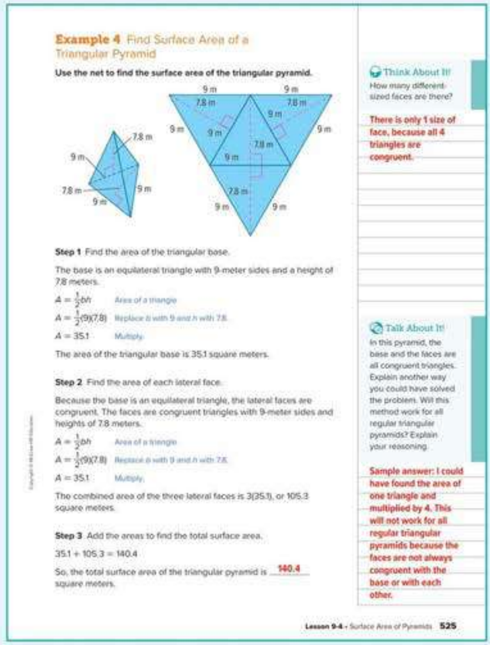
AL Why is it helpful to draw a net to find the surface area of this pyramid? Sample answer: When I created the net, I could see that all four faces of the pyramid were congruent triangles, and the dimensions were easily shown.

OL Think of the vocabulary used in this lesson and make a conjecture about what *lateral surface area* means. Sample answer: Lateral surface area is the total area of the lateral faces of a pyramid not including the base.

BL A quart of paint will cover about 9 square meters. If you only wanted to paint the *lateral surface area*, how many quarts of paint would you need? Explain. 12; Sample answer: The lateral surface area is the area of the lateral faces, or 105.3 m². If one quart covers 9 m², I would need to have 12 quarts of paint.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Find Surface Area of a Triangular Pyramid

Use the net to find the surface area of the triangular pyramid.

Step 1 Find the area of the triangular base.

The base is an equilateral triangle with 9-meter sides and a height of 7.8 meters.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(9)(7.8)$ Replace b with 9 and h with 7.8.

$A = 35.1$ Multiply.

The area of the triangular base is 35.1 square meters.

Step 2 Find the area of each lateral face.

Because the base is an equilateral triangle, the lateral faces are congruent. The faces are congruent triangles with 9-meter sides and heights of 7.8 meters.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(9)(7.8)$ Replace b with 9 and h with 7.8.

$A = 35.1$ Multiply.

The combined area of the three lateral faces is 3(35.1), or 105.3 square meters.

Step 3 Add the areas to find the total surface area.

$35.1 + 105.3 = 140.4$

So, the total surface area of the triangular pyramid is 140.4 square meters.

Think About It! How many different sized faces are there?

There is only 1 size of face, because all 4 triangles are congruent.

Talk About It! In this pyramid, the base and the faces are all congruent triangles. Explain another way you could have solved the problem. Will this method work for all regular triangular pyramids? Explain your reasoning.

Sample answer: I could have found the area of one triangle and multiplied by 4. This will not work for all regular triangular pyramids because the faces are not always congruent with the base or with each other.

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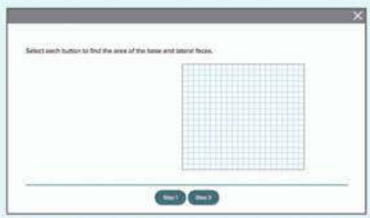
Interactive Presentation



Example 4, Find Surface Area of a Triangular Pyramid, Slide 2 of 5

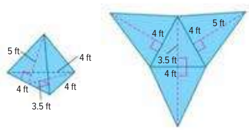
CLICK On Slide 2, students select the buttons to view the steps to determine the area of the base and lateral faces.

TYPE On Slide 3, students determine the surface area of the triangular pyramid.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

Use the net to find the surface area of the triangular pyramid. **37 ft^2**



Show your work here

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you make any errors when finding the surface area of triangular pyramids? What can you do to make sure you don't repeat that error in the future?

Record your observations here

See students' observations.

Apply Set Design

Objective

Students will come up with their own strategy to solve an application problem involving finding the price to construct pyramids for a school play.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What are the shapes that make up the faces and base of each pyramid?
- What is the formula used to find the area of a square? the area of a triangle?
- How does the price per square foot affect this problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Set Design

Morgan needs to construct three different square pyramids for the school play. The dimensions of the pyramids are shown in the table. The cost of materials to build the pyramids is $0.29 per square foot. How much will Morgan spend on materials for all three pyramids?

Pyramid	Base Edge (ft)	Height of Faces (ft)
A	2	5
B	5	12
C	3.5	9

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

$70.83; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Suppose Morgan needed to construct triangular pyramids instead of square pyramids. What information would we need to know to solve the problem?

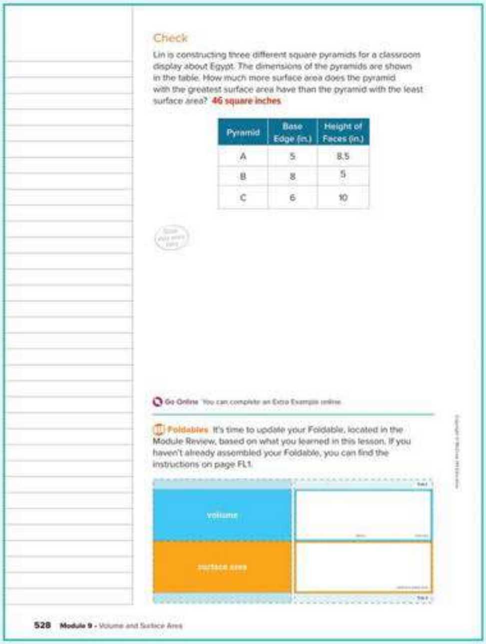
Sample answer: We would need to know the dimensions of the triangular base.

Interactive Presentation

Apply, Set Design

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Lin is constructing three different square pyramids for a classroom display about Egypt. The dimensions of the pyramids are shown in the table. How much more surface area does the pyramid with the greatest surface area have than the pyramid with the least surface area? **46 square inches**

Pyramid	Base Edge (in.)	Height of Faces (in.)
A	5	8.5
B	8	5
C	6	10

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

528 Module 9 • Volume and Surface Area

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add a description of how to find the surface area of a pyramid. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can you describe the size of a three-dimensional figure?

In this lesson, students learned how to use nets to find the surface area of pyramids. Encourage them to work with a partner to compare and contrast the surface area of prisms and pyramids – both rectangular and triangular. Some students may say that rectangular/triangular prisms and pyramids both have at least one rectangular/triangular base. While prisms have two parallel and congruent bases, pyramids have only one base.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you need to cover 3 of the 4 sides of a climbing section at the local park with non-slip paint. The section is shaped like a square pyramid with a base length of 6 feet and a slant height of 8.25 feet. How many square feet will be painted?
74.25 square feet

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 3, 5–9
- Extension: Surface Area of Cones
- ALEKS Surface Area

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–6, 9
- Extension: Surface Area of Cones
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

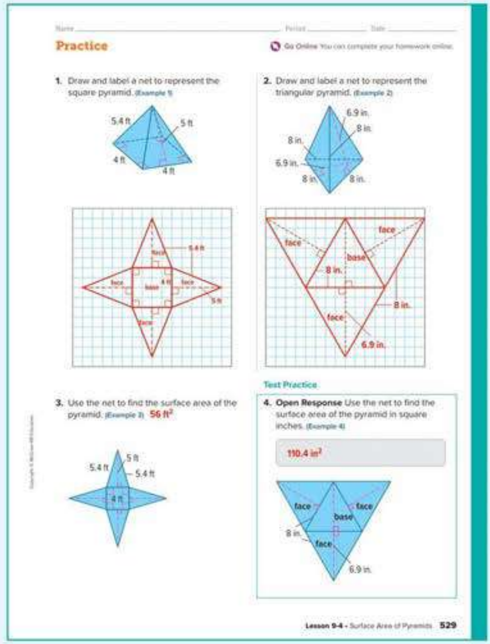
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	make a net to represent a pyramid	1, 2
2	use a net to find the surface area of a pyramid	3, 4
3	solve application problems involving nets of triangular pyramids	5
3	higher-order and critical thinking skills	6–9

Common Misconception

When finding the surface area of triangular pyramids, some students may find the sum of the areas incorrectly by multiplying the base by three, instead of the lateral faces. Consider having students clearly label each lateral face and the base on the net with the correct area, or perhaps make a table to record each face and base with the corresponding area.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Draw and label a net to represent the square pyramid. (Example 1)

2. Draw and label a net to represent the triangular pyramid. (Example 2)

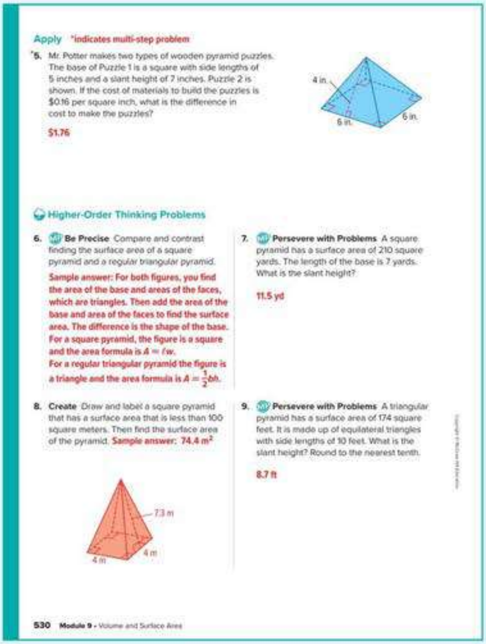
3. Use the net to find the surface area of the pyramid. (Example 3) $56\ ft^2$

Test Practice

4. **Open Response** Use the net to find the surface area of the pyramid in square inches. (Example 4)

$110.4\ in^2$

Lesson 9-4 • Surface Area of Pyramids 529



Apply *indicates multi-step problem

*5. Mr. Potter makes two types of wooden pyramid puzzles. The base of Puzzle 1 is a square with side lengths of 5 inches and a slant height of 7 inches. Puzzle 2 is shown. If the cost of materials to build the puzzles is $0.16 per square inch, what is the difference in cost to make the puzzles?

$1.76

Higher-Order Thinking Problems

6. MP Be Precise Compare and contrast finding the surface area of a square pyramid and a regular triangular pyramid.

Sample answer: For both figures, you find the area of the base and areas of the faces, which are triangles. Then add the area of the base and area of the faces to find the surface area. The difference is the shape of the base. For a square pyramid, the figure is a square and the area formula is $A = \ell w$. For a regular triangular pyramid the figure is a triangle and the area formula is $A = \frac{1}{2}bh$.

7. MP Persevere with Problems A square pyramid has a surface area of 210 square yards. The length of the base is 7 yards. What is the slant height?

11.5 yd

8. Create Draw and label a square pyramid that has a surface area that is less than 100 square meters. Then find the surface area of the pyramid. Sample answer: 74.4 m²

9. MP Persevere with Problems A triangular pyramid has a surface area of 174 square feet. It is made up of equilateral triangles with side lengths of 10 feet. What is the slant height? Round to the nearest tenth.

8.7 ft

530 Module 9 • Volume and Surface Area

Teaching the Mathematical Practices

6 Attend to Precision In Exercise 6, students compare and contrast finding the surface area of a triangular pyramid and a square pyramid by using precise mathematical vocabulary.

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 7, students solve for the missing slant height of a square pyramid given the surface area and the length of the base.

In Exercise 9, students solve for the missing slant height of a triangular pyramid formed with four equilateral triangles given the surface area and the length of the side of the equilateral triangles.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 5 Have students work together to prepare a brief demonstration that illustrates why this problem might require multiple steps to solve. For example, before they can find the difference in the costs, students must first find the surface area of each puzzle. Have each pair or group of students present their response to the class.

Create your own higher-order thinking problem.
Use with Exercises 6–9 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of how to calculate volume and surface area. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

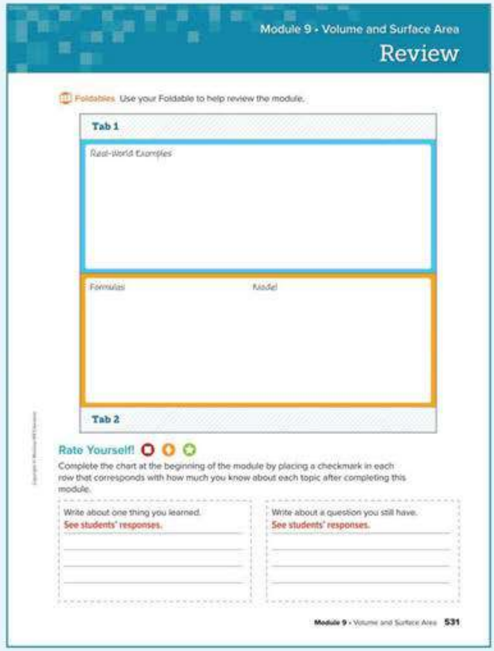
Assessment Resources

Put It All Together: Lessons 9-1 and 9-2
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Geometry**.

- Surface Area of Rectangular Prisms
- Surface Area of Solids with Triangular Faces
- Volume


Module 9 • Volume and Surface Area
Review

Foldables Use your Foldable to help review the module.

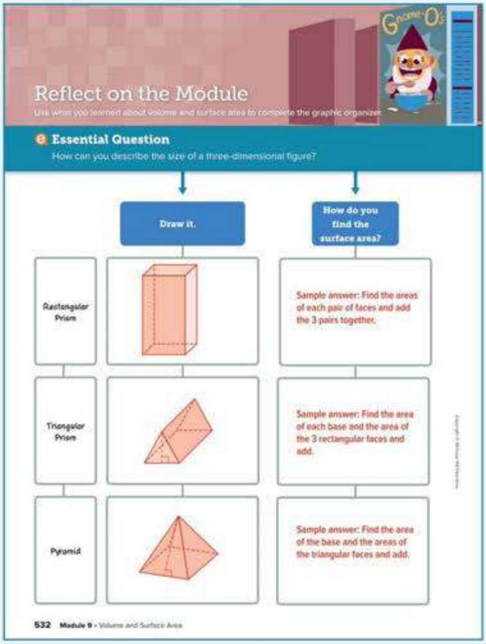
Tab 1
Real-World Examples
Formulas
Model
Tab 2

Rate Yourself!
Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 9 • Volume and Surface Area 531



Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can you describe the size of a three-dimensional figure?
See students' graphic organizers.

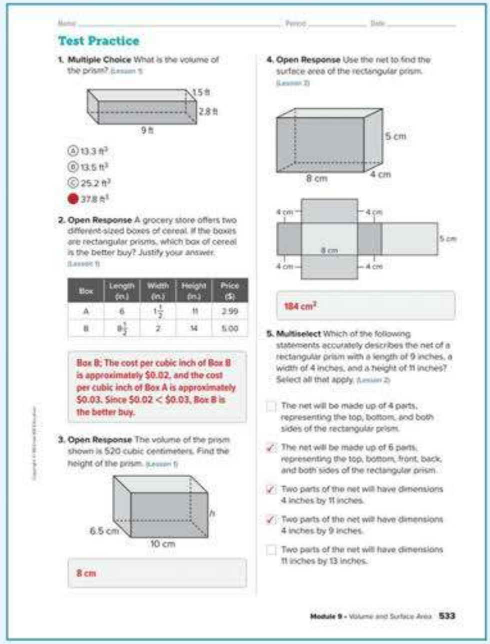
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–7 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 7
Multiselect	Multiple answers may be correct. Students must select all correct answers.	5
Table Item	Students complete a table by correctly classifying the information.	6
Open Response	Students construct their own response in the area provided.	2, 3, 4

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.G.A.2	9-1	1–3
6.G.A.4	9-2, 9-3, 9-4	4–7


Name Period Date

Test Practice

1. Multiple Choice What is the volume of the prism? (Lesson 1)

Ⓐ 13.3 ft³
Ⓑ 13.5 ft³
Ⓒ 25.2 ft³
● 37.8 ft³

2. Open Response A grocery store offers two different-sized boxes of cereal. If the boxes are rectangular prisms, which box of cereal is the better buy? Justify your answer. (Lesson 1)

Box	Length (in.)	Width (in.)	Height (in.)	Price ($)
A	6	$1\frac{1}{2}$	11	2.99
B	$8\frac{1}{2}$	2	14	5.00

Box B; The cost per cubic inch of Box B is approximately \$0.02, and the cost per cubic inch of Box A is approximately \$0.03. Since \$0.02 < \$0.03, Box B is the better buy.

3. Open Response The volume of the prism shown is 520 cubic centimeters. Find the height of the prism. (Lesson 1)

8 cm

4. Open Response Use the net to find the surface area of the rectangular prism. (Lesson 2)

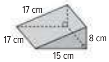
184 cm²

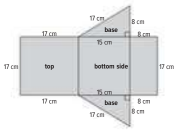
5. Multiselect Which of the following statements accurately describes the net of a rectangular prism with a length of 9 inches, a width of 4 inches, and a height of 11 inches? Select all that apply. (Lesson 2)

- [ ] The net will be made up of 4 parts, representing the top, bottom, and both sides of the rectangular prism.
- [x] The net will be made up of 6 parts, representing the top, bottom, front, back, and both sides of the rectangular prism.
- [x] Two parts of the net will have dimensions 4 inches by 11 inches.
- [x] Two parts of the net will have dimensions 4 inches by 9 inches.
- [ ] Two parts of the net will have dimensions 11 inches by 13 inches.

Module 9 • Volume and Surface Area 533

6. Table Item Consider the prism and the net shown. (Lesson 3)





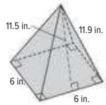
A. Indicate which of the following calculations are correct.

	Correct	Incorrect
Area of top face: 255 cm^2		x
Area of bottom face: 120 cm^2		x
Area of base: 70 cm^2		x
Area of base: 60 cm^2	x	
Area of face: 136 cm^2	x	

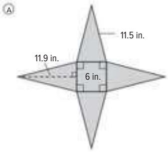
B. What is the surface area of the prism?

(A) 800 cm^2
(B) 885 cm^2
(C) 886 cm^2
(D) 2,040 cm^2

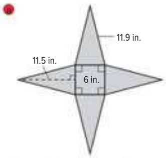
7 . Multiple Choice Select the net that represents the pyramid shown. (Lesson 4)



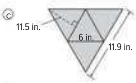
(A)



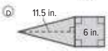
(B)



(C)



(D)



IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

Why is data collected and analyzed and how can it be displayed? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

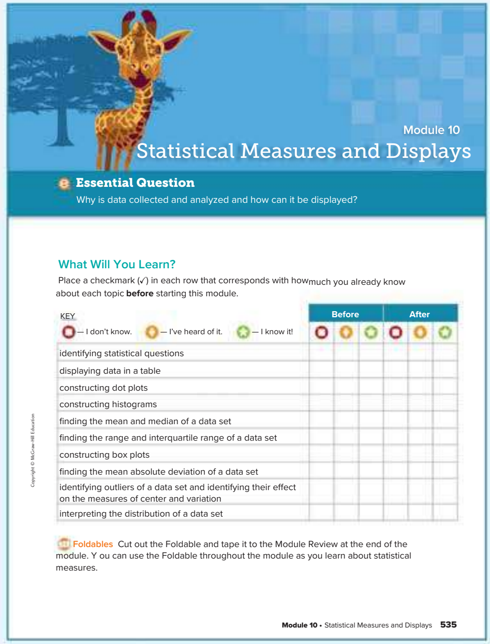
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of zoo attendance and types of animal species to introduce the idea of statistical measures. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 10

Statistical Measures and Displays

Essential Question

Why is data collected and analyzed and how can it be displayed?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
identifying statistical questions						
displaying data in a table						
constructing dot plots						
constructing histograms						
finding the mean and median of a data set						
finding the range and interquartile range of a data set						
constructing box plots						
finding the mean absolute deviation of a data set						
identifying outliers of a data set and identifying their effect on the measures of center and variation						
interpreting the distribution of a data set						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. Y ou can use the Foldable throughout the module as you learn about statistical measures.

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Module 10 • Statistical Measures and Displays 535

Interactive Student Presentation

Module 10

Statistical Measures and Displays

Module Goal

Find and use statistical measures.

Focus

Domain: Statistics and Probability

Additional Cluster(s):

6.SP.A Develop understanding of statistical variability.

6.SP.B Summarize and describe distributions.

Standards for Mathematical Content:

6.SP.A.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.B.5 Summarize numerical data sets in relation to their context.

Also addresses 6.SP.A.1, 6.SP.A.2, 6.SP.B.4, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C, 6.SP.B.5.D.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- fluently perform all four operations with positive rational numbers
- solve one-step equations
- graph positive rational numbers on the number line
- find the absolute value of integers

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students represented and interpreted data.
5.MD.B.2

Now
Students find and use statistical measures.
6.SP.A.1, 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5

Next
Students will use statistics to compare two populations.
7.SP.B.4

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of representing and interpreting data to develop *understanding* of statistical measures. They use this understanding to build *fluency* with finding measures of center and variation as well as identifying outliers. They also build fluency with constructing and interpreting dot plots, histograms, and box plots. They *apply* their understanding of statistical measures to solve real-world problems.

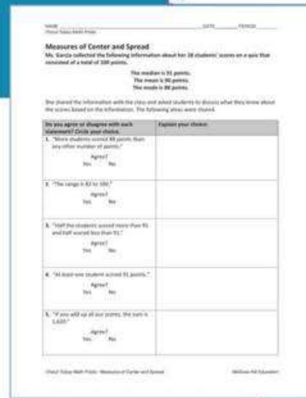
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
10-1	Statistical Questions	6.SP.A.1	1	0.5
10-2	Dot Plots and Histograms	6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A	1	0.5
10-3	Measures of Center	6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C	3	1.5
Put It All Together 1: Lessons 10-1, 10-2, and 10-3			0.5	0.25
10-4	Interquartile Range and Box Plots	6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C	1	0.5
10-5	Mean Absolute Deviation	6.SP.A.3, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B. 6.SP.B.5.C	1	0.5
10-6	Outliers	6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C, 6.SP.B.5.D	2	1
10-7	Interpret Graphical Displays	6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C, 6.SP.B.5.D	2	1
Put It All Together 2: Lessons 10-2, 10-3, 10-4, 10-5, 10-6, and 10-7			0.5	0.25
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	15	7.5

Formative Assessment Math Probe
Measures of Center and Spread

CHERYL TOBEY MATH PROBES


Measures of Center and Spread

Correct Answers: 1. Yes; **2.** No; **3.** Yes; **4.** No; **5.** Yes

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students state whether they agree or disagree with each statement about the measures of center, and explain their choice.

Targeted Concept Understand that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Targeted Misconceptions

- Students may view the measure of variation (range) as a fixed measure based on the measures of center, rather than a measure that can vary based on the entire data set.
- Students may view the measures of center as arbitrary numbers based on a procedure.

Assign the probe after Lesson 4.

Collect and Assess Student Work

If the student selects...	Then the student likely...
2. Yes	does not consider that different data sets can have the same measures of center. **Example:** For Exercise 2, the student simply adds and subtracts 9 to the median, without considering other possibilities.
4. Yes	views the measures of center as numbers without context. **Example:** For Exercise 3, the student understands the median to be the middle number, without considering that an even number of scores do not have a middle number. Or the student does not consider that more than one student can score 91, thus varying the number of scores below and/or above the median.
Various other patterns	misunderstands or confuses terms of measures of center with spread.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Data Analysis and Probability
- Lesson 3, Examples 1–4
- Lesson 4, Examples 1–3

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ average	☐ interquartile range (IQR)	☐ quartiles
☐ box plot	☐ mean	☐ range
☐ cluster	☐ mean absolute deviation	☐ second quartile
☐ distribution	☐ measures of center	☐ statistical question
☐ dot plot	☐ measures of variation	☐ statistics
☐ first quartile	☐ median	☐ symmetric distribution
☐ gap	☐ outlier	☐ third quartile
☐ histogram	☐ peak	

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Add rational numbers.

Find 11.83 + 8.76 + 13.28 + 16.38.

$$\begin{array}{r} 11.83 \\ 8.76 \\ 13.28 \\ +\ 16.38 \\ \hline 50.25 \end{array}$$

Add.

Example 2
Divide rational numbers.

Lydia typed 105.2 words in 4 minutes. How many words did Lydia average typing each minute?

$105.2 \div 4 = 26.3$ Divide the total number of words typed by the number of minutes.

Lydia averaged 26.3 words each minute.

Quick Check

1. Find 7 .68+ 5.25 + 2.99 + 3.18.
   19. 1

2. A pilot flew 1,308.3 miles this week. The pilot flew the same number of miles each of 3 days this week. How many miles did the pilot fly each day?
   436. 1 miles

How Did Y ou Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right.

1 2

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine.

Define The **measures of center** are numbers that are used to describe the center of a data set; these measures include the mean and median.

Example A data set consists of the numbers 2, 3, 3, 4, 18, and 3.

Ask Find the mean and median of the data set. Then compare them. mean: 5.5; median: 3; Sample answer: The mean is greater than the median, because of the data value 18 being far away from the other data values.

Are You Ready?

Students may need to review the following prerequisite skills to succeed.

- plotting points on a number line
- understanding bar diagrams
- understanding ratios and rates
- ordering rational numbers
- absolute value
- subtracting, multiplying, and dividing rational numbers

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Data Analysis and Probability** topic in order to adjust your instruction as appropriate.

Mindset Matters

Attitude Ownership

Part of developing a growth mindset involves acknowledging progress in growth thinking and sharing it with others. It's important for a student to own his or her mindset, attitude, and beliefs and be proud of the growth. Students should view themselves as people who have a growth mentality—not just in math, but with learning, in general.

How Can I Apply It?

Have students complete a math mindset project to share how they have grown throughout the year. They might choose their own delivery method, such as a poster, blog post, video, or podcast. Encourage them to give specific examples from their journey, such as times when they made a mistake and learned from it, times when they took a risk to solve a challenging problem, or times when they engaged in reflection. Students can share their mindset journey with their classmates, or might post their projects for others to see.

Learn Statistical Questions

Objective

Students will understand that statistical questions are answered by collecting data and anticipate a variety of responses.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* questions on Slide 3, encourage them to adhere to the definition of a statistical question in order to explain why the question is not a statistical question.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Why is *How many people attended last night's jazz concert?* not a statistical question? How can you rewrite the question so it is a statistical question? Sample answer: The question is not a statistical question because it does not anticipate a variety of answers from data. There is only one value that answers that question. In order to be a statistical question, I can rewrite it to be *How many concerts do typical students attend?* or *How early do people typically arrive before concerts begin?*

DIFFERENTIATE

Language Development Activity ELL

Some students may struggle to understand the difference between *statistical questions* and questions that are *not statistical*. Give students the following additional examples of each type of question and then have students create their own statistical question and a question that is not statistical.

Statistical Questions:

How many times a day do students in your school typically check their phone?

How many times a week do people typically check their email?

Not Statistical Questions:

How many customers attended the grand opening of the new restaurant last night?

How many registered voters voted in the last election?

Lesson 10-1

Statistical Questions

I Can... understand that a statistical question anticipates a variety of responses.

What Vocabulary Will You Learn?
statistical question
statistics

Learn Statistical Questions

Statistics involves collecting, organizing, and interpreting pieces of information, or data. One way to collect data is by asking statistical questions. A **statistical question** is a question that is answered by collecting data. Answers to a statistical question will vary based on the data collected.

The table gives some examples of statistical questions and examples that are not statistical questions.

Statistical Questions	Not Statistical Questions
How many text messages do middle school students typically send each day?	What is the height in feet of the tallest mountain in Colorado?
How many hours per night does a typical teenager spend watching television?	How many people attended last night's jazz concert?

In the table, the questions on the left are statistical questions because if you were to survey a group of students, you will likely get a variety of responses. The questions on the right are not statistical questions because each question has one specific response.

Constructing statistical questions is an important part of the process of using statistics to collect, organize, and interpret data. You will learn how to apply these steps in order to help answer a statistical question.

Step 1 Construct a statistical question.

Step 2 Use your question to collect data.

Step 3 Summarize the data using tables or graphical displays.

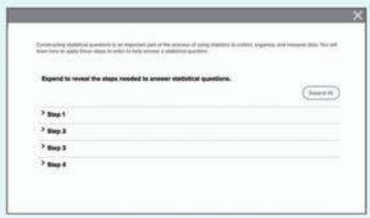
Step 4 Use the data to answer the statistical question.

Talk About It!
Why is *How many people attended last night's jazz concert?* not a statistical question? How can you rewrite the question so it is a statistical question?

Sample answer: The question is not a statistical question because it does not anticipate a variety of answers from data. There is only one value that answers that question. In order to be a statistical question, I can rewrite it to be *How many concerts do typical students attend?* or *How early do people typically arrive before concerts begin?*

Lesson 10-1 • Statistical Questions 537

Interactive Presentation



Learn, Statistical Questions, Slide 2 of 3

EXPAND

On Slide 2, students expand to reveal the steps needed to answer statistical questions.

Lesson 10-1

Statistical Questions

6.SP.A.1

LESSON GOAL

Students will identify and use statistical questions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Statistical Questions
Example 1: Identify Statistical Questions

Explore: Collect Data

Learn: Display Data in a Table
Example 2: Display Data in a Table

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

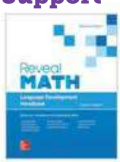
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Biased and Unbiased Samples		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 53 of the *Language Development Handbook* to help your students build mathematical language related to statistical questions.



ELL You can use the tips and suggestions on page T53 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability

Additional Cluster(s): In this lesson, students address additional cluster **6.SP.A** by identifying and using statistical questions.

Standards for Mathematical Content: **6.SP .A.1,** *Also addresses 6.SP.B.5.A*

Standards for Mathematical Practice: **MP2, MP3, MP6**

Coherence

Vertical Alignment

Previous
Students represented and interpreted data.
5.MD.B.2

Now
Students identify and use statistical questions.
6.SP.A.1

Next
Students will construct dot plots and histograms using collected data.
6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A

Rigor

The Three Pillars of Rigor

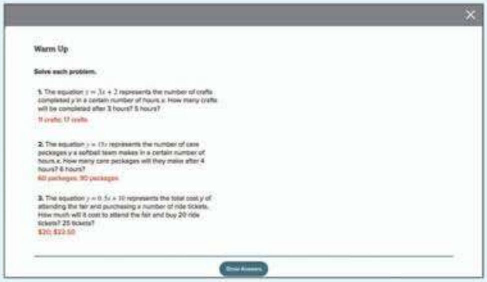
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of representing and interpreting data (gained in grade 5) to begin to develop *understanding* of statistical measures. They come to understand that statistical questions anticipate a variety of answers based on data. They also learn how to organize collected data in a table and analyze the results.

Mathematical Background

Statistics is the collection and analysis of data. Data are often collected using surveys with *statistical questions*. A statistical question is a question that has varying answers. Examples include wait times at an amusement park, heights of individuals in a school, or ages of trees in a forest. A table can be used to organize the results of a survey, where one column lists the possible survey responses and the other column records the number of respondents indicating a specific response.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- solving equations (Exercises 1–3)

Answers

1. 11 crafts; 17 crafts
2. 60 packages; 90 packages
3. \$20; \$22.50

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about wait times for rides at an amusement park.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

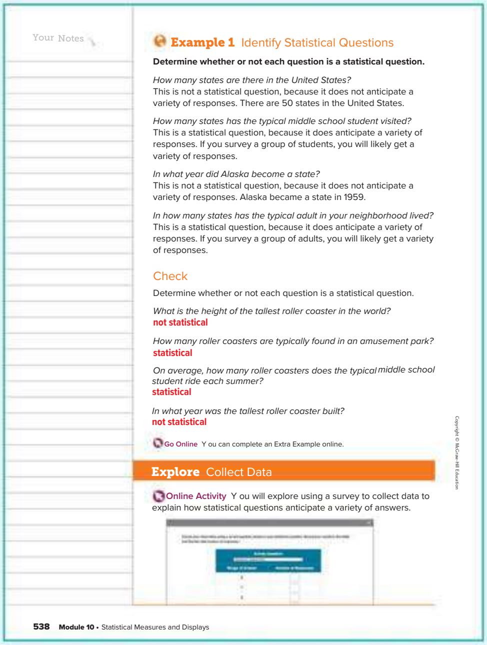
Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Statistics deals with collecting, analyzing, interpreting, and presenting data based on *statistical questions.* What might a *statistical question* ask? Sample answer: What are the heights of students in my class?
- *Statistics* is a branch of mathematics as is arithmetic, geometry, and algebra. What do you think makes statistics different than the other branches? Sample answer: Statistics deals with working with data, while the others concentrate on numbers, figures, and equations.



Your Notes

Example 1 Identify Statistical Questions

Determine whether or not each question is a statistical question.

How many states are there in the United States?
This is not a statistical question, because it does not anticipate a variety of responses. There are 50 states in the United States.

How many states has the typical middle school student visited?
This is a statistical question, because it does anticipate a variety of responses. If you survey a group of students, you will likely get a variety of responses.

In what year did Alaska become a state?
This is not a statistical question, because it does not anticipate a variety of responses. Alaska became a state in 1959.

In how many states has the typical adult in your neighborhood lived?
This is a statistical question, because it does anticipate a variety of responses. If you survey a group of adults, you will likely get a variety of responses.

Check

Determine whether or not each question is a statistical question.

What is the height of the tallest roller coaster in the world?
not statistical

How many roller coasters are typically found in an amusement park?
statistical

On average, how many roller coasters does the typical middle school student ride each summer?
statistical

In what year was the tallest roller coaster built?
not statistical

Go Online You can complete an Extra Example online.

Explore Collect Data

Online Activity You will explore using a survey to collect data to explain how statistical questions anticipate a variety of answers.

538 **Module 10** • Statistical Measures and Displays

Interactive Presentation

Example 1, Identify Statistical Questions, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to identify whether or not each question is a statistical question.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Identify Statistical Questions

Objective

Students will identify statistical questions.

Teaching the Mathematical Practices

6 Attend to Precision As students study each of the given questions, encourage them to adhere to the definition of a statistical question, in order to determine whether or not each question is a statistical question. For any question they determine is not a statistical question, have them explain why not.

Questions for Mathematical Discourse

SLIDE 1

AL How can you determine if a question is a statistical question? Sample answer: The question needs to have a variety of answers and I can answer it based on data I collect.

OL Why is *In what year did Alaska become a state?* not a statistical question? Sample answer: This question is not a statistical question because it has one answer.

BL Identify a topic that interests you and write a statistical question you might want to answer about that topic. Sample answer: *How many different video games does the typical middle school student own?*

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Display Data in a Table

Objective

Students will learn how to display the responses to a statistical question in a table.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the data in the table in order to make observations about the data.

Talk About It!

SLIDE 2

Mathematical Discourse

What are some other observations you can make about the data in the table? Sample answer: About half of the people exercise 2 or 3 hours each week.

Example 2 Display Data in a Table

Objective

Students will organize the responses to a statistical question in a table and analyze the results.

Questions for Mathematical Discourse

SLIDE 2

AL How will you record the data in the table? Sample answer: I will count the number of responses for each number of hours and put that number in the column on the right.

OL How is recording the responses in a table helpful? Sample answer: It shows me how many responses there were to each number of hours. The list does not show the total number of responses.

BL If someone reports 7 hours as a response, how could you alter the table? Sample answer: Add a row to the existing table and record the result.

(continued on next page)

Learn Display Data in a Table

A survey is one way to collect data to answer a statistical question. Once the data are collected, you can record the results in an organized way, such as a table, and then analyze the results.

Suppose a random group of adults were asked the question *How many hours do you exercise each week?* The results are shown in the table.

How many hours do you exercise each week?							
Number of Hours	0	1	2	3	4	5	6
Number of Responses	1	2	5	5	3	1	2

Based on the results in the table, one observation you can make is that more than half of the people who responded exercised fewer than four hours per week.

Talk About It!
What are some other observations you can make about the data in the table?

Sample answer: About half of the people exercise 2 or 3 hours each week.

Example 2 Display Data in a Table

Suppose you want to answer the statistical question *How many hours per week does the typical sixth grade math student study?* You survey students in your math class using the question *How many hours do you typically spend studying each week?* The responses were 2, 4, 5, 4, 2, 1, 3, 1, 1, 4, 6, 3, 5, 2, 2, 1, 1, and 4 hours.

Organize the data in a table. Then analyze the results.

Part A Organize the data in a table.

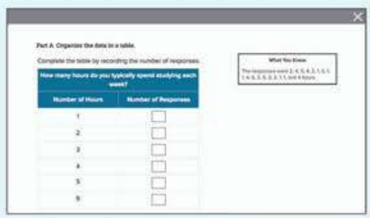
Complete the table by recording the number of responses.

How many hours do you typically spend studying each week?	
Number of Hours	Number of Responses
1	5
2	4
3	2
4	4
5	2
6	1

(continued on next page)

Lesson 10-1 • Statistical Questions 539

Interactive Presentation



Example 2, Display Data in a Table, Slide 2 of 6

TYPE

On Slide 2 of Example 2, students organize the data in a table.

TYPE

On Slide 3 of Example 2, students find the total number of students surveyed.

Explore Collect Data

Objective

Students will explore how statistical questions can produce a variety of answers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will pick a statistical question, survey their classmates, and then record the results and find any patterns in the data. Students should use their statistical question to explain how a good statistical question allows for a variety of responses.

Inquiry Question

How does a good statistical question allow for a variety of responses? Sample answer: A statistical question asks a group of people about a specific topic that can have a variety of answers. To create a good statistical question, anticipate and predict possible responses before collecting numerical data.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

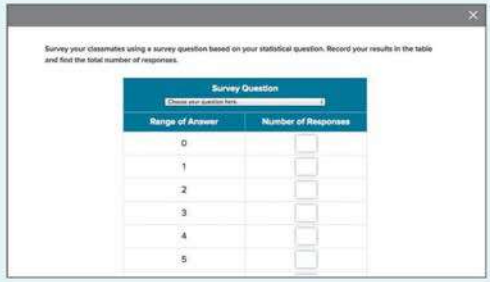
How do you know the question is a statistical question? Sample answer: The question anticipates a variety of answers based on data.

How do you plan to record the responses to the survey? Sample answer: I will create a table that includes possible answers, and a place for recording answers from survey participants.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 5

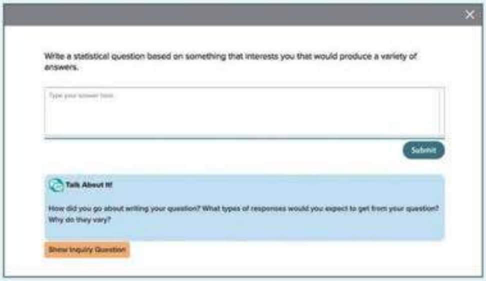


Explore, Slide 3 of 5

CLICK

On Slide 3, students select a statistical question.

Interactive Presentation



Explore, Slide 4 of 5

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Collect Data *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should pause, as needed, during the process to explain that a statistical question produces a variety of answers in order to write a new statistical question.

6 Attend to Precision Encourage students to use precision when recording their data and use the data to explain any trends.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

How did you go about writing your question? What types of responses would you expect to get from your question? Why do they vary? Sample answer: Using topics that interest me, I wrote a question that anticipates a variety of responses. For the statistical question *How many hours of television did the average middle school student watch last week?,* I expect responses to range from 0 hours to 10 or more hours. The responses will vary because students watch a varying amount of television each week.

Part B Analyze the results.

Step 1 Find the total number of students surveyed.

Find the sum of the number of responses.

5 + 4 + 2 + 4 + 2 + 1 = **18** students

Talk About It!
What are some other observations that can be made based on the data?

Sample answer: Most students study for fewer than 4 hours each week

Step 2 Summarize the data.

Study the responses to determine if there is an overall trend.

One observation you can make is that half of the students in the survey studied fewer than 3 hours per week.

Check

Suppose you want to answer the statistical question *How many times does the typical middle school student exercise each month?* You survey your friends using the question *How many times each month do you typically exercise?*

The responses were 14, 12, 6, 2, 1, 0, 10, 6, 3, 4, and 5 times.

Organize the data in a table. Then analyze the results.

Part A

Organize the data by completing the table.

Number of Times Spent Exercising	Number of Responses
fewer than 4	4
4–7	4
8–11	1
12 or more	2

Part B

Select the statement that best represents the data.

Ⓐ Most students surveyed typically exercise at least 8 times each month.

Ⓑ Most students surveyed typically exercise more than 7 times each month.

Ⓒ Most students surveyed typically exercise 7 or fewer times each month.

Ⓓ Exactly half of the students surveyed typically exercise 4 or more times each month.

Go Online You can complete an Extra Example online.

Example 2 Display Data in a Table *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL How will you find the total number of responses? I will add the numbers in the right side of the column.

OL What does the value of 4 mean when the number of hours is 2? There were four people surveyed that reported that they spent 2 hours studying last week.

BL How could you make sure that you correctly represented the data? Sample answer: I could check to make sure that each response is represented once in the left side of the table, and then make sure that each response is recorded in the table.

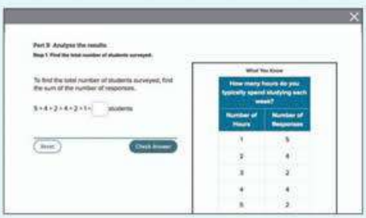
SLIDE 4

AL What values will you add when the statement says, "...fewer than 3 hours..."? Explain. 5 and 4; Sample answer: Fewer than 3 hours means 1 hour or 2 hours. The number of responses for 1 hour is 5, and the number of responses for 2 is 4.

OL Make another observation about the data. Sample answer: Less than half of the people studied more than 3 hours.

BL What percent of the people studied more than 3 hours? Round to the nearest percent. Does writing the value as a percent help you to make observations? Explain. 39%; yes; Sample answer: When I write a value as a percent, I can see how it relates to the entire group of people surveyed, or 100%.

Interactive Presentation



Example 2, Display Data in a Table, Slide 4 of 6

CLICK

On Slide 4 of Example 2, students select the correct phrase to summarize the data.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Essential Question Follow-Up

Why is data collected and analyzed and how can it be displayed?

In this lesson, students learned how to identify examples and non-examples of statistical questions, and how to collect and organize data from a survey. Have them discuss with a partner why collecting data is important. Some students may say collecting data is important because it helps answer questions about their world. Encourage students to be inquisitive about their everyday lives and how they can collect data to help them answer their questions.

Exit Ticket

Refer to the Exit Ticket slide. Brady does not have a pass and wants to know how many minutes he can expect to wait in line for the new roller coaster. He surveys a group of people who rode the roller coaster about the number of minutes they waited. The responses were 45, 30, 28, 35, 20, 60, 60, 60, 60, 90, 45, 30, and 45. Use a table to estimate how long Brady can expect to wait to ride the roller coaster. Write a mathematical argument that can be used to defend your solution. Sample answer: 60 minutes is the most common response from the survey participants, so Brady should expect to wait about that long.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify statistical questions	1–4
1	organize the responses to a statistical question in a table and analyze the results	5–7
2	extend concepts learned in class to apply them in new contexts	8
3	higher-order and critical thinking skills	9–12

Common Misconception

Students may confuse *statistical questions* and *survey questions.* A *survey question* is a question that students ask to answer a *statistical question*. In Exercise 6, *How many toppings do you like on an ice cream sundae?* is a survey question designed to answer the statistical question *How many toppings do students typically like on an ice cream sundae?*. Remind students that statistical questions anticipate a variety of responses.

Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Determine whether or not each question is a statistical question. (Example 1)

1. *How many continents are there?* not a statistical question
2. *How many continents has the average student visited?* statistical question
3. *How many sporting events did the average student attend last year?* statistical question
4. *In what year was the first World Series?* not a statistical question

5. Suppose you want to determine the number of siblings each of your classmates have. You survey them using the question *How many siblings do you have?*. The responses were 1, 4, 2, 3, 0, 1, 0, 5, 1, 2, 2, 3, 0, 1, 2, 0, 1, 1, 6, and 2 siblings. Organize the data by completing the table and analyze the results. (Example 2)

Number of Siblings	Number of Responses
0–1	10
2–3	7
4–5	2
6 or more	1

Sample answer: Half of the students have 0 or 1 siblings.

6. You survey your classmates using the question *How many toppings do you like on an ice cream sundae?*. The responses were 2, 3, 7, 4, 5, 5, 4, 4, 1, 2, 4, 3, 4, 3, 6, 0, 4, 5, 6, and 5 toppings. Organize the data by completing the table and analyze the results. (Example 2)

Number of Toppings	Number of Responses
0–1	2
2–3	5
4–5	10
6 or more	3

Sample answer: The most common response is 4–5 toppings.

7. You survey your classmates using the question *How many sports do you play?*. The responses were 2, 2, 1, 3, 1, 2, 4, 1, 2, 1, 3, 2, 2, and 2 sports. Organize the data by completing the table and analyze the results. (Example 2)

Number of Sports	Number of Responses
1	4
2	7
3	2
4	1

Sample answer: Half of the students that responded play 2 sports.

Lesson 10-1 • Statistical Questions 541

Interactive Presentation

Exit Ticket

T est Practice

8. Multiselect Which of the following are statistical questions? Select all that apply.

- ✓ How many DVDs does a typical student own?
- ✓ How many classes does each student take?
- ☐ How many oceans are there in the world?
- ✓ How many pets does a typical student own?
- ✓ How many times did a typical student go to the zoo last year?
- ☐ How many continents are there?

Higher-Order Thinking Problems

9. Create Write a survey question that is a statistical question. Then write a survey question that is *not* a statistical question. Explain why each question is or is not a statistical question.

Sample answers: How many smartphones does a typical family own?; In what year was the cell phone invented?; The first question is a statistical question because it anticipates a variety of responses. The second question is not a statistical question because it does not anticipate a variety of responses.

10. Find the Error Pete surveyed his friends as to the amount of their weekly allowance. The responses were $5, $0, $8, $10, $8, $10, $0, $0, and $1. Pete analyzed the results and stated that more than half of his friends earned $8 or more per week. Find his mistake and correct it.

Sample answer: Only 4 out of 9 friends earn $8 or more, which is less than half. Pete may not have been counting the $0 responses, but he must still include them in the results. A correct analysis would be that more than half of his friends earn $5 or less. Another correct analysis would be that a third of his friends do not earn an allowance.

11. Reason Abstractly Mara surveyed her friends as to the number of tablets their family owns. The responses were 1, 2, 2, 1, 0, 3, 1, 2, 4 and 2 tablets. Mara concludes that of her friends' families, most own 1 or 2 tablets. Is she correct? Explain.

yes; Sample answer: Of the 10 families, 3 own one tablet and 4 own two tablets. Since 3 + 4 is 7 and 7 is close to 10, this is a reasonable conclusion.

12. Refer to Exercise 8. Choose one of the questions that is not a statistical question and rewrite it so that it is a statistical question.

Sample answer: Revise the question *How many continents are there?* to be *How many continents has the typical adult in your community visited?*.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students will find the error and correct it. Encourage students to find the error and then explain how the error of the friend can be corrected.

2 Reason Abstractly and Quantitatively In Exercise 11, students explain whether Mara is correct or not. Encourage students to use reasoning to explain why Mara is correct.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Interview a student.

Use with Exercises 10–11 Have pairs of students interview each other as they complete this problem. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 10 might be, "Does the number of people that Pete surveyed affect the data? Why or why not?"

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or above on the Checks,
THEN assign:

- Practice, Exercises 1–7 odd, 9–12
- Extension: Biased and Unbiased Samples
- ALEKS Collecting Data

OL

IF students score 66–89% on the Checks,
THEN assign:

- Practice, Exercises 1–7, 10, 11
- Extension: Biased and Unbiased Samples
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Collecting Data

AL

IF students score 65% or below on the Checks,
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Collecting Data

Learn Construct Dot Plots

Objective

Students will learn how to construct a dot plot to represent a data set.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

How does using the visual representation allow you to make observations more easily? Sample answer: The dot plot helps to quickly see values and patterns, such as values that do not have responses, or values that have a lot of responses.

Example 1 Construct Dot Plots

Objective

Students will construct a dot plot to represent a data set and summarize the results.

Questions for Mathematical Discourse

SLIDE 2

AL How will you determine what numbers to label on the number line? Sample answer: I will look at the range of data values, then label the greatest value, the least value, and the others in between.

OL Why do you think negative numbers are not part of the number line? Sample answer: There are no negative data values because you can't have a negative number of pets.

BL Looking at the table, where do you think the tallest parts of the graph or *a peak* will occur? Explain. Sample answer: I think a peak will occur on 2 as it seems more people have 2 pets than any other number.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Lesson 10-2

Dot Plots and Histograms

I Can... use dot plots and histograms to display and analyze data.

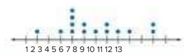
What Vocabulary Will You Learn?
dot plot
histogram

Learn Construct Dot Plots

One way to represent a data set is to construct a **dot plot**. A dot plot is a visual display of a distribution of data values where each data value is shown as a dot above a number line.

The number of wins in a recent year by several football teams is 10, 9, 6, 6, 5, 5, 2, 12, 12, 8, 8, 7, 5, 5, and 4 wins. The dot plot shown organizes the data and shows possible patterns.

Football Team Wins



Talk About It! How does using the visual representation allow you to make observations more easily?

Sample answer: The dot plot helps to quickly see values and patterns, such as values that do not have responses, or that have a lot of responses.

Example 1 Construct Dot Plots

Jasmine surveyed the students in her class using the question *How many pets do you own?*. The results are shown in the table.

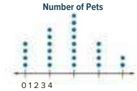
Number of Pets											
3	0	0	1	2	1	1	2	2	0	1	2
1	2	3	4	3	3	2	0	1	4	2	2

Construct a dot plot of the data. Then summarize the results.

Part A Construct a dot plot.

Draw and label a number line from 0 to 4, because the least data value is 0 and the greatest data value is 4. For each data value, place a dot above the corresponding number on the number line.

Number of Pets



Part B Summarize the results.

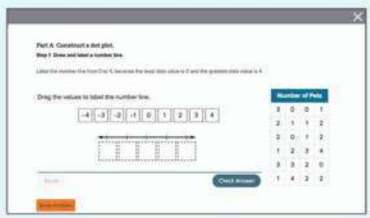
A total of 24 students responded to the survey. Students are more likely to own 1 or 2 pets than 3 or 4 pets. No student in the survey owned more than 4 pets.

Talk About It! Which representation – the table or the dot plot – helps you visualize the results of the survey? Explain.

Sample answer: The dot plot allows me to see an overall pattern in the data.

Lesson 10-2 • Dot Plots and Histograms 543

Interactive Presentation



Example 1, Construct Dot Plots, Slide 2 of 6

CLICK

On Slide 3 of Example 1, students select as many dots above each number as there are responses for that number to construct the dot plot.

DRAG & DROP

On Slide 2 of Example 1, students drag to label the number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Lesson 10-2

Dot Plots and Histograms

6.SP.B.4

LESSON GOAL

Students will construct dot plots and histograms using collected data.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Construct Dot Plots
Example 1: Construct Dot Plots
Learn: Construct Histograms
Example 2: Construct Histograms

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

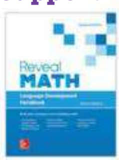
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Stem-and-Leaf Plots		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 54 of the *Language Development Handbook* to help your students build mathematical language related to dot plots and histograms.



ELL You can use the tips and suggestions on page T54 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability

Additional Cluster(s): In this lesson, students address additional cluster **6.SP.B** by constructing dot plots and histograms using collected data.

Standards for Mathematical Content: **6.SP .B.4, 6.SP.B.5, 6.SP.B.5.A**

Standards for Mathematical Practice: **MP2, MP3**

Coherence

Vertical Alignment

Previous
Students identified and used statistical questions.
6.SP.A.1

Now
Students construct dot plots and histograms using collected data.
6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A

Next
Students will understand and apply different measures of center.
6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C

Rigor

The Three Pillars of Rigor

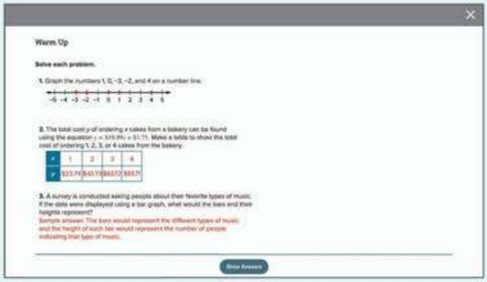
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of statistical measures as they learn about dot plots and histograms. They build *fluency* with constructing dot plots and histograms using collected data, and also with summarizing the results of real-world scenarios.

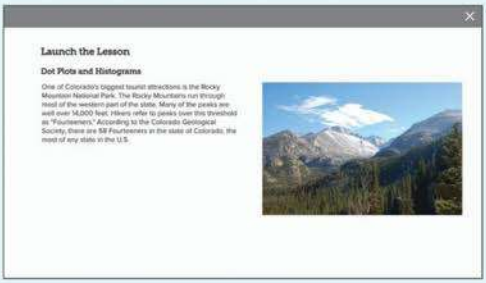
Mathematical Background

Two ways to display quantitative data are dot plots and histograms. A *dot plot* displays the distribution of data values by placing a dot above each data value on a number line. A *histogram* can be created from a frequency table by creating equal intervals spanning the range of the data, counting the number of data values falling in each interval, and plotting the results using a bar graph.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- plotting points on a number line (Exercise 1)
- creating tables (Exercise 2)
- understanding bar graphs (Exercise 3)

Answers

1–2. See Warm Up slide online for correct answers.

3. Sample answer: The bars would represent the different types of music and the height of each bar would represent the number of people indicating that type of music.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the height of the peaks of mountains in Rocky Mountain National Park.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Thinking about the meaning of the words *dot* and *plot*, what is a *dot plot*? Sample answer: A dot plot could represent data using a number line and dots that represent each answer to a survey question.
- A *histogram* is one type of graphical display. What other kinds of graphical displays have you seen? Sample answer: bar diagram, line plot

Jessica Byrne

Your Notes

Check

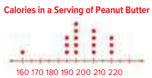
Leah researched the number of Calories in a serving of peanut butter from various brands of peanut butter. The results are shown in the table. Construct a dot plot of the data. Then summarize the results.

Calories in a Serving of Peanut Butter			
190	160	210	210
200	185	190	190
185	200	190	210
190	185	200	200

Part A

Construct a dot plot.

Show your work here



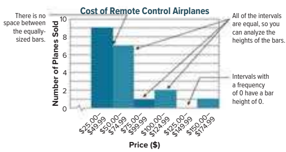
Part B

There are 16 brands of peanut butter. The brand with the greatest number of Calories per serving contained 210 Calories and the brand with the least contained 160 Calories. The most common number of Calories per serving was 190.

Learn Construct Histograms

Data from a frequency table can be displayed as a **histogram**, a type of bar graph used to display numerical data that have been organized into equal intervals. This allows you to see the frequency distribution of the data, or the quantity of data that are in each interval.

When constructing a histogram, it is important that the intervals are equal and consecutive, so that you can make accurate observations about the data, based on the heights of the bars. The intervals should leave no gaps so that the entire range of data can be represented.



Talk About It!

For which representation – a dot plot or a histogram – can you see all of the individual data values? When might you choose to use a histogram as opposed to a dot plot?

dot plot; Sample answer: If the number of data values is a large number, a histogram can be more efficient to construct.

Interactive Presentation

Learn, Construct Histograms, Slide 2 of 3

CLICK

On Slide 2, students select the markers to learn how histograms are structured.

Learn Construct Histograms

Objective

Students will learn how to construct a histogram to represent a data set.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others Encourage students to construct a plausible argument to explain why it is important that the intervals in a histogram are equally spaced. Have them think about how easy or difficult it would be to study the data displayed in a histogram if the intervals were not equally spaced.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

For which representation – a dot plot or a histogram – can you see all of the individual data values? When might you choose to use a histogram as opposed to a dot plot? dot plot; Sample answer: If the number of data values is a large number, a histogram can be more efficient to construct.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of dot plots and histograms, have them work with a partner to compare and contrast the structure and purpose of dot plots and histograms. Have them make predictions as to when it might be more useful to create one type of display instead of the other. Have them create a Venn diagram or other type of graphic organizer that compares the two types of displays. They should share their graphic organizer with another pair of students and discuss and resolve any differences.

Sample responses can include the following:

A dot plot shows all of the individual data values in a data set, while a histogram does not.

Both types of plots display numerical data.

A dot plot might be more useful to create when the total number of data values is relatively small, since each data value is plotted on the number line.

A histogram might be more useful to create when the total number of data values is relatively large, or there is a large range of data values.

Example 2 Construct Histograms

Objective

Students will construct a histogram to represent a data set.

Questions for Mathematical Discourse

SLIDE 2

AL After the intervals have been decided, what do you need to do? Sample answer: I need to determine the number of data values that are in each interval.

OL Why is 50 a good interval size to use for the histogram? Sample answer: If the interval was smaller, the histogram might be too long, and if the interval was larger, there would be too many data values in each interval.

BL Why aren't the intervals labeled 100–150, 150–200, 200–250, ...? Sample answer: Those intervals have 51 whole number data values, not 50. They also overlap, so a value of 150 would be in two intervals.

SLIDE 3

AL What intervals will you use for the *x*-axis? 100–149, 150–199, 200–249, 250–299, 300–349, 350–399

OL What are the labels for each axis? "Visitors" will be on the *x*-axis and "Frequency" will be on the *y*-axis.

BL If the scale of the vertical axis was increased by a factor of 2, how would the labels change? Sample answer: Each line on the *y*-axis would be labeled 0, 2, 4, 6, and 8.

SLIDE 4

AL How do you know how many sections to shade for each bar? Sample answer: Using the table, I will shade the frequency for each interval. For example, the interval 100–149 has a frequency of 2, so I will shade two sections above that interval.

OL How does the histogram help you detect patterns in the data? Sample answer: The histogram is a visual display of the data. I can see what intervals have a low frequency of occurrence and which have a high frequency.

BL How can you use the histogram to find the number of days the park had between 100–249 visitors? Sample answer: I can add the frequencies for the intervals 100–149, 150–199, and 200–249.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Construct Histograms

A park ranger at a state park was asked the question *How many daily visitors attended the park each day for 20 days?*. The table shows the results.

Daily Visitors				
108	209	171	152	236
165	244	263	212	161
327	185	192	226	137
193	235	207	382	241

Construct a histogram to represent the data.

Step 1 Make a frequency table.

Use a scale to include all of the values, 100 through 399, with equally-spaced intervals.

Complete the frequency table to organize the data.

Daily Visitors	
Visitors	Frequency
100–149	2
150–199	7
200–249	8
250–299	1
300–349	1
350–399	1

Step 2 Draw and label the axes.

When you construct the histogram, first draw the axes. Label the horizontal axis using the intervals from the frequency table, 100–149 through 350–399. Label the vertical axis with the frequencies, 1–10.

Step 3 Graph the intervals.

For each interval, draw a bar with a height that is indicated by the frequency table. Complete the histogram by drawing and shading the correct bar heights.

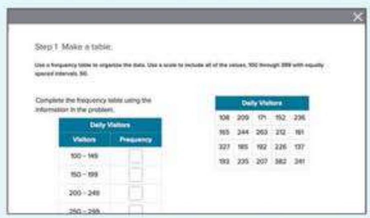
Think About It! What intervals will you select for the histogram?

See students' responses.

Talk About It! What observations can you make about the data by studying the histogram?

Sample answer: On most days, the number of visitors was between 150 and 249.

Interactive Presentation



Example 2, Construct Histograms, Slide 2 of 6

TYPE On Slide 2, students complete a frequency table to organize the data.

CLICK On Slides 3 and 4, students label the axes and graph the intervals.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

The students in Mrs. Angelo's class were asked the question *How many books did you read over summer vacation?*. The responses are shown in the table.

Number of Books Read
3 6 4 2 8 0
6 3 9 3 1 4
5 12 10 4 11 0
7 3 7 5 12 6
7 13 14 5 1 2

Construct a histogram to represent the data.

Show your work here

Sample answer:

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of when to use dot plots and histograms. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

Why is data collected and analyzed and how can it be displayed?

In this lesson, students learned how to display data using dot plots and histograms. Encourage them to work with a partner to compare and contrast how dot plots and histograms display data. For example, they may say that both kinds of graphs represent numerical data. While a dot plot shows every single data value, a histogram does not.

Exit Ticket

Refer to the Exit Ticket slide. Suppose in one certain region of the Rocky Mountains there are 7 peaks with elevations of 12,361 feet, 12,618 feet, 13,308 feet, 13,631 feet, 13,829 feet, 14,100 feet, and 14,440 feet. Describe the intervals you would use to make a histogram to represent the data.

Sample answer: I would use the intervals 12,000–12,499, 12,500–12,999, 13,000–13,499, 13,500–13,999, and 14,000–14,499. An interval of 1,000 would be too large and patterns in the data might not be seen.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1–5 odd, 6–9
- Extension: Stem-and-Leaf Plots
- ALEKS Graphs of Data

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–3, 5, 7, 8
- Extension: Stem-and-Leaf Plots
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Collecting Data

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Collecting Data

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

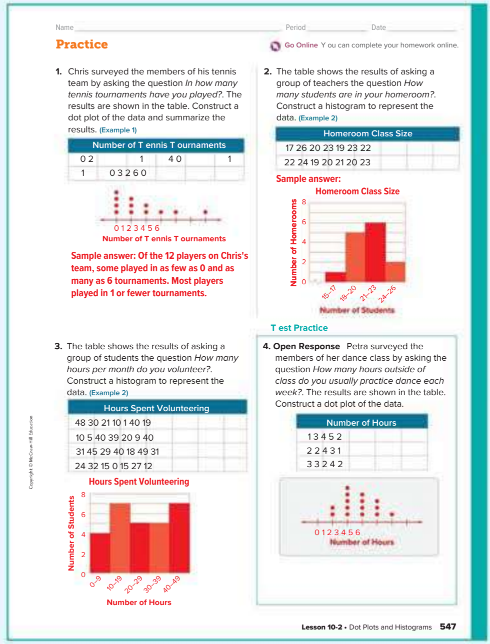
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	construct a dot plot to represent a data set and summarize the results	1
1	construct a histogram to represent a data set	2, 3
2	extend concepts learned in class to apply them in new contexts	4
3	solve application problems involving dot plots and histograms	5
3	higher-order and critical thinking skills	6–9

Common Misconception

In Exercise 1, some students may misinterpret the meaning of zero in the data set. Students may think that the zeros in the table should not be counted in the total number of tournaments. Encourage students to reread the problem and to reason about the given data values. Students should understand that even though there are four responses of zero, those four responses should still be counted in the total number of tournaments.



Name Period Date

Practice

Go Online You can complete your homework online.

1. Chris surveyed the members of his tennis team by asking the question *In how many tennis tournaments have you played?*. The results are shown in the table. Construct a dot plot of the data and summarize the results. (Example 1)

Number of Tennis Tournaments					
0 2		1	4 0		1
1	0 3 2 6 0				

Sample answer: Of the 12 players on Chris's team, some played in as few as 0 and as many as 6 tournaments. Most players played in 1 or fewer tournaments.

2. The table shows the results of asking a group of teachers the question *How many students are in your homeroom?*. Construct a histogram to represent the data. (Example 2)

Homeroom Class Size
17 26 20 23 19 23 22
22 24 19 20 21 20 23

Sample answer:

Test Practice

3. The table shows the results of asking a group of students the question *How many hours per month do you volunteer?*. Construct a histogram to represent the data. (Example 2)

Hours Spent Volunteering
48 30 21 10 1 40 19
10 5 40 39 20 9 40
31 45 29 40 18 49 31
24 32 15 0 15 27 12

4. Open Response Petra surveyed the members of her dance class by asking the question *How many hours outside of class do you usually practice dance each week?*. The results are shown in the table. Construct a dot plot of the data.

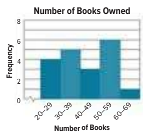
Number of Hours
1 3 4 5 2
2 2 4 3 1
3 3 2 4 2

Apply *indicates multi-step problem

*5. Lou wanted to determine how much his friends pay for video games. He surveyed them using the question *How much did you pay for the last video game you bought?* The responses were $29, $45, $50, $55, $34, $28, $35, $35, $45, $30, $34, and $55. How many more games cost between $30 and $39 than between $40 and $49?
3 video games

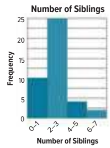
Higher-Order Thinking Problems

6. Provide a data set that can be represented by the histogram shown.



Sample answer: 21, 23, 20, 25, 35, 38, 34, 36, 39, 44, 42, 48, 50, 52, 57, 51, 50, 50, and 65 books

7. **Make a Conjecture** Refer to the histogram. In one or two sentences, write a conclusion you can make about the data.



Sample answer: Most students have 3 or fewer siblings. The most common number of siblings is 2 or 3.

8. **Reason Abstractly** Laura recorded the daily temperatures, in degrees Fahrenheit, during January in Minnesota. What changes might she have to make in a number line for a dot plot that starts at zero and goes to 20, so that it could be used to make a dot plot of the temperatures? Explain.

Sample answer: The number line might need to change to have numbers less than zero because the temperatures in January in Minnesota most likely will be below 0°F.

9. **Justify Conclusions** Determine if the statement is *true* or *false*. Justify your conclusion.

Histograms display individual data values.

false; Sample answer: Dot plots display individual data values. Histograms display data by equal intervals, not individual data values.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 7, students write a conclusion about the data. Encourage students to construct a conclusion that correctly illustrates the histogram.

2 Reason Abstractly and Quantitatively In Exercise 8, students explain changes that Laura might have to make in a number line for a dot plot that starts at zero and goes to 20. Encourage students to use reasoning to determine that the number line might need to have negative numbers.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students determine the validity of the statement. Encourage students to construct an explanation that correctly determines the statement is false.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.
Use with Exercise 5 Have students work in pairs. Have students individually read Exercise 5 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection.

Be sure everyone understands.
Use with Exercises 7–8 Have students work in groups of 3–4 to solve the problem in Exercise 7. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution with the class. Repeat the process for Exercise 8.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Measures of Center

Objective

Students will understand that the measures of center are used to represent a data set with a single value.

Teaching Notes

SLIDE 1

You may wish to ask students if they have heard of the measures of center before. Many students may be familiar with the terms *mean* and *median*. Be sure that students understand the purpose of describing a data set using a measure of center. You may wish to present students with a few examples of data sets that have many numerical values. Using a measure of center to describe the data set is an efficient way to summarize the data with a single number that best represents it.

SLIDE 2

Point out that the mean of 84 summarizes the data set with a single number. You may wish to ask students what this number means. For example, it does not mean that each of the four test scores were equivalent to 84. In fact, in this case, none of the data values are equivalent to the mean of 84. However, it does mean that the data is *centered around* the score of 84. The mean is often referred to as the *balance point* of the data. You may wish to have students create a line plot of the data values. Then have them visualize the number line as a tray that they need to carry with the palm of their hand supporting the tray. Ask them where they would position their hand in order to balance the tray. That location represents the mean of the data.

(continued on next page)

Lesson 10-3

Measures of Center

I Can... use the measures of center to summarize a numerical data set with a single number, and find a missing data value given the mean.

What Vocabulary Will Y ou Learn?
average
mean
measures of center
median

Explore Mean

Online Activity Y ou will use interactive workmats to explore how to find the mean of a data set.

Learn Measures of Center

A data set can contain many values, but sometimes it is beneficial to find a single value that can represent, or summarize, the entire data set. **Measures of center**are numbers used to describe the center of a numerical data set. The measures of center you will learn about in this lesson are the mean and median.

One measure of center used to describe a numerical data set is the **mean**. The mean, or**average**, of a data set is the sum of the data divided by the number of data values.

Suppose you have 4 test scores, 86%, 90%, 72%, and 88%. Y ou can find the mean by adding the test scores and then dividing by the total number of scores, 4.

$$\frac{86 + 90 + 72 + 88}{4} = 84$$

Add the test scores. Then divide by the total number of scores.

The mean score is 84%.

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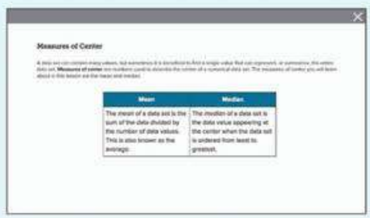
Interactive Presentation



Learn, Measures of Center, Slide 1 of 3

Lesson 10-3

Measures of Center

6.SP.A.3, 6.SP.B.4

LESSON GOAL

Students will understand and apply different measures of center.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Mean

Learn: Measures of Center
Example 1: Find the Mean
Learn: Find a Missing Data Value Using the Mean
Example 2: Find a Missing Data Value Using the Mean
Learn: Find the Median
Example 3: Find the Median Given an Odd Number of Data Values
Example 4: Find the Median Given an Even Number of Data Values
Apply: Track

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Weighted Average		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 55 of the *Language Development Handbook* to help your students build mathematical language related to the measures of center.

ELL You can use the tips and suggestions on page T55 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: Statistics and Probability

Major Cluster(s): In this lesson, students address major cluster **6.EE.B** and additional clusters **6.SP.A** and **6.SP.B** by understanding and applying different measures of center.

Standards for Mathematical Content: **6.SP .A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C,** *Also addresses 6.EE.B.6*

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous

Students constructed dot plots and histograms using collected data.
6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A

Now

Students understand and apply different measures of center.
6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C

Next

Students will understand interquartile range and create box plots.
6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of statistical measures as they explore measures of center. They use real-world scenarios to build *fluency* with finding the mean and median of a data set. They also build *fluency* with finding a missing data value given the mean.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation

Warm Up

Launch the Lesson

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- ordering rational numbers (Exercises 1–3)

Answers

1. 1.8 mi, 1.9 mi, 2.1 mi, 2.7 mi, 3.1 mi, 3.8 mi, and 4.7 mi
2. 0.31 in., 0.48 in., 0.68 in., 1.2 in., 1.88 in., 4.28 in., 4.55 in., 4.7 in., 6.42 in., 6.92 in., 7.18 in., and 12.1 in.
3. 7.9 oz, 8.8 oz, 9.4 oz, 9.8 oz, 10.5 oz, 10.8 oz, 11.1 oz, and 12.2 oz

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about measures of center, using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- In what real-world contexts have you used the word *average*? Sample answer: the average test score, the average height of students in the class
- The *mean* of the data set 2, 3, and 10 is 5. How would you describe what a mean is? Sample answer: A mean is the average value of a set of data.
- How might you measure or describe the center of a set of numbers? Sample answer: I would put the numbers in order from least to greatest, and then find the middle number.
- How does the *median* divide a road that you drive on? Sample answer: The median divides the road in half.

Explore Mean

Objective

Students will explore how to find the mean.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with different survey results. Students should use the results of the survey and rearrange them. Throughout this activity, students will use interactive workmats to determine if a single value can represent the data set.

Inquiry Question

How can you represent a data set with a single value? Sample answer: To find the average value of a data set I can use a model to evenly distribute the values, or I can find the sum of the values and divide the sum by the number of participants.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

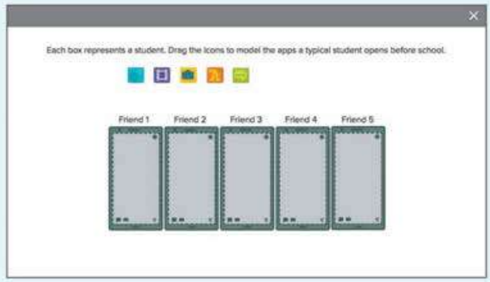
Mathematical Discourse

Compare your strategy with a partner. If your strategies were the same, is there another way to find the value? Sample answer: I could plot all of the data values on a number line and then find the number in the middle, or I could divide the values into an equal number of groups.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

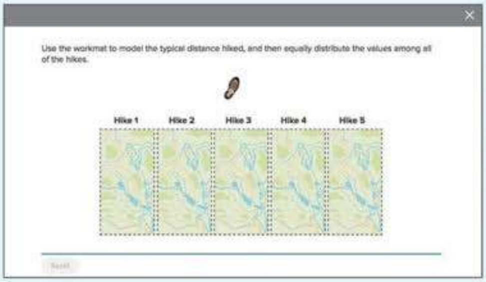


Explore, Slide 3 of 7

DRAG & DROP

On Slide 3, students drag the icons to model the apps a typical student opens before school.

Interactive Presentation



Explore, Slide 4 of 7

DRAG & DROP

On Slide 4, students drag the boot to equally distribute the values among all of the hikes.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Mean *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to ask statistical questions and record their observations using the workmat. Students should explore and deepen their understanding of data to determine if a single data value can represent the entire data set.

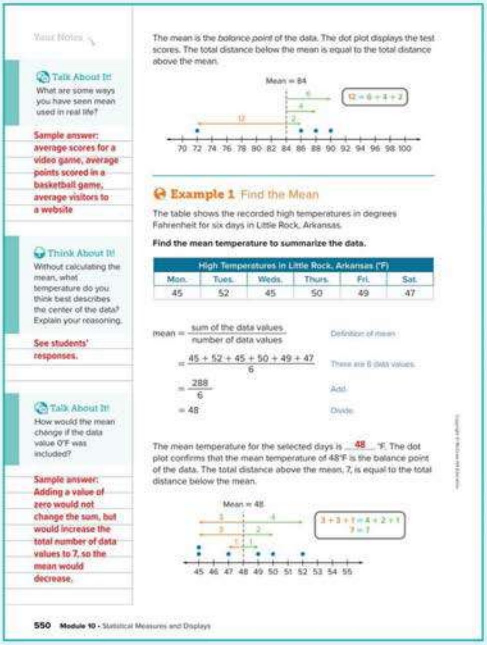
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Of the five distances, which value best represents the group? Is there a different value, not included in the group, that could represent the distance of a typical hike? If so, explain how you found this value.
See students' responses.



Your Notes

Talk About It!
What are some ways you have seen mean used in real life?

Sample answer: average scores for a video game, average points scored in a basketball game, average visitors to a website

The *mean* is the *balance point* of the data. The dot plot displays the test scores. The total distance below the mean is equal to the total distance above the mean.

Example 1 Find the Mean

The table shows the recorded high temperatures in degrees Fahrenheit for six days in Little Rock, Arkansas.

Find the mean temperature to summarize the data.

High Temperatures in Little Rock, Arkansas (°F)					
Mon.	Tues.	Weds.	Thurs.	Fri.	Sat.
45	52	45	50	49	47

$\text{mean} = \frac{\text{sum of the data values}}{\text{number of data values}}$ Definition of mean

$= \frac{45 + 52 + 45 + 50 + 49 + 47}{6}$ There are 6 data values.

$= \frac{288}{6}$ Add.

$= 48$ Divide.

Think About It!
Without calculating the mean, what temperature do you think best describes the center of the data? Explain your reasoning.

See students' responses.

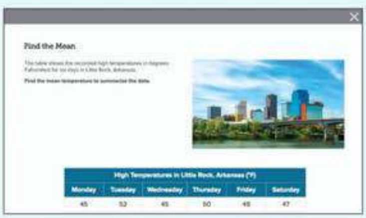
The mean temperature for the selected days is 48 °F. The dot plot confirms that the mean temperature of 48°F is the balance point of the data. The total distance above the mean, 7, is equal to the total distance below the mean.

Talk About It!
How would the mean change if the data value 0°F was included?

Sample answer: Adding a value of zero would not change the sum, but would increase the total number of data values to 7, so the mean would decrease.

550 Module 10 • Statistical Measures and Displays

Interactive Presentation



Example 1, Find the Mean, Slide 1 of 4

CLICK

On Slide 2 of Example 1, students move through the steps to find the mean of the data set.

TYPE

On Slide 2 of Example 1, students determine the average high temperature.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Measures of Center *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

What are some ways you have seen mean used in real life?
Sample answer: average scores for a video game, average points scored in a basketball game, average visitors to a website

Example 1 Find the Mean

Objective

Students will calculate the mean to summarize a data set with a single value.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to adhere to the definition of mean in order to calculate the mean accurately. Students should be able to explain how the mean summarizes the data set with a single value.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language to explain how the mean would chang e if an additional data value of 0°F was added to the set. They should note that while the sum of the values would not change, the mean would decrease since the total number of values would increase.

Questions for Mathematical Discourse

SLIDE 2

AL Do any of the data values need to equal the mean? Explain.
no; Sample answer: Because the mean is the sum of the data values divided by the number of data values, it is possible that the mean value will not appear in the data set.

OL Why do you think we add the data values and then divide by the number of data values to find the mean? Sample answer: Adding all of the data values gives a total of all the data combined. When you divide the total by the number of data values, you are equally distributing the data, so you find the average or mean.

BL A classmate found the mean temperature to be 57.6°F. How do you know that the classmate made a mistake? Sample answer: All of the data values are less than 57.6°F. The mean cannot be greater than all of the data.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Find a Missing Data Value Using the Mean

Objective

Students will understand how the mean can be applied to find a missing value in a data set.

Teaching Notes

SLIDE 1

Students will learn how to use the mean as a balance point. You might consider leading with a discussion about the meaning of the word balance. Ask students what it means to *balance* something, or how they might know something is in balance. Point out that in using the mean as a balance point, students are trying to balance the sum of the values of the distances from the mean, above and below the mean. In order to find the fifth quiz score, students need to balance the distances above and below the mean. They first need to determine if the quiz score will be above or below the mean of 90, and then use the difference in distances to determine the precise score.

(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

Some students may struggle with the concept of the *balance point* when using a dot plot to find the mean. If students need help, remind them that there does not have to be an even number of data values less than and greater than the mean. Have them picture a balance. When using a balance, the weight of the items on each side needs to be the same, but the number of items can be different. In this case, the number of data values on each side of the mean does not matter, it is the "weight" or the total distance that each value is from the mean, that needs to be the same.

Check

The table shows the number of headphones sold at an electronics store during a sale. Find the mean number of headphones sold to summarize the data.

Headphones Sold					
Mon.	Tues. Weds. Thurs.			Fri.	Sat.
9	18	7	7	10	15

Show your work here

11 headphones

Go Online You can complete an Extra Example online.

Learn Find a Missing Data Value Using the Mean

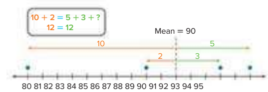
You can use dot plots and bar diagrams to find a missing data value given the mean and the other data values. Consider the following problem.

Caitlin's first four quiz scores are shown in the table. What score does Caitlin need to earn on her fifth quiz to have a mean quiz score of 90?

Caitlin's Quiz Scores				
88	95	93	80	?

Method 1 Use the mean as a balance point.

Plot the four known quiz scores and label the mean.



distance below the mean = **10 + 2**, or **12**

distance above the mean = **5 + 3**, or **8**

The distances are not the same because the fifth quiz score is not plotted. There is a greater distance below the mean. This means the missing value must be above the mean. In order for the total distance above the mean to equal 12, the missing value must be 4 units above the mean, because 8 + 4 = 12. The missing value is 90 + 4, or 94.

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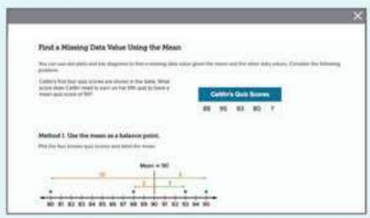
Interactive Presentation



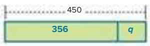
Learn, Find a Missing Data Value Using the Mean, Slide 1 of 2

Method 2 Use an equation.

Draw a bar diagram to represent the situation. T o find the total amount needed to achieve a mean score of 90, multiply the mean, 90, by the number of data values, 5.

$90(5) = 450$

The sum of the known data values is $88 + 95 + 93 + 80$, or 356. Let q represent Caitlin's score on her fifth quiz.



The equation that can be used to find the missing data value is $356 + q = 450$. Solve the equation.

$356 + q = 450$	Write the equation.
$-356 \quad -356$	Subtraction Property of Equality
$q = 94$	Simplify.

The missing value is 94.

So, using either method, Caitlin needs a score of 94 on her fifth quiz to have a mean quiz score of 90.

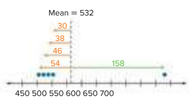
Example 2 Find a Missing Data Value Using the Mean

The number of messages Alex sent on her phone each month for the past five months were 494, 502, 486, 690, and 478. Suppose the mean for six months was 532 messages.

How many messages did Alex send during the sixth month?

Method 1 Use the mean as a balance point.

Plot the five known data values and label the mean.



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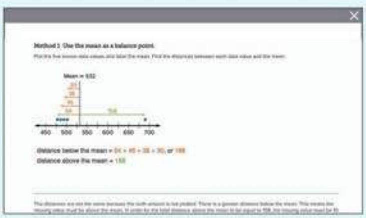
Think About It! Do you think the missing value is less than, greater than, or equal to the mean? Explain.

See students' responses.

Interactive Presentation



Example 2, Find a Missing Data Value Using the Mean, Slide 2 of 5

CLICK

On Slide 2 of Example 2, students use a dot plot to find the missing data value.

Learn Find a Missing Data Value Using the Mean *(continued)*

Teaching Notes

SLIDE 2

When creating the bar diagram to represent the situation, ask students why they need to multiply 90 by 5, not 4. Remind students that Caitlin wants to earn a mean of 90 after all 5 quizzes, not just on the four she has already taken. Encourage students to reason that 450 is the total number of points she would need to earn on all 5 quizzes, and she has already earned 356 of those points. Point out to students that by using this reasoning, and perhaps with the help of a bar diagram, they can easily set up an equation to find the missing quiz score. You might consider mentioning that if they know the total points they have earned in a class, they can use this method when determining what score they need to earn on an assignment to obtain a certain average in their class.

Example 2 Find a Missing Data Value Using the Mean

Objective

Students will apply the mean to find a missing value in a data set.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them As students discuss the *Talk About It!* question on Slide 4, encourage them to understand the benefits of each method and identify the correspondences between them.

6 Attend to Precision Students should calculate the mean precisely and accurately and make sense of the missing data using a bar diagram.

Questions for Mathematical Discourse

SLIDE 2

AL What values do you need to graph on the dot plot? 494, 502, 486, 690, and 478

AL Which value(s) are to the left of the mean on the dot plot? Which value(s) are to the right? 478, 486, 494, and 502; 690

OL What is the distance between each data value and the mean on the number line? 38, 30, 46, 158, and 54

BL Do you think the missing value is greater than or less than 532? Explain. greater than; Sample answer: Most of the values given are less than the mean, so I would expect the missing one to be greater than the mean.

BL The equation $532 = \frac{2{,}650 + m}{6}$ could also be used to find the mean. Explain how you could solve this equation to find the mean. Sample answer: First multiply each side of the equation by 6. Then subtract 2,650 from each side to find that $m = 542$.

(continued on next page)

Example 2 Find a Missing Data Value Using the Mean *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why do you need to multiply the mean by 6? Sample answer: Multiplying the mean by 6 gives you the total number of messages sent over 6 months.

OL How does the bar diagram help you write the equation? Sample answer: I can see that the sum of the known numbers of messages and the unknown number of messages is equal to the total messages.

BL If Alex sends 550 messages next month, would you expect the mean to increase or decrease? Explain your reasoning. Sample answer: I would expect the mean to increase since 550 is greater than the current mean of 532.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

distance below the mean = 54 + 46 + 38 + 30, or 168

distance above the mean = 158

The distances are not the same because the sixth amount is not plotted. There is a greater distance below the mean. This means the missing value must be above the mean. In order for the total distance above the mean to be equal to 158, the missing value must be 10 units above the mean, because 158 + 10 = 168.

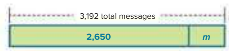
The missing value is 532 + 10, or 542.

Method 2 Use an equation.

Draw a bar diagram to represent the situation. To find the total amount needed for the mean number of messages to be 532, multiply the mean by the number of data values.

532(6) = 3,192

The sum of the known data values is 494 + 502 + 486 + 690 + 478 or 2,650. Let m represent the number of messages Alex sent during the sixth month.



The equation that can be used to find the missing data value is $2{,}650 + m = 3{,}192$. Solve the equation.

$2{,}650 + m = 3{,}192$	Write the equation.
$-2{,}650 \quad -2{,}650$	Subtraction Property of Equality
$m = 542$	Simplify.

The missing value is 542.

So, using either method, Alex sent 542 messages during the sixth month.

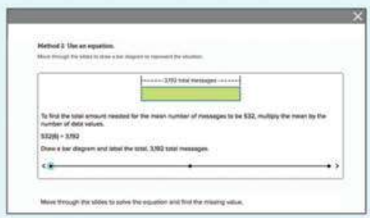
Talk About It!

Compare and contrast Method 1 and Method 2. When might it be more advantageous to use Method 2?

Sample answer: If the data set includes a large number of values, it would be more efficient to use Method 2.

Lesson 10-3 • Measures of Center 553

Interactive Presentation



Example 2, Find a Missing Data Value Using the Mean, Slide 3 of 5

CLICK

On Slide 3, students more through the slides to use a bar diagram to find the missing value.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The table shows the greatest depths of four of Earth's five oceans. If the average greatest depth is 8.094 kilometers, what is the greatest depth of the Southern Ocean? Round to the nearest hundredth.
7.24 km

Show your work here

Ocean	Greatest Depth (km)
Pacific	10.92
Atlantic	9.22
Indian	7.46
Arctic	5.63
Southern	d

Go Online You can complete an Extra Example online.

Learn Find the Median

Another measure of center used to describe a numerical data set is the **median**.

The median of a numerical data set is the middle value when the data are ordered from least to greatest. If there is an odd number of data values, the median is the middle data value. If there is an even number of data values, the median is the mean of the two values in the middle.

Just as the mean is a single value used to summarize a data set, the median also summarizes a data set with a single value.

Consider the following set of numerical data, which represents the ages of participants in a board game club.

8, 8, 8, 8, 9, 10, 10, 11, 12, 12, 16, 16, 19

There are 13 data values. Since the number of data values is odd, the median is the middle data value. Make sure the data values are ordered from least to greatest before finding the median.

The median is 10.

8, 8, 8, 8, 9, 10, 10, 11, 12, 12, 16, 16, 19

There are 6 data values below the median. There are 6 data values above the median.

Talk About It!

Why must the data be ordered from least to greatest before finding the median?

Sample answer: The median is the middle value. If the data are not in order, the value in the middle of the list might not be the actual middle value of the data set.

Interactive Presentation

Learn, Find the Median, Slide 2 of 3

CLICK

On Slide 2, students select the markers to learn how to find the median of a data set with an odd number of data values.

Learn Find the Median

Objective

Students will understand what the median of a data set represents.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of what the median of a data set represents in order to explain why it is important to write the values in order.

Teaching Notes

SLIDE 1

Students will learn that the *median* is the numerical value appearing at the center when the list is ordered from least to greatest. Students should pay close attention to the process used to find the median of an odd number of values and an even number of values.

SLIDE 2

When finding the median, if there is an even number of data values, the median may or may not be part of the data set. If there is an odd number of data values, the mean is always part of the data set. Have students determine the total number of data values and explain why the median is a data value from the data set. Students should recognize that there is an odd number of data values, which means that the median is part of the data set.

Talk About It!

SLIDE 3

Mathematical Discourse

Why must the data be ordered from least to greatest before finding the median? Sample answer: The median is the middle value. If the data are not in order, the value in the middle of the list might not be the actual middle value of the data set.

Example 3 Find the Median Given an Odd Number of Data Values

Objective

Students will find the median given an odd number of values in a data set.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively, 3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* questions on Slide 4, encourage them to reason about the relationship between mean and median and how the mean compares to the median for this data set. Students could also reason about whether this will be true for the mean and median of every data set. They should also be able to use their knowledge of mean and median to determine how changing a data value will affect each measure.

Questions for Mathematical Discourse

SLIDE 2

AL How many data values are in the data set? What is the least value? the greatest value? 7 data values; 2; 12

OL How could you make sure you don't miss any values? Sample answer: I can write the values in increasing order, and cross the numbers off of the original list as I go.

BL Would the median change if the values were put in decreasing order? Explain. no; Sample answer: It is the same group of numbers in order so the middle number is the same whether the order is increasing or decreasing.

SLIDE 3

AL Will any computations need to be done in order to find the median? Explain. no; Sample answer: There is an odd number of values.

OL In this data set do the median and the mean have the same value? Explain. no; Sample answer: The mean in this data set is about 6.3 which is not the same value as the median.

BL Add two data values to the set that do not change the median. Explain why they do not change the median. Sample answer: 5 and 7; If I add a value less than the median and a value greater than the median, the middle value remains the same.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

NASA's Terra satellite/MODIS

Example 3 Find the Median Given an Odd Number of Data Values

Between 2009 and 2015, the number of Atlantic hurricanes each year were 3, 12, 7, 10, 2, 6, and 4.

Find the median of the data.

There are 7 data values. Since the number of data values is odd, the median is the middle data value.

Step 1 Order the values from least to greatest.

2	3	4	6	7	10	12

least **greatest**

Step 2 Find the median.

How many data values are below the median? 3 values

How many data values are above the median? 3 values

What is the median? 6 hurricanes

The center of the data can be represented by the single value, 6. So, the median number of hurricanes from 2009 to 2015 is 6 hurricanes.

Check

Dina's scores on recent science tests were 86, 98, 85, 90, 85, 91, 89, 88, and 89 points. Find the median of her test scores. 89 points

Show your work here

Think About It! A classmate immediately stated the median is 10. What was the likely mistake?

Sample answer: The classmate likely did not order the data from least to greatest.

Talk About It! Find the mean of the data set to the nearest tenth. What do you notice about its value when compared to the median? Why do you think that is?

6.3 hurricanes; Sample answer: The mean is close to the median. It is slightly greater than the median because the data values 10 and 12 are farther from the mean than the data values 2 and 3.

Talk About It! If the data value of 12 was changed to 13, how would the mean be affected? the median?

The mean would be slightly greater because of the inclusion of a greater data value. The median would not be affected because the middle value would still be 6.

Go Online You can complete an Extra Example online.

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Lesson 10-3 • Measures of Center 555

Interactive Presentation

Example 3, Find the Mean Given an Odd Number of Data Values, Slide 1 of 5

DRAG & DROP

On Slide 2, students drag to place the numerical data in order from least to greatest.

TYPE

On Slide 3, students identify the median.

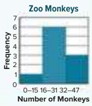
CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
A classmate stated that the median will not be one of the data values. Is this correct? Why or why not?

The classmate is correct; Sample answer: There is an even number of data values, so the median will not be one of the data values.

Talk About It!
The histogram represents the data from the table. Can you use the histogram to find the mean or median? Explain your reasoning.



no; Sample answer: The histogram does not show the individual data values. You can find the interval in which the median lies, but not the actual median.

Example 4 Find the Median Given an Even Number of Data Values

The table shows the number of monkeys at ten different zoos.

Find the median of the data.

Number of Monkeys				
27	36	18	25	12
18	42	34	16	30

There are 10 data values. Because the number of data values is even, the median is the mean (average) of the two middle data values.

Step 1 Order the values from least to greatest.

In order from least to greatest, the values are 12, 16, 18, 18, 25, 27 , 30, 34, 36, and 42.

Step 2 Find the median.

Because there is an even number of data values, find the two values closest to the middle.

The two values closest to the middle are 25 and 27.

Find the mean of the two middle data values.

$\text{mean} = \frac{25 + 27}{2}$ Find the mean of 25 and 27.

$= \frac{52}{2}$ Add.

$= 26$ Divide.

So, the median of the data is 26 monkeys. The data can be summarized by describing the center of the data as 26 monkeys.

Check

The table shows the prices of different packages of juice boxes at a local store. Find the median of the data.

Cost of Juice Boxes ($)			
1.65	1.97	2.45	2.87
2.35	3.75	2.49	2.87

Show your work here

$2.47

Go Online You can complete an Extra Example online.

556 Module 10 • Statistical Measures and Displays

Interactive Presentation

Example 4, Find the Median Given an Even Number of Data Values, Slide 1 of 5

WATCH

On Slide 1, students watch an animation that demonstrates the problem they are about to solve.

DRAG & DROP

On Slide 2, students drag to place the numerical data in order from least to greatest.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find the Median Given an Even Number of Data Values

Objective

Students will find the median given an even number of values in a data set.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the histogram and the range of values that represent numbers from a problem to explain why the histogram cannot be used to find the mean or median of the values.

6 Attend to Precision Students should represent the data from the table precisely, using small steps to find the median of the set of values by finding the mean of the two numbers in the middle of the set.

Questions for Mathematical Discourse

SLIDE 2

AL How will you order the numbers? Sample answer: I will write the least number, followed by the next greatest number. I will continue doing so until all of the numbers have been included.

OL Why do the numbers need to be in order? Sample answer: I need to order the numbers to find the middle two values.

BL There are 10 data values in the set. How can you determine without crossing off numbers, where the median will be? Sample answer: I can divide 10 in half and get 5. Five data values will be in the lower half and 5 data values will be in the upper half. The median will be between the fifth and sixth data values.

SLIDE 3

AL Why do you add the two central values and divide by 2? I need to find the value that is in the center of 25 and 27. I can find the mean or the average of those two numbers.

OL Why does the process for finding the median for an odd number of values not work with an even number of values? Sample answer: If a set has an even number of values, there are two values in the middle of the set, not one.

BL When will the median of a data set, with an even number of values, be a member of the data set? when the two central values are the same

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Track

Objective

Students will come up with their own strategy to solve an application problem involving the mean and median of 100-meter dash times.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you find the median of a data set?
- How do you find the mean of a data set?
- Would a greater or lesser value be used to show a faster time?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Track

The table shows Kendra's 100-meter dash times. Kendra wants to record the measure of center that describes her times as the fastest. Which measure should she use, the mean or median? Why?

Kendra's 100-meter Dash Times (seconds)			
15.1	17.2	14.6	16.2
17.9	16.5	17.8	17.1
14.7	17.1	19.5	13.8

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

the mean; Sample answer: The mean of the data is about 16.46 seconds and the median is 16.8 seconds. Since Kendra wants to use a measure that represents a faster time, she should choose the least of the two measures, the mean; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! In the next two races, Kendra had times of 14 seconds and 19 seconds. Does adding these times to the data set affect the measure that she should choose?

no; Sample answer: The median is still 16.8 seconds, and the mean is still about 16.46 seconds. Kendra should still choose the mean to show the faster time.

Interactive Presentation

Apply, Track

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Rosario recorded the number of hours she spent doing homework for five nights. She wants to use the greater measure of center to describe her time spent doing homework. Which measure should she use, the mean or median? Why?

Day	Time (h)
1	1.25
2	2.25
3	1.5
4	2
5	0.75

Show your work here

the mean; Sample answer: The mean of the data is 1.55 hours and the median is 1.5 hours. Since Rosario wants to use a measure that represents a greater number of hours spent on homework, she should choose the greater of the two measures, the mean.

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that compares and contrasts the two measures of center you studied in this lesson.

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

Why is data collected and analyzed and how can it be displayed?

In this lesson, students learned how the measures of center can be used to summarize numerical data. Encourage them to discuss with a partner the benefits of summarizing a set of numerical data with a single number. For example, using a measure of center to summarize a data set means that not every data value needs to be mentioned in order to have an overall picture of the data.

Exit Ticket

Refer to the Exit Ticket slide. Find the mean and median of the number of goals scored per game. Round to the nearest tenth if necessary. Write a mathematical argument that can be used to defend your solution.

The mean is 3.3 goals and the median is 2 goals. Sample answer: The mean is 3.3 goals because

$$\frac{4+3+9+1+2+1+2+4+1+2+3+3+7+2+1+1+2+5+10}{19} \approx 3.3.$$

The median is 2 because when the data are listed in order from least to greatest, 2 is the middle value of the data set.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 8–14
- Extension: Weighted Average
- ALEKS Mean, Median, and Mode

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 9, 12, 13
- Extension: Weighted Average
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Collecting Data

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Collecting Data

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- AL Practice Form B
- OL Practice Form A
- BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	calculate the mean of a data set	1, 2
1	apply the mean to find a missing value of a data set	3, 4
1	find the median of a data set	5–7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving measures of center	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconceptions

When finding the mean, students may not include all of the given data values, especially when data values repeat or are equal to zero. For example, in Exercise 2, students might say that there are only 6 data values because they did not include 0 in their calculation. Remind students that each data value must be included in the sum and the total number of data values.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The number of cans collected over the weekend by each sixth grade homeroom was 57, 59, 60, 58, 58, and 56 cans. Find the mean number of cans collected. (Example 1)

   58 cans

2. Grace and her friends are comparing the number of pets they own. They have 1, 2, 0, 5, 1, 1 and 4 pets. Find the mean number of pets owned. (Example 1)

   2 pets

3. The amount Lucy earned babysitting each month for the past five months was \$225, \$280, \$240, \$180, and \$200. Suppose the mean for six months was \$220. How much did Lucy earn babysitting during the sixth month? (Example 2)

   \$195

4. The average high temperature last week was 65 degrees Fahrenheit. The high temperatures for Sunday through Friday were 68, 70, 73, 45, 68, and 71 degrees Fahrenheit. What was the high temperature on Saturday? (Example 2)

   60° F

5. The table shows the results of a survey about the number of E-mails sent in one day. Find the median number of E-mails sent per day. (Example 3)

Number of E-mails Sent Per Day
20 24 22 27 21 27 20
27 22 23 20 22 24 26
23 26 27 22 27 20 25

   23 E-mails

6. The table shows the number of students in each group on a school field trip. Find the median size of a group. (Example 3)

Number of Students in Each Group				
5	7	8	7	6
4	4	5	6	9
7	5	7	8	6
9	7	5	4	5

   6 students

7. The table shows the number of points scored by a basketball team in each game last season. Find the median number of points scored. (Example 4)

Number of Points
64 41 52 63 44 54
42 67 44 68 43 61

   53 points

Test Practice

8. **Open Response** The number of points Seth has earned playing his favorite game is shown. Find the median of the data.

   40, 28, 24, 37, 43, 26, 30, 36

   33 points

Apply *indicates multi-step problem

*9. The table shows the number of minutes Kenny spent practicing the piano. Kenny wants to record the greater measure of center that describes his time spent practicing. Which measure should he use, the mean or median? Why?

Number of Minutes
38 30 26 25
20 24 25 60

the mean; Sample answer: The mean of the data is 31 minutes, and the median is 25.5 minutes. Since Kenny wants to use a measure that represents a greater number of minutes spent practicing, he should choose the greater of the two measures, the mean.

*10. The table shows the number of push-ups Jade completed each day this week. Jade wants to record the greater measure of center that describes her ability to do push-ups. Which measure should she use, the mean or median? Why?

Number of Push-ups
65 70 67 38
55 68 64

the median; Sample answer: The mean of the data is 61 push-ups, and the median is 65 push-ups. Since Jade wants to use a measure that represents a greater number of push-ups completed, she should choose the greater of the two measures, the median.

Higher-Order Thinking Problems

11. Create Generate a real-world data set that has a mean of 8.

Sample answer: Shoe sizes of the Holden family: 8, 10, 7 , 9, and 6

12. MP Use a Counterexample Determine if the following statement is *true* or *false*. If *false*, provide a counterexample.

The mean and median of a data set cannot be the same value.

false; Sample answer: The data set 4, 6, 9, 11, 8, 13, and 12 has a mean of 9 and a median of 9.

13. MP Find the Error A student said the mean of the data set shown is 17 . Find the student's error and correct it.

number of texts sent in an hour: 15, 11, 25, 19, 11, 27

Sample answer: The student found the median of the data set. The mean of the data set is 18 texts.

14. MP Reason Abstractly Ty worked 5 nights this week at an ice cream shop. He earned \$23, \$29, \$25, and \$16 in tips. The average amount he earned in tips for the 5 nights was \$22. Is the amount he earned in tips on night 5 more or less than the average amount? Explain.

less than; Sample answer: The sum of the five data values must equal 22×5 or 110. So, $110 - (23 + 29 + 25 + 16) = 17$. \$17 is less than \$22.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students determine the validity of the statement and provide a counterexample if the statement is false. Encourage students to construct a counterexample that shows the statement is false.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students find the student's error and correct it. Students should determine that the value the student found was actually the median. Students should then find the mean of the data set.

2 Reason Abstractly and Quantitatively In Exercise 14, students will determine if the amount Ty earned in tips on night 5 is more or less than the average amount. Encourage students to use reasoning to determine that the amount he earned in tips on night 5 is less than the average amount.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercises 9–10 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 9 might be, "The mean of the data set is 31 minutes" An example of a false statement might be, "The median of the data set is 24 minutes." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Create your own higher-order thinking problem.
Use with Exercises 11–14 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Learn Measures of Variation

Objective

Students will understand that the measures of variation describe the variation of a data set using a single value.

MP Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to relate the term *quartile* with other words that have the same prefix *quart-*, such as *quarter* or *quarterly*. This can help them remember that a data set can be divided into four parts.

As students discuss the *Talk About It!* question on Slide 4, encourage them to use clear and precise language, such as *variation*, *spread*, *middle 50%*, etc., in order to explain what the interquartile range of a data set describes.

Go Online to find additional teaching notes and the *Talk About It!* question on Slide 2.

Talk About It!

SLIDE 4

Mathematical Discourse

If the median describes the center of a data set, what does the interquartile range describe? Sample answer: The interquartile range describes how spread out the middle 50% of the values are around the median.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling to determine the interquartile range of a data set, encourage them to work with a partner to create a list of steps that they need to follow in order to find the interquartile range. Have them write a reason that justifies each step, so that they understand why they need to complete it. Then have them share their steps and reasons with a partner, and discuss and resolve any differences. Sample response shown.

1. Write the data in numerical order. *Reason:* Before I can find the first and third quartiles, I need to find the median.
2. Divide the data into quartiles. *Reason:* In order to find the interquartile range, I need to find the range from the first quartile to the third quartile.
3. Subtract the first quartile from the third quartile. This is the interquartile range. *Reason:* The interquartile range is the range from the first quartile to the third quartile.

Lesson 10-4

Interquartile Range and Box Plots

I Can... understand how a measure of variation describes the variability of a data set with a single value, display a numerical data set in a box plot, and summarize the data.

What Vocabulary Will You Learn?
box plot
first quartile
interquartile range
measures of variation
quartiles
range
second quartile
third quartile

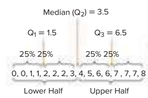
Learn Measures of Variation

Measures of variation are values that describe the variability, or spread, of a data set. They describe how the values of a data set vary with a single number.

One measure of variation is the **range**, which is the difference between the greatest and least data values in a data set. Consider the data set shown.

0, 0, 1, 1, 2, 2, 2, 3, 4, 5, 6, 6, 7, 7, 7, 8 The data values range from 0 to 8. The range is 8–0, or 8.

Another measure of variation is the *interquartile range*. Before you can find this measure, you first need to understand and find quartiles. **Quartiles** divide the data into four equal parts. The **first quartile**, Q_1, is the median of the data values less than the median. The **third quartile**, Q_3, is the median of the data values greater than the median. The median is also known as the **second quartile**, Q_2.



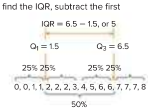
The median divides the data into two halves. The quartiles divide the data into fourths. Each fourth represents 25% of the data.

Talk About It!
How does knowing that the data is divided into four equal parts help you remember the vocabulary term quartile?

See students' responses.

The **interquartile range (IQR)** is the distance between the first and third quartiles of the data set. To find the IQR, subtract the first quartile from the third quartile.

The interquartile range represents the middle half, or middle 50%, of the data. The lower the IQR is for a data set, the closer the middle half of the data is to the median.



In the given data set, the IQR is 6.5 – 1.5, or 5.

Talk About It!
If the median describes the center of a data set, what does the interquartile range describe?

Sample answer: The interquartile range describes how spread out the middle 50% of the values are around the median.

Lesson 10-4 • Interquartile Range and Box Plots 561

Interactive Presentation

Learn, Measures of Variation, Slide 2 of 4

CLICK

On Slide 1, students select a button to show the range of the data set.

CLICK

On Slide 2, students move through the steps to see the data set divided into quartiles.

CLICK

On Slide 3, students select the buttons to show Q_1, Q_3, and the IQR.

Interquartile Range and Box Plots

6.SP.A.3, 6.SP.B.4

LESSON GOAL

Students will understand interquartile range and construct box plots.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Measures of Variation
Example 1: Find the Range and Interquartile Range
Learn: Construct Box Plots
Example 2: Interpret Box Plots
Example 3: Construct and Interpret Box Plots

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Constructing and Interpreting a Double Box Plot		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 56 of the *Language Development Handbook* to help your students build mathematical language related to interquartile range and box plots.

ELL You can use the tips and suggestions on page T56 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability
Additional Cluster(s): In this lesson, students address additional clusters **6.SP.A** and **6.SP.B** by understanding interquartile range and creating box plots.
Standards for Mathematical Content: **6.SP .A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C**
Standards for Mathematical Practice: **MP2, MP3, MP6**

Coherence

Vertical Alignment

Previous
Students understood and applied different measures of center.
6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C

Now
Students understand interquartile range and create box plots.
6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C

Next
Students will understand mean absolute deviation.
6.SP.A.3, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C

Rigor

The Three Pillars of Rigor

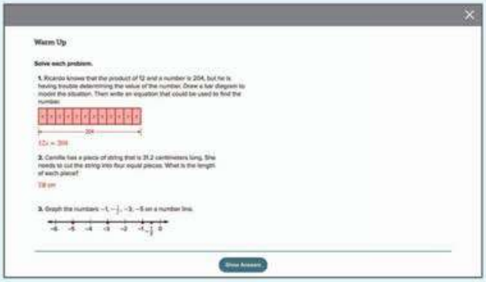
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students begin to expand on their *understanding* of statistical measures as they explore interquartile range and box plots. They learn about measures of variation, including range and interquartile range, to build *fluency* with describing the variation of a data set and constructing a box plot to represent a data set.

Mathematical Background

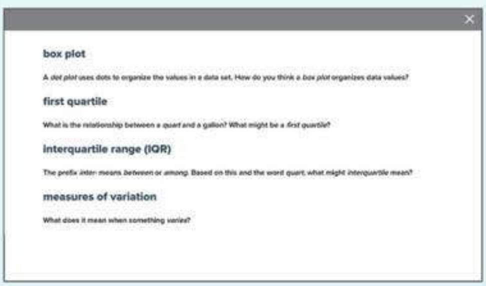
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding bar diagrams (Exercise 1)
- dividing rational numbers (Exercise 2)
- plotting rational numbers on a number line (Exercise 3)

Answers

1–3. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the large variation of life expectancy within the animal family Felidae.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- A *dot plot* uses dots to organize the values in a data set. How do you think a *box plot* organizes data values? Sample answer: A box plot could display data using boxes.
- What is the relationship between a *quart* and a gallon? What might be a *first quartile*? Sample answer: A quart is one fourth of a gallon. A first quartile might be the first of four parts of something.
- The prefix *inter-* means *between* or *among*. Based on this and the word *quart*, what might *interquartile* mean? Sample answer: a section between two quartiles of data
- What does it mean when something *varies*? Sample answer: When something varies, it differs in size or amount from other things in the same group.

Your Notes

Think About It!
Do the data values need to be in numerical order? Why?

yes; You need to find the middle number.

Talk About It!
Which value, the interquartile range, the first quartile, or the third quartile tells you more about the spread of the data values? Explain your reasoning.

Sample answer: The interquartile range tells you more about the spread of the data, specifically the spread of the middle half of the data.

Example 1 Find the Range and Interquartile Range

The table shows the approximate maximum speeds, in miles per hour, of different animals.

Animal Speed (mph)	
Housecat	30
Cheetah	70
Elephant	25
Lion	50
Mouse	8
Spider	1

Use the range and interquartile range to describe how the data vary.

Part A Describe the variation of the data using the range.

The greatest speed in the data set is 70 miles per hour. The least speed in the data set is 1 mile per hour.

The range is 70 − 1, or 69 miles per hour.

The speeds of animals vary by 69 miles per hour.

Part B Describe the variation of the data using the interquartile range.

Step 1 Find the median.

Write the speeds in order from least to greatest.

1 8 25 30 50 70

least **greatest**

The median is 27.5. Find the mean of the two middle numbers, 25 and 30.

Step 2 Find the first and third quartiles.

The first quartile is 8. Find the median of the lower half of the data.

The third quartile is 50. Find the median of the upper half of the data.

Step 3 Find the interquartile range.

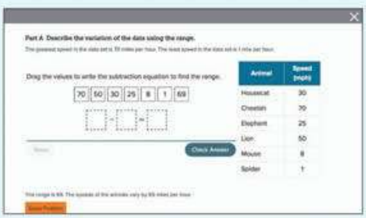
Interquartile range = $Q_3 - Q_1$

= 50 − 8 $Q_3 = 50$; $Q_1 = 8$

= 42   Subtract.

So, the spread of the middle 50% of the data is 42. This means that the middle half of the data values vary by 42 miles per hour.

Interactive Presentation



Example 1, Find the Range and Interquartile Range, Slide 2 of 7

DRAG & DROP

On Slide 2, students drag the values to write a subtraction equation in order to find the range.

DRAG & DROP

On Slide 3, students drag to arrange the values in order from least to greatest before finding the interquartile range.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find the Range and Interquartile Range

Objective

Students will describe the variation of a data set using the range and interquartile range.

Questions for Mathematical Discourse

SLIDE 2

AL How do you find the range? The range is found by subtracting the least value from the greatest value.

OL Why don't you subtract the greatest value from the least value? If you subtract the greatest value from the least value, the difference is a negative number.

BL What does the range mean? Sample answer: The difference between the least and greatest speeds is 69 mph. So, this means that the speeds vary by 69 miles per hour.

SLIDE 3

AL How do you find the median of a data set with an odd number of values? I order the values from least to greatest and then find the middle value.

OL Why do the numbers have to be in order from least to greatest when finding the median? The numbers have to be listed in order from least to greatest so that the middle value is found correctly.

BL Can the median be represented by a quartile? Explain. yes; Sample answer: If the first quartile represents the first quarter of the data set, the second quartile represents the second quarter or half of the data set. This is the same as the median.

SLIDE 4

AL How will you find the first quartile? Sample answer: I will find the median of 1, 8, and 25.

OL How would finding the first and third quartiles be different if each half had an even number of values? Sample answer: I would need to find the average of the middle two values to find the quartiles.

BL Do you think you could add a value to the data set that would have no effect on the first and third quartiles? Explain. yes; Sample answer: If I added a median, 26, that would not affect the lower half nor upper half of the data.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Construct Box Plots

Objective

Students will understand how to construct a box plot to represent a data set.

Teaching Notes

SLIDE 1

Have students use the interactive tool to see how to construct a box plot. Point out that they need to identify the lower extreme, first quartile (Q_1), median, third quartile (Q_3), and upper extreme values before constructing the plot. The median is also known as the second quartile (Q_2).

Some students may confuse the length of a box or whisker with the quantity of data represented by that section. It is important to stress to students that each section represents 25% of the data values. A box or whisker that is longer than the other sections means that the data is more spread out in that section, not that that section has more data values.

Check

The average wind speeds for several cities in Pennsylvania are given in the table. Use the range and interquartile range to describe how the data vary.

Wind Speed	
City	Speed (mph)
Allentown	8.9
Erie	11.0
Harrisburg	7.5
Middletown	7.7
Philadelphia	9.5
Pittsburgh	9.0
Williamsport	7.6

Part A

Describe the variation of the data using the range.

Show your work here

The data vary by a range of 3.5 miles per hour.

Part B

Describe the variation of the data using the interquartile range.

Show your work here

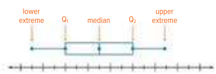
The middle half of the data values vary by 1.9 miles per hour.

Go Online You can complete an Extra Example online.

Learn Construct Box Plots

A **box plot**, or box-and-whisker plot, uses a number line to show the distribution of a data set by plotting the median, quartiles, and extreme values. The extreme values, or extremes, are the greatest and least values in the data set. The extremes, quartiles, and median are referred to as the *five-number summary*.

A box is drawn around the two quartile values. The whiskers extend from each quartile to the extreme data values, unless the extremes are very far apart from the rest of the data set. The median is marked with a vertical line, and separates the box into two boxes.



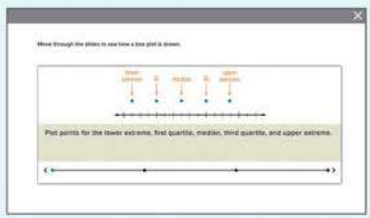
Box plots separate data into four sections. These sections are visual representations of quartiles. Even though the parts may differ in length, each contain 25% of the data. The two boxes represent the middle 50% of the data. A longer box or whisker indicates the data are more spread out in that section. A longer box or whisker does not mean there are more data values in that section. Each section contains the same number of values, 25% of the data.

Math History Minute

Florence Nightingale (1820–1910) used statistics to help improve the survival rates of hospital patients. She discovered that by improving sanitation, survival rates improved. She designed charts to display the data, as statistics had rarely been presented with charts before. She is known for inventing the *coxcomb graph*, which is a variation of the circle graph.

Lesson 10-4 • Interquartile Range and Box Plots 563

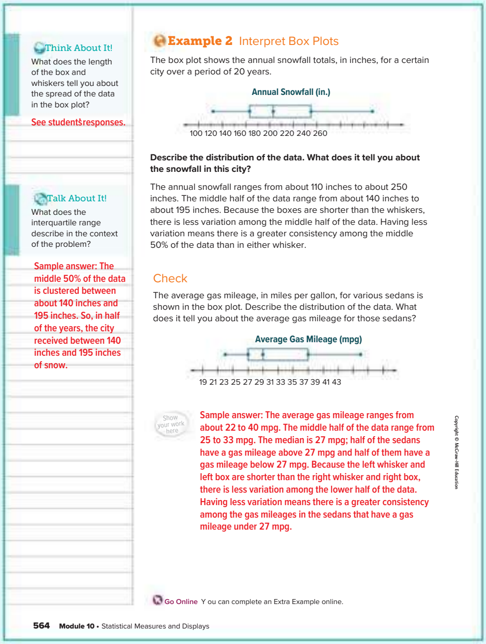
Interactive Presentation



Learn, Construct Box Plots

CLICK

Students move through the slides to see how to construct a box plot.



Think About It!
What does the length of the box and whiskers tell you about the spread of the data in the box plot?

See students' responses.

Talk About It!
What does the interquartile range describe in the context of the problem?

Sample answer: The middle 50% of the data is clustered between about 140 inches and 195 inches. So, in half of the years, the city received between 140 inches and 195 inches of snow.

Example 2 Interpret Box Plots

The box plot shows the annual snowfall totals, in inches, for a certain city over a period of 20 years.

Annual Snowfall (in.)

100 120 140 160 180 200 220 240 260

Describe the distribution of the data. What does it tell you about the snowfall in this city?

The annual snowfall ranges from about 110 inches to about 250 inches. The middle half of the data range from about 140 inches to about 195 inches. Because the boxes are shorter than the whiskers, there is less variation among the middle half of the data. Having less variation means there is a greater consistency among the middle 50% of the data than in either whisker.

Check

The average gas mileage, in miles per gallon, for various sedans is shown in the box plot. Describe the distribution of the data. What does it tell you about the average gas mileage for those sedans?

Average Gas Mileage (mpg)

19 21 23 25 27 29 31 33 35 37 39 41 43

Show your work here

Sample answer: The average gas mileage ranges from about 22 to 40 mpg. The middle half of the data range from 25 to 33 mpg. The median is 27 mpg; half of the sedans have a gas mileage above 27 mpg and half of them have a gas mileage below 27 mpg. Because the left whisker and left box are shorter than the right whisker and right box, there is less variation among the lower half of the data. Having less variation means there is a greater consistency among the gas mileages in the sedans that have a gas mileage under 27 mpg.

Go Online You can complete an Extra Example online.

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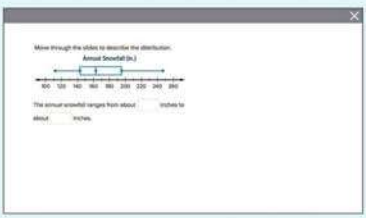
564 Module 10 • Statistical Measures and Displays

Interactive Presentation



Example 2, Interpret Box Plots, Slide 2 of 4

CLICK

On Slide 2, students more through the steps to describe the distribution of the data.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Interpret Box Plots

Objective

Students will analyze the distribution of data displayed in a box plot.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of what the interquartile range tells them about the data, given the context of the problem.

Questions for Mathematical Discourse

SLIDE 2

AL What do you know about the lower extreme and upper extreme? Sample answer: The lower extreme is the least value included in the box plot and the upper extreme is the greatest value included in the box plot.

OL Can you find the mean from the box plot? Explain. no; Sample answer: The box plot doesn't show all of the values in the data set. I need to know that information in order to find the mean.

BL Do you think the data are evenly distributed? Explain. no; Sample answer: If the data were evenly distributed, the whiskers and the two portions inside of the box would be the same length. The whiskers are not the same, and the portions inside of the box are shorter than the whiskers.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Construct and Interpret Box Plots

Objective

Students will construct a box plot to represent a data set and interpret the distribution of the data.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you need to order the numbers? Sample answer: I need to order the numbers because the box plot uses values in order and I need to calculate the median.

OL What strategy do you use to order the numbers? Sample answer: I identify the least number in the list and write it down, cross it out from the original list, and repeat the process until I've crossed all the numbers out.

BL If another car drove by at 42 miles per hour, where would this data value fall in the ordered list? It would be placed at the end of the list because it would be the greatest speed.

SLIDE 3

AL How will you find the first quartile? To find the first quartile, I will find the median of the lower half of the data set, 19, 20, 22, 23, and 25.

OL Does the order in which you graph the points matter? Explain. no; Sample answer: As long as I graph the points before I construct the box plot, the order doesn't matter. They will still be graphed in the same location.

BL The graphed numbers can be referred to as the five-number summary. Why do you think it is called this? Sample answer: There are five numbers and together, you can get an idea about what the data set looks like as a whole.

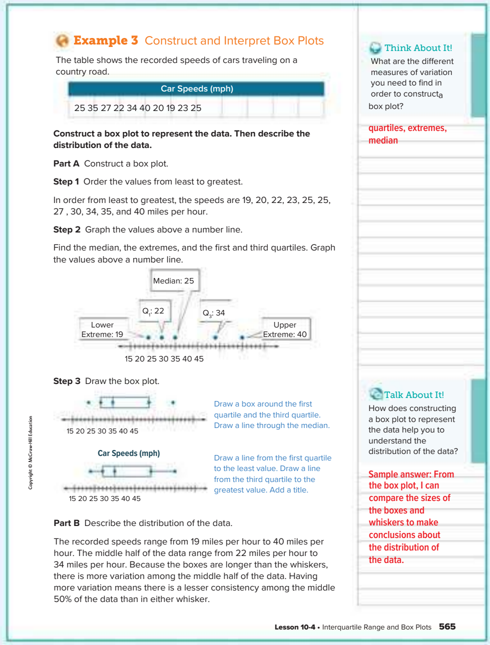
SLIDE 4

AL Why is it helpful to draw the box first? Sample answer: Drawing the box first helps me find the endpoints for the whiskers.

OL Why isn't the box drawn around the point for 19? 19 is the lower extreme and isn't included in the box. It is an endpoint for a whisker.

BL Looking at the finished box plot, make an observation about half of the speeds in the data set. Sample answer: Half of the speeds were between 22 and 34 miles per hour.

(continued on next page)



Example 3 Construct and Interpret Box Plots

The table shows the recorded speeds of cars traveling on a country road.

Car Speeds (mph)
25 35 27 22 34 40 20 19 23 25

Construct a box plot to represent the data. Then describe the distribution of the data.

Part A Construct a box plot.

Step 1 Order the values from least to greatest.

In order from least to greatest, the speeds are 19, 20, 22, 23, 25, 25, 27 , 30, 34, 35, and 40 miles per hour.

Step 2 Graph the values above a number line.

Find the median, the extremes, and the first and third quartiles. Graph the values above a number line.

Step 3 Draw the box plot.

Draw a box around the first quartile and the third quartile. Draw a line through the median.

Draw a line from the first quartile to the least value. Draw a line from the third quartile to the greatest value. Add a title.

Part B Describe the distribution of the data.

The recorded speeds range from 19 miles per hour to 40 miles per hour. The middle half of the data range from 22 miles per hour to 34 miles per hour. Because the boxes are longer than the whiskers, there is more variation among the middle half of the data. Having more variation means there is a lesser consistency among the middle 50% of the data than in either whisker.

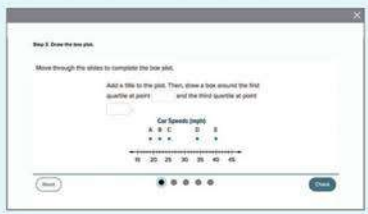
Think About It! What are the different measures of variation you need to find in order to construct a box plot?

quartiles, extremes, median

Talk About It! How does constructing a box plot to represent the data help you to understand the distribution of the data?

Sample answer: From the box plot, I can compare the sizes of the boxes and whiskers to make conclusions about the distribution of the data.

Interactive Presentation



Example 2, Construct and Interpret Box Plots, Slide 4 of 7

DRAG & DROP

On Slide 2, students drag to order the numbers from least to greatest.

CLICK

On Slide 3, students select the buttons to indicate the location of the median, extremes, and quartiles.

CLICK

On Slide 4, students move through the slides to construct the box plot.

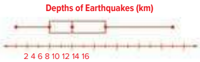
Check

Earthquakes occur at different depths below Earth's surface. Stronger earthquakes happen at depths that are closer to the surface. The table shows the depths of recent earthquakes, in kilometers.

Depth of Recent Earthquakes (km)
5 15 1 11 2 7 3
9 5 4 9 10 5 7

Part A Construct a box plot to represent the data.

Show your work here



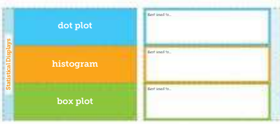
Part B Describe the distribution of the data.

Show your work here

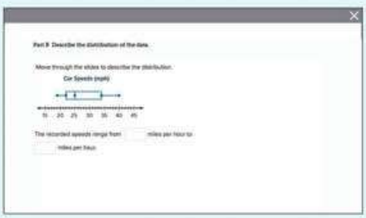
Sample answer: The earthquake depths range from 1 km to 15 km. The middle half of the data range from 4 km to 9 km. The median is 6 km; half of the earthquakes occurred at a depth greater than 6 km and half of them occurred at a depth less than 6 km. Because the left whisker and left box are shorter than the right whisker and right box, there is less variation among the lower half of the data. Having less variation means there is a greater consistency among the earthquakes that have a depth of less than 6 km.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation



Example 3, Construct and Interpret Box Plots, Slide 5 of 7

CLICK

On Slide 5, students more through the steps to describe the distribution.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Construct and Interpret Box Plots *(continued)*

Questions for Mathematical Discourse

SLIDE 5

AL Are the whiskers shorter or longer than the boxes? shorter

OL Does a shorter box or whisker indicate data that are more spread out or closer together? closer together

BL The next day, speeds of 21, 24, 34, and 39 miles per hour were recorded. How does adding these data values to the data set affect the box plot? Sample answer: Even though four values were added to the data set, the box plot is not affected because the extremes, quartiles, and median did not change.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of how to construct a box plot and when a box plot should be used to represent a data set. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

Why is data collected and analyzed and how can it be displayed?

In this lesson, students learned how the measures of variation can be used to describe the spread of a data set, and how to represent numerical data with a box plot. Encourage them to work with a partner to compare and contrast how box plots, dot plots, and histograms are used to display data. For example, they may say that all three kinds of graphs represent numerical data. While a dot plot shows every single data value, histograms and box plots do not.

Exit Ticket

Refer to the Exit Ticket slide. Find the median, first and third quartile, and extremes of the data set. median: 18, first quartile: 15, third quartile: 22, extremes: 11, 23

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

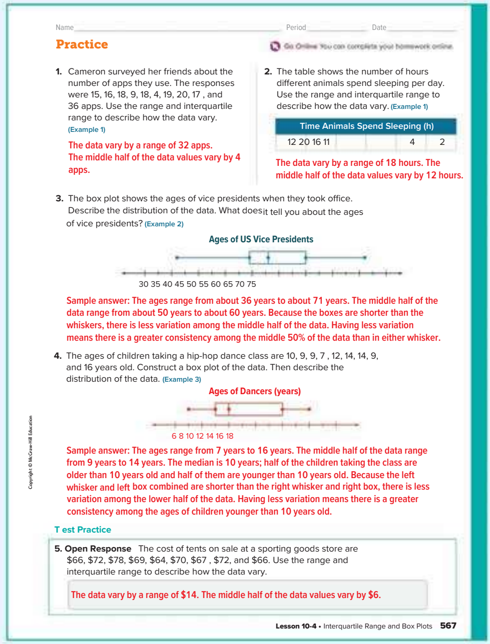
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	describe the variation of a data set using the range and interquartile range	1, 2
1	interpret box plots	3
1	construct and interpret a box plot to represent a data set	4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving box plots	6
3	higher-order and critical thinking skills	7–10

Common Misconception

In Exercise 1, students may find the range of the numbers by subtracting the first number from the last number without putting the values in order. Remind students that the range is found by finding the difference between the greatest and the least data values.



Name Period Date

Practice

Go Online You can complete your homework online.

1. Cameron surveyed her friends about the number of apps they use. The responses were 15, 16, 18, 9, 18, 4, 19, 20, 17 , and 36 apps. Use the range and interquartile range to describe how the data vary. (Example 1)

   The data vary by a range of 32 apps. The middle half of the data values vary by 4 apps.

2. The table shows the number of hours different animals spend sleeping per day. Use the range and interquartile range to describe how the data vary. (Example 1)

Time Animals Spend Sleeping (h)					
12 20 16 11				4	2

   The data vary by a range of 18 hours. The middle half of the data values vary by 12 hours.

3. The box plot shows the ages of vice presidents when they took office. Describe the distribution of the data. What does it tell you about the ages of vice presidents? (Example 2)

   Ages of US Vice Presidents

   30 35 40 45 50 55 60 65 70 75

   Sample answer: The ages range from about 36 years to about 71 years. The middle half of the data range from about 50 years to about 60 years. Because the boxes are shorter than the whiskers, there is less variation among the middle half of the data. Having less variation means there is a greater consistency among the middle 50% of the data than in either whisker.

4. The ages of children taking a hip-hop dance class are 10, 9, 9, 7 , 12, 14, 14, 9, and 16 years old. Construct a box plot of the data. Then describe the distribution of the data. (Example 3)

   Ages of Dancers (years)

   6 8 10 12 14 16 18

   Sample answer: The ages range from 7 years to 16 years. The middle half of the data range from 9 years to 14 years. The median is 10 years; half of the children taking the class are older than 10 years old and half of them are younger than 10 years old. Because the left whisker and left box combined are shorter than the right whisker and right box, there is less variation among the lower half of the data. Having less variation means there is a greater consistency among the ages of children younger than 10 years old.

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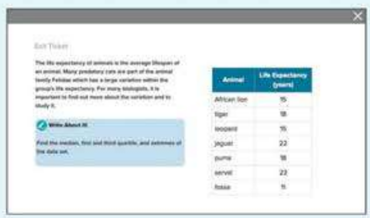
Test Practice

5. **Open Response** The cost of tents on sale at a sporting goods store are \$66, \$72, \$78, \$69, \$64, \$70, \$67 , \$72, and \$66. Use the range and interquartile range to describe how the data vary.

   The data vary by a range of \$14. The middle half of the data values vary by \$6.

Lesson 10-4 • Interquartile Range and Box Plots 567

Interactive Presentation

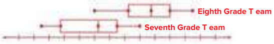


Exit Ticket

Apply *indicates multi-step problem

*6. The table shows the number of points scored by the seventh and eighth grade girls basketball teams in each of their games this season. Construct a box plot to represent the data for each team. Then use the box plots to compare the data.

Points Scored per Game	
Seventh Grade T eam	Eighth Grade T eam
39 36 40 27 34 36 47 40	
35 29 36 29 39 38 45 43	
31 38 30 34 42 41 45 42	



Sample answer: Overall, the ranges of the points scored by each team are the same, 13 points. However, the interquartile range for the eighth grade team is 5.5 points and the interquartile range for the seventh grade team is 7 .5 points. This means that the eighth grade team had a greater consistency among the middle half of the data than the seventh grade team.

Higher-Order Thinking Problems

7 . **Justify Conclusions** Determine if the following statement is *true* or *false*. If *false*, justify your reasoning.

Y ou can determine the mean of a data set from a box plot.

false; Sample answer: A box plot does not show individual data values, so you cannot find the mean of the data from a box plot alone.

8. **Create** Provide a set of real-world data and then construct a box plot of the data.

Sample answer: Marc and 7 friends played 9 holes of golf. Their combined scores on each hole were 45, 58, 52, 58, 40, 56, 61, and 47 .

9. **Make an Argument** A student said that, in a box plot, if the box to the right of the median is longer than the box to the left of the median, there are more data values represented by the longer box. Is the student's reasoning correct? Construct an argument to defend your solution.

no; Sample answer: Each section of the box plot represents 25% of the total values. This means that each whisker and each box represents the same amount of data values. The length of each section depends on the spread of the data.

10. **Reason Inductively** What can you conclude about a data set shown in a box plot where the whiskers and boxes are all the same length?

Sample answer: The data is spread out equally among each section.

568 **Module 10** • Statistical Measures and Displays

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 7, students determine the validity of the statement. Encourage students to use the structure and characteristics of a box plot to determine the statement is false.

In Exercise 9, students determine if the student's thinking is correct. Encourage students to explain why the student's thinking is correct.

In Exercise 10, students will determine a conclusion for the information provided. Encourage students to use reasoning to determine the data is equally spread out among each quartile.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Listen and ask clarifying questions.

Use with Exercises 9–10 Have students work in pairs. Have students individually read Exercise 9 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 10.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–5 odd, 6–10
- Extension: Constructing and Interpreting a Double Box Plot
- ALEKS Measures of Variation, Graphs of Data

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–4, 6, 7, 9
- Extension: Constructing and Interpreting a Double Box Plot
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Collecting Data

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ALEKS Collecting Data

Learn Mean Absolute Deviation

Objective

Students will understand what the mean absolute deviation of a data set represents, and how to calculate it.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to adhere to the definition of absolute value in order to explain how absolute value is related to mean absolute deviation.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 2. The animation illustrates how to calculate the mean absolute deviation.

Talk About It!

SLIDE 2

Mathematical Discourse

The term *absolute* in *mean absolute deviation* refers to the absolute value of a number. How do you think absolute value relates to mean absolute deviation? Sample answer: The absolute value of a number is the distance that number is from 0 on a number line. Distance is always positive and absolute value is always positive. When finding the mean absolute deviation, the distance each data value is from the mean is always positive.

(continued on next page)

DIFFERENTIATE

Language Development Activity ELL

To support students' understanding of the mean absolute deviation of a data set, and how to calculate it, have students brainstorm different strategies they can use to find the mean absolute deviation without needing to graph the data values on a number line. Have them work with a partner to describe other strategies they can use, and have them share their strategies with another pair of students, or with the entire class. Sample answer: After finding the mean of the data set, find the distance each data value is from the mean by finding the difference between the greater value and the lesser value for each data value in the set. Then find the average of these values.

Lesson 10-5

Mean Absolute Deviation

I Can... understand how the mean absolute deviation describes the variation in a data set and interpret its value within the context of a given real-world scenario.

What Vocabulary Will Y ou Learn?
mean absolute deviation

Learn Mean Absolute Deviation

Y ou have learned how the range and interquartile range describe the spread of a data set. Another measure of variation is the **mean absolute deviation** (MAD). The MAD of a data set is the average distance between each data value and the mean. The lower the MAD is for a data set, the closer the data values are to the mean.

Go Online Watch the animation to learn about mean absolute deviation. The animation shows a data set of the points scored in each game. Follow the steps to find the mean absolute deviation.

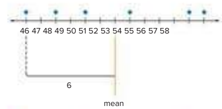
Points Scored per Game
46 58 50 53 48 57

Step 1 Find the mean.

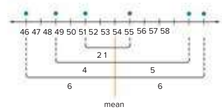
$$\frac{46 + 58 + 50 + 53 + 48 + 57}{6} = 52$$

Calculate the sum of the values in the data set and then divide by the total number of values.

Step 2 Find the distance between each data value and the mean.



Plot the data on a number line. Find the distance between the mean and the first value. Distance is always positive.



Continue to find the distance between the mean and each of the other data values.

Talk About It!
The term *absolute* in *mean absolute deviation* refers to the absolute value of a number. How do you think absolute value relates to mean absolute deviation?

Sample answer: The absolute value of a number is the distance that number is from 0 on a number line. Distance is always positive and absolute value is always positive. When finding the mean absolute deviation, the distance each data value is from the mean is always positive.

(continued on next page)

Lesson 10-5 • Mean Absolute Deviation 569

Interactive Presentation

Learn, Mean Absolute Deviation, Slide 2 of 3

WATCH

On Slide 2, students watch an animation that illustrates how to find the mean absolute deviation of a data set.

Lesson 10-5

Mean Absolute Deviation

6.SP.A.3, 6.SP.B.5

LESSON GOAL

Students will understand mean absolute deviation.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Mean Absolute Deviation

Example 1: Find Mean Absolute Deviation

Example 2: Compare Mean Absolute Deviations

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 57 of the *Language Development Handbook* to help your students build mathematical language related to the mean absolute deviation.

ELL You can use the tips and suggestions on page T57 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability

Additional Cluster(s): In this lesson, students address additional clusters **6.SP.A** and **6.SP.B** by understanding mean absolute deviation.

Standards for Mathematical Content: **6.SP .A.3, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B. 6.SP.B.5.C**

Standards for Mathematical Practice: **MP1, MP2, MP3, MP6**

Coherence

Vertical Alignment

Previous

Students understood interquartile range and created box plots.

6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C

Now

Students understand mean absolute deviation.

6.SP.A.3, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C

Next

Students will understand outliers and their effect on measures of center.

6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C, 6.SP.B.5.D

Rigor

The Three Pillars of Rigor

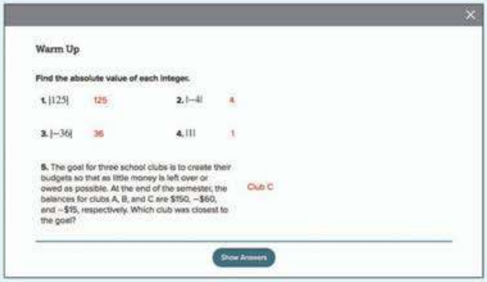
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their *understanding* of statistical measures as they learn about mean absolute deviation. They build *fluency* with finding the mean absolute deviation and explaining what it represents. They also build *fluency* with comparing the mean absolute deviation of two data sets.

Mathematical Background

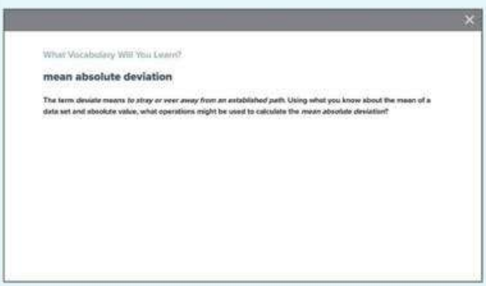
The spread of a data set can be described using the *mean absolute deviation*. This measure of spread describes the average distance between each data value and the mean. Higher values of the mean absolute deviation indicate higher levels of spread. To calculate the mean absolute deviation, first calculate the mean. Find the distance between each data value and the mean by subtracting. Finally, divide the sum of the results by the number of data values.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- absolute value (Exercises 1–5)

Answers

1. 125
2. 4
3. 36
4. 1
5. Club C

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the small variation of life expectancy within the animal family Canidae.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The term *deviate* means *to stray or veer away from an established path*. Using what you know about the mean of a data set and absolute value, what operations might be used to calculate the *mean absolute deviation*? Sample answer: For a number in the set, it might be compared to the mean by calculating its distance from the mean using absolute value.

belizar73/iStock/Getty Images

Your Notes

Talk About It!
The mean absolute deviation is a measure of variability that compares each data value's distance to the mean. In the animation, the MAD of the team scores is 4. Do you think the MAD indicates the data set has a great deal of variability? Explain.

no; Sample answer: If you look at the number line, the point totals could have been spread out much more than they were. I think the MAD of 4 means the data are fairly close together.

Talk About It!
Does the MAD indicate a large or small variation in the data? Explain your reasoning.

Sample answer: The mean absolute deviation indicates that data values are on average 12.5 miles per hour greater or less than the mean. This would indicate a large amount of variance within the data set since the spread of the data set is not that large.

Step 3 Find the mean of the distances.

$$\frac{6+4+2+1+5+6}{6} = 4$$

Calculate the sum of the distances and divide by the number of distances, 6.

So, the mean absolute deviation of the data set is 4. In other words, the average distance each score is from the mean score of 52 is 4 points.

Example 1 Find Mean Absolute Deviation

The table shows the maximum speeds of eight roller coasters.

Maximum Speeds (mph)			
58	88	40	60
72	66	80	48

Find the mean absolute deviation of the data set. Explain what the mean absolute deviation represents.

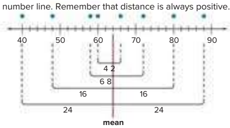
Part A Find the mean absolute deviation.

Step 1 Find the mean.

$$\frac{58+88+40+60+72+66+80+48}{8} = 64 \text{ mph}$$

Step 2 Find the distance between each data value and the mean.

Use a number line. Remember that distance is always positive.



Step 3 Find the mean of the distances.

$$\frac{24+16+6+4+2+8+16+24}{8} = \frac{100}{8}$$
$$= 12.5$$

So, the mean absolute deviation is 12.5 miles per hour.

Part B Explain what the mean absolute deviation represents.

The average distance each roller coaster's speed is from the mean is 12.5 miles per hour.

Interactive Presentation

Example 1, Find the Mean Absolute Deviation, Slide 3 of 7

CLICK

On Slide 3, students select the buttons to graph the data, and find the distance between each value and the mean.

TYPE

On Slide 4, students find the mean absolute deviation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Mean Absolute Deviation *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

The mean absolute deviation is a measure of variability that compares each data value's distance to the mean. In the animation, the MAD of the team scores is 4. Do you think the MAD indicates the data set has a great deal of variation? Explain. no; Sample answer: If you look at the number line, the point totals could have been spread out much more than they were. I think the MAD of 4 means the data are fairly close together.

Example 1 Find Mean Absolute Deviation

Objective

Students will find the mean absolute deviation of a data set and explain what it represents.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to use the mathematics they know to solve the real-world problem and find the mean absolute deviation. Students should make sense of the mean absolute deviation and explain what it represents in the context of the problem.

As students discuss the *Talk About It!* question on Slide 6, encourage them to make sense of the MAD to explain whether it indicates a large or small variation in this data set.

6 Attend to Precision Students should calculate the mean and the distance between each data value and the mean precisely and accurately.

Questions for Mathematical Discourse

SLIDE 2

AL How is the mean of a data set found? To find the mean, find the sum of the values and then divide by the total number of values.

OL When finding the MAD, why is the first step calculating the mean? Sample answer: The MAD is the average distance of the data values from the mean, so you need to find the mean before you can find the distance from each data value to the mean.

BL Without calculating the MAD, do you think there is a great deal of variation in the data set? Explain. yes; Sample answer: I can look at the range of values, 88 — 40 or 48, and see that there is a great deal of variation in the data set.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Compare Mean Absolute Deviations

Objective

Students will compare the mean absolute deviations of two data sets, in order to compare their variations.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the means and the MADs of the two data sets? The means are equal and the MAD of School A is greater than the MAD of School B.

OL What do you think the greater MAD for School A indicates about the two data sets? Sample answer: I think the scores were closer to the mean in School B because the MAD is smaller.

BL Suppose you were given the mean of 81.2 and the MAD of 9.8 for School C. What, if anything, could you determine about the individual values in the data set? Explain. Sample answer: I can only generalize about the variation in the data set from the given information. I can't determine the number of data values nor what individual data values are.

SLIDE 3

AL How does the MAD for School A compare to the MAD for School B? Sample answer: The MAD of School B means that the scores are grouped closer together than that of School A.

OL What does the difference in the MAD between School A and School B indicate? Sample answer: The data for School A are more spread out (farther from the mean) than the data for School B.

BL For School A, the range is 25 and the IQR is 14. For School B, the range is 6 and the IQR is 4. Does this information support the conclusion about the variation in the data for the two schools? Explain. yes; Sample answer: The range of the data for School B is less than the range for School A. This means the data are less spread out. The IQR of the data for School B is also much less than the IQR for School A. This means that half of the data is more clustered around another measure of center, the median.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

The table shows the number of daily visitors to a website on the Internet. Find the mean absolute deviation of the data set. Explain what the mean absolute deviation represents.

Number of Daily Visitors				
112	145	108	160	122

Part A Find the mean absolute deviation. Round to the nearest hundredth.

Show your work here

18.48 visitors

Part B Explain what the mean absolute deviation represents.

Sample answer: The average distance each number of daily visitors is from the mean is 18.48 visitors.

Go Online Y ou can complete an Extra Example online.

Example 2 Compare Mean Absolute Deviations

Two driving schools use the same practice driver's test. Out of 100, School A had scores of 70, 79, 80, 82, and 95. School B had scores of 77 , 83, 83, 81, and 82.

Find the mean absolute deviations. Then compare the variations.

Part A Find the means and the mean absolute deviations.

	School A	School B
Mean:	81.2	81.2
MAD:	5.84	1.76

Show your work here

Part B Compare the variations.

The mean absolute deviation for School A is greater than that for School B. This means the scores for School B are closer together and clustered around the mean. The scores for School A are more spread out and not as clustered around the mean.

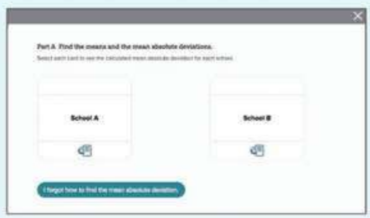
Think About It! What does a small value for the MAD tell you about the data set? What does a large value for the MAD tell you about the data set?

See students' responses.

Talk About It! Based on what you know about the mean absolute deviations, explain which school had more consistent test score results.

Sample answer: School B had more consistent test score results because the MAD was closer to zero than the MAD for School A.

Interactive Presentation



Example 2, Compare Mean Absolute Deviations, Slide 2 of 5

FLASHCARDS

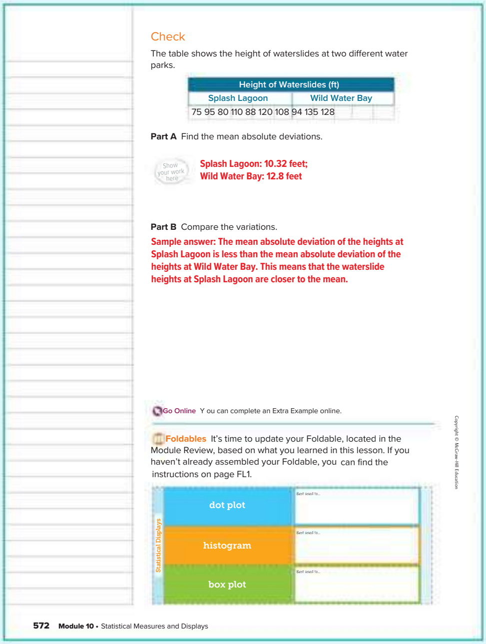
On Slide 2, students use Flashcards to view the mean and MAD for each school.

CLICK

On Slide 3, students compare the mean absolute deviations.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows the height of waterslides at two different water parks.

Height of Waterslides (ft)	
Splash Lagoon	Wild Water Bay
75 95 80 110 88 120 108 94 135 128	

Part A Find the mean absolute deviations.

Show your work here

Splash Lagoon: 10.32 feet; Wild Water Bay: 12.8 feet

Part B Compare the variations.

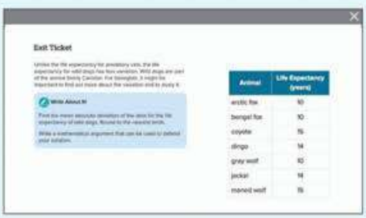
Sample answer: The mean absolute deviation of the heights at Splash Lagoon is less than the mean absolute deviation of the heights at Wild Water Bay. This means that the waterslide heights at Splash Lagoon are closer to the mean.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

572 **Module 10** • Statistical Measures and Displays

Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record an example of how to use a dot plot to find the mean absolute deviation of a data set. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Find the mean absolute deviation of the data for the life expectancy of wild dogs. Round to the nearest tenth.
Write a mathematical argument that can be used to defend your solution.
2.2 years; Sample answer: The mean of the data set is about 12.6 years. So, the mean absolute deviation is $\frac{2.6 + 2.6 + 2.4 + 1.4 + 2.6 + 1.4 + 2.4}{7}$ or 2.2 years.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–5 odd, 6–10
- ALEKS Measures of Variation

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 6, 8, 9
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Collecting Data

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS Collecting Data

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

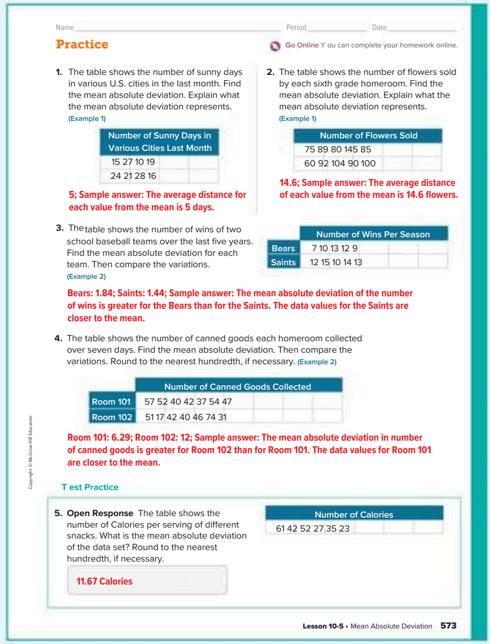
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the mean absolute deviation of a data set and explain what it represents	1, 2
1	compare the mean absolute deviations of two data sets in order to compare their variations	3, 4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving mean absolute deviation	6
3	higher-order and critical thinking skills	7–10

Common Misconception

Some students struggle with remembering to complete all of the steps needed to find the mean absolute deviation. They may forget to use the absolute value to represent the distance from each data value to the mean, or they may forget to find the mean of these distances. Encourage them to understand what the mean absolute deviation of a data set actually means. If they understand that it is the average distance each data value is from the mean, they are more likely to remember to complete all of the steps necessary to find that average.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The table shows the number of sunny days in various U.S. cities in the last month. Find the mean absolute deviation. Explain what the mean absolute deviation represents. (Example 1)

Number of Sunny Days in Various Cities Last Month			
15	27	10	19
24	21	28	16

5; Sample answer: The average distance for each value from the mean is 5 days.

2. The table shows the number of flowers sold by each sixth grade homeroom. Find the mean absolute deviation. Explain what the mean absolute deviation represents. (Example 1)

Number of Flowers Sold				
75	89	80	145	85
60	92	104	90	100

14.6; Sample answer: The average distance of each value from the mean is 14.6 flowers.

3. The table shows the number of wins of two school baseball teams over the last five years. Find the mean absolute deviation for each team. Then compare the variations. (Example 2)

	Number of Wins Per Season				
Bears	7	10	13	12	9
Saints	12	15	10	14	13

Bears: 1.84; Saints: 1.44; Sample answer: The mean absolute deviation of the number of wins is greater for the Bears than for the Saints. The data values for the Saints are closer to the mean.

4. The table shows the number of canned goods each homeroom collected over seven days. Find the mean absolute deviation. Then compare the variations. Round to the nearest hundredth, if necessary. (Example 2)

	Number of Canned Goods Collected						
Room 101	57	52	40	42	37	54	47
Room 102	51	17	42	40	46	74	31

Room 101: 6.29; Room 102: 12; Sample answer: The mean absolute deviation in number of canned goods is greater for Room 102 than for Room 101. The data values for Room 101 are closer to the mean.

Test Practice

5. **Open Response** The table shows the number of Calories per serving of different snacks. What is the mean absolute deviation of the data set? Round to the nearest hundredth, if necessary.

Number of Calories					
61	42	52	27	35	23

11.67 Calories

Lesson 10-5 • Mean Absolute Deviation 573

Apply *indicates multi-step problem

*6. The table shows the number of laps Candice and her two friends ran each day for five days. Which friend ran the most consistent number of laps each day? Use the mean absolute deviation to construct an argument to justify your response.

Girl	Day 1	Day 2	Day 3	Day 4	Day 5
Candice	5	6	8	5	7
Malaya	4	5	3	3	5
Zoe	7	8	6	8	8

Zoe; Sample answer: The MAD for Candice is 1.04 laps, for Malaya is 0.8 laps, and for Zoe is 0.72 laps. Since 0.72< 0.8 < 1.04, Zoe ran the most consistent number of laps each day.

Higher-Order Thinking Problems

7. MP **Persevere with Problems** The table shows the highway fuel economy of various popular vehicles. Find the mean absolute deviation. How many data values are closer than one mean absolute deviation away from the mean?

Fuel Economy (miles per gallon)
34 48 25 35 33
37 32 34 23 30

4.5 miles per gallon; 7 data values

8. MP **Justify Conclusions** The table shows the high temperatures for the last 6 days. If today's high temperature was 61°F, how is the mean absolute deviation affected? Justify your response.

High T emperature (F)
75 58 72 68 69 66

Sample answer: The mean absolute deviation increases from 4 to about 4.6. Since the mean is affected, then the mean absolute deviation is also affected.

9. MP **Make an Argument** Use the meanings of the terms *mean*, *absolute*, and *deviation* to make an argument for why the mean absolute deviation of a data set is named using these terms.

Sample answer: The term absolute refers to the absolute value of a number , which is the distance a number is from 0 on a number line and distance is always positive. T o deviate means to vary or change. So, the mean absolute deviation of a data set is the average (mean) distance from each data value to the mean, which is a description of how the data values deviate or vary from the mean.

10. MP **Reason Inductively** If the distance between the mean and a data value on a number line is 0, what do you know about the data value? Explain.

The data value must be equal to the mean; Sample answer: For example, if the mean is 7 and the data value is 7 , the distance between the two points is 0 units.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 7, students find the mean absolute deviation and determine how many values are closer than one mean absolute deviation away from the mean. Encourage students to plan a solution pathway that can be implemented to solve the problem.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 8, students determine how adding another value to the data set affects the mean and mean absolute deviation. Encourage students to support their answer with a logical explanation.

In Exercise 9, students explain why the mean absolute deviation is named as such. Encourage students to explain that the term absolute refers to the absolute value, which is the distance a number is from 0, and to deviate means to vary or change. So, the mean absolute deviation is a description of how the data values vary from the mean.

In Exercise 10, students will determine what they know about the data value. Encourage students to use reasoning to form an explanation that concludes the data value must be equal to the mean.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 6 Have students work together to prepare a brief demonstration that illustrates why this problem might require multiple steps to solve. For example, before they can order the students, they have to find the mean and mean absolute deviation for each student. Have each pair or group of students present their response to the class.

Clearly explain your strategy.

Use with Exercise 7 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would find the number of data values that are closer than one mean absolute deviation from the mean, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Learn Outliers

Objective

Students will understand what an outlier is and how to determine if a data value is an outlier.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It*! question on Slide 2, encourage them to understand that outliers can be values that are much less than the other data values. They should be able to reason that if there was a value that was 262.5 units less than Q_1, it would also be considered an outlier.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

If the outlier was removed from the data set, will the median still be 387.5? Why or why not? no; Sample answer: There would now be an odd number of data values. The median would be 387.

Lesson 10-6

Outliers

I Can... understand how an outlier may affect a measure of center, and determine which measure of center is most appropriate to use when describing a data set that does or does not contain an outlier.

What Vocabulary Will You Learn?
outlier

Learn Outliers

An **outlier** is a data value that is very far away from the other data values. It can be much greater in value or much less than the other values. Consider the data set shown.

225, 245, 295, 305, 360, 387 , 388, 420, 470, 480, 625, 780

How do you know if either of the extreme values, 225 or 780, are considered outliers?

An outlier is defined as a value that lies more than 1.5 times the interquartile range either above Q_3 or below Q_1.

Q_1 = 300 Median = 387.5 Q_3 = 475

225, 245, 295, 305, 360, 387 , 388, 420, 470, 480, 625, 780

IQR = 475 − 300, or 175

Determine the upper and lower limits for the outliers.

Upper Limit		Lower Limit
$Q_3 + (1.5 \cdot IQR)$		$Q_1 - (1.5 \cdot IQR)$
$= 475 + (1.5 \cdot 175)$	Substitute.	$= 300 - (1.5 \cdot 175)$
$= 475 + 262.5$	Multiply.	$= 300 - 262.5$
$= 737.5$	Simplify.	$= 37.5$

Any data values that are greater than 737 .5 or less than 37 .5 are outliers. So, the value 780 is an outlier. Because the data set does not contain any values that are less than 37 .5, the only outlier is 780.

The box plot represents the data set. Outliers are indicated with an asterisk (*).

Talk About It!
If the outlier was removed from the data set, will the median still be 387 .5? Why or why not?

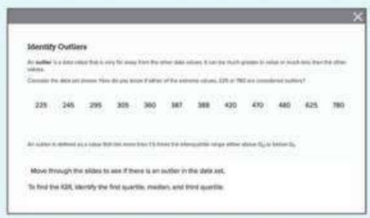
no; Sample answer: There would now be an odd number of data values. The median would be 387 .

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of outliers, have them work with a partner to create their own data sets. Each pair of students should create one data set that has an outlier, and one data set that does not have an outlier, according to the definition of outlier presented in the Learn. Remind students that an outlier does not always have to be much greater than the other data values; it can be much less. Then have students trade data sets with another pair of students. Each pair should determine which data set has an outlier, and what that value is. Have pairs check each other's work and discuss and resolve any differences.

Interactive Presentation



Learn, Outliers, Slide 1 of 2

CLICK

On Slide 1, students move through the slides to determine whether or not an outlier exists in the data set.

Lesson 10-6

Outliers

6.SP.A.3, 6.SP.B.5

LESSON GOAL

Students will understand outliers and their effect on measures of center.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Outliers
Example 1: Identify Outliers

Explore: Mean, Median, and Outliers

Learn: Describe the Effect of Outliers
Example 2: Describe the Effect of Outliers

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

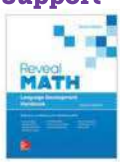
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 58 of the *Language Development Handbook* to help your students build mathematical language related to outliers.



ELL You can use the tips and suggestions on page T58 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Statistics and Probability

Additional Cluster(s): In this lesson, students address additional clusters **6.SP.A** and **6.SP.B** by understanding outliers and their effects on measures of center.

Standards for Mathematical Content: **6.SP .A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C, 6.SP.B.5.D**

Standards for Mathematical Practice: **MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous

Students understood mean absolute deviation.
6.SP.A.3, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C

Now

Students understand outliers and their effect on measures of center.
6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C, 6.SP.B.5.D

Next

Students will interpret dot plots, histograms, and box plots.
6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C, 6.SP.B.5.D

Rigor

The Three Pillars of Rigor

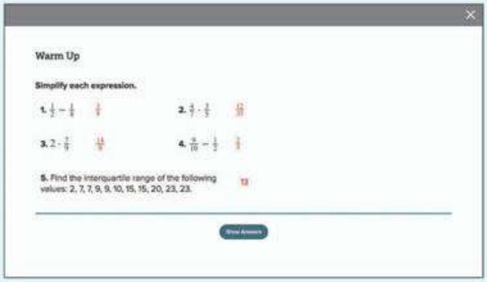
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students further expand their *understanding* of statistical measures as they explore outliers. They come to understand that outliers affect measures of center, as they build *fluency* with identifying outliers and describing the effect outliers have on the mean and median of a data set from real-world scenarios.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- subtracting and multiplying rational numbers (Exercises 1–4)
- finding the interquartile range (Exercise 5)

Answers

1. $\frac{3}{8}$

2. $\frac{12}{35}$

3. $\frac{14}{9}$ or $1\frac{5}{9}$

4. $\frac{2}{5}$

5. 13

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about outliers of the populations of Illinois and Florida.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Based on words you recognize within *outlier*, what do you think an outlier might be in a data set? Sample answer: I think an outlier is a number that is outside most of the other numbers in a data set.

Your Notes

Think About It!
What measures of variation do you need to find in order to identify any outliers?

median, quartiles, interquartile range

Talk About It!
What does it mean that 23 is an outlier?

Sample answer: The age of the youngest person is an extreme value in the data set and is significantly less than the other data values.

Example 1 Identify Outliers

The ages, in years, of the candidates in an election are 55, 49, 48, 57, 23, 63, and 72.

Identify any outliers in the data set.

Step 1 Find the quartiles and interquartile range.

List the data values from least to greatest.

23 48 49 55			57	63	72

least … greatest

Find the quartiles and interquartile range.

Median = **55** Q_1 = **48** Q_3 = **63** IQR = **15**

Step 2 Determine the upper and lower limits for the outliers.

Upper Limit		Lower Limit
$Q_3 + (1.5 \cdot \text{IQR})$		$Q_1 - (1.5 \cdot \text{IQR})$
= **63** + (1.5 • 15)	Substitute.	= **48** − (1.5 • 15)
= **63** + **22.5**	Multiply.	= **48** − **22.5**
= **85.5**	Simplify.	= **25.5**

Step 3 Identify any outliers.

Any data values that are greater than 85.5 or less than 25.5 are outliers. So, the value 23 is an outlier. Because the data set does not contain any values that are greater than 85.5, the only outlier is 23.

Check

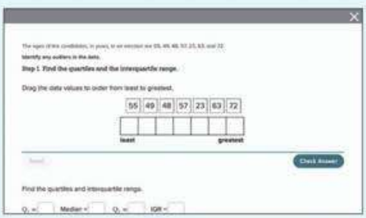
The lengths, in feet, of various bridges are 354, 88, 251, 275, 727, and 1,121. Identify any outliers in the data.

Show your work here

There are no outliers in the data set.

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Identify Outliers, Slide 2 of 6

DRAG & DROP

On Slide 2, students drag to order the values from least to greatest and identify the quartiles.

CLICK

On Slide 3, students move through the steps to determine the boundary for any outliers.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Identify Outliers

Objective

Students will use the definition of an outlier to identify any outliers in a data set.

Questions for Mathematical Discourse

SLIDE 2

AL How does ordering the values help you determine Q_1 and Q_3? Sample answer: I need to order the values so I can separate the values into two halves. The quartiles are the medians of each half.

OL How is the first quartile found? The first quartile is the median of the lower half of the data.

BL Is there a value you think may be an outlier? Explain. yes; Sample answer: I think 23 may be an outlier because it seems much less than the other data values.

SLIDE 3

AL Why do you subtract 22.5 from 48, and add 22.5 to 63? Sample answer: Subtracting the number I calculated from the IQR from the lower quartile and adding it to the upper quartile helps me set the boundaries for the outliers.

OL Based on the limits for outliers, would 85 be considered an outlier? Explain. no; Sample answer: 85 is between the lower limit, 25.5, and the upper limit, 85.5, so it is not an outlier.

BL Why do you need to establish a boundary for outliers? The boundary will help me determine if a data value is actually an outlier. I may not be able to detect an outlier just by looking at the data set.

SLIDE 4

AL How will you determine if there are any outliers? Sample answer: I will look at the lower boundary, 25.5, and see if there are values in the data set less than that. Then I will look at the upper boundary, 85.5, and see if there are values in the data set greater than that.

OL How do you know there is not an outlier other than 23? There are no other values less than 25.5 or greater than 85.5.

BL If a new candidate with the age of 25 is added, how would the outlier change? Sample answer: there wouldn't be an outlier in the set anymore, because the lower limit would become 1.25 and the upper limit would become 95.25.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Describe the Effect of Outliers

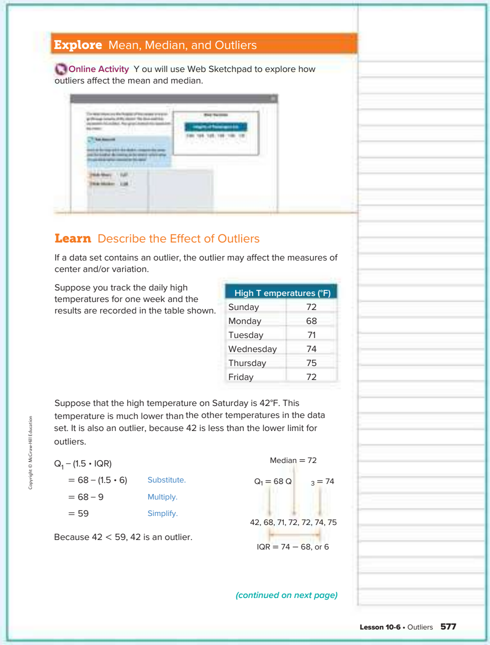
Objective

Students will understand the effects an outlier can have on the measures of center.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use clear and precise mathematical language to explain which measure(s) of center and variation were and were not affected by adding Saturday's temperature, as an outlier, to the data set. If Saturday's temperature was not technically an outlier, have them explain the effects of adding that data value on the mean and median.

(continued on next page)



Explore Mean, Median, and Outliers

Online Activity Y ou will use Web Sketchpad to explore how outliers affect the mean and median.

Learn Describe the Effect of Outliers

If a data set contains an outlier, the outlier may affect the measures of center and/or variation.

Suppose you track the daily high temperatures for one week and the results are recorded in the table shown.

High T emperatures (°F)	
Sunday	72
Monday	68
Tuesday	71
Wednesday	74
Thursday	75
Friday	72

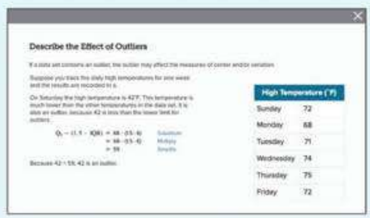
Suppose that the high temperature on Saturday is 42°F. This temperature is much lower than the other temperatures in the data set. It is also an outlier, because 42 is less than the lower limit for outliers.

$Q_1 - (1.5 \cdot \text{IQR})$
$= 68 - (1.5 \cdot 6)$ Substitute.
$= 68 - 9$ Multiply.
$= 59$ Simplify.

Because 42 < 59, 42 is an outlier.

(continued on next page)

Interactive Presentation



Learn, Describe the Effect of Outliers, Slide 1 of 3

Explore Mean, Median, and Outliers

Objective

Students will use Web Sketchpad to explore how outliers affect the mean and median.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the heights of people in a line. Students will use the Web Sketchpad to explore how the outliers will change the measures of center. Encourage students to share their predictions with one another.

Inquiry Question

How does an outlier affect measures of central tendency? Sample answer: The outlier can have an effect on both the mean and the median, but the mean is more likely to be affected. The mean is affected more by the outlier because it uses the sum of all data values.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

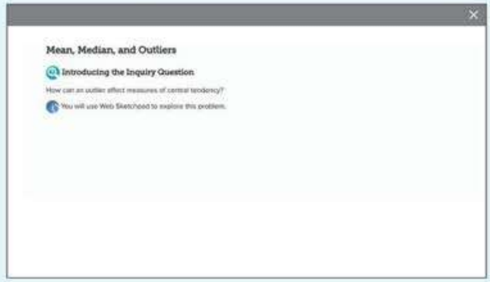
SLIDE 3

Mathematical Discourse

Look at the data set in the sketch to compare the mean and the median. By looking at the sketch, which value do you think better represents the data? Sample answer: The mean and median are close together, with the mean being slightly lower. I think the mean better represents the data because it seems to be more centrally located on the number line.

(continued on next page)

Interactive Presentation



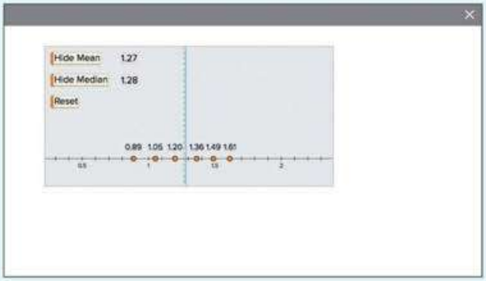
Explore, Slide 1 of 7

Explore, Slide 3 of 7

WEB SKETCHPAD

On Slide 5, students use Web Sketchpad to explore how outliers affect the mean and median.

Interactive Presentation



Explore, Slide 5 of 7

WEB SKETCHPAD

On Slide 6, students use Web Sketchpad to explore how outliers affect the mean and median.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Mean, Median, and Outliers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use Web Sketchpad to explore and deepen their understanding between outliers and the measures of center.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Do the results match your prediction? Why do you think the mean was more affected by the outlier? Sample answer: Yes; the mean was more affected because I took away a small value and added a much greater value.

Talk About It!
Suppose Saturday's temperature had been 59°F, which does not qualify as an outlier, but is cooler than the rest. How does this affect the mean? the median?

Sample answer: The mean and median without Saturday's temperature was 72, when Saturday's temperature was added, the mean decreased to 70, but the median was unchanged.

To see how an outlier affects the measures of center and variation, calculate the measures both with and without the outlier.

Calculate the measures with the outlier.

Mean

$$\frac{42 + 68 + 71 + 72 + 72 + 74 + 75}{7} \approx 67.7$$

Median = 72

$Q_1 = 68$ $Q_3 = 74$

42, 68, 71, 72, 72, 74, 75

IQR = 74 − 68, or 6

Mean Absolute Deviation (MAD)

Show your work here

To the nearest tenth, the MAD is **7.4**.

Median

The median is **72**.

Interquartile Range (IQR)

The IQR is **6**.

Calculate the measures without the outlier.

Mean

$$\frac{68 + 71 + 72 + 72 + 74 + 75}{6} = 72$$

Median = 72

$Q_1 = 71$ $Q_3 = 74$

68, 71, 72, 72, 74, 75

IQR = 74 − 71, or 3

Mean Absolute Deviation (MAD)

Show your work here

To the nearest tenth, the MAD is **1.7**.

Median

The median is **72**.

Interquartile Range (IQR)

The IQR is **3**.

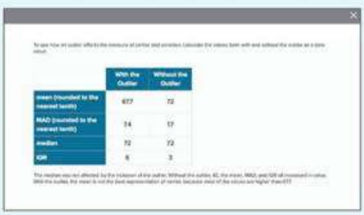
The median was not affected by the inclusion of the outlier. Without the outlier, the mean, MAD, and IQR all increased in value. With the outlier, the mean is not the best representation of center, because most of the values are higher than 67.7.

Use either the mean or median when the data does not contain any outliers. Use only the median when the data contains an outlier. While the median might change a little when an outlier is included or removed, it does not change as much as the mean.

Use the corresponding measure of variation to describe the spread of the data.

- If you choose the mean to describe the center, choose the MAD to describe the variation.
- If you choose the median to describe the center, choose the IQR to describe the variation.

Interactive Presentation



Learn, Describe the Effect of Outliers, Slide 2 of 3

Learn Describe the Effect of Outliers *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

Suppose Saturday's temperature had been 59°F, which does not qualify as an outlier, but is cooler than the rest. How does this affect the mean? the median? Sample answer: The mean and median without Saturday's temperature was 72, when Saturday's temperature was added, the mean decreased to 70, but the median was unchanged.

DIFFERENTIATE

Enrichment Activity BL

To support students' understanding of how an outlier affects the mean, have students think of a time when they earned a low score on a test or assignment. Ask students, "How did that single score affect your overall grade in the class?" Then have students think of a time when they earned a high score on a test or assignment. Ask students, "How did that single score affect your overall grade in the class?" Have students refer to the Learn to notice the change in mean with and without the outlier.

Example 2 Describe the Effect of Outliers

Objective

Students will describe the effect outliers can have on measures of center.

Questions for Mathematical Discourse

SLIDE 2

AL How are the mean and median found? The mean is found by finding the sum of the data and dividing by the total number of values. The median is found by finding the value in the middle of the data set once the values are listed in increasing order.

OL How could you confirm that 200 is an outlier? Sample answer: Determine the upper and lower limits for the outliers by adding 1.5 times the IQR to Q_3 and subtracting 1.5 times the IQR from Q_1. The only data value greater than or less than the limits is 200.

BL One of the boundaries for finding an outlier is −30. Why is that not relevant to this problem? Sample answer: The lifespan of an animal in years cannot be negative.

SLIDE 3

AL Do you think the mean will increase or decrease when the outlier is removed from the data set? Explain. decrease; Sample answer: The outlier is much greater than the other data values, so removing it will cause the mean to decrease.

OL When finding the mean with the outlier you divided by 7 and without the outlier, you divided by 6. Why are those divisors different? Sample answer: The data set has seven data values with the outlier. When I remove the outlier, I only have 6 data values, which is why the divisors are different.

BL Compare the mean and the median with and without the outlier. What do you notice about them when you remove the outlier? How is this related to the concept of *measure of center*? Sample answer: With the outlier, the mean and median have very different values. When I remove the outlier, the mean changes to a value that is much closer to the median. When there are outliers in a data set, the median represents the center, while the mean will be a value pulled away from the center by the outlier.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Describe the Effect of Outliers

The table shows the average lifespans of selected animals.

Calculate the mean and median with and without the outlier, 200. Then choose the measure that best describes the center.

Average Lifespan	
Animal	**Lifespan (years)**
Elephant	35
Dolphin	30
Chimpanzee	50
Tortoise	200
Gorilla	30
Gray Whale	70
Horse	20

Step 1 Calculate the mean and median with the outlier. Round to the nearest tenth, if necessary.

Mean

$\frac{35 + 30 + 50 + 200 + 30 + 70 + 20}{7} \approx 62.1$

The mean lifespan is about 62 years.

Median

Show your work here

The median lifespan is 35 years.

Step 2 Calculate the mean and median without the outlier. Round to the nearest tenth, if necessary.

Mean

$\frac{35 + 30 + 50 + 30 + 70 + 20}{6} \approx 39.2$

The mean lifespan is about 39 years.

Median

Show your work here

The median lifespan is 32.5 years.

Step 3 Choose the measure that best describes the center.

The mean was most affected by the inclusion of the outlier.
The median changed very little.

So, the median best describes the center of the data.

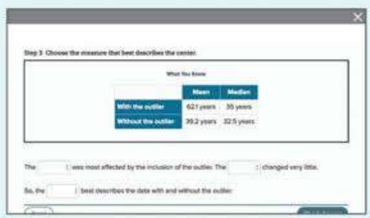
Think About It! Will the outlier affect the mean or the median more? Explain your reasoning.

See students' responses.

Talk About It! Explain why it makes sense that the lifespan of the animals listed in the table are centered around 32.5 or 35 years, rather than around 39 or 62 years.

Sample answer: Most of the animals have lifespans that are 50 years or less.

Interactive Presentation



Example 2, Describe the Effect of Outliers, Slide 4 of 6

CLICK

On Slides 2 and 3, students move through the steps to calculate the mean and median with and without the outlier.

CLICK

On Slide 4, students choose the measure that best describes the data.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The table shows the cooking temperatures for different recipes. Calculate the mean and median with and without the outlier. Round to the nearest tenth, if necessary. Then choose the measure that best describes the center.

Cooking Temperature (°F)			
175	325	325	350
350	350	400	450

Show your work here

mean with outlier: ≈ 340.6
median with outlier: 350
mean without outlier: ≈ 364.3
median without outlier: 350
Sample answer: The mean was affected the most by the outlier. The median temperature did not change when the outlier was removed, so it would be the best measure of center to represent the data set.

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that will help you study the concepts you learned today in class.

Record your observations here

See students' observations.

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. The populations, rounded to the nearest thousand, of five cities in Florida are shown. Is there an outlier in the data set? If so, what value is the outlier? Write a mathematical argument that can be used to defend your solution. no outliers; Sample answer: The upper and lower limits for the outliers are −285,250 and 1,184,750. There are no values that are less than the lower limit or greater than the upper limit, so there are no outliers.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

BL

IF students score 90% or above on the Checks,
THEN assign:

- Practice, Exercises 1–7 odd, 9–12
- ALEKS® Graphs of Data

OL

IF students score 66–89% on the Checks,
THEN assign:

- Practice, Exercises 1–7, 11, 12
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Collecting Data

AL

IF students score 65% or below on the Checks,
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Collecting Data

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify outliers in a data set	1–4
1	describe the effect outliers can have on the measures of center	5–7
2	extend concepts learned in class to apply them in new contexts	8
3	higher-order and critical thinking skills	9–12

Common Misconception

Students might indicate that a data value is an outlier simply by observing the values in the data set. For example, in Exercise 4, they might indicate that 96 is an outlier because it appears to be much greater than the other values in the data set. Remind students that a data value is only an outlier if it is 1.5 times greater than or less than the interquartile range. In Exercise 4, 96 is greater than the other data values, but it is not 1.5 times greater than the interquartile range, so it is not an outlier.

Name ____________ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Last week, Joakim spent 40, 25, 60, 30, 35, and 40 minutes practicing the piano. Identify any outliers in the data. (Example 1)

   60 minutes

2. Last month, a basketball team scored 83, 84, 85, 87, 89, 88, 67, 79, and 81 points in their games. Identify any outliers in the data. (Example 1)

   67 points

3. Abrianna sold 20, 23, 18, 4, 17, 21, 15, and 56 boxes of cookies after different football games. Identify any outliers in the data. (Example 1)

   4 boxes and 56 boxes are both outliers

4. Last week a certain pet store had 52, 72, 96, 21, 58, 40, and 75 paying customers. Identify any outliers in the data. (Example 1)

   no outliers

5. The prices of trees that Sahana bought are \$46, \$39, \$40, \$45, \$44, \$68, and \$51. Calculate the mean and median with and without the outlier. Round to the nearest tenth, if necessary. Choose the measure that best describes the center. (Example 2)

   mean with outlier: ≈ 47.6
   median with outlier: 45
   mean without outlier: ≈ 44.2
   median without outlier: 44.5
   The median best describes the center.

6. The prices of backpacks are \$37, \$43, \$41, \$36, \$44, and \$70. Calculate the mean and median with and without the outlier. Round to the nearest tenth, if necessary. Choose the measure that best describes the center. (Example 2)

   mean with outlier: ≈ 45.2
   median with outlier: 42
   mean without outlier: ≈ 40.2
   median without outlier: 41
   The median best describes the center.

7. The table shows the number of points scored by a football team. Calculate the mean and median with and without the outlier. Round to the nearest tenth, if necessary. Choose the measure that best describes the center. Explain. (Example 2)

Points Scored by a Football Team			
14	20	3	9
18	35	21	24
7	12	31	68

   mean with outlier: ≈ 21.8
   median with outlier: 19
   mean without outlier: ≈ 17.6
   median without outlier: 18
   The median best describes the center because the mean was affected the most with the outlier.

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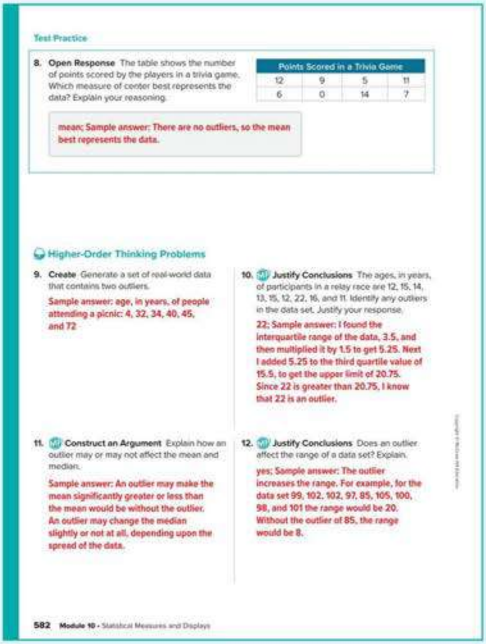

Test Practice

8. **Open Response** The table shows the number of points scored by the players in a trivia game. Which measure of center best represents the data? Explain your reasoning.

Points Scored in a Trivia Game			
12	9	5	11
6	0	14	7

mean; Sample answer: There are no outliers, so the mean best represents the data.

Higher-Order Thinking Problems

9. **Create** Generate a set of real-world data that contains two outliers.

Sample answer: age, in years, of people attending a picnic: 4, 32, 34, 40, 45, and 72

10. **Justify Conclusions** The ages, in years, of participants in a relay race are 12, 15, 14, 13, 15, 12, 22, 16, and 11. Identify any outliers in the data set. Justify your response.

22; Sample answer: I found the interquartile range of the data, 3.5, and then multiplied it by 1.5 to get 5.25. Next I added 5.25 to the third quartile value of 15.5, to get the upper limit of 20.75. Since 22 is greater than 20.75, I know that 22 is an outlier.

11. **Construct an Argument** Explain how an outlier may or may not affect the mean and median.

Sample answer: An outlier may make the mean significantly greater or less than the mean would be without the outlier. An outlier may change the median slightly or not at all, depending upon the spread of the data.

12. **Justify Conclusions** Does an outlier affect the range of a data set? Explain.

yes; Sample answer: The outlier increases the range. For example, for the data set 99, 102, 102, 97, 85, 105, 100, 98, and 101 the range would be 20. Without the outlier of 85, the range would be 8.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students identify an outlier in a data set and then justify their response. Encourage students to support their answer with a precise and logical explanation.

In Exercise 11, students explain the effect an outlier may or may not have on measures of center. Encourage students to construct a logical argument for each measure of center.

In Exercise 12, students determine if an outlier affects the range of a data set. Encourage students to support their answer with a logical explanation.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Solve the problem another way.

Use with Exercise 10 Have students work in groups of 3–4. After completing Exercise 10, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Learn Interpret Dot Plots

Objective

Students will understand that a dot plot can be described by its overall shape.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to clearly and precisely explain why the mean and median will be the same value for data that is symmetric. Encourage them to use the definitions of *mean* and *median* to help support their explanation.

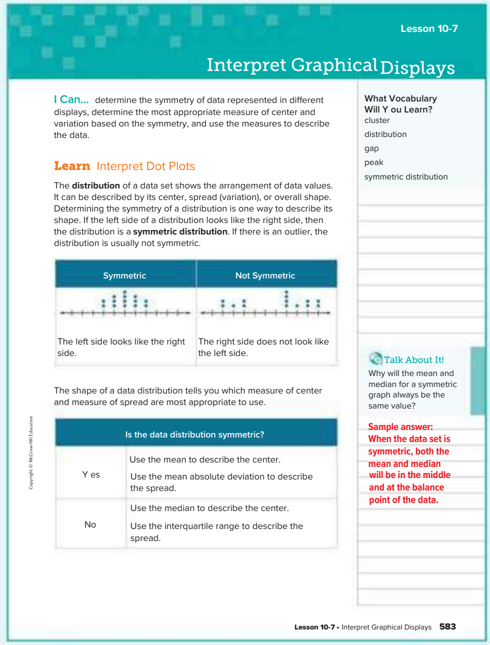
Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Why will the mean and median for a symmetric graph always be the same value? Sample answer: When the data set is symmetric, both the mean and median will be in the middle and at the balance point of the data.



Lesson 10-7

Interpret Graphical Displays

I Can... determine the symmetry of data represented in different displays, determine the most appropriate measure of center and variation based on the symmetry, and use the measures to describe the data.

What Vocabulary Will You Learn?
cluster
distribution
gap
peak
symmetric distribution

Learn Interpret Dot Plots

The **distribution** of a data set shows the arrangement of data values. It can be described by its center, spread (variation), or overall shape. Determining the symmetry of a distribution is one way to describe its shape. If the left side of a distribution looks like the right side, then the distribution is a **symmetric distribution**. If there is an outlier, the distribution is usually not symmetric.

Symmetric	Not Symmetric
The left side looks like the right side.	The right side does not look like the left side.

The shape of a data distribution tells you which measure of center and measure of spread are most appropriate to use.

Is the data distribution symmetric?	
Yes	Use the mean to describe the center. Use the mean absolute deviation to describe the spread.
No	Use the median to describe the center. Use the interquartile range to describe the spread.

Talk About It! Why will the mean and median for a symmetric graph always be the same value?

Sample answer: When the data set is symmetric, both the mean and median will be in the middle and at the balance point of the data.

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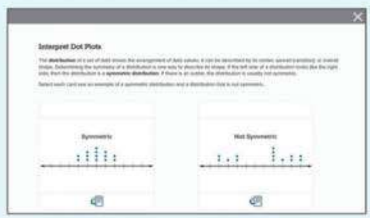
Interactive Presentation



Learn, Interpret Dot Plots, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to see an example of a symmetric distribution and a distribution that is not symmetric.

DIFFERENTIATE

Enrichment Activity BL

For students that need more of a challenge, use the following exercise.

Have students work with a partner to generate a set of values that will have a symmetric distribution and a set of values that will not have a symmetric distribution. Have students construct the dot plot for each set and then explain why the sets are symmetric and not symmetric.

Lesson 10-7

Interpret Graphical Displays

6.SP.B.4, 6.SP.B.5

LESSON GOAL

Students will interpret dot plots, histograms, and box plots.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Interpret Dot Plots
Example 1: Interpret Dot Plots
Learn: Interpret Histograms
Example 2: Interpret Histograms

Explore: Interpret Box Plots

Learn: Interpret Box Plots
Example 3: Interpret Box Plots
Apply: Travel

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Select an Appropriate Display		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 59 of the *Language Development Handbook* to help your students build mathematical language related to interpreting graphical displays.

ELL You can use the tips and suggestions on page T59 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Statistics and Probability

Major Cluster(s): In this lesson, students address major cluster **6.RP.A** and additional clusters **6.SP.A** and **6.SP.B** by interpreting dot plots, histograms, and box plots.

Standards for Mathematical Content: **6.SP .A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B,6.SP.B.5.C, 6.SP.B.5.D,** *Also addresses 6.RP.A.1, 6.RP.A.3*

Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students understood outliers and their effect on measures of center.
6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.C, 6.SP.B.5.D

Now
Students interpret dot plots, histograms, and box plots.
6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.A, 6.SP.B.5.B, 6.SP.B.5.C, 6.SP.B.5.D

Next
Students will use statistics to compare two populations.
7.SP.B.4

Rigor

The Three Pillars of Rigor

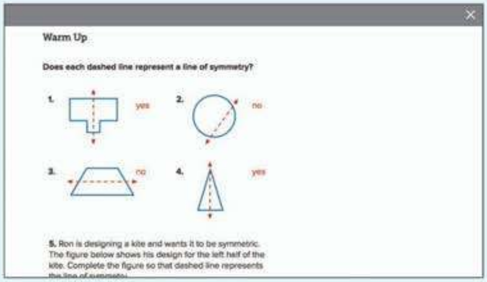
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students deepen their *understanding* of statistical measures as they interpret graphical displays. They use measures of center and variation to build *fluency* with describing data sets represented in dot plots, histograms, and box plots. They *apply* their understanding of graphical displays to solve multi-step, real-world problems.

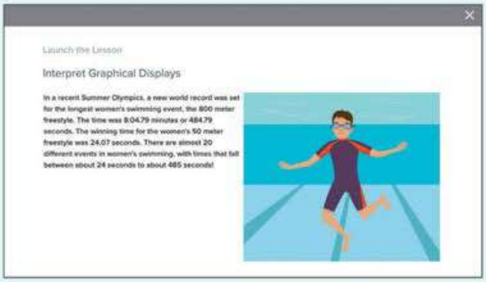
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

• understanding symmetry (Exercises 1–5)

Answers

1. yes

2. no

3. no

4. yes

5. See Warm Up slide online for correct answer.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about different winning times for different events in a recent Summer Olympics.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

• A synonym for the word *cluster* is a bunch or a mass. What do you think is a *cluster* in a data set? Sample answer: I think a cluster is a group of data values bunched together.

• What does it mean to *distribute* papers to the class? Sample answer: to pass them out to students in the classroom.

• What does the word *gap* mean in everyday life? Sample answer: A gap is an empty spot.

• How would you describe the *peak* of a mountain? Sample answer: The peak is the highest point.

• How can you determine if a figure is *symmetric*? Sample answer: A figure is symmetric if it can be folded and both sides are the same.

Your Notes

Think About It!
What do you notice about the shape of the distribution?

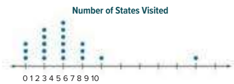
See students' responses.

Talk About It!
What do you notice about the measure of center's location on the dot plot?

Sample answer: The measure of center is 2 where the data values are piled up on the dot plot. Most of the students in the class responded near this value.

Example 1 Interpret Dot Plots

The results of a class survey about the number of states visited by students are shown in the dot plot.



Choose the appropriate measure of center and variation. Then use the measures to describe the distribution.

Part A Choose the appropriate measures.

The data are not evenly distributed between the left side and the right side.

There appears to be an outlier.

So, the distribution is not symmetric.

Which measure of center should you use? median

Which measure of variation should you use? interquartile range

Part B Describe the distribution.

A total of 19 students responded to the survey.

Find the measure of center you chose in Part A.

Show your work here

The median is 2 states.

The measure of center indicates that the number of states visited by the students can be summarized by the single value of 2 states.

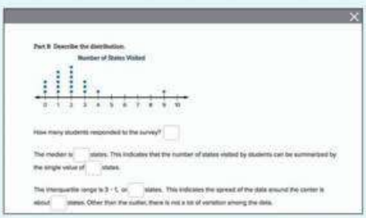
Find the measure of variation you chose in Part A.

Show your work here

The interquartile range is 3 − 1, or 2 states.

The measure of variation indicates that the spread of the data around the center is 2 states. Other than the outlier, there is not a lot of variation among the data.

Interactive Presentation



Example 1, Interpret Dot Plots, Slide 3 of 5

TYPE

On Slide 3, students describe the data set.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Interpret Dot Plots

Objective

Students will choose the appropriate measure of center and variation to describe a data set represented by a dot plot.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to describe the data set using clear and precise mathematical language, including the most appropriate measures of center and variation.

As students discuss the *Talk About It!* question on Slide 4, encourage them to clearly identify where the measure of center falls on the dot plot.

7 Look For and Make Use of Structure Encourage students to study the structure of the dot plot to determine whether or not the distribution is symmetric. This will help them determine the most appropriate measures of center and variation to use.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the distribution of the data in the dot plot? Sample answer: The data are mostly found on the left side.

OL What do you think the data value 9 represents? Explain. Sample answer: I think this value is an outlier because it is so far away from the other data values.

BL Do you notice any gaps in the data? Explain what that might represent. Sample answer: There is a gap between 4 and 9. This means that no students reported visiting 5, 6, 7, or 8 states.

SLIDE 3

AL What do the values 3 and 1 represent? Why are they subtracted? 3 is the third quartile and 1 is the first quartile. You subtract them to find the interquartile range.

OL How can you find the median without listing the values in order? Sample answer: The dot plot shows the values in order. I can cross off dots, alternating between the lower end and the greater end, until I arrive at the middle dot.

BL If more students were surveyed and they responded with a majority of answers above 6, how would this affect the data? Sample answer: The data would be affected in terms of symmetry so the appropriate measures of center and variation may change.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Interpret Histograms

Objective

Students will understand that a histogram can be described by its overall shape, including clusters, gaps, and peaks.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to use the structure of the histogram to describe, in their own words, the heights of the buildings in Seattle.

Go Online to find additional teaching notes.

Talk About It!

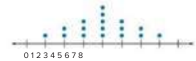
SLIDE 2

Mathematical Discourse

Use the histogram to describe the heights of the buildings in Seattle. Sample answer: Most of the buildings are between 400–599 feet tall, and only one of the twenty-one buildings is taller than 799 feet. The measure of center that best describes the data is in the 500–599 feet range.

Check

The results of a class survey about the number of hours spent on the Internet each week by students are shown in the dot plot.



Part A Choose the appropriate measure of center and variability.

Sample answer: There are no outliers, and the distribution is symmetric. I should use the mean to describe the measure of center and the mean absolute deviation to describe the measure of spread.

Part B Use the chosen measures to describe the distribution.

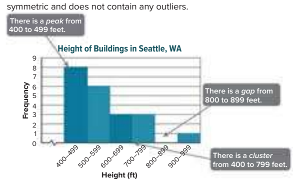
Sample answer: The data are centered around the mean of 4 hours. The spread of the data around the center is about 1.2 hours, which is the mean absolute deviation.

Go Online You can complete an Extra Example online.

Learn Interpret Histograms

You can also describe the distribution of histograms, including symmetry, clusters, gaps, and peaks. A **cluster** occurs when data values are grouped together. A **gap** occurs where there are no data values. A **peak** is the most frequently occurring value or interval of values in a data set.

Data were collected on the heights of some buildings in Seattle, Washington and are displayed in the histogram. The graph shows an example of a peak, a gap, and a cluster. This distribution is not symmetric and does not contain any outliers.



Talk About It! Use the histogram to describe the heights of the buildings in Seattle.

Sample answer: Most of the buildings are between 400–599 feet tall, and only one of the twenty–one buildings is taller than 799 feet. The measure of center that best describes the data is in the 500–599 feet range.

Interactive Presentation

Learn, Interpret Histograms, Slide 1 of 2

CLICK

On Slide 1, students select each button to see an example of a peak, a gap, and a cluster.

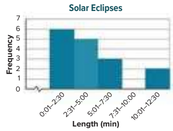
Think About It!
How many solar eclipses are represented in the data set?

16 eclipses

Example 2 Interpret Histograms

The histogram shows the duration, in minutes and seconds, of solar eclipses over a 10-year period.

Use clusters, gaps, peaks, outliers, and symmetry to describe the shape of the distribution.



Step 1 Identify any symmetry, clusters, and outliers. The distribution is not symmetric. There is a cluster from 0:01–7:30. There are no outliers.

Step 2 Identify any peaks.
There is a peak from 0:01–2:30.

Step 3 Identify any gaps.
There is a gap from 7:31–10:00.

Step 4 Describe the distribution.

Summarize the information you found.

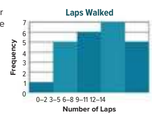
The distribution is not symmetric and does not contain any outliers. The data cluster around 1 second to 7 minutes and 30 seconds and have a peak at 1 second to 2 minutes and 30 seconds. There is a gap at 7 minutes and 31 seconds to 10 minutes.

Talk About It!
What can you infer about solar eclipses using the cluster of data values?

Sample answer: Most solar eclipses last for at most 7 minutes and 30 seconds.

Check

The histogram shows the number of laps each student walked while exercising. Use clusters, gaps, peaks, outliers, and symmetry to describe the shape of the distribution.



Show your work here

Sample answer: The distribution is not symmetric, and does not contain any outliers. The data cluster around 3 laps to 14 laps, and have a peak from 9 to 11 laps. There are no gaps in the distribution.

Go Online You can complete an Extra Example online.

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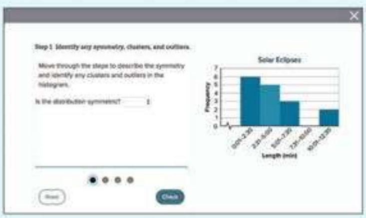
Interactive Presentation



Example 2, Interpret Histograms, Slide 2 of 7

CLICK

On Slides 2 and 3, students move through the steps to identify symmetry, clusters, and peaks.

CLICK

On Slides 4 and 5, students identify gaps and summarize the data.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Interpret Histograms

Objective

Students will describe the shape of a distribution, displayed in a histogram.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know the histogram is not symmetric? Sample answer: If I draw a line through the middle of the histogram, the left side does not match the right side.

OL How can you determine if there is a cluster? Sample answer: I can look for places where the data values are grouped together.

BL What do the clusters in the histogram represent? Sample answer: The clusters in this histogram represent most of the eclipses, and they last between one second and 7 minutes and 30 seconds.

SLIDE 3

AL How will you determine if there is a peak? Sample answer: I will check to see if there is a bar that is the tallest.

OL Does every histogram have a peak? Explain. no; Sample answer: If a histogram has all of the bars at the same height, there will not be a peak.

BL Why isn't the peak from 0:01 to 5:00? Sample answer: The bar for 0:01-2:30 is taller than the bar for 2:31-5:00, so it is the only peak.

SLIDE 4

AL How will you determine if there is a gap? Sample answer: I will check to see if there is an interval with no data.

OL What does the gap at 7:31—10:00 mean? The gap means that there were no eclipses that lasted between 7 minutes and 31 seconds and 10 minutes.

BL If there was one eclipse that lasted 16 minutes, would there be another gap? If yes, where would it be located? yes; Sample answer: If the intervals were extended, the category 12:31 to 15:00 would represent a gap.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Interpret Box Plots

Objective

Students will understand how to use the structure of a box plot to interpret the data that it represents.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of what it means for one whisker on a box plot to be longer than the other whisker. Students should reason that a longer whisker means the data are more spread out in that section, than in the shorter whisker.

Teaching Notes

SLIDE 1

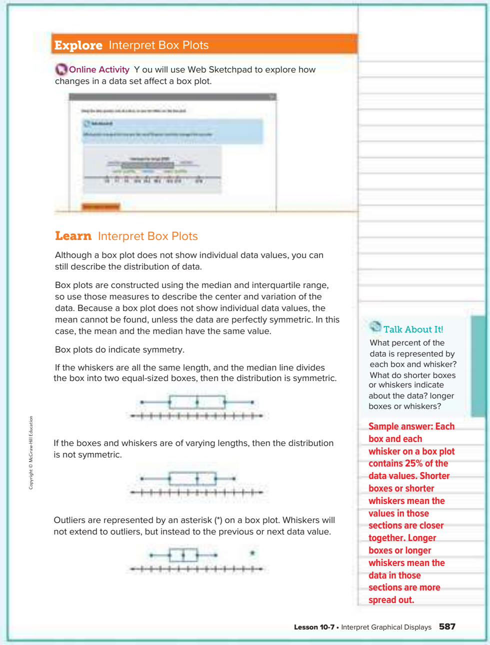
Remind students that a box plot is constructed using the lower extreme, first quartile, median, third quartile, and upper extreme. These values separate the data into quartiles, so each section of a box plot represents 25% of the data. Some students think that a longer box or a longer whisker mean that there are more data values in that section. Remind students that each section contains the same number of data values. A longer section indicates that the data within that section are more spread out.

Talk About It!

SLIDE 2

Mathematical Discourse

What percent of the data is represented by each box and whisker? What do shorter boxes or whiskers indicate about the data? longer boxes or whiskers? Sample answer: Each box and each whisker on a box plot contains 25% of the data values. Shorter boxes or shorter whiskers mean the values in those sections are closer together. Longer boxes or longer whiskers mean the data in those sections are more spread out.



Explore Interpret Box Plots

Online Activity You will use Web Sketchpad to explore how changes in a data set affect a box plot.

Learn Interpret Box Plots

Although a box plot does not show individual data values, you can still describe the distribution of data.

Box plots are constructed using the median and interquartile range, so use those measures to describe the center and variation of the data. Because a box plot does not show individual data values, the mean cannot be found, unless the data are perfectly symmetric. In this case, the mean and the median have the same value.

Box plots do indicate symmetry.

If the whiskers are all the same length, and the median line divides the box into two equal-sized boxes, then the distribution is symmetric.

If the boxes and whiskers are of varying lengths, then the distribution is not symmetric.

Outliers are represented by an asterisk (*) on a box plot. Whiskers will not extend to outliers, but instead to the previous or next data value.

Talk About It!

What percent of the data is represented by each box and whisker? What do shorter boxes or whiskers indicate about the data? longer boxes or whiskers?

Sample answer: Each box and each whisker on a box plot contains 25% of the data values. Shorter boxes or shorter whiskers mean the values in those sections are closer together. Longer boxes or longer whiskers mean the data in those sections are more spread out.

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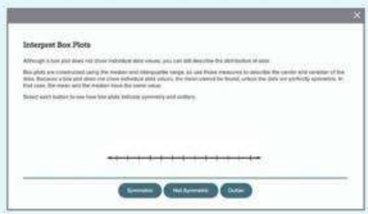
Lesson 10-7 • Interpret Graphical Displays 587

Interactive Presentation



Learn, Interpret Box Plots, Slide 1 of 2

CLICK

On Slide 1, students select the buttons to learn about symmetry in box plots.

CLICK

On Slide 2, students move through the steps to learn more about the structure of box plots.

Explore Interpret Box Plots

Objective

Students will use Web Sketchpad to explore how changes in a data set affect a box plot.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will explore how changing data values affects the different parts of a box plot.

Inquiry Question

How does a box plot reflect changes in a data set? Sample answer: The whiskers and the boxes can change lengths depending on the changes in the values.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

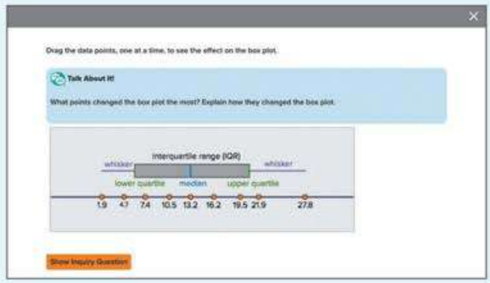
Mathematical Discourse

What points changed the box plot the most? Explain how they changed the box plot. Sample answer: The fifth largest data point directly affects the median. The minimum and maximum values directly affect the whiskers. The second and third largest, and the seventh and eighth largest indirectly affect the whiskers by changing the values of the quartiles.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 10

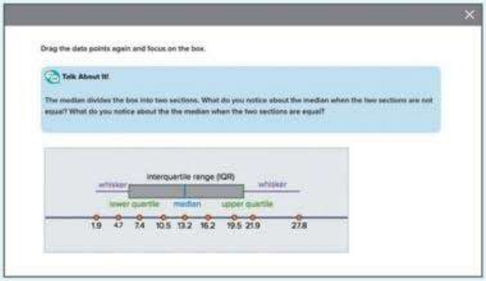


Explore, Slide 3 of 10

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how changes in a data set affect a box plot.

Interactive Presentation



Explore, Slide 7 of 10

TYPE

On Slide 10, students respond to the Inquiry Question and view a sample answer.

Explore Interpret Box Plots *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and deepen their understanding about the box plots and observe how changing the data values in a set will impact the box plot.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 7 are shown.

Talk About It!

SLIDE 7

Mathematical Discourse

The median divides the box into two sections. What do you notice about the median when the two sections are not equal? What do you notice about the median when the two sections are equal? Sample answer: The two sections of the box are unequal when the median is closer to one quartile than the other. The two sections of the box are equal when the distance between the median and the lower quartile is equal to the distance between the median and the upper quartile.

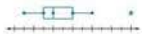
Think About It!
What are the key parts of the box plot you will need to examine?

See students' responses.

Example 3 Interpret Box Plots

The box plot shows the daily attendance at a fitness club.

Fitness Club Attendance



Describe the distribution of the data, including any symmetry, outliers, measures of center , and measures of variation.

The distribution is not symmetric.

The data contain an outlier, 110 people, indicated by the asterisk.

The median is 70 people. This means that for half of the days, the daily attendance at the fitness club was below 70, and for half of the days, the daily attendance was above 70.

The interquartile range is 80–65, or 15. This means that the middle 50% of the data vary by 15.

The left box is the shortest. This means that 25% of the data is between 65 and 70 people, and these data values are closer together than the data values in the other box or whiskers.

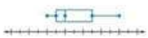
Talk About It!
What does the shape of the box plot tell you about the attendance at the fitness club?

Sample answer: On most days you can expect 65–80 guests. Sometimes the club will be busier, but you shouldn't expect to see more than 90 guests in one day and there has never been fewer than 55 daily guests. The measure of center that best describes the attendance is 70 people.

Check

The average gas mileage for various sedans is shown in the box plot. Describe the distribution of the data, including any symmetry, outliers, measures of center, and measures of variation.

Average Gas Mileage



Show your work here

The distribution is not symmetric.

The data do not contain any outliers.

The median is 27 mpg. Half of the sedans had a gas mileage greater than 27 mpg, and half had a gas mileage less than 27 mpg.

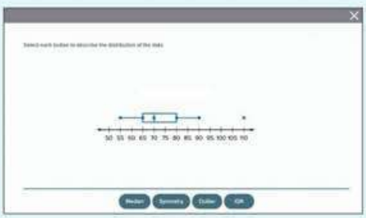
The interquartile range is 8. This means that the middle 50% of the data vary by 8 mpg.

The left box and left whisker are the shortest. This means that 50% of the data is between 23 and 27 mpg, and these data values are closer together than the data values in the other box or whisker.

Go Online You can complete an Extra Example online.

588 **Module 10** • Statistical Measures and Displays

Interactive Presentation



Example 3, Interpret Box Plots, Slide 2 of 4

CLICK

On Slide 2, students find the median, range, and interquartile range.

TYPE

On Slide 2, students describe the data.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Interpret Box Plots

Objective

Students will use the median and measures of variation to describe a data set represented by a box plot.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to describe the data set using clear and precise mathematical language, including what the lengths of the boxes and whiskers indicate about the data.

As students discuss the *Talk About It!* question on Slide 3, encourage them to clearly explain what the box plot tells them about the attendance at the fitness club.

7 Look for and Make Use of Structure Encourage students to study the structure of the box plot to determine the median, range, and interquartile range.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the lengths of the whiskers? Sample answer: The whiskers are approximately the same length.

OL What does it mean when the lengths of the whiskers are the same? Sample answer: The lengths of the two whiskers indicate that they are similarly distributed.

OL Why is the outlier included when finding the range? Sample answer: The outlier is still part of the data set. The range is a measure of variation, so it is important to know that the range is 55, including the outlier, and not 35, without the outlier.

BL How could you check that 110 is definitely an outlier? Sample answer: I could use the interquartile range to make sure that 110 is an outlier.

BL Make a conjecture about the attendance of the fitness club on a daily basis. Sample answer: The club usually has between 65 and 80 members that attend each day.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Travel

Objective

Students will come up with their own strategy to solve an application problem involving travel distances of a volleyball team.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

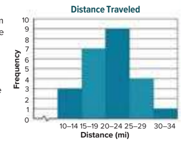
- How will you use the frequency data for the bar labeled 20–24?
- How can you determine the total number of times the team traveled?
- Notice which number is the part and which is the whole. How can you express these as a fraction in order to find the percent of games for which the team traveled 20–24 miles?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Travel

The histogram shows the distances a volleyball team travels to their games. One player claimed that because the peak of the distribution is from 20-24 miles, that the team traveled 20-24 miles more than 50% of the time. Is the player correct?



1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. Y ou may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

no; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

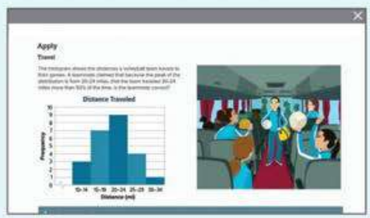
See students' arguments.

Talk About It!

Give an example of how the data may have been gathered. Could this have affected the results? Explain your reasoning.

Sample answer: One way to gather the information would be to survey the members of the team. This could have affected the results. If the students all live in a rural area they likely traveled farther to their games. If the students live in an urban area, most teams they played were probably closer.

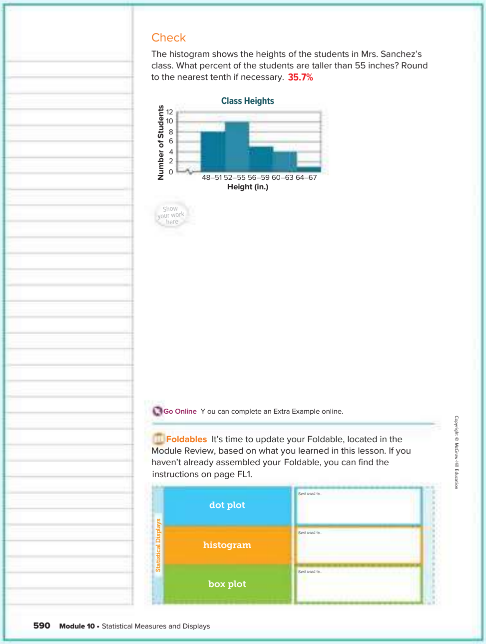
Interactive Presentation



Apply, Travel

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

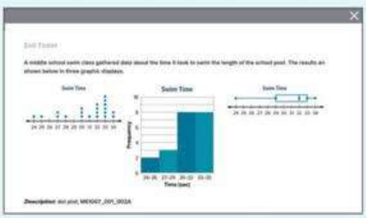
The histogram shows the heights of the students in Mrs. Sanchez's class. What percent of the students are taller than 55 inches? Round to the nearest tenth if necessary. 35.7%

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

590 Module 10 • Statistical Measures and Displays

Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of the measures of center and variation that can be used to describe different data displays. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

Why is data collected and analyzed and how can it be displayed?

In this lesson, students learned how to interpret data represented in dot plots, histograms, and box plots. Encourage them to work with a partner to compare and contrast how they can interpret these displays. For example, they may see they can find the median from a dot plot or a box plot, but not a histogram. They can find the mean from a dot plot, but not from a histogram or box plot.

Exit Ticket

Refer to the Exit Ticket slide. Choose the appropriate measure of center and variation and use the measures to describe the data set. Sample answer: The median is 32 and the interquartile range is 4. This means that the time to swim the length of the school pool is centered on 32 seconds. The interquartile range means the spread of the data around the center is about 4 seconds.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 1–7 odd, 8–11
- Extension: Select an Appropriate Display
- ALEKS Graphs of Data

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–4, 7–9
- Extension: Select an Appropriate Display
- Personal Tutor
- Extra Examples 1–3
- ALEKS Collecting Data

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- ArriveMATH Take Another Look
- ALEKS Collecting Data

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

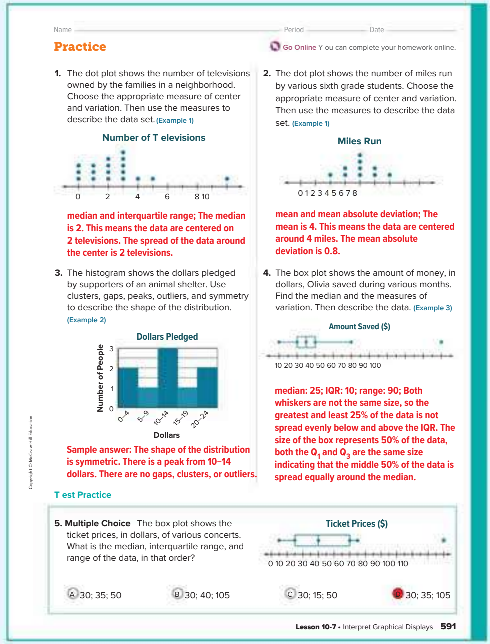
The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	choose the appropriate measure of center and variation to describe a data set represented by a dot plot	1, 2
1	describe the shape of a distribution, displayed in a histogram	3
1	use the median and measures of variation to describe a data set represented by a box plot	4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving interpreting graphical displays	6, 7
3	higher-order and critical thinking skills	8–11


Name Period Date

Practice

Go Online You can complete your homework online.

1. The dot plot shows the number of televisions owned by the families in a neighborhood. Choose the appropriate measure of center and variation. Then use the measures to describe the data set. (Example 1)

median and interquartile range; The median is 2. This means the data are centered on 2 televisions. The spread of the data around the center is 2 televisions.

2. The dot plot shows the number of miles run by various sixth grade students. Choose the appropriate measure of center and variation. Then use the measures to describe the data set. (Example 1)

mean and mean absolute deviation; The mean is 4. This means the data are centered around 4 miles. The mean absolute deviation is 0.8.

3. The histogram shows the dollars pledged by supporters of an animal shelter. Use clusters, gaps, peaks, outliers, and symmetry to describe the shape of the distribution. (Example 2)

Sample answer: The shape of the distribution is symmetric. There is a peak from 10–14 dollars. There are no gaps, clusters, or outliers.

4. The box plot shows the amount of money, in dollars, Olivia saved during various months. Find the median and the measures of variation. Then describe the data. (Example 3)

median: 25; IQR: 10; range: 90; Both whiskers are not the same size, so the greatest and least 25% of the data is not spread evenly below and above the IQR. The size of the box represents 50% of the data, both the Q_1 and Q_3 are the same size indicating that the middle 50% of the data is spread equally around the median.

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Test Practice

5. **Multiple Choice** The box plot shows the ticket prices, in dollars, of various concerts. What is the median, interquartile range, and range of the data, in that order?

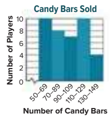
Ⓐ 30; 35; 50 Ⓑ 30; 40; 105 Ⓒ 30; 15; 50 Ⓓ 30; 35; 105

Lesson 10-7 • Interpret Graphical Displays 591

Apply *indicates multi-step problem

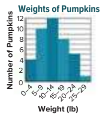
*6. The histogram shows the number of candy bars each player on a football team sold. One player claimed that more than 50% of the players sold 90 or more candy bars. Is the player correct? Write an argument that can be used to defend your solution.

yes; Sample answer: There were 21 players that sold 90 or more candy bars, out of 39 total players. Since $\frac{21}{39} \approx 54\%$, which is greater than 50%, the player was correct.



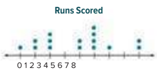
*7. The histogram shows the weights of pumpkins picked by students on a pumpkin farm. One student claimed that more than 25% of the pumpkins picked weighed 20 pounds or more. Is the student correct? Write an argument that can be used to defend your solution.

no; Sample answer: There were 6 pumpkins that weighed 20 pounds or more, out of 40 total pumpkins picked. $\frac{6}{40} = 15\%$, which is less than 25%, so the student was not correct.



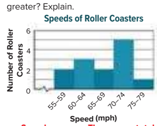
Higher-Order Thinking Problems

8. **Be Precise** The dot plot shows the number of runs scored by a baseball team for last season. Use clusters, gaps, peaks, outliers, and symmetry to describe the shape of the distribution.



Sample answer: The shape is not symmetric. There are gaps from 2–4 and 6–8. There is a peak at 5. There are clusters from 0–2 and 4–6. There are no outliers.

9. **Justify Conclusions** According to the histogram, do more than 50% of the roller coasters have a speed of 70 mph or greater? Explain.



no; Sample answer: There are a total of 13 roller coasters. There are 6 roller coasters that have speeds 70 mph or greater. $\frac{6}{13}$ is about 46.2%. 46.2% is less than 50%.

10. **Create** Draw a dot plot that is not symmetric.

Sample answer:

11. **Persevere with Problems** If a box plot's distribution is symmetric, which measure of center and measures of spread are most appropriate to use?

mean; mean absolute deviation

Teaching the Mathematical Practices

6 Attend to Precision In Exercise 8, students describe the shape of the distribution shown in the dot plot. Encourage students to use precision when explaining the data displayed.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students determine if more than 50% of the roller coasters have a speed of 70 mph or greater. Encourage students to use information from the graph to support their answer.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 11, students determine which measure of center and measures of spread are the most appropriate to use. Encourage students to check each measure of center and spread before determining an answer.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 7 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Solve the problem another way.

Use with Exercise 9 Have students work in groups of 3–4. After completing Exercise 9, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include a review of dot plots, histograms, and box plots. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

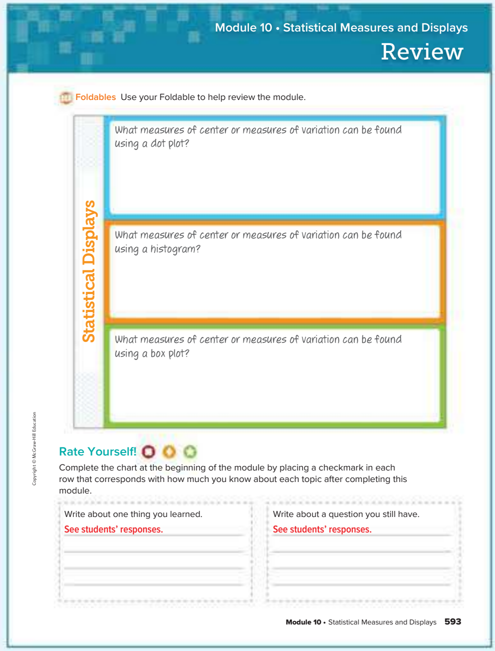
Assessment Resources

Put It All Together 1: Lessons 10-1, 10-2, and 10-3
Put It All Together 2: Lessons 10-2, 10-3, 10-4, 10-5, 10-6, and 10-7
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Statistics and Probability**.

- Statistical Questions and Frequency Distributions
- Dot Plots
- Measure of Center
- Measure of Variability
- Measure of Center and Variability
- Histograms
- Box Plots


Module 10 • Statistical Measures and Displays

Review

Foldables Use your Foldable to help review the module.

Statistical Displays

What measures of center or measures of variation can be found using a dot plot?

What measures of center or measures of variation can be found using a histogram?

What measures of center or measures of variation can be found using a box plot?

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.	Write about a question you still have.
See students' responses.	See students' responses.

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Module 10 • Statistical Measures and Displays 593

Reflect on the Module

Use what you learned about statistical measures to complete the graphic organizer.

Essential Question

Why is data collected and analyzed and how can it be displayed?

How are the mean and median helpful in describing data?		
	Mean	**Median**
Definition	The sum of the numbers in a data set divided by the number of data values.	The middle value when a list of numerical values is ordered from least to greatest.
When is it appropriate to use?	when there are no extreme values	in a large data set with extreme values
How does an outlier affect it?	can alter the mean significantly	usually minimal, if at all

How can data be displayed?			
	Dot Plot	**Histogram**	**Box Plot**
Definition	A visual display of a distribution of data values where each data value is shown as a dot above a number line.	A type of bar graph used to display numerical data that have been organized into equal intervals.	A display that uses a number line to show the distribution by using the median, quartiles, and extreme values.
Explain how to describe the data.	If the left side of a distribution looks like the right side, then the distribution is symmetric. If there is an outlier, the distribution is usually not symmetric.	Data that are evenly distributed between the left side and the right side have a symmetric distribution. A cluster occurs when data values are grouped. A gap is where there are no data. A peak is the most frequently occurring value.	Use the length of the box and whiskers to describe the characteristics of the data set.

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

Why is data collected and analyzed and how can it be displayed?
See students' graphic organizers.

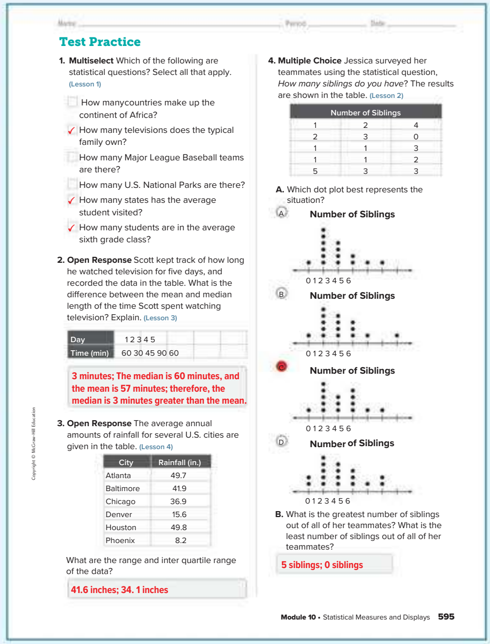
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–8 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	4, 6
Multiselect	Multiple answers may be correct. Students must select all correct answers.	1, 7
Table Item	Students complete a table by correctly classifying the information.	5
Open Response	Students construct their own response in the area provided.	2, 3, 8

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
6.SP.A.1	10-1	1
6.SP.A.2	10-4, 10-7	2, 3, 5
6.SP.A.3	10-3, 10-4, 10-5, 10-6, 10-7	2, 3, 5, 6
6.SP.B.4	10-2, 10-3, 10-4, 10-6, 10-7	4, 5, 8
6.SP.B.5	10-2, 10-3, 10-4, 10-5, 10-6, 10-7	2–8
6.SP.B.5.A	10-2, 10-3, 10-5, 10-7	4, 8
6.SP.B.5.B	10-3, 10-5, 10-7	2, 6
6.SP.B.5.C	10-3, 10-4, 10-5, 10-6, 10-7	2, 3, 5–7
6.SP.B.5.D	10-6, 10-7	8



Name ______ Period ______ Date ______

Test Practice

1. **Multiselect** Which of the following are statistical questions? Select all that apply. (Lesson 1)

- [ ] How manycountries make up the continent of Africa?
- [x] How many televisions does the typical family own?
- [ ] How many Major League Baseball teams are there?
- [ ] How many U.S. National Parks are there?
- [x] How many states has the average student visited?
- [x] How many students are in the average sixth grade class?

2. **Open Response** Scott kept track of how long he watched television for five days, and recorded the data in the table. What is the difference between the mean and median length of the time Scott spent watching television? Explain. (Lesson 3)

Day	1	2	3	4	5
Time (min)	60	30	45	90	60

3 minutes; The median is 60 minutes, and the mean is 57 minutes; therefore, the median is 3 minutes greater than the mean.

3. **Open Response** The average annual amounts of rainfall for several U.S. cities are given in the table. (Lesson 4)

City	Rainfall (in.)
Atlanta	49.7
Baltimore	41.9
Chicago	36.9
Denver	15.6
Houston	49.8
Phoenix	8.2

What are the range and inter quartile range of the data?

41.6 inches; 34. 1 inches

4. **Multiple Choice** Jessica surveyed her teammates using the statistical question, *How many siblings do you have*? The results are shown in the table. (Lesson 2)

Number of Siblings		
1	2	4
2	3	0
1	1	3
1	1	2
5	3	3

A. Which dot plot best represents the situation?

(A) Number of Siblings
0 1 2 3 4 5 6

(B) Number of Siblings
0 1 2 3 4 5 6

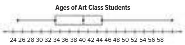
(C) Number of Siblings
0 1 2 3 4 5 6

(D) Number of Siblings
0 1 2 3 4 5 6

B. What is the greatest number of siblings out of all of her teammates? What is the least number of siblings out of all of her teammates?

5 siblings; 0 siblings

5. Table Item The ages of the current students attending an art class at a local community center are shown in the box plot. Consider the parts of the box plot and indicate which of the parts are correctly named. (Lesson 4)



	Correct	Incorrect
Lower Extreme = 24		X
Median = 39	X	
Q_1 = 33	X	
Q_3 = 44		X
Upper Extreme = 58	X	

6. Multiple Choice The table shows the top ten test scores of the students in Ms. Schneider's science class. (Lesson 5)

T est Scores		
102 100 95	93	88
96 100 99	90	97

A. Which of the following represents the mean absolute deviation of the data?

- (A) 3.2 points
- (B) 3.4 points
- (C) 3.6 points
- (D) 4.1 points

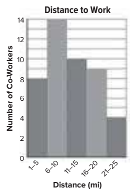
B. Describe what the mean absolute deviation represents.

Sample answer: The average distance each test score is from the mean is 3.6 points.

7 .Multiselect The heights, in feet, of various trees in the park are 32, 10, 70, 40, 34, 44, and 36. Identify any outliers in the data set. Select all that apply. (Lesson 6)

- [x] 10 feet
- [ ] 34 feet
- [ ] 36 feet
- [ ] 40 feet
- [x] 70 feet

8. Open Response The histogram shows the distances Jerome's co-workers have to commute to work each morning. What percent of his co-workers travel more than 10 miles to work? Round to the nearest percent. (Lesson 7)



51%

596 Module 10 • Statistical Measures and Displays

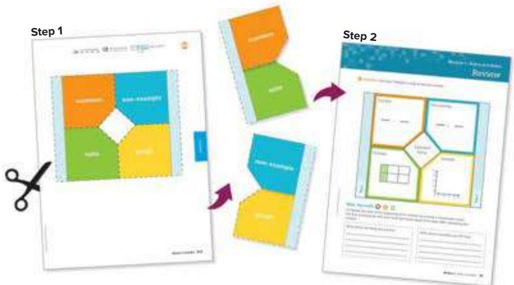
Foldables Study Organizers

What Are Foldables and How Do I Create Them?

Foldables are three-dimensional graphic organizers that help you create study guides for each module in your book.

Step 1 Go to the back of your book to find the Foldable for the module you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Module Review at the end of the module you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.



How Will I Know When to Use My Foldable?

You will be directed to work on your Foldable at the end of selected lessons. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the module test.

How Do I Complete My Foldable?

No two Foldables in your book will look alike. However , some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

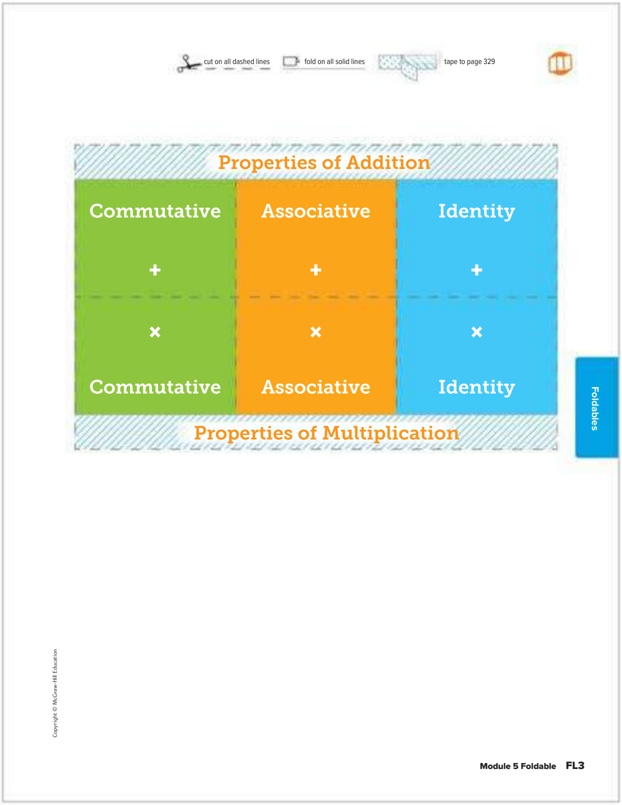
Instructions and What They Mean

Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. Y ou may use one already in the text or you can make up your own.
Example	Write an example about the concept. Y ou may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. Y ou may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. Dinah is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

FL2 Foldables Study Organizers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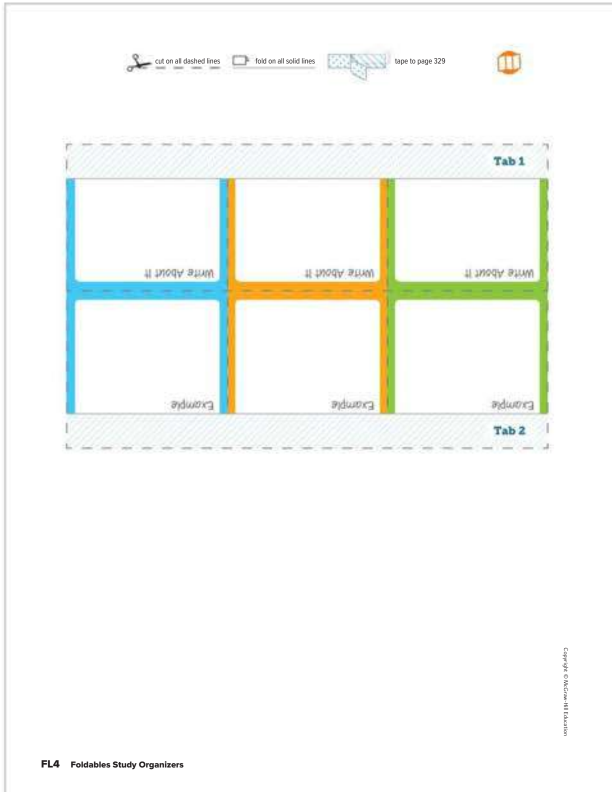
Properties of Addition

Commutative	Associative	Identity
+	+	+
×	×	×
Commutative	Associative	Identity

Properties of Multiplication

Foldables

Foldables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Tab 1
Write About It
Write About It
Write About It
Example
Example
Example
Tab 2
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FL4 Foldables Study Organizers

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equations

Models   Symbols

addition (+)

Models   Symbols

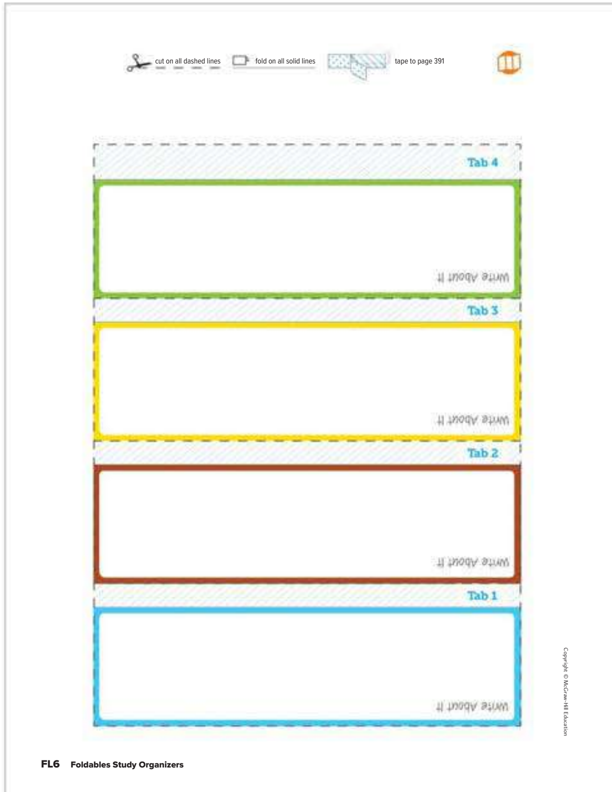
subtraction (–)

Models   Symbols

multiplication (×)

Foldables

Foldables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Write About It
Tab 3
Write About It
Tab 2
Write About It
Tab 1
Write About It
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FL6 Foldables Study Organizers

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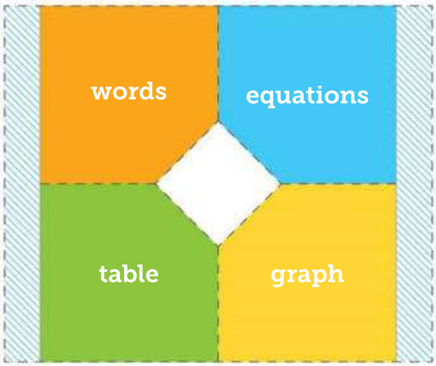
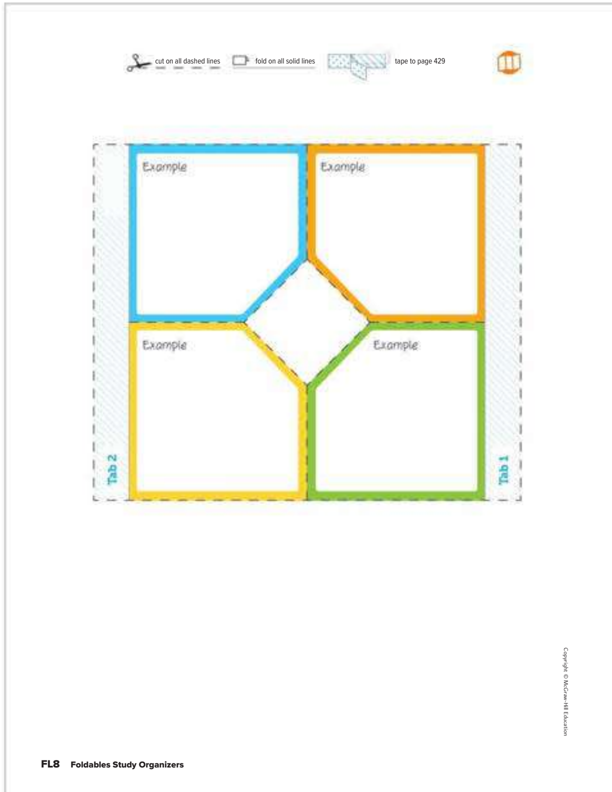


Foldables

Foldables

Module 7 Foldable FL7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Example
Example
Example
Example
Tab 2
Tab 1
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FL8 Foldables Study Organizers

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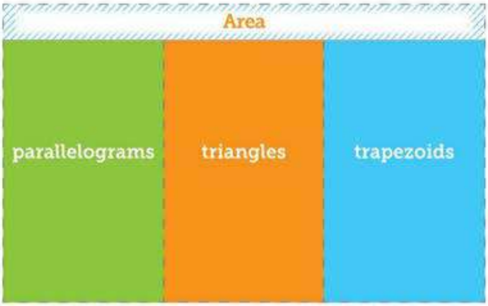
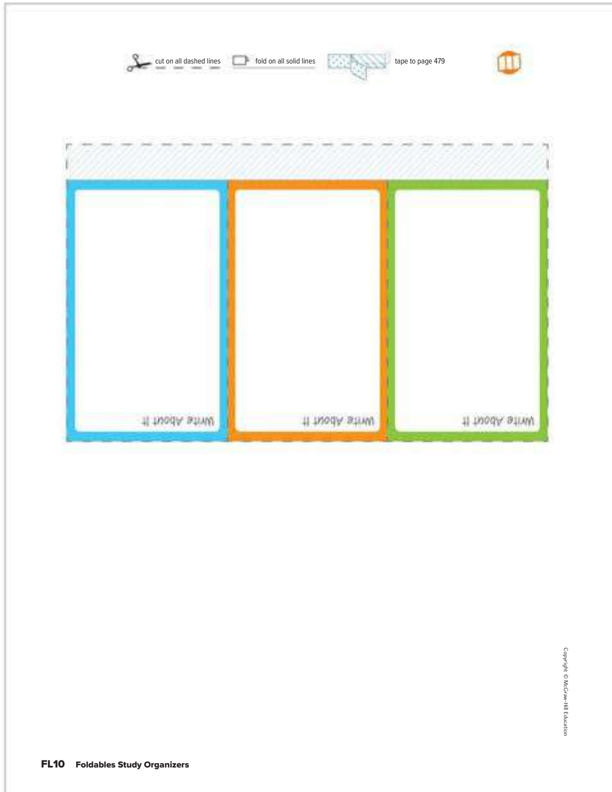


Foldables

Foldables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Write About It
Write About It
Write About It
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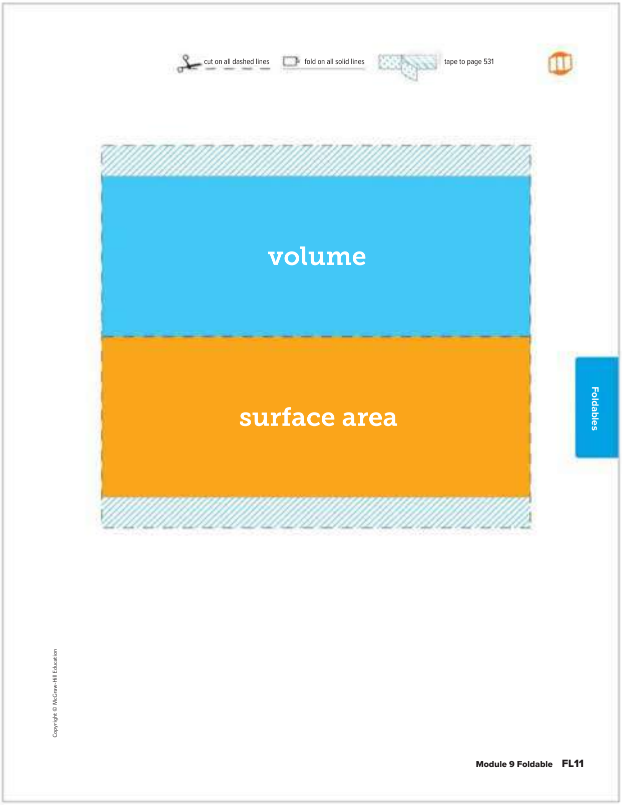
FL10 Foldables Study Organizers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volume

surface area

Foldables

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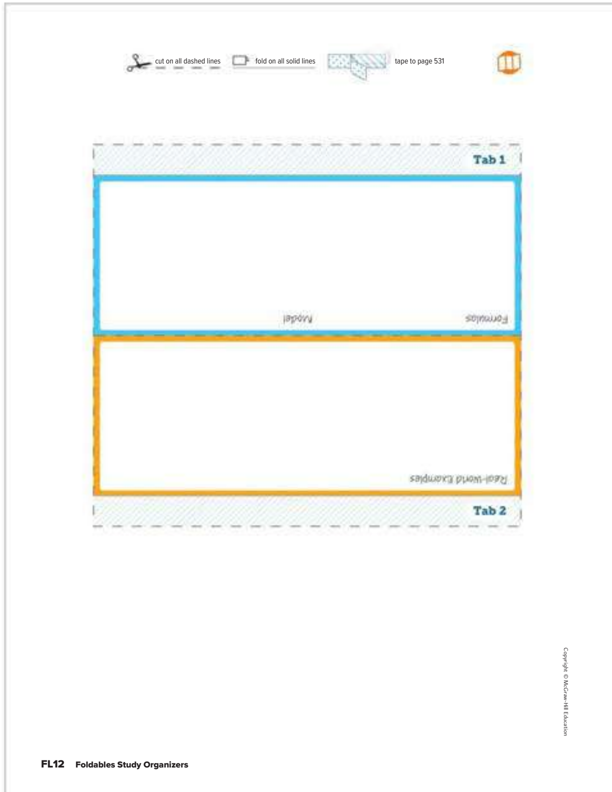
Module 9 Foldable FL11

Foldables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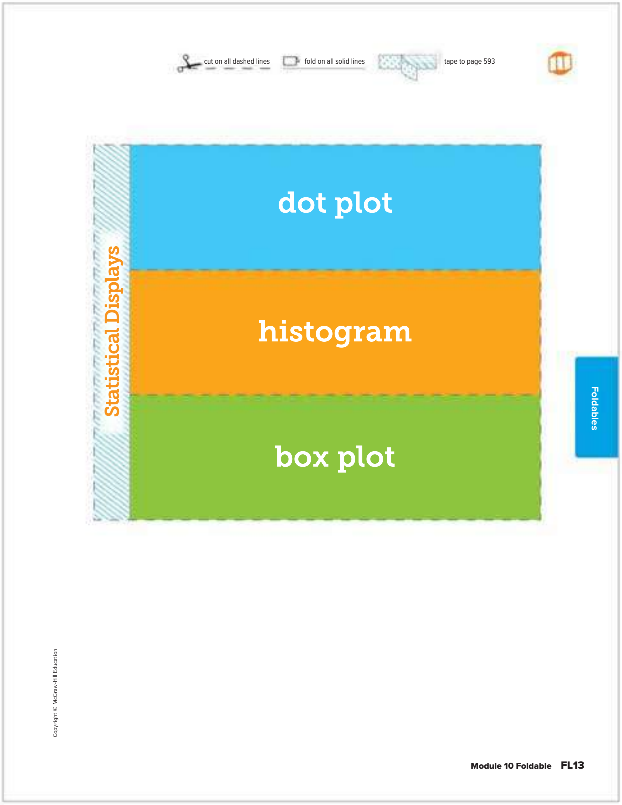
Tab 1

Model
Formulas

Real-World Examples

Tab 2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Statistical Displays

dot plot

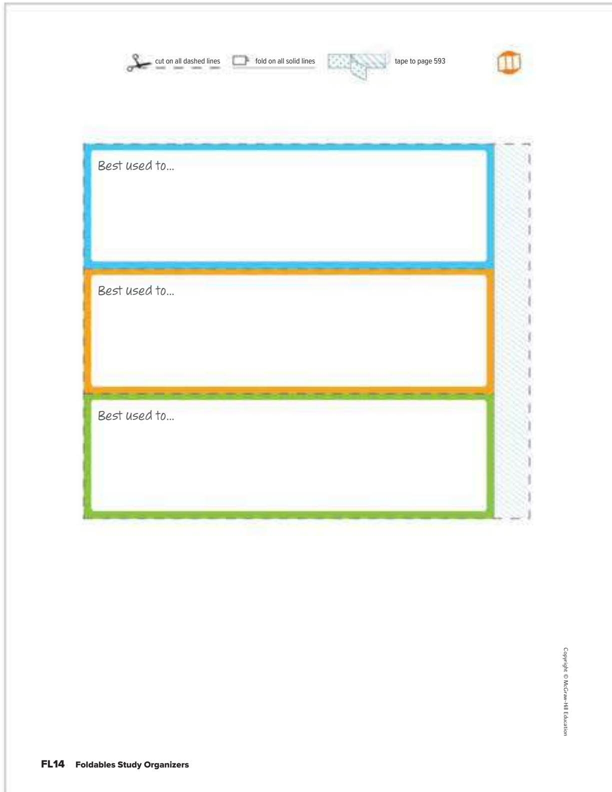
histogram

box plot

Foldables

Module 10 Foldable FL13

Foldables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Best used to...

Best used to...

Best used to...

Glossary

The eGlossary contains words and definitions in the following 14 languages:

Arabic	English	Hmong	Russian	Urdu
Bengali	French	Korean	Spanish	Vietnamese
Brazilian Portuguese	Haitian Creole	Mandarin	Tagalog	

English	Español
A	
absolute value (Lesson 4-2) The distance between a number and zero on a number line.	valor absoluto Distancia entre un número y cero en la recta numérica.
Addition Property of Equality (Lesson 6-8) If you add the same number to each side of an equation, the two sides remain equal.	propiedad de adición de la igualdad Si sumas el mismo número a ambos lados de una ecuación, los dos lados permanecen iguales.
algebra (Lesson 5-3) A mathematical language of symbols, including variables.	álgebra Lenguaje matemático que usa símbolos, incluyendo variables.
algebraic expression (Lesson 5-3) A combination of variables, numbers, and at least one operation.	expresión algebraica Combinación de variables, números y, por lo menos, una operación.
analyze (Lesson 10-1) To use observations to describe and compare data.	analizar Usar observaciones para describir y comparar datos.
area (Lesson 8-1) The measure of the interior surface of a two-dimensional figure.	área La medida de la superficie interior d una figura bidimensional.
Associative Property (Lesson 5-7) The way in which numbers are grouped does not change the sum or product.	propiedad asociativa La forma en que se agrupan tres números al sumarlos o multiplicarlos no altera su suma o producto.
average (Lesson 10-3) The sum of two or more quantities divided by the number of quantities; the mean.	promedio La suma de dos o más cantidades dividida entre el número de cantidades; la media.
B	
base (Lesson 8-1) Any side of a parallelogram or any side of a triangle.	base Cualquier lado de un paralelogramo o cualquier lado de un triángulo.
base (Lesson 9-1) One of the two parallel congruent faces of a prism.	base Una de las dos caras paralelas congruentes de un prisma.

base (Lesson 5-1) In a power, the number used as a factor. In 10^3, the base is 10. That is, $10^3 = 10 \times 10 \times 10$.

base En una potencia, el número usado como factor. En 10^3, la base es 10. Es decir, $10^3 = 10 \times 10 \times 10$.

bases (Lesson 8-3) The bases of a trapezoid are the two parallel sides.

bases Las bases de un trapecio son los dos lados paralelos.

benchmark percent (Lesson 2-5) A common percent used when estimating part of a whole.

porcentaje de referencia Porcentaje común utilizado para estimar parte de un todo.

box plot (Lesson 10-4) A diagram that is constructed using five values.

diagrama de caja Diagrama que se construye usando cinco valores.

C

cluster (Lesson 10-7) Data that are grouped closely together.

agrupamiento Conjunto de datos que se agrupan.

coefficient (Lesson 5-3) The numerical factor of a term that contains a variable.

coeficiente El factor numérico de un término que contiene una variable.

common factor (Lesson 5-5) A number that is a factor of two or more numbers.

factor común Un número que es un factor de dos o más números.

Commutative Property (Lesson 5-7) The order in which numbers are added or multiplied does not change the sum or product.

propiedad commutativa La forma en que se suman o multiplican dos números no altera su suma o producto.

congruent (Lesson 8-2) Having the same measure.

congruente Ques tienen la misma medida.

congruent figures (Lesson 8-2) Figures that have the same size and same shape; corresponding sides and angles have equal measures.

figuras congruentes Figuras que tienen el mismo tamaño y la misma forma; los lados y los ángulos correspondientes con igual medida.

constant (Lesson 5-3) A term without a variable.

constante Un término sin una variable.

coordinate plane (Lesson 1-3) A plane in which a horizontal number line and a vertical number line intersect at their zero points.

plano de coordenadas Plano en que una recta numérica horizontal y una recta numérica vertical se intersecan en sus puntos cero.

cubic units (Lesson 9-1) Used to measure volume. Tells the number of cubes of a given size it will take to fill a three-dimensional figure.

unidades cúbicas Se usan para medir el volumen. Indican el número de cubos de cierto tamaño que se necesitan para llenar una figura tridimensional.

D

data (Lesson 10-1) Information, often numerical, which is gathered for statistical purposes.

datos Información, con frecuencia numérica, que se recoge con fines estadísticos.

defining the variable (Lesson 5-3) Choosing a variable and deciding what the variable represents.

definir la variable Elegir una variable y decidir lo que representa.

dependent variable (Lesson 7-1) The variable in a relation with a value that depends on the value of the independent variable.

variable dependiente La variable en una relación cuyo valor depende del valor de la variable independiente.

distribution (Lesson 10-7) The arrangement of data values.

distribución El arreglo de valores de datos.

Distributive Property (Lesson 5-6) To multiply a sum by a number, multiply each addend by the number outside the parentheses.

propiedad distributiva Para multiplicar una suma por un número, multiplica cada sumando por el número fuera de los paréntesis.

dividend (Lesson 3-1) The number that is divided in a division problem.

dividendo El número que se divide en un problema de división.

Division Property of Equality (Lesson 6-4) If you divide each side of an equation by the same nonzero number, the two sides remain equal.

propiedad de igualdad de la división Si divides ambos lados de una ecuación entre el mismo número no nulo, los lados permanecen iguales.

divisor (Lesson 3-1) The number used to divide another number in a division problem.

divisor El número utilizado para dividir otro número en un problema de división.

double number line (Lesson 1-2) A double number line consists of two number lines, in which the coordinated quantities are equivalent ratios.

línea doble Una línea numérica doble consta de dos líneas numéricas, en las cuales las cantidades coordinadas son proporciones equivalentes.

dot plot (Lesson 10-2) A diagram that shows the frequency of data on a number line. Also known as a line plot.

diagrama de puntos Diagrama que muestra la frecuencia de los datos sobre una recta numérica.

E

equals sign (Lesson 6-1) A symbol of equality, =.

signo de igualdad Símbolo que indica igualdad, =.

equation (Lesson 6-1) A mathematical sentence showing two expressions are equal. An equation contains an equals sign, =.

ecuación Enunciado matemático que muestra que dos expresiones son iguales. Una ecuación contiene el signo de igualdad, =.

equivalent expressions (Lesson 5-7) Expressions that have the same value, regardless of the values of the variable(s).

expresiones equivalentes Expresiones que poseen el mismo valor, sin importer los valores de la(s) variable(s).

equivalent ratios (Lesson 1-2) Ratios that express the same relationship between two quantities.

razones equivalentes Razones que expresan la misma relación entre dos cantidades.

evaluate (Lesson 5-2) To find the value of an algebraic expression by replacing variables with numbers.

evaluar Calcular el valor de una expresión algebraica sustituyendo las variables por número.

exponent (Lesson 5-1) In a power, the number that tells how many times the base is used as a factor. In 5^3, the exponent is 3. That is, $5^3 = 5 \times 5 \times 5$.

exponente En una potencia, el número que indica las veces que la base se usa como factor. En 5^3, el exponente es 3. Es decir, $5^3 = 5 \times 5 \times 5$.

F

face (Lesson 9-1) A flat surface of a prism or pyramid.

cara Una superficie plana de un prisma o pirámide.

factoring the expression (Lesson 5-6) The process of writing numeric or algebraic expressions as a product of their factors.

factorizar la expresión El proceso de escribir expresiones numéricas o algebraicas como el producto de sus factores.

first quartile (Lesson 10-4) The first quartile is the median of the data values less than the median.

primer cuartil El primer cuartil es la mediana de los valores menores que la mediana.

G

gap (Lesson 10-7) An empty space or interval in a set of data.

laguna Espacio o intervalo vacío en un conjunto de datos.

graph (Lesson 1-3) To place a dot on a number line, or on the coordinate plane at a point named by an ordered pair.

graficar Colocar una marca puntual en una línea numérica, o en el plano de coordenadas en el punto que corresponde a un par ordenado.

greatest common factor (GCF) (Lesson 5-5) The greatest of the common factors of two or more numbers.

máximo común divisor (MCD) El mayor de los factores comunes de dos o más números.

guess, check, and revise strategy (Lesson 6-1) A strategy used to solve a problem which involves narrowing in on the correct answer using educated guesses.

adivinar, comprobar y revisar la estrategia Una estrategia utilizada para resolver un problema que implica el estrechamiento en la respuesta correcta usando conjeturas educadas.

H

height (Lesson 8-1) The height of a parallelogram is the perpendicular distance between the base and its opposite side.

altura La altura de un paralelogramo es la distancia perpendicular entre la base y su lado opuesto.

height (Lesson 8-2) The height of a triangle is the perpendicular distance from the base to the opposite vertex.

altura La altura de un triángulo es la distancia perpendicular de la base al vértice opuesto.

height (Lesson 8-3) The height of a trapezoid is the perpendicular distance between the two bases.

altura La altura de un trapecio es la distancia perpendicular entre las dos bases.

histogram (Lesson 10-2) A type of bar graph used to display numerical data that have been organized into equal intervals.

histograma Tipo de gráfica de barras que se usa para exhibir datos que se han organizado en intervalos iguales.

I

Identity Properties (Lesson 5-7) Properties that state that the sum of any number and 0 equals the number and that the product of any number and 1 equals the number.

propiedades de identidad Propiedades que establecen que la suma de cualquier número y 0 es igual al número y que el producto de cualquier número y 1 es igual al número.

independent variable (Lesson 7-1) The variable in a relationship with a value that is subject to choice.

variable independiente Variable en una relación cuyo valor está sujeto a elección.

inequality (Lesson 6-6) A mathematical sentence indicating that two quantities are not equal.

desigualdad Enunciado matemático que indica que dos cantidades no son iguales.

integer (Lesson 4-1) Any number from the set {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...} where ... means *continues without end.*

entero Cualquier número del conjunto {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...} donde ... significa que *continúa sin fin.*

interquartile range (IQR) (Lesson 10-4) A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set.

rango intercuartil (RIQ) El rango intercuartil, una medida de la variación en un conjunto de datos numéricos, es la distancia entre el primer y el tercer cuartil del conjunto de datos.

interval (Lesson 10-2) The difference between successive values on a scale.

intervalo La diferencia entre valores sucesivos de una escala.

inverse operations (Lesson 6-2) Operations which undo each other. For example, addition and subtraction are inverse operations.

operaciones inversas Operaciones que se anulan mutuamente. La adición y la sustracción son operaciones inversas.

Inverse Property of Multiplication (Lesson 3-3) A property that states that the product of a number and its multiplicative inverse is 1.

propiedad inversa de la multiplicación Una propiedad que indica que el producto de un número y su inverso multiplicativo es 1.

L

lateral face (Lesson 9-4) Any face that is not a base.

cara lateral Cualquier superficie plana que no sea la base.

least common multiple (LCM) (Lesson 5-5) The smallest whole number greater than 0 that is a common multiple of each of two or more numbers.

mínimo común múltiplo (mcm) El menor número entero, mayor que 0, múltiplo común de dos o más números.

like terms (Lesson 5-3) Terms that contain the same variable(s) to the same power.

términos semejantes Términos que contienen la misma variable o variables elevadas a la misma potencia.

M

mean (Lesson 10-3) The sum of the numbers in a set of data divided by the number of pieces of data.

media La suma de los números en un conjunto de datos dividida entre el número total de datos.

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Glossary · Glosario

mean absolute deviation (MAD) (Lesson 10-5) A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.

desviación media absoluta (DMA) Una medida de variación en un conjunto de datos numéricos que se calcula sumando las distancias entre el valor de cada dato y la media, y luego dividiendo entre el número de valores.

measures of center (Lesson 10-3) Numbers that are used to describe the center of a data set. These measures include the mean and median.

medidas del centro Numéros que se usan para describir el centro de un conjunto de datos. Estas medidas incluyen la media, la mediana y la moda.

measures of variation (Lesson 10-4) A measure used to describe the distribution of data.

medidas de variación Medida usada para describir la distribución de los datos.

median (Lesson 10-3) A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list, or the mean of the two central values, if the list contains an even number of values.

mediana Una medida del centro en un conjunto de datos numéricos. La mediana de una lista de valores es el valor que aparece en el centro de una versión ordenada de la lista, o la media de los dos valores centrales si la lista contiene un número par de valores.

Multiplication Property of Equality (Lesson 6-5) If you multiply each side of an equation by the same nonzero number, the two sides remain equal.

propiedad de multiplicación de la igualdad Si multiplicas ambos lados de una ecuación por el mismo número no nulo, lo lados permanecen iguales.

multiplicative inverses (Lesson 3-3) Any two numbers that have a product of 1.

inversos multiplicativos Cualquier dos números que tengan un producto de 1.

N

negative integer (Lesson 4-1) A number that is less than zero. It is written with a − sign.

entero negativo Número que es menor que cero y se escribe con el signo −.

net (Lesson 9-2) A two-dimensional figure that can be used to build a three-dimensional figure.

red Figura bidimensional que sirve para hacer una figura tridimensional.

numerical expression (Lesson 5-2) A combination of numbers and operations.

expresión numérica Una combinación de números y operaciones.

O

opposites (Lesson 4-2) Two integers are opposites if they are represented on the number line by points that are the same distance from zero, but on opposite sides of zero. The sum of two opposites is zero.

opuestos Dos enteros son opuestos si, en la recta numérica, están representados por puntos que equidistan de cero, pero en direcciones opuestas. La suma de dos opuestos es cero.

order of operations (Lesson 5-2) The rules that tell which operation to perform first when more than one operation is used.

1. Simplify the expressions inside grouping symbols.
2. Find the value of all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

orden de las operaciones Reglas que establecen cuál operación debes realizar primero, cuando hay más de una operación involucrada.

1. Ejecuta todas las operaciones dentro de los símbolos de agrupamiento.
2. Evalúa todas las potencias.
3. Multiplica y divide en orden de izquierda a derecha.
4. Suma y resta en orden de izquierda a derecha.

ordered pair (Lesson 1-3) A pair of numbers used to locate a point on the coordinate plane. The ordered pair is written in the form (*x*-coordinate, *y*-coordinate).

par ordenado Par de números que se utiliza para ubicar un punto en un plano de coordenadas. Se escribe de la forma (coordenada *x*, coordenada *y*).

origin (Lesson 1-3) The point of intersection of the *x*-axis and *y*-axis on a coordinate plane.

origen Punto de intersección de los ejes axiales en un plano de coordenadas.

outlier (Lesson 10-6) A value that is much greater than or much less than the other values in a set of data.

valor atípico Dato que se encuentra muy separado de los otros valores en un conjunto de datos.

P

parallelogram (Lesson 8-1) A quadrilateral with opposite sides parallel and opposite sides congruent.

paralelogramo Cuadrilátero cuyos lados opuestos son paralelos y congruentes.

part-to-part ratio (Lesson 1-1) A ratio that compares one part of a group to another part of the same group.

proporción de parte a parte Una proporción que compara una parte de un grupo con otra parte del mismo grupo.

part-to-whole ratio (Lesson 1-1) A ratio that compares one part of a group to the whole group.

proporción de parte a total Una proporción que compara una parte de un grupo con todo el grupo.

peak (Lesson 10-7) The most frequently occurring value in a line plot.

pico El valor que ocurre con más frecuencia en un diagrama de puntos.

percent (Lesson 2-1) A ratio, or rate, that compares a number to 100.

por ciento Una relación, o tasa, que compara un número a 100.

positive integer (Lesson 4-1) A number that is greater than zero. It can be written with or without a + sign.

entero positivo Número que es mayor que cero y se puede escribir con o sin el signo +.

powers (Lesson 5-1) A number expressed using an exponent. The power 3^2 is read *three to the second power,* or *three squared.*

potncias Números que se expresan usando exponentes. La potencia 3^2 se lee *tres a la segunda potencia* o *tres al cuadrado*.

prism (Lesson 9-1) A three-dimensional figure with at least three rectangular lateral faces and top and bottom faces parallel.

prisma Figura tridimensional que tiene por lo menos tres caras laterales rectangulares y caras paralelas superior e inferior.

pyramid (Lesson 9-4) A three-dimensional figure with at least three triangular sides that meet at a common vertex and only one base that is a polygon.

pirámide Una figura de tres dimensiones con que es en un un polígono y tres o mas caras triangulares que se encuentran en un vértice común.

Q

quadrants (Lesson 4-5) The four regions in a coordinate plane separated by the *x*-axis and *y*-axis.

cuadrantes Las cuatro regiones de un plano de coordenadas separadas por el eje *x* y el eje *y*.

quartiles (Lesson 10-4) Values that divide a data set into four equal parts.

cuartiles Valores que dividen un conjunto de datos en cuatro partes iguales.

quotient (Lesson 3-1) The result when one number is divided by another.

cociente El resultado cuando un número es dividido por otro.

R

range (Lesson 10-4) The difference between the greatest number and the least number in a set of data.

rango La diferencia entre el número mayor y el número menor en un conjunto de datos.

rate (Lesson 1-7) A special kind of ratio in which the units are different.

tasa Un tipo especial de relación en el que las unidades son diferentes.

ratio (Lesson 1-1) A comparison between two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity.

razón Una comparación entre dos cantidades, en la que por cada *a* unidades de una cantidad, hay unidades *b* de otra cantidad.

ratio table (Lesson 1-2) A collection of equivalent ratios that are organized in a table.

table de razones Una colección de proporciones equivalentes que se organizan en una tabla.

rational number (Lesson 4-4) A number that can be written as a fraction.

número racional Número que se puede expresar como fracción.

reciprocals (Lesson 3-3) Any two numbers that have a product of 1. Since $\frac{5}{6} \times \frac{6}{5} = 1$, $\frac{5}{6}$ and $\frac{6}{5}$ are reciprocals.

recíproco Cualquier par de números cuyo producto es 1. Como $\frac{5}{6} \times \frac{6}{5} = 1$, $\frac{5}{6}$ y $\frac{6}{5}$ son recíprocos.

rectangular prism (Lesson 9-1) A prism that has rectangular bases.

prisma rectangular Una prisma que tiene bases rectangulares.

reflection (Lesson 4-6) The mirror image produced by flipping a figure over a line.

reflexión Transformación en la cual una figura se voltea sobre una recta. También se conoce como simetría de espejo.

regular polygon (Lesson 8-4) A polygon with all congruent sides and all congruent angles.

polígono regular Un polígono con todos los lados congruentes y todos los ángulos congruentes.

S

scaling (Lesson 1-2) The process of multiplying each quantity in a ratio by the same number to obtain equivalent ratios.

homotecia El proceso de multiplicar cada cantidad en una proporción por el mismo número para obtener relaciones equivalentes.

second quartile (Lesson 10-4) Another name for the median, or the center of a set of numerical data.

segundo cuartil Otro nombre para la mediana, o el centro de un conjunto de datos numéricos.

simplest form (Lesson 5-4) The status of an expression when it has no like terms and no parentheses.

forma más simple El estado de una expresión cuando no tiene términos iguales y no hay paréntesis.

slant height (Lesson 9-4) The height of each lateral face of a pyramid.

altura oblicua Altura de cada cara lateral de un pirámide.

solution (Lesson 6-1) The value of a variable that makes an equation true.

solución Valor de la variable de una ecuación que hace verdadera la ecuación.

solve (Lesson 6-1) To replace a variable with a value that results in a true sentence.

resolver Reemplazar una variable con un valor que resulte en un enunciado verdadero.

statistical question (Lesson 10-1) A question that anticipates and accounts for a variety of answers.

cuestión estadística Una pregunta que se anticipa y da cuenta de una variedad de respuestas.

statistics (Lesson 10-1) Collecting, organizing, and interpreting data.

estadística Recopilar, ordenar e interpretar datos.

Subtraction Property of Equality (Lesson 6-2) If you subtract the same number from each side of an equation, the two sides remain equal.

propiedad de sustracción de la igualdad Si sustraes el mismo número de ambos lados de una ecuación, los dos lados permanecen iguales.

surface area (Lesson 9-2) The sum of the areas of all the surfaces (faces) of a three-dimensional figure.

área de superficie La suma de las áreas de todas las superficies (caras) de una figura tridimensional.

survey (Lesson 10-1) A question or set of questions designed to collect data about a specific group of people, or population.

encuesta Pregunta o conjunto de preguntas diseñadas para recoger datos sobre un grupo específico de personas o población.

symmetric (Lesson 10-7) Data that are evenly distributed.

simétrica Datos que están distribuidos.

T

term (Lesson 5-3) Each part of an algebraic expression separated by a plus or minus sign.

término Cada parte de un expresión algebraica separada por un signo más o un signo menos.

third quartile (Lesson 10-4) The third quartile is the median of the data values greater than the median.

tercer cuartil El tercer cuartil es la mediana de los valores mayores que la mediana.

Glossary · Glosario

three-dimensional figure (Lesson 9-1) A figure with length, width, and height.

figura tridimensional Una figura que tiene largo, ancho y alto.

trapezoid (Lesson 8-3) A quadrilateral with one pair of parallel sides.

trapecio Cuadrilátero con un único par de lados paralelos.

triangular prism (Lesson 9-3) A prism that has triangular bases.

prisma triangular Prisma con bases triangulares.

U

unit price (Lesson 1-7) The cost per unit of an item.

precio unitario El costo por unidad de un artículo.

unit rate (Lesson 1-7) A rate in which the first quantity is compared to 1 unit of the second quantity.

tasa unitaria Una tasa en la que la primera cantidad se compara con 1 unidad de la segunda cantidad.

unit ratio (Lesson 1-6) A ratio in which the first quantity is compared to 1 unit of the second quantity.

razón unitaria Una relación en la que la primera cantidad se compara con 1 unidad de la segunda cantidad.

V

variable (Lesson 5-3) A symbol, usually a letter, used to represent a number.

variable Un símbolo, por lo general, una letra, que se usa para representar un número.

volume (Lesson 9-1) The amount of space inside a three-dimensional figure. Volume is measured in cubic units.

volumen Cantidad de espacio dentro de una figura tridimensional. El volumen se mide en unidades cúbicas.

X

x-axis (Lesson 1-3) The horizontal line of the two perpendicular number lines in a coordinate plane.

eje *x* La recta horizontal de las dos rectas numéricas perpendiculares en un plano de coordenadas.

x-coordinate (Lesson 1-3) The first number of an ordered pair. The *x*-coordinate corresponds to a number on the *x*-axis.

coordenada *x* El primer número de un par ordenado, el cual corresponde a un número en el eje *x*.

Y

y-axis (Lesson 1-3) The vertical line of the two perpendicular number lines in a coordinate plane.

eje *y* La recta vertical de las dos rectas numéricas perpendiculares en un plano de coordenadas.

y-coordinate (Lesson 1-3) The second number of an ordered pair. The *y*-coordinate corresponds to a number on the *y*-axis.

coordenada *y* El segundo número de un par ordenado, el cual corresponde a un número en el eje *y*.

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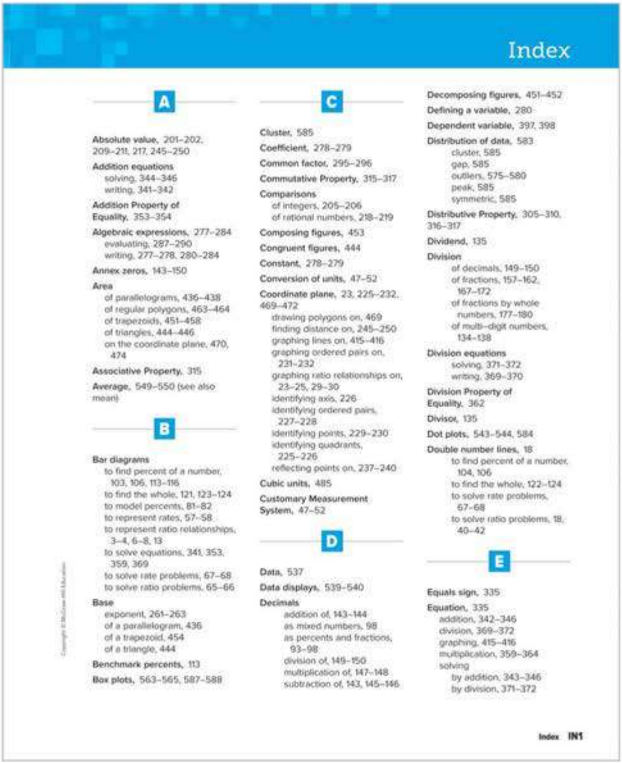

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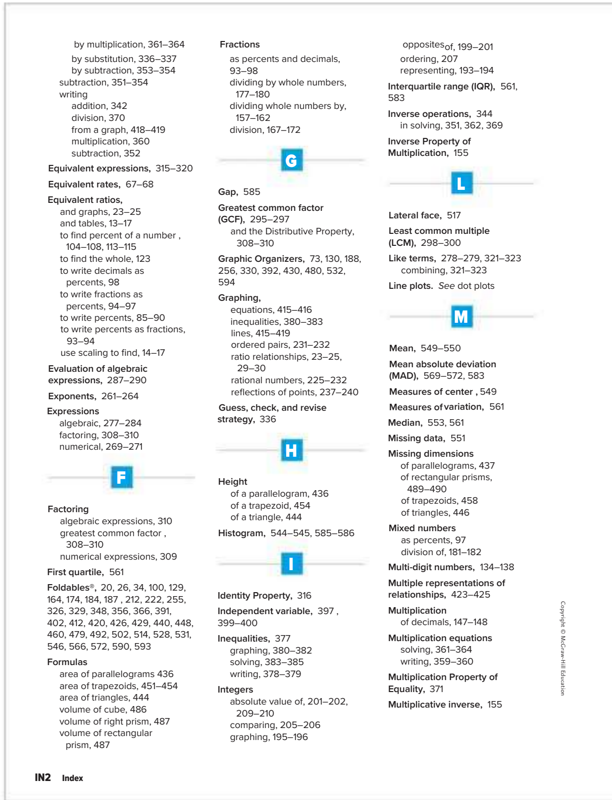

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Selected Answers

Lesson 5-1 Powers and Exponents, Practice Pages 267–268

1. 4^3 **3.** 15^4 **5.** $\left(\frac{1}{3}\right)^7$ **7.** 3,125 **9.** 10,000 **11.** $\frac{8}{125}$ **13.** 3.375 **15.** 0.064 **17.** 7^4; 2,401 **19.** 1,024 cells **21.** Sample answer: The student used the exponent as the base. The base should be 2 and the exponent is 3. The power evaluated should be $2 \times 2 \times 2 = 8$. **23.** Sample answer: Exponential form is repeated multiplication of a common factor.

Lesson 5-2 Numerical Expressions, Practice Pages 275–276

1. 7 **3.** 46 **5.** 23 **7.** 436 **9.** $\frac{3}{5}$ or 0.6 **11.** Sample expression: $(6 \times 1.49) + (2^2) + (3 \times 3.50)$; \$23.44 **13.** $8(1.25 + 0.85)$; $8(1.25) + 8(0.85)$ **15.** 294 muffins **17.** Sample answer: The student did not follow the order of operations. The student added first before dividing. The division should have been performed first. $42 + 6 \div 2 = 42 + 3$ or 45 **19.** Sample answer: Frankie and his two sisters each order a hamburger, a fruit cup, and a bottled water for lunch. A hamburger costs \$3, a fruit cup costs \$0.75, and a bottled water costs \$1.25.; $3^2 + (3 \times 0.75) + (3 \times 1.25)$; \$15

Lesson 5-3 Write Algebraic Expressions, Practice Pages 285–286

1. terms: $4e$, $7e$, 5, $2e$; like terms: $4e$, $7e$, $2e$; coefficients: 4, 7, 2; constant: 5 **3.** terms: 4, $4y$, y, 3; like terms: $4y$, y; 4, 3; coefficients: 4, 1; constants: 4, 3 **5.** Sample answer: Let q represent the number of questions on the first test; $q - 12$ **7.** Sample answer: Let y represent the number of yards; $\frac{1}{3}y$ **9.** Sample answer: Let c represent the cost of a pizza; $\frac{1}{4}c + 2.5$ **11.** Sample answer: Let c represent the number of classes; $35 + 20c$ **13.** Sample answer: Let ℓ represent the length of one of the equal sides; $\ell + \ell + 1.5\ell$ **15.** Sample answer: $2x + 8 + x + 6$; like terms: $2x$, x; 8, 6; coefficients: 2, 1; constants: 8, 6 **17.** $8 + 0.25c$

Lesson 5-4 Evaluate Algebraic Expressions, Practice Pages 293–294

1. 6 **3.** 4 **5.** $\frac{26}{5}$ or $5\frac{1}{5}$ **7.** 2 **9.** 2 **11.** 24.2 ft^2 **13.** 13 **15.** \$80 **17.** Sample answer: The student replaced the variables with the incorrect values. The correct value should be $4(2) + 3$, or 11. **19.** Sample answer: If $a = 2$, then $a + 10 = 12$; $(15 + 5) - 8 = 12$

Lesson 5-5 Factors and Multiples, Practice Pages 303–304

1. 6 **3.** 9 **5.** 14 **7.** 20 visits **9.** 12 **11.** 5 flowers **13.** \$65 **15.** Sample answer: The bottom row of the factor trees may not show the factors listed in order from least to greatest. I can use the Commutative Property to write the factors in order from least to greatest. **17.** false; Sample answer: 25 and 50; 50 is a multiple of 25, 50 is the LCM, and 50 is the greater number. The LCM is the greater of the two numbers.

Lesson 5-6 Use the Distributive Property, Practice Pages 313–314

1. $3x + 24$ **3.** $27 + 9x$ **5.** 40 **7.** $16(1 + 3)$ **9.** $13(2 + 3)$ **11.** $6(4 + x)$ **13.** $5x + 120$ **15.** \$5.40 **17.** Sample answer: $8\left(4\frac{3}{4}\right) = 8\left(4 + \frac{3}{4}\right)$ **19.** no; Sample answer: The Distributive Property combines addition and multiplication. The expression $2(6x)$ is one term with three factors and does not contain addition. $2(6x)$ is equal to $12x$.

Selected Answers

Lesson 5-7 Equivalent Algebraic Expressions, Practice Pages 327–328

1. equivalent **3.** not equivalent **5.** $8x + 3$ **7.** $4x^2 + 7x + 10$ **9.** $10x + 8$ **11.** $76.4x + 8$ **13.** Sample answer: $2y^2 + y^2 + y + y + \frac{1}{2}$ **15.** Sample answer: $3x + 0$ and $3x$

Module Review Pages 331–332

1. $5 \times 5 \times 5$; 125 **3a.** $(2 \times 0.75) + (5 \times 1.79) + (3 \times 3)$ **3b.** 19.45 **5.** terms: $8p$, $6q$, 5, $9q$, $12p$; like terms: $8p$ and $12p$, $6q$ and $9q$; coefficients: 8, 6, 9, 12; constant: 5 **7.** C **9.** C

11.

	Equivalent	Not Equivalent
$5x + 2$ and $4x + 1 + x + 2$		X
$(8y + 4x + 4y + 5)$ and $4(3y + x) + 5$	X	
$y^2 + 4y + 5 + 3y$ and $y^2 + y + 5$	X	

Lesson 6-1 Use Substitution to Solve One-Step Equations, Practice Pages 339–340

1. 6 **3.** 23 **5.** 4 **7.** 22 **9.** 8 headbands **11.** 7 batches **13.** Sample answer: Jack had \$7.50. His mother gave him his allowance at the end of the week. Now Jack has \$16. Solve the equation $7.5 + x = 16$ to find how much money his mother gave him. **15.** Sample answer: $x + 1$ is an algebraic expression and is not equal to a specific value. So, there are no restrictions placed on the value of x. $x + 1 = 2$ is an algebraic equation. Each side of an algebraic equation must be equal, so x can only be equal to one value. In this case, $x = 1$.

Lesson 6-2 One-Step Addition Equations, Practice Pages 349–350

1. Sample answer: $320 + c = 647.5$ **3.** Sample answer: $m + 19.5 = 38.25$ **5.** 6 **7.** $3\frac{1}{2}$ **9.** 13.25 **11.** Sample equation: $8.99 + 2(5.75) + 2(1.15) + 3.45 + x = 35$; \$8.76 **13.** The value of b must be decreased by 1. **15.** 4, 5, 6

Lesson 6-3 One-Step Subtraction Equations, Practice Pages 357–358

1. Sample answer: $c - 17 = 64$ **3.** Sample answer: $f - 1\frac{1}{4} = 1\frac{1}{2}$ **5.** 29 **7.** $10\frac{8}{9}$ **9.** 72.35 **11.** \$667.92 **13.** yes; Sample answer: Solve the equation $x - 7 = 18$ to find the height of Devon's rocket. Devon's rocket reached a height of $18 + 7$ or 25 yards. Since $25 > 23$, Devon's rocket reached a height greater than 23 yards **15a.** Sample answer: Today's high temperature is 64°F. This is 9°F less than yesterday's high temperature. What was yesterday's high temperature? **15b.** $x - 9 = 64$ **15c.** 73°F

Lesson 6-4 One-Step Multiplication Equations, Practice Pages 367–368

1. Sample answer: $46.75p = 374$ **3.** Sample answer: $2.6t = 18.2$ **5.** 2 **7.** $\frac{8}{9}$ **9.** 6.58 **11.** caramel popcorn; 38 Calories **13.** no; Sample answer: Solve the equation $52.5x = 367.50$ to find the number of weeks she needs to save. She needs to save for 7 weeks. Since $7 > 6$, she will not have enough money in 6 weeks. **15.** yes; Sample answer: If you solve each equation you get a value of $x = \frac{1}{9}$. If you replace x with $\frac{1}{9}$ for each equation it makes the equation true. So, $\frac{1}{3} = 3 \times \frac{1}{9}$ or $\frac{1}{3}$ and $\frac{1}{3} \div \frac{1}{9} = 3$.

Lesson 6-5 One-Step Division Equations, Practice Pages 375–376

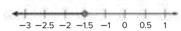
1. Sample answer: $c \div 284.5 = 6$ **3.** Sample answer: $d \div 5.25 = 3$ **5.** 48 **7.** $\frac{8}{3}$ or $2\frac{2}{3}$ **9.** 48.852 **11.** cheese crackers: 112.5 oz; pretzels: 227.5 oz; 115 oz

13. 200 miles; Sample answer: Write and solve the division equation $\frac{m}{5} = 40$; 5×40 is 200. So, $m = 200$ miles. **15.** 186 in.; Sample answer: The length of the actual car c divided by 24, the scale, equals the length of the model car: $\frac{c}{24} = 7.75$; So, $c = 186$ in.

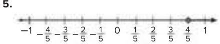
Lesson 6-6 Inequalities, Practice Pages 489–490

1. $a \geq 75$

3.



5.

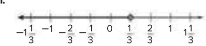


7. 4, 5 **9.** $\frac{1}{4}, \frac{1}{3}$ **11.** Jessica can buy no more than 6 tickets. **13.** China, Maria; $8.25h \geq 74.50$ **15.** Sample answer: More than 2,500 people attended the game; $x > 2{,}500$ **17a.** 4 **17b.** 12 **17c.** 6

Module 6 Review Pages 493–494

1. D **3.** $m + 225 = 478.50$; $225 + m = 478.50$ **5.** $c - 6.50 = 12.99$ **7.** B **9.** $\frac{y}{11}(11) = 28(11)$; $y = 308$

11.



13. 13 teammates; 12 teammates

Lesson 7-1 Relationships Between Two Variables, Practice Pages 403–404

1.

Input, Cost of Pizza (\$), p	Rule $p + 3.50$	Output, Total Cost (\$), c
9.75	9.75 + 3.50	13.25
12.00	12.00 + 3.50	15.50
14.50	14.50 + 3.50	18.00

3.

Input, Cost of Sundae (\$), s	Rule $s - 0.75$	Output, Total Cost (\$), c
2.79	2.79 − 0.75	2.04
3.55	3.55 − 0.75	2.80
4.25	4.25 − 0.75	3.50

5.

Input, Number of Pies, p	Rule $9.50p$	Output, Total Cost (\$), c
2	9.50(2)	19.00
3	9.50(3)	28.50
5	9.50(5)	47.50

7.

Original Price (\$), p	Rule, $p - 15$	Total Cost (\$), c
65	65 − 15	50
73	73 − 15	58
79	79 − 15	64

She could buy the pair that originally cost \$65 or the pair that originally cost \$73.

9.

Input, x	Rule, $2x - 2.5$	Output, y
5	2(5) − 2.5	7.5
6.5	2(6.5) − 2.5	10.5
8	2(8) − 2.5	13.5

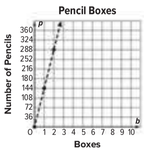
11. \$14.25; Sample answer: In the equation $c = 2.75p + 1.5d$, replace p with 3 and d with 4 and then simplify. $c = 2.75 \times 3 + 1.5 \times 4$ or 14.25.

Lesson 7-2 Write Equations to Represent Relationships Represented in Tables, Practice Pages 413–414

1. $c = 7t$ **3.** $c = 4g + 2$ **5.** $c = 15m + 10$ **7.** $y = \frac{1}{3}x + 3$ **9.** Sample answer: The student switched the coefficient and the constant. The coefficient is 12 and the constant is 20. The equation should be $c = 12h + 20$.

Lesson 7-3 Graphs of Relationships, Practice Pages 421–422

1.



3. $c = 4d + 8$ **5.** 2 more hours **7.** Sample answer: The student switched the coefficient and constant. The correct equation is $s = 5w + 10$. **9.** Sample answer: a straight line through the origin; (0, 0), (2, 1), (4, 2)

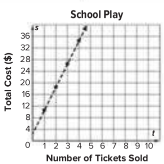
Lesson 7-4 Multiple Representations, Practice Pages 427–428

1a. $c = 8t + 2.5$

1b.

Number of Tickets, t	Total Cost (\$), c
1	10.50
2	18.50
3	26.50
4	34.50

1c.



3. $e = 6p$ **5.** $p = 5b + 5$; 55 points
7. no; Sample answer: The graphs of the lines will never meet other than at zero hours.

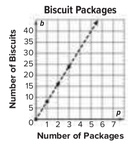
Module 7 Review Pages 431–432

1. 5.1 **3.** $c = 6b$ **5.** \$61

7a.

Number of Packages, p	Number of Biscuits, b
0	0
1	8
2	16
3	24

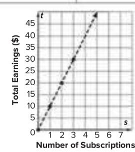
7b.



9a. $t = 10s$;

Number of Subscriptions, s	Total Earnings, t
1	10
2	20
3	30

9b.



Lesson 8-1 Area of Parallelograms, Practice Pages 441–442

1. 50.02 in² **3.** $b = 7$ m **5.** 13.6 in² **7.** 41 tiles **9.** 480 mm² **11.** Sample answer: Because the area of a parallelogram is found by multiplying the base and height, the area of each of the three parallelograms would be 20 square units, because $5 \times 4 = 20$.

Lesson 8-2 Area of Triangles, Practice Pages 449–450

1. 15 yd² **3.** $11\frac{3}{8}$ ft² **5.** $b = 9$ km **7.** 144.5 cm² **9.** \$13.58 **11.** Sample answer: The formula for the area of a triangle is $A = \frac{1}{2}bh$, not bh. $\frac{1}{2}(17)h = 68$; $h = 8$ m **13.** no; Sample answer: The area of her lawn is 125 ft² because the area of a triangle is $A = \frac{1}{2}bh$. So, $\frac{1}{2}(25 \times 10) = 125$.

Lesson 8-3 Area of Trapezoids, Practice Pages 461–462

1. 105 cm² **3.** 36 in² **5.** 155,000 km² **7.** 4 in. **9.** The cost of the patio is \$1,443.75. Since this is less than \$1,500, Greta has budgeted enough money. **11.** Start with the area formula: $A = \frac{1}{2}h(b_1 + b_2)$. Multiply each side by 2: $2A = h(b_1 + b_2)$. Multiply each side by $\frac{1}{h}$: $\frac{2A}{h} = b_1 + b_2$. Subtract b_1 from each side: $\frac{2A}{h} - b_1 = b_2$, or $b_2 = \frac{2A}{h} - b_1$. **13.** 4 cm and 12 cm

Lesson 8-4 Area of Regular Polygons, Practice Pages 467–468

1. 31.82 in² **3.** 281.92 cm² **5.** \$120.03 **7.** 473.2 cm² **9.** 492 in²; the base length of each triangle is $80 \div 10$ or 8 in. So, $10\left(\frac{1}{2} \times 8 \times 12.3\right) = 492$.

Lesson 8-5 Polygons on the Coordinate Plane, Practice Pages 477–478

1. 38 units **3.** 22 units **5.** 9 square units **7.** Space A; The monthly rental price of Space A is \$4,720. The monthly rental price of Space B is \$4,756. \$4,720 is less than \$4,756. **9.** Sample answer: (3, 4) and (3, 7) **11.** Sample answer: The student subtracted 10 − 7 and 7 − 2 to find lengths 3 and 5. The student should have subtracted 7 − 1 and 10 − 2 to find lengths 6 and 8. The perimeter is 28 units.

Module 8 Review Pages 481–482

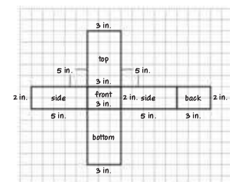
1. 12 cm² **3.** 45 in² **5.** 7 **7a.** Sample answer: I would decompose the hexagon into two trapezoids. **7b.** 261 cm² **9.** 26

Lesson 9-1 Volume of Rectangular Prisms, Practice Pages 493–494

1. 15 ft³ **3.** 4 m **5.** 6 ft **7.** medium dumpster **9.** Sample answer: The classmate switched the volume measurement and h in the formula. The correct value for the height is 0.5 centimeter. **11.** 90 in³; Sample answer: The volume of the pan is $9 \times 5 \times 3$ or 135 cubic inches. Multiply that by two-thirds to find the volume that is filled with batter. $134 \times \frac{2}{3} = 90$.

Lesson 9-2 Surface Area of Rectangular Prisms, Practice Pages 503–504

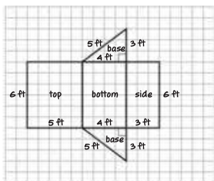
1.



3. 1,120 cm² **5.** Sample answer: Let ℓ = length, w = width, and h = height; $S.A. = 2\ell w + 2\ell h + 2wh$ **7.** Block A: 94 in²; 60 in³; Block B: 104 in²; 60 in³; Block B has a greater surface area. No, the volumes of Blocks A and B are the same.

Lesson 9-3 Surface Area of Triangular Prisms, Practice Pages 515–516

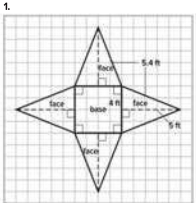
1.



3. 811.2 m² **5.** $106.79 **7.** 75 ft²

Lesson 9-4 Surface Area of Pyramids, Practice Pages 529–530

1.



3. 56 ft² **5.** $1.76 **7.** 11.5 yd **9.** 8.7 ft

Module 9 Review Pages 533–534

1. D **3.** 8 cm **5.** The net will be made up of 6 parts, representing the top, bottom, front, back, and both sides of the rectangular prism; Two parts of the net will have dimensions 4 in. by 11 in.; Two parts of the net will have dimensions 4 in. by 9 in. **7.** B

Lesson 10-1 Statistical Questions, Practice Pages 541–542

1. not a statistical question **3.** statistical question
5.

Number of Siblings	Number of Responses
0-1	10
2-3	7
4-5	2
6 or more	1

Sample answer: Half of the students have 0 or 1 siblings.

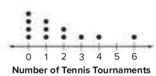
7.

Number of Sports	Number of Responses
1	4
2	7
3	2
4	1

Sample answer: Half of the students that responded play 2 sports. **9.** Sample answers: How many smartphones does a typical family own?; In what year was the cell phone invented?; The first question is a statistical question because it anticipates a variety of responses. The second question is not a statistical question because it does not anticipate a variety of responses. **11.** yes; Sample answer: Of the 10 families, 3 own one tablet and 4 own two tablets. Since 3 + 4 is 7 and 7 is close to 10, this is a reasonable conclusion.

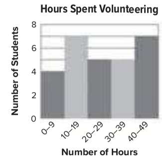
Lesson 10-2 Dot Plots and Histograms, Practice Pages 567–568

1.



Sample answer: Of the 12 players on Chris's team, some played in as few as 0 and as many as 6 tournaments. Most players played in 1 or fewer tournaments.

3.



5. 3 video games **7.** Sample answer: Most students have 3 or fewer siblings. The most common number of siblings is 2 or 3. **9.** false; Sample answer: Dot plots display individual data values. Histograms display data by equal intervals, not individual data values.

Lesson 10-3 Measures of Center, Practice Pages 559–560

1. 58 cans **3.** $195 **5.** 23 E-mails
7. 53 points **9.** the mean; Sample answer: The mean of the data is 31 minutes and the median is 25.5 minutes. Since Kenny wants to use a measure that represents a greater number of minutes spent practicing, he should choose the greater of the two measures, the mean.
11. Sample answer: Shoe sizes of the Holden family: 8, 10, 7, 9, and 6. **13.** Sample answer: The student found the median of the data set. The mean of data set is 18 texts.

Lesson 10-4 Interquartile Range and Box Plots, Practice Pages 567–568

1. The data vary by a range of 32 apps. The middle half of the data values vary by 4 apps.
3. Sample answer: The ages range from about 36 years to about 71 years. The middle half of the data range from about 50 years to about 60 years. Because the boxes are shorter than the whiskers, there is less variation among the middle half of the data. Having less variation means there is a greater consistency among the middle 50% of the data than in either whisker.

5. The data vary by a range of $14. The middle half of the data values vary by $6. **7.** false; Sample answer: A box plot does not show individual data values so you cannot find the mean of the data from a box plot alone. **9.** no; Sample answer: Each section of the box plot represents 25% of the total values. This means that each whisker and each box represents the same amount of data values. The length of each section depends on the spread of the data.

Lesson 10-5 Mean Absolute Deviation, Practice Pages 573–574

1. 5; Sample answer: The average distance for each value from the mean is 5 days.
3. Bears: 1.84; Saints: 1.44; Sample answer: The mean absolute deviation of the number of wins is greater for the Bears than for the Saints. The data values for the Saints are closer to the mean. **5.** 11.67 Calories **7.** 4.5 miles per gallon; 7 data values **9.** Sample answer: The term absolute refers to the absolute value of a number, which is the distance a number is from 0 on a number line and distance is always positive. To deviate means to vary or change. So, the mean absolute deviation of a data set is the average (mean) distance from each data value to the mean, which is a description of how the data values deviate or vary from the mean.

Lesson 10-6 Outliers, Practice Pages 581–582

1. 60 minutes **3.** 4 boxes and 56 boxes are both outliers **5.** mean with outlier: ≈ 47.6, mean without outlier: ≈ 44.2; median with outlier: 45, median without outlier: 44.5; The median best describes the center. **7.** mean with outlier: ≈ 21.8, mean without outlier: ≈ 17.6; median with outlier: 19, median without outlier: 18; The median best describes the center because the mean was affected the most by the outlier.
9. Sample answer: age, in years, of people attending a picnic: 4, 32, 34, 40, 45, and 72
11. Sample answer: An outlier may make the mean significantly greater or less than the mean would be without the outlier.

Lesson 10-7 Interpret Graphical Displays, Practice Pages 591–592

1. median and interquartile range; The median is 2. This means the data are centered on 2 televisions. The spread of the data around the center is 2 televisions. **3.** Sample answer: The shape of the distribution is symmetric. There is a peak from 10–14 dollars. There are no gaps, clusters, or outliers. **5.** D **7.** no; Sample answer: There were 6 pumpkins that weighed 20 pounds or more, out of 40 total pumpkins picked. $\frac{6}{40} = 15\%$, which is less than 25%, so the student was not correct. **9.** no; Sample answer: There are a total of 13 roller coasters. There are 6 roller coasters that have speeds 70 mph or greater. $\frac{6}{13}$ is about 46.2%. 46.2% is less than 50%. **11.** mean; mean absolute deviation.

Module 10 Review Pages 595–596

1. How many televisions does the typical family own?; How many states has the average student visited?; How many students are in the average sixth grade class? **3.** 41.6 inches; 34.1 inches.

5.

	Correct	Incorrect
Lower Extreme = 24		X
Median = 39	X	
$Q_1 = 33$	X	
$Q_3 = 44$		X
Upper Extreme = 58	X	

7. 10 feet; 70 feet

Mathematics Reference Sheet

Formulas

Perimeter	Square	$P = 4s$	Rectangle	$P = 2\ell + 2w$ or $P = 2(\ell + w)$
Area	Square	$A = s^2$	Rectangle	$A = \ell w$
	Parallelogram	$A = bh$	Triangle	$A = \frac{1}{2}bh$
	Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$		
Volume	Cube	$V = s^3$	Prism	$V = \ell wh$ or Bh
Temperature	Fahrenheit to Celsius	$C = \frac{5}{9}(F - 32)$	Celsius to Fahrenheit	$F = \frac{9}{5}C + 32$

Measurement Conversions

Length	1 kilometer (km) = 1,000 meters (m) 1 meter (m)= 100 centimeters (cm) 1 centimeter = 10 millimeters (mm)	1 foot (ft) = 12 inches (in.) 1 yard (yd) = 3 feet or 36 inches 1 mile (mi) = 1,760 yards or 5,280 feet
Volume and Capacity	1 liter (L) = 1,000 milliliters (mL) 1 kiloliter (kL) = 1,000 liters	1 cup (c) = 8 fluid ounces (fl oz) 1 pint (pt) = 2 cups 1 quart (qt) = 2 pints 1 gallon (gal) = 4 quarts
Weight and Mass	1 kilogram (kg) = 1,000 grams (g) 1 gram = 1,000 milligrams (mg) 1 metric ton = 1,000 kilograms	1 pound (lb) = 16 ounces (oz) 1 ton (T) = 2,000 pounds
Time	1 minute (min) = 60 seconds (s) 1 hour (h) = 60 minutes 1 day (d) = 24 hours	1 week (wk) = 7 days 1 year (yr) = 12 months (mo) or 52 weeks or 365 days 1 leap year = 366 days
Metric to Customary	1 meter ≈ 39.37 inches 1 kilometer ≈ 0.62 mile 1 centimeter ≈ 0.39 inch	1 kilogram ≈ 2.2 pounds 1 gram ≈ 0.035 ounce 1 liter ≈ 1.057 quarts

Reveal
MATH
Course 1

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MHID 0-07-899720-8
978-0-07-899720-4

McGraw Hill